Palatini formulation of pure R^2 gravity yields Einstein gravity with no massless scalar

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Pure R^2 gravity has been shown to be equivalent to Einstein gravity with nonzero cosmological constant and a massless scalar field when restricted Weyl symmetry is spontaneously broken. We show that the Palatini formulation of pure R^2 gravity is equivalent to Einstein gravity with a nonzero cosmological constant as before but with no massless scalar field when the Weyl symmetry is spontaneously broken. This is an important new development because the massless scalar field is not readily identifiable with any known particle in nature or unknown particles like cold dark matter which are expected to be massive. We then include a nonminimally coupled Higgs field as well as fermions to discuss how the rest of the standard model fields fit into this paradigm. With Higgs field, Weyl invariance is maintained by using a hybrid formalism that includes both the Palatini curvature scalar $\mathcal{R}$ and the usual Ricci scalar R.

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I. INTRODUCTION

Pure R^2 gravity (i.e., R^2 gravity with no extra R term) possesses a symmetry that is larger than scale symmetry and smaller than full Weyl symmetry. This was dubbed restricted Weyl symmetry [1] as the action is invariant under $g_{\mu\nu} \to \Omega^2(x) g_{\mu\nu}$ with the condition that $\Box\Omega = 0$. This theory was shown to be equivalent to Einstein gravity with nonzero cosmological constant and a massless scalar field [2–6] (see also [7] for a Weyl geometry approach.) The massless scalar is identified as the Nambu-Goldstone boson of the spontaneously broken restricted Weyl symmetry [6].

The Palatini formulation [8,9][1] of pure R^2 gravity is Weyl invariant [39] and we show in this paper that when the Weyl symmetry is spontaneously broken it is equivalent to Einstein gravity with nonzero cosmological constant as before but no massless scalar field appears in contrast to the metric formulation. This is an important and positive development; the presence of a massless scalar is not an issue theoretically but as far as we know, there is no evidence for such a particle in nature. In particular, it cannot act as a cold dark matter candidate which is expected to be massive.

We then introduce into the action a nonminimally coupled massless Higgs field. To maintain Weyl invariance, we use a hybrid formalism where the action includes both the Palatini curvature scalar $\mathcal{R}$ and the usual Ricci scalar R. We show that this action is equivalent to Einstein gravity with cosmological constant and a nonminimally coupled *massive* Higgs field. Again, no massless scalar field appears. The original massless Higgs becomes massive and can now be taken to be the doublet of the standard model. The rest of the standard model fields can be readily included into this paradigm. In particular fermions can be included via the torsion-free Levi-Civita spin connection ω_μ^{ab} constructed from the vielbein. A separate Palatini spin connection Ω_μ^{ab} is also introduced and applied to the gravity sector. The main difference between the two is that the Levi-Civita spin connection changes under a Weyl transformation while the Palatini spin connection remains unchanged. The use of these two connections in the action is prescribed by the principle that the action is Weyl invariant.

II. PALATINI FORMULATION OF PURE R^2 GRAVITY

We show that the Palatini formulation of pure R^2 gravity, upon regarding the Weyl symmetry as a gauge symmetry, is equivalent to Einstein gravity with nonzero cosmological constant. First note that pure R^2 gravity in the Palatini formulation in four dimensions is Weyl invariant [39] unlike in the metric formulation, which has only restricted Weyl symmetry [1,4]. The Palatini action is given by

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[1]For use of the Palatini formalism in various contexts see Refs. [10–38].

$$S = \int d^4x\sqrt{-g}\alpha\mathcal{R}^2. \quad (1)$$

Here, the Palatini scalar curvature $\mathcal{R}$ is given by

$$\mathcal{R} = g^{\mu\sigma}(\partial_\nu\Gamma^\nu_{\mu\sigma} - \partial_\sigma\Gamma^\nu_{\mu\nu} + \Gamma^\nu_{\alpha\nu}\Gamma^\alpha_{\mu\sigma} - \Gamma^\nu_{\alpha\sigma}\Gamma^\alpha_{\mu\nu}), \quad (2)$$

where we treat $g_{\mu\nu}$ and $\Gamma^\sigma_{\mu\nu}$ as independent variables (i.e., $\Gamma^\sigma_{\mu\nu}$ is not the Christoffel connection at this point). We assume that the Palatini connection is symmetric $\Gamma^\sigma_{\mu\nu} = \Gamma^\sigma_{\nu\mu}$.[2] The above action is invariant under the Weyl transformation

$$g_{\mu\nu} \to \Omega^2(x)g_{\mu\nu}$$
$$\Gamma^\mu_{\nu\rho} \to \Gamma^\mu_{\nu\rho}. \quad (3)$$

Note that in contrast to the metric formulation, the connection is chosen to be unchanged under a Weyl transformation. Thus the scalar curvature transforms as

$$\mathcal{R} \to \Omega^{-2}(x)\mathcal{R} \quad (4)$$

while the metric Ricci scalar, denoted by R, transforms as

$$R \to \Omega^{-2}R - 6\Omega^{-3}\Box\Omega \quad (5)$$

in four dimensions. It is now obvious that the pure $\mathcal{R}^2$ action is Weyl invariant in the Palatini formulation because we use $\mathcal{R}$ instead of R.

While the equations of motion of the model were studied e.g., in [43] (as reviewed in [44]), we do not follow their approach. Instead we would like to find an equivalent expression at the action level. Let us rewrite the Palatini R^2 action by introducing an auxiliary field $\varphi(x)$ as

$$S_1 = \int d^4x\sqrt{-g}(-\alpha(c_1\varphi + \mathcal{R})^2 + \alpha\mathcal{R}^2)$$
$$= \int d^4x\sqrt{-g}(-c_1^2\alpha\varphi^2 - 2\alpha c_1\varphi\mathcal{R}), \quad (6)$$

where c_1 is an arbitrary (nonzero) number. The above action is still Weyl invariant if φ transforms as $\varphi \to \varphi/\Omega^2$ when $g_{\mu\nu} \to \Omega^2(x)g_{\mu\nu}$.

We now perform the following field redefinition to go to the Einstein frame:

$$g_{\mu\nu} = \varphi^{-1}g^{(E)}_{\mu\nu}$$
$$\sqrt{-g} = \varphi^{-2}\sqrt{-g^{(E)}}$$
$$\mathcal{R} = \varphi\mathcal{R}^{(E)}. \quad (7)$$

This is not the Weyl transformation we introduced in the last paragraph, which involves the change of φ. The transformation here acts only on the metric and it changes the appearance of the action.

After the change of the field variables, the resulting action is

$$S_E = \int d^4x\sqrt{-g^{(E)}}(-c_1^2\alpha - 2\alpha c_1\mathcal{R}^{(E)}). \quad (8)$$

This is nothing but Einstein gravity with nonzero cosmological constant in the Palatini formulation in the Einstein frame. Varying the action with respect to $\Gamma^\mu_{\rho\sigma}$ gives the metric compatibility condition

$$D_\rho g^{(E)}_{\mu\nu} = 0, \quad (9)$$

which now identifies $\Gamma^\mu_{\rho\sigma}$ with the Christoffel connection (with the assumption of the symmetric connection)

$$\Gamma^\mu_{\rho\sigma} = \frac{1}{2}g^{\mu\nu(E)}(\partial_\rho g^{(E)}_{\sigma\nu} + \partial_\sigma g^{(E)}_{\rho\nu} - \partial_\nu g^{(E)}_{\rho\sigma}). \quad (10)$$

Varying the action with respect to $g^{(E)}_{\mu\nu}$ then gives the Einstein equations with cosmological constant.

At this point, we would like to discuss the fate of the Weyl symmetry. The Einstein action (8) is clearly not Weyl invariant in the conventional sense of $g^{(E)}_{\mu\nu} \to \Omega^2(x)g^{(E)}_{\mu\nu}$. The original Weyl symmetry, after the change of field variables (7), does not act on the Einstein-frame metric $g^{(E)}_{\mu\nu}(x)$ in (8). The original Weyl symmetry, now acting only on the scalar φ as

$$\varphi \to \Omega^{-2}\varphi, \quad (11)$$

is still a symmetry but it is trivial because $\varphi(x)$ does not appear in the action.

Note that in order to make sense of the field redefinition (7), φ cannot be zero. With nonzero φ, the original Weyl symmetry is spontaneously broken because $\varphi \neq 0$ has a scale dimension [which is equivalent to $\mathcal{R} \neq 0$ in the original action (1)].[3] In the Einstein metric R^2 gravity,

[2] Alternatively one may impose the projective (gauge) symmetry $\Gamma^\sigma_{\mu\nu} \to \Gamma^\sigma_{\mu\nu} + \delta^\sigma_\mu U_\nu$ and make it torsion-free (i.e., symmetric in μ and ν) by fixing the gauge (see e.g., [40–42]). Without matter, there is no inconsistency about this torsion-free assumption. We will later show that our hybrid theory coupled with Higgs and fermions does not couple to the torsion unlike the conventional Einstein-Palatini theory thanks to the Weyl invariance we impose as our guiding principle.

[3] The notion of "spontaneous symmetry breaking" is slightly different from "dynamical symmetry breaking." We do not claim our vacuum is dynamically favored over the unbroken vacuum solution $\varphi = 0$ (or $\mathcal{R} = 0$). However, once we choose our spontaneously broken vacuum solution, the unbroken solution is infinite distance away in the field theory space and we cannot restore the symmetry with finite time and energy.

such spontaneous breaking of the (restricted) Weyl symmetry led to the emergence of the Nambu-Goldston boson [4]. The distinct point here is that the auxiliary scalar φ disappears completely due to the Weyl invariance.[4] We should regard φ as a would-be Nambu-Goldstone boson for the spontaneous breaking of the Weyl symmetry [6]. However, because of the Weyl invariance, it does not appear in the final action.

Since it does not appear in the action, we may just leave it as it is and forget the field φ, but theoretically a more satisfactory treatment is to declare that the Weyl symmetry is a gauge symmetry (as first advocated by Weyl himself). Then we can get rid of φ completely by fixing the gauge (say $\varphi = 1$), and there is no Nambu-Goldstone boson for spontaneously broken gauge symmetry. Thus, the R^2 gravity in the Palatini formulation is completely equivalent to the Einstein gravity with (arbitrary but nonzero) cosmological constant once we regard the Weyl symmetry as a gauge symmetry.

To complete our analysis, we would like to discuss the case with unbroken Weyl symmetry with $\varphi = 0$ or $\mathcal{R} = 0$ in the original action (1). In the Einstein R^2 gravity, it is known that when the restricted Weyl symmetry is not spontaneously broken in the Minkowski vacuum (i.e., $R = 0$), the theory does not gravitate and the theory is uninteresting [5]. In the Palatini $\mathcal{R}^2$ gravity, when the Weyl symmetry is not spontaneously broken the Palatini connection is undetermined [43] and the theory is pathological and again uninteresting. In this paper, we only focus on the physically relevant solutions of $\mathcal{R}^2$ gravity that are equivalent to the Einstein gravity with nonzero cosmological constant.

III. INCLUSION OF HIGGS FIELD AND FERMIONS

We may generalize the construction by introducing the Higgs field as we did in our previous papers [4,6]. Given the mechanism of decoupling of φ in the pure R^2 case, the crucial assumption here is we impose the Weyl invariance and regard it as a gauge symmetry. For our purpose of reproducing the standard model of particle physics coupled with Einstein gravity, we focus on the following Weyl invariant action:

$$S_1 = \int d^4x\sqrt{-g}\bigg(\alpha\mathcal{R}^2 - \xi\mathcal{R}|\Phi|^2 - \frac{1}{6}R|\Phi|^2 - (\partial_\mu\bar{\Phi}\partial^\mu\Phi) - \lambda|\Phi|^4\bigg). \tag{12}$$

We stress that $\mathcal{R}$ is the Palatini scalar curvature while R is the Ricci scalar constructed out of the metric tensor (and its derivatives) and they are different (at this point). The nonminimal coupling of the Higgs field to R is fixed (i.e., $1/6$) by the Weyl invariance while the nonminimal coupling constant ξ, which couples to the Palatini curvature $\mathcal{R}$, is arbitrary.

At this point, we should stress that we depart slightly from the original philosophy of Palatini that avoids the use of the (Riemann) curvature in the action. Our philosophy rather is to impose the Weyl invariance, and with the Palatini curvature, we claim (12) is Weyl invariant and ghost-free.[5] In particular, it is impossible to introduce the Weyl invariant kinetic term for scalar fields with only the Palatini curvature.

We introduce the auxiliary field φ to rewrite the action as

$$S_1 = \int d^4x\sqrt{-g}\bigg(-\alpha\Big(c_1\varphi + \mathcal{R} + \frac{c_2}{\alpha}|\Phi|^2\Big)^2 + \alpha\mathcal{R}^2 - \xi\mathcal{R}|\Phi|^2 - \frac{1}{6}R|\Phi|^2 - (\partial_\mu\bar{\Phi}\partial^\mu\Phi) - \lambda|\Phi|^4\bigg) \tag{13}$$

with arbitrary constants c_1 and c_2. After the field redefinition,

$$\begin{aligned} g_{\mu\nu} &\to \varphi^{-1}g_{\mu\nu} \\ \sqrt{-g} &\to \varphi^{-2}\sqrt{-g} \\ \mathcal{R} &\to \varphi\mathcal{R} \\ \Phi &\to \varphi^{1/2}\Phi \\ R &\to \varphi R - 6\varphi^{3/2}\Box\varphi^{-1/2}, \end{aligned} \tag{14}$$

the action becomes[6]

$$S = \int d^4x\sqrt{-g}\bigg(-\alpha c_1^2 - 2\alpha c_1\mathcal{R} - \partial_\mu\bar{\Phi}\partial^\mu\Phi - 2c_1c_2|\Phi|^2 - (\alpha^{-1}c_2^2 + \lambda)|\Phi|^4 - (\xi + 2c_2)\mathcal{R}|\Phi|^2 - \frac{1}{6}R|\Phi|^2\bigg). \tag{15}$$

[4]When we break the Weyl invariance e.g., by adding the linear $\mathcal{R}$ term in the action, φ does not disappear [45]. The matter sector also has to respect the Weyl symmetry to make φ disappear as we will do in the following section.

[5]The Weyl invariance allows other terms such as Weyl squared, but generally it predicts propagating ghost modes. See e.g., [42,46] for a recent study of such propagating ghost modes in the Palatini formulations. The full analysis of all the possible Weyl invariant terms is not yet completed and there is a chance that specific linear combinations of Palatini/Ricci curvature terms (e.g., difference of Palatini Weyl squared and Riemann Weyl squared) may result in unitary theories, but this is beyond the scope of our paper. We also note that there are further Weyl invariant terms if we abandon the torsion-free condition or nonmetricity. The Weyl invariant terms constructed out of torsion or nonmetricity result in propagating extra degrees of freedom and typically cause a ghost. See e.g., [41,42] for a recent study.

[6]While we have put the subscript (E) to the Einstein frame fields in the last section, we hereafter omit the subscript after the field redefinition.

Note that φ does not appear in the final action thanks to the Weyl invariance of the original action. Now, we declare that we regard the Weyl symmetry as a gauge symmetry and discard φ. To make the action simpler, we choose $\xi = -2c_2$ so that the Palatini scalar curvature $\mathcal{R}$ does not couple to $|\Phi|^2$. Since c_2 is arbitrary we can always make this happen, but this choice is particularly useful because then the variation of the Palatini connection $\Gamma^{\mu}_{\rho\sigma}$ just gives the condition that the Palatini connection is identified with the Christoffel connection. With this choice, $\mathcal{R} = R$ as a consequence of the equations of motion and we recover the Einstein action with cosmological constant coupled with the Higgs field $|\Phi|^2$.[7]

This formulation of the standard model may be of aesthetic interest because the starting point of the action is completely dimension free. In addition, our formulation has a property that the nonminimal gravitational coupling of the Higgs field is classically fixed to be 1/6.

In order to obtain the full standard model of the particle physics within our setup, we need to introduce gauge fields and fermion fields. The introduction of fermion fields requires a little bit of care because (1) it uses a spin connection and (2) we probably cannot introduce the mass for the fermions. With the fermion fields Ψ and the gauge fields, the action of the matter is schematically given by

$$\int d^4x\sqrt{-g}\left(\frac{1}{2g_{\rm YM}^2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} + \theta\mathrm{Tr}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} + \bar{\Psi}D_\mu\gamma^\mu\Psi \right.$$
$$\left. + y(\Psi\Psi\Phi) + \bar{y}(\bar{\Psi}\bar{\Psi}\bar{\Phi}) + \mathrm{Higgs}\right). \tag{16}$$

The Higgs sector is essentially given in (12) by replacing the derivative ∂_μ with the gauge covariant one. Without violating the Weyl symmetry, one may use the standard spin connection constructed out of the vielbein e^a_μ for fermion kinetic terms. More precisely, we introduce the torsion-free Levi-Civita spin connection constructed out of the vielbein:

$$\omega^{ab}_\mu = \frac{1}{2}e^{\nu a}(\partial_\mu e^b_\nu - \partial_\nu e^b_\mu) - \frac{1}{2}e^{\nu b}(\partial_\mu e^a_\nu - \partial_\nu e^a_\mu)$$
$$- \frac{1}{2}e^{\rho a}e^{\sigma b}(\partial_\rho e_{\sigma c} - \partial_\sigma e_{\rho c})e^c_\mu. \tag{17}$$

We also introduce the Palatini spin connection Ω^{ab}_μ and we treat it as an independent variable.

Under the Weyl transformation, the vielbein transforms as $e^a_\mu \to \Omega(x)e^a_\mu$ so that $g_{\mu\nu} = e^a_\mu e_{\nu a} \to \Omega^2(x)g_{\mu\nu}$. Then the Levi-Civita spin connection transforms as

$$\omega^{ab}_\mu \to \omega^{ab}_\mu - e^{\nu a}e^b_\mu\partial_\nu(\ln\Omega) + e^{\nu b}e^a_\mu\partial_\nu(\ln\Omega) \tag{18}$$

while the Palatini spin connection transforms as

$$\Omega^{ab}_\mu \to \Omega^{ab}_\mu. \tag{19}$$

In the gravitational part of the action, the Weyl invariant curvature term, i.e., $\mathcal{R}^2$, is now constructed out of the Palatini spin connection Ω^{ab}_μ:

$$\mathcal{R} = e^\mu_a e^\nu_b(\partial_\mu\Omega^{ab}_\nu - \partial_\nu\Omega^{ab}_\mu + \Omega^{ac}_\mu\Omega_{\nu c}{}^b - \Omega^{ac}_\nu\Omega_{\mu c}{}^b). \tag{20}$$

The nonminimal coupling to the Higgs field contains both the Palatini curvature and the Ricci scalar as in (12). On the other hand, in the fermionic kinetic term, we exclusively use the Levi-Civita spin connection ω^{ab}_μ (in addition to the Yang-Mills gauge field A_μ) as

$$D_\mu\Psi = (\partial_\mu + iA_\mu)\Psi + \frac{1}{2}\omega^{ab}_\mu\Sigma_{ab}\Psi, \tag{21}$$

where $\Sigma_{ab} = \frac{1}{2}[\gamma_a, \gamma_b]$. In (21) we use ω^{ab}_μ rather than the Palatini spin connection Ω^{ab}_μ in order to preserve the Weyl invariance because we assume the Palatini spin connection does not transform under the Weyl transformation (like the Palatini connection $\Gamma^\mu_{\rho\sigma}$) so the Weyl transformation of the fermion must be canceled by the Levi-Civita spin connection.

Phenomenologically this is very appealing because if we used the Palatini spin connection in the fermionic kinetic terms (as we usually do in the conventional Palatini formulation), the equations of motion for the Palatini spin connection Ω^{ab}_μ would dictate that it has an additional torsional contribution from the fermionic spin currents, resulting in a spin-spin interaction in the final matter action as in the Einstein-Cartan theory. In our case, after rewriting the action in the equivalent Einstein form, the variation of the Palatini spin connection makes it identified with the Levi-Civita spin connection and hence we do not generate such (so-far) unobserved four-Fermi interactions.

Note also that the Weyl invariance does not allow fermion mass terms or cubic self-interaction terms for scalar fields (even if they were allowed by gauge symmetry in the more general matter action). In this sense, it is remarkable that our standard model of particle physics as well as all the gravitational physics we observe in the Universe is compatible with our formulation of Weyl invariant Palatini gravity.[8]

[7]When the Palatini curvature has nonminimal couplings to the matter, the Palatini connection is not necessarily equal to the Christoffel connection [47].

[8]While it is beyond the scope of our paper which focuses on classical physics, quantum effects including renormalizations can be taken care of in our formulation possibly except for the quantum gravity sector. For example, we may employ the Pauli-Villars regularization because all the Pauli-Villars fields (in the Einstein frame) can be embedded in our setup in the Weyl invariant way. With the quantum corrections, the classical property of our theory that the coupling between Higgs and the Ricci scalar is 1/6 will be modified.

IV. CONCLUSION

In this work, we showed that the Palatini formulation of pure R^2 gravity is equivalent to Einstein gravity with nonzero cosmological constant. No massless scalar field appears in contrast to the metric formalism where it appears as a Nambu-Goldstone boson [6]. We regarded the Weyl symmetry as a gauge symmetry and the gauge symmetry is spontaneously broken. There is no Nambu-Goldstone boson for spontaneously broken gauge symmetry. This is a positive development because there is no evidence of such a massless scalar in nature (including cold dark matter which is expected to be massive). The equivalence is of aesthetic appeal because the original action is Weyl invariant and has no scale.

We then construct a Weyl invariant unitary action where we include a nonminimally coupled massless Higgs field. The Weyl invariance requires a hybrid action that includes both the Palatini curvature scalar $\mathcal{R}$ and the usual Ricci scalar R. We show that this action is equivalent to Einstein gravity with cosmological constant and a nonminimally coupled massive Higgs with coefficient fixed to be $1/6$ at the classical level. So the theory, besides its aesthetic appeal, has a classical property that the coupling of the Higgs to the Ricci scalar comes with a coupling constant of $1/6$. In other words, the original Weyl invariance which is subsequently spontaneously broken, leaves an imprint on the final action and determines one of the coupling constants. This is pertinent for work on inflationary models which couple the Higgs boson to curvature in Einstein gravity (as a recent example see [48]). Typically, in such work, one considers the coupling to be an arbitrary constant ξ instead of $1/6$.

Remarkably, one can incorporate into this paradigm all the other standard model fields, including fermions. We showed that this can be accomplished by working with two spin connections instead of only one: the torsion-free Levi-Civita connection ω_μ^{ab} constructed out of the vielbein and the Palatini spin connection Ω_μ^{ab} as an independent variable. The former is used for the fermions and the latter for the $\mathcal{R}^2$ gravitational action. Therefore our final action reproduces the standard model as we know it (with the added benefit of fixing the coupling constant between the Higgs and the Ricci scalar to be $1/6$ at the classical level).

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