Quasitopological magnetic brane coupled to nonlinear electrodynamics

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In this paper, we are eager to construct a new class of $(n+1)$-dimensional static magnetic brane solutions in quasitopological gravity coupled to nonlinear electrodynamics such as exponential and logarithmic forms. The solutions of this magnetic brane are horizonless and have no curvature. For ρ near r_+, the solution $f(\rho)$ is dependent on the values of parameters q and n, and for larger ρ, it depends on the coefficients of LoveLock and quasitopological gravities λ, μ, and c. The obtained solutions also have a conic singularity at $r=0$ with a deficit angle that is only dependent on the parameters q, n, and β. We should remind the reader that the two forms of nonlinear electrodynamics theory have similar behaviors on the obtained solutions. At last, by using the counterterm method, we obtain conserved quantities such as mass and electric charge. The value of the electric charge for this static magnetic brane is obtained as zero.

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I. INTRODUCTION

Both modified theories of gravity and nonlinear electrodynamics theory are marvelous subjects that can solve many problems. Models of modified gravities can find a way to unify the early-time inflation [1] and late-time cosmic speed-up [2,3], and also they can make a natural gravitational alternative to dark energy. These models also describe four cosmological phases [4,5], the hierarchy problem, and the unification of grand unified theories with gravity, which are theories in high-energy physics [6]. $f(R)$ gravity is one model of modified gravities in which we can find the galactic dynamics of massive test particles without the need of dark matter [7,8]. In another kind of modified gravities in which violation of the equivalence principle is obvious, the matter Lagrangian density is coupled to an arbitrary function of scalar curvature R [9]. Another new kind of modified gravities is quasitopological gravity, which is similar to Lovelock theory with more benefits. As Einstein's equations are not the most complete ones in higher dimensions $(n>4)$ and they cannot satisfy Einstein's assumptions [10,11], so quasitopological gravity is a higher-derivative theory that can solve these problems. This gravity consists of cubic and quartic terms of the Riemann tensor and has no limitation on dimensions higher than 5 because its terms are not true topological invariants. As this gravity yields to at most second-order field equations, this results in the quantization of linearized quasitopological theory being free of ghosts.

On the other side, there are some motivations to consider nonlinear electrodynamics theory. First, it can remove infinite self-energy of point-like charges. Second, it can describe complex systems and chaotic phenomena and behaviors of the compact astrophysical objects such as neutron stars and pulsars. Third, it has compatibility with AdS/CFT correspondence and string theory frames and fourth, it can describe pair creation for Hawking radiation [12–14]. Nonlinear electrodynamics theory has also been used in applications in cosmological models [15], such as the description of the inflationary epoch and the late-time accelerated expansion of the Universe [16]. This theory has also been successful in finding the first exact regular black hole solutions with a nonlinear electrodynamic source satisfying the weak energy condition [17]. Nonlinear electrodynamics theory has different types, the Born-Infeld type of which is the first one. The Born-Infeld form is so important because it naturally comes in the low-energy limit of the open string theory and has applications to the description of D-branes and AdS/CFT correspondence [18]. We can name exponential [19] and logarithmic [20] Lagrangians as the other types of nonlinear electrodynamics theory, which are defined as

$$\mathcal{L}(F) = \begin{cases} 4\beta^2[\exp(-\frac{F}{4\beta^2})-1], & \text{EN} \\ -8\beta^2\ln[1+\frac{F}{8\beta^2}]. & \text{LN.} \end{cases} \tag{1}$$

β is the nonlinear parameter with dimension of mass, and $F=F_{\mu\nu}F^{\mu\nu}$, where $F_{\mu\nu}$ is the electromagnetic field tensor that is determined as $F_{\mu\nu}=\partial_\mu A^\nu-\partial_\nu A^\mu$ and A_μ is the vector potential. We should note that these two Lagrangians reduce to the linear Maxwell Lagrangian as $\beta\to\infty$. Like Born-Infeld nonlinear electrodynamics, the logarithmic

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form can remove the infinity of the electric field at the origin [21]; however, the exponential form cannot cancel this infinity, but it causes a weaker singularity than the one in Einstein-Maxwell theory [22].

The idea of using modified gravities with nonlinear electrodynamics theory can be so interesting and has been studied in many papers. For example, $f(R)$ gravity in the presence of nonlinear electrodynamics has been successful in describing the phenomena such as power-law inflation and late-time cosmic accelerated expansion due to breaking conformal invariance of the electromagnetic field through a nonminimal gravitational coupling [23]. Similar investigation has been also done in Ref. [24], in which the conformal invariance is not broken. Dilaton black holes and dilaton black branes with nonlinear electrodynamics in four and higher dimensions have been studied in Refs. [21,22]. Topological and anti-de Sitter (AdS) black holes in Lovelock-Born-Infeld gravity have also been studied in Refs. [25,26]. Third-order Lovelock black branes in the presence of a nonlinear electromagnetic field have been also investigated in Ref. [27]. Magnetic brane solutions of Lovelock gravity with nonlinear electrodynamics have been obtained in Ref. [28].

Recently, quasitopological gravity in the presence of nonlinear electrodynamics has been studied in many papers. For example, Lifshitz quartic quasitopological black holes in the presence of Born-Infeld electrodynamics have been studied in Ref. [29]. A review of quartic quasitopological black holes with the nonlinear electromagnetic Born-Infeld field is also presented in Ref. [30]. Some of us have also studied the solutions of cubic quasitopological magnetic branes in the presence of Maxwell and Born-Infeld electromagnetic field in Ref. [31]. Magnetic branes are attractive because their solutions are horizonless and have a conical geometry. They are also flat everywhere except at the location of the line source. Now, we have a decision to take a further step and study the solutions of $(n+1)$-dimensional magnetic branes with exponential and logarithmic nonlinear electrodynamics in quartic quasitopological gravity.

The structure of this paper is as follows. In Sec. II, we begin with the metric of a horizonless spacetime and an action including nonlinear electrodynamics and quartic quasitopological theories. Then, we obtain equations and solutions. In Sec. III, we investigate physical structure and behavior of the obtained solutions, and in Sec. IV, we obtain conserved quantities using the counterterm method. At last, in Sec. V, we write a brief result of the obtained data from this magnetic brane.

II. GENERAL FORMALISM

To have magnetic solutions with no horizons, we start with a metric with characteristics $(g_{\rho\rho})^{-1} \propto g_{\phi\phi}$ and $g_{tt} \propto -\rho^2$ instead of $(g_{\rho\rho})^{-1} \propto g_{tt}$ and $g_{\phi\phi} \propto -\rho^2$. So, the $(n+1)$-dimensional metric of a horizonless spacetime with a magnetic brane interpretation is written

$$ds^2 = -\frac{\rho^2}{l^2}dt^2 + \frac{d\rho^2}{f(\rho)} + l^2 g(\rho)d\phi^2 + \frac{\rho^2}{l^2}dX^2, \qquad (2)$$

where l is a scale factor that is related to the cosmological constant Λ. $dX^2 = \sum_{i=1}^{n-2}$ is a $(n-2)$-dimensional hypersurface with the form of a Euclidean metric in volume V_{n-2}. ρ and ϕ are, respectively, the radial and angular coordinates at which ϕ is dimensionless and has the range $0 \le \phi < 2\pi$.

The $(n+1)$-dimensional action in the presence of quartic quasitopological gravity and nonlinear electrodynamics theory is

$$I_{\mathrm{bulk}} = \frac{1}{16\pi}\int d^{n+1}x\sqrt{-g}\{-2\Lambda + \mathcal{L}_1 + \hat{\lambda}\mathcal{L}_2 + \hat{\mu}\mathcal{L}_3 + \hat{c}\mathcal{L}_4 + \mathcal{L}(F)\}, \qquad (3)$$

where $\Lambda = -n(n-1)/2l^2$ and g is the determinant of the metric (2). Einstein-Hilbert, Gauss-Bonnet, cubic, and quartic quasitopological Lagrangians are, respectively, defined as

$$\mathcal{L}_1 = R, \qquad (4)$$

$$\mathcal{L}_2 = R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2, \qquad (5)$$

$$\mathcal{L}_3 = R_a{}^c{}_b{}^d R_c{}^e{}_d{}^f R_e{}^a{}_f{}^b + \frac{1}{(2n-1)(n-3)}\left(\frac{3(3n-5)}{8}R_{abcd}R^{abcd}R - 3(n-1)R_{abcd}R^{abc}{}_e R^{de} + 3(n+1)R_{abcd}R^{ac}R^{bd} + 6(n-1)R_a{}^b R_b{}^c R_c{}^a - \frac{3(3n-1)}{2}R_a{}^b R_b{}^a R + \frac{3(n+1)}{8}R^3\right), \qquad (6)$$

$$\mathcal{L}_4 = c_1 R_{abcd}R^{cdef}R^{hg}{}_{ef}R_{hg}{}^{ab} + c_2 R_{abcd}R^{abcd}R_{ef}{}^{ef} + c_3 R R_{ab}R^{ac}R_c{}^b + c_4(R_{abcd}R^{abcd})^2 + c_5 R_{ab}R^{ac}R_{cd}R^{db} + c_6 R R_{abcd}R^{ac}R^{db} + c_7 R_{abcd}R^{ac}R^{be}R^d{}_e + c_8 R_{abcd}R^{acef}R^b{}_e R^d{}_f + c_9 R_{abcd}R^{ac}R_{ef}R^{bedf} + c_{10}R^4 + c_{11}R^2 R_{abcd}R^{abcd} + c_{12}R^2 R_{ab}R^{ab} + c_{13}R_{abcd}R^{abef}R_{ef}{}^c{}_g R^{dg} + c_{14}R_{abcd}R^{aecf}R_{gehf}R^{gbhd}, \qquad (7)$$

where

$$\begin{aligned}
c_1 &= -(n-1)(n^7 - 3n^6 - 29n^5 + 170n^4 - 349n^3 + 348n^2 - 180n + 36) \\
c_2 &= -4(n-3)(2n^6 - 20n^5 + 65n^4 - 81n^3 + 13n^2 + 45n - 18) \\
c_3 &= -64(n-1)(3n^2 - 8n + 3)(n^2 - 3n + 3) \\
c_4 &= -(n^8 - 6n^7 + 12n^6 - 22n^5 + 114n^4 - 345n^3 + 468n^2 - 270n + 54) \\
c_5 &= 16(n-1)(10n^4 - 51n^3 + 93n^2 - 72n + 18) \\
c_6 &= -32(n-1)^2(n-3)^2(3n^2 - 8n + 3) \\
c_7 &= 64(n-2)(n-1)^2(4n^3 - 18n^2 + 27n - 9) \\
c_8 &= -96(n-1)(n-2)(2n^4 - 7n^3 + 4n^2 + 6n - 3) \\
c_9 &= 16(n-1)^3(2n^4 - 26n^3 + 93n^2 - 117n + 36) \\
c_{10} &= n^5 - 31n^4 + 168n^3 - 360n^2 + 330n - 90 \\
c_{11} &= 2(6n^6 - 67n^5 + 311n^4 - 742n^3 + 936n^2 - 576n + 126) \\
c_{12} &= 8(7n^5 - 47n^4 + 121n^3 - 141n^2 + 63n - 9) \\
c_{13} &= 16n(n-1)(n-2)(n-3)(3n^2 - 8n + 3) \\
c_{14} &= 8(n-1)(n^7 - 4n^6 - 15n^5 + 122n^4 - 287n^3 + 297n^2 - 126n + 18).
\end{aligned} \tag{8}$$

$\hat{\lambda}$, $\hat{\mu}$, and $\hat{c}$ are, respectively, the parameters of Gauss-Bonnet, cubic, and quartic quasitopological Lagrangians

$$\hat{\lambda} = \frac{\lambda L^2}{(n-2)(n-3)}, \tag{9}$$

$$\hat{\mu} = \frac{8\mu(2n-1)l^4}{(n-2)(n-5)(3n^2-9n+4)}, \tag{10}$$

$$\hat{c} = \frac{cl^6}{n(n-1)(n-3)(n-7)(n-2)^2(n^5 - 15n^4 + 72n^3 - 156n^2 + 150n - 42)}. \tag{11}$$

The magnetic field is associated with the angular component A_ϕ. So, we introduce the gauge potential for the static solutions as

$$A_\mu = h(\rho)\delta_\mu^\phi. \tag{12}$$

Using the above relations in action (3) and integrating by parts, we can get the action

$$S = \frac{n-1}{16\pi l^2} \times \int d^n x \int d\rho N(\rho)\{[\rho^n(1 + \Psi + \lambda\Psi^2 + \mu\Psi^3 + c\Psi^4)]' + \begin{cases} \frac{4\beta^2 l^2 \rho^{n-1}}{n-1}\left[\exp\left(-\frac{h'^2}{2l^2\beta^2 N^2(\rho)}\right) - 1\right]\}, & \text{EN}, \\ -\frac{8\beta^2 l^2 \rho^{n-1}}{n-1}\ln\left[1 + \frac{h'^2}{4\beta^2 l^2 N^2(\rho)}\right]\}, & \text{LN}, \end{cases} \tag{13}$$

where $\Psi(\rho) = -\frac{l^2}{\rho^2}f(\rho)$, $g(\rho) = N^2(\rho)f(\rho)$, and the prime shows the first derivative with respect to ρ. EN and LN are, respectively, the abbreviation of exponential and logarithmic nonlinear. Varying this action with respect to function $\Psi(\rho)$ leads to the equation

$$\{1 + 2\lambda\Psi(\rho) + 3\mu\Psi^2(\rho) + 4c\Psi^3(\rho)\}N'(\rho) = 0. \tag{14}$$

The above equation shows that $N(\rho)$ must be a constant value, which we choose $N(\rho) = 1$. By varying the action (13) with respect to the functions $N(\rho)$ and $h(\rho)$ and using the obtained condition $N(\rho) = 1$ [or $f(\rho) = g(\rho)$], we get the equations

$$\{(n-1)\rho^n(1 + \Psi + \lambda\Psi^2 + \mu\Psi^3 + c\Psi^4)\}' + \begin{cases} 4\rho^{n-1}(l^2\beta^2 + h'^2)\exp\left(-\frac{h'^2}{2l^2\beta^2}\right) - 4l^2\beta^2\rho^{n-1} = 0, & \text{EN}, \\ -8\beta^2 l^2 \rho^{n-1}\ln\left(1 + \frac{h'^2}{4\beta^2 l^2}\right) + 4\rho^{n-1}h'^2\left(1 + \frac{h'^2}{4\beta^2 l^2}\right)^{-1} = 0, & \text{LN}, \end{cases} \tag{15}$$

and

$$\begin{cases} \left(\rho^{n-1}h'\exp\left[-\frac{h'^2}{2l^2\beta^2}\right]\right)' = 0, & \text{EN}, \\ \left(\rho^{n-1}h'\left(1+\frac{h'^2}{4\beta^2 l^2}\right)^{-1}\right)' = 0, & \text{LN}. \end{cases} \tag{16}$$

If we solve Eq. (16), we get the electromagnetic field

$$F_{\phi\rho} = h' = \begin{cases} l\beta\sqrt{-L_W(-\eta)}, & \text{EN}, \\ \frac{2ql^{n-2}}{\rho^{n-1}}(1+\sqrt{1-\eta})^{-1}, & \text{LN}, \end{cases} \tag{17}$$

where $\eta = \frac{q^2 l^{2n-6}}{\beta^2\rho^{2n-2}}$ and q is the constant of integration. L_W is the Lambert function that has the following series expansion:

$$L_W(x) = x - x^2 + \frac{3}{2}x^3 + \cdots. \tag{18}$$

If we expand $F_{\phi\rho}$ for large β, we get

$$F_{\phi\rho} = \frac{ql^{n-2}}{\rho^{n-1}} + \begin{cases} \frac{q^3 l^{3n-8}}{2\beta^2\rho^{3n-3}} + \mathcal{O}\left(\frac{1}{\beta^4}\right), & \text{EN}, \\ \frac{q^3 l^{3n-8}}{4\beta^2\rho^{3n-3}} + \mathcal{O}\left(\frac{1}{\beta^4}\right), & \text{LN}, \end{cases} \tag{19}$$

where the first term is the electromagnetic field of the magnetic brane in the presence of linear Maxwell theory in higher dimensions [31] and the next terms are the corrections to the electromagnetic field, in the presence of nonlinear electrodynamics. By remembering that the vector potential A_ϕ is only dependent on coordinate ρ, we get to the relation $F_{\phi\rho} = -\partial_\rho A_\phi$ that by solving it, we obtain

$$A_\phi = \begin{cases} -\frac{n-1}{n-2}l\beta\left(\frac{l^{n-3}q}{\beta}\right)^{\frac{1}{n-1}}(-L_W(-\eta))^{\frac{n-2}{2(n-1)}}\left\{{}_2F_1\left(\left[\frac{n-2}{2(n-1)}\right],\left[\frac{3n-4}{2(n-1)}\right],-\frac{1}{2(n-1)}L_W(-\eta)\right)-\frac{n-2}{n-1}\exp\left(-\frac{1}{2(n-1)}L_W(-\eta)\right)\right\}, & \text{EN}, \\ \frac{ql^{n-2}}{(n-2)\rho^{n-2}}\,{}_3F_2\left(\left[\frac{n-2}{2(n-1)},\frac{1}{2},1\right],\left[\frac{3n-4}{2(n-1)},2\right],\eta\right). & \text{LN}. \end{cases} \tag{20}$$

As $\beta \to \infty$, we get

$$A_\phi = \frac{ql^{n-2}}{(n-2)\rho^{n-2}}, \tag{21}$$

which is the $(n+1)$-dimensional vector potential of Maxwell theory [31]. Using Eq. (17) in Eq. (15) leads to the relation

$$c\Psi^4 + \mu\Psi^3 + \lambda\Psi^2 + \Psi + \kappa = 0, \tag{22}$$

where κ is

$$\kappa = 1 - \frac{M}{(n-1)\rho^n} + \begin{cases} -\frac{4l^2\beta^2}{n(n-1)} - \frac{4(n-1)\beta q l^{n-1}}{n(n-2)\rho^n}\left(\frac{l^{n-3}q}{\beta}\right)^{\frac{1}{n-1}}(-L_W(-\eta))^{\frac{n-2}{2(n-1)}} \times {}_2F_1\left(\left[\frac{n-2}{2(n-1)}\right],\left[\frac{3n-4}{2(n-1)}\right],-\frac{1}{2(n-1)}L_W(-\eta)\right) \\ +\frac{4\beta q l^{n-1}}{(n-1)\rho^{n-1}}[-L_W(-\eta)]^{\frac{1}{2}} \times [1+\frac{1}{n}(-L_W(-\eta))^{-1}], & \text{EN}, \\ \frac{8(2n-1)}{n^2(n-1)}\beta^2 l^2[1-\sqrt{1-\eta}] - \frac{8(n-1)q^2 l^{2n-4}}{n^2(n-2)\rho^{2n-2}}\,{}_2F_1\left(\left[\frac{n-2}{2(n-1)},\frac{1}{2}\right],\left[\frac{3n-4}{2(n-1)}\right],\eta\right) - \frac{8}{n(n-1)}l^2\beta^2\ln\left[\frac{2-2\sqrt{1-\eta}}{\eta}\right], & \text{LN}, \end{cases} \tag{23}$$

and M is the integration constant and is related to the mass of this magnetic brane. In the above solution, we have used the following relation for the Lambert function:

$$L_W(x)e^{L_W(x)} = x. \tag{24}$$

To have real solutions for Eq. (22), the condition

$$\Delta = \frac{H^2}{4} + \frac{P^3}{27} > 0 \tag{25}$$

should be satisfied, and P and H are defined as

$$P = -\frac{\alpha^2}{12} - \gamma, \qquad H = -\frac{\alpha^3}{108} + \frac{\alpha\gamma}{3} - \frac{\beta^2}{8}, \tag{26}$$

where α, β, and γ are

$$\alpha = \frac{-3\mu^2}{8c^2} + \frac{\lambda}{c}, \qquad \beta = \frac{\mu^3}{8c^3} - \frac{\mu\lambda}{2c^2} + \frac{1}{c},$$
$$\gamma = \frac{-3\mu^4}{256c^4} + \frac{\lambda\mu^2}{16c^3} - \frac{\mu}{4c^2} + \frac{\kappa}{c}. \tag{27}$$

If we define the definitions

$$U = \left(-\frac{H}{2} \pm \sqrt{\Delta}\right)^{\frac{1}{3}}, \tag{28}$$

$$y = \begin{cases} -\frac{5}{6}\alpha + U - \frac{P}{3U}, & U \neq 0, \\ -\frac{5}{6}\alpha + U - \sqrt[3]{H} & , U = 0, \end{cases} \tag{29}$$

$$W = \sqrt{\alpha + 2y}, \tag{30}$$

the solution $f(\rho)$ for Eq. (22) is obtained as

$$f(\rho) = \frac{-\rho^2}{l^2}\left(-\frac{\mu}{4c} + \frac{\pm_s W \mp_t \sqrt{-(3\alpha + 2y \pm_s \frac{2\beta}{W})}}{2}\right). \tag{31}$$

In the above equation, two $\pm_s$ should both have the same sign, while the sign of $\pm_t$ is independent. It is noteworthy to say that in order to have the cubic quasitopological or Gauss-Bonnet solutions we should replace $\mu = 0$ or $\lambda = 0$ in Eq. (22) and not find the solutions in the above relations because we get vague values.

III. PHYSICAL PROPERTIES OF THE SOLUTIONS

In this section, we aim to investigate the geometric and physical properties of the solutions like horizons, singularity, and behaviors of the function $f(\rho)$. As we know, to find the horizons of the obtained solutions, the condition $f(r_+) = 0$ should be satisfied where r_+ is the horizon. Suppose that r_+ is the largest real root of $f(\rho) = 0$, which leads to the function $f(\rho)$ being positive for $\rho > r_+$ and negative for $\rho < r_+$. The range $0 < \rho < r_+$ is not acceptable as $g_{\rho\rho}$ cannot be negative (which occurs for $\rho < r_+$, because of the change of signature of the metric from $(n-1)+$ to $(n-2)+$. Therefore, we delete this unacceptable range $0 < \rho < r_+$, and so the function $f(\rho)$ is limited to the acceptable range $\rho > r_+$. For ease, we can use the suitable transformation

$$r = \sqrt{\rho^2 - r_+^2} \Rightarrow d\rho^2 = \frac{r^2}{r^2 + r_+^2} dr^2, \tag{32}$$

which results in

$$ds^2 = -\frac{r^2 + r_+^2}{l^2} dt^2 + \frac{r^2 dr^2}{(r^2 + r_+^2) f(r)} + l^2 g(r) d\phi^2$$
$$+ \frac{r^2 + r_+^2}{l^2} dX^2. \tag{33}$$

By this transformation, r has the range $0 \leq r < \infty$, for which $f(r)$ is positive and real for $0 < r < \infty$ and zero for $r = 0$. This transformation also leads to the changes for $F_{\phi r}$ and κ as

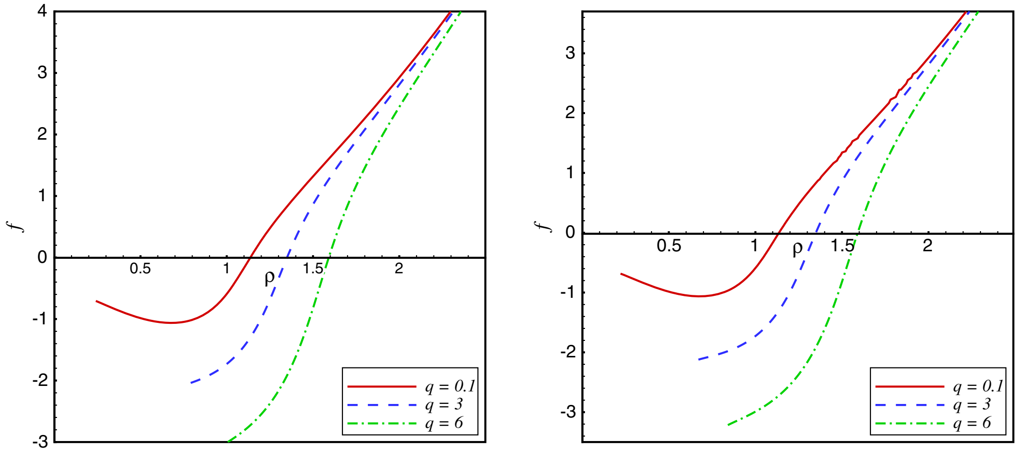
$$F_{\phi r} = \begin{cases} l\beta\sqrt{-L_W(-\eta)}, & \text{EN} \\ \frac{2ql^{n-2}}{(r^2+r_+^2)^{\frac{n-1}{2}}}(1 + \sqrt{1-\eta})^{-1}, & \text{LN}, \end{cases} \tag{34}$$

$$\kappa = 1 - \frac{M}{(n-1)(r^2 + r_+^2)^{\frac{n}{2}}}$$
$$+ \begin{cases} -\frac{4l^2\beta^2}{n(n-1)} - \frac{4(n-1)\beta q l^{n-1}}{n(n-2)(r^2+r_+^2)^{\frac{n}{2}}}\left(\frac{l^{n-3}q}{\beta}\right)^{\frac{1}{n-1}}(-L_W(-\eta))^{\frac{n-2}{2(n-1)}} \times {}_2F_1\left(\left[\frac{n-2}{2(n-1)}\right], \left[\frac{3n-4}{2(n-1)}\right], \right. \\ \left. -\frac{1}{2(n-1)} L_W(-\eta)\right) + \frac{4\beta q l^{n-1}}{(n-1)(r^2+r_+^2)^{\frac{n-1}{2}}}[-L_W(-\eta)]^{\frac{1}{2}} \times [1 + \frac{1}{n}(-L_W(-\eta))^{-1}], & \text{EN} \\ \frac{8(2n-1)}{n^2(n-1)}\beta^2 l^2 [1 - \sqrt{1-\eta}] - \frac{8(n-1)q^2 l^{2n-4}}{n^2(n-2)(r^2+r_+^2)^{n-1}} {}_2F_1\left(\left[\frac{n-2}{2(n-1)}, \frac{1}{2}\right], \left[\frac{3n-4}{2(n-1)}\right], \eta\right) - \frac{8}{n(n-1)} l^2\beta^2 \ln\left[\frac{2-2\sqrt{1-\eta}}{\eta}\right], & \text{LN} \end{cases} \tag{35}$$

where $\eta = \frac{q^2 l^{2n-6}}{\beta^2(r^2+r_+^2)^{n-1}}$. To find the singularity of the obtained solutions, we calculate Kretschmann scalar,

$$\mathcal{K} = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} = f''^2 + \frac{2(n-1)}{\rho^2} f'^2 + \frac{2(n-1)(n-2)}{\rho^4} f^2, \tag{36}$$

where the double prime is the second derivative of the function f with respect to ρ. It seems that the solutions have a singularity at $\rho = 0$ because the Kretschmann scalar diverges at this point, but as we found out, the point $\rho = 0$ is not in the acceptable range of ρ. So, this magnetic brane has no singularity.



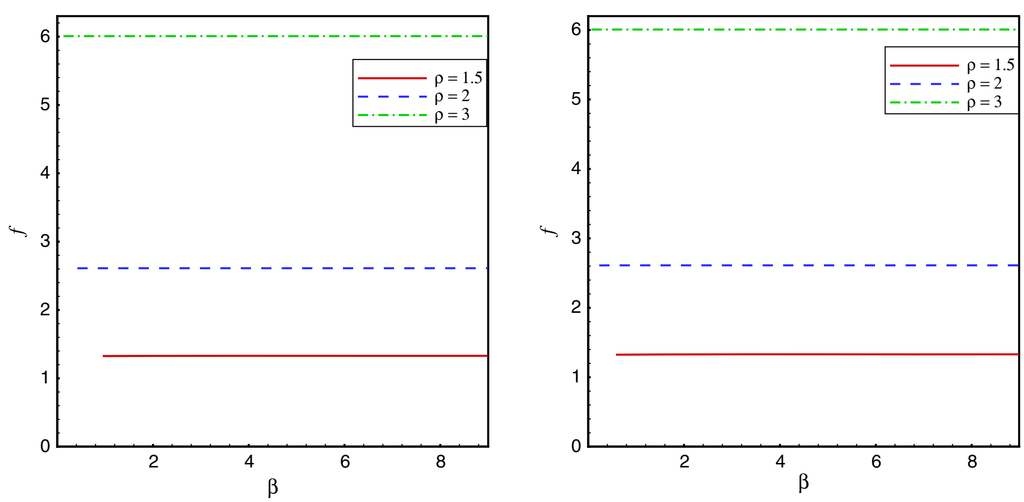
(a) EN (b) LN

FIG. 1. $f(\rho)$ vs ρ with $M = 5$, $\beta = 10$, $n = 4$, $\lambda = -0.01$, $\mu = 0.4$, and $c = -0.01$.

To know more about this magnetic brane, we have studied the behavior of $f(\rho)$ in Figs. 1–4. For this purpose, we have considered $l = 1$ without losing the issue. In Fig. 1, we have plotted $f(\rho)$ vs ρ for different values of q and for exponential nonlinear (EN) and logarithmic nonlinear (LN) electrodynamics. According to our previous sayings, the region $\rho < r_+$ is not acceptable, and so $f(\rho)$ is not seen in this range in the figures. In Fig. 1, for constant values of parameters M, β, n, λ, μ, and c, the value of r_+ depends on the value of q, and it increases by increasing the value of q. Also, for a definite value of q, the value of r_+ is independent of the kinds of the nonlinear electrodynamics, exponential or logarithmic. The function f is also not sensitive to the value of q for large r_+ and has a constant value for each value of ρ, but in the region near r_+, it decreases as q increases.

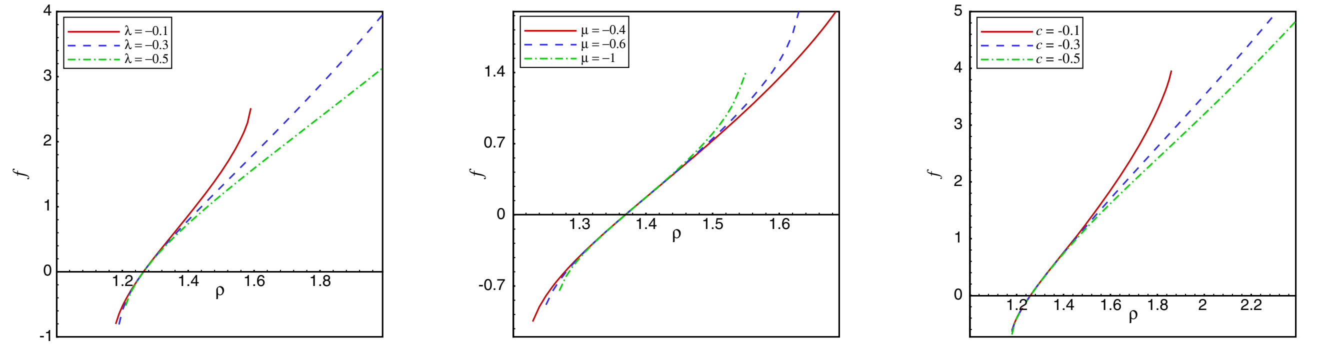
In Fig. 2, we are eager to know the behavior of f vs β for different values of ρ. In Figs. 2(a) and 2(b), for fixed parameters M, q, n, λ, μ, and c, there is a β_{min} for each value of ρ for which the function f is not real for $\beta < \beta_{\text{min}}$, and it has a constant value for $\beta > \beta_{\text{min}}$. The value of β_{min} depends on the value of ρ and increases by decreasing the value of ρ. It is also clear that Fig. 2(b) is similar to Fig. 2(a). This shows that the kinds of nonlinear electrodynamics cannot affect the values of β_{min} nor f. This led us to avoid using the same figures of f for LN electrodynamics in Figs. 3 and 4.

We have studied the behaviors of $f(\rho)$ vs ρ for different values of λ, μ, and c for EN electrodynamics in Fig. 3. It is obvious that the value of r_+ is independent of the values of λ, μ, and c. We can also realize this statement by extracting the constant M using $f(r_+) = 0$,



(a) EN (b) LN

FIG. 2. $f(\rho)$ vs β with $M = 1$, $q = 2$, $n = 4$, $\lambda = 0.01$, $\mu = 1.1$, and $c = -0.02$.



(a) $M = 2$, $q = 3$, $\beta = 5$, $n = 4$, $\mu = -0.4$, and $c = -0.01$ (b) $M = 10$, $q = 1$, $\beta = 2$, $n = 4$, $\lambda = -0.01$, and $c = -0.01$ (c) $M = 5$, $q = 2$, $\beta = 5$, $n = 4$, $\lambda = -0.2$, and $\mu = -0.5$

FIG. 3. $f(\rho)$ vs ρ for EN electrodynamics.

$$M = (n-1)r_+^n + \begin{cases} -\frac{4l^2\beta^2}{n} r_+^n - \frac{4(n-1)^2\beta q l^{n-1}}{n(n-2)} \left(\frac{l^{n-3}q}{\beta}\right)^{\frac{1}{n-1}} (-L_W(-\eta_+))^{\frac{n-2}{2(n-1)}} \times {}_2F_1\left(\left[\frac{n-2}{2(n-1)}\right], \left[\frac{3n-4}{2(n-1)}\right], \right. \\ \left. -\frac{1}{2(n-1)} L_W(-\eta_+)\right) + 4\beta q l^{n-1} r_+ [-L_W(-\eta_+)]^{\frac{1}{2}} \times [1 + \frac{1}{n}(-L_W(-\eta_+))^{-1}], & \text{EN} \\ \frac{8(2n-1)}{n^2}\beta^2 l^2 r_+^n [1 - \sqrt{1-\eta}] - \frac{8(n-1)^2 q^2 l^{2n-4}}{n^2(n-2) r_+^{n-2}} {}_2F_1\left(\left[\frac{n-2}{2(n-1)}, \frac{1}{2}\right], \left[\frac{3n-4}{2(n-1)}\right], \eta\right) - \frac{8}{n} l^2\beta^2 r_+^n \ln\left[\frac{2-2\sqrt{1-\eta}}{\eta}\right], & \text{LN} \end{cases} \quad (37)$$

where $\eta_+ = \eta(r=0) = \frac{q^2 l^{2n-6}}{\beta^2 r_+^{2n-2}}$. We can find out from the above equation that the value of r_+ is not related to the values of parameters λ, μ, and c. Also, for ρ near r_+, f has similar behavior and is independent of these parameters.

However, for larger ρ and fixed values for parameters q, M, n, and β, the function f depends on the values of λ, μ, and c. In this region, by choosing small values for λ and c and a large value for μ, we can have a larger region of ρ for which f is real. For example, in Fig. 3(a), f is real for a larger region of ρ in diagrams with $\lambda = -0.3, -0.5$ than in the one with $\lambda = -0.1$.

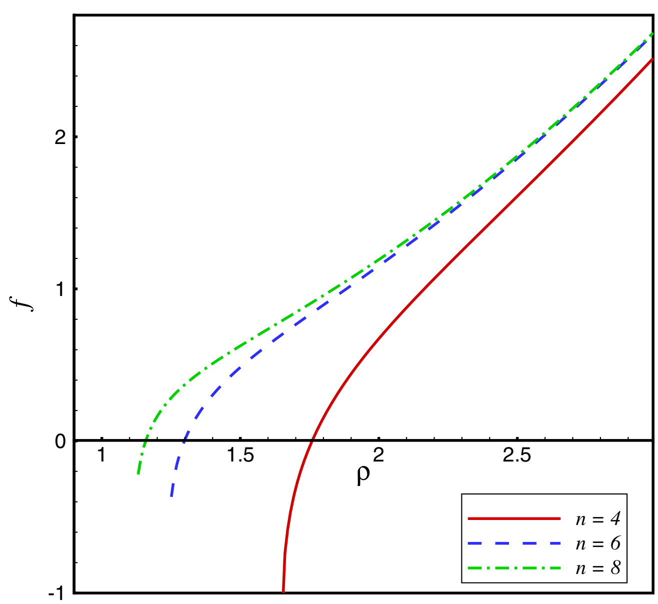


FIG. 4. $f(\rho)$ vs ρ with $M = 5$, $q = 3$, $\beta = 10$, $\lambda = -0.01$, $\mu = 0.4$, and $c = -0.1$ for EN electrodynamics.

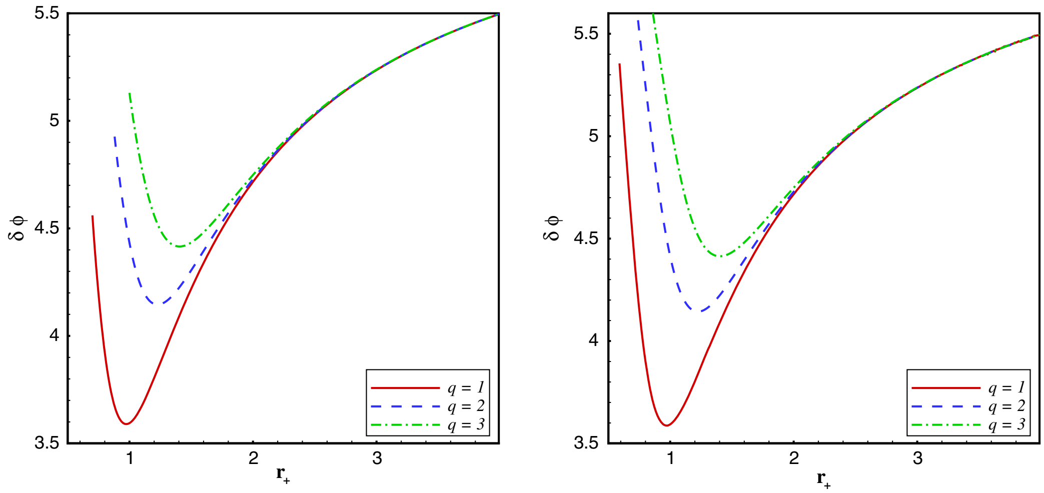
In Fig. 4, we compare the behavior of function f for different values of dimension n. In this case, f has behavior similar to that in Fig. 1 for ρ near r_+ or larger than it. But we can see that for different values of n the value of r_+ is variable and it decreases as n increases. Also, near r_+, the function f increases as the value of n increases. In the limit $r_+ \to \infty$, the function f goes to a constant value for each value of r_+.

Although the Kretschmann scalar does not diverge in the range $r = [0, \infty)$, this spacetime has a canonical singularity at $r = 0$. That is, the limit of the ratio "circumference/radius" is not 2π as the radius r goes to zero. We can prove this by evaluating

$$\left(\lim_{r\to 0}\left(\frac{1}{r}\sqrt{\frac{g_{\phi\phi}}{g_{rr}}}\right)\right)^{-1} = \left(\lim_{r\to 0}\frac{\sqrt{r^2 + r_+^2 l f(r)}}{r^2}\right)^{-1} = \frac{2}{l r_+}\left(\frac{d^2 f(r)}{dr^2}\bigg|_{r=0}\right)^{-1} \neq 1, \quad (38)$$

where we have used Taylor expansion for $f(r)$ at $r = 0$ (or r_0),

$$f(r) = f(r)|_{r_0} + r\frac{df(r)}{dr}\bigg|_{r_0} + \frac{r^2}{2}\frac{d^2 f(r)}{dr^2}\bigg|_{r_0} + \mathcal{O}(r^3), \quad (39)$$



(a) EN (b) LN

FIG. 5. $\delta\phi$ vs r_+ with $\beta = 5$ and $n = 4$.

that $f(r)|_{r_0} = \frac{df(r)}{dr}|_{r_0} = 0$. We can remove this canonical singularity at $r = 0$, if we recognize the coordinate ϕ with the period

$$\mathrm{Period}_\phi = 2\pi\left(\lim_{r\to 0}\left(\frac{1}{r}\sqrt{\frac{g_{\phi\phi}}{g_{rr}}}\right)\right)^{-1} = 2\pi(1-4\tau), \quad (40)$$

where by using Eqs. (38) and (22) and the fact the $f(r_0) = 0$, τ is given by,

$$\tau = \frac{1}{4}\left[1 - \frac{2l}{r_+^3}\left(\frac{d^2\kappa}{dr^2}\bigg|_{r_0}\right)^{-1}\right]. \quad (41)$$

So, metric (33) describes a locally flat spacetime that has a conical singularity at $r = 0$ with a deficit angle $\delta\phi = 8\pi\tau$.

Now, we tend to investigate the behavior of $\delta\phi$. The first point is that, according to the relation (41), the deficit angle parameter is independent of the coefficients of Gauss-Bonnet and third- and fourth-order quasitopological gravities, and it is only dependent on the parameters q, β, and n. So, we have plotted $\delta\phi$ vs r_+ for different values of q, β, and n in Figs. 5–7, respectively. In Fig. 5, for each value of q, there is a minimum value for r_+ (we call it $r_{+\min}$) that $\delta\phi$ is determined for the range $r_{+\min} < r_+$. Also, there is a $r_{+\max}$ that for $r_+ > r_{+\max}$, $\delta\phi$ is independent of the value q and has a constant value for each value of r_+. But for $r_{+\min} < r_+ < r_{+\max}$, $\delta\phi$ depends on the value of q, and it increases as q increases. In this region, there is also a r_{+0} for which $\delta\phi$ has a minimum value.

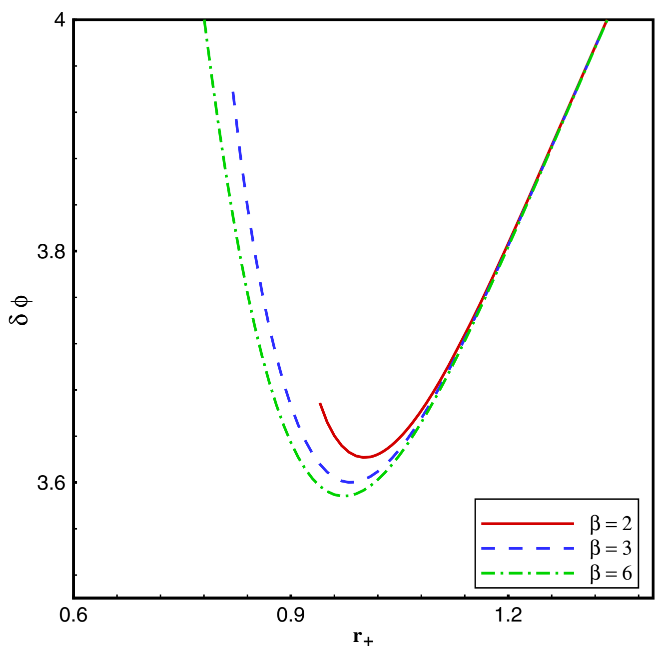


FIG. 6. $\delta\phi$ vs r_+ with $q = 1$ and $n = 4$ for EN electrodynamics.

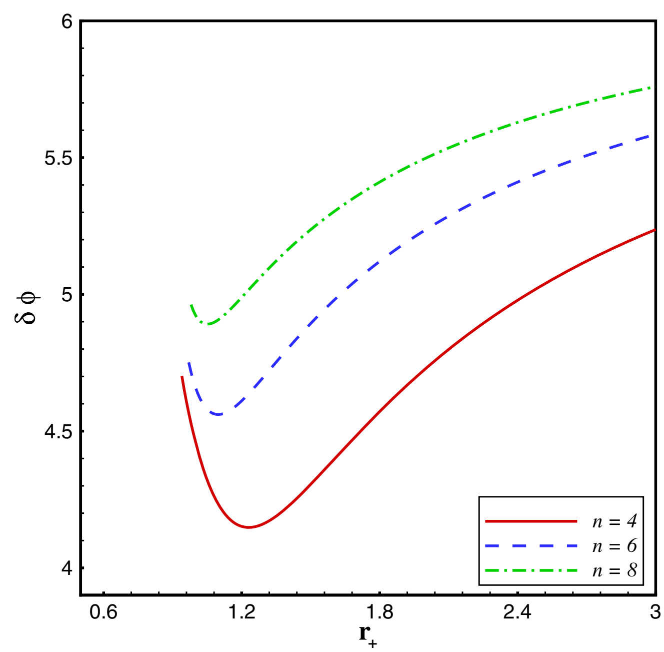


FIG. 7. $\delta\phi$ vs r_+ with $q = 2$ and $\beta = 4$ for EN electrodynamics.

Although for the same parameters β and n the values of $r_{+\min}$ in the LN form in Fig. 5(b) are smaller than the ones in the EN form, $\delta\phi$ has similar behavior in both of them. So, this led us to refuse to investigate $\delta\phi$ for the LN form in the next figures.

In Fig. 6 and for different values of β, the general behavior of $\delta\phi$ is almost similar to that in Figs. 5(a) and 5(b) with a little difference. For constant values of parameters q and n, the value of $\delta\phi$ in the region $r_{+\min} < r_+ < r_{+\max}$ is related to the parameter β and decreases as β increases. Also, by increasing β, the value of $r_{+\min}$ decreases. Figure 7 shows different behavior than the two previous figures. In this figure, for each value of $r_+ > r_{+\min}$, the deficit angle increases as the dimension n increases.

IV. CONSERVED QUANTITIES

According to our previous statements, we cannot define thermodynamic quantities for magnetic branes because they are without any event horizons. Now, we would like to obtain conserved quantities of this magnetic brane such as the mass density and charge. Using AdS/CFT correspondence [32], we can derive the action and then the conserved quantities. For this purpose, we define the finite action

$$I_1 = I_{\text{bulk}} + I_b, \tag{42}$$

where I_b is a boundary term. I_b makes the variational principle well defined if we choose it as

$$I_b = I_b^{(1)} + I_b^{(2)} + I_b^{(3)} + I_b^{(4)}, \tag{43}$$

where $I_b^{(1)}$, $I_b^{(2)}$, $I_b^{(3)}$, and $I_b^{(4)}$ are, respectively, the proper surface terms for Hilbert-Einstein [33], Gauss-Bonnet [34,35], third-order [36] and fourth-order quasitopological [37] gravities, which are obtained as

$$I_b^{(1)} = \frac{1}{8\pi}\int_{\partial\mathcal{M}} d^n x\sqrt{-\gamma}K, \tag{44}$$

$$I_b^{(2)} = \frac{1}{8\pi}\int_{\partial\mathcal{M}} d^n x\sqrt{-\gamma}\frac{2\lambda l^2}{3(n-2)(n-3)}(3KK_{ac}K^{ac} - 2K_{ac}K^{cd}K^a_d - K^3), \tag{45}$$

$$\begin{aligned} I_b^{(3)} = \frac{1}{8\pi}\int_{\partial\mathcal{M}} d^n x\sqrt{-\gamma}\bigg\{&\frac{3\mu l^4}{5n(n-2)(n-1)^2(n-5)}(nK^5 - 2K^3K_{ab}K^{ab} + 4(n-1)K_{ab}K^{ab}K_{cd}K^d_eK^{ec} \\ &- (5n-6)KK_{ab}[nK^{ab}K^{cd}K_{cd} - (n-1)K^{ac}K^{bd}K_{cd}])\bigg\}, \end{aligned} \tag{46}$$

$$\begin{aligned} I_b^{(4)} = \frac{1}{8\pi}\int_{\partial\mathcal{M}} d^n x\sqrt{-\gamma}&\frac{2cl^6}{7n(n-1)(n-2)(n-7)(n^2-3n+3)}\{\alpha_1K^3K^{ab}K_{ac}K_{bd}K^{cd} + \alpha_2K^2K^{ab}K_{ab}K^{cd}K^e_cK_{de} \\ &+ \alpha_3K^2K^{ab}K_{ac}K_{bd}K6ceK^d_e + \alpha_4KK^{ab}K_{ab}K^{cd}K^e_cK^f_dK_{ef} + \alpha_5KK^{ab}K^c_aK_{bc}K^{de}K^f_dK_{ef} \\ &+ \alpha_6KK^{ab}K_{ac}K_{bd}K^{ce}K^{df}K_{ef} + \alpha_7K^{ab}K^c_aK_{bc}K^{de}K_{df}K_{eg}K^{fg}\}. \end{aligned} \tag{47}$$

In the above terms, $\gamma_{\mu\nu}$ is the induced metric on the boundary $\partial\mathcal{M}$, and K^{ab} is the extrinsic curvature of this boundary with the trace K.

The evaluated conserved quantities of the action (42) have the problem that they are divergent. To solve this problem and define a finite action for asymptotically AdS solutions with flat boundary, $\hat{R}_{abcd}(\gamma) = 0$, we use the counterterm method inspired by AdS/CFT correspondence. In this method, we add a new term I_{ct} to the action (42) to have a divergence-free stress-energy tensor [38]. I_{ct} is defined as

$$I_{ct} = -\frac{1}{8\pi}\int_{\partial\mathcal{M}} d^n x\sqrt{-\gamma}\frac{(n-1)}{l_{\text{eff}}}, \tag{48}$$

where l_{eff} is a scale length factor that is related to l and the coefficients of Gauss-Bonnet and quasitopological gravities. It also reduces to l as these coefficients go to zero.

To compute the conserved quantities, we first choose a spacelike surface $\mathcal{B}$ in $\partial\mathcal{M}$ with metric σ_{ij} and then write the boundary metric in ADM form:

$$\gamma^{ab}dx^a dx^b = -N^2dt^2 + \sigma_{ij}(d\phi^i + V^i dt)(d\phi^j + V^j dt). \tag{49}$$

N and V^i are, respectively, the lapse and shift functions, and the coordinates ϕ^i are the angular variables parametrizing the hypersurface of constant r around the origin. If we evaluate the finite stress tensor T_{ab} by the new finite action, we can obtain the quasilocal conserved quantities

$$\mathcal{Q}(\xi) = \int_{\mathcal{B}} d^{n-1}\phi\sqrt{\sigma}T_{ab}n^a\xi^b, \tag{50}$$

where n^a is the timelike unit normal vector to the boundary $\mathcal{B}$ and σ is the determinant of the metric σ_{ij}. ξ^b is a Killing vector field on the boundary for which we can obtain the total mass per unit volume V_{n-1} by its dedicated Killing vector $\xi = \partial/\partial t$ as

$$M_{\text{total}} = \frac{M}{4(n-1)}. \tag{51}$$

We can be sure that the obtained mass is finite because we have used the limit in which the boundary $\mathcal{B}$ becomes infinite. The next step is to determine the electric charge of this magnetic brane. To obtain the electric charge of the spacetimes with a longitudinal magnetic field, we should consider the projections of the electromagnetic field tensors on special hypersurfaces with normal $u^0 = \frac{1}{N}$, $u^r = 0$, and $u^i = \frac{N^i}{N}$. So, the electric field is described as

$$E^u = g^{\mu\rho}F_{\rho\nu}u^\nu. \tag{52}$$

By calculating the flux of the electromagnetic field at infinity, the electric charge per unit volume V_{n-1} is obtained as zero. The zero value of the electric charge returns to the zero value of the electric field. The electric field is obtained when the magnetic brane has at least one rotation. As we have considered a magnetic brane with no rotation, so it has no electric field and followed by, no electric charge.

V. CONCLUDING RESULTS

At last, we give brief conclusion on this magnetic brane. We started our theory with an $(n+1)$-dimensional action in quartic quasitopological gravity that is coupled to the exponential and logarithmic forms of the nonlinear electrodynamics. Quasitopological gravity is a comprehensive higher-derivative theory that leads to, at most, second-order field equations and has no limitations on dimensions. The theory reduces to Einstein's theory, as we choose the coefficients of quasitopological gravity zero ($\lambda = \mu = c = 0$). Nonlinear electrodynamics theory is also a nonlinear theory to remove some problems such as the divergence of the electromagnetic field of Maxwell theory in the origin. This theory reduces to the linear Maxwell one, as the nonlinearity parameter β goes to infinity.

For our purpose, we used the metric of the spacetime that has a magnetic brane interpretation with characteristics $(g_{\rho\rho})^{-1} \propto g_{\phi\phi}$ and $g_{tt} \propto -\rho^2$. The obtained solutions included an electromagnetic field ($F_{\phi\rho}$) that is related to the only nonzero component of the vector potential $A_\phi(r)$. The other solution $[f(\rho)]$ was made from a fourth-order field equation and was without any horizons and curvature singularities. The allowed region for f is defined in the interval $r_+ < \rho < \infty$, where does not contain the point $\rho = 0$. We then investigated the behaviors of the function f for different parameters. In these figures, exponential and logarithmic forms of nonlinear electrodynamics theory had the same effects on the function f. We also proved that the value of r_+ is independent of the values of the coefficients of Love-Lock and quasitopological gravities (μ, λ, and c) and showed this in the figures. r_+ increases as q increases, or n decreases separately. For ρ near r_+, the behavior of f is dependent on the parameters q and n and independent of the values of λ, μ, and c. In this region and for each value of ρ, by increasing the value of q or decreasing the value of n separately, the function f decreases. At larger ρ, the function f behaves in the opposite way, and it is independent of the values of parameters q, β, and n but is related to the parameters λ, μ, and c. Also, for constant values of q, β, and n, the function f is real for more regions of ρ, if we choose small values for λ and c and a large value for μ. For each value of ρ, there is a β_{min}, where for $\beta > \beta_{\text{min}}$, the function f has a constant value. The value of β_{min} also depends on the value of ρ and increases by decreasing the value of ρ.

The solutions of this magnetic brane have a conic singularity at $r = 0$ with a deficit angle $\delta\phi$. We proved that the deficit angle is not related to the coefficients of Love-Lock and quasitopological gravities and is dependent only on the parameters q, β, and n. So, we investigated the behavior of $\delta\phi$ vs r_+ for different values of q, β, and n in some figures. There are two $r_{+\text{min}}$ and $r_{+\text{max}}$ that for $r_{+\text{min}} < r_+ < r_{+\text{max}}$, the function f is dependent on the values of q and β. It increases as q increases, or separately, it increases as βdecreases. But for $r_+ > r_{+\text{max}}$, the function f is independent of the values of q and β. For different n, there is no $r_{+\text{max}}$, and for all $r_+ > r_{+\text{min}}$, the function f increases, as n increases. The figures also showed that, although the kinds of nonlinearity cannot change the general behaviors of $\delta\phi$, they cause a few changes. For example, for a constant value of q, they cause different values for r_+.

As this magnetic brane did not have any horizons, we could not consider thermodynamics for it. We just obtained conserved quantities such as the mass density and electric charge by using the counterterm method. The mass per unit volume V_{n-1} has a finite value, and the electric charge is zero because there is no electric field. It is clear that both of these quantities were independent of the nonlinearity parameter β.

In our next study, we would like to generalize this static spacetime to the case of rotating solutions with one and more rotation parameters.

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[1] A. A. Starobinsky, Phys. Lett. **91B**, 99 (1980).
[2] S. M. Carroll, V. Duvvuri, M. Trodden, and M. S. Turner, Phys. Rev. D **70**, 043528 (2004).
[3] S. Fay, R. Tavakol, and S. Tsujikawa, Phys. Rev. D **75**, 063509 (2007).
[4] S. Nojiri and S. D. Odintsov, Phys. Rev. D **74**, 086005 (2006); J. Phys. Conf. Ser. **66**, 012005 (2007).
[5] S. Capozziello, S. Nojiri, S. D. Odintsov, and A. Troisi, Phys. Lett. B **639**, 135 (2006).
[6] G. Cognola, E. Elizalde, S. Nojiri, S. D. Odintsov, and S. Zerbini, Phys. Rev. D **73**, 084007 (2006).
[7] C. G. Boehmer, T. Harko, and F. S. N. Lobo, Astropart. Phys. **29**, 386 (2008); S. Capozziello, V. F. Cardone, and A. Troisi, J. Cosmol. Astropart. Phys. 08 (2006) 001.
[8] A. Borowiec, W. Godlowski, and M. Szydlowski, Int. J. Geom. Methods Mod. Phys. **04**, 183 (2007); C. G. Boehmer, T. Harko, and F. S. N. Lobo, J. Cosmol. Astropart. Phys. 03 (2008) 024.
[9] O. Bertolami, C. G. Bohmer, T. Harko, and F. S. N. Lobo, Phys. Rev. D **75**, 104016 (2007); V. Faraoni, Phys. Rev. D **76**, 127501 (2007); G. Allemandi, A. Borowiec, M. Francaviglia, and S. D. Odintsov, Phys. Rev. D **72**, 063505 (2005); T. Koivisto, Classical Quantum Gravity **23**, 4289 (2006).
[10] M. H. Dehghani, A. Bazrafshan, R. B. Mann, M. R. Mehdizadeh, M. Ghanaatian, and M. H. Vahidinia, Phys. Rev. D **85**, 104009 (2012).
[11] P. Kovtun, D. T. Son, and A. O. Starinets, Phys. Rev. Lett. **94**, 111601 (2005); J. High Energy Phys. 10 (2003) 064; A. Buchel, Phys. Lett. B **609**, 392 (2005); K. Landsteiner and J. Mas, J. High Energy Phys. 07 (2007) 088.
[12] R. Aros, M. Romo, and N. Zamorano, Phys. Rev. D **75**, 067501 (2007).
[13] F. Chen, K. Dasgupta, J. M. Lapan, J. Seo, and R. Tatar, Phys. Rev. D **88**, 066003 (2013).
[14] M. Fukuma, S. Matsuura, and T. Sakai, Prog. Theor. Phys. **105**, 1017 (2001).
[15] V. V. Dyadichev, D. V. Galtsov, A. G. Zorin, and M. Yu. Zotov, Phys. Rev. D **65**, 084007 (2002); R. Garcia-Salcedo and N. Breton, Classical Quantum Gravity **20**, 5425 (2003); D. N. Vollick, Gen. Relativ. Gravit. **35**, 1511 (2003); R. Garcia-Salcedo and N. Breton, Classical Quantum Gravity **22**, 4783 (2005).
[16] M. Novello, S. E. Perez Bergliaffa, and J. Salim, Phys. Rev. D **69**, 127301 (2004).
[17] E. Ayon-Beato and A. Garca, Phys. Lett. B **464**, 25 (1999); Gen. Relativ. Gravit. **31**, 629 (1999).
[18] E. Fradkin and A. Tseytlin, Phys. Lett. **163B**, 123 (1985); O. Andreev and A. Tseytlin, Nucl. Phys. **B311**, 205 (1988); C. Callan, C. Lovelace, C. Nappi, and S. Yost, Nucl. Phys. **B308**, 221 (1988); E. Bergshoeff, E. Sezgin, C. Pope, and P. Townsend, Phys. Lett. B **188**, 70 (1987).
[19] S. H. Hendi, J. High Energy Phys. 03 (2012) 065; S. H. Hendi and A. Sheykhi, Phys. Rev. D **88**, 044044 (2013).
[20] H. H. Soleng, Phys. Rev. D **52**, 6178 (1995).
[21] A. Sheykhi, F. Naeimipour, and S. M. Zebarjad, Phys. Rev. D **91**, 124057 (2015); **92**, 124054 (2015); Gen. Relativ. Gravit. **48**, 33 (2016); **48**, 96 (2016).
[22] A. Sheykhi and S. Hajkhalili, Phys. Rev. D **89**, 104019 (2014).
[23] K. Bamba and S. D. Odintsov, J. Cosmol. Astropart. Phys. 04 (2008) 024.
[24] L. Hollenstein and F. S. Lobo, Phys. Rev. D **78**, 124007 (2008).
[25] M. H. Dehghani, N. Alinejadi, and S. H. Hendi, Phys. Rev. D **77**, 104025 (2008).
[26] S. H Hendi and A. Dehghani, Phys. Rev. D **91**, 064045 (2015).
[27] S. H. Hendi, S. Panahiyan, and H. Mohammadpour, Eur. Phys. J. C **72**, 2184 (2012).
[28] S. H. Hendi, B. Eslam Panah, and S. Panahiyan, Phys. Rev. D **91**, 084031 (2015).
[29] M. Ghanaatian, A. Bazrafshan, and W. G. Brenna, Phys. Rev. D **89**, 124012 (2014).
[30] M. Ghanaatian, Gen. Relativ. Gravit. **47**, 105 (2015).
[31] M. Ghanaatian, A. Bazrafshan, S. Taghipoor, and R. Tawoosi, Can. J. Phys. **96**, 1209 (2018).
[32] M. H. Dehghani and R. B. Mann, Phys. Rev. D **64**, 044003 (2001).
[33] G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15**, 2752 (1977).
[34] R. C. Myers, Phys. Rev. D **36**, 392 (1987); S. C. Davis, Phys. Rev. D **67**, 024030 (2003).
[35] M. H. Dehghani and R. B. Mann, Phys. Rev. D **73**, 104003 (2006); M. H. Dehghani, N. Bostani, and A. Sheykhi, Phys. Rev. D **73**, 104013 (2006).
[36] M. H. Dehghani and M. H. Vahidinia, Phys. Rev. D **84**, 084044 (2011).
[37] A. Bazrafshan, M. H. Dehghani, and M. Ghanaatian, Phys. Rev. D **86**, 104043 (2012).
[38] M. Henningson and K. Skenderis, J. High Energy Phys. 07 (1998) 023; S. Nojiri and S. D. Odintsov, Phys. Lett. B **444**, 92 (1998); V. Balasubramanian and P. Kraus, Commun. Math. Phys. **208**, 413 (1999).