pp-waves as exact solutions to ghost-free infinite derivative gravity

Ercan Kilicarslan*
Department of Physics, Usak University, 64200, Usak, Turkey

(Received 17 March 2019; revised manuscript received 6 May 2019; published 26 June 2019)

We construct exact pp-wave solutions of ghost-free infinite derivative gravity. These waves described in the Kerr-Schild form also solve the linearized field equations of the theory. We also find an exact gravitational shock wave with nonsingular curvature invariants and with a finite limit in the ultraviolet regime of nonlocality which is in contrast to the divergent limit in Einstein's theory.

DOI: 10.1103/PhysRevD.99.124048

I. INTRODUCTION

Among the small scale modifications of Einstein's theory of general relativity (GR), infinite derivative gravity (IDG) [1–3] seems to be a viable candidate to have a complete theory in the UV scale (short distances). A particular form of IDG is free from the Ostragradsky type instabilities and black hole or cosmological type singularities. The theory is described by a Lagrangian density built from analytic form factors which lead to nonlocal interactions. The propagator of ghost and singularity free IDG in flat background is obtained by the modification of a pure GR propagator via an exponential of an entire function that has no roots in the finite domain [2,4]. This modification provides that the theory does not have ghostlike instabilities and an extra degree of freedom (DOF) other than the massless graviton. On the other hand, an infinite derivative extension of GR describes nonsingular Newtonian potential for a pointlike source at small distances [2,5]. This result is extended to the case where pointlike sources also have velocities, spins, and orbital motion which leads to spin-spin and spin-orbit interactions in addition to mass-mass interactions [6]. It was shown that not only mass-mass interaction but also spin-spin and spin-orbit interactions are nonsingular in the UV regime of nonlocality. Hence, the theory is well behaved in the small scale unlike GR. Furthermore, power counting arguments have been recently studied for the renormalizability discussion and it is shown that loop diagrams beyond one-loop may give finite results with dressed propagators [3,7–11]. Moreover, IDG may be devoid of black hole and cosmological big bang type singularities at a linear and nonlinear level [1,2,9,12–19]. These encouraging developments led us to study exact solutions of the theory.

There are many works and some books on finding and classifying the exact solutions of Einstein's gravity [20]. Furthermore, some exact solutions are studied in detail in some specific modified gravity theories, such as the quadratic gravity [21–26], higher order theories of gravity [27], $f(Riemann)$ theories [28], $f(R_{\mu\nu})$ theories [29], and $f(R)$ theories [30]. On the other hand, although IDG received attention in the recent literature, exact solutions of the theory have not been studied at a nonlinear level[1] since the field equations are very lengthy and complicated. At the linearized level around a flat background, some specific solutions have been found: a nonsingular rotating solution without ring singularity was studied in [32], a solution for an electric point charge was found in [33], a conformally flat static metric was constructed in [34], and a metric for the nonlocal star was given in [35]. However, at the nonlinear level, we are not aware of any known exact solution for the theory. Nevertheless, since Kundt Einstein spacetimes of Petrov (Weyl) type N are universal [36–40], these spacetimes are exact solutions of IDG.

In this work, we would like to construct exact pp-wave solutions of the IDG. Therefore, we consider the pp-wave metric in the Kerr-Schild form which leads to a remarkable simplification in finding exact solutions. We show that pp-wave spacetimes are exact solutions of the IDG. We also show that these waves solve not only generic nonlinear field equations but also the linearized ones. Furthermore, pp-wave solutions of Einstein's theory also solve the IDG since they are Kundt spacetimes of Petrov type N with zero curvature scalar [36–40]. We also discuss the pp-wave solution of the theory in the presence of the null matter which contains the Dirac delta type singularity; namely, we construct an exact nonsingular gravitational shock-wave solution at the nonlinear level. We show that curvature tensors are regular at the origin. Although, the exact gravitational shock-wave solution of Einstein's theory generated by a massless point particle is singular at the origin, gravitational nonlocal interactions in IDG leads to the cancellation of such a singularity at the nonlinear level.

*ercan.kilicarslan@usak.edu.tr

[1]Some exact solutions of weakly nonlocal gravity theories are discussed in [31].

The layout of the paper is as follows: In Sec. II, we will briefly review the IDG. Section III is devoted to some mathematical preliminaries of the pp-wave metrics in the Kerr-Schild form. In that section, we write the generic field equations of IDG for pp-wave spacetimes. In Sec. IV, we give the explicit form of the exact solution for ghost-free IDG. In addition to the nonlinear theory, we show that pp-wave solutions of the generic theory also satisfy the linearized field equations. In Sec. V, we construct the exact nonsingular gravitational shock-wave solutions of IDG for the proper choice of form factors [see Eq. (29)].

II. INFINITE DERIVATIVE GRAVITY

The most general quadratic, parity-invariant and torsion-free Lagrangian density of IDG is [1–3]

$$\mathcal{L} = \frac{1}{16\pi G}\sqrt{-g}[R + \alpha_c(R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + C_{\mu\nu\rho\sigma}\mathcal{F}_3(\Box)C^{\mu\nu\rho\sigma})], \tag{1}$$

where $G = \frac{1}{M_p^2}$ is the Newton's gravitational constant and $\alpha_c = \frac{1}{M_s^2}$ is a dimensionful parameter where M_s is the scale of the nonlocality, R is the scalar curvature, $R_{\mu\nu}$ is the Ricci tensor, and $C_{\mu\nu\rho\sigma}$ is the Weyl tensor. We work with the $(-,+,+,+)$ signature. In the $\alpha_c \to 0$ (or $M_s \to \infty$) limit, the theory reduces to Einstein's gravity with a massless spin-two graviton. Note that IDG is a special case of ghost-free quadratic curvature theories of gravity. On the other hand, the three form factors $\mathcal{F}_i(\Box)$'s containing infinite derivative functions are defined as[2]

$$\mathcal{F}_i(\Box) \equiv \sum_{n=0}^{\infty} f_{i_n}\frac{\Box^n}{M_s^{2n}}, \tag{3}$$

in which f_{i_n} are dimensionless coefficients. The form factors lead to nonlocal gravitational interactions and f_{i_n} play an important role to avoid ghostlike instabilities. The source-free field equations are [12]

$$\begin{aligned} &G^{\alpha\beta} + \frac{\alpha_c}{2}(4G^{\alpha\beta}\mathcal{F}_1(\Box)R + g^{\alpha\beta}R\mathcal{F}_1(\Box)R - 4(\nabla^\alpha\nabla^\beta - g^{\alpha\beta}\Box)\mathcal{F}_1(\Box)R + 4R^\alpha{}_\nu\mathcal{F}_2(\Box)R^{\nu\beta} - g^{\alpha\beta}R_\nu{}^\mu\mathcal{F}_2(\Box)R_\mu{}^\nu \\ &\quad - 4\nabla_\nu\nabla^\beta(\mathcal{F}_2(\Box)R^{\nu\alpha}) + 2\Box(\mathcal{F}_2(\Box)R^{\alpha\beta}) + 2g^{\alpha\beta}\nabla_\mu\nabla_\nu(\mathcal{F}_2(\Box)R^{\mu\nu}) - g^{\alpha\beta}C^{\mu\nu\rho\sigma}\mathcal{F}_3(\Box)C_{\mu\nu\rho\sigma} + 4C^\alpha{}_{\mu\nu\sigma}\mathcal{F}_3(\Box)C^{\beta\mu\nu\sigma} \\ &\quad - 4(R_{\mu\nu} + 2\nabla_\mu\nabla_\nu)(\mathcal{F}_3(\Box)C^{\beta\mu\nu\alpha}) - 2\Omega_1^{\alpha\beta} + g^{\alpha\beta}(\Omega_{1\rho}{}^\rho + \bar{\Omega}_1) - 2\Omega_2^{\alpha\beta} + g^{\alpha\beta}(\Omega_{2\rho}{}^\rho + \bar{\Omega}_2) \\ &\quad - 4\Delta_2^{\alpha\beta} - 2\Omega_3^{\alpha\beta} + g^{\alpha\beta}(\Omega_{3\gamma}{}^\gamma + \bar{\Omega}_3) - 8\Delta_3^{\alpha\beta}) = 0. \end{aligned} \tag{4}$$

Here, the symmetric tensors are given as [12]

$$\begin{aligned} &\Omega_1^{\alpha\beta} = \sum_{n=1}^{\infty} f_{1_n}\sum_{l=0}^{n-1}\nabla^\alpha R^{(l)}\nabla^\beta R^{(n-l-1)}, \qquad \bar{\Omega}_1 = \sum_{n=1}^{\infty} f_{1_n}\sum_{l=0}^{n-1} R^{(l)}R^{(n-l)}, \\ &\Omega_2^{\alpha\beta} = \sum_{n=1}^{\infty} f_{2_n}\sum_{l=0}^{n-1} R_\nu{}^{\mu;\alpha(l)}R_\mu{}^{\nu;\beta(n-l-1)}, \qquad \bar{\Omega}_2 = \sum_{n=1}^{\infty} f_{2_n}\sum_{l=0}^{n-1} R_\nu{}^{\mu(l)}R_\mu{}^{\nu(n-l)} \\ &\Delta_2^{\alpha\beta} = \frac{1}{2}\sum_{n=1}^{\infty} f_{2_n}\sum_{l=0}^{n-1}[R_\sigma{}^{\nu(l)}R^{(\beta|\sigma|;\alpha)(n-l-1)} - R_\sigma{}^{\nu;(\alpha(l)}R^{\beta)\sigma(n-l-1)}]_{;\nu} \\ &\Omega_3^{\alpha\beta} = \sum_{n=1}^{\infty} f_{3_n}\sum_{l=0}^{n-1} C^{\mu;\alpha(l)}{}_{\nu\rho\sigma}C_\mu{}^{\nu\rho\sigma;\beta(n-l-1)}, \qquad \bar{\Omega}_3 = \sum_{n=1}^{\infty} f_{3_n}\sum_{l=0}^{n-1} C^{\mu(l)}{}_{\nu\rho\sigma}C_\mu{}^{\nu\rho\sigma(n-l)} \\ &\Delta_3^{\alpha\beta} = \frac{1}{2}\sum_{n=1}^{\infty} f_{3_n}\sum_{l=0}^{n-1}[C^{\rho\nu(l)}{}_{\sigma\mu}C_\rho{}^{(\beta|\sigma\mu|;\alpha)(n-l-1)} - C^{\rho\nu}{}_{\sigma\mu}{}^{;(\alpha(l)}C_\rho{}^{\beta)\sigma\mu(n-l-1)}]_{;\nu} \end{aligned} \tag{5}$$

where we used the notation $R^{(n)} = \Box^n R$ for the tensors which are built from the curvature tensors and their derivatives and semicolon denote the covariant derivative. Note that since the field equations are highly complicated and nonlinear, finding

[2]These three form factors are not independent and are constrained. For example, in flat background to conserve general covariance and the massless spin-two nature of graviton, these form factors satisfy the following constraint equation [2,12]:

$$6\mathcal{F}_1(\Box) + 3\mathcal{F}_2(\Box) + 2\mathcal{F}_3(\Box) = 0, \tag{2}$$

which provides that theory has only a transverse-traceless massless spin-two graviton degree of freedom.

exact solutions to the theory might seem hopeless. In the next section, we will give some mathematical preliminaries of the pp-wave spacetimes and show that these spacetimes are the exact solution of the theory for a proper choice of the profile function.

III. *pp*-WAVE SPACETIMES IN IDG

Here we want to find the pp-wave solution of the theory. For this purpose, let us consider the pp-wave (or plane-fronted parallel waves) metric described in the Kerr-Schild form as[3]

$$g_{\mu\nu} = \eta_{\mu\nu} + 2H\lambda_\mu\lambda_\nu. \tag{6}$$

Here $\eta_{\mu\nu}$ denotes the flat metric and the covariantly constant null vector λ_μ satisfies the following relations:

$$\lambda^\mu\lambda_\mu = 0, \qquad \nabla_\mu\lambda_\nu = 0, \tag{7}$$

which give $\lambda^\mu\partial_\mu H = 0$. The null vector λ_μ is nonexpanding $\nabla_\mu\lambda^\mu = 0$, nontwisting $\nabla_\mu\lambda^\nu\nabla_{[\mu}\lambda_{\nu]} = 0$, and shear-free $\nabla_\mu\lambda^\nu\nabla_{(\mu}\lambda_{\nu)} = 0$; hence, the pp-wave metrics belong to class of the Kundt spacetimes [20]. The inverse metric reads as

$$g^{\mu\nu} = \eta^{\mu\nu} - 2H\lambda^\mu\lambda^\nu. \tag{8}$$

To find the pp-wave solution of IDG, one needs to calculate relevant tensors (such as the Riemann, Ricci, and scalar curvature) corresponding to metric. For this purpose, let us note that the Christoffel connection can be computed to be

$$\Gamma^\sigma_{\mu\nu} = \lambda^\sigma\lambda_\mu\partial_\nu H + \lambda^\sigma\lambda_\nu\partial_\mu H - \lambda_\mu\lambda_\nu\eta^{\sigma\beta}\partial_\beta H, \tag{9}$$

which satisfies $\lambda_\sigma\Gamma^\sigma_{\mu\nu} = 0$, $\lambda^\mu\Gamma^\sigma_{\mu\nu} = 0$. Now we are ready to calculate Riemann, Ricci, and Weyl tensors. The Riemann tensor can be found as [38]

$$\begin{aligned} R_{\rho\sigma\mu\nu} = {} & \lambda_\rho\lambda_\nu\partial_\sigma\partial_\mu H + \lambda_\sigma\lambda_\mu\partial_\rho\partial_\nu H \\ & - \lambda_\rho\lambda_\mu\partial_\sigma\partial_\nu H - \lambda_\sigma\lambda_\nu\partial_\rho\partial_\mu H, \end{aligned} \tag{10}$$

with which one gets the Ricci tensor as

$$R_{\mu\nu} = -\lambda_\mu\lambda_\nu\partial^2 H, \tag{11}$$

where ∂^2 is a flat space Laplace operator defined as $\partial^2 = \eta^{\mu\nu}\partial_\mu\partial_\nu$. It is straightforward to see that the scalar curvature is zero as a consequence of the fact that λ_μ is null. Note that any contraction of λ_μ with Weyl, Riemann, and Ricci tensors vanishes:

$$\lambda^\mu C_{\rho\sigma\mu\nu} = 0, \qquad \lambda^\mu R_{\rho\sigma\mu\nu} = 0, \qquad \lambda^\mu R_{\mu\nu} = 0. \tag{12}$$

Furthermore, all the curvature scalars vanish for the pp-wave metric [44,45]. On the other hand, the pp-waves have some remarkable algebraic properties which provide simplicity in calculations. For example, any nontrivial second rank tensor built from the Riemann tensor or its covariant derivatives can be described by a linear combination of traceless Ricci[4] and higher orders of traceless-Ricci ($\Box^n S_{\mu\nu}$'s) tensors [27]. With this property and vanishing of all scalar invariants, the pp-wave spacetimes are Weyl type N. Another remarkable property of the pp-wave metric is that the contraction λ_μ vector with $\nabla^n H$'s vanish [27]

$$\lambda^{\mu_1}\nabla_{\mu_1}\nabla_{\mu_2}\ldots\nabla_{\mu_n}H = 0, \tag{13}$$

which will be frequently used in the paper. Therefore, the λ contraction with other λ's or with $\nabla^n H$'s give zero. Finally, let us consider the structure of a nonzero term given in the form

$$\nabla_{\nu_1}\nabla_{\nu_2}\ldots\nabla_\alpha\ldots\nabla_\beta\ldots\nabla_{\nu_{2n-2}}C^{\beta\mu\alpha\nu} = \frac{1}{2}\nabla_{\nu_1}\nabla_{\nu_2}\ldots\nabla_{\nu_{2n-2}}\Box R^{\mu\nu}, \tag{14}$$

where we have used the following twice-contracted Bianchi identity of the Weyl tensor for the pp-wave metric (6):

$$\nabla_\alpha\nabla_\beta C^{\beta\mu\alpha\nu} = \frac{1}{2}\Box R^{\mu\nu}. \tag{15}$$

A. Field equations of the IDG for *pp*-wave spacetime

Now we are ready to write the field equations of the IDG for the pp-wave spacetimes. By using relations obtained above for each term in the field equations, thanks to the fact that pp-waves have a Riemann tensor of type N together with all its derivatives (and also $R = 0$), only terms linear in the curvature give nonzero contribution in (4) [38,40], and the field equations take the form

$$[1 + \alpha_c(\Box\mathcal{F}_2(\Box) + 2\mathcal{F}_3(\Box)\Box)]R_{\mu\nu} = 0. \tag{16}$$

Note that the pp-wave metrics which satisfy $R_{\mu\nu} = 0$ also solve IDG field equations (16). Using the Ricci tensor definition (11) for the pp-wave metric ansatz, the complete field equations (16) can be recast as

$$[1 + \alpha_c(\Box\mathcal{F}_2(\Box) + 2\mathcal{F}_3(\Box)\Box)]\partial^2 H = 0, \tag{17}$$

[3]For the detailed properties of pp-waves, see [27,41–43].

[4]By traceless-Ricci tensor, we mean $S_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{4}g_{\mu\nu}R$ where $S_{\mu\nu}$ is the traceless-Ricci tensor.

where we also used the fact that the null vector is covariantly constant. Since the form factors $\mathcal{F}_2$ and $\mathcal{F}_3$ can be described in terms of generic operator of d'Alembert as

$$\mathcal{F}_2(\Box) = \sum_{n=0}^{\infty} f_{2_n} \frac{\Box^n}{M_s^{2n}}, \quad \mathcal{F}_3(\Box) = \sum_{n=0}^{\infty} f_{3_n} \frac{\Box^n}{M_s^{2n}}, \tag{18}$$

one needs to evaluate the $\Box^n H$. For this purpose, first let us consider the box operator acting on H

$$\Box H = g^{\mu\nu}\nabla_\mu\nabla_\nu H = \eta^{\mu\nu}\partial_\mu\partial_\nu H - \eta^{\mu\nu}\Gamma^\sigma_{\mu\nu}\partial_\sigma H. \tag{19}$$

By using Eq. (9), it can be easily shown that the last term vanishes since $\eta^{\mu\nu}\Gamma^\sigma_{\mu\nu} = 0$. Then Eq. (19) takes the form

$$\Box H = \partial^2 H. \tag{20}$$

Consequently, one can show that $\Box^n \partial^2 H = \partial^{2n}(\partial^2 H)$, with which the field equations of IDG (17) reduce to

$$[1 + \alpha_c(\partial^2\mathcal{F}_2(\partial^2) + 2\partial^2\mathcal{F}_3(\partial^2))]\partial^2 H = 0, \tag{21}$$

whose most general solution can be given as

$$H_{\mathrm{IDG}} = H_E + \Re(H_I), \tag{22}$$

where H_E refers to the solution of pure Einstein gravity and satisfies the equation $\partial^2 H_E = 0$, H_I is the solution to IDG theory solving equation $[1 + \alpha_c(\partial^2\mathcal{F}_2(\partial^2) + 2\partial^2\mathcal{F}_3(\partial^2))]H_I = 0$, and $\Re$ denotes the real part of the solution of H_I. Here, one should notice that the pp-wave metric solution of Einstein's theory also solves IDG theory.

For the choice of the form factor $\mathcal{F}_2 = \mathcal{F}_3 = 0$ which yields the theory

$$\mathcal{L} = \frac{1}{16\pi G}\sqrt{-g}[R + \alpha_c(R\mathcal{F}_1(\Box)R], \tag{23}$$

which has the nonsingular bouncing solution which may avoid cosmological singularity problem [1]. The associated field equations for the pp-wave spacetimes reduce to $\partial^2 H = 0$. This shows that the pp-wave solutions of Einstein's theory are the exact solution of the theory.

IV. *pp*-WAVE SOLUTIONS

In order to obtain the explicit form of solution (21), one can describe the pp-wave metric in null coordinates with the appropriate choice of λ_μ as [20]

$$ds^2 = 2dudv + 2H(u,x,y)du^2 + dx^2 + dy^2, \tag{24}$$

in which u and v are light-cone background coordinates defined as $u = \frac{1}{\sqrt{2}}(x - t)$ and $v = \frac{1}{\sqrt{2}}(x + t)$. Here, since $\lambda_\mu = \delta^u_\mu$ which yields $\lambda^\mu = \delta^\mu_v$, we have

$$\lambda_\mu dx^\mu = du, \qquad \lambda^\mu\partial_\mu H = \partial_v H = 0. \tag{25}$$

With these properties and using the Laplacian for the metric (24) as $\partial^2 = 2\frac{\partial^2}{\partial u\partial v} + \partial^2_\perp$, here $\partial^2_\perp = \partial^2_x + \partial^2_y$, and Eq. (20) takes the form

$$\Box H = \partial^2_\perp H, \tag{26}$$

where we used the fact that $\partial_v H = 0$, and similarly one has

$$\Box^n H = \partial^{2n}_\perp H \tag{27}$$

and (21) reduces to

$$[1 + \alpha_c(\partial^2_\perp\mathcal{F}_2(\partial^2_\perp) + 2\partial^2_\perp\mathcal{F}_3(\partial^2_\perp))]\partial^2_\perp H = 0, \tag{28}$$

which is the general equation that we want to solve. To proceed further we need the explicit form of form factors $\mathcal{F}_2(\Box)$ and $\mathcal{F}_3(\Box)$.

A. Explicit solutions

For the sake of simplicity, one can choose the following form factors that satisfy ghost freedom [1,2]:

$$\mathcal{F}_2(\Box) = -2\mathcal{F}_1(\Box) = \frac{-1 + e^{-\frac{\Box}{M_s^2}}}{\frac{\Box}{M_s^2}}, \quad \mathcal{F}_3(\Box) = 0, \tag{29}$$

which satisfies the constraint equation (2). With this setting, the theory has only a massless spin-two graviton about the flat background. The corresponding field equation (28) takes the form

$$e^{-\frac{\partial^2_\perp}{M_s^2}}\partial^2_\perp H = 0. \tag{30}$$

To solve this differential equation, use the redefinition as $e^{-\frac{\partial^2_\perp}{M_s^2}} H = V$[5] and later, plugging this into Eq. (30), one obtains

$$\partial^2_\perp V = 0, \tag{31}$$

which is the same field equation satisfied by the pp-wave solutions of Einstein's gravity. All analytic solutions are known [20] for Eq. (31). For the sake of simplicity, we consider the following solution[6]:

[5]To solve this type of differential equation, see [33].

[6]This solution is the gravitational plane wave solution of Einstein's theory [46]. Observe that this solution is not singular at the origin unlike the gravitational shock-wave solutions produced by the point particle.

$$V(u,x,y) = A(u)(x^2 - y^2) + B(u)xy, \qquad (32)$$

where $A(u)$ and $B(u)$ are any arbitrary smooth functions of null coordinate u. By using this solution, one has

$$H(u,x,y) = e^{\frac{\partial_\perp^2}{M_s^2}}(A(u)(x^2 - y^2) + B(u)xy). \qquad (33)$$

To find the solution, one must calculate the action of $e^{\frac{\partial_\perp^2}{M^2}}$ on $V(u,x,y)$ via the Fourier transform. Therefore, after using the Fourier transform and calculating related integrals, the pp-wave solution for IDG can be found as

$$H(u,x,y) = A(u)M_s\left(\left(\frac{2}{M_s^2} + x^2\right)e^{-\frac{M_s^2 y^2}{4}} - \left(\frac{2}{M_s^2} + y^2\right)e^{-\frac{M_s^2 x^2}{4}}\right) + B(u)xy. \qquad (34)$$

B. Linearized field equations of IDG as exact field equations

In this part, we wish to consider the pp-wave solutions of the linearized form of IDG. In fact, one can recognize from (16) that the pp-wave metric solves both the full IDG field equations and the linearized version. In other words, by defining the metric perturbation $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu} = 2H\lambda_\mu\lambda_\nu$, the exact field equations of the IDG take the form of the linearized field equations. To show this explicitly, let us turn our attention to the source-free linearized field equations of the IDG around the flat background of $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$a(\Box)R^L_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}c(\Box)R^L - \frac{1}{2}f(\Box)\partial_\mu\partial_\nu R^L = 0, \qquad (35)$$

where L denotes the linearization and infinite derivative nonlinear functions are described as

$$a(\Box) = 1 + M_s^{-2}(\mathcal{F}_2(\Box) + 2\mathcal{F}_3(\Box))\Box,$$
$$c(\Box) = 1 - M_s^{-2}(4\mathcal{F}_1(\Box) + \mathcal{F}_2(\Box) - \frac{2}{3}\mathcal{F}_3(\Box))\Box,$$
$$f(\Box) = M_s^{-2}(4\mathcal{F}_1(\Box) + 2\mathcal{F}_2(\Box) + \frac{4}{3}\mathcal{F}_3(\Box)), \qquad (36)$$

which yield the constraint $a(\Box) - c(\Box) = f(\Box)\Box$. In the metric perturbation $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu} = 2H\lambda_\mu\lambda_\nu$ for the Kerr-Schild form, after using the linearized form of curvature tensors [47], the linearized Ricci and scalar curvature will read, respectively,

$$R^L_{\mu\nu} = -\frac{1}{2}\partial^2 h_{\mu\nu} = -H\lambda_\mu\lambda_\nu, \qquad R^L = 0. \qquad (37)$$

Observe that the metric perturbation $h_{\mu\nu}$ is transverse traceless: $h = 0$ and $\nabla^\mu h_{\mu\nu} = 0$, hence the linearized scalar curvature R^L vanishes. Furthermore, the theory describes only massless transverse-traceless spin-two DOF. Accordingly, by plugging the linearized tensors (37) into the linearized field equations, one gets

$$a(\Box)(\Box H) = 0. \qquad (38)$$

To further reduce (38), using the definition of nonlinear function $a(\Box)$ (36), one gets

$$[1 + \alpha_c(\mathcal{F}_2(\Box) + 2\mathcal{F}_3(\Box))\Box](\Box H) = 0. \qquad (39)$$

This shows that all solutions of the linearized field equations for the metric perturbation $h_{\mu\nu}$ satisfy the nonlinear field equations of the IDG. Furthermore, the field equations of linearized theory coincide with nonlinear theory for the pp-wave metric. Moreover, in order to have ghost freedom, $a(\Box)$ should be an entire function. The simplest choice is $a(\Box) = e^{-\frac{\Box}{M_s^2}}$ [2]. Thus, the field equations reduce to

$$e^{-\frac{\Box}{M_s^2}}(\Box H) = 0. \qquad (40)$$

For the metric (24), the final result for the linearized field equations is

$$e^{-\frac{\partial_\perp^2}{M_s^2}}\partial_\perp^2 H = 0. \qquad (41)$$

V. EXACT NONSINGULAR GRAVITATIONAL SHOCK WAVE SOLUTION OF IDG FOR THE SPECIFIC CHOICE OF FORM FACTORS

In this section, we would like to extend the pp-wave solutions in the presence of the pure radiation sources (null dust). Gravitational shock-wave solution can provide understanding of the gravitational interactions between high energy massless particles in IDG. Shock waves are a special class of axisymmetric pp-waves and its metric produced by a fast moving massless point particle can be described as follows [46,48][7]

$$ds^2 = -dudv + \delta(u)g(x_\perp)du^2 + dx_\perp^2, \qquad (42)$$

where $u = t - z$ and $v = t + z$ are the null-cone background coordinates,[8] $(x^i) = x_\perp$ where $i = 1, 2$ are the transverse coordinates to the wave propagation and $g(x_\perp)$ is

[7] In fact we can use the pp-wave metric given in the form (24), but the form of Eq. (42) is commonly used in the literature. Therefore, we use this form. Note that the metric (42) can also be described in the Kerr-Schild form as $g_{\mu\nu} = \eta_{\mu\nu} + V\lambda_\mu\lambda_\nu$ which leads to $R_{\mu\nu} = -\frac{1}{2}\lambda_\mu\lambda_\nu\partial^2 V$ where $V = \delta(u)g(x_\perp)$.

[8] $(t, x_\perp, z)$ are the coordinates in the Minkowski space.

the wave profile function. To find the exact shock-wave solution of IDG, one needs to find the form of the wave profile function. For this purpose, let us consider the massless point particle traveling in the positive z direction with momentum $p^\mu = |p|(\delta^\mu_t + \delta^\mu_z)$. The associated null source creating the shock-wave geometry can be described as $T_{uu} = |p|\delta(x_\perp)\delta(u)$. For the shock-wave ansatz (42), the only nonvanishing components of the Ricci tensor is

$$R_{uu} = G_{uu} = -\frac{\delta(u)}{2}\frac{\partial^2}{\partial^2_\perp} g(x_\perp). \tag{43}$$

On the other hand, the energy-momentum tensor in the Kerr-Schild form can be written as $T_{\mu\nu} = |p|\delta(x_\perp)\delta(u)\lambda_\mu\lambda_\nu$. Therefore, the null source coupled IDG field equations (28) reduce to the much simpler form

$$[1+\alpha_c(\partial^2_\perp \mathcal{F}_2(\partial^2_\perp) + 2\partial^2_\perp \mathcal{F}_3(\partial^2_\perp))]\partial^2_\perp g(x_\perp) = -16\pi G|p|\delta(x_\perp). \tag{44}$$

For the simplest choice of the form factors as in (29), Eq. (44) becomes a modified Poisson type equation

$$e^{-\frac{\partial^2_\perp}{M_s^2}}\partial^2_\perp g(x_\perp) = -16\pi G|p|\delta(x_\perp). \tag{45}$$

After using the Fourier transform and evaluating related integrals, the axial symmetric solution can be obtained as

$$g(r) = -8G|p|\left(\ln\left(\frac{r}{r_0}\right) - \frac{1}{2}\mathrm{Ei}\left(-\frac{r^2M_s^2}{4}\right)\right), \tag{46}$$

where r is the distance to the origin defined as $r=\sqrt{x^2_\perp}$ and r_0 is integral constant. Here, Ei is the exponential integral function.[9] Note that in the $M_s \to \infty$ limit, the profile function becomes [51–53]

$$g(r) = -8G|p|\ln\left(\frac{r}{r_0}\right), \tag{48}$$

which is the Einstein's gravity result as expected. Thus, the exact gravitational shock-wave solution metric for IDG is

$$ds^2 = -dudv - 4G|p|\delta(u)\left(\ln\left(\frac{r^2}{r_0^2}\right) - \mathrm{Ei}\left(-\frac{r^2M_s^2}{4}\right)\right)du^2 + dx^2_\perp. \tag{49}$$

[9]The exponential integral function for negative arguments defined by the integral [49,50]

$$\mathrm{Ei}(r) = -\int_{-r}^{\infty}\frac{e^{-t}}{t}dt, \tag{47}$$

and its derivative is $\mathrm{Ei}'(r) = \frac{d}{dr}\mathrm{Ei}(r) = \frac{e^r}{r}$.

Note that there is a distributional term in the null coordinate u, but this discontinuity can be removed by redefining new coordinates [51]. On the other hand, for small distances (in the UV regime of nonlocality), expanding the exponential integral function into the Puiseux series around $r=0$ gives [49,54]

$$\mathrm{Ei}(r) = \gamma + \ln|r| + r + \mathcal{O}(r^2), \tag{50}$$

where γ is Euler-Mascheroni constant. In the nonlocal regime $M_s r \ll 2$, the profile function is nonsingular and reduces to

$$\lim_{M_s r\to 0} g(r) = g_0 = 4G\gamma|p|, \tag{51}$$

which is a constant. Here, for the sake of simplicity we set $r_0 = \frac{2}{M_s}$. It is important to note that this choice does not affect the result in (51) to be constant. Interestingly, the gravitational shock-wave solution of IDG is nonsingular in the UV regime of nonlocality $M_s r \ll 2$ while the result of pure GR diverges. Even though the shock wave is produced by the null matter source which contains the Dirac delta function type singularity in the radial direction, the solution is nonsingular at the origin due to the improved behavior of the propagator in the UV scale.

In fact, the discussion given above is not enough to conclude that the singularity disappears. One must also analyze whether the curvature tensor diverges at the origin or not. Even if some modified gravity theories which contain four derivatives or less such as quadratic gravity have a nonsingular profile function,[10] some component of the Riemann tensor diverges logarithmically [57,58]. Now, let us show that curvature tensors and invariants are nonsingular at the position of the particle for the nonsingular metric (49) in the ghost-free IDG. One can demonstrate that the only nonzero components of the Riemann tensor are

$$R^v{}_{rur} = 8G|p|\delta(u)\left(\frac{(1-e^{-\frac{r^2}{4M_s^2}})}{r^2} - \frac{e^{-\frac{r^2}{4M_s^2}}}{2M_s^2}\right),$$
$$R^v{}_{\phi u\phi} = 8G|p|\delta(u)(1-e^{-\frac{r^2}{4M_s^2}}),$$
$$R^\phi{}_{uu\phi} = 4G|p|\delta(u)\frac{(-1+e^{-\frac{r^2}{4M_s^2}})}{r^2},$$
$$R^r{}_{uur} = 4G|p|\delta(u)\left(\frac{(1-e^{-\frac{r^2}{4M_s^2}})}{r^2} - \frac{e^{-\frac{r^2}{4M_s^2}}}{2M_s^2}\right), \tag{52}$$

wherein the components for the $M_s r \to 0$ limit behave as

[10]For regularity properties of higher derivative gravity theories which contain at least six derivatives, see [55,56].

$$R^{v}{}_{rur} \sim -\frac{2G|p|\delta(u)}{M_s^2}, \qquad R^{v}{}_{\phi u \phi} \sim 0,$$
$$R^{\phi}{}_{u u \phi} \sim -\frac{G|p|\delta(u)}{M_s^2} \qquad R^{r}{}_{uur} \sim -\frac{G|p|\delta(u)}{M_s^2}, \qquad (53)$$

which are finite at the origin. So, all the nonzero components of the Riemann tensor are nonsingular in the UV regime of nonlocality $M_s r \ll 2$. On the other hand, the only nonvanishing component of the Ricci tensor is

$$R_{uu} = 2G|p|\delta(u)\frac{e^{-\frac{r^2}{4M_s^2}}}{M_s^2}, \qquad (54)$$

which approaches to a constant in the nonlocal region. Finally, the scalar curvature vanishes, all components of Weyl tensor are zero ($C_{\rho\sigma\mu\nu} \sim 0$) in the in $M_s r \to 0$ limit and all the curvature invariants squared are given by

$$R^2 = 0, \qquad R_{\mu\nu}R^{\mu\nu} = 0,$$
$$\mathcal{K} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = 0, \qquad C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} = 0, \qquad (55)$$

where $\mathcal{K}$ is the Kretschmann scalar. In fact, the results given in (55) are a direct consequence of the fact that all the curvature scalars vanish for the pp-wave metric [44,45]. With this discussion, we have shown that the gravitational shock-wave solution of IDG is nonsingular at the origin. It is also important to note that to investigate the nonsingular nature, one usually chooses a geodesic and constructs a frame parallelly transported along the geodesic completeness [59,60]. For this purpose, say $e^{\mu}_{(a)}$ are such parallelly transported frames, then one needs to compute $R_{abcd} = e^{\mu}_{(a)}e^{\nu}_{(b)}e^{\rho}_{(c)}e^{\sigma}_{(d)}R_{\mu\nu\rho\sigma}$ and show the finiteness of R_{abcd}. But, since this is beyond scope of the core of the current study, we will not do this here.

VI. CONCLUSIONS

In this work, we studied exact pp-wave metrics of the ghost and singularity-free IDG and showed that these metrics are exact solutions. The pp-wave metrics also solve linearized field equations of the IDG. That is, the field equations of nonlinear theory coincide with the linearized field equations for the pp-wave metrics. Undoubtedly, finding the exact solution is not an easy task since the field equations of the theory are highly nontrivial and nonlinear. But, writing the metric in the Kerr-Schild form leads to a remarkable simplification on the field equations.

We have also concentrated on the special class of axisymmetric pp-waves. Here, we studied the nonperturbative solution of the theory in the presence of the null source and found the exact nonsingular gravitational shock-wave solution of the theory. We have shown that unlike the case in Einstein's gravity, although gravitational shock-wave solution are created by a source having a Dirac delta type singularity, the solution and curvature tensors are regular in the nonlocal regime due to gravitational nonlocal interactions. Even though some nonsingular solutions of the IDG at the linearized level are known [32–34], we find a nonsingular gravitational shock-wave solution for the theory at the nonlinear level.

Although, we considered the exact solutions in the ghost-free IDG with a zero cosmological constant, this work can be extended to the case of the nonzero cosmological constant as was done for quadratic gravity [27]. For example, anti–de Sitter plane waves are potential exact solutions of the theory. On the other hand, studying the Kerr-Schild class of metrics in nonlocal gravity models [61–63] which are the infrared modification of GR, where the form factors are nonanalytic functions of the d'Alembert operator, would also be interesting.

ACKNOWLEDGMENTS

We would like to thank B. Tekin and S. Dengiz for useful discussions, suggestions, and critical readings of the manuscript. We would also like to thank T. de Paula Netto for suggestions and comments. We would like to thank the referee for bringing the point about parallelly transported frames to our attention.

[1] T. Biswas, A. Mazumdar, and W. Siegel, Bouncing universes in string-inspired gravity, J. Cosmol. Astropart. Phys. 03 (2006) 009.
[2] T. Biswas, E. Gerwick, T. Koivisto, and A. Mazumdar, Towards Singularity and Ghost Free Theories of Gravity, Phys. Rev. Lett. **108**, 031101 (2012).
[3] L. Modesto, Super-renormalizable Quantum Gravity, Phys. Rev. D **86**, 044005 (2012).
[4] T. Biswas, T. Koivisto, and A. Mazumdar, Nonlocal theories of gravity: The flat space propagator, arXiv:1302.0532.
[5] J. Edholm, A. S. Koshelev, and A. Mazumdar, Behavior of the Newtonian potential for ghost-free gravity and singularity-free gravity, Phys. Rev. D **94**, 104033 (2016).
[6] E. Kilicarslan, Weak field limit of infinite derivative gravity, Phys. Rev. D **98**, 064048 (2018).

[7] S. Talaganis, T. Biswas, and A. Mazumdar, Towards understanding the ultraviolet behavior of quantum loops in infinite-derivative theories of gravity, Classical Quantum Gravity **32**, 215017 (2015).
[8] S. Talaganis and A. Mazumdar, High-energy scatterings in infinite-derivative field theory and ghost-free gravity, Classical Quantum Gravity **33**, 145005 (2016).
[9] E. T. Tomboulis, Superrenormalizable gauge and gravitational theories, arXiv:hep-th/9702146.
[10] L. Buoninfante, G. Lambiase, and A. Mazumdar, Ghost-free infinite derivative quantum field theory, Nucl. Phys. **B944**, 114646 (2019).
[11] G. Calcagni and L. Modesto, Nonlocal quantum gravity and M-theory, Phys. Rev. D **91**, 124059 (2015).
[12] T. Biswas, A. Conroy, A. S. Koshelev, and A. Mazumdar, Generalized ghost-free quadratic curvature gravity, Classical Quantum Gravity **31**, 015022 (2014).
[13] L. Modesto, J. W. Moffat, and P. Nicolini, Black holes in an ultraviolet complete quantum gravity, Phys. Lett. B **695**, 397 (2011).
[14] T. Biswas, T. Koivisto, and A. Mazumdar, Towards a resolution of the cosmological singularity in non-local higher derivative theories of gravity, J. Cosmol. Astropart. Phys. 11 (2010) 008.
[15] T. Biswas, A. S. Koshelev, A. Mazumdar, and S. Y. Vernov, Stable bounce and inflation in non-local higher derivative cosmology, J. Cosmol. Astropart. Phys. 08 (2012) 024.
[16] V. P. Frolov, Mass-Gap for Black Hole Formation in Higher Derivative and Ghost Free Gravity, Phys. Rev. Lett. **115**, 051102 (2015).
[17] A. S. Koshelev, K. S. Kumar, and A. A. Starobinsky, R^2 inflation to probe non-perturbative quantum gravity, J. High Energy Phys. 03 (2018) 071.
[18] A. S. Koshelev, J. Marto, and A. Mazumdar, Schwarzschild $1/r$-singularity is not permissible in ghost free quadratic curvature infinite derivative gravity, Phys. Rev. D **98**, 064023 (2018).
[19] J. Boos, V. P. Frolov, and A. Zelnikov, Gravitational field of static p-branes in linearized ghost-free gravity, Phys. Rev. D **97**, 084021 (2018).
[20] H. Stephani, D. Kramer, M. MacCallum, C. Hoenselaers, and E. Herlt, *Exact Solutions of Einstein's Field Equations* (Cambridge University Press, Cambridge, England, 2003).
[21] A. S. Eddington, *The Mathematical Theory of Relativity* (Cambridge University Press, Cambridge, England, 1924).
[22] H. A Buchdahl, On Eddington's higher order equations of the gravitational field, Proc. Edinb. Math. Soc. **8**, 89 (1948).
[23] H. A. Buchdahl, On the gravitational field equations arising from the square of the gaussian curvature, Nuovo Cimento **23**, 141 (1962).
[24] I. Gullu, M. Gurses, T. C. Sisman, and B. Tekin, AdS waves as exact solutions to quadratic gravity, Phys. Rev. D **83**, 084015 (2011).
[25] M. Gurses, T. C. Sisman, and B. Tekin, New exact solutions of quadratic curvature gravity, Phys. Rev. D **86**, 024009 (2012).
[26] M. Gurses, T. C. Sisman, and B. Tekin, Gravity waves in three dimensions, Phys. Rev. D **92**, 084016 (2015).
[27] M. Gurses, T. C. Sisman, and B. Tekin, AdS-plane wave and pp-wave solutions of generic gravity theories, Phys. Rev. D **90**, 124005 (2014).
[28] M. Gurses, T. C. Sisman, B. Tekin, and S. Hervik, Anti–de Sitter–Wave Solutions of Higher Derivative Theories, Phys. Rev. Lett. **111**, 101101 (2013).
[29] M. Gurses, T. C. Sisman, and B. Tekin, Some exact solutions of all $f(R_{/mu/nu})$ theories in three dimensions, Phys. Rev. D **86**, 024001 (2012).
[30] T. P. Sotiriou and V. Faraoni, f(R) theories of gravity, Rev. Mod. Phys. **82**, 451 (2010).
[31] Y. D. Li, L. Modesto, and L. Rachwal, Exact solutions and spacetime singularities in nonlocal gravity, J. High Energy Phys. 12 (2015) 173.
[32] A. S. Cornell, G. Harmsen, G. Lambiase, and A. Mazumdar, Rotating metric in nonsingular infinite derivative theories of gravity, Phys. Rev. D **97**, 104006 (2018).
[33] L. Buoninfante, G. Harmsen, S. Maheshwari, and A. Mazumdar, Nonsingular metric for an electrically charged point-source in ghost-free infinite derivative gravity, Phys. Rev. D **98**, 084009 (2018).
[34] L. Buoninfante, A. S. Koshelev, G. Lambiase, J. Marto, and A. Mazumdar, Conformally-flat, non-singular static metric in infinite derivative gravity, J. Cosmol. Astropart. Phys. 06 (2018) 014.
[35] L. Buoninfante and A. Mazumdar, Nonlocal star as a blackhole mimicker, arXiv:1903.01542.
[36] R. Gueven, Plane waves in effective field theories of superstrings, Phys. Lett. B **191**, 275 (1987).
[37] D. Amati and C. Klimcik, Nonperturbative computation of the Weyl anomaly for a class of nontrivial backgrounds, Phys. Lett. B **219**, 443 (1989).
[38] G. Horowitz and A. R. Steif, Spacetime Singularities in String Theory, Phys. Rev. Lett. **64**, 260 (1990).
[39] A. A. Coley, G. W. Gibbons, S. Hervik, and C. N. Pope, Metrics with vanishing quantum corrections, Classical Quantum Gravity **25**, 145017 (2008).
[40] S. Hervik, V. Pravda, and A. Pravdova, Type III and N universal spacetimes, Classical Quantum Gravity **31**, 215005 (2014).
[41] M. Blau, Plane waves and Penrose limits, Lecture Notes for the ICTP School on Mathematics in String and Field Theory, 2003.
[42] M. Ortaggio, V. Pravda, and A. Pravdova, Algebraic classification of higher dimensional spacetimes based on null alignment, Classical Quantum Gravity **30**, 013001 (2013).
[43] J. D. Edelstein, G. Giribet, C. Gomez, E. Kilicarslan, M. Leoni, and B. Tekin, Causality in 3D massive gravity theories, Phys. Rev. D **95**, 104016 (2017).
[44] A. Peres, Some Gravitational Waves, Phys. Rev. Lett. **3**, 571 (1959).
[45] V. Pravda, A. Pravdova, A. Coley, and R. Milson, All spacetimes with vanishing curvature invariants, Classical Quantum Gravity **19**, 6213 (2002).
[46] J. B. Griffiths and J. Podolský, *Exact Space-Times in Einstein's General Relativity* (Cambridge University Press, Cambridge, England, 2009).
[47] S. Deser and B. Tekin, Energy in generic higher curvature gravity theories, Phys. Rev. D **67**, 084009 (2003).

[48] R. Penrose, in *General relativity, Papers in Honour of J. L. Synge*, edited by L. O'Raifeartaigh (Clarendon Press, Oxford, 1972), pp. 101–115.

[49] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 7th ed. edited by A. Jeffrey and D. Zwillinger (Academic Press, New York, 2007).

[50] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions* (Dover Publications, New York, 1972).

[51] T. Dray and G. 't Hooft, The gravitational shock wave of a massless particle, Nucl. Phys. **B253**, 173 (1985).

[52] W. B. Bonnor, The gravitational field of light, Commun. Math. Phys. **13**, 163 (1969).

[53] P. C. Aichelburg and R. U. Sexl, On the gravitational field of a massless particle, Gen. Relativ. Gravit. **2**, 303 (1971).

[54] F. E. Harris, Tables of the exponential integral Ei(x), Math. Comp. **11**, 9 (1957).

[55] V. P. Frolov, A. Zelnikov, and T. de Paula Netto, Spherical collapse of small masses in the ghost-free gravity, J. High Energy Phys. 06 (2015) 107.

[56] B. L. Giacchini and T. de Paula Netto, Weak-field limit and regular solutions in polynomial higher-derivative gravities, Eur. Phys. J. C **79**, 217 (2019).

[57] M. Campanelli and C. O. Lousto, Exact gravitational shock wave solution of higher order theories, Phys. Rev. D **54**, 3854 (1996).

[58] C. O. Lousto and F. D. Mazzitelli, Exact selfconsistent gravitational shock wave in semiclassical gravity, Phys. Rev. D **56**, 3471 (1997).

[59] C. B. Collins and G. F. R. Ellis, Singularities in Bianchi cosmologies, Phys. Rep. **56**, 65 (1979).

[60] A. A. Coley, S. Hervik, W. C. Lim, and M. A. H. MacCallum, Properties of kinematic singularities, Classical Quantum Gravity **26**, 215008 (2009).

[61] A. O. Barvinsky and G. A. Vilkovisky, The generalized Schwinger-Dewitt technique in gauge theories and quantum gravity, Phys. Rep. **119**, 1 (1985).

[62] S. Deser and R. P. Woodard, Nonlocal Cosmology, Phys. Rev. Lett. **99**, 111301 (2007).

[63] A. Conroy, T. Koivisto, A. Mazumdar, and A. Teimouri, Generalized quadratic curvature, non-local infrared modifications of gravity and Newtonian potentials, Classical Quantum Gravity **32**, 015024 (2015).