Updated limits on the *CP* violating $\eta\pi\pi$ and $\eta'\pi\pi$ couplings derived from the neutron EDM

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(Received 22 April 2019; published 6 June 2019)

We complete our derivation of upper limits on the *CP* violating $\eta\pi\pi$ and $\eta'\pi\pi$ couplings from an analysis of their two-loop contributions to the neutron electric dipole moment (nEDM). We use a phenomenological Lagrangian approach which is formulated in terms of hadronic degrees of freedom—nucleons and pseudoscalar mesons. The essential part of the Lagrangian contains the *CP* violating couplings between $\eta(\eta')$ and pions. Previously, we included photons using minimal substitution in case of the proton and charged pions. Now we extend our Lagrangian by adding the nonminimal couplings, i.e., anomalous magnetic couplings of nucleons with the photon. The obtained numerical upper limits for the $\eta\pi\pi$ and $\eta'\pi\pi$ couplings $|f_{\eta\pi\pi}(M_\eta^2)| < 4.4\times10^{-11}$ and $|f_{\eta'\pi\pi}(M_{\eta'}^2)| < 3.8\times10^{-11}$ can be useful for the related, planned experiments at the JLab Eta Factory. Using present experimental limits on the nEDM, we derive upper limits on the *CP* violating $\bar\theta$ parameter of $\bar\theta < 4.7\times10^{-10}$.

DOI: 10.1103/PhysRevD.99.115004

I. INTRODUCTION

Since the 1950s the study of *T* or *CP* violation in hadronic processes is a relevant topic in particle physics since it helps to shed light on the entries of the Cabibbo-Kobayashi-Maskawa mixing matrix and the related oscillations of neutral kaons, *D* and *B* mesons. Some phenomena, like *CP* violation in processes involving *K* and *B* mesons, have been explained in the framework of the Standard Model (SM). The study of other *CP* violating effects, such as strong *CP* violation, a neutron electric dipole moment (nEDM), decays of η and η' mesons into two pions, etc., clearly call for the search of possible new physics mechanisms, which are outside the scope of the SM. In particular, SM predictions for the nEDM are up to several orders of magnitude lower than existing experimental limits. Clearly the study of EDMs of hadrons and nuclei could probe new physics beyond the SM (for a review see, e.g., Refs. [1,2]). Our interest in hadron EDMs is motivated by the possibility to extract limits on the *CP* violating strong coupling between hadrons and the $\bar\theta$ parameter (*CP* violating gluon-gluon coupling). From the study of the nEDM one can estimate the QCD $\bar\theta$ parameter and the πNN, $\eta^{(\prime)}NN$, and $\eta^{(\prime)}\pi\pi$ couplings. In a series of papers [3–9] we gave several analyses of *CP* violating physics focused on the nEDM and strong *CP* violating phenomena with the relation to the QCD $\bar\theta$ term, aspects of the phenomenology of axions, *CP* violating hadronic couplings, intrinsic electric and chromoelectric dipole moments of quarks, *CP* violating quark-gluon, three-gluon, four-quark couplings, etc.

The study of rare processes of the η and η' mesons with emphasis on neutral modes ($\eta(\eta')\to2\pi$, $\eta\to\pi^0 2\gamma$, $\eta\to2\pi^0\gamma$, $\eta\to3\gamma$, etc.) is planned by the recently approved JLab Eta Factory (JEF) experiment [10], which is an extension of the physical program of the GlueX Collaboration at JLab (Hall D). The JEF experiment will offer an opportunity to reduce the background by a factor of ~2 orders of magnitude in comparison with all other existing or planned experiments. The $\eta(\eta')$ mesons will be produced via the photoproduction reaction on a proton target $\gamma+p\to\eta(\eta')+p$ with a 12 GeV photon beam. The main objectives of the JEF experiment are (1) to search for a leptophobic gauge boson V_B in the mass region of 140–540 MeV and in the decay channels $V_B\to\pi^0\gamma$ and

$\eta \to V_B\gamma \to \pi^0 2\gamma$. V_B is associated with a new $U_B(1)$ baryon number gauge symmetry and couples predominantly to quarks; (2) to study C and CP violation in strong and electromagnetic decays of $\eta(\eta')$ mesons. Therefore, the study of the CP violating processes $\eta(\eta') \to \pi\pi$ has a further strong motivation.

In Refs. [8,9] we focused on the determination of the CP violating couplings $\eta\pi\pi$ and $\eta'\pi\pi$ from the nEDM. Our formalism was based on a phenomenological Lagrangian describing the interaction of nucleons with pseudoscalar mesons [pions and $\eta(\eta')$]. In Ref. [9] the interaction of charged particles with photons has been introduced by the minimal substitution. In the previous calculation we did not include the nonminimal coupling because in chiral perturbation theory [11] it is subleading in comparison with minimal coupling due to power counting arguments. In particular, the contribution due to nonminimal coupling comes with a factor $\delta = M_P/M_N$ equal to the ratio of pseudoscalar meson and nucleon masses. The suppression factor δ is small for pions, but this is not necessarily the case for heavier states—η and η' mesons. In the present paper we extend our previous analysis by calculating additional effects of nonminimal coupling induced by the anomalous magnetic moments of nucleons.

The paper is structured as follows. In Sec. II review our approach. In particular, we specify Lagrangian generating matrix elements of the $\eta(\eta') \to \pi\pi$ transition. In Sec. III we discuss calculation of the neutron EDM induced by minimal and nonminimal electromagnetic couplings of pions and nucleons with photon and CP violating $\eta(\eta')\pi\pi$ couplings. In Sec. IV we present numerical results from which we extract information about $\eta(\eta')\pi\pi$ couplings using data limits on nEDM. Also here we summarize the results of the paper.

II. FRAMEWORK

In this section we review our formalism to link the nEDM to the CP violating couplings. It is based on a phenomenological Lagrangian $\mathcal{L}_{\rm eff}$ formulated in terms of hadronic degrees of freedom (nucleons $N = (p, n)$, pions $\pi = (\pi^\pm, \pi^0)$, $H = (\eta, \eta')$ mesons) and photons A_μ which separates into a free $\mathcal{L}_0$ and interaction part $\mathcal{L}_{\rm int}$ with

$$\mathcal{L}_{\rm eff} = \mathcal{L}_0 + \mathcal{L}_{\rm int}. \tag{1}$$

$\mathcal{L}_0$ includes the usual free terms of nucleons, mesons, and photons

$$\mathcal{L}_0 = \bar{N}(i\not\partial - M_N)N + \frac{1}{2}\vec{\pi}(\Box - M_\pi^2)\vec{\pi} + \frac{1}{2}H(\Box - M_H^2)H - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \tag{2}$$

where $\Box = -\partial_\mu\partial^\mu$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the stress tensor of the electromagnetic field, M_N, M_π, and M_H are the masses of nucleons, pions, and $\eta(\eta')$ mesons, respectively. The interaction Lagrangian $\mathcal{L}_{\rm int}$ is given by a sum of two parts. The first part contains the strong interaction terms, which describe the CP-even couplings of nucleons with pions $\mathcal{L}_{\pi NN}$ and $\eta(\eta')$ mesons $\mathcal{L}_{HNN}$ and the CP violating $\eta(\eta')\pi\pi$ coupling $\mathcal{L}^{CP}_{H\pi\pi}$. The second part includes the electromagnetic interaction terms, describing the coupling of charged pions and nucleons with the photon ($\mathcal{L}_{\gamma NN}$ and $\mathcal{L}_{\gamma\pi\pi}$, respectively):

$$\begin{aligned}
\mathcal{L}_{\rm int} &= \mathcal{L}_{\pi NN} + \mathcal{L}_{\eta(\eta')NN} + \mathcal{L}^{CP}_{\eta(\eta')\pi\pi} + \mathcal{L}_{\gamma NN} + \mathcal{L}_{\gamma\pi\pi},\\
\mathcal{L}_{\pi NN} &= g_{\pi NN}\bar{N}i\gamma_5\vec{\pi}\,\vec{\tau}N,\\
\mathcal{L}_{HNN} &= g_{HNN}H\bar{N}i\gamma_5 N,\\
\mathcal{L}^{CP}_{H\pi\pi} &= f_{H\pi\pi}M_H H\vec{\pi}^2,\\
\mathcal{L}_{\gamma NN} &= eA_\mu\bar{N}\left(\gamma^\mu Q_N + \frac{i\sigma^{\mu\nu}q_\nu}{2M_N}k_N\right)N,\\
\mathcal{L}_{\gamma\pi\pi} &= eA_\mu(\pi^- i\partial^\mu\pi^+ - \pi^+ i\partial^\mu\pi^-) + e^2A_\mu A^\mu\pi^+\pi^-,
\end{aligned} \tag{3}$$

where $g_{\pi NN} = \frac{g_A}{F_\pi}M_N$, $g_A = 1.275$ is the nucleon axial charge, $F_\pi = 92.4$ MeV is the pion decay constant, g_{HNN} and $f_{H\pi\pi}$ are corresponding CP-even and CP-odd couplings between pions and $\eta(\eta')$, γ^μ and γ^5 are the Dirac matrices, and $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$. The values of $g_{\eta NN}$ and $g_{\eta' NN}$ are taken from Ref. [12]: $g_{\eta NN} = g_{\eta' NN} = 0.9$. Note that in the case of nucleons we include both minimal (first term in $\mathcal{L}_{\gamma NN}$) and nonminimal (second term in $\mathcal{L}_{\gamma NN}$) electromagnetic couplings. Here $Q_N = {\rm diag}(1, 0)$ and $k_N = {\rm diag}(k_p, k_n)$ are the diagonal matrices of nucleon charges and anomalous magnetic moments, respectively, where $k_p = 1.793$ and $k_n = -1.913$. For the CP-even interactions of nucleons with pseudoscalar mesons we use the pseudoscalar (PS) coupling [13], which is equivalent to the pseudovector (PV) coupling as demonstrated in Refs. [13–15]. As we have shown in Ref. [9], matrix elements in the two theories can differ by a divergent term, which can always be absorbed by an appropriative choice of a counterterm. In particular, in the case of the matrix element describing the nEDM the PS theory does not contain a logarithmic divergence, while it occurs in the PV theory.

The CP violating term $\mathcal{L}^{CP}_{H\pi\pi}$ induces a contribution to the nEDM. The $\eta(\eta')\pi\pi$ couplings define the corresponding two-body decay branching ratios as

$${\rm Br}(H \to \pi\pi) = n_\Gamma \frac{\sqrt{M_H^2 - 4M_\pi^2}}{4\pi\Gamma_H^{\rm tot}} f^2_{H\pi\pi}, \tag{4}$$

where $\Gamma_H^{\rm tot}$ is the total width of H; n_Γ is a final-state factor, which equals 1/2 for the $\pi^0\pi^0$ and 1 for the $\pi^+\pi^-$ final states. Upper limits for these decays are set by the LHCb results [16]

$$\mathrm{Br}(\eta(\eta') \to \pi\pi) < \begin{cases} 1.3(1.8) \times 10^{-5}, & \pi^+\pi^- \\ 3.5(4.0) \times 10^{-4}, & \pi^0\pi^0 \end{cases}. \quad (5)$$

As discussed in Ref. [9], there are two possible mechanisms for the generation of the $\eta(\eta')\pi\pi$ effective couplings. In the first mechanism this coupling is generated by the QCD $\bar{\theta}$-term [17,18]

$$f^{\bar{\theta}}_{\eta\pi\pi} = -\frac{1}{\sqrt{3}} \frac{\bar{\theta} M_\pi^2 R}{F_\pi M_\eta (1+R)^2}, \qquad f^{\bar{\theta}}_{\eta'\pi\pi} = \sqrt{2} f^{\bar{\theta}}_{\eta\pi\pi} \frac{M_\eta}{M_{\eta'}}, \quad (6)$$

where $\bar{\theta}$ is the QCD vacuum angle and $R = m_u/m_d$ is the ratio of u and d current quark masses. In this scenario, the $\eta(\eta')\pi\pi$ couplings are proportional to $\bar{\theta}$, which in turn is originally constrained by the experimental bounds on the neutron EDM [19,20]. In the second scenario the nEDM and the *CP* violating $\eta \to \pi\pi$ vertices are generated by two distinct mechanisms, without specifying details of a particular model in which this scenario would be realized. Thereby one can expect that the yet unknown mechanisms due to new physics could enlarge the $\eta(\eta')\pi\pi$ couplings, which would induce a contribution to the nEDM at the two-loop level (see details in Ref. [9]).

III. NEUTRON EDM INDUCED BY THE *CP* VIOLATING $\eta(\eta')\pi\pi$ COUPLINGS AT THE TWO-LOOP LEVEL

In our approach a neutron EDM is described by a set of two-loop diagrams shown in Figs. 1 and 2. In Ref. [9] we already evaluated the diagrams generated by the minimal electromagnetic coupling (see Fig. 1), i.e., by the coupling of virtual charged pions and the proton to the electromagnetic field from the $\mathcal{L}_{\gamma\pi\pi}$ interaction Lagrangian and the first term from $\mathcal{L}_{\gamma NN}$ in Eq. (3). Here we extend our analysis by inclusion of the nonminimal couplings of the nucleon to the electromagnetic field due to the anomalous magnetic moments k_N (see Fig. 2) contained in the interaction Lagrangian $\mathcal{L}_{\gamma NN}$.

The matrix element corresponding to the diagrams of Figs. 1 and 2 or the electromagnetic vertex function of the neutron is expanded in terms of four relativistic form factors F_E (electric), F_M (magnetic), F_D (electric dipole), and F_A (anapole) as

$$\begin{aligned} M^\mu_{\mathrm{inv}} &= \bar{u}_N(p_2)\Gamma^\mu(p_1,p_2)u_N(p_1), \\ \Gamma^\mu(p_1,p_2) &= \gamma^\mu F_E(q^2) + \frac{i}{2M_N}\sigma^{\mu\nu}q_\nu F_M(q^2) \\ &\quad + \frac{1}{2M_N}\sigma^{\mu\nu}q_\nu\gamma^5 F_D(q^2) \\ &\quad + \frac{1}{M_N^2}(\gamma^\mu q^2 - 2M_N q^\mu)\gamma^5 F_A(q^2), \end{aligned} \quad (7)$$

where p_1 and p_2 are the momenta of the initial and final neutron states, $q^2 = (p_2 - p_1)^2$ is the transfer momentum squared. The nEDM is defined as $d_n^E = -F_D(0)/(2M_N)$. Note that the anapole form factor is generated by both minimal and nonminimal couplings. It vanishes for real photons: when $q^2 = 0$ and due to Lorenz transversity condition $q^\mu \epsilon_\mu = 0$, where ϵ_μ is the photon polarization vector.

To proceed we have to evaluate two-loop diagrams in the framework of the PS approach for the coupling between nucleons and pseudoscalar mesons. We want to point out again that the diagrams generated by the minimal coupling of charged pions and the proton with the electromagnetic field have been calculated in Ref. [9]. Also in Ref. [9] one can find details of the calculational technique, which is the same for the diagrams in Fig. 2 involving the nonminimal electromagnetic couplings of nucleons (anomalous magnetic moments). The diagrams in Fig. 2 can be grouped into sets with the same topology: Figs. 2(a) and 2(b), 2(c) and 2(d), 2(e) and 2(f), 2(g) and 2(h), and 2(i) and 2(k).

The generic contribution of these diagrams to the nEDM is written as

$$\begin{aligned} & -\bar{u}_N(p_2) d^i_N \sigma^{\mu\nu} q_\nu \gamma^5 u_N(p_1) + \dots \\ &\quad = f_{H\pi\pi} g_{HNN} g^2_{\pi NN} M_H \frac{k_N}{2M_N} I_{\mathrm{loop}}. \end{aligned} \quad (8)$$

I_{loop} is the sum of two topologically equivalent Feynman diagrams,

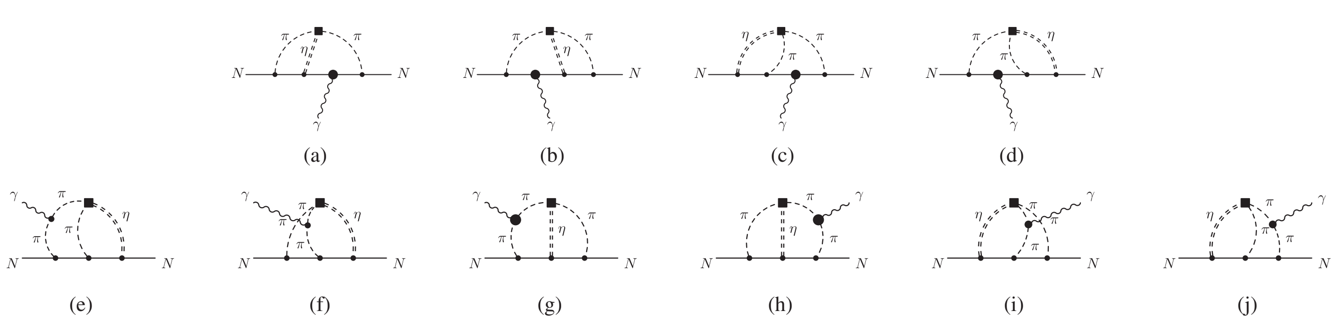

FIG. 1. Diagrams contributing to the nEDM which are induced by the minimal electromagnetic couplings of proton and charged pions. The solid square denotes the *CP* violating $\eta\pi^+\pi^-$ vertex.

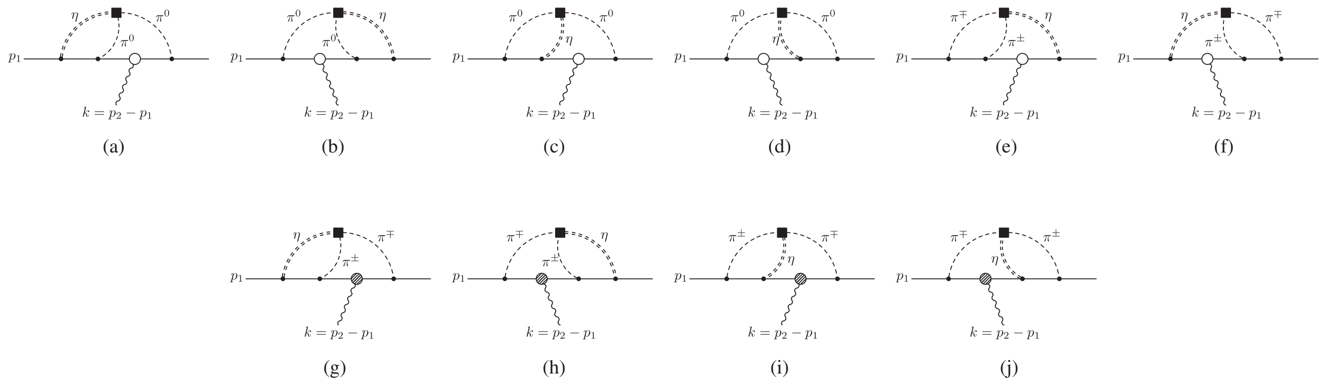

FIG. 2. Diagrams contributing to the nEDM which are induced by the nonminimal electromagnetic couplings (anomalous magnetic moments) of nucleons. The solid square denotes the CP violating $\eta(\eta')\pi\pi$ vertex. Empty and shaded circles correspond to the nonminimal electromagnetic couplings of neutron and proton, respectively.

$$
\begin{aligned}
I_{\rm loop} &= \int \frac{d^4q_1 d^4q_2}{(2\pi)^8} S_{M_1}(q_1) S_{M_1}(q_2) S_{M_3}(q_2 - q_1) \\
&\quad \times [\bar{u}_N(p_2)(\gamma_5 S_N(p_2+q_2)\sigma^{\mu\nu}q_\nu S_N(p_1+q_2)\gamma_5 S_N(p_1+q_1)\gamma_5 \\
&\quad + \gamma_5 S_N(p_2+q_2)\gamma_5 S_N(p_2+q_1)\sigma^{\mu\nu}q_\nu S_N(p_1+q_1)\gamma_5)u_N(p_1)] \\
&= -\bar{u}_N(p_2)\sigma^{\mu\nu}q_\nu\gamma^5 u_N(p_1) I_d(M_1, M_2, M_3), \\
I_d(M_1, M_2, M_3) &= \frac{2M_N}{(4\pi)^4}\int_0^1 d\alpha_1 \cdots \int_0^1 d\alpha_6 \frac{\delta(1-\sum_{i=1}^6 \alpha_i)}{(\det A)^2(\Delta + BA^{-1}B)}\left[-1 + 3A_{12}^{-1}\beta_2 + \frac{M_N^2\beta_1\beta_2}{\Delta + BA^{-1}B}(2-\beta_2)\right],
\end{aligned} \tag{9}
$$

where $S_N(k) = (\not{k} - M_N)^{-1}$ and $S_{M_i}(k) = (k^2 - M_i^2)^{-1}$ are the nucleon and meson (with mass M_i) propagators, respectively. Here A_{ij} is the 2×2 matrix

$$
A_{ij} = \begin{pmatrix} \alpha_{146} & -\alpha_6 \\ -\alpha_6 & \alpha_{2356} \end{pmatrix}, \qquad \alpha_{i_1\cdots i_k} = \alpha_{i_1} + \cdots + \alpha_{i_k}, \tag{10}
$$

A^{-1} and $\det A$ are its inverse and determinant, respectively, and $B_1 = p_1\alpha_1$, $B_2 = p_1\alpha_3 + p_2\alpha_2$, $\Delta = M_1^2\alpha_4 + M_2^2\alpha_5 + M_3^2\alpha_6$, $\beta_1 = \alpha_1 A_{11}^{-1} + \alpha_{23}A_{12}^{-1}$, and $\beta_2 = \alpha_1 A_{12}^{-1} + \alpha_{23}A_{22}^{-1}$. $I_d(M_1, M_2, M_3)$ is the scalar function deduced after calculation of the generic two-loop diagram from the set in Fig. 2.

Summing all graphs of Fig. 2 we obtain the resulting expression for the nEDM

$$
\begin{aligned}
d_N = 2M_\eta f_{\eta\pi\pi} g_{NN\eta} g_{NN\pi}^2 \bigg[& k_n \left(I_d(M_\pi, M_\pi, M_\eta) + \frac{1}{2} I_d(M_\pi, M_\eta, M_\pi) + \frac{1}{2} I_d(M_\eta, M_\pi, M_\pi) \right) \\
& + k_p (I_d(M_\pi, M_\eta, M_\pi) + I_d(M_\eta, M_\pi, M_\pi)) \bigg].
\end{aligned} \tag{11}
$$

Here the factor $1/2$ corresponds to the graphs with neutral pion loops.

IV. RESULTS AND DISCUSSION

Our numerical result for the nEDM induced by the CP violating $\eta(\eta')\pi\pi$ couplings and the anomalous magnetic moments of nucleons is

$$
\begin{aligned}
d_n^{E,k} &\simeq (c_\eta f_{\eta\pi\pi} + c_{\eta'} f_{\eta'\pi\pi}) \times 10^{-16}\ \mathrm{e\cdot cm}, \\
c_\eta &= -0.14, \qquad c_{\eta'} = -0.22.
\end{aligned} \tag{12}
$$

The full result including both minimal and nonminimal electromagnetic couplings of nucleons can easily be computed by taking into account our previous results of Ref. [9] restricted to the case of minimal coupling:

$$
\begin{aligned}
d_n^{E} &\simeq (c_\eta f_{\eta\pi\pi} + c_{\eta'} f_{\eta'\pi\pi}) \times 10^{-16}\ \mathrm{e\cdot cm}, \\
c_\eta &= 6.62, \qquad c_{\eta'} = 7.64.
\end{aligned} \tag{13}
$$

In Table I we present the detailed numerical results for the contribution of each diagram and their total contribution to

TABLE I. Numerical results for the c_η and $c_{\eta'}$ couplings.

	Coupling c_η				Coupling $c_{\eta'}$			
Diagram (Figs. 1 and 2)	PP	MC	NC	PP + MC + NC	PP	MC	NC	PP + MC + NC
1(a) + 1(b)/2(i) + 2(k)	⋯	0.58	−0.92	−0.34	⋯	0.71	−1.57	−0.86
1(c) + 1(d)/2(g) + 2(h)	⋯	0.56	−0.88	−0.32	⋯	0.67	−1.5	−0.83
1(e) + 1(k)	1.12	⋯	⋯	1.12	1.29	⋯	⋯	1.29
1(g) + 1(h)	1.02	⋯	⋯	1.02	1.13	⋯	⋯	1.01
1(f) + 1(i)	0.1	⋯	⋯	0.1	0.13	⋯	⋯	0.13
2(a) + 2(b)	⋯	⋯	0.77	0.77	⋯	⋯	1.32	1.32
2(c) + 2(d)	⋯	⋯	0.47	0.47	⋯	⋯	0.8	0.8
2(e) + 2(f)	⋯	⋯	0.49	0.49	⋯	⋯	0.84	0.84
Total	4.48	2.28	−0.14	6.62	5.1	2.76	−0.22	7.64

the couplings c_η and $c_{\eta'}$. For each diagram we specify (if it occurs) the contribution of charged pion-photon (PP) coupling, nucleon-photon minimal (MC) and nonminimal (NC) couplings and also indicate their total contribution (PP + MC + NC). The contributions coming from the nonminimal coupling of proton and neutron to the electromagnetic field have the same order of magnitude as the one induced by minimal coupling of the proton, but they compensate each other due to their opposite sign. The total numerical contribution of the nonminimal couplings of the nucleon is relatively suppressed (by 1 order of magnitude) compared to the total contribution of the minimal coupling of the proton.

The bounds for the branching ratios of the rare decays $\Gamma_{\eta\pi\pi}$ and $\Gamma_{\eta'\pi\pi}$ are strongly suppressed when compared to existing data [16]

$$\mathrm{Br}(\eta \to \pi^+\pi^-) < 5.54 \times 10^{-17},$$
$$\mathrm{Br}(\eta' \to \pi^+\pi^-) < 5.33 \times 10^{-19}, \tag{14}$$

$$\mathrm{Br}(\eta \to \pi^0\pi^0) < 2.27 \times 10^{-17},$$
$$\mathrm{Br}(\eta' \to \pi^0\pi^0) < 2.17 \times 10^{-19}. \tag{15}$$

When we deduce these new bounds we suppose that the *CP* violating $\eta\pi\pi$ and $\eta'\pi\pi$ couplings are independent and use the current experimental bound on the nEDM: $|d_n^E| < 2.9 \times 10^{-26}$ e · cm. These limits are about ~12–14 orders of magnitude more stringent than given by the recent data from the LHCb Collaboration [16]. The planned study of the $\eta(\eta')\pi\pi$ decays at the JEF [10] could shed light on the possible impact of new physics on these *CP* violating processes.

The *CP* violating $\eta\pi\pi$ and $\eta'\pi\pi$ couplings are estimated using Eq. (13) and limits on the nEDM [21]:

$$|f_{\eta\pi\pi}(m_\eta^2)| < 4.4 \times 10^{-11}, \qquad |f_{\eta'\pi\pi}(m_{\eta'}^2)| < 3.8 \times 10^{-11}. \tag{16}$$

Note that these results are very close to the ones obtained in Ref. [9] restricted to the minimal coupling of charged pions and proton with the photon:

$$|f_{\eta\pi\pi}(m_\eta^2)| < 4.3 \times 10^{-11}, \qquad |f_{\eta'\pi\pi}(m_{\eta'}^2)| < 3.7 \times 10^{-11}. \tag{17}$$

Using Eq. (6) we also derive upper limits for the $\bar\theta$ parameter,

$$\bar\theta^{\eta} < 9.1 \times 10^{-10}, \qquad \bar\theta^{\eta'} < 9.7 \times 10^{-10}, \tag{18}$$

for a current quark mass ratio $R = 0.556$ taken from chiral perturbation theory (ChPT) at 1 GeV scale [22] and

$$\bar\theta^{\eta} < 8.6 \times 10^{-10}, \qquad \bar\theta^{\eta'} < 9.1 \times 10^{-10}, \tag{19}$$

for $R = 0.468$ taken from QCD lattice (LQCD) data at a scale of 2 GeV [21].

One can also pursue another way to obtain an upper estimate for the $\bar\theta$ parameter. In particular, one can extract $\bar\theta$ substituting the QCD relations between $f^{\bar\theta}_{\eta(\eta')\pi\pi}$ and $\bar\theta$ (6) into the expression for the nEDM (13). For the quark mass ratios taken from ChPT and LQCD we get

$$d_n^E \simeq (c_n^E \times 10^{-16}) \cdot \bar\theta \ \mathrm{e \cdot cm},$$
$$c_n^E = 0.65 \ (\mathrm{ChPT}), \qquad c_n^E = 0.62 \ (\mathrm{LQCD}). \tag{20}$$

Using data on the nEDM [23] we extract the following upper limits for $\bar\theta$:

$$\bar\theta = 4.4 \times 10^{-10} \ (\mathrm{ChPT}), \qquad \bar\theta = 4.7 \times 10^{-10} \ (\mathrm{LQCD}). \tag{21}$$

The second predictions of upper limits for $\bar\theta$ are by a factor 2 smaller than the first ones and are closer to the prediction done in Ref. [24]. In particular, in Ref. [24] the value $\bar\theta \sim 1.1 \times 10^{-10}$ was extracted using data for the nEDM with $|d_n^E| < 1.6 \times 10^{-26}$ e · cm. For this limit on the nEDM we deduce $\bar{\theta} = 2.4 \times 10^{-10}$ (ChPT) and $\bar{\theta} = 2.6 \times 10^{-10}$ (LQCD).

In conclusion, we studied limits on the QCD CP violating parameter $\bar{\theta}$ and branchings of the CP violating rare decays $\eta \to \pi\pi$ and $\eta' \to \pi\pi$ using a phenomenological Lagrangian approach. We particularly took into account both minimal and nonminimal couplings of the nucleon to the photon. The nEDM was induced by the CP violating $\eta(\eta') \to \pi\pi$ couplings. The obtained results will be important for the planned experiments on rare η and η' meson decays at JEF [10]. In a continuation of our previous papers [8,9] we analyze the impact of the nonmimal coupling. We find that the separate contributions of specific diagrams generated by the nonminimal coupling of photon with nucleons are comparable with the ones generated by the minimal coupling. But total contribution of diagrams with noniminal coupling is suppressed due to the compensation of contributions with opposite signs.

ACKNOWLEDGMENTS

The work of A. S. Zh. was funded by a Russian Science Foundation grant (Project No. 18-72-00046), in particular part of the numerical calculation. The work of V. E. L. was funded by the Carl Zeiss Foundation under Project "Kepler Center für Astro- und Teilchenphysik: Hochsensitive Nachweistechnik zur Erforschung des unsichtbaren Universums (Gz: 0653-2.8/581/2)," and by CONICYT (Chile) under Grant No. PIA/Basal FB0821.

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