Runaway potentials and a massive Goldstino

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We study $N = 1$ globally supersymmetric theories on runaway backgrounds arising from scalar potentials with a slope characterized by a scale $M = |V/V'|$. We find that, under mild assumptions, there always exists a massive Goldstino in the low-energy effective theory. In the simplest models, the effective mass of such a fermion is of order $\sqrt{V}/M$ or of order $V/M^2\mu$ when a seesaw mechanism takes place, where μ is a scale that characterizes the masses of other heavy fermions.

I. INTRODUCTION

Runaway potentials have received a renewed interest because of the controversy surrounding (meta)stable de Sitter vacua, not only in string theory [1,2] but also within quantum gravity in general [3,4]. Moreover, a *de Sitter conjecture* was put forward in Ref. [5], restricting the form of the scalar potential V in low-energy effective theories and excluding de Sitter critical points. For generic runaway potentials, for a *positive V*, one has

$$\frac{|\nabla_\phi V|}{V} = \frac{1}{M(\phi)} > 0. \qquad (1)$$

In Eq. (1), $M(\phi)$ is some field-dependent mass scale, and $\nabla_\phi V$ refers to the derivative with respect to scalars with canonically normalized kinetic terms. In particular, the authors of Ref. [5] have put a bound on $M(\phi)$ and have correlated it explicitly with M_P. However, if one simply studies runaway potentials, such restriction is unwarranted. Indeed, it was subsequently pointed out that the bound on M set by Ref. [5] may be difficult to reconcile with the Standard Model Higgs potential [6], and further *refinements* of the de Sitter conjecture have been proposed [7–11] (for recent reviews, see Refs. [12,13]). The implications of such a conjecture on inflation have been summarized in Refs. [14,15], and the study of the implications on late-time cosmology [14,16–19] supports a quintessence-type behavior [20–22] for the low-energy supergravity theory [23–26]. For further recent developments on the de Sitter Kachru-Kallosh-Linde-Trivedi construction [27],

see Refs. [28–36], and for discussions on other possible counterexamples to the de Sitter conjectures, see Refs. [37–40].

Our approach in this article is to remain agnostic about the validity of the de Sitter conjectures and study instead directly the implications of such a runaway background on the low-energy $N = 1$ supersymmetric theory. In other words, we are solely interested in the runaway behavior implied by (1) and its implications. In particular, as M_P does not explicitly appear in (1), this condition may hold also when gravity is decoupled, assuming that M is not explicitly correlated to M_P. Let us therefore assume that the condition (1) holds for a globally supersymmetric field theory. Then, at least one scalar (say ϕ) of some supermultiplet will have a runaway behavior, implying that the scalar potential part of the Lagrangian has the form

$$\mathcal{L}_V = -V = -V_0 - c\phi + \dots \qquad (2)$$

Here, we assume for simplicity that ϕ and c are real without loss of generality. Clearly, if this background is to satisfy (1) in a nontrivial way (e.g., $V \neq 0$), then the scalar background will develop a runaway time-dependent behavior, and supersymmetry will be generically broken [41,42]. However, assuming the breaking is spontaneous, then such a theory will still be invariant under the supersymmetry transformations. For the scalar ϕ, the supersymmetry transformations will generically have the form

$$\phi \to \phi + \epsilon\psi + \dots, \qquad (3)$$

where the fermion ψ is the superpartner of ϕ and ϵ is the supersymmetry parameter. Then, the scalar potential part of the Lagrangian, that is (2), will produce the following term under a supersymmetry variation:

$$-c\epsilon\psi - c\bar{\epsilon}\,\bar{\psi}\,. \qquad (4)$$

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The only term that can cancel (4) is a term of the form

$$\mathcal{L}_{G-\psi} = -cG^\alpha \psi_\alpha - c\bar{G}_{\dot\alpha}\bar\psi^{\dot\alpha}, \tag{5}$$

where G is the Goldstino, that is, a linear combination of the physical fermions of the low-energy theory, that transforms under supersymmetry as

$$G_\alpha \to G_\alpha + \epsilon_\alpha + \dots \tag{6}$$

The linear combination of the physical fermions that contribute to the Goldstino is uniquely fixed by requiring that the latter is the only fermion with a nontrivial shift in its supersymmetry transformation, when it is evaluated on the background [41]. In (5) and in (6), for simplicity, we have set the supersymmetry breaking scale to unity, but we will restore it when we study supersymmetric chiral models in the next sections.

Let us now inspect the meaning of a term of the form (5) in more detail. Generically, either the Goldstino will be identified with ψ, and therefore have a *Majorana mass*, or, if it is not completely aligned with ψ, then the two fermions will share a *Dirac mass*. In both cases, however, independent of the details, a term of the form (5) will signal that the Goldstino has acquired an effective mass. As we will see in the rest of this article, the existence of a massive Goldstino in runaway dynamical backgrounds is a generic feature of globally supersymmetric theories. Our findings can be contrasted to vacua with stabilized scalars (or with flat directions), which violate (1), and would give rise to a *massless* Goldstino. Therefore, supersymmetric theories on runaway backgrounds lead to distinct phenomenological implications that deserve a dedicated study.

Before closing this section, let us note that other types of *massive Goldstini* have been studied in supersymmetry within different contexts in Refs. [43–48]. These massive fermions have a physical origin different from the massive Goldstino we study here. For example, the massive fermions studied in Refs. [45,46] are *not* true Goldstini (they do not shift); rather, they are the orthogonal fermions to the Goldstino that generically become massive, whereas the Goldstino itself remains massless. The breaking of supersymmetry in Refs. [45,46] is of course spontaneous, and it originates from multiple sectors. Moreover, the pseudo-Goldstini studied in Ref. [48] are related to the explicit breaking of supersymmetry.

This article is organized as follows. In Sec. II, we discuss generic time-dependent scalar backgrounds and show how Yukawa couplings give rise to effective fermionic masses. In Sec. III, we elaborate on chiral models with a single chiral superfield, and we exemplify our general discussion from the Introduction with two explicit models that contain effective Goldstino mass terms. In Sec. IV, we discuss chiral models with multiple chiral superfields and study the fermion mass matrix on time-dependent scalar backgrounds. We close in Sec. V, with a short discussion of our findings and future directions.

II. EFFECTIVE FIELD THEORY ON TIME-DEPENDENT SCALAR BACKGROUNDS

In supersymmetric field theories, the fermions generically appear with Yukawa-type couplings to the scalars [42,49]. As a result, nontrivial time-dependent scalar backgrounds will generically give rise to effective fermion masses, even for the Goldstino [41]. Therefore, before we introduce supersymmetric theories, we would like to set the stage for the study of these effective fermionic mass terms. To this end, consider a field theory with a fermion χ and a real scalar ϕ described by a Lagrangian,[1]

$$\mathcal{L} = -\frac{1}{2}\partial_m\phi\partial^m\phi - i\bar\chi\bar\sigma^m\partial_m\chi$$
$$\quad - \frac{1}{2}m(\phi)\chi^2 - \frac{1}{2}m(\phi)\bar\chi^2 - V(\phi). \tag{7}$$

Let us split the scalar ϕ as

$$\phi = \phi_\mathbb{B} + \delta\phi, \tag{8}$$

where $\phi_\mathbb{B}$ will serve as the background for the scalar, while $\delta\phi$ describes the fluctuations around such background. By varying the Lagrangian, we find that the equations satisfied by the scalar background have the form

$$\partial^m\partial_m\phi_\mathbb{B} = V'(\phi_\mathbb{B}). \tag{9}$$

Clearly, if we wish to study the theory around a critical point, then the background will satisfy $V'(\phi_\mathbb{B}) = 0$. However, if we are not explicitly interested in the critical points of the scalar potential, we can still replace the split form of the scalar (9) into the Lagrangian (7) and study the effective theory of the fluctuations around the background $\phi_\mathbb{B}$. Then, the total action describing the system will have the form

$$S = S|_\mathbb{B} + S_{\text{eff}}. \tag{10}$$

The background part is given by

$$S|_\mathbb{B} = \int d^4x \left(-\frac{1}{2}\partial_m\phi_\mathbb{B}\partial^m\phi_\mathbb{B} - V(\phi_\mathbb{B})\right). \tag{11}$$

The effective part of the action $S_{\text{eff}} = \int d^4x\mathcal{L}_{\text{eff}}$, containing the fluctuations, is given in terms of the Lagrangian

$$\mathcal{L}_{\text{eff}} = -\frac{1}{2}\partial_m\delta\phi\partial^m\delta\phi - i\bar\chi\bar\sigma^m\partial_m\chi$$
$$\quad - \frac{1}{2}m_{\text{eff}}\chi^2 - \frac{1}{2}m_{\text{eff}}\bar\chi^2 - V_{\text{eff}}, \tag{12}$$

where

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[1]We use the conventions of Ref. [49].

$$V_{\text{eff}} = \sum_{n=2}^{\infty} \frac{1}{n!} V^{(n)}|_{\phi_{\mathbb{B}}} \delta\phi^n, \qquad (13)$$

and

$$m_{\text{eff}} = m(\phi_{\mathbb{B}}) + \sum_{n=1}^{\infty} \frac{1}{n!} m^{(n)}|_{\phi_{\mathbb{B}}} \delta\phi^n. \qquad (14)$$

Here, we used the notation $V^{(n)} = \partial^n V/\partial\phi^n$ and similarly for the mass m, and with "$|_{\mathbb{B}}$," we mean that the quantity is evaluated on the background. Notice that there are no linear terms with respect to $\delta\phi$ in the effective Lagrangian except of the Yukava couplings. This is happening because the background satisfies the bosonic equations of motion. Therefore, because of the splitting (10), the effective action will contain the Gaussian terms and higher-order interactions and can be treated within perturbation theory.

Let us turn to the fermionic mass matrix on such a background. The fermionic equations of motion can be now derived from the effective action S_{eff}. We vary S_{eff} with respect to $\bar{\chi}^{\dot{\alpha}}$ and find

$$-i\bar{\sigma}^m_{\dot{\alpha}\beta}\partial_m\chi^\beta = \bar{\chi}_{\dot{\alpha}}\left[m(\phi_{\mathbb{B}}) + \sum_{n=1}^{\infty} \frac{1}{n!} m^{(n)}|_{\phi_{\mathbb{B}}} \delta\phi^n \right]. \qquad (15)$$

Equation (15) describes the propagation of a massive spinor with mass given by evaluating the $m(\phi)$ on the background, whereas the rest of the terms that appear describe interactions of the Yukawa type with the scalar fluctuations.

We conclude that the effective theory that arises from (7) when it is studied around a scalar field background $\phi_{\mathbb{B}}$ that satisfies the bosonic equations of motion describes a massive spinor with effective mass given by

$$m(\phi_{\mathbb{B}}). \qquad (16)$$

Clearly, our discussion here can also be extended to theories with multiple fermions, and we will study such supersymmetric theories in the fourth section.

III. SINGLE CHIRAL SUPERFIELD

Let us first discuss a supersymmetric field theory with a single chiral superfield $\Phi = A + \sqrt{2}\theta\chi + \theta^2 F$, where A is a complex scalar, χ is the fermion superpartner of A, and F is the complex scalar auxiliary field. We study the Lagrangian

$$\mathcal{L} = \int d^4\theta\, \Phi\bar{\Phi} + \left\{ \int d^2\theta\, W(\Phi) + \text{c.c.} \right\}. \qquad (17)$$

Here, $W(\Phi)$ is the superpotential, which is a holomorphic function of Φ. Once we reduce (17) to components and integrate out the auxiliary field, we find

$$\mathcal{L} = -\partial_m A \partial^m \bar{A} - i\bar{\chi}\bar{\sigma}^m\partial_m\chi$$
$$-\frac{1}{2}\left(W''\chi^2 + \bar{W}''\bar{\chi}^2 \right) - V, \qquad (18)$$

where the scalar potential has the form

$$V = W'\bar{W}', \qquad (19)$$

with $W' = \partial W/\partial A$.

We wish to study the effective theory of the fluctuations around a nontrivial time-dependent background $A_{\mathbb{B}}(t)$ that satisfies the scalar equations of motion. To this end, we can split A as

$$A = A_{\mathbb{B}}(t) + \delta A, \qquad (20)$$

where $\partial^2 A_{\mathbb{B}} = V'(A_{\mathbb{B}})$. Assuming that the scalar potential satisfies the condition (1) gives generically

$$\frac{W''(A_{\mathbb{B}})}{W'(A_{\mathbb{B}})} = \mathcal{C}(A_{\mathbb{B}}), \qquad (21)$$

with $\mathcal{C}(A_{\mathbb{B}})$ related to the scale M as $|\mathcal{C}(A_{\mathbb{B}})| = M^{-1}$. As a result, for the scalar field background $A_{\mathbb{B}}$, we should assume

$$W'(A_{\mathbb{B}}) \neq 0, \qquad W''(A_{\mathbb{B}}) \neq 0. \qquad (22)$$

Clearly, such background will break supersymmetry as the energy density of the system evaluated on the background is nonvanishing and given by [41]

$$\rho|_{\mathbb{B}} = \left| \frac{\partial A_{\mathbb{B}}}{\partial t} \right|^2 + |W'(A_{\mathbb{B}})|^2 > 0. \qquad (23)$$

The supersymmetry transformation of the fermion χ evaluated on such background is a shift,

$$\delta_{\text{SUSY}}\chi_\alpha|_{\mathbb{B}} = -\sqrt{2}\bar{W}'(A_{\mathbb{B}})\epsilon_\alpha + i\sqrt{2}\sigma^0_{\beta\dot{\beta}}\bar{\epsilon}^{\dot{\beta}}\frac{\partial A_{\mathbb{B}}}{\partial t}, \qquad (24)$$

and it describes the Goldstino.

As we have explained in the previous section, to find the effective mass of the fermion, we only have to evaluate its mass term given in (18) on the bosonic background. We find

$$m_\chi(A_{\mathbb{B}}) = W''(A_{\mathbb{B}}) \neq 0; \qquad (25)$$

therefore, the Goldstino is massive. Evaluating (21) on the background gives

$$|m_\chi|^2 = \left| \frac{W''W'}{W'} \right|^2 = V|\mathcal{C}|^2. \qquad (26)$$

As we see, the mass of the Goldstino is related only to the value of the scalar potential evaluated on the background and to the scale M (via $\mathcal{C}$) that characterizes the slope. Identifying $|\mathcal{C}(A_\mathbb{B})| = M^{-1}$, we find

$$|m_\chi| = \sqrt{V}/M. \qquad (27)$$

The relation (27) will appear also in models that include more than one fermion, as we will see later.

Let us now stress that if we take the limit

$$W'(A_\mathbb{B}) = \text{finite}, \qquad W''(A_\mathbb{B}) \to 0, \qquad (28)$$

which is equivalent to

$$V(A_\mathbb{B}) = \text{finite}, \qquad \mathcal{C}(A_\mathbb{B}) \to 0, \qquad (29)$$

we find

$$m_\chi(A_\mathbb{B}) \to 0. \qquad (30)$$

This happens because in such background the scalar is stabilized while supersymmetry is broken; therefore, the Goldstino essentially becomes massless. In the example we studied here, as there is only one fermionic mode in the effective theory, this observation is trivial. However, when we study models with multiple fermions, we will identify the eigenvalue of the fermionic mass matrix corresponding to the Goldstino as the one that vanishes in the limit $\mathcal{C}(A_\mathbb{B}) \to 0$. Such a limit might not be physical in a generic setup; however, we will only employ it as a *formal* limit that will help us identify the Goldstino effective mass on a nontrivial background.

The Goldstino was also studied in a nontrivial background in Ref. [50],[2] directly within a nonlinear realization of supersymmetry, in which it was assumed that terms including $\dot{\phi}$ acquire nonvanishing *vacuum expectation values*. The Goldstino was found to be massless in such a setup. Nevertheless, terms contributing to a Goldstino effective mass may appear in Ref. [50] with order $\ddot{\phi}$. The latter terms were ignored in Ref. [50] as being highly suppressed; therefore, a detailed study of these terms is required to compare with our analysis here. Our findings, however, are in complete agreement with Ref. [41], in which it is explained how the Goldstino acquires an effective mass during reheating.

Before we turn to generalizations with multiple fermions, let us discuss two simple examples with a single chiral superfield.

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[2]Supergravity models with a time-dependent background have been also studied in Ref. [51] within a different context.

A. Displacement from supersymmetric vacuum

For a simple example, we can discuss the superpotential

$$W = \frac{1}{2}m\Phi^2. \qquad (31)$$

This model has a massive fermion with mass

$$m_\chi = m, \qquad (32)$$

which is *independent* of the background. Clearly, on the vacuum where $\langle A \rangle = 0$, supersymmetry is unbroken; thus, the fermion is not the Goldstino, and it is massive, forming a massive multiplet with the scalar. However, we can assume that the scalar is displaced from its true vacuum to some position A^* at time t^*, and therefore an effective theory can be constructed for the nontrivial background of the form

$$\ddot{A}_\mathbb{B} = -m^2 A_\mathbb{B}, \qquad A_\mathbb{B}(t = t^*) = A^* \neq 0. \qquad (33)$$

We then find

$$V|_\mathbb{B} = m^2 |A_\mathbb{B}|^2, \qquad \mathcal{C}|_\mathbb{B} = \frac{1}{A_\mathbb{B}}. \qquad (34)$$

In such a case, we have the general relation (26) satisfied trivially,

$$|m_\chi(A_\mathbb{B})|^2 = m^2 = m^2 |A_\mathbb{B}|^2 \frac{1}{|A_\mathbb{B}|^2}, \qquad (35)$$

giving a background with broken supersymmetry and a massive Goldstino.

B. Runaway potential

The second example we would like to study is a supersymmetric theory with a runaway potential that is given by

$$W = W_0 e^{-\Phi/m}, \qquad (36)$$

where m is a real positive constant. In this setup, the scalar potential has a runaway behavior,

$$V = \frac{|W_0|^2}{m^2} e^{-(A+\bar{A})/m}, \qquad (37)$$

and supersymmetry is in a spontaneously broken phase. For the slope parameter, we find

$$\mathcal{C} = -\frac{1}{m}. \qquad (38)$$

Assuming that we study the theory around a nontrivial background $A_\mathbb{B}$, we find

$$|m_\chi(A_\mathbb{B})|^2 = \frac{|W_0|^2}{m^4} e^{-(A_\mathbb{B}+\bar{A}_\mathbb{B})/m} = V(A_\mathbb{B})|\mathcal{C}|^2, \quad (39)$$

in agreement with our general discussion.

IV. MULTIPLE CHIRAL SUPERFIELDS

We can now extend our discussion to a theory with multiple chiral multiplets that are described by the superfields Φ^I with θ expansion,

$$\Phi^I = A^I + \sqrt{2}\theta\chi^I + \theta^2 F^I. \quad (40)$$

The A^I describe the physical complex scalars, the χ^I are Weyl spinors that describe the fermions, and the F^I are the auxiliary fields. The supersymmetry transformations of the fermions are

$$\delta\chi_\alpha^I = \sqrt{2}F^I\epsilon_\alpha + i\sqrt{2}\sigma_{\beta\dot{\beta}}^a\bar{\epsilon}^{\dot{\beta}}\partial_a A^I. \quad (41)$$

When we study the theory around a background where $F^I \neq 0$ or $\dot{A}^I \neq 0$, these fermions contribute collectively to the Goldstino.

The most general chiral model (up to two derivatives) has the form [49]

$$\mathcal{L} = \int d^4\theta K(\Phi, \bar{\Phi}) + \left\{ \int d^2\theta W(\Phi) + \text{c.c.} \right\}. \quad (42)$$

The holomorphic function $W(\Phi)$ is the superpotential, and the function K is real, and it is the Kähler potential. The Kähler metric is defined as

$$g_{I\bar{J}} = K_{I\bar{J}} = \frac{\partial^2 K}{\partial A^I \partial \bar{A}^{\bar{J}}}. \quad (43)$$

The connection is given by $\Gamma_{IJ}^K = g^{\bar{L}K}\partial_I g_{J\bar{L}}$. Clearly, one can choose a specific set of A^I such that at a given point A_*^I in field space it will hold $\Gamma_{IJ}^K|_* = 0$; however, the derivatives of the connection evaluated at that point will not generically vanish. Note that the derivatives of the connection Γ_{IJ}^K contribute only to the four-fermion interactions in the component form of the Lagrangian (42). To proceed with our discussion, we assume from now on that

$$g_{I\bar{J}} = \delta_{I\bar{J}}, \quad (44)$$

such that the Kähler manifold is flat. As we have explained, this is not a very strict requirement, as one can generically go to a coordinate system in which (44) will hold for small field excursions and higher-order terms will be highly suppressed. However, for clarity, we will impose (44) in general, which then leads to a Kähler potential of the form

$$K = \delta_{I\bar{J}}\Phi^I\bar{\Phi}^{\bar{J}}. \quad (45)$$

The component form of the Lagrangian (42), with Kähler potential (45), reads

$$\mathcal{L} = -\delta_{I\bar{J}}\partial A^I\partial\bar{A}^{\bar{J}} - i\delta_{I\bar{J}}\bar{\chi}^{\bar{J}}\bar{\sigma}^m\partial_m\chi^I$$
$$- \frac{1}{2}W_{IJ}\chi^I\chi^J - \frac{1}{2}\bar{W}_{\bar{I}\bar{J}}\bar{\chi}^{\bar{I}}\bar{\chi}^{\bar{J}} - V, \quad (46)$$

where

$$V = \delta^{I\bar{J}}W_I\bar{W}_{\bar{J}}. \quad (47)$$

In (46), the auxiliary fields have been integrated out and take the values

$$F^I = \delta^{I\bar{J}}\bar{W}_{\bar{J}}. \quad (48)$$

Notice finally that the fermion kinetic terms in (46) are canonical.

Let us now study the effective theory around a nontrivial background for the complex scalars A^I. Without loss of generality, we can assume that the scalar that gives rise to the nontrivial background is A^o, while all other scalars are constant (on the background). We first split the scalars in their background values and their fluctuations as

$$A^I = A_\mathbb{B}^I(t) + \delta A^I, \quad (49)$$

and we see that the equations that define the bosonic background have the form

$$\ddot{A}_\mathbb{B}^I(t) = -\delta^{I\bar{J}}\frac{\partial V}{\partial\bar{A}^{\bar{J}}}\bigg|_\mathbb{B}. \quad (50)$$

Then, we split the scalar manifold coordinates as $I = (o, A)$, and we have

$$\ddot{A}_\mathbb{B}^A(t) = 0, \qquad \ddot{A}_\mathbb{B}^o(t) = -\frac{\partial V}{\partial\bar{A}^o}\bigg|_\mathbb{B} \neq 0. \quad (51)$$

In accordance to our previous discussions, we set

$$\frac{\partial V}{\partial A^o}\bigg|_\mathbb{B} = V_o|_\mathbb{B} = c \neq 0. \quad (52)$$

The property of the background then leads to the condition

$$\delta^{I\bar{J}}W_{Io}\bar{W}_{\bar{J}}\big|_\mathbb{B} = c \neq 0, \quad (53)$$

but because the other scalars A^A are all stabilized, we also have

$$V_A|_\mathbb{B} = 0 \rightarrow \delta^{I\bar{J}}W_{IA}\bar{W}_{\bar{J}}\big|_\mathbb{B} = 0. \quad (54)$$

We have used here the notation $V_A = \partial V/\partial A^A$. In addition, for (53) to hold, we have

$$W_I|_\mathbb{B} \neq 0, \qquad V|_\mathbb{B} \neq 0, \qquad (55)$$

and therefore supersymmetry is spontaneously broken.

We wish now to evaluate the mass of the Goldstino m_G on the nontrivial background (51). One way to proceed would be to identify the Goldstino as in Ref. [41], then define the orthogonal fermions to the Goldstino, and finally calculate the eigenvalue of the mass matrix corresponding to the Goldstino on the nontrivial background. However, since we are interested *only* in identifying the effective mass of the Goldstino, there is a simpler method which we can follow. As we have explained, the fermionic mass matrix will have an eigenvalue m_G that vanishes in the formal limit $c \to 0$. This limit of course has to be taken with care because m_G will not generically vanish if in such a limit supersymmetry is allowed to be restored. We will therefore proceed by ascribing to the effective Goldstino mass the mass matrix eigenvalue in the nontrivial background (51) that has the property

$$m_G|_{c\to 0} \to 0, \qquad V|_{c\to 0} \neq 0. \qquad (56)$$

With this strategy in mind, we will proceed to write down the fermion mass matrix. The fermionic masses read

$$-\frac{1}{2}\hat{W}_{oo}\chi^o\chi^o - \hat{W}_{oA}\chi^o\chi^A - \frac{1}{2}\hat{W}_{AB}\chi^A\chi^B, \qquad (57)$$

where the $\hat{W}_{IJ}$ are evaluated on the background (51), that is,

$$\hat{W}_{IJ} = W_{IJ}|_\mathbb{B}, \qquad \hat{W}_I = W_I|_\mathbb{B}. \qquad (58)$$

There are now three possibilities depending on the type of supersymmetry breaking:
  (A) We have $\hat{W}_o = f \neq 0$ and $\hat{W}_A = 0$.
  (B) We have $\hat{W}_o = 0$ and $\hat{W}_A = f_A \neq 0$.
  (C) We have $\hat{W}_o = f_o \neq 0$ and $\hat{W}_A = f_A \neq 0$.
From now on, without loss of generality, we will assume that f and f_A are real and that c is real and positive. Notice that the c is related to the scale M appearing in (1) via

$$c = \frac{V_\mathbb{B}}{M} \qquad (59)$$

and that

$$V_\mathbb{B} = \delta^{IJ}\hat{W}_I\bar{\hat{W}}_{\bar{J}}. \qquad (60)$$

We now turn to the study of the three possibilities A, B, and C.

A. $\hat{W}_o = f \neq 0$ and $\hat{W}_A = 0$

We start with the possibility A, which also means that the fermion χ^o is completely aligned with the Goldstino. The conditions (53) and (54) imply

$$\hat{W}_{oo} = \frac{c}{f} \neq 0, \qquad \hat{W}_{oA} = 0. \qquad (61)$$

Here, the effective mass matrix is

$$m_{IJ} = \begin{pmatrix} \frac{c}{f} & 0 \\ 0 & \hat{W}_{AB} \end{pmatrix}. \qquad (62)$$

Clearly, the Goldstino is massive, with mass proportional to c, whereas the masses of the other fermions are independent and given by $\hat{W}_{AB}$. Notice that, in particular, for the Goldstino mass, we have

$$m_G = \frac{c}{f} = \frac{V_\mathbb{B}}{M}\frac{1}{\sqrt{V_\mathbb{B}}} = \frac{\sqrt{V_\mathbb{B}}}{M}, \qquad (63)$$

delivering the same result as the single chiral multiplet model.

B. $\hat{W}_o = 0$ and $\hat{W}_A = f_A \neq 0$

We now turn to the possibility B and rotate the superfields Φ^A such that only one of them has a nonvanishing value for the auxiliary field evaluated on the background. Namely, we set

$$f_o = 0, \qquad f_1 = f \neq 0, \qquad f_a = 0, \qquad (64)$$

where we further split the scalar manifold coordinates as $A = (1, a)$. The conditions (53) and (54) imply

$$\hat{W}_{oo} = \mathrm{m}, \qquad \hat{W}_{o1} = \frac{c}{f} \neq 0, \qquad \hat{W}_{11} = 0, \quad (65)$$

and

$$\hat{W}_{1a} = 0, \qquad \hat{W}_{oa} = v_a. \qquad (66)$$

We stress that the values of m and v_a are unconstrained; we choose them, however, to be real and positive. Therefore, we have the effective mass matrix for the fermions (χ^o, χ^1, χ^a) given by

$$m_{IJ} = \begin{pmatrix} \mathrm{m} & \frac{c}{f} & v_a \\ \frac{c}{f} & 0 & 0 \\ v_b & 0 & \hat{W}_{ab} \end{pmatrix}. \qquad (67)$$

The equation that defines the eigenvalues λ reads

$$\frac{c^2}{f^2}\det[\hat{W}_{ab} - \lambda\delta_{ab}]$$
$$+ \lambda\det\left[\begin{pmatrix} \mathrm{m} - \lambda & v_a \\ v_b & \hat{W}_{ab} - \lambda\delta_{ab} \end{pmatrix}\right] = 0. \qquad (68)$$

Equation (68) is a higher-order polynomial equation. Notice, however, that (68) does describe an eigenvalue that goes to zero as c goes to zero. This is the eigenvalue that corresponds to the Goldstino. To illustrate the behavior of this eigenvalue, we will focus on two limiting cases.

1. Case $v_a \to 0$

Let us first assume that the fermions belonging to the runaway/Goldstino system do not mix with the other fermions, that is,

$$v_a \to 0. \tag{69}$$

In this case, the two fermions take part in a *seesaw* mechanism. Indeed, we find the c-dependent eigenvalues of (68) to be

$$\lambda_\pm = \frac{1}{2}\left(\mathrm{m} \pm \sqrt{\mathrm{m}^2 + 4(c/f)^2}\right), \tag{70}$$

along with the eigenvalues of $\hat{W}_{ab}$ that are independent of c. Clearly, the effective Goldstino mass will be related to $\lambda_\pm$. Let us see which of the two, λ_+ or λ_-, is to be ascribed to the Goldstino.

For $|m| \gg |c/f|$, we have

$$\lambda_+ \simeq \mathrm{m} + \frac{c^2}{\mathrm{m}f^2}, \qquad \lambda_- \simeq -\frac{c^2}{\mathrm{m}f^2}, \tag{71}$$

up to order $\mathcal{O}(c^4)$. The effective Goldstino mass is identified with the eigenvalue that vanishes when $c \to 0$ and is thus given by

$$|m_G| \equiv |\lambda_-| = \frac{c^2}{\mathrm{m}f^2} = \frac{V_\mathbb{B}}{M^2}\frac{1}{\mathrm{m}}. \tag{72}$$

We see that (72) is different from (63). This happens because the Goldstino is a linear combination of both fermions χ^o and χ^1. The reader may wonder why the Goldstino would receive contribution from the fermion χ^o when $f_o = 0$. This happens because on the nontrivial background that we are studying we have

$$\dot{A}^o|_\mathbb{B} \neq 0, \tag{73}$$

and therefore the fermion χ^o will also shift under supersymmetry, as can be seen from (41). As a result, the scale m also enters into the eigenvalue m_G, characterizing the effective Goldstino mass.

By studying the limit $c \to 0$, we thus found that it is the λ_- eigenvalue of (70) that relates to the Goldstino. We can then also consider a different limit, namely, $|m| \ll |c/f|$, and we see that the effective Goldstino mass is

$$\lambda_- = -\frac{c}{f} + \frac{\mathrm{m}}{2} + \frac{c}{f}\mathcal{O}(\mathrm{m}^2 f^2/c^2). \tag{74}$$

Interestingly, this limit reproduces the result of a single chiral superfield we studied earlier. Indeed, we find

$$|m_G| \simeq \frac{|c|}{|f|} = \frac{\sqrt{V_\mathbb{B}}}{M}. \tag{75}$$

The mass (75) seems not to vanish in the limit $c \to 0$. This happens because to derive (75) we have assumed that $|m| \ll |c/f|$; therefore, we cannot apply the limit $c \to 0$ here unless m itself goes to zero, in which case $|m_G|$ would go to zero as well.

2. Case $\frac{c}{f} \ll \mu$

Another natural scenario that can allow us to study the mass matrix (67) is to assume that

$$\frac{c}{f} \ll \mu, \tag{76}$$

where μ is a mass scale that characterizes collectively the *eigenvalues* of the matrices $\hat{W}_{ab}$ and $\left(\begin{smallmatrix} \mathrm{m} & v_a \\ v_b & \hat{W}_{ab} \end{smallmatrix}\right)$. This is not an innocent assumption because there is no guarantee this will hold in realistic models, because of seesaw mechanisms, e.g., but it is a way that allows us to proceed here. In this limit, we can recast Eq. (68) into the form

$$\lambda_G = -\frac{c^2}{f^2}\frac{\det[\hat{W}_{ab} - \lambda_G\delta_{ab}]}{\det\left[\left(\begin{matrix} \mathrm{m} - \lambda_G & v_a \\ v_b & \hat{W}_{ab} - \lambda_G\delta_{ab} \end{matrix}\right)\right]} \tag{77}$$

and solve iteratively for λ_G up to any order in c/f. This can be done by expanding the determinant in the numerator as

$$\det[\hat{W}_{ab} - \lambda_G\delta_{ab}] = \prod_i(\mu_i - \lambda_G), \tag{78}$$

where the μ_i, with $\mu_i \sim \mu$, are the eigenvalues of the matrix $\hat{W}_{ab}$. A similar expansion can be performed for the determinant appearing in the denominator of (77). Clearly, the eigenvalue λ_G in (77) describes the Goldstino eigenvalue as it does go to zero when c goes to zero. Let us stress that the iterative procedure is valid only because we are assuming the eigenvalues of the matrices appearing in (77) to be much larger than c/f; otherwise, the procedure would not be *a priori* justifiable to give the correct λ_G. After the first step in the iterative procedure, we see that the λ_G eigenvalue will take the form

$$\lambda_G = -\frac{c^2}{f^2}\frac{\det[\hat{W}_{ab}]}{\det\left[\left(\begin{matrix} \mathrm{m} & v_a \\ v_b & \hat{W}_{ab} \end{matrix}\right)\right]} + \frac{c^2}{f^2\mu}\mathcal{O}(c^2/f^2\mu^2). \tag{79}$$

Then, the Goldstino mass is of order

$$m_G \sim \frac{c^2}{f^2 \mu} \sim \frac{V_{\mathbb{B}}}{M^2 \mu}.$$ (80)

We see that here the Goldstino mass behaves as in (72). If we also assume the v_a to be much smaller than m and the eigenvalues of $\hat{W}_{ab}$, then the latter drop out, and we find the λ_- mass of (71).

C. $\hat{W}_o = f_o \neq 0$ and $\hat{W}_A = f_A \neq 0$

The possibility C means that all the fermions would contribute to the Goldstino. However, we can always rotate the superfields Φ^A such that only one of the rotated χ^A eventually contributes; therefore, we have

$$f_o \neq 0, \qquad f_1 \neq 0, \qquad f_a = 0.$$ (81)

The conditions (53) and (54) imply

$$\hat{W}_{oo} f_o + \hat{W}_{1o} f_1 = c, \qquad \hat{W}_{oA} f_o = -\hat{W}_{1A} f_1.$$ (82)

Once we set

$$\hat{W}_{oo} = m, \qquad \hat{W}_{oa} = v_a,$$ (83)

the mass matrix takes the form

$$m_{IJ} = \begin{pmatrix} m & \frac{c - m f_o}{f_1} & v_a \\ \frac{c - m f_o}{f_1} & \frac{m f_o^2 - c f_o}{f_1^2} & -\frac{f_o}{f_1} v_a \\ v_b & -\frac{f_o}{f_1} v_b & \hat{W}_{ab} \end{pmatrix}.$$ (84)

We search again for the mass eigenstate that will vanish in the limit $c \to 0$. We will study here only two limiting cases and leave a detailed account of the properties of (84) for a future work.

1. Case $v_a \to 0$

Let us first assume that the parameters that induce the mixing of the supersymmetry breaking sector with the matter fermions can be ignored, which means we set

$$v_a \to 0.$$ (85)

In this case, the effective mass matrix eigenvalues for the supersymmetry breaking sector read

$$\lambda_\pm = \frac{\mathcal{B}}{2} \pm \frac{\sqrt{\mathcal{B}^2 - 4[\frac{c f_o}{f_1^2} m - \frac{c^2}{f_1^2}]}}{2},$$ (86)

where

$$\mathcal{B} = \frac{f_o^2}{f_1^2} m + m - \frac{c f_o}{f_1^2}.$$ (87)

The Goldstino effective mass is given by the λ_- because it goes to zero as c goes to zero. For small c/f_1, we have

$$\lambda_- = \frac{c f_o}{f_1^2 + f_o^2} + \mathcal{O}(c^2),$$ (88)

which gives for the Goldstino effective mass

$$m_G \simeq \frac{f_o}{M}.$$ (89)

Again, we see that the mass of the Goldstino is proportional to the slope of the scalar potential.

2. Case $\frac{c}{f} \ll \mu$

A way to proceed without assuming that the v_a are small is to shift the fermion χ^o as

$$\chi^o \to \chi^o + \frac{f_o}{f_1} \chi^1,$$ (90)

which brings the mass matrix to the form

$$m_{IJ} = \begin{pmatrix} m & \frac{c}{f_1} & v_a \\ \frac{c}{f_1} & \frac{c f_o}{f_1^2} & 0 \\ v_b & 0 & \hat{W}_{ab} \end{pmatrix}.$$ (91)

Because of the shift (90), the kinetic matrix of the fermions also changes and takes the form

$$k_{IJ} = \begin{pmatrix} 1 & \frac{f_o}{f_1} & 0 \\ \frac{f_o}{f_1} & 1 + \frac{f_o^2}{f_1^2} & 0 \\ 0 & 0 & \delta_{ab} \end{pmatrix}.$$ (92)

To find the effective mass eigenvalues, we have to solve the eigenvalue equation $\det[m_{IJ} - \lambda k_{IJ}] = 0$, that takes the form

$$\left(\frac{c}{f_1} - \lambda \frac{f_o}{f_1}\right)^2 \mathcal{A}(\lambda) + \left(\lambda + \lambda \frac{f_o^2}{f_1^2} - \frac{c f_o}{f_1^2}\right) \mathcal{B}(\lambda) = 0,$$ (93)

where

$$\mathcal{A}(\lambda) = \det[\hat{W}_{ab} - \lambda \delta_{ab}],$$

$$\mathcal{B}(\lambda) = \det\left[\begin{pmatrix} m - \lambda & v_a \\ v_b & \hat{W}_{ab} - \lambda \delta_{ab} \end{pmatrix}\right]. \qquad (94)$$

To proceed, we will assume that (76) holds also here for the scale μ that characterizes the eigenvalues μ_i of the matrices $\hat{W}_{ab}$ and $\begin{pmatrix} m & v_a \\ v_b & \hat{W}_{ab} \end{pmatrix}$. In this limit, we can solve (93) iteratively by first bringing the determinants to the form (78) and assuming $\mu_i \sim \mu$. Indeed, we find

$$\lambda_G = \frac{c f_o \mathcal{B}(\lambda_G) - c^2 \mathcal{A}(\lambda_G)}{(f_1^2 + f_o^2)\mathcal{B}(\lambda_G) + (\lambda_G f_o^2 - 2 f_o c)\mathcal{A}(\lambda_G)}. \qquad (95)$$

Clearly, λ_G reproduces the correct limit for $c \to 0$, and we can solve it up to any order in c. The eigenvalue λ_G then reads

$$\lambda_G = \frac{c f_o}{(f_1^2 + f_o^2)} + \mathcal{O}(c^2), \qquad (96)$$

giving again (89) as the leading-order contribution to the effective Goldstino mass.

V. DISCUSSION

Let us end this article with a discussion on the implications of our findings. We have demonstrated that in supersymmetric field theories with runaway potentials (or generically in time-dependent backgrounds) the Goldstino instead of being massless becomes massive. This happens because the system is not stabilized *on* the vacuum; rather, it is described by an effective theory around a nontrivial scalar background that evolves in time. As we have shown, the effective mass of such a fermion has a specific order of magnitude under rather general assumptions. In the simplest examples, it is controlled by the value of the scalar potential V evaluated on the background and the scale M that characterizes the slope

$$m_G \sim \frac{\sqrt{V}}{M}, \qquad (97)$$

as we found in (27). In a setup where the runaway fermion mixes with some other fermion via a Dirac mass, we would find a seesaw mechanism taking place, thus delivering

$$m_G \sim \frac{V}{M^2 \mu}, \qquad (98)$$

as, e.g., we found in (72), where μ is a mass scale that enters the fermion mass matrix.

We did not study here gauged chiral models, but our results are expected to hold also when gaugings are introduced. The $N = 1$ gauge multiplets do not have

physical scalars; therefore, the mass terms of the gaugini will be similar to those of the χ^A fermions appearing in (57). This is, however, an interesting generalization that we leave for future work.

Let us also discuss if it is natural to have a small c (we remind the reader that $c = V'$). Recall that the pure Volkov-Akulov Goldstino model enjoys a global R symmetry, if we ignore higher-order terms. However, the global R symmetry is explicitly broken by a Goldstino mass. Then, because the Goldstino becomes strictly massless in the vanishing c limit,

$$m_G|_{c \to 0} \to 0, \qquad (99)$$

the R symmetry may in some cases be restored. Therefore, a small value for the Goldstino mass would fall under the *technical naturalness* arguments [52] as the symmetry of the system increases when $c \to 0$. As a result, for models where the R symmetry is mildly broken by c, the massive Goldstino will be protected from receiving a large mass from quantum corrections, and c is naturally small.

An interesting question is how to proceed in the study of generic matter couplings and in particular how to construct low-energy effective actions in the spirit of the nonlinear realizations [53–57]. Such low-energy effective theories can help to classify the generic couplings of the massive Goldstino and point to possible distinct signals in experiments. In particular, such very light fermions could serve as dark matter that is weakly interacting with the matter particles of the Standard Model in the spirit of Ref. [48]. Extrapolating the properties of the massless Goldstino, one would expect that the massive Goldstino will couple to the Standard Model sector in a universal manner via the energy-momentum tensor [53–55]. Namely, we would have couplings of the form

$$\frac{i}{\mathcal{F}^4} (G\sigma^m \partial^n \bar{G} - \partial^n G \sigma^m \bar{G}) T_{mn}^{(\mathrm{SM})}, \qquad (100)$$

where $T_{mn}^{(\mathrm{SM})}$ is the energy-momentum tensor of the Standard Model particles and $\mathcal{F}$ is some effective supersymmetry breaking scale. A detailed study of such a fermion serving as a dark matter candidate is in order.

Finally, our findings are expected to hold also for extended globally supersymmetric theories. In particular, if the real scalar that appears in (2) transforms as

$$\delta \phi = \epsilon^i \psi_i + \dots, \qquad (101)$$

then the arguments presented in the Introduction would lead us to conclude that there exist nontrivial mass terms of the form

$$-c \psi_i G^i - c \bar{\psi}_i \bar{G}^i, \qquad (102)$$

where G^i are the Goldstini of the extended supersymmetric theory [56], transforming as $G^i \to G^i + \epsilon^i + \cdots$ As a

result, massive Goldstini would also arise in extended supersymmetry on runaway backgrounds. A detailed discussion of the fermion mass matrix of such theories is, however, beyond the scope of this work, and we leave it for future research.

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