Large mass hierarchy from a small nonminimal coupling

Christophe Ringeval,[1,*] Teruaki Suyama,[2,†] and Masahide Yamaguchi[2,‡]
[1]*Cosmology, Universe and Relativity at Louvain, Institute of Mathematics and Physics, Louvain University, 2 Chemin du Cyclotron, 1348 Louvain-la-Neuve, Belgium*
[2]*Department of Physics, Tokyo Institute of Technology, 2-12-1 Ookayama, Meguro-ku, Tokyo 152-8551, Japan*

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We propose a simple but novel cosmological scenario where both the Planck mass and the dark energy scale emerge from the same super-Hubble quantum fluctuations of a nonminimally coupled ultralight scalar field during primordial inflation. The current cosmic and Solar System observations constrain the nonminimal coupling to be small.

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I. INTRODUCTION

The standard model (SM) of particle physics and general relativity (GR) are two pillars of the current elementary theory of physics. Apart for nonzero neutrino masses and dark matter which, under the new particle hypothesis, require an extension beyond the SM, there are no observations that manifestly contradict the SM and GR. Yet, the wide separations among the four energy scales appearing in the SM and GR—which are the Planck scale $M_{\mathrm{Pl}} = (8\pi G)^{-1/2} \simeq 10^{18}$ GeV, the electroweak scale $M_{\mathrm{EW}} \simeq 10^2$ GeV, the neutrino mass scale $m_\nu \simeq 0.1$ eV, and the dark energy scale $\rho_\Lambda^{1/4} \simeq 10^{-3}$ eV—should provide enough motivation to search for a dynamical explanation. One possible method is to assume that at least some of these quantities are not fundamental constants, but rather fields that evolved together with the cosmological evolution [1].

The idea that the gravitational constant G (namely, the Planck scale) evolves in time has long been a topic of investigation, and many different proposals have been made in the literature in various contexts. For instance, Dirac was the first to conjecture that G could vary with the cosmic time as $G \propto t^{-1}$ based on his large-number hypothesis [2]. Later, scalar-tensor theories that consistently implement the variation of G were formulated by Jordan and by Brans and Dicke [3,4]. Similarly, in the context of dark energy, quintessence models have been proposed to explain the apparent smallness of the measured dark energy density (assuming a zero cosmological constant) and/or the coincidence problem [5–7].

In Ref. [8], we have shown that cosmic inflation occurring at TeV energy scales—and therefore relatively close to M_{EW}—could provide a natural answer to the smallness of the cosmological constant today. The mechanism advocated there relies on the growth of super-Hubble quantum fluctuations for an ultralight scalar field during primordial inflation [9–14], which manifest themselves as a universal quantum-generated variance after inflation. A similar mechanism for cosmological vector fields has also been presented in Refs. [15,16], again predicting an inflationary era at the TeV scale. Various other works have since confirmed the robustness of the mechanism and proposed extensions to scalar-tensor theories of gravity as well as to gravitational vector fields [17–19].

In this paper, we show that cosmic inflation can *simultaneously* explain both the largeness of the Planck scale and the smallness of the cosmological constant by the very same mechanism: super-Hubble quantum fluctuations of a unique nonminimally coupled ultralight scalar field. Proposals of an emerging Planck scale have provoked continuous theoretical constructions within scalar-tensor theories, but only a few have been concerned with the generation of an effective Planck mass from inflation [20–26]. As far as we are aware, the scenario we propose is a new way to address the dark energy scale and the value of the Planck mass simultaneously while providing a potential link to the physics around the electroweak energy scale. Let us finally mention that such a scenario is fundamentally different compared to induced gravity theories [27,28], in which the Einstein-Hilbert term emerges from the quantum fluctuations of matter fields immersed in the curved spacetime. In our case, the Einstein-Hilbert term is already present at the classical level, although inflation makes it become negligibly small compared to the nonminimal coupling term.

The paper is organized as follows. In the next section we present the main idea and basic model requirements needed for the scenario to work, before turning to a more detailed

*christophe.ringeval@uclouvain.be
†suyama@phys.titech.ac.jp
‡gucci@phys.titech.ac.jp

calculation in Sec. III. In Sec. IV we enumerate the observational constraints, in Sec. V we discuss other aspects of the scenario, and we conclude in Sec. VI.

II. MAIN IDEA

The idea relies on an ultralight scalar field ϕ which only couples to gravity with a nonminimal coupling to the Ricci scalar R. During an extended period of inflation, it undergoes a significant growth and could acquire a quantum-generated super-Planckian variance. In the next section we will explain this process in more detail, but here we describe the model requirements. The relevant part of the Lagrangian is given by

$$\mathcal{L} = \frac{1}{2}(M^2 + \xi\phi^2)R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m_0^2\phi^2, \tag{1}$$

where ξ represents the strength of the nonminimal coupling. The hypothesis that forms the basis of this paper is that the *bare* gravitational energy scale M is much smaller than the measured Planck mass $M \ll M_{\mathrm{Pl}}$ and could be as low as or even smaller than the electroweak scale. Since the main result does not depend on the concrete value of M, we leave M unspecified aside from the condition $M \ll M_{\mathrm{Pl}}$. As Eq. (1) shows, nonvanishing and time-independent vacuum expectation values (VEVs) for $\langle\phi^2\rangle$ contribute to the effective gravitational energy scale by $\xi\langle\phi^2\rangle$. We therefore require that ξ be positive; otherwise, our scenario does not work.

Once $\xi\langle\phi^2\rangle$ settles to Planck-like values, the potential energy of the field today can source the acceleration of the Universe by the mechanism of Ref. [8]. For this to happen, it should match the cosmological constant energy scale,

$$\frac{1}{2}m_0^2\langle\phi^2\rangle \simeq 3H_0^2 M_{\mathrm{Pl}}^2 \Omega_\Lambda. \tag{2}$$

Moreover, ϕ behaves as dark energy provided it remains (quasi)frozen in the Hubble flow and, as discussed in Sec. IV, this implies some constraints on ξ.

During inflation, due to the nonminimal coupling, the effective mass of the field is given by

$$m^2 = m_0^2 - 12\xi H_{\mathrm{inf}}^2, \tag{3}$$

where we have taken the de Sitter value for the Ricci scalar $R = 12H_{\mathrm{inf}}^2$. There are *a priori* three possible regimes.

In the limit $\xi \ll m_0^2/(12H_{\mathrm{inf}}^2)$, the nonminimal coupling is so small that it has essentially no effect during inflation. The model matches the one of Ref. [8], and for sufficiently long inflation one gets the de Sitter variance of a test scalar field, $\langle\phi^2\rangle \to 3H_{\mathrm{inf}}^4/(8\pi^2 m_0^2)$. Dark energy is explained by satisfying Eq. (2), namely, for inflation occurring at the TeV scale, $H_{\mathrm{inf}}^2 = 4\pi\sqrt{\Omega_\Lambda}H_0 M_{\mathrm{Pl}}$. As a result, one gets

$$\frac{\xi\langle\phi^2\rangle}{M_{\mathrm{Pl}}^2} \to \frac{3H_0\sqrt{\Omega_\Lambda}}{2\pi M_{\mathrm{Pl}}}\frac{\xi H_{\mathrm{inf}}^2}{m_0^2} \ll 1, \tag{4}$$

and thus the super-Hubble quantum fluctuations of $\xi\phi^2$ are always deeply sub-Planckian and the model cannot explain the measured Planck mass.

One could then consider the massless limit of Eq. (3), obtained by taking quite fine-tuned values of $\xi \to m_0^2/(12H_{\mathrm{inf}}^2)$. Because $m^2 \to 0$ during inflation, $\langle\phi^2\rangle \to 3H_{\mathrm{inf}}^4/(8\pi^2 m^2)$ can become very large. Plugging these values into Eq. (2), and requiring $\xi\langle\phi^2\rangle \simeq M_{\mathrm{Pl}}^2$, one obtains a condition for the energy scale of inflation which, after some algebra, reads $H_{\mathrm{inf}}^2 \simeq (\Omega_\Lambda/2)H_0^2$, and the model is also ruled out.

The only remaining possibility is $m_0^2 < 12\xi H_{\mathrm{inf}}^2$ and we are in the presence of an ultralight tachyonic field *during* inflation. Such a situation is not problematic and has been considered as a dark energy candidate in Ref. [19]. Indeed, because of the bare mass of the field $m_0^2 > 0$, the tachyonic instability generated by the expansion of the Universe through the nonminimal coupling is only transient. As we detail below, such a transient instability is actually a virtue and allows the mechanism to generate both the Planck mass and the actual value of dark energy.

III. QUANTUM-GENERATED FIELD VARIANCE

Let us now consider the limit $m_0^2 \ll 12\xi H_{\mathrm{inf}}^2$ to perform a more detailed calculation of the quantum-generated variance for ϕ. We moreover assume that inflation lasted for a very long time in the sense that the total number of e-folds of accelerated expansion can be a large number. For a slowly evolving Hubble parameter H during inflation, the ϕ field undergoes a stochastic process on super-Hubble scales, which effectively pushes its variance to larger amplitudes [14,29–31]. Then, under the slow-motion approximation, the coarse-grained field (which we still denote by ϕ) follows the Langevin equation

$$\frac{\mathrm{d}\phi}{\mathrm{d}N} = 4\xi\phi + \frac{H}{2\pi}\eta(N), \tag{5}$$

where $N = \int H\mathrm{d}t$ is the number of e-folds and we have used $m^2 \simeq -12\xi H^2$. The second term on the right-hand side represents a stochastic noise arising from the transition of the sub-Hubble modes to the super-Hubble modes. The quantity η is a Gaussian white noise whose two-point correlation function is given by

$$\langle\eta(N_1)\eta(N_2)\rangle = \delta(N_1 - N_2), \tag{6}$$

with $\langle\eta(N)\rangle = 0$. The Hubble parameter H is determined by the Friedmann-Lemaître equation stemming from Eq. (1), plus other terms coming from the field driving inflation. If we denote the inflaton field by ψ, where $V(\psi)$ is its potential, one gets

$$H^2 = \frac{V(\psi)}{3(M^2+\xi\phi^2) - \frac{1}{2}{\phi_{,N}}^2 - 6\xi\phi\phi_{,N} - \frac{1}{2}\psi_{,N}^2} \simeq \frac{V(\psi)}{3(M^2+\xi\phi^2)}, \tag{7}$$

where a comma denotes a derivative. The second line is obtained by assuming slow-roll and keeping only the leading term. Assuming $V(\psi) = V_{\rm inf}$ to be almost constant during a plateau-like inflationary era, we can solve the Langevin equation to determine the stochastic motion of ϕ. Since M is the fundamental scale in the present scenario, we assume $M^4 \gtrsim V_{\rm inf}$ in the following analysis.

The dependence of H on ϕ prevents us from solving Eq. (5) exactly, but the solution can be approximated in two domains. Defining

$$\phi_{\rm crit} \equiv \frac{M}{\sqrt{\xi}}, \tag{8}$$

one sees that the behavior of H changes at $\phi = \phi_{\rm crit}$. We exploit this observation and consider the two limiting cases $\phi \ll \phi_{\rm crit}$ and $\phi \gg \phi_{\rm crit}$ separately, and then combine them to obtain the (approximate) final result. In order to give a conservative estimate, we assume that ϕ, as well its classical value, are initially vanishing.

Let us first investigate the motion of ϕ for $\phi \ll \phi_{\rm crit}$. During this phase, we can ignore the term $\xi\phi^2$ in the Friedmann-Lemaître equation, and the Langevin equation for ϕ can be solved analytically. One gets

$$\phi(N) = \frac{1}{2\pi M}\sqrt{\frac{V_{\rm inf}}{3}} e^{4\xi N}\int_0^N e^{-4\xi N'}\eta(N')\mathrm{d}N'. \tag{9}$$

Thus, the expectation value of ϕ^2 is given by

$$\langle\phi^2(N)\rangle = \frac{V_{\rm inf}}{96\pi^2 M^2\xi}(e^{8\xi N} - 1). \tag{10}$$

As it should be, this solution incorporates the features of both the stochastic motion and the tachyonic instability. For $\xi N \ll 1$, picking up the leading term, we obtain $\langle\phi^2\rangle \approx V_{\rm inf}/(12\pi^2 M^2)N$ and recover Brownian motion. For $\xi N \gg 1$, we have $\langle\phi^2\rangle \propto e^{8\xi N}$ and its exponential growth represents the tachyonic instability. Let us notice that had we started from a nonvanishing VEV for ϕ, Eq. (10) would still apply but for the variance, i.e., $\langle\delta\phi^2\rangle = \langle\phi^2\rangle - \langle\phi\rangle^2$. If we further add the fluctuations of ϕ at the initial time $\delta\phi(0)$, Eq. (10) contains an additional term evolving as $\langle\delta\phi(0)\rangle^2 e^{8\xi N}$. As a result, for all possible initial conditions, a long-enough inflationary period always induces an exponential growth of the field variance.

However, Eq. (10) becomes invalid when $\sqrt{\langle\phi^2\rangle}$ reaches $\phi_{\rm crit}$. In terms of the number of e-folds, this happens at $N = N_{\rm crit}$, where $N_{\rm crit}$ is given by

$$N_{\rm crit} = \frac{1}{8\xi}\ln\left(1 + \frac{96\pi^2 M^4}{V_{\rm inf}}\right). \tag{11}$$

Thus, $N_{\rm crit} = \mathcal{O}(\xi^{-1})$ and becomes very large for small ξ. Next, let us investigate the opposite regime, $\phi \gg \phi_{\rm crit}$. In this limit, we can ignore the term M^2 in the Friedmann-Lemaître equation and we can solve the Langevin equation analytically for ϕ^2. The result is given by

$$\phi^2(N) = e^{8\xi(N-N_{\rm crit})}\phi_{\rm crit}^2 + e^{8\xi N}\sqrt{\frac{V}{3\pi^2\xi}}\int_{N_{\rm crit}}^N \eta(N')e^{-8\xi N'}\mathrm{d}N'. \tag{12}$$

The second term on the right-hand side is directly sourced by the stochastic noise $\eta(N')$ and disappears by taking the statistical average. Hence, one obtains

$$\xi\langle\phi^2(N)\rangle = \frac{M^2}{1 + \frac{96\pi^2 M^4}{V_{\rm inf}}} e^{8\xi N}. \tag{13}$$

From this equation, we can estimate the typical number of e-folds required for the ϕ field to generate a large gravitational energy scale, say $\bar{M}_{\rm Pl}$, as

$$\bar{N}_{\rm Pl} = N_{\rm crit} + \frac{1}{4\xi}\ln\left(\frac{\bar{M}_{\rm Pl}}{M}\right). \tag{14}$$

Thus, $\bar{N}_{\rm Pl}$ is also $\mathcal{O}(\xi^{-1})$. Here we have introduced the new mass scale $\bar{M}_{\rm Pl}$ instead of the usual Planck mass $M_{\rm Pl}$ because, as explained in the next section, the gravitational coupling $M^2 + \xi\langle\phi^2\rangle$ appearing in the Lagrangian (1) does not necessarily equal the one measured by Cavendish-like experiments due to the existence of a fifth force. To summarize, for all possible initial conditions of ϕ, a Planck-like energy scale $\bar{M}_{\rm Pl}$ can be generated by $\xi\langle\phi^2\rangle$ provided primordial inflation lasts for about $\mathcal{O}(\xi^{-1})$ e-folds.[1]

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[1]Strictly speaking, the number of e-folds $\mathcal{N}$ along each trajectory is a stochastic quantity and another possible route for deriving the result is to calculate its mean stochastic value $\langle\mathcal{N}\rangle$ [31–34]. For the regime $\phi < \phi_{\rm crit}$, one finds

$$\langle\mathcal{N}\rangle \simeq \frac{1}{8\xi}\ln\left(\frac{192\pi^2 M^4}{V_{\rm inf}}\right) + \frac{\gamma}{8\xi}, \tag{15}$$

which matches $N_{\rm crit}$ up to a factor of $\mathcal{O}(1)$ correction. Here $\gamma \simeq 0.5772$ is Euler's constant. For the regime $\phi > \phi_{\rm crit}$, one finds

$$\langle\mathcal{N}\rangle \simeq \frac{1}{4\xi}\left[\ln\left(\frac{\bar{M}_{\rm Pl}}{M}\right) + \frac{V_{\rm inf}}{384\pi^2 M^4}\right], \tag{16}$$

which matches the second term of Eq. (14) up to a factor of $\mathcal{O}(1)$ correction.

Let us stress that the inflationary period relevant for observations is only about 60 e-folds before the end and we have found that the time scale for the variation of ϕ is ξ^{-1} (in e-folds). As a result, and provided inflation can end (see Sec. V), the variation of ϕ during the last 60 e-folds of inflation is thus negligibly small. Standard GR is perfectly recovered during the inflationary era relevant to observations. Let us now examine the experimental bounds on such a mechanism.

IV. EXPERIMENTAL BOUNDS

The existence of an ultralight massive field ϕ today leaves various observational signatures from which we can place bounds on both ξ and m_0.

Although ϕ is not directly coupled to matter, the nonminimal coupling of the ultralight scalar field induces a fifth force among bodies, in addition to the pure GR gravitational terms. This effect can be made manifest by making a conformal transformation [35] from the present frame with the metric $g_{\mu\nu}$ to the Einstein frame with the metric $\tilde{g}_{\mu\nu}$ verifying

$$g_{\mu\nu} = A^2(\phi)\tilde{g}_{\mu\nu}, \tag{17}$$

where

$$A^2 \equiv \frac{1}{(1+\xi\frac{\phi^2}{M^2})}. \tag{18}$$

The action can be canonically normalized from the field redefinition $\phi \to \chi$ with [36]

$$e^{\bar{\chi}} \equiv \left[\frac{\sqrt{1+\xi\bar{\phi}^2}}{\sqrt{1+\xi(1+6\xi)\bar{\phi}^2}+\sqrt{6}\xi\bar{\phi}}\right]^{\sqrt{6}} \times\left[\sqrt{1+(\xi+6\xi^2)\bar{\phi}^2}+\sqrt{\xi(1+6\xi)\bar{\phi}^2}\right]^{\sqrt{6+\frac{1}{\xi}}}, \tag{19}$$

where we have defined the dimensionless fields $\bar{\phi}\equiv\phi/M$ and $\bar{\chi}\equiv\chi/M$. The original action is transformed as

$$S = \frac{M^2}{2}\int \mathrm{d}^4x\sqrt{-\tilde{g}}\left[\tilde{R}-(\tilde{\nabla}\bar{\chi})^2-2\frac{W(\chi)}{M^2}\right] + S_\mathrm{m}[A^2(\chi)\tilde{g}_{\mu\nu},\psi_\mathrm{m}], \tag{20}$$

where the potential is given by

$$\frac{W(\chi)}{M^2} = \frac{A^4(\chi)}{2}m_0^2\bar{\phi}^2. \tag{21}$$

The field redefinition (19) cannot be straightforwardly inverted, but we can take the limit we are interested in, namely, $\xi\bar{\phi}^2 = \bar{M}_\mathrm{Pl}^2/M^2 \gg 1$ and $\xi \ll 1$. We obtain

$$\bar{\phi}\simeq\frac{1}{2\sqrt{\xi}}e^{\sqrt{\xi}\bar{\chi}}, \qquad A^2\simeq\frac{1}{1+\frac{1}{4}e^{2\sqrt{\xi}\bar{\chi}}}. \tag{22}$$

As it should be, the coupling between χ and matter disappears in the minimal coupling limit ($\xi\to 0$). Such a fifth force changes the parametrized post-Newtonian (PPN) parameters compared to the values in GR as [37–39]

$$\beta_\mathrm{PPN}-1=\frac{1}{2}\frac{\alpha^2\beta}{(1+\alpha^2)^2}, \qquad \gamma_\mathrm{PPN}-1=-2\frac{\alpha^2}{1+\alpha^2}, \tag{23}$$

where α and β are defined by

$$\alpha=\sqrt{2}\frac{\partial\ln A}{\partial\bar{\chi}}\simeq-\frac{\sqrt{2\xi}}{4}\frac{e^{2\sqrt{\xi}\bar{\chi}}}{1+\frac{1}{4}e^{2\sqrt{\xi}\bar{\chi}}},$$
$$\beta=2\frac{\partial^2\ln A}{\partial\bar{\chi}^2}\simeq-\xi\frac{e^{2\sqrt{\xi}\bar{\chi}}}{(1+\frac{1}{4}e^{2\sqrt{\xi}\bar{\chi}})^2}. \tag{24}$$

Using the limit $\xi\bar{\phi}^2=\bar{M}_\mathrm{Pl}^2/M^2\gg 1$ for $\bar{\chi}$,

$$e^{\sqrt{\xi}\bar{\chi}}=2\frac{\bar{M}_\mathrm{Pl}}{M}, \tag{25}$$

one gets

$$\beta_\mathrm{PPN}=1+\mathcal{O}\left(\xi^2\frac{M^2}{\bar{M}_\mathrm{Pl}^2}\right),$$
$$\gamma_\mathrm{PPN}=1-\frac{4\xi}{1+2\xi}+\mathcal{O}\left(\xi\frac{M}{\bar{M}_\mathrm{Pl}}\right). \tag{26}$$

Thus, γ_PPN becomes slightly smaller than unity. The most stringent bound on γ_PPN comes from the Shapiro time delay measurement using the Cassini spacecraft [40]: $-0.03<(\gamma_\mathrm{PPN}-1)\times 10^5<4.4$. This limit translates into an upper limit on ξ as

$$\xi<7.5\times 10^{-8}. \tag{27}$$

As mentioned in the previous section, the gravitational coupling as measured by Cavendish-like experiments is $M_\mathrm{Pl}^2=1/(8\pi G)$, where G is the measured Newton's constant. It is slightly different from $\bar{M}_\mathrm{Pl}^2$ due to the fifth force induced by ϕ and reads

$$M_\mathrm{Pl}^2=\frac{M^2}{A^2(1+\alpha^2)}=\frac{\bar{M}_\mathrm{Pl}^2}{1+2\xi}+\mathcal{O}(M^2)\simeq\bar{M}_\mathrm{Pl}^2. \tag{28}$$

For the values of ξ compatible with the Cassini constraints of Eq. (27), M_Pl^2 is therefore indistinguishable from $\bar{M}_\mathrm{Pl}^2$ and both quantities will be identified in the following.

Another effect comes from demanding that the potential energy of the field sources the current acceleration of the Universe. From Eq. (2) and $\xi\langle\phi^2\rangle=\bar{M}_\mathrm{Pl}^2\simeq M_\mathrm{Pl}^2$, one gets

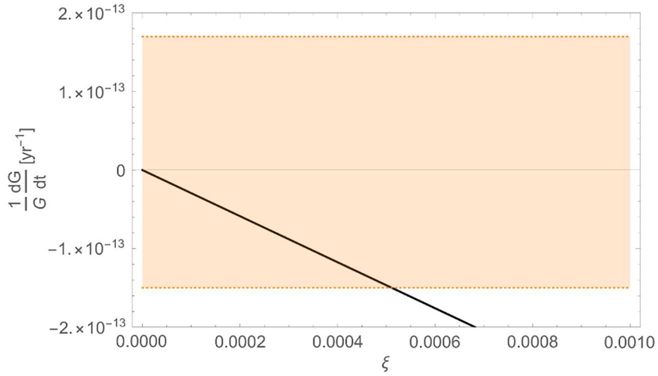

FIG. 1. Time variation of G at present day as a function of ξ. The bare mass of the field has been set to $m_0 = \sqrt{6\xi\Omega_\Lambda}H_0$, the value explaining dark energy today. The orange region, which is obtained from improvements in the ephemeris of Mars [45], is the observationally allowed region.

$$m_0^2 \simeq 6\xi H_0^2 \Omega_\Lambda. \tag{29}$$

Therefore, the mass is not a free parameter and for values of ξ satisfying the Cassini bound we get $m_0 < 7\times10^{-4}H_0$, i.e., the field is extremely light. Let us notice that, because it is not coupled to other sectors, such a tiny mass is not *a priori* problematic. Moreover, dynamical mechanisms able to generate small masses have been proposed; see, for instance, Ref. [41]. The ultralight scalar field is thus compatible with all limits associated with an evolution of the equation of state of dark energy and its perturbations [42,43].

Finally, there are constraints coming from the cosmological time variation of ϕ which also drives the time variation of the gravitational constant. The equation of motion of ϕ on the cosmological background is given by

$$\ddot{\phi} + 3H\dot{\phi} + m_0^2\phi - 6\xi(2H^2 + \dot{H})\phi = 0, \tag{30}$$

where a dot stands for a derivative with respect to the cosmic time and where m_0^2 is given by Eq. (29). Nondetections of the time variation of G imply that ϕ has not moved significantly from the initial value until the present epoch. In the slow-roll regime, the Hubble parameter is approximately given by that of the standard ΛCDM model [44]. Using this Hubble parameter, we can solve the above equation of motion and derive the relative time variation of G at present day for different values of ξ. At leading order in $M/\bar{M}_{\rm Pl}$, we have

$$\frac{\dot{G}}{G} = -2\frac{\dot{\phi}}{\phi}. \tag{31}$$

The result is shown as a thick line in Fig. 1. Interestingly, contrary to the minimally coupled case, the nonminimal coupling term makes ϕ grow, which explains the negative sign of $\dot{G}$. The orange region is the observationally allowed region obtained by the improvements in the ephemeris of Mars [45]. From this figure, we obtain the upper bound $\xi < 5\times10^{-4}$, which is weaker than the one coming from the Shapiro effect in Eq. (27).

V. DISCUSSION

In the previous sections, we have seen that the ultralight scalar field can dynamically generate the large measured value of the Planck mass $M_{\rm Pl}$ from a much lower gravitational energy scale M, which could be as low as or even smaller than the electroweak scale. Once its VEV generates the observed Planck mass, the same field can also source dark energy from its small, but nonvanishing mass term. However, the mechanism requires a very long period of inflation, of the order of $\mathcal{O}(\xi^{-1})$ e-folds. For $\xi < 10^{-7}$, this means that the scale factor a should have grown during inflation by a factor of at least the tetration ${}^4e$. Accurate observations of the cosmic microwave background (CMB) anisotropies in the last decade strongly support the idea that inflation occurred in the very early Universe [46–48]. Although only the last ∼60 e-folds of inflation can be probed observationally, it is legitimate to suppose that the total period of inflation that the Universe has experienced may be much longer. This can happen if the inflaton had a nearly flat potential over a sufficiently large field range and started its motion far from the end point of inflation, as this could very well be the case for the plateau inflationary models favored by the data [49]. Another possibility is that the observable inflation was preceded by a false vacuum phase of the same field as the one relevant to the last ∼60 e-folds of inflation. It is also equally possible that the very long inflation is sourced by a different field than the inflaton responsible for the observable inflation.

In the following, we describe in more detail the primordial inflationary part of the model in the presence of the two fields. The dynamics is easier to understand in the Einstein frame. The equations of motion for the inflaton field ψ and the canonically normalized gravity field χ read [50]

$$\frac{\bar{\psi}_{,NN} + \sqrt{2}\alpha(\chi)\bar{\chi}_{,N}\bar{\psi}_{,N}}{3-\epsilon_1} + \bar{\psi}_{,N} = -\frac{1}{A^2(\chi)}\frac{{\rm d}\ln U}{{\rm d}\bar{\psi}},$$
$$\frac{\bar{\chi}_{,NN} - [\alpha(\chi)/\sqrt{2}]A^2(\chi)\bar{\psi}_{,N}^2}{3-\epsilon_1} + \bar{\chi}_{,N} = -\frac{{\rm d}\ln U}{{\rm d}\bar{\chi}}, \tag{32}$$

where ϵ_1 is the first Hubble flow function in the Einstein frame,

$$\epsilon_1 \equiv -\frac{{\rm d}\ln H}{{\rm d}N} = \frac{1}{2}\bar{\chi}_{,N}^2 + \frac{1}{2}A^2(\chi)\bar{\psi}_{,N}^2. \tag{33}$$

We have introduced the two-field potential $U(\chi,\psi)$ as

$$U(\chi,\psi) \equiv \frac{W(\chi)}{M^2} + \frac{A^4(\chi)V(\psi)}{M^2}. \tag{34}$$

These equations can be simplified by taking the limits we are interested in, $\xi \ll 1$ and $\xi\bar{\phi}^2 \gg 1$, together with Eq. (22). One gets

$$A^2(\chi) \simeq 4e^{-2\sqrt{\xi}\bar{\chi}}, \qquad \alpha(\chi) \simeq -\sqrt{2\xi},$$
$$U(\chi,\psi) \simeq 2e^{-2\sqrt{\xi}\bar{\chi}}\left[\frac{m_0^2}{\xi} + \frac{8V(\psi)}{M^2}e^{-2\sqrt{\xi}\bar{\chi}}\right]. \tag{35}$$

From these equations, with $m_0^2/\xi = \mathcal{O}(H_0^2)$, one gets

$$U(\chi,\psi) \simeq 16e^{-4\sqrt{\xi}\bar{\chi}}\frac{V(\psi)}{M^2}, \tag{36}$$

which from Eq. (32) gives

$$\frac{\bar{\psi}_{,NN} - 2\sqrt{\xi}\bar{\chi}_{,N}\bar{\psi}_{,N}}{3-\epsilon_1} + \bar{\psi}_{,N} \simeq -\frac{e^{2\sqrt{\xi}\bar{\chi}}}{4}\frac{\mathrm{d}\ln V}{\mathrm{d}\bar{\psi}}, \tag{37}$$

$$\frac{\bar{\chi}_{,NN} + 4\sqrt{\xi}e^{-2\sqrt{\xi}\bar{\chi}}\bar{\psi}_{,N}^2}{3-\epsilon_1} + \bar{\chi}_{,N} \simeq 4\sqrt{\xi}. \tag{38}$$

Under the slow-roll approximation, one can find an approximate solution of Eqs. (37) and (38). Let us first assume in Eq. (38) that

$$4\sqrt{\xi}e^{-2\sqrt{\xi}\bar{\chi}}\bar{\psi}_{,N}^2 \ll \bar{\chi}_{,N}. \tag{39}$$

The slow-roll solution for $\bar{\chi}$ reads

$$\bar{\chi}_{,N} \simeq 4\sqrt{\xi} \ll 1. \tag{40}$$

This equation implies that $\bar{\chi}(N) \propto 4\sqrt{\xi}N$. As can be explicitly checked by using Eq. (22), this is the Einstein-frame manifestation of the tachyonic growth of ϕ. Plugging the above equation into Eq. (37), we get the slow-roll solution for the inflaton ψ (with $\xi \ll 1$),

$$\bar{\psi}_{,N} \simeq -\frac{e^{2\sqrt{\xi}\bar{\chi}}}{4}\frac{\mathrm{d}\ln V}{\mathrm{d}\bar{\psi}}. \tag{41}$$

This allows us to estimate the first Hubble flow function from Eq. (33),

$$\epsilon_1 \simeq 8\xi + \frac{e^{2\sqrt{\xi}\bar{\chi}}}{8}\left(\frac{\mathrm{d}\ln V}{\mathrm{d}\bar{\psi}}\right)^2. \tag{42}$$

Under our hypothesis (39), the second term

$$\epsilon_\psi \equiv \frac{e^{2\sqrt{\xi}\bar{\chi}}}{8}\left(\frac{\mathrm{d}\ln V}{\mathrm{d}\bar{\psi}}\right)^2 \tag{43}$$

is small, and for $\xi \ll 1$ we recover the condition of slow-roll inflation, $\epsilon_1 \ll 1$. Let us mention that reversing the inequality in our working hypothesis of Eq. (39) is not acceptable as one would get a value larger than unity for ϵ_1 and no inflation at all.

From Eq. (42), we see that the tachyonic growth of ϕ induces corrections to the inflaton dynamics, compared to what one would have obtained in standard GR. The factor $e^{2\sqrt{\xi}\bar{\chi}}$ in Eq. (43) increases with χ and this implies that the term ϵ_ψ will ultimately dominate in Eq. (42). When this happens, the kinetic energy of the ψ field will drive inflation towards its graceful ending, as needed. Let us notice that, even if the first Hubble flow function ϵ_1 has an additional term 8ξ, a more detailed calculation shows that the tensor-to-scalar ratio is given by $r = 16\epsilon_{\psi *}$, which passes current constraints for plateau-like potentials [47,51].

A last comment is in order concerning the very large-scale structure of the Universe generated in this scenario. Although not explicit in the above description, the fact that the inflaton potential $V(\psi)$ should be asymptotically very flat implies that not only ϕ but also ψ is expected to develop large super-Hubble fluctuations. In that situation, the earliest phase of inflation is certainly chaotic, and possibly eternal, depending on the shape of $V(\psi)$ [52–56]. Determining the probability that the chaotic regime ends in a classical evolution matching our scenario is still an open and relevant question, which we leave to future work [22,33].

VI. CONCLUSION

We have proposed a novel scenario where both the Planck scale and dark energy are dynamically generated by the stochastic and tachyonic motion of a weakly nonminimally coupled ultralight scalar field, which alleviates the large hierarchy between the Planck, electroweak, neutrino mass, and cosmological constant scales. According to this scenario, such an ultralight field is still present in the current Universe and mediates a long-range fifth force among bodies. Cosmological observations and Solar System experiments require ξ to be small. The stronger bound comes from the Shapiro effect measured by the Cassini spacecraft and $\xi < \mathcal{O}(10^{-7})$. Generically, all improvements on the bounds of a possible nonminimal coupling in a terrestrial or Solar System environment will be relevant in constraining, or proving, our model [57].

However, we could think of other means to test the scenario. A possible route of detection could be through the cosmological motion of the scalar field, which is not exactly static. The equation-of-state parameter w for dark energy differs from -1 due to the slow motion of the field as

$$w = -1 + \mathcal{O}(\xi). \tag{44}$$

According to Ref. [58], one could expect the future Euclid satellite [59] and SKA radio telescope [60], combined with Planck CMB data, to constrain the deviation of $w+1$ down to 10^{-3}. This will certainly not be enough to reach the current bound $\xi < \mathcal{O}(10^{-7})$ and one may have to wait for the next generation of giant radio telescopes [61]. However, let us remark that as soon as the field ϕ starts to evolve on cosmological scales, the effective gravitational coupling given by $\xi\phi^2$ is also modified. We have not assessed the possible joint constraints from varying dark energy and a varying Newton's constant, but it may be another interesting route to explore.

Recent detections of gravitational waves (GWs) by the LIGO/VIRGO observatory [62] have opened a new era for GW astronomy. In the future, various types of GW detectors will be launched and the physics of the gravity sector will be probed much more widely and deeply. It has been shown in Ref. [63] that it is possible to place an upper limit on the Brans-Dicke parameter $\omega_{\mathrm{BD}} \gtrsim 4\times 10^8$ using the Deci-hertz Interferometer Gravitational wave Observatory (DECIGO), which is a planned space-based GW detector consisting of four constellations of three satellites forming a triangular shape [64]. In the massless limit, the nonminimal coupling parameter is related to ω_{BD} as $\xi = 1/(4\omega_{\mathrm{BD}})$. From the DECIGO limit, we obtain $\xi \lesssim 6\times 10^{-10}$, which is a roughly 2 orders-of-magnitude improvement over the current bound. Hence, there is a window that can be probed by future GW experiments such as DECIGO or LISA [65].

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