Rapid Communications | Editors' Suggestion

Dark matter decaying in the late Universe can relieve the H_0 tension

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(Received 21 March 2019; published 10 June 2019)

We study the cosmological effects of two-body dark matter decays in which the products of the decay include a massless and a massive particle. We show that if the massive daughter particle is slightly warm it is possible to relieve the tension between distance ladder measurements of the present-day Hubble parameter with measurements from the cosmic microwave background.

DOI: 10.1103/PhysRevD.99.121302

In the standard ΛCDM cosmological model, the expansion history of the Universe is driven by the presence of dark matter and dark energy. Dark matter dominates the growth of structure and deceleration at early times, while dark energy is responsible for the accelerated expansion of the Universe at the present epoch. Numerous experimental probes independently confirm the predictions of this model over many cosmological scales and epochs [1].

Despite these successes, there are hints for possible caveats to the basic assumptions of ΛCDM. The potential discrepancies between different experimental probes seem to remain, despite ongoing efforts to understand known or control the presence of unknown systematics. The most prominent is the discrepancy between the value of the Hubble parameter at the present time H_0 as it is inferred from cosmic microwave background (CMB) measurements [1] and the direct measurement in the local Universe using supenovae type Ia (SNIa) [2].

The discrepancy between the CMB measurement of H_0 and the distance ladder estimates from SNIa has evolved in the last few years from 2.5σ [3] to 3.6σ [2].[1] More recently, the Dark Energy Survey [5] found H_0 to be consistent with the current measurement from the CMB [1]. An independent measurement of H_0 [6] was recently made possible using gravitational waves produced from a binary neutron star merger [7]. However, the uncertainty in this measurement is large (due to the single event) and cannot be used to resolve the tension. Nevertheless, future observations should be able to reduce the uncertainty to the percent level [8–13].

The origin of this discrepancy is unknown. There have been suggestions that systematics are at play; however, those claims were quickly dismissed [14–17]. Alternatively, attempts to relieve the tension focus on modifying either the dark energy equation of state and its dynamics or the dark matter model. For example, a negative cosmological constant model still consistent with the data was investigated in Ref. [18], while dark energy with a dynamical equation of state was considered in Refs. [19,20]. In Ref. [21], the authors showed that a model with a minimally coupled and slowly or moderately rolling quintessence field cannot alleviate the discrepancy, while a more general approach was taken in Ref. [22], in which multiple models of dark energy were considered. Other proposals have been based on an early period of dark energy domination [23] or vacuum phase transitions [24–26].

On the other hand, modifications to the dark matter sector in order to resolve the discrepancy include partially acoustic dark matter models [27], charged dark matter with chiral photons [28], dissipative dark matter models [29], cannibal dark matter [30], and axions [31]. Decaying dark matter models were also considered in combination with solving other problems [32–39]. Finally, models of interacting dark matter–dark energy [40–42] as well as modifications of the general relativity theory [43–45] were discussed.

Here, we concentrate on the dark matter component of the cosmological model. Instead of a simple cold fluid, we allow dark matter to decay into multiple components. Such a model must account for a proper treatment of the cosmological evolution of the massive decaying products [46].

We focus on two-body decays, of the form $\psi \to \chi + \gamma'$, i.e., a parent particle decays to a massless photonlike particle (e.g., a dark photon [47–49]—we denote this particle as γ' in order to distinguish from the electromagnetic photon in the standard model) and a massive daughter particle. Below, we label the massive parent particle with a

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[1]Recently, we learned of an improved SNIa determination of H_0 (using updated Cepheids calibration in the Large Magellanic Cloud) that increases the significance to 4.4σ [4].

subscript 0, the massless daughter with subscript 1, and the massive daughter particle with subscript 2. Such decays have been proposed in the literature in the context of superweakly interacting massive particles (super WIMPs) [50] or decays that can explain several observables in the late Universe [51]. In addition, it has been recently shown that small deviations from a completely cold dark matter could be present in the late Universe [52].

The proposed dark matter decays are modeled using two free parameters, namely, the lifetime $\tau = 1/\Gamma$ (where Γ is the associated decay rate) and the fraction of rest mass energy of the parent particle transferred to the massless particle, ϵ. In this scenario, the 4-momenta of the three particles involved are given by $p_{\mu,0} = (m_0, \mathbf{0})$, $p_{\mu,1} = (\epsilon m_0, \mathbf{p}_1)$, $p_{\mu,2} = ([1-\epsilon]m_0, \mathbf{p}_2)$, and the equations that govern the cosmological evolution of the massive parent and the massless daughter particles are

$$\frac{d\rho_0}{dt} + 3\frac{\dot{a}}{a}\rho_0 = -\Gamma\rho_0, \qquad \frac{d\rho_1}{dt} + 4\frac{\dot{a}}{a}\rho_1 = \epsilon\Gamma\rho_0, \quad (1)$$

where a is the scale factor and we assume no decays occurred prior to the redshift of recombination $z_{\rm rec} \approx 1090$ [1]. We assign initial conditions of the dark matter density at recombination, $\rho_0(a_{\rm rec}) = \rho_{\rm crit}\Omega_{\rm DM}a_{\rm rec}^{-3}$, for any given set of ΛCDM values of $\Omega_{\rm DM}$ and H_0.

The evolution of the massive daughter particle is more complex for two reasons. First, this particle has a dynamic equation of state $w_2(a)$. It is possible for it to be born relativistic at some early time $a_D < 1$ (when the expansion rate is given by H_D) but behave like matter as the Universe evolves. Second, at any time, the collective behavior of these particles needs to be averaged over all particles that were born prior to that interval. This means the redshift evolution of the energy density of the massive daughter particle depends on the sum of all contributions of particles born during the interval $1 \geq a \geq a_D$, some of which were born relativistic and redshifted away by $a = 1$ and some of which were born at late times but had no time to be redshifted. This collective behavior can be expressed as (for details, see Sec. II in Ref. [46])

$$\rho_2(a) = \frac{\mathcal{C}}{a^3}\int_{a_*}^{a}\frac{e^{-\Gamma t(a_D)}}{a_D H_D}\left[\frac{\beta_2^2}{1-\beta_2^2}\left(\frac{a_D}{a}\right)^2 + 1\right]^{1/2} da_D, \quad (2)$$

where $\beta_2 \equiv v_2/c = \epsilon/(1-\epsilon)$ and the constant $\mathcal{C}$ is obtained from the initial conditions [46] as $\mathcal{C} = \rho_{\rm crit}\Omega_{\rm DM}\Gamma \exp[\Gamma t(a_{\rm rec})]\sqrt{1-2\epsilon}$.

It is important to emphasize that this approach is different from the models considered in Refs. [34,53], in which the assumption was that dark matter decays only to radiation; thus, previously derived constraints and conclusions do not apply here.

We can understand qualitatively the effects of a decay in this scenario in the following way. For a fixed dark matter density $\Omega_{\rm DM}$ and lifetime τ, increasing ϵ lowers the value of $H(z=0)$ at low redshifts as more nonrelativistic energy density is transferred to radiation of which the energy density is diluted at a faster rate as the Universe expands. On the other hand, keeping the value of ϵ constant and decreasing the lifetime of the parent particle τ shifts the matter–dark energy equality to earlier times and also decreases the value of $H(z=0)$ as more dark matter will decay by the present epoch. Therefore, a combination of ϵ and τ could bring the measured value of the expansion rate at $z = 0$ in agreement with the evolution of $H(z)$ at higher redshifts as measured at recombination.

Given the above considerations, we use the Friedmann equation for a flat geometry,

$$H^2(a) \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\sum_i \rho_i(a), \quad (3)$$

where

$$\sum_i \rho_i(a) = \rho_0(a) + \rho_1(a) + \rho_2(a) + \rho_r(a) + \rho_\nu(a) + \rho_b(a) + \rho_\Lambda, \quad (4)$$

and perform a Monte Carlo Markov chain (MCMC) analysis of a decaying dark matter cosmology under the assumption that no decays have taken place prior to recombination (in other words, the Universe at recombination is correctly described by CMB measurements [1], and $\tau \gg 400\,000$ yr).

We allow four free parameters, τ, ϵ, $\Omega_{\rm DM}$, and $h = H_0/(100\ {\rm km/s\,Mpc^{-1}})$. We assume flat logarithmic priors for the dark matter decaying parameters in the ranges $-4 \leq \log_{10}\epsilon < \log_{10}(0.5)$, $-3 \leq \log_{10}\tau \leq 4$, and flat priors in the ranges $0 \leq \Omega_{\rm DM} \leq 1$ and $0.5 \leq h \leq 1$ for the matter density and the hubble parameter. A sample of $\Omega_{\rm DM}$ and h (thus $\rho_{\rm crit}$) give the initial conditions for the dark matter energy density at recombination (scale factor $a_{\rm rec}$) as $\rho_0 = \Omega_{\rm DM}\rho_{\rm crit}a_{\rm rec}^{-3}$ and $\rho_1 = \rho_2 = 0$ (taking $a_{\rm rec} = 1/(1+z_{\rm rec})$, where $z_{\rm rec} \approx 1090$ [1]). Note that the sampled choice of h (in setting the initial conditions) is not the same as the value of $H(z=0)$ derived *after* evolving the Universe from recombination to $z = 0$. For the remainder of the species, we use the Planck TT,TE,EE +lowE+lensing model to obtain their densities at recombination with the exception of dark energy, which is chosen by enforcing flatness. To take into account the neutrino mass, we follow Sec. 3.3 of Ref. [54].

The aforementioned procedure guarantees the Universe behaves like ΛCDM at recombination and sets the initial conditions from which we then solve Eq. (3) [with Eqs. (1) and (2)] from $a = a_{\rm rec}$ to $a = 1$. We run the MCMC analysis against the following late Universe measurements of $H(z)$: the distance-ladder Hubble measurement [2], the baryon acoustic oscillation (BAO) measurements from BOSS DR12 [55] and BOSS DR14 [56] quasars, as well as

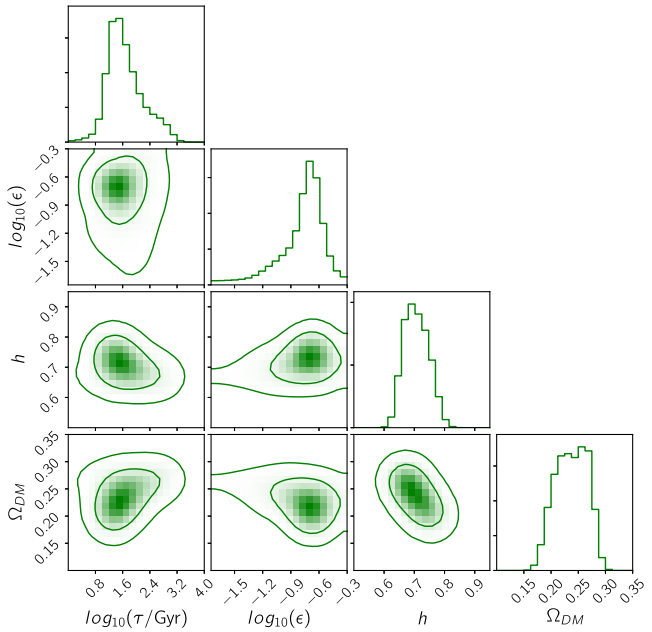

FIG. 1. Results of the MCMC analysis of the decaying dark matter scenario with energy fraction ϵ and decay time τ using the late Universe data together with effective data points between $z = 3$ and 1090. The Hubble parameter h and matter density Ω_{DM} represent the values used in the ΛCDM Universe that sets the initial conditions for the decaying dark matter model and not the values obtained in the latter (see the text for details).

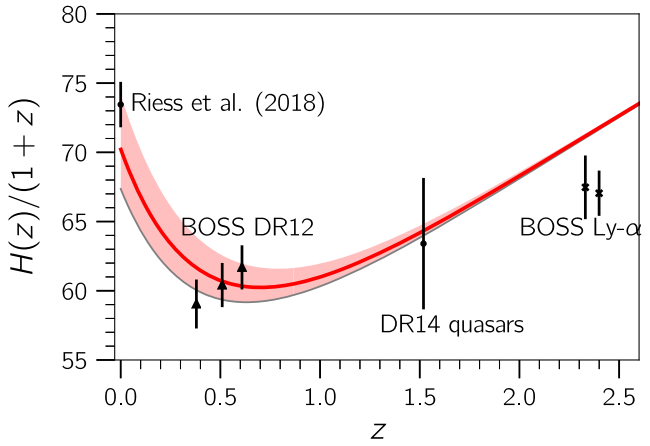

FIG. 2. The evolution of the Hubble parameter as a function of redshift for ΛCDM (thin gray line) and the proposed decaying dark matter scenario. Thick red line depicts the median, while the red shaded area represents the 68% allowed interval. The units of the y axis are km s^{-1} Mpc^{-1}. The decaying dark matter scenario proposed here can ease the tension between Planck18 [1] and the SHOES measurement of H_0 [2] while matching the Planck18 ΛCDM Universe at high redshifts.

the Ly-α auto-correlation function at z = 2.33 [57] and the joint constraint from the Ly-α autocorrelation and cross-correlation function at z = 2.4 [58].

The result of this analysis is shown in Fig. 1. The inner and outer contours correspond to the 68th and 96th percentiles of the projected two-dimensional space of each panel. As expected, h and Ω_{DM} are anticorrelated, in agreement with the earlier analysis of Ref. [59]. This is to be expected as increasing h requires the Universe to expand faster at late times, which means the matter–dark energy equality must move to earlier times, which necessitates a lower value of Ω_{DM}. This effect can also be seen as the correlation between the lifetime of the particle τ, Ω_{DM}, and h—for smaller lifetimes and a larger fraction energy transferred to radiation, h and Ω_{DM} need to adjust accordingly by increasing and lowering their values, respectively, to maintain agreement with the data. In addition, for lower values of τ, higher values of ϵ are required, implying that more energy is needed to be transferred from matter to radiation, and thus the deviation from ΛCDM occurs over a shorter period of time. Note that h and Ω_{DM} in Fig. 1 are not the derived values of the Hubble expansion rate and matter density obtained from the evolved decaying dark matter scenario but the sampled values that are used to set the initial conditions (i.e., the values used to obtain the dark matter density at recombination).

The elongation of the 96th percentile contour of ϵ is due to the fact the data used seem to be fitted well by Planck's ΛCDM model with the only exception being the distance-ladder measurement of the Hubble parameter [2], and therefore this extension to very small values of ϵ encapsulates the tendency of the allowed parameter space to approach ΛCDM (a Universe with $\epsilon = 0$ is identical to ΛCDM).

In Fig. 2, we show the redshift evolution of the Hubble parameter. At early times (prior to the onset of decays), the Universe behaves identically to the ΛCDM Universe inferred from CMB measurements. At late times, decays (and the resultant transfer of energy from matter to radiation) speed up the expansion and result in a late Universe measurement of h that is higher than the one obtained from the CMB under ΛCDM, thus potentially alleviating the tension.

The derived 68% confidence limits for each one of the free parameters are shown in Table I, which is the main result of this paper—a two-body decay with a relatively high value of ϵ and a long lifetime (significantly longer than the age of the Universe) can relieve the tension between measurements of the present expansion rate measured from the CMB and the local Universe. These results are in agreement with the lower limits on the lifetime set by the analysis of a similar model against the BOSS BAO measurements [60] as well as the joint analysis of CMB, BAOs, and SNIa in Ref. [59]. The model works because it allows for a fraction of the rest mass energy of the parent

TABLE I. 68% confidence limits.

$\log_{10}\epsilon$	$\log_{10}(\tau/\mathrm{Gyr})$	Ω_{DM}	h
$-0.78^{+0.14}_{-2.10}$	$1.55^{+0.63}_{-0.25}$	$0.24^{+0.03}_{-0.03}$	$0.70^{+0.04}_{-0.03}$

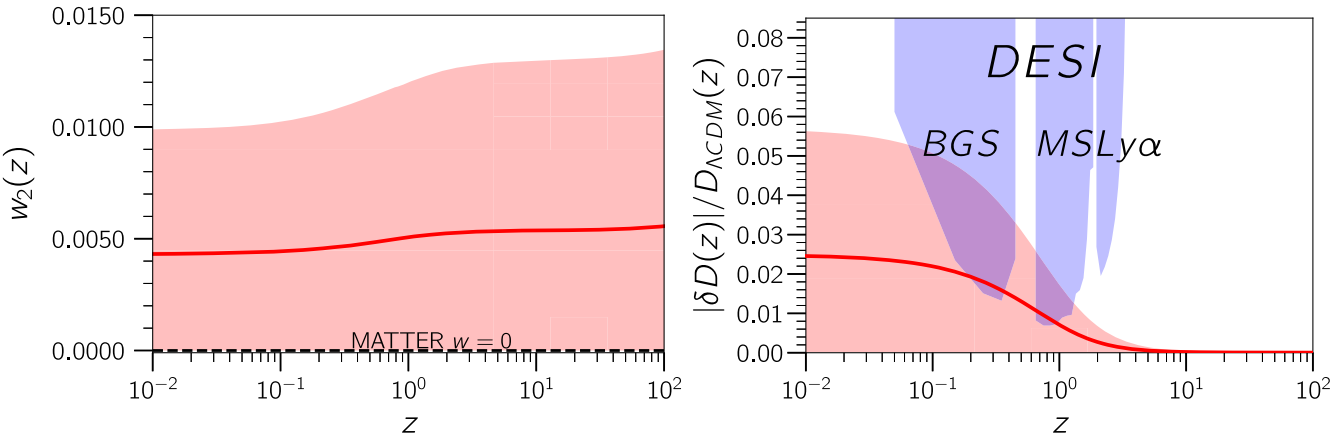

FIG. 3. Left: The evolution of the equation of state w_2 of the massive daughter particle as a function of redshift. The shaded area represents the 68th percentile region. Right: Ratio of the difference for the linear growth factor between the decaying dark matter scenario proposed here and ΛCDM. Future surveys such as the Dark Energy Spectroscopic Instrument (DESI) [68] will be able to test the proposed decaying dark matter model. At lower redshifts, DESI will constrain the growth factor with the Bright Galaxy Survey, around $z \approx 1$, with the Main Survey and at higher redshifts with the Lyman α survey (Ly-α) [68].

particle to go into radiation, with the remainder going to the speed of the massive daughter particle. The left panel in Fig. 2 shows the evolution of the equation of state of the massive daughter particle as a function of redshift, which shows that it evolves as $\rho_2(a) \propto a^{-3.15}$, as expected from a particle that is not fully relativistic or fully cold.

Note that part of the allowed parameter space (lifetimes less than or equal to 40 Gyr) has already been constrained by numerical simulations of decaying dark matter together with the Sloan Digital Sky Survey (SDSS) Lyman-α power spectrum [61]. These constraints are obtained by comparing dark matter–only simulations together with the fluctuation Gunn-Peterson approximation [62] against the SDSS Lyman-α power spectrum [63,64] (see, however, the difficulties in understanding the systematics relevant to damped Lyman-α systems [65]). However, the analysis presented here is not simply an estimate of the lifetime of the dark matter parent particle but a combination of the four free parameters that can alleviate the H_0 tension—in other words, the preferred value of the lifetime of the particle may not correspond to the largest change in the value of H_0.

The allowed parameter space encompasses dark matter decays that can alleviate some of the small-scale problems that exist in the context of galaxy formation [66] as well as the amplitude of linear fluctuations [67]. For example, a dark matter particle with a long lifetime (approximately 150 Gyr) and a value of $\epsilon \sim 10^{-2}$ provides a boost of $\gamma_2 = [1-\epsilon^2/(1-\epsilon)^2]^{-1/2} \approx 1.00005$, a characteristic velocity dispersion of order $\mathcal{O}(10^3)$ km/s, and a fraction of dark matter that has already decayed by few percent.

Dark matter decays will also affect the growth of structure. We can quantify the effects of decays on structure by calculating the linear growth factor $D(z)$ as the growing mode solution to the differential equation that governs the linear evolution of matter perturbations,

$$\frac{d^2D}{da^2} + \left(\frac{d\ln H}{da} + \frac{3}{a}\right)\frac{dD}{da} - \frac{4\pi G\rho_m}{a^2} = 0, \tag{5}$$

where $\rho_m = \rho_0 + \rho_2 + \rho_b$ and D is normalized to unity today. Here, we assume that the massive daughter contributes to the matter content of the Universe; in reality (as we showed above), this is not entirely correct as the particle is warm, and therefore the derived result is a conservative upper bound to the effect. The right panel in Fig. 3 shows the deviation of the linear growth factor from the standard Λ cosmology. Future surveys, such as DESI [68] will be able to test the predictions of this decaying dark matter scenario (see also Ref. [69]).

Finally, we can put the derived constraints on τ and ϵ in the context of particle physics models that include super WIMPs or exited dark fermions that can decay to a lighter fermion and a photon via a magnetic dipole transition [50,51]. In general, the rate will be given by $\Gamma \sim \delta m^3/\Lambda^2$, where Λ is some high scale. The mass difference can be obtained from the kinematics of this two-point decay and is given by $\delta m = 1 - \sqrt{(1-2\epsilon)}$ in units of the parent particle. Therefore, a lower limit of $\epsilon \approx 0.17$ and a lifetime of the parent particle of $\tau \approx 20$ Gyr implies that for a giga electron volt–scale particle $\delta m \approx 180$ MeV with $\Lambda \approx 10^{16}$ GeV.

In summary, we have shown that dark matter decays of the form of $\psi \to \chi + \gamma'$ can potentially relieve the H_0 tension between the value obtained by the local measurements using the distance ladder and the value obtained by observations of the CMB. Further analysis of the related effects on large-scale structure formation and the CMB power spectrum is required to confirm to extent to which decays can solve the H_0 problem.

ACKNOWLEDGMENTS

We acknowledge useful conversations with Manuel Buen-Abad, Jatan Busch, Ian Dell'Antonio, JiJi Fan,

John Leung, David Pinner, Vivian Poulin, Robert Sims, Michael Turner, Terry Walker, and Andrew Zentner. K. V. and S. M. K. are supported by U. S. Department of Energy (DE-SC0017993). A. L. was supported in part by a John Templeton Foundation grant awarded to the Black Hole Initiative at Harvard University.

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