Direct CP violation for $\bar{B}_s^0 \to \phi\pi^+\pi^-$ in perturbative QCD

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In the perturbative QCD approach, we study the direct CP violation in the decay of $\bar{B}_s^0 \to \rho(\omega)\phi \to \pi^+\pi^-\phi$ via isospin symmetry breaking. An interesting mechanism involving the charge symmetry violating between ρ and ω is applied to enlarge the CP violating asymmetry. We find that the CP violation can be enhanced by the ρ-ω mixing mechanism when the invariant masses of the $\pi^+\pi^-$ pairs are in the vicinity of the ω resonance. For the decay process of $\bar{B}_s^0 \to \rho(\omega)\phi \to \pi^+\pi^-\phi$, the maximum CP violation can reach 5.98%. The possibility of detecting the CP violation is also presented.

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I. INTRODUCTION

CP violation has obtained extensive attention even since its first discovery in the neutral kaon systems [1]. Within the standard model, CP violation originates from a nonzero weak phase angle from the complex Cabibbo-Kobayashi-Maskawa (CKM) matrix, which describes the mixing of weak interaction and mass eigenstates of the quarks [2,3]. Although the source of CP violation has not been well understood up to now, physicists are striving to increase their knowledge of the mechanism for the CP violation. Many theoretical studies [4–6] (within and beyond the standard model) and experimental investigations have been conducted since 1964. According to theoretical predictions, large CP violation may be expected in B meson decay process due to the large mass of b quarks. In recent years, the LHCb Collaboration observed the large CP violation in the three-body decay channels of $B^\pm \to \pi^\pm\pi^+\pi^-$ and $B^\pm \to K^\pm\pi^+\pi^-$ [7–9]. Hence, more attention has been focused on the nonleptonic B meson three-body decays channels in searching for CP violation.

Direct CP violation in the B meson decay process occurs through the interference of at least two amplitudes with a different weak phase ϕ and strong phase δ. The weak phase difference ϕ is determined by the CKM matrix elements, while the strong phase can be produced by the hadronic matrix elements and interference between the intermediate states. However, one can know that the strong phase δ is not still well determined from the theoretical approach. The nonleptonic weak decay amplitudes of the B meson involve the hadronic matrix elements of $\langle M_1 M_2|O_i|B\rangle$, which can be calculated from the different factorization methods. However, the different methods may present different strong phases so as to affect the value of the CP violation. Currently, there are three popular theoretical approaches to study the dynamics of the two-body hadronic decays, which are the naive factorization approach [10–13], the QCD factorization [14–18], perturbative QCD (pQCD) [19–21], and soft-collinear effective theory [22–24]. Based on the power expansion in $1/m_b$ (m_b is the b-quark mass), all of the theories of factorization are shown to deal with the hadronic matrix elements in the leading power of $1/m_b$. However, these methods pertain to whether one takes into account the collinear degrees of freedom or the transverse momenta. Meanwhile, in order to have a large signal of CP violation, we need to appeal to some phenomenological mechanism to obtain a large strong phase δ. ρ-ω mixing has been used for this purpose in the past few years and focuses on the naive factorization and QCD factorization approaches [25–30]. Recently, Lü *et al.* attempted to generalize the pQCD approach to the three-body nonleptonic decay via ρ-ω mixing in $B^{0,\pm} \to \pi^{0,\pm}\pi^+\pi^-$ and $B_c \to D^+_{(s)}\pi^+\pi^-$ decays [31,32]. In this paper, we will focus on the CP violation of the decay process $\bar{B}_s^0 \to \rho(\omega)\phi \to \pi^+\pi^-\phi$ via ρ-ω mixing in the pQCD approach.

Isospin symmetry breaking plays a significant role in the ρ-ω mixing mechanism. The mixing between the u and d flavors leads to the breaking of isospin symmetry for the ρ-ω system [33,34]. In Refs. [35,36], the authors studied the ρ-ω mixing and the pion form factor in the timelike

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region, where ρ-ω mixing was used to obtain the (effective) mixing matrix element $\tilde{\Pi}_{\rho\omega}(s)$, which consists of two part contributions: one from the direct coupling of $\omega \to 2\pi$ and the other from $\omega \to \rho \to 2\pi$ mixing [37–39]. The magnitude has been determined by the pion form factor through the data for the cross section of $e^+e^- \to \pi^+\pi^-$ in the ρ and ω resonance region [36,39–42]. Recently, isospin symmetry breaking was discussed by incorporating the vector meson dominance model in the weak decay process of the meson [27,32,43–45]. However, one can find that ρ-ω mixing produces the large CP violation from the effect of isospin symmetry breaking in the three- and four-body decay processes. Hence, in this paper, we shall follow the method of Refs. [27,32,43–45] to investigate the decay process of $\bar{B}_s^0 \to \rho(\omega)\phi \to \pi^+\pi^-\phi$ via isospin symmetry breaking.

The remainder of this paper is organized as follows. In Sec. II we will briefly introduce the pQCD framework and present the form of the effective Hamiltonian and wave functions. In Sec. III we give the calculating formalism and details of the CP violation from ρ-ω mixing in the decay process $\bar{B}_s^0 \to \rho(\omega)\phi \to \pi^+\pi^-\phi$. In Sec. IV we show the input parameters. We present the numerical results in Sec. V. Summary and discussion are included in Sec. VI. The related function defined in the text are given in the Appendix.

II. THE FRAMEWORK

For the decay process of $\bar{B}_s \to M_2M_3$, integrated over the longitudinal and the transverse momenta, the emitted or annihilated particle M_2 can be factored out. The rest of the amplitude can be expressed as the convolution of the wave functions ϕ_{B_s}, ϕ_{M_3} and the hard scattering kernel T_H. The pQCD factorization theorem has been developed for nonleptonic heavy meson decays, based on the formalism of Lepage, Brodsky, Botts, and Sterman [46–49]. The basic idea of the pQCD approach is that it takes into account the transverse momentum of the valence quarks in the hadrons which results in the Sudakov factor in the decay amplitude. Then, it is conceptually written as the following:

$$\text{Amplitude} \sim \int d^4k_1 d^4k_2 d^4k_3 \mathrm{Tr}[C(t)\phi_{B_s}(k_1)\phi_{M2}(k_2) \times \phi_{M3}(k_3)T_H(k_1,k_2,k_3,t)], \quad (1)$$

where k_i ($i = 1, 2, 3$) are momenta of light quarks in the mesons. Tr denotes the trace over Dirac and color indices. $C(t)$ is the Wilson coefficient which comes from the radiative corrections at short distance. ϕ_M ($m = 2, 3$) is teh wave function which describes the nonperturbative contribution during the hadronization of mesons, which should be universal and channel independent. The hard part T_H is rather process dependent.

With the operator product expansion, the effective weak Hamiltonian in bottom hadron decays is [50]

$$\mathcal{H}_{\mathrm{eff}} = \frac{G_F}{\sqrt{2}}\left\{V_{ub}V_{us}^*[C_1(\mu)Q_1^u(\mu) + C_2(\mu)Q_2^u(\mu)] - V_{tb}V_{ts}^*\left[\sum_{i=3}^{10} C_i(\mu)Q_i(\mu)\right]\right\} + \text{H.c.}, \quad (2)$$

where G_F is the Fermi constant, $C_i(\mu)$ (i = 1, …, 10) are the Wilson coefficients, and $V_{q_1q_2}$ (q_1 and q_2 represent quarks) is the CKM matrix element. The operators O_i have the following forms:

$$\begin{aligned}
O_1^u &= \bar{s}_\alpha\gamma_\mu(1-\gamma_5)u_\beta\bar{u}_\beta\gamma^\mu(1-\gamma_5)b_\alpha,\\
O_2^u &= \bar{s}\gamma_\mu(1-\gamma_5)u\bar{u}\gamma^\mu(1-\gamma_5)b,\\
O_3 &= \bar{s}\gamma_\mu(1-\gamma_5)b\sum_{q'}\bar{q}'\gamma^\mu(1-\gamma_5)q',\\
O_4 &= \bar{s}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{q'}\bar{q}'_\beta\gamma^\mu(1-\gamma_5)q'_\alpha,\\
O_5 &= \bar{s}\gamma_\mu(1-\gamma_5)b\sum_{q'}\bar{q}'\gamma^\mu(1+\gamma_5)q',\\
O_6 &= \bar{s}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{q'}\bar{q}'_\beta\gamma^\mu(1+\gamma_5)q'_\alpha,\\
O_7 &= \frac{3}{2}\bar{s}\gamma_\mu(1-\gamma_5)b\sum_{q'}e_{q'}\bar{q}'\gamma^\mu(1+\gamma_5)q',\\
O_8 &= \frac{3}{2}\bar{s}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{q'}e_{q'}\bar{q}'_\beta\gamma^\mu(1+\gamma_5)q'_\alpha,\\
O_9 &= \frac{3}{2}\bar{s}\gamma_\mu(1-\gamma_5)b\sum_{q'}e_{q'}\bar{q}'\gamma^\mu(1-\gamma_5)q',\\
O_{10} &= \frac{3}{2}\bar{s}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{q'}e_{q'}\bar{q}'_\beta\gamma^\mu(1-\gamma_5)q'_\alpha, \quad (3)
\end{aligned}$$

where α and β are color indices, the sum index q' runs over the "active" flavors quarks at the scale m_b, and $e_{q'}$ is the electric charge of the quark q' ($q' = u$, d, s, c or b quarks). In Eq. (3) O_1^u and O_2^u are tree operators, O_3–O_6 are QCD penguin operators, and O_7–O_{10} are the operators associated with electroweak penguin diagrams.

The Wilson coefficients, $C_i(\mu)$, represent the power contributions from scales higher than μ (which refer to the long-distance contributions) [51]. Since the QCD has the property of asymptotic freedom, they can be calculated in perturbation theory. The Wilson coefficients include the contributions of all heavy particles, such as the top quark, the $W^\pm$ bosons, and so on. Usually, the scale μ is chosen to be of order $\mathcal{O}(m_b)$ for B meson decays. Since we work in the leading order of perturbative QCD ($O(\alpha_s)$), it is consistent to use the leading order Wilson coefficients. So, we use numerical values of $C_i(m_b)$ as follows [19,21]:

$$\begin{aligned} &C_1 = -0.2703, \qquad C_2 = 1.1188,\\ &C_3 = 0.0126, \qquad C_4 = -0.0270,\\ &C_5 = 0.0085, \qquad C_6 = -0.0326,\\ &C_7 = 0.0011, \qquad C_8 = 0.0004,\\ &C_9 = -0.0090, \qquad C_{10} = 0.0022. \end{aligned} \tag{4}$$

The Wilson coefficients a_1–a_{10} are defined as usual [52–55]:

$$\begin{aligned} &a_1 = C_2 + C_1/3, \qquad a_2 = C_1 + C_2/3,\\ &a_3 = C_3 + C_4/3, \qquad a_4 = C_4 + C_3/3,\\ &a_5 = C_5 + C_6/3, \qquad a_6 = C_6 + C_5/3,\\ &a_7 = C_7 + C_8/3, \qquad a_8 = C_8 + C_7/3,\\ &a_9 = C_9 + C_{10}/3, \qquad a_{10} = C_{10} + C_9/3. \end{aligned} \tag{5}$$

For the decay channel of $\bar{B}_s^0 \to M_2 M_3$, we denote the emitted meson as M_2 while the recoiling meson is M_3. The M_2 (ρ or ω) and the final state M_3 (ϕ) move along the direction of $n_+ = (1, 0, \mathbf{0}_T)$ and $n_- = (0, 1, \mathbf{0}_T)$ in the light-cone coordinates, respectively. We denote the ratios $r_\phi = \frac{M_\phi}{M_{B_s}}$, $r_\rho = \frac{M_\rho}{M_{B_s}}$, and $r_\omega = \frac{M_\omega}{M_{B_s}}$. In the limit M_ϕ, M_ρ, $M_\omega \to 0$, one can drop the terms of proportional to r_ϕ^2, r_ρ^2, r_ω^2 safely. The symbols P_B, P_2, and P_3 refer to the $\bar{B}_s$ meson momentum, the $\rho(\omega)$ meson momentum, and the final-state ϕ momentum, respectively. Under the above approximation, the momenta can be written as

$$P_B = \frac{M_{B_s}}{\sqrt{2}}(1, 1, \mathbf{0}_T), \qquad P_2 = \frac{M_{B_s}}{\sqrt{2}}(1, 0, \mathbf{0}_T),$$
$$P_3 = \frac{M_{B_s}}{\sqrt{2}}(0, 1, \mathbf{0}_T). \tag{6}$$

One can denote the light (anti-)quark momenta k_1, k_2, and k_3 for the mesons B_s, $\rho(\omega)$, and ϕ, respectively. We can write

$$k_1 = \left(x_1 \frac{M_{B_s}}{\sqrt{2}}, 0, \mathbf{k}_{1\perp}\right), \qquad k_2 = \left(x_2 \frac{M_{B_s}}{\sqrt{2}}, 0, \mathbf{k}_{2\perp}\right),$$
$$k_3 = \left(0, x_3 \frac{M_{B_s}}{\sqrt{2}}, \mathbf{k}_{3\perp}\right), \tag{7}$$

where x_1, x_2,and x_3 are the momentum fraction. $\mathbf{k}_{1\perp}$, $\mathbf{k}_{2\perp}$, and $\mathbf{k}_{3\perp}$ refer to the transverse momentum of the quark, respectively. The longitudinal polarization vectors of the $\rho(\omega)$ and ϕ are given as

$$\epsilon_2(L) = \frac{P_2}{M_{\rho(\omega)}} - \frac{M_{\rho(\omega)}}{P_2 \cdot n_-} n_-, \quad \epsilon_3(L) = \frac{P_3}{M_\phi} - \frac{M_\phi}{P_3 \cdot n_+} n_+, \tag{8}$$

which satisfy the orthogonality relationship of $\epsilon_2(L) \cdot P_2 = \epsilon_3(L) \cdot P_3 = 0$, and the normalization of $\epsilon_2^2(L) = \epsilon_3^2(L) = -1$. The transverse polarization vectors can be adopted directly as

$$\epsilon_2(T) = (0, 0, \mathbf{1}_T), \qquad \epsilon_3(T) = (0, 0, \mathbf{1}_T). \tag{9}$$

The wave function of the B_s meson can be expressed as

$$\phi_{B_s} = \frac{i}{\sqrt{6}}(\not{P}_{B_s} + M_{B_s})\gamma_5 \phi_{B_s}(k_1), \tag{10}$$

where the distribution amplitude ϕ_{B_s} is shown in Refs. [56–58]:

$$\phi_{B_s}(x, b) = N_{B_s} x^2 (1-x)^2 \exp\left[-\frac{M_{B_s}^2 x^2}{2\omega_b^2} - \frac{1}{2}(\omega_b b)^2\right]. \tag{11}$$

The shape parameter ω_b is a free parameter. Based on lattice QCD and the light-cone sum rule [59], we take $\omega_b = 0.50$ GeV for the B_s meson. The normalization factor N_{B_s} depends on the values of ω_b and the decay constant f_{B_s}, which is defined through the normalization relation $\int_0^1 dx \phi_{B_s}(x, 0) = f_{B_s}/(2\sqrt{6})$.

The distribution amplitudes of the vector meson ($V = \rho$, ω or ϕ), ϕ_V, ϕ_V^T, ϕ_V^t, ϕ_V^s, ϕ_V^v, and ϕ_V^a, are calculated using the light-cone QCD sum rule [60,61]:

$$\phi_\rho(x) = \frac{3f_\rho}{\sqrt{6}} x(1-x)[1 + 0.15 C_2^{3/2}(t)], \tag{12}$$

$$\phi_\omega(x) = \frac{3f_\omega}{\sqrt{6}} x(1-x)[1 + 0.15 C_2^{3/2}(t)], \tag{13}$$

$$\phi_\phi(x) = \frac{3f_\phi}{\sqrt{6}} x(1-x)[1 + 0.18 C_2^{3/2}(t)], \tag{14}$$

$$\phi_V^T(x) = \frac{3f_V^T}{\sqrt{6}} x(1-x)[1 + 0.14 C_2^{3/2}(t)], \tag{15}$$

$$\phi_V^t(x) = \frac{3f_V^T}{2\sqrt{6}} t^2, \tag{16}$$

$$\phi_V^s(x) = \frac{3f_V^T}{2\sqrt{6}}(-t), \tag{17}$$

$$\phi_V^v(x) = \frac{3f_V}{8\sqrt{6}}(1 + t^2), \tag{18}$$

$$\phi_V^a(x) = \frac{3f_V}{4\sqrt{6}}(-t), \tag{19}$$

where $t = 2x - 1$. Here f_V is the decay constant of the vector meson with longitudinal polarization. The Gegenbauer polynomials $C_n^{\nu}(t)$ can be found easily in Refs. [62,63].

III. *CP* VIOLATION IN $\bar{B}_s^0 \to \rho(\omega)\phi \to \pi^+\pi^-\phi$ DECAY PROCESS

A. Formalism

The hadronic decay rate for the process of $\bar{B}_s^0 \to \rho(\omega)\phi$ is written as

$$\Gamma = \frac{P_c}{8\pi M_{B_s}^2} \sum_{\sigma=L,T} A^{(\sigma)\dagger} A^{(\sigma)}, \tag{20}$$

where $P_c = |P_{2z}| = |P_{3z}|$ is the momentum of the vector meson. The superscript σ denotes the helicity states of the two vector mesons with the longitudinal (transverse) components L (T). The amplitude $A^{(\sigma)}$ is decomposed into [63–65]

$$A^{(\sigma)} = M_{B_s}^2 A_L + M_{B_s}^2 A_N \epsilon_2^*(\sigma = T) \cdot \epsilon_3^*(\sigma = T) + i A_T \epsilon^{\alpha\beta\gamma\rho} \epsilon_{2\alpha}^*(\sigma) \epsilon_{3\beta}^*(\sigma) P_{2\gamma} P_{3\rho}, \tag{21}$$

with the convention $\epsilon^{0123} = 1$. The amplitude A_i [i refer to the three kind of polarizations, longitudinal (L), normal (N), and transverse (T)] can be written as

$$M_{B_s}^2 A_L = a \epsilon_2^*(L) \cdot \epsilon_3^*(L) + \frac{b}{M_2 M_3} \epsilon_2^*(L) \cdot P_3 \epsilon_3^*(L) \cdot P_2,$$
$$M_{B_s}^2 A_N = a,$$
$$A_T = \frac{c}{M_2 M_3}, \tag{22}$$

where a, b, and c are the Lorentz-invariant amplitudes. M_2, M_3 refer to the masses of the vector mesons $\rho(\omega)$ and ϕ, respectively.

The longitudinal H_0, transverse $H_{\pm}$ of helicity amplitudes can be expressed

$$H_0 = M_{B_s}^2 A_L,$$
$$H_{\pm} = M_{B_s}^2 A_N \mp M_2 M_3 \sqrt{\kappa^2 - 1} A_T, \tag{23}$$

where H_0, H_+, and H_- are the tree-level and penguin-level helicity amplitudes of the decay process $\bar{B}_s^0 \to \rho(\omega)\phi \to \pi^+\pi^-\phi$ from the three kind of polarizations, respectively. The helicity summation satisfy the relation,

$$\sum_{\sigma=L,R} A^{(\sigma)\dagger} A^{(\sigma)} = |H_0|^2 + |H_+|^2 + |H_-|^2. \tag{24}$$

In the vector meson dominance model [66,67], the vacuum polarization of the photons are assumed to be coupled through the vector meson (ρ meson). Based on the same mechanism, ρ-ω mixing was proposed and later gradually applied to B meson physics. The formalism for the CP violation in B hadronic decays can be generalized to B_s in a straightforward manner [25,27,43]. According to the effective Hamiltonian, the amplitude A ($\bar{A}$) for the decay process of $\bar{B}_s^0 \to \pi^+\pi^-\phi$ ($B_s^0 \to \pi^+\pi^-\bar{\phi}$) can be written as [43]

$$A = \langle \pi^+\pi^-\phi | H^T | \bar{B}_s^0 \rangle + \langle \pi^+\pi^-\phi | H^P | \bar{B}_s^0 \rangle, \tag{25}$$

$$\bar{A} = \langle \pi^+\pi^-\bar{\phi} | H^T | B_s^0 \rangle + \langle \pi^+\pi^-\bar{\phi} | H^P | B_s^0 \rangle, \tag{26}$$

where H^T and H^P are the Hamiltonian for the tree and penguin operators, respectively.

The relative magnitude and phases between the tree and penguin operator contribution are defined as follows:

$$A = \langle \pi^+\pi^-\phi | H^T | \bar{B}_s^0 \rangle [1 + r e^{i(\delta+\phi)}], \tag{27}$$

$$\bar{A} = \langle \pi^+\pi^-\bar{\phi} | H^T | B_s^0 \rangle [1 + r e^{i(\delta-\phi)}], \tag{28}$$

where δ and ϕ are strong and weak phases, respectively. The weak phase difference ϕ can be expressed as a combination of the CKM matrix elements, and it is $\phi = \arg[(V_{tb}V_{ts}^*)/(V_{ub}V_{us}^*)]$ for the $b \to s$ transition. The parameter r is the absolute value of the ratio of tree and penguin amplitudes:

$$r \equiv \left| \frac{\langle \pi^+\pi^-\phi | H^P | \bar{B}_s^0 \rangle}{\langle \pi^+\pi^-\phi | H^T | \bar{B}_s^0 \rangle} \right|. \tag{29}$$

The parameter of CP violating asymmetry, A_{CP}, can be written as

$$A_{CP} = \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} = \frac{-2(T_0^2 r_0 \sin\delta_0 + T_+^2 r_+ \sin\delta_+ + T_-^2 r_- \sin\delta_-)\sin\phi}{\sum_{i=0+-} T_i^2 (1 + r_i^2 + 2 r_i \cos\delta_i \cos\phi)}, \tag{30}$$

where $T_i (i = 0, +, -)$ are the tree-level helicity amplitudes of the decay process $\bar{B}_s^0 \to \pi^+\pi^-\phi$ from H_0, H_+, and H_- of Eq. (23), respectively. $r_j (j = 0, +, -)$ refer to the absolute value of the ratio of the tree and penguin amplitudes for the three kind of polarizations, respectively. $\delta_k (k = 0, +, -)$ represent the relative strong phases between the tree and penguin operator contributions from three kinds of helicity amplitudes. We can see explicitly from Eq. (30) that both weak and strong phase differences are responsible for CP violation. In order to obtain a large signal for direct CP violation, we need some mechanism to change either $\sin\delta$ or r. With this mechanism, working at the first order of isospin violation, we have the following results when the invariant mass of $\pi^+\pi^-$ is near the ω resonance mass [26,43]:

$$\langle \pi^+\pi^-\phi|H^T|\bar{B}_s^0\rangle = \frac{g_\rho}{s_\rho s_\omega}\tilde{\Pi}_{\rho\omega}t_\omega^i + \frac{g_\rho}{s_\rho}t_\rho^i, \tag{31}$$

$$\langle \pi^+\pi^-\phi|H^P|\bar{B}_s^0\rangle = \frac{g_\rho}{s_\rho s_\omega}\tilde{\Pi}_{\rho\omega}p_\omega^i + \frac{g_\rho}{s_\rho}p_\rho^i, \tag{32}$$

where $t_\rho^i(p_\rho^i)$ and $t_\omega^i(p_\omega^i)$ are the tree (penguin)-level helicity amplitudes for $\bar{B}_s \to \rho^0\phi$ and $\bar{B}_s \to \omega\phi$, respectively. The amplitudes t_ρ^i, p_ρ^i, t_ω^i, and p_ω^i can be found in Sec. III B. g_ρ is the coupling for $\rho^0 \to \pi^+\pi^-$. $\tilde{\Pi}_{\rho\omega}$ is the effective ρ-ω mixing amplitude which also effectively includes the direct coupling $\omega \to \pi^+\pi^-$ [39]. s_V, m_V, and Γ_V ($V = \rho$ or ω) is the inverse propagator, mass, and decay rate of the vector meson V, respectively. s_V can be expressed as

$$s_V = s - m_V^2 + im_V\Gamma_V, \tag{33}$$

with $\sqrt{s}$ being the invariant masses of the $\pi^+\pi^-$ pairs. The numerical values for the ρ-ω mixing parameter $\tilde{\Pi}_{\rho\omega}(s) = \mathrm{Re}\tilde{\Pi}_{\rho\omega}(m_\omega^2) + \mathrm{Im}\tilde{\Pi}_{\rho\omega}(m_\omega^2)$ are [68]

$$\mathrm{Re}\tilde{\Pi}_{\rho\omega}(m_\omega^2) = -4760 \pm 440\ \mathrm{MeV}^2, \quad \mathrm{Im}\tilde{\Pi}_{\rho\omega}(m_\omega^2) = -6180 \pm 3300\ \mathrm{MeV}^2. \tag{34}$$

From Eqs. (25), (27), (31), and (32) one has

$$re^{i\delta_i}e^{i\phi} = \frac{\tilde{\Pi}_{\rho\omega}p_\omega^i + s_\omega p_\rho^i}{\tilde{\Pi}_{\rho\omega}t_\omega^i + s_\omega t_\rho^i}. \tag{35}$$

Defining [25,69]

$$\frac{p_\omega^i}{t_\rho^i} \equiv r'e^{i(\delta_q^i+\phi)}, \qquad \frac{t_\omega^i}{t_\rho^i} \equiv \alpha e^{i\delta_\alpha^i}, \qquad \frac{p_\rho^i}{p_\omega^i} \equiv \beta e^{i\delta_\beta^i}, \tag{36}$$

where δ_α^i, δ_β^i, and δ_q^i are strong phases form the three kinds of polarizations, respectively. One finds the following expression from Eqs. (35) and (36):

$$re^{i\delta_i} = r'e^{i\delta_q^i}\frac{\tilde{\Pi}_{\rho\omega} + \beta e^{i\delta_\beta^i}s_\omega}{\tilde{\Pi}_{\rho\omega}\alpha e^{i\delta_\alpha^i} + s_\omega}. \tag{37}$$

$\alpha e^{i\delta_\alpha^i}$, $\beta e^{i\delta_\beta^i}$, and $r'e^{i\delta_q^i}$ will be calculated later. In order to obtain the *CP* violating asymmetry in Eq. (30), A_{cp}, $\sin\phi$ and $\cos\phi$ are needed, where ϕ is determined by the CKM matrix elements. In the Wolfenstein parametrization [70], the weak phase ϕ comes from $[V_{tb}V_{ts}^*/V_{ub}V_{us}^*]$. One has

$$\sin\phi = -\frac{\eta}{\sqrt{\rho^2+\eta^2}}, \quad \cos\phi = -\frac{\rho}{\sqrt{\rho^2+\eta^2}}, \tag{38}$$

where the same result has been used for the $b \to s$ transition from Ref. [44].

B. Calculation details

We can decompose the decay amplitude for the decay process $\bar{B}_s^0 \to \rho^0(\omega)\phi$ in terms of tree-level and penguin-level contributions depending on the CKM matrix elements of $V_{ub}V_{us}^*$ and $V_{tb}V_{ts}^*$. From Eqs. (30), (35), and (36), in order to obtain the formulas of the *CP* violation, we need calculate the amplitudes t_ρ, t_ω, p_ρ, and p_ω in the perturbative QCD approach. The F and M functions can be found in the Appendix from the perturbative QCD approach.

There are four types of Feynman diagrams contributing to the $\bar{B}_s^0 \to \rho^0(\omega)\phi$ emission decay mode. The leading order diagrams in the pQCD approach are shown in Fig. 1. The first two diagrams in Fig. 1 [(a),(b)] are called factorizable diagrams and the last two diagrams in Fig. 1 [(c),(d)] are called nonfactorizable diagrams [71,72]. The relevant decay amplitudes can be easily obtained by these hard gluon exchange diagrams and the Lorenz structures of the mesons wave functions. Through calculating these diagrams, the formulas of $\bar{B}_s^0 \to \rho\phi$ or $\bar{B}_s^0 \to \omega\phi$ are similar to those of $B \to \phi K^*$ and $B_s \to K^{*-}K^{*+}$ [72,73]. We just need to replace some corresponding wave functions, Wilson coefficients, and corresponding parameters.

With the Hamiltonian equation (2), depending on the CKM matrix elements of $V_{ub}V_{us}^*$ and $V_{tb}V_{ts}^*$, the electroweak penguin dominant decay amplitudes $A^{(i)}$ for $\bar{B}_s^0 \to \rho^0\phi$ in pQCD can be written as

$$\sqrt{2}A^{(i)}(\bar{B}_s^0 \to \rho^0\phi) = V_{ub}V_{us}^*T_\rho^i - V_{tb}V_{ts}^*P_\rho^i, \tag{39}$$

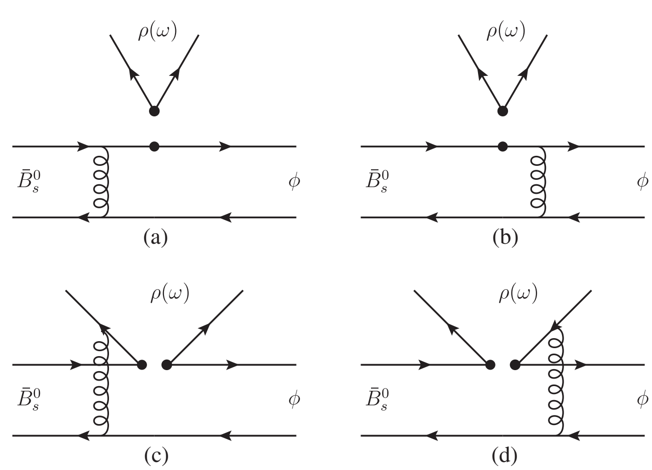

FIG. 1. Leading order Feynman diagrams for $\bar{B}_s^0 \to \rho(\omega)\phi$. (a), (b): Factorizable diagrams; (c),(d): nonfactorizable diagrams.

where the superscript i denotes different helicity amplitudes L, N, and T. The longitudinal $t^0_{\rho(\omega)}$, transverse $t^{\pm}_{\rho(\omega)}$ of the helicity amplitudes satisfy the relationship of $t^0_{\rho(\omega)} = t^L_{\rho(\omega)}$ and $t^{\pm}_{\rho(\omega)} = (t^N_{\rho(\omega)} \mp t^T_{\rho(\omega)})/(\sqrt{2})$. The amplitudes from tree and penguin diagrams can be written as $T^i_\rho = t^i_\rho / V_{ub}V^*_{us}$ and $P^i_\rho = p^i_\rho / V_{tb}V^*_{ts}$, respectively. The formula for the tree-level amplitude is

$$T^i_\rho = \frac{G_F}{\sqrt{2}} \{ f_\rho F^{LL,i}_{B_s\to\phi}[a_2] + M^{LL,i}_{B_s\to\phi}[C_2] \}, \tag{40}$$

where f_ρ refers to the decay constant of the ρ meson. The penguin-level amplitudes are expressed in the following

$$P^i_\rho = \frac{G_F}{\sqrt{2}} \left\{ f_\rho F^{LL,i}_{B_s\to\phi}\left[\frac{3}{2}(a_9 + a_7)\right] + M^{LL,i}_{B_s\to\phi}\left[\frac{3}{2}C_{10}\right] - M^{SP,i}_{B_s\to\phi}\left[\frac{3}{2}C_8\right] \right\}. \tag{41}$$

The QCD penguin dominant decay amplitude for $\bar{B}^0_s \to \omega\phi$ can be written as

$$\sqrt{2}A^i(\bar{B}^0_s \to \omega\phi) = V_{ub}V^*_{us}T^i_\omega - V_{tb}V^*_{ts}P^i_\omega, \tag{42}$$

where $T^i_\omega = t^i_\omega / V_{ub}V^*_{us}$ and $P^i_\omega = p^i_\omega / V_{tb}V^*_{ts}$, which refer to the tree and penguin amplitude, respectively. We can give the tree-level contribution in the following:

$$T^i_\omega = \frac{G_F}{\sqrt{2}} \{ f_\omega F^{LL,i}_{B_s\to\phi}[a_2] + M^{LL,i}_{B_s\to\phi}[C_2] \}, \tag{43}$$

where f_ω refers to the decay constant of the ω meson. The penguin-level contributions are given as the following

$$P^i_\omega = \frac{G_F}{\sqrt{2}} \left\{ f_\omega F^{LL,i}_{B_s\to\phi}\left[2a_3 + 2a_5 + \frac{1}{2}a_7 + \frac{1}{2}a_9\right] + M^{LL,i}_{B_s\to\phi}\left[2C_4 + \frac{1}{2}C_{10}\right] - M^{SP,i}_{B_s\to\phi}\left[2C_6 + \frac{1}{2}C_8\right] \right\}. \tag{44}$$

Based on the definition of Eq. (36), we can get

$$\alpha e^{i\delta^i_\alpha} = \frac{t^i_\omega}{t^i_\rho}, \tag{45}$$

$$\beta e^{i\delta^i_\beta} = \frac{p^i_\rho}{p^i_\omega}, \tag{46}$$

$$r' e^{i\delta^i_q} = \frac{P^i_\omega}{T^i_\rho} \times \left| \frac{V_{tb}V^*_{ts}}{V_{ub}V^*_{us}} \right|, \tag{47}$$

where

$$\left| \frac{V_{tb}V^*_{ts}}{V_{ub}V^*_{us}} \right| = \frac{\sqrt{\rho^2 + \eta^2}}{\lambda^2(\rho^2 + \eta^2)}. \tag{48}$$

From the above equations, the new strong phases δ^i_α, δ^i_β, and δ^i_q are obtained from tree and penguin diagram contributions by the ρ-ω interference. The total strong phase δ_i is obtained by Eqs. (36) and (37) in the framework of pQCD.

IV. INPUT PARAMETERS

The CKM matrix, whose elements are determined from experiments, can be expressed in terms of the Wolfenstein parameters A, ρ, λ, and η [70,74]:

$$\begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - \mathrm{i}\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - \mathrm{i}\eta) & -A\lambda^2 & 1 \end{pmatrix}, \tag{49}$$

where $\mathcal{O}(\lambda^4)$ corrections are neglected. The latest values for the parameters in the CKM matrix are [74]

$$\lambda = 0.22453 \pm 0.00044, \qquad A = 0.836 \pm 0.015,$$
$$\bar{\rho} = 0.122^{+0.018}_{-0.017}, \qquad \bar{\eta} = 0.355^{+0.012}_{-0.011}. \tag{50}$$

where

$$\bar{\rho} = \rho\left(1 - \frac{\lambda^2}{2}\right), \qquad \bar{\eta} = \eta\left(1 - \frac{\lambda^2}{2}\right). \tag{51}$$

From Eqs. (50) and (51) we have

$$0.108 < \rho < 0.144, \qquad 0.353 < \eta < 0.377. \tag{52}$$

The other parameters and the corresponding references are listed in Table I.

V. THE NUMERICAL RESULTS OF *CP* VIOLATION IN $\bar{B}^0_s \to \rho^0(\omega)\phi \to \pi^+\pi^-\phi$

We have investigated the *CP* violating asymmetry, A_{CP}, for the $\bar{B}^0_s \to \rho^0(\omega)\phi \to \pi^+\pi^-\phi$ decay process. The numerical results of the *CP* violating asymmetry are shown for the decay process in Fig. 2. It is found that the *CP* violation can be enhanced via ρ-ω mixing for the decay channel $\bar{B}^0_s \to \rho^0(\omega)\phi \to \pi^+\pi^-\phi$ when the invariant mass of $\pi^+\pi^-$ is in the vicinity of the ω resonance within the perturbative QCD scheme.

The *CP* violation depends on the weak phase difference from CKM matrix elements and the strong phase difference. The CKM matrix elements, which relate to ρ, η, and λ, are given in Eq. (50). The uncertainties due to the CKM matrix elements are mostly from ρ and η since λ is well determined. Hence we take the central value of $\lambda = 0.224$

TABLE I. Input parameters.

Parameters	Input data	References
Fermi constant (in GeV^{-2})	$G_F = 1.16638 \times 10^{-5}$.	[74]
Masses and decay widths (in MeV)	$m_{B_s^0} = 5366.89, \tau_{B_s^0} = 1.509 \times 10^{-12} s$, $m_{\rho^0(770)} = 775.26, \Gamma_{\rho^0(770)} = 149.1$, $m_{\omega(782)} = 782.65, \Gamma_{\omega(782)} = 8.49$, $m_\pi = 139.57, m_\phi = 1019.461$.	[74]
Decay constants (in MeV)	$f_\rho = 215.6 \pm 5.9, f_\rho^T = 165 \pm 9$, $f_\omega = 196.5 \pm 4.8, f_\omega^T = 145 \pm 10$, $f_\phi = 231 \pm 4, f_\phi^T = 200 \pm 10$.	[75,76]

in Eq. (52). In our numerical calculations, we let ρ, η, and $\lambda = 0.224$ vary among the limiting values. The numerical results are shown from Figs. 2–4 with the different parameter values of CKM matrix elements. The dash line, dot line, and solid line correspond to the maximum, middle, and minimum CKM matrix element for the decay channel of $\bar{B}_s^0 \to \rho^0(\omega)\phi \to \pi^+\pi^-\phi$, respectively. We find the CP violation is not sensitive to the CKM matrix elements for the different values of ρ and η. In Fig. 2, we give the plot of CP violating asymmetry as a function of $\sqrt{s}$. From Fig. 2, one can see that the CP violation parameter is dependent on $\sqrt{s}$ and changes rapidly due to ρ-ω mixing when the invariant mass of $\pi^+\pi^-$ is in the vicinity of the ω resonance. From the numerical results, it is found that the maximum CP violating parameter reaches 5.98% for the decay channel of $\bar{B}_s^0 \to \pi^+\pi^-\phi$ in the case of $(\rho_{\max}, \eta_{\max})$.

From Eq. (30), one can see that the CP violating parameter is related to $\sin\delta$ and r. The plots of $\sin\delta_0$ ($\sin\delta_+$ and $\sin\delta_-$) and r_0 (r_+ and r_-) as a function of $\sqrt{s}$ are shown in Figs. 3 and 4. We can see that the ρ-ω mixing mechanism produces a large $\sin\delta_0$ ($\sin\delta_+$ and $\sin\delta_-$) at the ω resonance. As can be seen from Fig. 3, the plots vary sharply in the cases of $\sin\delta_0$, $\sin\delta_+$, and $\sin\delta_-$. Meanwhile, $\sin\delta_0$ and $\sin\delta_-$ change weakly compared with the $\sin\delta_+$. It can be seen from Fig. 4 that r_+ change more rapidly than r_0 and r_- when the $\pi^+\pi^-$ pairs in the vicinity of the ω resonance.

The Large Hadron Collider (LHC) is a proton-proton collider which has currently started at the European Organization for Nuclear Research (CERN). In order to achieve the required energy and luminosity, the technology and equipment has been upgraded many times. The LHC run I first data-taking period lasted from 2010 to 2013 [77]. In the next few years, there were two major detector (ATLAS and CMS) upgrades happening after run II and run III. With a series of upgrades and modifications, the LHC provides a TeV-level high energy frontier and an opportunity to further improve conformance testing of the CKM matrix. The production rates for heavy quark flavors will be high at the LHC, and the $b\bar{b}$ production cross section will be of the order of 0.5 mb, providing about 10^{12} bottom quark events per year [77,78]. The heavy flavor physics experiment is one of the main projects of LHC experiments. Especially, LHCb is a specialized B-physics

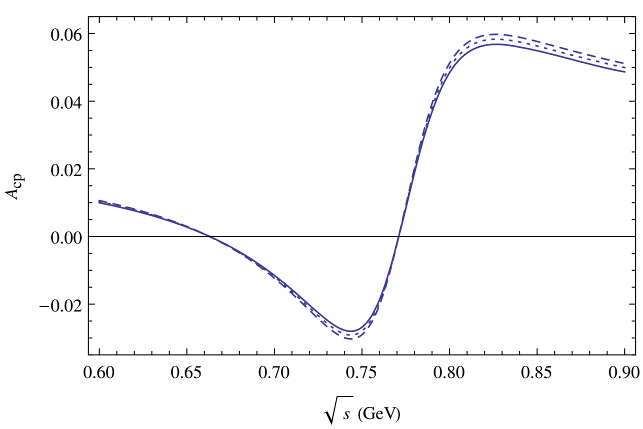

FIG. 2. The CP violating asymmetry, A_{cp}, as a function of $\sqrt{s}$ for different CKM matrix elements. The dash line, dot line, and solid line correspond to the maximum, middle, and minimum CKM matrix elements for the decay channel of $\bar{B}_s^0 \to \rho^0(\omega)\phi \to \pi^+\pi^-\phi$, respectively.

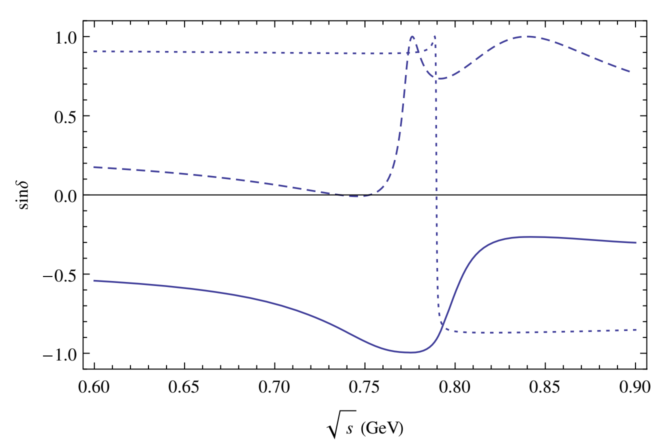

FIG. 3. $\sin\delta$ as a function of $\sqrt{s}$ corresponding to the central parameter values of CKM matrix elements for $\bar{B}_s^0 \to \rho^0(\omega)\phi \to \pi^+\pi^-\phi$. The dash line, dot line, and solid line correspond to $\sin\delta_0$, $\sin\delta_+$, and $\sin\delta_-$, respectively.

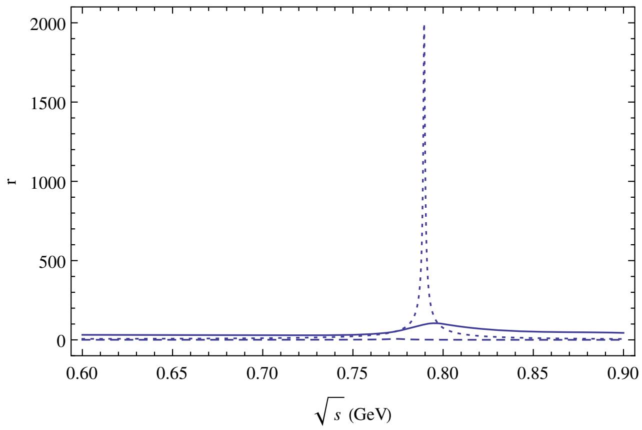

FIG. 4. Plot of r as a function of $\sqrt{s}$ corresponding to the central parameter values of CKM matrix elements for $\bar{B}_s^0 \to \rho^0(\omega)\phi \to \pi^+\pi^-\phi$. The dash line, dot line, and solid line correspond to r_0, r_+, and r_-, respectively.

experiment, designed primarily to precisely measure the parameters of new physics in CP violation and rare decays in the interactions of beauty and charm hadrons systems. Such studies can help to explain the matter-antimatter asymmetry of the Universe. Recently, the LHCb Collaboration found clear evidence for direct CP violation in some three-body decay channels in charmless decays of the B meson. Meanwhile, large CP violation is obtained in $B^\pm \to \pi^\pm\pi^+\pi^-$ in the region $0.6\ \mathrm{GeV}^2 < m^2_{\pi^+\pi^-\mathrm{low}} < 0.8\ \mathrm{GeV}^2$ and $m^2_{\pi^+\pi^-\mathrm{high}} > 14\ \mathrm{GeV}^2$ [8,79]. A zoom of the $\pi^+\pi^-$ invariant mass from the $B^\pm \to \pi^\pm\pi^+\pi^-$ decay process shows the region $0.6\ \mathrm{GeV}^2 < m^2_{\pi^+\pi^-\mathrm{low}} < 0.8\ \mathrm{GeV}^2$ zone in Fig. 8 of Ref. [79]. In addition, the branching fractions are probed in the $\pi^+\pi^-$ invariant mass range $400 < m(\pi^+\pi^-) < 1600\ \mathrm{MeV/c}^2$ for $\bar{B}_s^0 \to \pi^+\pi^-\phi$ [80]. In the next years, we expect the LHCb Collaboration to focus our prediction of CP violation from the $\bar{B}_s^0 \to \rho^0(\omega)\phi \to \pi^+\pi^-\phi$ decay process when the invariant mass of $\pi^+\pi^-$ is in the vicinity of the ρ resonance [80]. Theoretically, the LHC achieves the current experiments on b-hadrons, which can only provide about 10^7 $B\bar{B}$ pairs [81]. Therefore, it is very convenient to observe the CP violation for $\bar{B}_s^0 \to \rho^0(\omega)\phi \to \pi^+\pi^-\phi$ when the invariant masses of $\pi^+\pi^-$ pairs are in the vicinity of the ω resonance at the LHC experiments.

VI. SUMMARY AND CONCLUSION

In this paper, we study the CP violation for the decay process of $\bar{B}_s^0 \to \rho^0(\omega)\phi \to \pi^+\pi^-\phi$ due to the interference of ρ-ω mixing in perturbative QCD. It has been found that the CP violation can be enhanced at the area of ρ-ω resonance. There is the resonance effect via ρ-ω mixing which can produce large strong phase in this decay process. As a result, one can find that the maximum CP violation can reach 5.98% when the invariant mass of the $\pi^+\pi^-$ pair is in the vicinity of the ω resonance.

In the calculation, we need the renormalization scheme independent Wilson coefficients for the tree and penguin operators at the scale m_b. The major uncertainties is from the input parameters. In particular, these include the CKM matrix element parameters, the perturbative QCD approach, and the hadronic parameters (the shape parameters, decay constants, the wave function, etc.). We expect that our predictions will provide useful guidance for future investigations in B_s decays.

ACKNOWLEDGMENTS

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APPENDIX: RELATED FUNCTIONS DEFINED IN THE TEXT

In this Appendix we present explicit expressions of the factorizable and nonfactorizable amplitudes in Perturbative QCD [19–21,56]. The factorizable amplitudes $F^{LL,i}_{B_s\to\phi}(a_i)$ $(i = L, N, T)$ are written as

$$f_{M_2}F^{LL,L}_{B_s\to\phi}(a_i) = 8\pi C_F M^4_{B_s} f_{M_2}\int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_{B_s}(x_1,b_1)\{a_i(t_a)E_e(t_a) \times [(1+x_3)\phi_3(x_3) + r_3(1-2x_3)(\phi_3^s(x_3)+\phi_3^t(x_3))]h_e(x_1,x_3,b_1,b_3) + 2r_3\phi_3^s(x_3)a_i(t'_a)E_e(t'_a)h_e(x_3,x_1,b_3,b_1)\}, \tag{A1}$$

$$f_{M_2}F^{LL,N}_{B_s\to\phi}(a_i) = 8\pi C_F M^4_{B_s} f_{M_2} r_2\int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_{B_s}(x_1,b_1)\{h_e(x_1,x_3,b_1,b_3) \times E_e(t_a)a_i(t_a)[\phi_3^T(x_3) + 2r_3\phi_3^v(x_3) + r_3x_3(\phi_3^v(x_3)-\phi_3^a(x_3))] + r_3[\phi_3^v(x_3)+\phi_3^a(x_3)]E_e(t'_a)a_i(t'_a)h_e(x_3,x_1,b_3,b_1)\}, \tag{A2}$$

$$f_{M_2} F^{LL,T}_{B_s\to\phi}(a_i) = 16\pi C_F M^4_{B_s} f_{M_2} r_2 \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_{B_s}(x_1,b_1)\{h_e(x_1,x_3,b_1,b_3) \times [\phi_3^T(x_3) + 2r_3\phi_3^v(x_3) - r_3 x_3(\phi_3^v(x_3) - \phi_3^a(x_3))]E_e(t_a)a_i(t_a) + r_3[\phi_3^v(x_3) + \phi_3^a(x_3)]E_e(t'_a)a_i(t'_a)h_e(x_3,x_1,b_3,b_1)\}, \tag{A3}$$

with the color factor $C_F = 3/4$, f_{M_2}, f_{B_s} referring to the decay constant of M_2 (ρ or ω) and $\bar{B}_s$ mesons and a_i represents the corresponding Wilson coefficients for emission decay channels. In the above functions, $r_2(r_3) = m_{V_2}(m_{V_3})/m_{B_s}$ and $\phi_2(\phi_3) = \phi_{\rho/\omega}(\phi_\phi)$, with m_{B_s} and $m_{V_2}(m_{V_3})$ being the masses of the initial and final states.

The nonfactorizable amplitudes $M^{LL,i}_{B_s\to\phi}(a_i)$, and $M^{SP,i}_{B_s\to\phi}(a_i)$ ($i = L, N, T$) are written as

$$M^{LL,L}_{B_s\to\phi}(a_i) = 32\pi C_F M^4_{B_s}/\sqrt{6}\int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_s}(x_1,b_1)\phi_2(x_2) \times \{[(1-x_2)\phi_3(x_3) - r_3 x_3(\phi_3^s(x_3) - \phi_3^t(x_3))]a_i(t_b)E'_e(t_b) \times h_n(x_1,1-x_2,x_3,b_1,b_2) + h_n(x_1,x_2,x_3,b_1,b_2) \times [-(x_2+x_3)\phi_3(x_3) + r_3 x_3(\phi_3^s(x_3) + \phi_3^t(x_3))]a_i(t'_b)E'_e(t'_b)\}, \tag{A4}$$

$$M^{LL,N}_{B_s\to\phi}(a_i) = 32\pi C_F M^4_{B_s} r_2/\sqrt{6}\int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_s}(x_1,b_1) \times \{[x_2(\phi_2^v(x_2) + \phi_2^a(x_2))\phi_3^T(x_3) - 2r_3(x_2+x_3)(\phi_2^v(x_2)\phi_3^v(x_3) + \phi_2^a(x_2)\phi_3^a(x_3))] \times h_n(x_1,x_2,x_3,b_1,b_2)E'_e(t'_b)a_i(t'_b) + (1-x_2)(\phi_2^v(x_2) + \phi_2^a(x_2))\phi_3^T(x_3)E'_e(t_b)a_i(t_b)h_n(x_1,1-x_2,x_3,b_1,b_2)\}, \tag{A5}$$

$$M^{LL,T}_{B_s\to\phi}(a_i) = 64\pi C_F M^4_{B_s} r_2/\sqrt{6}\int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_s}(x_1,b_1)\{E'_e(t'_b)a_i(t'_b) \times [x_2(\phi_2^v(x_2) + \phi_2^a(x_2))\phi_3^T(x_3) - 2r_3(x_2+x_3)(\phi_2^v(x_2)\phi_3^a(x_3) + \phi_2^a(x_2)\phi_3^v(x_3))]h_n(x_1,x_2,x_3,b_1,b_2) + (1-x_2)[\phi_2^v(x_2) + \phi_2^a(x_2)]\phi_3^T(x_3)E'_e(t_b)a_i(t_b)h_n(x_1,1-x_2,x_3,b_1,b_2)\}, \tag{A6}$$

$$M^{SP,L}_{B_s\to\phi}(a_i) = 32\pi C_F M^4_{B_s}/\sqrt{6}\int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_s}(x_1,b_1)\phi_2(x_2) \times \{[(x_2 - x_3 - 1)\phi_3(x_3) + r_3 x_3(\phi_3^s(x_3) + \phi_3^t(x_3))] \times a_i(t_b)E'_e(t_b)h_n(x_1,1-x_2,x_3,b_1,b_2) + a_i(t'_b)E'_e(t'_b) \times [x_2\phi_3(x_3) + r_3 x_3(\phi_3^t(x_3) - \phi_3^s(x_3))]h_n(x_1,x_2,x_3,b_1,b_2)\}. \tag{A7}$$

$$M^{SP,N}_{B_s\to\phi}(a_i) = 32\pi C_F M^4_{B_s}/\sqrt{6}\int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_s}(x_1,b_1) r_2 \times \{x_2(\phi_2^v(x_2) - \phi_2^a(x_2))\phi_3^T(x_3)E'_e(t'_b)a_i(t'_b)h_n(x_1,x_2,x_3,b_1,b_2) + h_n(x_1,1-x_2,x_3,b_1,b_2)[(1-x_2)(\phi_2^v(x_2) - \phi_2^a(x_2))\phi_3^T(x_3) - 2r_3(1-x_2+x_3)(\phi_2^v(x_2)\phi_3^v(x_3) - \phi_2^a(x_2)\phi_3^a(x_3))]E'_e(t_b)a_i(t_b)\}, \tag{A8}$$

$$M_{B_s\to\phi}^{SP,T}(a_i) = 64\pi C_F M_{B_s}^4/\sqrt{6}\int_0^1 dx_1dx_2dx_3\int_0^\infty b_1db_1b_2db_2\phi_{B_s}(x_1,b_1)r_2 \times\{x_2(\phi_2^v(x_2)-\phi_2^a(x_2))\phi_3^T(x_3)E_e'(t_b')a_i(t_b')h_n(x_1,x_2,x_3,b_1,b_2) + h_n(x_1,1-x_2,x_3,b_1,b_2)[(1-x_2)(\phi_2^v(x_2)-\phi_2^a(x_2))\phi_3^T(x_3) - 2r_3(1-x_2+x_3)(\phi_2^v(x_2)\phi_3^a(x_3)-\phi_2^a(x_2)\phi_3^v(x_3))]E_e'(t_b)a_i(t_b)\}. \quad (A9)$$

The hard scale t is chosen as the maximum of the virtuality of the internal momentum transition in the hard amplitudes, including $1/b_i$:

$$t_a = \max\{\sqrt{x_3}M_{B_s}, 1/b_1, 1/b_3\}, \quad (A10)$$

$$t_a' = \max\{\sqrt{x_1}M_{B_s}, 1/b_1, 1/b_3\}, \quad (A11)$$

$$t_b = \max\{\sqrt{x_1x_3}M_{B_s}, \sqrt{|1-x_1-x_2|x_3}M_{B_s}, 1/b_1, 1/b_2\}, \quad (A12)$$

$$t_b' = \max\{\sqrt{x_1x_3}M_{B_s}, \sqrt{|x_1-x_2|x_3}M_{B_s}, 1/b_1, 1/b_2\}, \quad (A13)$$

$$t_c = \max\{\sqrt{1-x_3}M_{B_s}, 1/b_2, 1/b_3\}, \quad (A14)$$

$$t_c' = \max\{\sqrt{x_2}M_{B_s}, 1/b_2, 1/b_3\}, \quad (A15)$$

$$t_d = \max\{\sqrt{x_2(1-x_3)}M_{B_s}, \sqrt{1-(1-x_1-x_2)x_3}M_{B_s}, 1/b_1, 1/b_2\}, \quad (A16)$$

$$t_d' = \max\{\sqrt{x_2(1-x_3)}M_{B_s}, \sqrt{|x_1-x_2|(1-x_3)}M_{B_s}, 1/b_1, 1/b_2\}. \quad (A17)$$

The function h, coming from the Fourier transform of the hard part H, are written as [82]

$$h_e(x_1,x_3,b_1,b_3) = [\theta(b_1-b_3)I_0(\sqrt{x}_3M_{B_s}b_3)K_0(\sqrt{x}_3M_{B_s}b_1) + \theta(b_3-b_1)I_0(\sqrt{x}_3M_{B_s}b_1)K_0(\sqrt{x}_3M_{B_s}b_3)]K_0(\sqrt{x_1x_3}M_{B_s}b_1)S_t(x_3), \quad (A18)$$

$$h_n(x_1,x_2,x_3,b_1,b_2) = [\theta(b_2-b_1)K_0(\sqrt{x_1x_3}M_{B_s}b_2)I_0(\sqrt{x_1x_3}M_{B_s}b_1) + \theta(b_1-b_2)K_0(\sqrt{x_1x_3}M_{B_s}b_1)I_0(\sqrt{x_1x_3}M_{B_s}b_2)]\times\begin{cases}\frac{i\pi}{2}H_0^{(1)}(\sqrt{(x_2-x_1)x_3}M_{B_s}b_2), & x_1-x_2<0\\ K_0(\sqrt{(x_1-x_2)x_3}M_{B_s}b_2), & x_1-x_2>0\end{cases}, \quad (A19)$$

$$h_a(x_2,x_3,b_2,b_3) = \left(\frac{i\pi}{2}\right)^2 S_t(x_3)[\theta(b_2-b_3)H_0^{(1)}(\sqrt{x_3}M_{B_s}b_2)J_0(\sqrt{x_3}M_{B_s}b_3) + \theta(b_3-b_2)H_0^{(1)}(\sqrt{x_3}M_{B_s}b_3)J_0(\sqrt{x_3}M_{B_s}b_2)]H_0^{(1)}(\sqrt{x_2x_3}M_{B_s}b_2), \quad (A20)$$

$$h_{na}(x_1,x_2,x_3,b_1,b_2) = \frac{i\pi}{2}[\theta(b_1-b_2)H_0^{(1)}(\sqrt{x_2(1-x_3)}M_{B_s}b_1)J_0(\sqrt{x_2(1-x_3)}M_{B_s}b_2) + \theta(b_2-b_1)H_0^{(1)}(\sqrt{x_2(1-x_3)}M_{B_s}b_2)J_0(\sqrt{x_2(1-x_3)}M_{B_s}b_1)] \times K_0(\sqrt{1-(1-x_1-x_2)x_3}M_{B_s}b_1), \quad (A21)$$

$$h'_{na}(x_1,x_2,x_3,b_1,b_2) = \frac{i\pi}{2}[\theta(b_1-b_2)H_0^{(1)}(\sqrt{x_2(1-x_3)}M_{B_s}b_1)J_0(\sqrt{x_2(1-x_3)}M_{B_s}b_2) + \theta(b_2-b_1)H_0^{(1)}(\sqrt{x_2(1-x_3)}M_{B_s}b_2)J_0(\sqrt{x_2(1-x_3)}M_{B_s}b_1)] \times \begin{cases} \frac{i\pi}{2}H_0^{(1)}(\sqrt{(x_2-x_1)(1-x_3)}M_{B_s}b_1), & x_1-x_2<0 \\ K_0(\sqrt{(x_1-x_2)(1-x_3)}M_{B_s}b_1), & x_1-x_2>0 \end{cases}, \tag{A22}$$

where J_0 and Y_0 are the Bessel function with $H_0^{(1)}(z) = \mathrm{J}_0(z) + i\mathrm{Y}_0(z)$.

The threshold resums factor S_t follows the parameterized [83]

$$S_t(x) = \frac{2^{1+2c}\Gamma(3/2+c)}{\sqrt{\pi}\Gamma(1+c)}[x(1-x)]^c, \tag{A23}$$

where the parameter $c = 0.4$. In the nonfactorizable contributions, $S_t(x)$ gives a very small numerical effect to the amplitude [84]. Therefore, we drop $S_t(x)$ in h_n and h_{na}.

The evolution factors $E_e^{(\prime)}$ and $E_a^{(\prime)}$ entering in the expressions for the matrix elements are given by

$$E_e(t) = \alpha_s(t)\exp[-S_B(t) - S_3(t)], \qquad E'_e(t) = \alpha_s(t)\exp[-S_B(t) - S_2(t) - S_3(t)]|_{b_1=b_3}, \tag{A24}$$

$$E_a(t) = \alpha_s(t)\exp[-S_2(t) - S_3(t)], \qquad E'_a(t) = \alpha_s(t)\exp[-S_B(t) - S_2(t) - S_3(t)]|_{b_2=b_3}, \tag{A25}$$

in which the Sudakov exponents are defined as

$$S_B(t) = s\left(x_1\frac{M_{B_s}}{\sqrt{2}}, b_1\right) + \frac{5}{3}\int_{1/b_1}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu})), \tag{A26}$$

$$S_2(t) = s\left(x_2\frac{M_{B_s}}{\sqrt{2}}, b_2\right) + s\left((1-x_2)\frac{M_{B_s}}{\sqrt{2}}, b_2\right) + 2\int_{1/b_2}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu})), \tag{A27}$$

where $\gamma_q = -\alpha_s/\pi$ is the anomalous dimension of the quark. The explicit form for the function $s(Q,b)$ is

$$s(Q,b) = \frac{A^{(1)}}{2\beta_1}\hat{q}\ln\left(\frac{\hat{q}}{\hat{b}}\right) - \frac{A^{(1)}}{2\beta_1}(\hat{q}-\hat{b}) + \frac{A^{(2)}}{4\beta_1^2}\left(\frac{\hat{q}}{\hat{b}}-1\right) - \left[\frac{A^{(2)}}{4\beta_1^2} - \frac{A^{(1)}}{4\beta_1}\ln\left(\frac{e^{2\gamma_E-1}}{2}\right)\right]\ln\left(\frac{\hat{q}}{\hat{b}}\right) + \frac{A^{(1)}\beta_2}{4\beta_1^3}\hat{q}\left[\frac{\ln(2\hat{q})+1}{\hat{q}} - \frac{\ln(2\hat{b})+1}{\hat{b}}\right] + \frac{A^{(1)}\beta_2}{8\beta_1^3}[\ln^2(2\hat{q}) - \ln^2(2\hat{b})], \tag{A28}$$

where the variables are defined by

$$\hat{q} \equiv \ln[Q/(\sqrt{2}\Lambda)], \qquad \hat{b} \equiv \ln[1/(b\Lambda)], \tag{A29}$$

and the coefficients $A^{(i)}$ and β_i are

$$\beta_1 = \frac{33-2n_f}{12}, \qquad \beta_2 = \frac{153-19n_f}{24}, \qquad A^{(1)} = \frac{4}{3}, \qquad A^{(2)} = \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27}n_f + \frac{8}{3}\beta_1\ln\left(\frac{1}{2}e^{\gamma_E}\right), \tag{A30}$$

where n_f is the number of the quark flavors and γ_E is the Euler constant. We will use the one-loop expression of the running coupling constant.

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