Origin of a two-loop neutrino mass from SU(5) grand unification

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In this work we propose a renormalizable model based on the $SU(5)$ gauge group where neutrino mass originates at the two-loop level without extending the fermionic content of the Standard Model (SM). Unlike the conventional $SU(5)$ models, in this proposed scenario, neutrino mass is intertwined with the charged fermion masses. In addition to correctly reproducing the SM charged fermion masses and mixings, neutrino mass is generated at the quantum level, hence naturally explains the smallness of neutrino masses. In this setup, we provide examples of gauge coupling unification that simultaneously satisfy the proton decay constraints. This model has the potential to be tested experimentally by measuring the proton decay in the future experiments. Scalar leptoquarks that are naturally contained within this framework can accommodate the recent B-physics anomalies.

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I. INTRODUCTION

Even though the Standard Model (SM) of particle physics is highly successful in describing the interactions of the fundamental particles, it has several drawbacks, such as not being able to explain the neutrino mass, the existence of dark matter (DM), and the origin of matter-antimatter asymmetry of the universe. Among them, one of the most important downsides of the SM is that the neutrinos remain massless. However, experimentally neutrino oscillation has been observed, hence the neutrinos must have acquired mass in some unspecified mechanism yet to be discovered. Due to these shortcomings, the SM begs for extensions. Grand unified theories (GUTs) [1–3] are the leading candidates beyond the Standard Model (BSM) since they are ultraviolet complete theories and come with many aesthetic features. Among different possibilities, $\boldsymbol{SU(5)}$ GUT is the simplest choice, this is the only simple group that contains SM gauge group as a subgroup and has the same rank as the SM gauge group. $\boldsymbol{SU(5)}$ GUT can incorporate gauge coupling unification, it relates quarks with leptons and quantization of the electric charge can also be understood.

In the first proposed $\boldsymbol{SU(5)}$ GUT by Georgi and Glashow [2], the three families of fermions of the SM belong to the $\bar{\mathbf{5}}_{\boldsymbol{F}i} + \mathbf{10}_{\boldsymbol{F}i}$ ($i = 1$–3 is the generation index) representations of the $\boldsymbol{SU(5)}$. Quarks and leptons are unified in these representations as can be seen from their decompositions under the SM:

$$\bar{\mathbf{5}}_{\boldsymbol{F}i} = \ell_i\left(1, 2, -\frac{1}{2}\right) \oplus d_i^c\left(\bar{3}, 1, \frac{1}{3}\right), \tag{1.1}$$

$$\mathbf{10}_{\boldsymbol{F}i} = q_i\left(3, 2, \frac{1}{6}\right) \oplus u_i^c(\bar{3}, 1, -2/3) \oplus e_i^c(1, 1, 1), \tag{1.2}$$

where, $\ell_i = (\nu_i \quad e_i)^T$ and $q_i = (u_i \quad d_i)^T$. Interestingly, these multiplets contain only the fermions that are present in the SM, no additional fermions need to be introduced to cancel the gauge anomalies.

To describe our universe, the $\boldsymbol{SU(5)}$ gauge symmetry needs to be broken to that of the SM at the high scale: $SU(5) \to SU(3)_C \times SU(2)_L \times U(1)_Y$ that can be achieved by employing a Higgs field in the $\mathbf{24}_{\boldsymbol{H}}$-dimensional[1] representation [2]. When this field acquires a vacuum expectation value (VEV) in the SM singlet direction, the GUT symmetry is spontaneously broken down to the SM. Then at the low energy scale, the SM gauge symmetry is spontaneously broken by the $\mathbf{5}_{\boldsymbol{H}}$-dimensional representation: $SU(3)_C \times SU(2)_L \times U(1)_Y \to SU(3)_C \times U(1)_{em}$. As a result, the SM Higgs contained in the $\mathbf{5}_{\boldsymbol{H}}$-Higgs generates masses to all the charged fermions. However, this scenario predicts special mass relations $m_e = m_d$, $m_\mu = m_s$ [6] at the GUT scale that is ruled out by the experimental data. The shortcomings of the Georgi-Glashow model are

(i) it predicts wrong mass relations among the charged fermions.

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[1]Alternatively, instead of $\mathbf{24}_{\boldsymbol{H}}$ Higgs, $\boldsymbol{SU(5)}$ breaking to the SM group can also be achieved by using $\mathbf{75}_{\boldsymbol{H}}$ Higgs [4,5].

(ii) it fails to achieve gauge coupling unification.
(iii) neutrinos remain massless.

In the literature, several different attempts are made to solve the aforementioned problems of the Georgi-Glashow model. In all these works, neutrinos receive mass either at the tree-level or at the one-loop level by extending the scalar and/or fermion sectors. Here on the contrary, we construct a viable model where neutrino mass appears at the two-loop level without introducing any new fermions to the SM. So in our model, neutrinos are Majorana in nature. Realistic charged fermion masses are generated at the tree-level and the neutrino masses are originated due to the quantum corrections, hence naturally explains the lightness of the neutrino masses. We also show that the neutrino mass in this setup does not decouple but gets entangled with the charged fermion masses. We construct the scalar potential and compute the Higgs spectrum that are relevant for the study of the gauge coupling unification. Successful scenarios of gauge coupling unification are presented by properly taking into account the proton decay constraints. Proton decay rate in this scenario is expected to be within the experimental observable range. The novelty of this work is, our proposed setup is the first construction of a renormalizable model based on $\boldsymbol{SU(5)}$ gauge symmetry without imposing any additional symmetries and without introducing any additional fermions where the neutrinos receive mass at the two-loop level. We compare our proposed model with the existing realistic $\boldsymbol{SU(5)}$ GUT models in the literature in great details. Scalar leptoquarks naturally contained within the representations required to generate realistic charged fermion and neutrino masses in our framework can accommodate the recent B-physics anomalies.

II. NEUTRINO MASS IN RENORMALIZABLE $\boldsymbol{SU(5)}$ GUTS

The first shortcoming of the Georgi-Glashow model listed above can be fixed in two different ways: one approach to correct the bad mass relations is to add higher-dimensional operators [7], which we do not pursue. The alternative approach that we are interested in, is to work within the renormalizable framework that requires extension of the minimal Higgs sector. As aforementioned, the SM fermions belong to the $\bar{\mathbf{5}}_F + \mathbf{10}_F$-multiplets of $\boldsymbol{SU(5)}$:

$$\bar{\mathbf{5}}_F = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ -\nu \end{pmatrix}, \quad \mathbf{10}_F = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_2^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^c \\ -d_1 & -d_2 & -d_3 & -e^c & 0 \end{pmatrix}. \tag{2.1}$$

The Higgs fields that can generate masses for the charged fermions can be identified from the fermion bilinears [8]:

$$\bar{\mathbf{5}} \times \mathbf{10} = \mathbf{5} + \mathbf{45}, \tag{2.2}$$

$$\mathbf{10} \times \mathbf{10} = \bar{\mathbf{5}}_s + \overline{\mathbf{45}}_a + \overline{\mathbf{50}}_s, \tag{2.3}$$

where the subscripts “s” and “a” represent symmetric and antisymmetric combinations. So the possible set of Higgs fields that can have Yukawa couplings is $\{\mathbf{5}_H, \mathbf{45}_H, \mathbf{50}_H\}$. However, among them, only $\mathbf{5}_H$ and $\mathbf{45}_H$ contain SM like Higgs that can break the electroweak (EW) symmetry and generate charged fermion masses. Hence, the only possibility is to add a $\mathbf{45}_H$ [6] in the Georgi-Glashow model to correct the bad mass relations. With this addition, the Yukawa part of the Lagrangian is given by [9]:

$$\begin{aligned} \mathcal{L}_Y = {} & Y_{1,ij} \bar{\mathbf{5}}_{F\,i}^{\alpha} \mathbf{10}_{F\alpha\beta,j} \mathbf{5}_H^{*\beta} + Y_{2,ij} \bar{\mathbf{5}}_{F\,i}^{\delta} \mathbf{10}_{F\alpha\beta,j} \mathbf{45}_{H\delta}^{*\,\alpha\beta} \\ & + \epsilon^{\alpha\beta\gamma\delta r} (Y_{3,ij} \mathbf{10}_{F\alpha\beta,i} \mathbf{10}_{F\gamma\delta,j} \mathbf{5}_{Hr} \\ & + Y_{4,ij} \mathbf{10}_{F\alpha\beta,i} \mathbf{10}_{Fm\gamma,j} \mathbf{45}_{H\,\delta r}^{\,m}), \end{aligned} \tag{2.4}$$

where $\boldsymbol{SU(5)}$ group indices are explicitly shown and i, $j = 1$–3 are the generation indices. From this Lagrangian, the down-type quark and the charged-lepton mass matrices are given by:

$$M_D = Y_1 v_5^* + Y_2 v_{45}^*, \tag{2.5}$$

$$M_E = Y_1^T v_5^* - 3 Y_2^T v_{45}^*. \tag{2.6}$$

Here we have defined $v_5 = \langle \phi_1^0 \rangle / (\sqrt{2})$ and $v_{45} = \langle \Sigma_1^0 \rangle / (-2\sqrt{3})$, where the SM like weak doublets from $\mathbf{5}_H$ and $\mathbf{45}_H$ are identified as $\phi_1 = (\phi_1^+ \quad \phi_1^0)^T$ and $\Sigma_1 = (\Sigma_1^+ \quad \Sigma_1^0)^T$. This normalization follows the relation: $2v_5^2 + 12 v_{45}^2 = v^2$, with $v = 174$ GeV. The above relations clearly violate the simple mass relations of the Georgi-Glashow model and, in the Yukawa sector, there are enough parameters to fit all the charged fermions masses and mixings. In Eq. (2.4), Y_1, Y_2, Y_3, Y_4 are arbitrary 3×3 Yukawa matrices, and the up-quark mass matrix is not related to the down-quark and charged lepton mass matrices and is a symmetric complex matrix, $M_U = M_U^T$. In this renormalizable model, one can also achieve gauge coupling unification and the model is safe from too rapid proton decay [9]. In Sec. V, we reproduce the result of Ref. [9] and present a plot to demonstrate successful gauge coupling unification in this scenario. So the minimal model extended by $\mathbf{45}_H$ Higgs can simultaneously solve the first two shortcomings of the Georgi-Glashow model listed above except the last one. This renormalizable model consists of fermion fields given in Eqs. (1.1)–(1.2) and scalar fields as given below in Eqs. (2.7)–(2.9), and for brevity we refer to this model as MRSU5 (minimal renormalizable $\boldsymbol{SU(5)}$ GUT) for the rest of the text.

$$\mathbf{5}_{\boldsymbol{H}} \equiv \phi = \phi_1\left(1,2,\frac{1}{2}\right) \oplus \phi_2\left(3,1,-\frac{1}{3}\right), \quad (2.7)$$

$$\mathbf{24}_{\boldsymbol{H}} \equiv \Phi = \Phi_1(1,1,0) \oplus \Phi_2(1,3,0) \oplus \Phi_3(8,1,0) \oplus \Phi_4\left(3,2,-\frac{5}{6}\right) \oplus \Phi_5\left(3,2,+\frac{5}{6}\right), \quad (2.8)$$

$$\mathbf{45}_{\boldsymbol{H}} \equiv \Sigma = \Sigma_1\left(1,2,\frac{1}{2}\right) \oplus \Sigma_2\left(3,1,-\frac{1}{3}\right) \oplus \Sigma_3\left(\bar{3},1,\frac{4}{3}\right) \oplus \Sigma_4\left(\bar{3},2,-\frac{7}{6}\right) \oplus \Sigma_5\left(3,3-\frac{1}{3}\right) \oplus \Sigma_6\left(\bar{6},1,-\frac{1}{3}\right) \oplus \Sigma_7\left(8,2,\frac{1}{2}\right). \quad (2.9)$$

Note however that in this minimal model the neutrinos are still massless just like the SM. Extension must be made to make this setup phenomenologically viable. Here we briefly review the different possibilities of generating nonzero neutrino mass[2] in the context of renormalizable $\boldsymbol{SU}(\mathbf{5})$ grand unified theory (GUT).

(i) Tree-level:

To incorporate neutrino mass, the simplest possibility is to add at least two right-handed Majorana neutrinos $\nu^c(1,1,0)$ to MRSU5 model that are singlets of $\boldsymbol{SU}(\mathbf{5})$. This possibility can give rise to neutrino masses by using the type-I seesaw mechanism [11–14]. Since this extension involves GUT group singlets, this approach may not be aesthetic and it is preferable to have multiplets that are nonsinglets under the gauge group. Neutrino mass can be generated via type-II seesaw scenario [15–18] if a Higgs in the $\mathbf{15}_{\boldsymbol{H}}$-dimensional representation[3] is added to the MRSU5 [22], it is because the $\mathbf{15}_{\boldsymbol{H}}$-Higgs contains a isospin triplet $(1,3,1) \subset \mathbf{15}_{\boldsymbol{H}}$. Another possibility is to add at least two copies of fermion multiplets in the adjoint $\mathbf{24}_{\boldsymbol{F}}$-dimensional representation[4] to the MRSU5 [24], this scenario generates neutrino mass in a combination of type-III [25] and type-I seesaw mechanisms. This scenario makes use of the fermionic weak triplet lies in the adjoint representation, $(1,3,0) \subset \mathbf{24}_{\boldsymbol{F}}$. These are the simple possibilities to incorporate neutrino mass at the tree-level by extending the MRSU5 model by one (type-II) or more (type-I and type-III) multiplets.

(ii) Loop-level:

Neutrino mass can also be generated at the quantum level within the $\boldsymbol{SU}(\mathbf{5})$ GUT framework and is a very interesting alternative possibility. The first radiative model[5] of neutrino mass generation for Majorana type particles was proposed by Zee [28] in the context of the SM gauge group by extending the SM by another Higgs doublet and a singly charged scalar singlet. In the Zee model, neutrino mass is generated at the one loop level as shown in Fig. 1 (Feynman diagram on the left). The $\boldsymbol{SU}(\mathbf{5})$ GUT embedding of Zee mechanism with the use of $\mathbf{10}_{\boldsymbol{H}}$-dimensional Higgs was first proposed in Ref. [29], also pointed out in Ref. [30] and just recently studied in the renormalizable context in Ref. [31]. In this realization, the two SM like Higgs doublets are contained in the $\mathbf{5}_{\boldsymbol{H}}$ and $\mathbf{45}_{\boldsymbol{H}}$ and the singly charged Higgs $h^+(1,1,1)$ which is singlet under the SM lies in $\mathbf{10}_{\boldsymbol{H}}$. Many different variations of the original Zee model by extending the SM are proposed in the literature. A particular model proposed in Ref. [32] uses a real scalar triplet instead of the second Higgs doublet. In addition to the singly charged scalar, this model needs three copies of vectorlike lepton doublets to incorporate Zee mechanism. Just recently this one-loop Zee-type model is also embedded in a renormalizable $\boldsymbol{SU}(\mathbf{5})$ GUT [33] by extending the MRSU5 by both scalar and fermion multiplets. In their work, the real scalar triplet is embedded in $\mathbf{24}_{\boldsymbol{H}}$, the singly charged scalar in $\mathbf{10}_{\boldsymbol{H}}$ and the vectorlike lepton doublets in three generations of $\mathbf{5}_{\boldsymbol{F}i} + \bar{\mathbf{5}}_{\boldsymbol{F}i}$ matter fields. Hence, the Georgi-Glashow model is extended by $\mathbf{10}_{\boldsymbol{H}}$ Higgs and three generations of vectorlike leptons $\mathbf{5}_{\boldsymbol{F}} + \bar{\mathbf{5}}_{\boldsymbol{F}}$. In the models mentioned above, the particle running in the loop are colorless, however, neutrino mass can also be generated via the Zee mechanism while colored particles run through the loop. For such leptoquark mechanism of neutrino masses at the one-loop level within the $\boldsymbol{SU}(\mathbf{5})$ GUT framework, see for example Ref. [34].

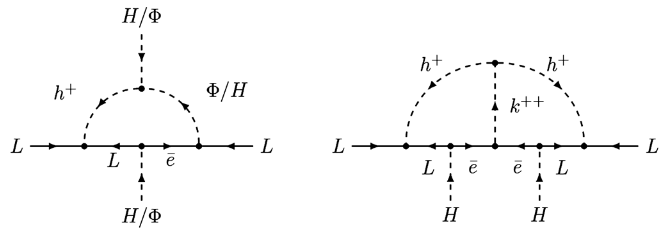

FIG. 1. Left: Zee mechanism to generate neutrino mass at one-loop level. Right: Zee-Babu mechanism to generate neutrino mass at two-loop level.

[2]For a recent general review on neutrino mass generation mechanisms for Majorana type neutrinos see Ref. [10].

[3]Extension by $\mathbf{15}_{\boldsymbol{H}}$ Higgs was first considered within the nonrenormalizable $\boldsymbol{SU}(\mathbf{5})$ context [19–21].

[4]Extension by $\mathbf{24}_{\boldsymbol{F}}$ fermions was first considered within the nonrenormalizable $\boldsymbol{SU}(\mathbf{5})$ context [23].

[5]First radiative model was proposed for Dirac type neutrinos [26]. Without introducing exotic fermions in the SM, generating Dirac mass for neutrinos are studied in great details recently in Ref. [27].

III. THE PROPOSED MODEL

In the models discussed above, the neutrino mass can be generated at the tree-level via seesaw mechanism or at the one-loop level via Zee mechanism within the context of renormalizable $\boldsymbol{SU(5)}$ GUT. In this work, we for the first time construct a realistic $\boldsymbol{SU(5)}$ GUT, where the origin of the neutrino mass is realized at the two-loop level. In our construction, we restrict ourselves by demanding the following requirements:

(i) the model should be renormalizable.
(ii) the only symmetry of the theory is $\boldsymbol{SU(5)}$ gauge symmetry.
(iii) no additional fermions are added compared to the already existing ones in the SM.
(iv) automatic vanishing of the neutrino mass at the tree-level and at the one-loop level.
(v) no $\boldsymbol{SU(5)}$ singlet is allowed.
(vi) in our search, we restrict ourselves to the representations of dimension <100.

As aforementioned, the Higgs fields that can have Yukawa interactions can be determined from the fermion bilinears presented in Eqs. (2.2)–(2.3). In search of the simplest renormalizable $\boldsymbol{SU(5)}$ GUT where two-loop neutrino mass mechanism can be realized, we take a closer look at these fermion bilinears. As noted above, out of $\{\mathbf{5_H}, \mathbf{45_H}, \mathbf{50_H}\}$ Higgs multiplets, $\mathbf{5_H}$ and $\mathbf{45_H}$-multiplets contain the SM like doublets, hence contribute to the generation of charged fermion masses. Due to the absence of a right-handed neutrino, no such Dirac mass term is allowed for the neutrinos. However, in this work, we show that the presence of the Yukawa coupling of the $\mathbf{50}_H$ Higgs to the fermions in Eq. (2.3) plays an important role in generating nonzero neutrino mass via two-loop mechanism. We show that this new Yukawa interaction:

$$\mathcal{L}_Y \supset Y_{5ij}\mathbf{10}_{\boldsymbol{F}\alpha\beta,i}\mathbf{10}_{\boldsymbol{F}\gamma\delta,j}\mathbf{50}_{\boldsymbol{H}}{}^{\alpha\beta\gamma\delta}, \tag{3.1}$$

along with the already existing Yukawa interactions given in Eq. (2.4) combinedly determine the neutrino mass. Hence, neutrino mass does not appear to be completely detached, rather it gets intertwined with the charged fermion masses. Here the Yukawa coupling Y_5 is a symmetric 3×3 matrix in the generation space. Since neutrino mass appears at the two-loop level in this model, the neutrino masses are highly suppressed compared to the charged fermions, hence naturally explains the smallness of the neutrino masses. The decomposition of this Higgs fields under the SM is as follows:

$$\mathbf{50_H} \equiv \chi = \chi_1(1,1,-2) \oplus \chi_2\left(3,1,-\frac{1}{3}\right) \oplus \chi_3\left(\bar{3},2,-\frac{7}{6}\right) \oplus \chi_4\left(6,1,\frac{4}{3}\right) \oplus \chi_5\left(\bar{6},3,-\frac{1}{3}\right) \oplus \chi_6\left(8,2,\frac{1}{2}\right). \tag{3.2}$$

In the context of the SM gauge group, the possibility of generating neutrino mass via two-loop is well known [35]. The simplest possibility is to add a singly charged scalar and a doubly charged scalar, both singlets under the SM group and commonly known as the Zee-Babu model [36] as shown in Fig. 1 (Feynman diagram on the right). Many variations of the Zee-Babu model are proposed in the literature by extending the SM particle content. Note that in both the one-loop (Zee model) and the two-loop (Zee-Babu model) neutrino mass mechanism, at least two new multiplets need to be added to the theory to generate nonzero neutrino mass. In the original Zee model, in addition to a second SM-like Higgs doublet, a singly charged scalar singlet needs to be added. In the original version of the Zee-Babu model, again two BSM multiplets, one singly charged singlet and one doubly charged singlet need to be introduced. Below, we show that to realize two-loop neutrino mass in the context of renormalizable $\boldsymbol{SU(5)}$ GUT, at least two new multiplets need to be added to the MRSU5.

Our framework incorporates the Zee-Babu mechanism to explain the extremely small neutrino mass. The Yukawa coupling given in Eq. (3.1) has doubly charged scalar couplings to two charged leptons (suppressing the group indices):

$$Y_{5ij}\mathbf{10}_{\boldsymbol{F}i}\mathbf{10}_{\boldsymbol{F}j}\mathbf{50_H} \supset Y_{5ij}\ell^c_i\ell^c_j\chi_1^{--}, \quad (\ell^c_i = e^c, \mu^c, \tau^c). \tag{3.3}$$

Now to complete the loop-diagram, one must introduce at least one more Higgs multiplet, which is however not arbitrary but unambiguously determined by the group theory. The simplest possibility is to add a $\mathbf{40_H}$-dimensional representation that has the following decomposition under the SM:

$$\mathbf{40_H} \equiv \eta = \eta_1\left(1,2,-\frac{3}{2}\right) \oplus \eta_2\left(\bar{3},1,-\frac{2}{3}\right) \oplus \eta_3\left(3,2,\frac{1}{6}\right) \oplus \eta_4\left(\bar{3},3,-\frac{2}{3}\right) \oplus \eta_5\left(\bar{6},2,\frac{1}{6}\right) \oplus \eta_6(8,1,1). \tag{3.4}$$

Note that $\mathbf{40_H}$ Higgs has an isospin doublet $\eta_1 = (\eta_1^-\eta_1^{--})^T$ with a hypercharge of $Y = -3/2$ that is necessary to close the loop-diagram. The $\boldsymbol{SU(5)}$ invariant scalar potential contains cubic terms relevant for neutrino mass generation that are of the form:

$$\begin{aligned} V &\supset \mu_1 \mathbf{5}_{\boldsymbol{H}\gamma}\mathbf{50}_{\boldsymbol{H}}{}^{\alpha\beta\gamma\delta}\mathbf{40}_{\boldsymbol{H}}{}^*{}_{\alpha\beta\gamma} + \mu_2 \mathbf{45}_{\boldsymbol{H}\gamma\delta}^{\rho}\mathbf{50}_{\boldsymbol{H}}{}^{\alpha\beta\gamma\delta}\mathbf{40}_{\boldsymbol{H}}{}^*{}_{\alpha\beta\rho} \\ &\supset \chi_1^{--}\eta_1^{+}\left(\left(\frac{1}{\sqrt{2}}\right)\mu_1\phi_1^{+} + \left(-\frac{\sqrt{3}}{2}\right)\mu_2\Sigma_1^{+}\right) \\ &\quad + \chi_1^{--}\eta_1^{++}\left(\left(-\frac{1}{\sqrt{2}}\right)\mu_1\phi_1^{0} + \left(\frac{\sqrt{3}}{2}\right)\mu_2\Sigma_1^{0}\right). \end{aligned} \tag{3.5}$$

Where $\phi_1^+ \subset \phi_1(1,2,\frac{1}{2})$ and $\Sigma_1^+ \subset \Sigma_1(1,2,\frac{1}{2})$ are the singly charged scalars from the SM like doublets. And the relevant quartic terms in the potential to complete the loop-diagram are of the form:

$$\begin{aligned} V &\supset \mathbf{40}_H{}^{\alpha\beta\gamma}(\lambda \mathbf{5}_{H\gamma}\mathbf{5}_{H\delta}\mathbf{45}_{H\alpha\beta}^{\delta} + \lambda' \mathbf{5}_{H\sigma}\mathbf{45}_{H\alpha\beta}^{\delta}\mathbf{45}_{H\gamma\delta}^{\sigma}) \\ &\supset \eta_1^-\phi_1^+\Sigma_1^0\left(\left(\frac{\sqrt{3}}{2}\right)\lambda\phi_1^0 + \left(-\frac{3}{4\sqrt{2}}\right)\lambda'\Sigma_1^0\right) \\ &\quad + \eta_1^-\Sigma_1^+\phi_1^0\left(\left(\frac{-\sqrt{3}}{2}\right)\lambda\phi_1^0 + \left(\frac{3}{4\sqrt{2}}\right)\lambda'\Sigma_1^0\right) \\ &\quad + \eta_1^{--}\phi_1^+\Sigma_1^0\left(\left(\frac{\sqrt{3}}{2}\right)\lambda\phi_1^+ + \left(-\frac{3}{4\sqrt{2}}\right)\lambda'\Sigma_1^+\right) \\ &\quad + \eta_1^{--}\Sigma_1^+\phi_1^0\left(\left(\frac{-\sqrt{3}}{2}\right)\lambda\phi_1^+ + \left(\frac{3}{4\sqrt{2}}\right)\lambda'\Sigma_1^+\right). \end{aligned} \tag{3.6}$$

With the simultaneous presence of the Yukawa coupling Eq. (3.1), the scalar cubic couplings Eq. (3.5) and the scalar quartic couplings Eq. (3.6), the accidental global $U(1)_{B-L}$ is broken that is required to generate nonzero neutrino mass. With these relevant cubic and quartic terms in the scalar potential, the diagrams responsible for generating neutrino mass in our model is presented are Fig. 2. The scalar multiplets beyond the MRSU5 running in the loop belonging to the $\mathbf{40}_H$ and $\mathbf{50}_H$ representations are shown in red.

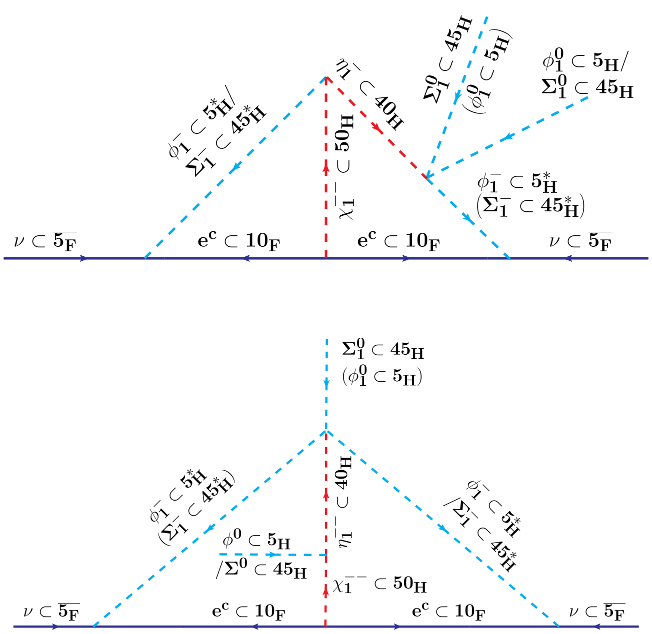

FIG. 2. Two-loop Feynman diagrams responsible for neutrino mass generation in our proposed renormalizable $SU(5)$ GUT. The propagators in red are the multiplets that belong to the $\mathbf{40}_H$ and $\mathbf{50}_H$ representations. For each of these diagrams, there is a second set of diagrams that can be achieved by replacing the multiplet by the associated multiplet shown in the parenthesis.

These BSM particle contributing to the generation of neutrino mass are expected to live at scales much below the GUT scale. Note that similar diagrams with colored particles running in the loop can also be drawn. For example, instead of $(1,2,\frac{1}{2}) \subset \mathbf{5}_H$ running in the loop, one can replace it by $(3,1,-\frac{1}{3}) \subset \mathbf{5}_H$. However, since these colored triplets mediate dangerous proton decay, their masses are assumed to be of the order of GUT scale to suppress the proton decay rate, hence we do not consider such diagrams, but can be trivially included. In the context of the SM, similar diagrams as shown in Fig. 2 are realized recently by extending the SM with three new fields, a doubly charged scalar singlet and two doublets, a second SM-like doublet with hypercharge of $1/2$, and a third doublet with hypercharge $3/2$ in two different works [37,38].

Here we compute the neutrino mass matrix. First note that the breaking of the EW symmetry allows mixings of the particles carrying the same electric charge. Mixing among the singly charged fields are induced by the quartic terms of Eq. (3.6), whereas for the doubly charged particles are induced by the cubic terms of Eq. (3.5). After the breaking of the EW symmetry, the part of the scalar potential containing the relevant mixing terms are given by:

$$\begin{aligned} V &\supset (\,\phi_1^0 \quad \Sigma_1^0\,)\begin{pmatrix} m_\phi^{0\,2} & m_{12}^{0\,2} \\ m_{12}^{0\,2} & m_\Sigma^{0\,2} \end{pmatrix}\begin{pmatrix} \phi_1^{0*} \\ \Sigma_1^{0*} \end{pmatrix} \\ &\quad + (\,\chi_1^{--} \quad \eta_1^{--}\,)\begin{pmatrix} m_\chi^{++2} & m_{12}^{++2} \\ m_{12}^{++2} & m_\eta^{++2} \end{pmatrix}\begin{pmatrix} \chi_1^{++} \\ \eta_1^{++} \end{pmatrix} \\ &\quad + (\,\phi_1^+ \quad \Sigma_1^+ \quad \eta_1^+\,)\begin{pmatrix} m_\phi^{+2} & m_{12}^{+2} & m_{13}^{+2} \\ m_{12}^{+2} & m_\Sigma^{+2} & m_{23}^{+2} \\ m_{13}^{+2} & m_{23}^{+2} & m_\eta^{+2} \end{pmatrix}\begin{pmatrix} \phi_1^- \\ \Sigma_1^- \\ \eta_1^- \end{pmatrix}. \end{aligned} \tag{3.7}$$

Here the off-diagonal entries are the mixing terms as already mentioned above and the diagonal entries are the mass terms which for simplicity we do not write down explicitly, however can be computed straightforwardly from the full potential. In the next section, we will construct part of the scalar potential that is relevant for the study of the gauge coupling unification. For simplicity, treating all the parameters of the scalar potential appearing in Eq. (3.7) to be real, the transformation between the weak basis and the mass basis for the CP-even neutral fields, the doubly charged scalars and the singly charged scalars can be written as:

$$\begin{pmatrix} \phi_1^0 \\ \Sigma_1^0 \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}\begin{pmatrix} H_1^0 \\ H_2^0 \end{pmatrix}, \tag{3.8}$$

$$\begin{pmatrix} \chi_1^{--} \\ \eta_1^{--} \end{pmatrix} = \begin{pmatrix} \cos\omega & \sin\omega \\ -\sin\omega & \cos\omega \end{pmatrix}\begin{pmatrix} H_1^{--} \\ H_2^{--} \end{pmatrix}, \tag{3.9}$$

$$\begin{pmatrix}\phi_1^+\\ \Sigma_1^+\\ \eta_1^+\end{pmatrix} = \begin{pmatrix}1 & 0 & 0\\ 0 & \cos\theta_{23} & \sin\theta_{23}\\ 0 & -\sin\theta_{23} & \cos\theta_{23}\end{pmatrix}\begin{pmatrix}\cos\theta_{13} & 0 & \sin\theta_{13}\\ 0 & 1 & 0\\ -\sin\theta_{13} & 0 & \cos\theta_{13}\end{pmatrix} \times \begin{pmatrix}\cos\theta_{12} & \sin\theta_{12} & 0\\ -\sin\theta_{12} & \cos\theta_{12} & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}G^+\\ H_1^+\\ H_2^+\end{pmatrix}. \tag{3.10}$$

Here, the fields labeled with H (H_i^+, H_i^{++}) represent the mass eigenstates and G^+ is the Goldstone boson. This leads to:

$$\mathcal{L} \supset Y_1 \bar{\mathbf{5}}_{\boldsymbol{F}} \mathbf{10}_{\boldsymbol{F}} \mathbf{5}^*_{\boldsymbol{H}} + Y_1 \bar{\mathbf{5}}_{\boldsymbol{F}} \mathbf{10}_{\boldsymbol{F}} \mathbf{45}^*_{\boldsymbol{H}} + Y_5 \mathbf{10}_{\boldsymbol{F}} \mathbf{10}_{\boldsymbol{F}} \mathbf{50}_{\boldsymbol{H}} \tag{3.11}$$

$$\supset \nu_{Li}\ell^c_j(H_1^- Y^+_{1\,ij} + H_2^- Y^+_{2\,ij}) + \ell^c_i\ell^c_j(H_1^{--}Y_1^{++}{}_{ij} + H_2^{--}Y_2^{++}{}_{ij}), \tag{3.12}$$

where we have defined,

$$Y_1^+ = \frac{1}{\sqrt{2}}Y_1(c_{13}s_{12}) + \frac{\sqrt{3}}{2}Y_2(c_{12}c_{23} - s_{12}s_{13}s_{23}), \tag{3.13}$$

$$Y_2^+ = \frac{1}{\sqrt{2}}Y_1(s_{13}) + \frac{\sqrt{3}}{2}Y_2(c_{13}s_{23}), \tag{3.14}$$

$$Y_1^{++} = Y_5 c_\omega, \tag{3.15}$$

$$Y_2^{++} = Y_5 s_\omega, \tag{3.16}$$

and we have made use of the notation: $c_\omega = \cos\omega$, $s_\omega = \sin\omega$, $c_{ij} = \cos_{\theta_{ij}}$, $s_{ij} = \sin_{\theta_{ij}}$.

Furthermore, we get:

$$V \supset \mu_1 \mathbf{5}_{\boldsymbol{H}\gamma} \mathbf{50}_{\boldsymbol{H}}{}^{\alpha\beta\gamma\delta} \mathbf{40}_{\boldsymbol{H}}{}^*{}_{\alpha\beta\gamma} + \mu_2 \mathbf{45}_{\boldsymbol{H}}{}^{\rho}_{\gamma\delta} \mathbf{50}_{\boldsymbol{H}}{}^{\alpha\beta\gamma\delta} \mathbf{40}_{\boldsymbol{H}}{}^*{}_{\alpha\beta\rho}$$
$$\supset (\mu_{11}H_1^+H_1^+ + \mu_{22}H_2^+H_2^+ + \mu_{12}H_1^+H_2^+ + \mu_{21}H_2^+H_1^+) \times (c_\omega H_1^{--} + s_\omega H_2^{--}). \tag{3.17}$$

Where,

$$\mu_{11} = (-c_{12}s_{23} - s_{12}c_{23}s_{13})\tilde{\mu}_1, \quad \mu_{22} = (c_{23}c_{13})\tilde{\mu}_2, \tag{3.18}$$

$$\mu_{12} = (-c_{12}s_{23} - s_{12}c_{23}s_{13})\tilde{\mu}_2, \quad \mu_{21} = (c_{23}c_{13})\tilde{\mu}_1, \tag{3.19}$$

$$\tilde{\mu}_1 = \frac{\mu_1}{\sqrt{2}}(c_{13}s_{12}) - \frac{\sqrt{3}\mu_2}{2}(c_{12}c_{23} - s_{12}s_{13}s_{23}), \tag{3.20}$$

$$\tilde{\mu}_2 = \frac{\mu_1}{\sqrt{2}}(s_{13}) - \frac{\sqrt{3}\mu_2}{2}(c_{13}s_{23}). \tag{3.21}$$

Then, the neutrino mass matrix is evaluated to be

$$\mathcal{M}_{\nu ij} = \mu_{AB} Y^+_{A\,ik} Y^*_{5kl} Y^+_{B\,lj}[c^2_\omega I_{1ABkl} + s^2_\omega I_{2ABkl}] + \text{transpose}. \tag{3.22}$$

It contains four terms corresponding to $AB = \{11, 22, 12, 21\}$. Here the sum over the repeated indices k and l is understood. The loop function is $I_{cabkl} = I(m_a^+, m_b^+, m_c^{++}, m_k, m_l)$, where $m_{k,l}$ are the mass of the SM fermions, m^+ and m^{++} are the mass of the singly and the doubly charged scalars running inside the loop. We evaluate this loop function as follows. To make life simple, we assume $m_1^+ = m_2^+$ and furthermore use the approximation $m_{c,a,b,} \gg m_{k,l}$, which is valid since charged lepton masses are small compared to the BSM charged scalars running in through the loop. Then one finds [38],

$$I_{cabkl} \approx I_{ca} = \frac{1}{(16\pi^2)^2}\int_0^1 dx \int_0^1 dy \left[\frac{-\ln(\Delta_{ca})}{1 - \Delta_{ca}}\right], \tag{3.23}$$

with,

$$\Delta_{ca} = \frac{(1-y)r^2_{ca} + y(1-x)}{y(1-y)}; \qquad r_{ca} = \frac{m_c^{++}}{m_a^+}. \tag{3.24}$$

Note that the loop function corresponding to our diagram is different from the one that appears in the conventional Zee-Babu model, it is due to different chirality structure. Unlike the conventional Zee-Babu model, chirality flip of the fermions does not take place inside the loop. The behavior of the loop function as a function of r^2_{ca} is presented in Fig. 3.

Here we note that the neutrino masses do not decouple from that of the charged fermion masses, rather they get entangled with them. To find the correlation, we express the Yukawa couplings Y_1 and Y_2 in terms of the down-quark and charged-lepton masses matrices from Eqs. (2.5)–(2.6) as:

$$Y_1 = \frac{\sqrt{2}}{4v_1}(3M_D + M_E^T), \tag{3.25}$$

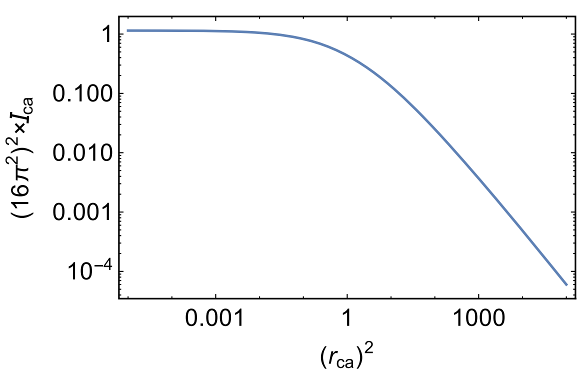

FIG. 3. The behavior of the loop function given in Eq. (3.23) as a function of $r^2_{ca} = (\frac{m_c^{++}}{m_a^+})^2$.

$$Y_2 = \frac{-\sqrt{3}}{2v_2}(M_D - M_E^T). \tag{3.26}$$

As a result, $Y_{1,2}^{+}$ can also be expressed in terms of down-quark and charged-lepton masses matrices:

$$Y_1^{+} = \frac{3}{4}\left(\frac{c_{13}s_{12}}{v_1} - \frac{c_{12}c_{23} - s_{12}s_{13}s_{23}}{v_2}\right) D^c M_D^{\mathrm{diag}} V_{\mathrm{CKM}}^{\dagger} + \frac{1}{4}\left(\frac{c_{13}s_{12}}{v_1} + \frac{3(c_{12}c_{23} - s_{12}s_{13}s_{23})}{v_2}\right) M_E^{\mathrm{diag}}, \tag{3.27}$$

$$Y_2^{+} = \frac{3}{4}\left(\frac{s_{13}}{v_1} - \frac{c_{13}s_{23}}{v_2}\right) D^c M_D^{\mathrm{diag}} V_{\mathrm{CKM}}^{\dagger} + \frac{1}{4}\left(\frac{s_{13}}{v_1} - \frac{3c_{13}s_{23}}{v_2}\right) M_E^{\mathrm{diag}}. \tag{3.28}$$

Here we have gone to a basis where the up-quark and charged lepton mass matrices are diagonal. In this rotated basis, the Y_5 matrix takes the form: $E^c Y_5 E^{cT}$. To get to these relations, we used the following convention for diagonalization of the charged fermion masses:

$$M_U = U^c M_U^{\mathrm{diag}} U, \qquad M_D = D^c M_D^{\mathrm{diag}} D,$$
$$M_E = E^c M_E^{\mathrm{diag}} E, \quad (U^c = U^T). \tag{3.29}$$

Here we provide an example of how correct order of neutrino mass can be achieved. The neutrino mass has the from, $m_\nu \sim y^3 \mu I$. To reproduce the correct tau mass, the biggest entry (33 entry) needs to be of order $Y_{1,2} \sim 10^{-2}$. So assuming Yukawa couplings of this order, for $\mu \sim 1$ TeV, one can get $m_\nu \sim 10^{-10}$ GeV for $r_{ca} \sim 40$. However, this choice is not unique and presented only for a demonstration for natural values of the Yukawa couplings.

To the best of the author's knowledge, the presented model in this work is the only *true two-loop* neutrino mass in the context of $\boldsymbol{SU(5)}$ GUT. Note that, one can think of embedding the original two-loop Zee-Babu mechanism [36] (right diagram in Fig. 1) in $\boldsymbol{SU(5)}$ GUT. Zee-Babu mechanism requires a singly charged singlet (1, 1, 1) and a doubly charged singlet (1, 1, 2) under the SM. These multiplets can be embedded in $(1,1,1) \subset \mathbf{10_H}$ and $(1,1,-2) \subset \mathbf{50_H}$ representations of $\boldsymbol{SU(5)}$. Introduction of $\mathbf{10_H}$ brings new Yukawa couplings into the theory contained in the following bilinear:

$$\bar{\mathbf{5}} \times \bar{\mathbf{5}} = \overline{\mathbf{10}} + \overline{\mathbf{15}}. \tag{3.30}$$

Hence the requirement of both $\mathcal{L} \supset Y_5 \mathbf{10_F 10_F 50_H}$ and $\mathcal{L} \supset Y_6 \bar{\mathbf{5}}_F \bar{\mathbf{5}}_F \mathbf{10_H}$ Yukawa couplings into the theory are required, where Y_5 is a symmetric 3×3 matrix where as Y_6 is antisymmetric 3×3 matrix in the flavor space. However, the presence of the Yukawa coupling Y_6 and the allowed gauge invariant cubic term $\mu \mathbf{5_H 5_H 10_H^*}$ in the scalar potential automatically leads to one-loop diagram via Zee-mechanism [29] (left diagram in Fig. 1). This is why, such an embedding which was realized in [39], cannot be a *true two-loop* neutrino mass model. Similar conclusion can be reached for the $\boldsymbol{SU(5)}$ model presented in [40], due to the presence of $\mathbf{10_H}$ Higgs, in addition to their two-loop diagram, one-loop diagram of the Zee-type automatically appears. So the model presented in this work is unique in its features.

IV. SCALAR POTENTIAL AND THE HIGGS BOSONS MASS SPECTRUM

As aforementioned, the minimal model consists of the Higgs set $\mathbf{24_H}$, $\mathbf{5_H}$, and $\mathbf{45_H}$. However, this model is still defective since neutrinos remain massless. In the previous section it is shown that to build a *true two-loop* neutrino mass model the minimal model needs to be extended by two more Higgs multiplet $\mathbf{40_H}$ and $\mathbf{50_H}$. In this section, we compute the Higgs mass spectrum of the set $\mathbf{5_H} + \mathbf{24_H} + \mathbf{40_H} + \mathbf{45_H} + \mathbf{50_H}$ after the GUT symmetry is broken spontaneously. This analysis is performed to find the Higgs mass relationships, which will be relevant for our study of the gauge coupling unification performed in the next section. The tensorial properties of all the Higgs multiplets in our framework are presented in Table I.

We are interested in the mass spectrum of the Higgs bosons as a result of breaking of the GUT symmetry down to the SM gauge group. The only field that acquires a vacuum expectation value (VEV) at this stage is the $\mathbf{24_H}$ as a result, the mass of the Higgs multiplets come from the interaction with the $\mathbf{24_H}$ representation. For our purpose of

TABLE I. Particle content of our model and their relevant properties.

Fields	Notation	Properties
$\mathbf{24_H}$	$\boldsymbol{\Phi}_\alpha^\beta$	adjoint 24-dimensional real traceless, $\Phi_\alpha^\alpha = 0$
$\mathbf{5_H}$	$\boldsymbol{\phi}_\alpha$	fundamental 5-dimensional complex
$\mathbf{45_H}$	$\boldsymbol{\Sigma}_{\alpha\beta}^\gamma$	45-dimensional complex, antisymmetric under, $\boldsymbol{\Sigma}_{\alpha\beta}^\gamma = -\boldsymbol{\Sigma}_{\beta\alpha}^\gamma$ and traceless, $\boldsymbol{\Sigma}_{\alpha\beta}^\alpha = 0$
$\mathbf{40_H}$	$\boldsymbol{\eta}^{\alpha\beta\gamma}$	40-dimensional complex, antisymmetric under, $\boldsymbol{\eta}^{\alpha\beta\gamma} = -\boldsymbol{\eta}^{\beta\alpha\gamma}$ and $\boldsymbol{\eta}^{\alpha\beta\gamma}\epsilon_{\alpha\beta\gamma\rho\sigma} = 0$
$\mathbf{50_H}$	$\boldsymbol{\chi}^{\alpha\beta\gamma\delta}$	50-dimensional complex, symmetric under, $\boldsymbol{\chi}^{\alpha\beta\gamma\delta} = \boldsymbol{\chi}^{\gamma\delta\alpha\beta}$ antisymmetric under, $\boldsymbol{\chi}^{\alpha\beta\gamma\delta} = -\boldsymbol{\chi}^{\beta\alpha\gamma\delta} = -\boldsymbol{\chi}^{\alpha\beta\delta\gamma}$ and additionally $\boldsymbol{\chi}^{\alpha\beta\gamma\delta}\epsilon_{\xi\alpha\beta\gamma\delta} = 0$

this analysis, the effect of the EW scale VEVs of the $\mathbf{5_H}$ and $\mathbf{45_H}$ fields can be completely ignored. Then the relevant part of the scalar potential contributing to their masses is given by:

$$V = V_{24} + V_{24,5} + V_{24,45} + V_{24,40} + V_{24,50} + V_{\rm mix}, \qquad (4.1)$$

where,

$$V_{24} = \frac{1}{2} m_{24}^2 \Phi^\beta_\alpha \Phi^\alpha_\beta + \mu_{24} \Phi^\beta_\alpha \Phi^\gamma_\beta \Phi^\alpha_\gamma + \lambda_1 (\Phi^\beta_\alpha \Phi^\alpha_\beta)^2 + \lambda_2 \Phi^\beta_\alpha \Phi^\gamma_\beta \Phi^\delta_\gamma \Phi^\alpha_\delta, \qquad (4.2)$$

$$V_{24,5} = m_5^2 \phi_\alpha \phi^{*\alpha} + \mu_5 \phi_\alpha \phi^{*\beta} \Phi^\alpha_\beta + \alpha_1 (\phi_\alpha \phi^{*\alpha})(\Phi^\beta_\gamma \Phi^\gamma_\beta) + \alpha_2 \phi_\alpha \phi^{*\beta} \Phi^\gamma_\beta \Phi^\alpha_\gamma, \qquad (4.3)$$

$$V_{24,45} = m_{45}^2 \Sigma^\gamma_{\alpha\beta} \Sigma^{*\alpha\beta}_\gamma + \mu_{45} \Sigma^\gamma_{\alpha\beta} \Sigma^{*\alpha\beta}_\delta \Phi^\delta_\gamma + \mu'_{45} \Sigma^\gamma_{\alpha\beta} \Sigma^{*\beta\delta}_\gamma \Phi^\alpha_\delta + \xi_1 (\Sigma^\gamma_{\alpha\beta} \Sigma^{*\alpha\beta}_\gamma)(\Phi^\sigma_\rho \Phi^\rho_\sigma) + \xi_2 \Sigma^\gamma_{\alpha\beta} \Sigma^{*\alpha\beta}_\delta \Phi^\rho_\gamma \Phi^\delta_\rho + \xi_3 \Sigma^\gamma_{\alpha\beta} \Sigma^{*\alpha\delta}_\gamma \Phi^\rho_\delta \Phi^\beta_\rho + \xi_4 \Sigma^\gamma_{\alpha\beta} \Sigma^{*\alpha\delta}_\rho \Phi^\beta_\gamma \Phi^\rho_\delta + \xi_5 \Sigma^\gamma_{\alpha\beta} \Sigma^{*\alpha\delta}_\rho \Phi^\rho_\gamma \Phi^\beta_\delta + \xi_6 \Sigma^\gamma_{\alpha\beta} \Sigma^{*\delta\rho}_\gamma \Phi^\alpha_\delta \Phi^\beta_\rho, \qquad (4.4)$$

$$V_{24,40} = m_{40}^2 \eta^{\alpha\beta\gamma} \eta^*_{\alpha\beta\gamma} + \mu_{40} \eta^{\alpha\beta\gamma} \eta^*_{\alpha\beta\delta} \Phi^\delta_\gamma + \mu'_{40} \eta^{\alpha\beta\gamma} \eta^*_{\delta\beta\gamma} \Phi^\delta_\alpha + \omega_1 (\eta^{\alpha\beta\gamma} \eta^*_{\alpha\beta\gamma})(\Phi^\sigma_\rho \Phi^\rho_\sigma) + \omega_2 \eta^{\alpha\beta\gamma} \eta^*_{\alpha\beta\delta} \Phi^\sigma_\gamma \Phi^\delta_\sigma + \omega_3 \eta^{\alpha\beta\gamma} \eta^*_{\alpha\delta\gamma} \Phi^\sigma_\beta \Phi^\delta_\sigma + \omega_4 \eta^{\alpha\beta\gamma} \eta^*_{\delta\rho\gamma} \Phi^\delta_\alpha \Phi^\rho_\beta + \omega_5 \eta^{\alpha\beta\gamma} \eta^*_{\delta\beta\rho} \Phi^\delta_\gamma \Phi^\rho_\gamma, \qquad (4.5)$$

$$V_{24,50} = m_{50}^2 \chi^{\alpha\beta\gamma\delta} \chi^*_{\alpha\beta\gamma\delta} + \mu_{50} \chi^{\alpha\beta\gamma\delta} \chi^*_{\rho\beta\gamma\alpha} \Phi^\rho_\delta + \zeta_1 (\chi^{\alpha\beta\gamma\delta} \chi^*_{\alpha\beta\gamma\delta})(\Phi^\sigma_\rho \Phi^\rho_\sigma) + \zeta_2 \chi^{\alpha\beta\gamma\delta} \chi^*_{\delta\gamma\alpha\beta} \Phi^\sigma_\delta \Phi^\rho_\sigma + \zeta_3 \chi^{\alpha\beta\gamma\delta} \chi^*_{\gamma\delta\rho\sigma} \Phi^\rho_\alpha \Phi^\sigma_\beta, \qquad (4.6)$$

$$V_{\rm mix} = \mu_1 \Sigma^\gamma_{\alpha\beta} \Phi^\alpha_\gamma \phi^{*\beta} + \kappa_1 \Sigma^\gamma_{\alpha\beta} \Phi^\alpha_\gamma \Phi^\beta_\delta \phi^{*\delta} + \kappa_2 \Sigma^\gamma_{\alpha\beta} \Phi^\delta_\gamma \Phi^\alpha_\delta \phi^{*\beta} + \mu_2 \Phi^\beta_\alpha \chi^{\rho\sigma\alpha\delta} \Sigma^{*\tau\kappa}_\delta \epsilon_{\rho\sigma\tau\kappa\beta} + \kappa_3 \Phi^\alpha_\rho \Phi^\beta_\sigma \phi_\nu \chi^*_{\alpha\beta\tau\kappa} \epsilon^{\rho\sigma\tau\kappa\nu} + \kappa_4 \Phi^\beta_\alpha \Phi^\gamma_\beta \chi^{\rho\sigma\alpha\delta} \Sigma^{*\tau\kappa}_\delta \epsilon_{\rho\sigma\tau\kappa\gamma} + \text{H.c.}. \qquad (4.7)$$

The SM singlet component, $\Phi_1(1,1,0)$ of the adjoint Higgs acquires VEV, $\langle \mathbf{\Phi_H} \rangle \equiv V_{\rm GUT}$ and breaks the GUT symmetry down to the SM group. The minimization condition demands:

$$m_{24}^2 = -\lambda_1 V_{\rm GUT}^2 - \frac{7}{30} \lambda_2 V_{\rm GUT}^2 - \frac{1}{4}\sqrt{\frac{3}{5}} \mu_{24} V_{\rm GUT}. \qquad (4.8)$$

The multiplets $(3, 2, -\frac{5}{6})$ and $(3, 2, \frac{5}{6})$ from $\mathbf{24_H}$ field correspond to the Goldstone bosons and hence eaten up by the massive gauge bosons. The masses of the other multiplets in $\mathbf{24_H}$ are given by:

$$m_{\Phi_1}^2 = \frac{1}{60} V_{\rm GUT}(120\lambda_1 V_{\rm GUT} + 28\lambda_2 V_{\rm GUT} + 3\sqrt{15}\mu_{24}), \qquad (4.9)$$

$$m_{\Phi_2}^2 = \frac{1}{12} V_{\rm GUT}(8\lambda_2 V_{\rm GUT} + 3\sqrt{15}\mu_{24}), \qquad (4.10)$$

$$m_{\Phi_3}^2 = \frac{1}{12} V_{\rm GUT}(2\lambda_2 V_{\rm GUT} - 3\sqrt{15}\mu_{24}). \qquad (4.11)$$

The mass spectrum of the multiplets residing in $\mathbf{5_H}$ Higgs by neglecting mixing with other fields are given by:

$$m_{\phi_1}^2 = \frac{1}{30}(15\alpha_1 V_{\rm GUT}^2 + 2\alpha_2 V_{\rm GUT}^2 - 2\sqrt{15}\mu_5 V_{\rm GUT} + 30 m_5^2), \qquad (4.12)$$

$$m_{\phi_2}^2 = \frac{1}{30}(15\alpha_1 V_{\rm GUT}^2 + 2\alpha_2 V_{\rm GUT}^2 - 2\sqrt{15}\mu_5 V_{\rm GUT} + 30 m_5^2). \qquad (4.13)$$

Similarly, ignoring the mixing of the fields, the mass spectrum of the multiplets contained in the $\mathbf{45_H}$ Higgs are given by:

$$m_{\Sigma_1}^2 = \frac{1}{480}(28\sqrt{15}\mu_{45} V_{\rm GUT} - 38\sqrt{15}\mu'_{45} V_{\rm GUT} + 240\xi_1 V_{\rm GUT}^2 + 62\xi_2 V_{\rm GUT}^2 + 67\xi_3 V_{\rm GUT}^2 + 75\xi_4 V_{\rm GUT}^2 + 52\xi_5 V_{\rm GUT}^2 + 42\xi_6 V_{\rm GUT}^2 + 480 m_{45}^2), \qquad (4.14)$$

$$m_{\Sigma_2}^2 = \frac{1}{240}(4\sqrt{15}\mu_{45} V_{\rm GUT} + 6\sqrt{15}\mu'_{45} V_{\rm GUT} + 120\xi_1 V_{\rm GUT}^2 + 26\xi_2 V_{\rm GUT}^2 + 21\xi_3 V_{\rm GUT}^2 + 50\xi_4 V_{\rm GUT}^2 + 11\xi_5 V_{\rm GUT}^2 - 4\xi_6 V_{\rm GUT}^2 + 240 m_{45}^2), \qquad (4.15)$$

$$m_{\Sigma_3}^2 = \frac{1}{60}(-4\sqrt{15}\mu_{45} V_{\rm GUT} - 6\sqrt{15}\mu'_{45} V_{\rm GUT} + 30\xi_1 V_{\rm GUT}^2 + 4\xi_2 V_{\rm GUT}^2 + 9\xi_3 V_{\rm GUT}^2 - 6\xi_5 V_{\rm GUT}^2 + 9\xi_6 V_{\rm GUT}^2 + 60 m_{45}^2), \qquad (4.16)$$

$$m_{\Sigma_4}^2 = \frac{1}{60}(6\sqrt{15}\mu_{45} V_{\rm GUT} + 4\sqrt{15}\mu'_{45} V_{\rm GUT} + 30\xi_1 V_{\rm GUT}^2 + 9\xi_2 V_{\rm GUT}^2 + 4\xi_3 V_{\rm GUT}^2 - 6\xi_5 V_{\rm GUT}^2 + 4\xi_6 V_{\rm GUT}^2 + 60 m_{45}^2), \qquad (4.17)$$

$$m_{\Sigma_5}^2 = \frac{1}{120}(12\sqrt{15}\mu_{45} V_{\rm GUT} - 2\sqrt{15}\mu'_{45} V_{\rm GUT} + 60\xi_1 V_{\rm GUT}^2 + 18\xi_2 V_{\rm GUT}^2 + 13\xi_3 V_{\rm GUT}^2 + 3\xi_5 V_{\rm GUT}^2 - 12\xi_6 V_{\rm GUT}^2 + 120 m_{45}^2), \qquad (4.18)$$

$$m_{\Sigma_6}^2 = \frac{1}{30}(-2\sqrt{15}\mu_{45} V_{\rm GUT} + 2\sqrt{15}\mu'_{45} V_{\rm GUT} + 15\xi_1 V_{\rm GUT}^2 + 2\xi_2 V_{\rm GUT}^2 + 2\xi_3 V_{\rm GUT}^2 + 2\xi_5 V_{\rm GUT}^2 + 2\xi_6 V_{\rm GUT}^2 + 30 m_{45}^2), \qquad (4.19)$$

$$m^2_{\Sigma_7} = \frac{1}{120}(-8\sqrt{15}\mu_{45}V_{\rm GUT} - 2\sqrt{15}\mu'_{45}V_{\rm GUT} + 60\xi_1 V^2_{\rm GUT} + 8\xi_2 V^2_{\rm GUT} + 13\xi_3 V^2_{\rm GUT} - 2\xi_5 V^2_{\rm GUT} - 12\xi_6 V^2_{\rm GUT} + 120 m^2_{45}). \tag{4.20}$$

Mass spectrum of the multiplets of $\mathbf{40}_H$ field:

$$m^2_{\eta_1} = \frac{1}{20}(2\sqrt{15}\mu_{40}V_{\rm GUT} + 2\sqrt{15}\mu'_{40}V_{\rm GUT} + 10\omega_1 V^2_{\rm GUT} + 3\omega_2 V^2_{\rm GUT} + 3\omega_3 V^2_{\rm GUT} + 3\omega_4 V^2_{\rm GUT} + 3\omega_5 V^2_{\rm GUT} + 20 m^2_{40}), \tag{4.21}$$

$$m^2_{\eta_2} = \frac{1}{360}(-4\sqrt{15}\mu_{40}V_{\rm GUT} + 26\sqrt{15}\mu'_{40}V_{\rm GUT} + 180\omega_1 V^2_{\rm GUT} + 34\omega_2 V^2_{\rm GUT} + 49\omega_3 V^2_{\rm GUT} + 24\omega_4 V^2_{\rm GUT} - 21\omega_5 V^2_{\rm GUT} + 360 m^2_{40}), \tag{4.22}$$

$$m^2_{\eta_3} = \frac{1}{360}(16\sqrt{15}\mu_{40}V_{\rm GUT} - 14\sqrt{15}\mu'_{40}V_{\rm GUT} + 180\omega_1 V^2_{\rm GUT} + 44\omega_2 V^2_{\rm GUT} + 29\omega_3 V^2_{\rm GUT} + 4\omega_4 V^2_{\rm GUT} - 26\omega_5 V^2_{\rm GUT} + 360 m^2_{40}), \tag{4.23}$$

$$m^2_{\eta_4} = \frac{1}{120}(12\sqrt{15}\mu_{40}V_{\rm GUT} + 2\sqrt{15}\mu'_{40}V_{\rm GUT} + 60\omega_1 V^2_{\rm GUT} + 18\omega_2 V^2_{\rm GUT} + 13\omega_3 V^2_{\rm GUT} - 12\omega_4 V^2_{\rm GUT} + 3\omega_5 V^2_{\rm GUT} + 120 m^2_{40}), \tag{4.24}$$

$$m^2_{\eta_5} = \frac{1}{120}(-8\sqrt{15}\mu_{40}V_{\rm GUT} + 2\sqrt{15}\mu'_{40}V_{\rm GUT} + 60\omega_1 V^2_{\rm GUT} + 8\omega_2 V^2_{\rm GUT} + 13\omega_3 V^2_{\rm GUT} - 12\omega_4 V^2_{\rm GUT} - 2\omega_5 V^2_{\rm GUT} + 120 m^2_{40}), \tag{4.25}$$

$$m^2_{\eta_6} = \frac{1}{30}(-2\sqrt{15}\mu_{40}V_{\rm GUT} - 2\sqrt{15}\mu'_{40}V_{\rm GUT} + 15\omega_1 V^2_{\rm GUT} + 2\omega_2 V^2_{\rm GUT} + 2\omega_3 V^2_{\rm GUT} + 2\omega_4 V^2_{\rm GUT} + 2\omega_5 V^2_{\rm GUT} + 30 m^2_{40}). \tag{4.26}$$

And finally the mass spectrum of the multiplets residing in $\mathbf{50}_H$ field:

$$m^2_{\chi_1} = \frac{1}{20}(10\zeta V^2_{\rm GUT} - 3\zeta_1 V^2_{\rm GUT} + 3\zeta_2 V^2_{\rm GUT} + \sqrt{15}\mu_{50}V_{\rm GUT} + 20 m^2_{50}), \tag{4.27}$$

$$m^2_{\chi_2} = \frac{1}{360}(180\zeta V^2_{\rm GUT} - 39\zeta_1 V^2_{\rm GUT} + 14\zeta_2 V^2_{\rm GUT} + 3\sqrt{15}\mu_{50}V_{\rm GUT} + 360 m^2_{50}), \tag{4.28}$$

$$m^2_{\chi_3} = \frac{1}{240}(120\zeta V^2_{\rm GUT} - 31\zeta_1 V^2_{\rm GUT} + 6\zeta_2 V^2_{\rm GUT} + 7\sqrt{15}\mu_{50}V_{\rm GUT} + 240 m^2_{50}), \tag{4.29}$$

$$m^2_{\chi_4} = \frac{1}{30}(15\zeta V^2_{\rm GUT} - 2\zeta_1 V^2_{\rm GUT} + 2\zeta_2 V^2_{\rm GUT} - \sqrt{15}\mu_{50}V_{\rm GUT} + 30 m^2_{50}), \tag{4.30}$$

$$m^2_{\chi_5} = \frac{1}{120}(60\zeta V^2_{\rm GUT} - 13\zeta_1 V^2_{\rm GUT} - 12\zeta_2 V^2_{\rm GUT} + \sqrt{15}\mu_{50}V_{\rm GUT} + 120 m^2_{50}), \tag{4.31}$$

$$m^2_{\chi_6} = \frac{1}{240}(120\zeta V^2_{\rm GUT} - 21\zeta_1 V^2_{\rm GUT} - 4\zeta_2 V^2_{\rm GUT} - 3\sqrt{15}\mu_{50}V_{\rm GUT} + 240 m^2_{50}). \tag{4.32}$$

These mass spectrum helps one to understand whether splitting among different multiplets originating from the same field is possible or not. Splitting among different multiplets for some of the fields is necessary to achieve unification to be discussed in the next section. From the mass spectrum computed above, it can be realized that, due to enough number of parameters, there is no mass relationship among the multiplets of $\mathbf{40}_H$. This is also true for $\mathbf{5}_H$, $\mathbf{24}_H$ and $\mathbf{45}_H$. However, which is not true for the multiplets contained in $\mathbf{50}_H$ and from the above calculation we find:

$$m^2_{\chi_4} = 3m^2_{\chi_2} - 2m^2_{\chi_3}, \tag{4.33}$$

$$m^2_{\chi_5} = 2m^2_{\chi_3} - m^2_{\chi_1}, \tag{4.34}$$

$$m^2_{\chi_6} = \frac{3}{2}m^2_{\chi_2} - \frac{1}{2}m^2_{\chi_1}. \tag{4.35}$$

For the study of the gauge coupling unification, we impose the mass relations as derived above.

Till now, we have ignored the mixings among the multiplets having the same quantum number coming from different Higgs representations. For completeness here we take into account such mixings. Note that the relevant mixing terms are contained in the $V_{\rm mix}$ term given in Eq. (4.7). Now taking these mixed terms into consideration, the mixing between the isospin doublets, $(1, 2, \frac{1}{2})$ present in $\mathbf{5}_H$ and $\mathbf{45}_H$ representations are given by:

$$\begin{pmatrix} \phi_1^{(D)} & \Sigma_1^{(D)} \end{pmatrix} \begin{pmatrix} m^2_{\phi_1} & m^2_{D_{12}} \\ m^2_{D_{12}} & m^2_{\Sigma_1} \end{pmatrix} \begin{pmatrix} \phi_1^{(D)*} \\ \Sigma_1^{(D)*} \end{pmatrix}, \tag{4.36}$$

with,

$$m^2_{D_{12}} = \frac{1}{24\sqrt{2}} V_{\rm GUT}(3\sqrt{3}\kappa_1 V_{\rm GUT} + \sqrt{3}\kappa_2 V_{\rm GUT} + 6\sqrt{5}\mu_1). \tag{4.37}$$

Furthermore, the color triplets, $(3,1,-1/3)$ contained in $\mathbf{5_H}$, $\mathbf{45_H}$ and $\mathbf{50_H}$ mix with each and we find:

$$(\phi_2^{(T)}\ \ \Sigma_2^{(T)}\ \ \chi_2^{(T)})\begin{pmatrix} m^2_{\phi_2} & m^2_{T_{12}} & m^2_{T_{13}} \\ m^2_{T_{12}} & m^2_{\Sigma_2} & m^2_{T_{23}} \\ m^2_{T_{13}} & m^2_{T_{23}} & m^2_{\chi_2}\end{pmatrix}\begin{pmatrix}\phi_2^{(T)*} \\ \Sigma_2^{(T)*} \\ \chi_2^{(T)*}\end{pmatrix}, \quad (4.38)$$

with,

$$m^2_{T_{12}} = -\frac{1}{12\sqrt{2}}V_{\mathrm{GUT}}(2\kappa_1 V_{\mathrm{GUT}} - \kappa_2 V_{\mathrm{GUT}} - 2\sqrt{15}\mu_1), \quad (4.39)$$

$$m^2_{T_{13}} = -\frac{5}{6\sqrt{3}}\kappa_3 V^2_{\mathrm{GUT}}, \quad (4.40)$$

$$m^2_{T_{23}} = -\frac{1}{9\sqrt{2}}V_{\mathrm{GUT}}(6\sqrt{5}\mu_2 + \sqrt{3}\kappa_4 V_{\mathrm{GUT}}). \quad (4.41)$$

Note that, due to the breaking of the GUT symmetry, all the multiplets acquire mass of the order of the GUT scale. However, to break the SM symmetry, the SM like Higgs doublet needs to be kept at the EW scale. This can be achieved by imposing the well known fine-tuning condition in the doublet mass matrix Eq. (4.36). This fine-tuning does not leave any color triplet Higgs light that can be seen from the corresponding mass matrix given in Eq. (4.38).

V. GAUGE COUPLING UNIFICATION AND PROTON DECAY CONSTRAINTS

In this section we present a few different scenarios where successful gauge coupling unification within our framework can be achieved which are also in agreement with the proton decay bounds. For the gauge couplings the renormalization group equations can be written as:

$$\alpha_i^{-1}(M_Z) = \alpha^{-1}_{\mathrm{GUT}} + \frac{B_i}{2\pi}\ln\left(\frac{M_{\mathrm{GUT}}}{M_Z}\right), \quad (5.1)$$

where,

$$b_i^{\mathrm{SM}} = \left(\frac{41}{10}, -\frac{19}{6}, -7\right), \quad (5.2)$$

$$B_i = b_i^{\mathrm{SM}} + \Delta b_{i,k} r_k, \quad (5.3)$$

$$r_k = \frac{\ln(M_{\mathrm{GUT}}/M_k)}{\ln(M_{\mathrm{GUT}}/M_Z)}. \quad (5.4)$$

Here, b_i^{SM} are the SM β-coefficients and r_k represents the threshold weight factor of the BSM multiplet k of mass M_k. To affect the coupling running, the BSM multiplet k needs to live in a scale that is in between the electroweak scale and the GUT scale, here we assume that the rest of the mutiplets are degenerate with the unification scale. $\Delta b_{i,k} = b_{i,k} - b_{i,k-1}$ is the increase in the renormalization group equation coefficient at the threshold, M_k for a BSM multiplet. It is convenient to rewrite the equations for the running of the gauge couplings in terms of the low energy observables at the electroweak scale and the differences in the coefficients $B_{ij} = B_i - B_j$ [41]. In this way, the equations become:

$$\frac{B_{23}}{B_{12}} = \frac{5}{8}\left(\frac{\sin^2\theta_W(M_Z) - \alpha(M_Z)/\alpha_s(M_Z)}{3/8 - \sin^2\theta_W(M_Z)}\right), \quad (5.5)$$

$$\ln\left(\frac{M_{\mathrm{GUT}}}{M_Z}\right) = \frac{16\pi}{5\alpha(M_Z)}\left(\frac{3/8 - \sin^2\theta_W(M_Z)}{B_{12}}\right). \quad (5.6)$$

From the experimental measurements, $\alpha(M_Z)^{-1} = 127.94$, $\sin^2\theta_W(M_Z) = 0.231$, and $\alpha_s(M_Z) = 0.1185$ [42] which infers

$$\frac{B_{23}}{B_{12}} = 0.718, \quad (5.7)$$

$$M_{\mathrm{GUT}} = M_Z \exp\left(\frac{184.87}{B_{12}}\right). \quad (5.8)$$

TABLE II. B_{ij} coefficients of the multiplets present in our theory.

Fields	ΔB_{12}	ΔB_{23}
$\Phi_2(1,3,0)$	$-\frac{1}{3}r_{\Phi_2}$	$\frac{1}{3}r_{\Phi_2}$
$\Phi_3(8,1,0)$	0	$-\frac{1}{2}r_{\Phi_3}$
$\phi_1(1,2,\frac{1}{2})$	$-\frac{1}{15}r_{\phi_1}$	$\frac{1}{6}r_{\phi_1}$
$\phi_2(3,1,-\frac{1}{3})$	$\frac{1}{15}r_{\phi_2}$	$-\frac{1}{6}r_{\phi_2}$
$\Sigma_1(1,2,\frac{1}{2})$	$-\frac{1}{15}r_{\Sigma_1}$	$\frac{1}{6}r_{\Sigma_1}$
$\Sigma_2(3,1,-\frac{1}{3})$	$\frac{1}{15}r_{\Sigma_2}$	$-\frac{1}{6}r_{\Sigma_2}$
$\Sigma_3(\bar{3},1,\frac{4}{3})$	$\frac{16}{15}r_{\Sigma_3}$	$-\frac{1}{6}r_{\Sigma_3}$
$\Sigma_4(\bar{3},2,-\frac{7}{6})$	$\frac{17}{15}r_{\Sigma_4}$	$\frac{1}{6}r_{\Sigma_4}$
$\Sigma_5(3,3,-\frac{1}{3})$	$-\frac{9}{5}r_{\Sigma_5}$	$\frac{3}{2}r_{\Sigma_5}$
$\Sigma_6(\bar{6},1,-\frac{1}{3})$	$\frac{2}{15}r_{\Sigma_6}$	$-\frac{5}{6}r_{\Sigma_6}$
$\Sigma_7(8,2,\frac{1}{2})$	$-\frac{8}{15}r_{\Sigma_7}$	$-\frac{2}{3}r_{\Sigma_7}$
$\eta_1(1,2,-\frac{3}{2})$	$\frac{11}{15}r_{\eta_1}$	$\frac{1}{6}r_{\eta_1}$
$\eta_2(\bar{3},1,-\frac{2}{3})$	$\frac{4}{15}r_{\eta_2}$	$-\frac{1}{6}r_{\eta_2}$
$\eta_3(3,2,\frac{1}{6})$	$-\frac{7}{15}r_{\eta_3}$	$\frac{1}{6}r_{\eta_3}$
$\eta_4(\bar{3},3,-\frac{2}{3})$	$-\frac{6}{5}r_{\eta_4}$	$\frac{3}{2}r_{\eta_4}$
$\eta_5(\bar{6},2,\frac{1}{6})$	$-\frac{14}{15}r_{\eta_5}$	$-\frac{2}{3}r_{\eta_5}$
$\eta_6(8,1,1)$	$\frac{8}{5}r_{\eta_6}$	$-r_{\eta_6}$
$\chi_1(1,1,-2)$	$\frac{4}{5}r_{\chi_1}$	0
$\chi_2(3,1,-\frac{1}{3})$	$\frac{1}{15}r_{\chi_2}$	$-\frac{1}{6}r_{\chi_2}$
$\chi_3(\bar{3},2,-\frac{7}{6})$	$\frac{17}{15}r_{\chi_3}$	$\frac{1}{6}r_{\chi_3}$
$\chi_4(6,1,\frac{4}{3})$	$\frac{32}{15}r_{\chi_4}$	$-\frac{5}{6}r_{\chi_4}$
$\chi_5(\bar{6},3,-\frac{1}{3})$	$-\frac{18}{5}r_{\chi_5}$	$\frac{3}{2}r_{\chi_5}$
$\chi_6(8,2,\frac{1}{2})$	$-\frac{8}{15}r_{\chi_6}$	$-\frac{2}{3}r_{\chi_6}$

So to achieve unification, $\frac{B_{23}}{B_{12}}$ ratio needs to be 0.718 ± 0.005 ($\pm 1\sigma$ range), whereas in the SM model this ratio is equal to 0.528, hence it fails badly to unify gauge couplings. The corresponding scale for the SM from Eq. (5.8) is found to be 10^{13} GeV. So threshold corrections from the BSM multiplets are needed to modify the B_{12} and B_{23} to get the correct ratio. These B_{ij} coefficients for all the BSM multiplets for our model with $\mathbf{5_H} + \mathbf{24_H} + \mathbf{40_H} + \mathbf{45_H} + \mathbf{50_H}$ are presented in Table II and are used in our study of the gauge coupling unification.

The main experimental test of the existence of GUTs is via the detection of the proton decay yet to be observed. In GUT models in the nonsupersymmetric framework, the leading contribution to the proton decay is due to the gauge mediated $d = 6$ operators. In $\boldsymbol{SU(5)}$ GUT, the gauge bosons that are responsible for the proton to decay are $(3, 2, -\frac{5}{6}) + (\bar{3}, 2, \frac{5}{6}) \subset \mathbf{24_G}$. The most stringent experimental bound on the proton lifetime comes from the gauge mediated proton decay mode: $p \to \pi^0 e^+$ and the corresponding decay width is given by [43–45]:

$$\Gamma(p\to\pi^0e^+) = \frac{\pi m_p A^2}{2\alpha_{\rm GUT}^{-2} M_{\rm GUT}^4} |\langle \pi^0 | (ud)_R u_L | p\rangle|^2 (|c(e^c, d)|^2 + |c(e, d^c)|^2), \quad (5.9)$$

here, m_p is the proton mass, the running factor of the relevant operators give $A \approx 1.8$ and,

$$c(e_i, d_j^c) = V_1^{11} V_3^{ji},$$
$$c(e_i^c, d_j) = V_1^{11} V_2^{ij} + (V_1 V_{UD})^{1j} (V_2 V_{UD}^\dagger)^{i1}, \quad (5.10)$$

$$V_1 = U_c^T U^\dagger, \qquad V_2 = E_c^T D^\dagger,$$
$$V_3 = D_c^T E^\dagger, \qquad V_{UD} = U D^\dagger. \quad (5.11)$$

All the $d = 6$ proton decay operators including Eq. (5.9) conserve $B - L$, as a result a nucleon can decay into a meson and an antilepton. Operators that contribute to proton decay are in general model dependent. For example, in supersymmetric (SUSY) theories the most dominating contributions of proton decay originate from $d = 5$ operators. Determination of proton decay in such cases require the knowledge of SUSY spectrum, the details of the Higgs potential, and the fermion masses. However, non-SUSY models are more predictive in this sense, because the aforementioned gauge mediated $d = 6$ operators mainly depend on the fermion mixings. There can be additional contributions to the proton decay originating from Higgs mediated $d = 6$ operators in non-SUSY models that are highly model dependent and less predictive, since *a priori* the couplings entering in the scalar potential are not known. This is why we only discuss the gauge mediated $d = 6$ proton decay operator of Eq. (5.9) as they have the least model dependence. The c-coefficients given in Eq. (5.10) depend on the detail of the flavor structure. Here to estimate the proton lifetime we take the most conservative scenario, $c(e^c, d) = 2$ and $c(e, d^c) = 1$ for the $p \to \pi^0 e^+$ channel. This is a very good assumption since the leading entries in the mixing matrices given in Eq. (5.11) that participate in the computation of $p \to \pi^0 e^+$ decay are expected to have the similar structure as that of the Cabibbo-Kobayashi-Maskawa matrix which to a very good approximation is given by $V_{\rm CKM} \approx \mathbb{1}$. Deviation from this will only increase the proton lifetime and requires cancellations utilizing fine-tuned Yukawa couplings (see for example Ref. [46]) that we do not consider here. Consequently, the proton lifetime is rather very sensitive to the unification scale $M_{\rm GUT}$ and the associated unified coupling constant $\alpha_{\rm GUT}$. The relevant nuclear matrix element needed in Eq. (5.9) is taken from Ref. [47]: $|\langle \pi^0 | (ud)_R u_L | p\rangle| = -0.118$. In the gauge coupling unification analysis presented below, we will demonstrate a few different scenarios and estimate the corresponding proton lifetime using Eq. (5.9). The current experimental upper bound on the proton lifetime is $\tau_p(p \to \pi^0 e^+) > 1.6 \times 10^{34}$ years [48] and the Hyper-Kamiokande experiment after 10 years of exposure can make a 3σ discovery of $p \to \pi^0 e^+$ process up to 6.3×10^{34} years [49].

However as aforementioned, the gauge mediated processes are not the only source for the protons to decay, some of the scalar leptoquarks present in the unified theories can also lead to proton decay. Note that light scalars must be present for gauge coupling unification to realize, recall that coupling unification does not happen in the Georgi-Glashow model. The scalars that mediate proton decay in our theory are $\phi_2(3, 1, -\frac{1}{3}) \subset \mathbf{5_H}$, $\Sigma_2(3, 1, -\frac{1}{3})$, $\Sigma_3(\bar{3}, 1, \frac{4}{3})$, $\Sigma_5(3, 3, -\frac{1}{3}) \subset \mathbf{45_H}$ and $\chi_2(3, 1, -\frac{1}{3}) \subset \mathbf{50_H}$. To suppress proton decay one would expect these fields to have masses of the order of the GUT scale. For our analysis we assume that these fields are sufficiently heavy so that the corresponding dangerous proton decay operators are suppressed. If any of these fields is assumed to be much lighter than the GUT scale, the associated Yukawa couplings need to be somewhat suppressed to avoid dangerous proton decay.

For the purpose of comparison, at first we discuss the coupling unification scenario within the minimal renormalizable model. Note that to achieve high GUT scale value, one needs to keep scalars light that provide negative contribution to the B_{12}. In the MRSU5 model, other than the SM like doublets, such negative contribution is provided by the Φ_2, Σ_5, Σ_7 multiplets, see Table II. Among these three, Σ_5 mediate proton decay, on the other hand, Φ_2 and Σ_7 do not and can be very light. However, keeping Σ_5 at the GUT scale and the other two fields light fails unification test, so Σ_5 must be light as well within this scenario. To avoid proton decay bounds, this multiplet needs to be heavier than about 10^{10} GeV

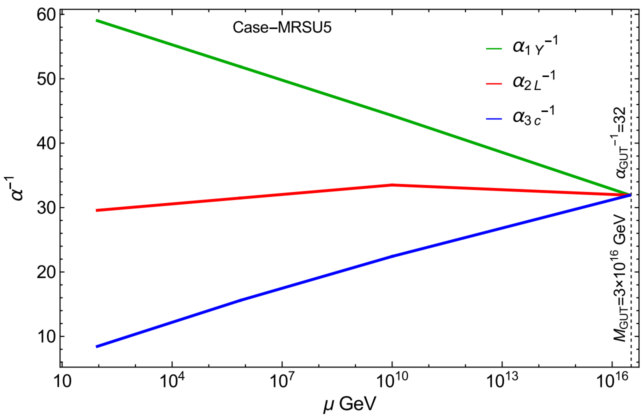

FIG. 4. Here we present the plot of the gauge coupling unification in the MRSU5 model as discussed in the text.

by assuming natural values of the Yukawa couplings [9], however, for smaller values of the Yukawa couplings, this multiplet can be kept at lower scale. In this minimal scenario, by fixing $m_{\Sigma_5} = 10^{10}$ GeV and $m_{\phi_1} = m_{\Sigma_1} = m_{\Sigma_7} = m_{\Phi_2} = M_Z$ we find, to achieve unification at the one-loop, one needs $m_{\Phi_3} = 7.28 \times 10^5$ GeV and the corresponding unification scale is 3.02×10^{16} GeV which agrees with [9]. In Fig. 4, we present the corresponding plot of the gauge coupling unification in this model.

However, this simple realization of the gauge coupling unification does not remain valid in our proposed model. The required scalar multiplets $\eta_1(1,2,-\frac{3}{2}) \subset \mathbf{40_H}$ and $\chi_1(1,1,-2) \subset \mathbf{50_H}$ running in the loop to generate neutrino mass are expected to reside in scales much smaller than the GUT scale. The presence of these additional light scalar multiplets completely ruins the successful coupling unification of the MRSU5 model as shown above, hence, threshold corrections from other scalar fields need to be taken into account to restore the gauge coupling unification. We find that in this setup it gets difficult to achieve very high scale gauge coupling unification without taking into account threshold correction from quite a few scalar multiplets living in between the EW and the GUT scales.

With five different scenarios (we label them as A, B, C, D, E) we demonstrate how gauge coupling unification can be restored in our model. For this analysis, we fix the masses of the two SM like isospin doublets as $m_{\phi_1,\Sigma_1} = v_{\rm EW}$. We also fix $m_{\Sigma_7} = 3.5$ TeV, since from the collider bounds it is required that $m_{\Sigma_7} > 3.1$ TeV provided that the Yukawa couplings take natural values [50]. We also take $m_{\Phi_2} = 1$ TeV and furthermore, for the cases A, C, D: $m_{\chi_1} = 1$ TeV, for case B: $m_{\eta_1} = 1$ TeV and for case E: $m_{\eta_1} = m_{\chi_1} = 1$ TeV are assumed. With these assumptions and using the mass relations derived in Sec. IV, the results

TABLE III. Successful gauge coupling unification scenarios within our framework. For all these five cases, the two isospin doublet masses are taken to be $m_{\phi_1,\Sigma_1} = v_{\rm EW}$ and the masses of the Φ_2 and Σ_7 multiplets are fixed at $m_{\Phi_2} = 1$ TeV and $m_{\Sigma_7} = 3.5$ TeV. It should be pointed out that for the cases C and D where the proton life times are estimated to be somewhat smaller than the current upper bound $\tau_p(p \to \pi^0 e^+) > 1.6 \times 10^{34}$ years, small threshold corrections near the GUT scale can make these scenarios viable. The corresponding gauge coupling unification plots are presented in Fig. 5.

Case	Multiplets	Mass (in GeV)	$M_{\rm GUT}$ (GeV)	$\alpha_{\rm GUT}^{-1}$	$\tau_p(p \to \pi^0 e^+)$ in years
	Φ_3, χ_1, $\eta_{3,4,5}$	10^3			
A	$\eta_{1,2,6}$	3.90×10^6	6.2×10^{15}	16.2	2.43×10^{34}
	Σ_5	1.58×10^{11}			
	Φ_3, $\eta_{1,3,4,5}$	10^3			
B	χ_1, $\eta_{2,6}$	2.33×10^6	5.37×10^{15}	15.9	1.32×10^{34}
	Σ_5	1.58×10^{11}			
	Φ_3, χ_1	10^3			
C	Σ_5	2×10^7	1.88×10^{15}	30.3	7.2×10^{32}
	η_1	2.9×10^{14}			
	Φ_3, χ_1, $\eta_{3,4,5}$	10^3			
	η_1	4.14×10^4			
D	η_6	7.68×10^6	4.25×10^{15}	16.33	5.46×10^{33}
	η_2	2.72×10^7			
	Σ_5	1.58×10^{11}			
	χ_1, η_1	10^3			
	Φ_3, η_5	3.5×10^3			
E	$\eta_{2,6}$	7.5×10^4	2.2×10^{17}	10.9	1.74×10^{40}
	η_3	4.8×10^7			
	Σ_5	1.58×10^9			

are presented in the Table III with the corresponding unified value of the gauge coupling constant, the scale of the unification, and the estimation of the associated proton lifetime for each scenario. It should be pointed out that for cases C and D, even though the proton lifetimes are estimated to be somewhat below the current upper bound $\tau_p(p \to \pi^0 e^+) > 1.6 \times 10^{34}$ years, small threshold corrections near to the GUT scale can make these scenario viable. Even though these choices are not unique, but clearly demonstrate how successful gauge coupling unification consistent with proton decay bounds can be achieved within our setup. Due to the presence of the light scalars that play role in neutrino mass generation, unification scale cannot be made arbitrarily large and the proton decay rate is expected to be within the observable range. Though no firm prediction can be made about the proton decay within this framework, but for most of the examples provided here, the proton decay rate is very close to the current experimental bound and has the potential to be tested in near future.

For completeness, in Fig. 5 we present the plots of the gauge coupling unification for the aforementioned five scenarios that are summarized in Table III. As already

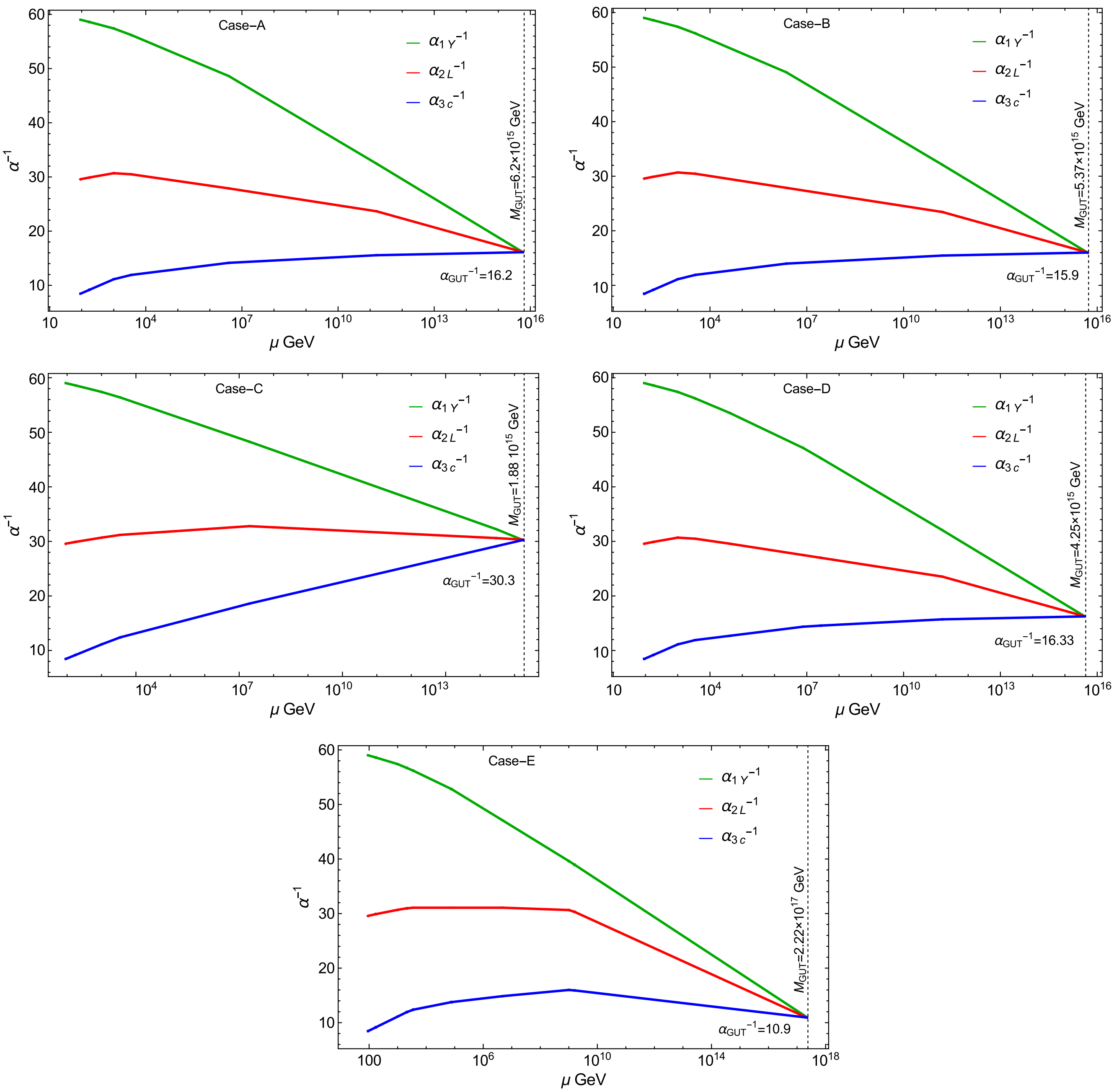

FIG. 5. Here we present the plots of the gauge coupling unification scenarios within our framework that are summarized in Table III.

pointed out, successful gauge coupling unification in our setup requires more number of light scalar multiplets compared to the minimal model (MRSU5). In coherence with the MRSU5 case, to achieve unification we keep the fields $\Phi_{2,3}$, Σ_7 around the TeV range and the multiplet Σ_5 not too far from 10^{10} GeV. However, since χ_1 and η_1 fields are expected to be much smaller than the GUT scale in our framework, to compensate for their effects on the running of the coupling constants, few more additional fields must reside in between the EW and the GUT scales. For most of the cases (A, B, C, D) considered here, even with quite a few new multiplets (different set of multiplets for different cases) living at the low energies, the scales of unification are found to be an order of magnitude less compared to the minimal set-up. We also demonstrate a scenario (case E) where unification scale can be achieved which is an order higher compared to the case of MRSU5.

VI. CONCLUSIONS

Grand unification based on the $\boldsymbol{SU(5)}$ gauge group is one of the leading candidates for the ultraviolet completion of the SM. The minimal $\boldsymbol{SU(5)}$ GUT has many attractive features, however fails to incorporate neutrino mass. In this work, we have proposed a renormalizable $\boldsymbol{SU(5)}$ GUT where neutrino mass originates at the two-loop level. By detailed analysis we have shown that this proposed model is the only *true two-loop* model of neutrino mass generation based on $\boldsymbol{SU(5)}$ GUT in the existing literature. This realization requires two Higgs representations beyond the minimal renormalizable model and no additional fermion beyond the SM is introduced. Within this setup, in addition to correctly reproducing the charged fermion and neutrino mass spectrum, successful gauge coupling unification can be realized while simultaneously satisfying the proton decay bounds. It is shown that the neutrino masses are not completely independent but are correlated with the charged fermion masses. By constructing the relevant scalar potential, the Higgs mass spectrum is computed and few examples of gauge coupling unification are demonstrated. Proton decay rate is expected to be within the observable range in our framework. The Higgs representations required for generating realistic fermions masses contain leptoquarks that can accommodate the recent B-physics anomalies.[6]

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[6]Recently, the B-physics anomalies have gained a lot of attention in the high energy physics community, particularly the lepton flavor universality ratios $R_{K^{(*)}}$ and $R_{D^{(*)}}$. The deviations of the measurements on $R_{K^{(*)}}$ [51,52] and $R_{D^{(*)}}$ [53–55] from the SM are within $2-3\sigma$ confidence level. In Ref. [56] it is pointed out that a TeV scale LQ with quantum number $(\bar{3}, 2, -\frac{7}{6})$ originating from $\mathbf{45}_H$ and $\mathbf{50}_H$ and a second LQ with quantum number $(3, 3, -\frac{1}{3})$ residing at the sub-TeV scale originating from $\mathbf{45}_H$ can simultaneously explain both these anomalies. Since our setup has both the $\mathbf{45}_H$ and $\mathbf{50}_H$ representations, we expect that the proposed model in this work along with explaining the origin of neutrino mass can also incorporate the observed B-anomalies. Whereas establishing such a link among these seemingly different phenomena in a unified framework is interesting, however is beyond the scope of the present work. For a detailed analysis on how to accommodate B-physics anomalies in the context of the framework discussed in this article, we refer the readers to this work [56].

[1] J. C. Pati and A. Salam, Lepton number as the fourth color, Phys. Rev. D **10**, 275 (1974); Erratum, Phys. Rev. D **11**, 703 (E) (1975).
[2] H. Georgi and S. L. Glashow, Unity of All Elementary Particle Forces, Phys. Rev. Lett. **32**, 438 (1974).
[3] H. Georgi, H. R. Quinn, and S. Weinberg, Hierarchy of Interactions in Unified Gauge Theories, Phys. Rev. Lett. **33**, 451 (1974).
[4] T. Hubsch and S. Pallua, Symmetry breaking mechanism in an alternative SU(5) model, Phys. Lett. **138B**, 279 (1984).
[5] T. Hubsch, S. Meljanac, and S. Pallua, A nonminimal SU(5) model, Phys. Rev. D **31**, 2958 (1985).
[6] H. Georgi and C. Jarlskog, A new lepton–quark mass relation in a unified theory, Phys. Lett. **86B**, 297 (1979).
[7] J. R. Ellis and M. K. Gaillard, Fermion masses and Higgs representations in SU(5), Phys. Lett. **88B**, 315 (1979).
[8] R. Slansky, Group theory for unified model building, Phys. Rep. **79**, 1 (1981).
[9] I. Dorsner and P. Fileviez Perez, Unification versus proton decay in SU(5), Phys. Lett. B **642**, 248 (2006).
[10] Y. Cai, J. Herrero-Garcia, M. A. Schmidt, A. Vicente, and R. R. Volkas, From the trees to the forest: A review of radiative neutrino mass models, Front. Phys. **5**, 63 (2017).
[11] P. Minkowski, , $\mu \to e\gamma$ at a rate of one out of 10^9 muon decays?, Phys. Lett. **67B**, 421 (1977); T. Yanagida, in *Proceedings of the Workshop on Unified Theories and Baryon Number in the Universe, Tsukuba, 1979*, edited by, A. Sawada and A. Sugamoto (KEK, Tsukuba, 2004).
[12] S. L. Glashow, *The future of elementary particle physics*, edited by M. Lévy, J. L. Basdevant, D. Speiser, J. Weyers, R. Gastmans, and M. Jacob, Quarks and Leptons. NATO Advanced Study Institutes Series (Series B. Physics) Vol. 61 (Springer, Boston, MA, 1980).
[13] M. Gell-Mann, P. Ramond, and R. Slansky, in *Proceedings of the Supergravity Stony Brook Workshop, New York, 1979*,

edited by P. Van Niewenhuizen and D. Freeman (North Holland Publishing Co., Holland, 1979).

[14] R. Mohapatra and G. Senjanović, Neutrino Mass and Spontaneous Parity Violation, Phys. Rev. Lett. **44**, 912 (1980).

[15] M. Magg and C. Wetterich, Neutrino mass problem and gauge hierarchy, Phys. Lett. **94B**, 61 (1980).

[16] J. Schechter and J. W. F. Valle, Neutrino masses in $SU(2)\times U(1)$ theories, Phys. Rev. D **22**, 2227 (1980).

[17] G. Lazarides, Q. Shafi, and C. Wetterich, Proton lifetime and fermion masses in an SO(10) model, Nucl. Phys. **B181**, 287 (1981).

[18] R. N. Mohapatra and G. Senjanovic, Neutrino masses and mixings in gauge models with spontaneous parity violation, Phys. Rev. D **23**, 165 (1981).

[19] I. Dorsner and P. Fileviez Perez, Unification without supersymmetry: Neutrino mass, proton decay and light leptoquarks, Nucl. Phys. **B723**, 53 (2005).

[20] I. Dorsner, P. Fileviez Perez, and R. Gonzalez Felipe, Phenomenological and cosmological aspects of a minimal GUT scenario, Nucl. Phys. **B747**, 312 (2006).

[21] I. Dorsner, P. Fileviez Perez, and G. Rodrigo, Fermion masses and the UV cutoff of the minimal realistic SU(5), Phys. Rev. D **75**, 125007 (2007).

[22] I. Dorsner and I. Mocioiu, Predictions from type II see-saw mechanism in SU(5), Nucl. Phys. **B796**, 123 (2008).

[23] B. Bajc and G. Senjanovic, Seesaw at LHC, J. High Energy Phys. 08 (2007) 014.

[24] P. Fileviez Perez, Renormalizable adjoint SU(5), Phys. Lett. B **654**, 189 (2007).

[25] R. Foot, H. Lew, X. G. He, and G. C. Joshi, Seesaw neutrino masses induced by a triplet of leptons, Z. Phys. C **44**, 441 (1989).

[26] T. P. Cheng and L. F. Li, On weak interaction induced neutrino oscillations, Phys. Rev. D **17**, 2375 (1978).

[27] S. Saad, Simplest radiative dirac neutrino mass models, Nucl. Phys. **B943**, 114636 (2019).

[28] A. Zee, A theory of lepton number violation, neutrino majorana mass, and oscillation, Phys. Lett. **93B**, 389 (1980); Erratum, Phys. Lett. **95B**, 461(E) (1980).

[29] L. Wolfenstein, Neutrino mixing in grand unified theories, eConf C **801002**, 116 (1980).

[30] R. Barbieri, D. V. Nanopoulos, and D. Wyler, Hierarchical fermion masses in SU(5), Phys. Lett. **103B**, 433 (1981).

[31] P. Fileviez Perez and C. Murgui, Renormalizable SU(5) unification, Phys. Rev. D **94**, 075014 (2016).

[32] V. Brdar, I. Picek, and B. Radovcic, Radiative neutrino mass with scotogenic scalar triplet, Phys. Lett. B **728**, 198 (2014).

[33] K. Kumericki, T. Mede, and I. Picek, Renormalizable SU(5) completions of a zee-type neutrino mass model, Phys. Rev. D **97**, 055012 (2018).

[34] I. Dorsner, S. Fajfer, and N. Kosnik, Leptoquark mechanism of neutrino masses within the grand unification framework, Eur. Phys. J. C **77**, 417 (2017).

[35] T. P. Cheng and L. F. Li, Neutrino masses, mixings and oscillations in $SU(2)\times U(1)$ models of electroweak interactions, Phys. Rev. D **22**, 2860 (1980).

[36] K. S. Babu, Model of 'Calculable' majorana neutrino masses, Phys. Lett. B **203**, 132 (1988).

[37] S. S. C. Law and K. L. McDonald, The simplest models of radiative neutrino mass, Int. J. Mod. Phys. A **29**, 1450064 (2014).

[38] Q. H. Cao, S. L. Chen, E. Ma, B. Yan, and D. M. Zhang, New class of two-loop neutrino mass models with distinguishable phenomenology, Phys. Lett. B **779**, 430 (2018).

[39] K. Tamvakis and J. D. Vergados, Neutrino masses and oscillations in an unconventional model of lepton number violation, Phys. Lett. **155B**, 373 (1985).

[40] D. D. Wu, The masses of the neutrinos in the unification gauge theories and the neutrino–anti-neutrino oscillations, Phys. Rev. D **23**, 2038 (1981).

[41] A. Giveon, L. J. Hall, and U. Sarid, SU(5) unification revisited, Phys. Lett. B **271**, 138 (1991).

[42] K. A. Olive *et al.* (Particle Data Group), Review of particle physics, Chin. Phys. C **38**, 090001 (2014).

[43] A. De Rujula, H. Georgi, and S. L. Glashow, Flavor Goniometry By Proton Decay, Phys. Rev. Lett. **45**, 413 (1980).

[44] P. Fileviez Perez, Fermion mixings versus $d=6$ proton decay, Phys. Lett. B **595**, 476 (2004).

[45] P. Nath and P. Fileviez Perez, Proton stability in grand unified theories, in strings and in branes, Phys. Rep. **441**, 191 (2007).

[46] I. Dorsner and P. Fileviez Perez, How long could we live?, Phys. Lett. B **625**, 88 (2005).

[47] Y. Aoki, T. Izubuchi, E. Shintani, and A. Soni, Improved lattice computation of proton decay matrix elements, Phys. Rev. D **96**, 014506 (2017).

[48] K. Abe *et al.* (Super-Kamiokande Collaboration), Search for proton decay via $p\to e^+\pi^0$ and $p\to\mu^+\pi^0$ in 0.31 megaton years exposure of the super-kamiokande water cherenkov detector, Phys. Rev. D **95**, 012004 (2017).

[49] K. Abe *et al.* (Hyper-Kamiokande Collaboration), Hyper-Kamiokande design report, arXiv:1805.04163.

[50] V. Khachatryan *et al.* (CMS Collaboration), Search for Narrow Resonances Decaying to Dijets in Proton-Proton Collisions at $\sqrt{(s)}=13$ TeV, Phys. Rev. Lett. **116**, 071801 (2016).

[51] R. Aaij *et al.* (LHCb Collaboration), Test of Lepton Universality using $B^+\to K^+\ell^+\ell^-$ Decays, Phys. Rev. Lett. **113**, 151601 (2014).

[52] R. Aaij *et al.* (LHCb Collaboration), Test of lepton universality with $B^0\to K^{*0}\ell^+\ell^-$ decays, J. High Energy Phys. 08 (2017) 055.

[53] J. P. Lees *et al.* (*BABAR* Collaboration), Measurement of an excess of $\bar{B}\to D^{(*)}\tau^-\bar{\nu}_\tau$ decays and implications for charged Higgs bosons, Phys. Rev. D **88**, 072012 (2013).

[54] S. Hirose *et al.* (Belle Collaboration), Measurement of the τ Lepton Polarization and $R(D^*)$ in the Decay $\bar{B}\to D^*\tau^-\bar{\nu}_\tau$, Phys. Rev. Lett. **118**, 211801 (2017).

[55] R. Aaij *et al.* (LHCb Collaboration), Measurement of the Ratio of Branching Fractions $\mathcal{B}(\bar{B}^0\to D^{*+}\tau^-\bar{\nu}_\tau)/\mathcal{B}(\bar{B}^0\to D^{*+}\mu^-\bar{\nu}_\mu)$, Phys. Rev. Lett. **115**, 111803 (2015); Erratum, Phys. Rev. Lett. **115**, 159901(E) (2015).

[56] D. Becirevic, I. Dorsner, S. Fajfer, N. Kosnik, D. A. Faroughy, and O. Sumensari, Scalar leptoquarks from grand unified theories to accommodate the B-physics anomalies, Phys. Rev. D **98**, 055003 (2018).