White dwarfs in de Rham-Gabadadze-Tolley like massive gravity

B. Eslam Panah[1,2,3,*] and H. L. Liu[4,†]

[1] *Physics Department and Biruni Observatory, College of Sciences, Shiraz University, Shiraz 71454, Iran*
[2] *Research Institute for Astronomy and Astrophysics of Maragha (RIAAM), P.O. Box 55134-441 Maragha, Iran*
[3] *ICRANet, Piazza della Repubblica 10, I-65122 Pescara, Italy*
[4] *School of Physical Science and Technology, Xinjiang University, Urumqi 830046, China*

(Received 10 January 2019; revised manuscript received 3 April 2019; published 28 May 2019)

The existence of possible massive white dwarfs more than the Chandrasekhar limit ($1.45\,M_\odot$, in which $M_\odot$ is mass of the sun) is a challenging topic. In this regard, and motivated by the important effect of massive graviton on the structure of white dwarfs, we study the white dwarfs in Vegh's massive gravity which is known as one of theories of de Rham, Gabadadze, and Tolley (dRGT) like massive gravity. First, we consider the modified Tolman-Oppenheimer-Volkoff equation in this theory of massive gravity and solve it numerically by using the Chandrasekhar's equation of state. Our results show that the maximum mass of white dwarfs in massive gravity can be more than the Chandrasekhar limit ($M > 1.45\,M_\odot$), and this result imposes some constraints on parameters of massive gravity. Then, we investigate the effects of various parameters on other properties of the white dwarfs such as mass-radius relation, mass-central density relation, Schwarzschild radius, average density, and Kretschmann scalar. Next, we study dynamical stability condition for super-Chandrasekhar white dwarfs and show that these massive compact objects enjoy dynamical stability. Finally, in order to have a better insight, we compare the super-Chandrasekhar white dwarfs with the obtained massive neutron stars in dRGT like massive theory of gravity.

DOI: 10.1103/PhysRevD.99.104074

I. INTRODUCTION

General relativity (GR) is a successful theory of gravity that contains gravitons as massless spin-2 particles. GR predicted some phenomena such as the gravitational deflection of light around gravitational sources such as the Sun, which was confirmed by Arthur Eddington. Nowadays, its direct application in the form of gravitational lensing is one of the indispensable tools in astrophysics and cosmology. Another powerful prediction of GR is the presence of gravitational waves, which was detected by the advanced LIGO/Virgo collaboration [1]. In spite of these successes at the large scales, GR cannot explain why our universe is undergoing an accelerated cosmic expansion. Therefore, GR without the cosmological constant term needs to be modified. Among these modified theories of gravity, massive gravity can explain the late-time acceleration without considering dark energy [2–6]. In order to build up a massive theory with a massive spin-2 particle propagation, one can add an interaction term to the Einstein-Hilbert action. In addition, Chamseddine and Volkov found that the effect of graviton mass is equivalent to introducing a matter source in the Einstein equations which can consist of several different types of matter; a cosmological term, quintessence and also nonrelativistic cold matter [3]. Massive gravity modifies gravitational effects by weakening it at the large scale comparing to GR. This allows the universe to accelerate, but its predictions at small scales are the same as GR. On the other hand, massive gravity will result in the graviton having a mass of m which in the case of $m \to 0$, the effect of massive gravity is vanished and this theory reduces to GR. In addition, it was shown that the graviton mass is very small in the usual weak gravity environments, but becomes much larger in the strong gravity regime such as black holes and compact objects [7]. Accordingly, there were numerous developments in the massive gravity theories in recent years [8–13]. On the other hand, recent observations by the advanced LIGO/Virgo collaboration have put a tight bound on the graviton mass [1,14], however cannot rule out the possibility of nonzero mass. Also, there are other theoretical and empirical limits on the graviton's mass (see Refs. [15–19], for more details). Thus one may motivate to investigate the effects of considering the massive gravitons on various branches related to gravitation.

Fierz and Pauli in 1939 introduced a class of massive gravity theory in flat background [8]. In other words, Fierz and Pauli added the interaction terms at the linearized level of GR; this theory is known as Fierz and Pauli massive (FP massive) gravity. Then van Dam, Veltman and Zakharov found out that FP massive gravity suffers from discontinuity

*beslampanah@shirazu.ac.ir
†heleiliu@xju.edu.cn

which is known as van Dam-Veltman-Zakharov (vDVZ) discontinuity [9–11]. In order to remove the vDVZ discontinuity, Vainshtein showed that such discontinuity appears as a consequence of working with the linearized theory of GR, and then a mechanism for the nonlinear massive gravity was introduced by him [12]. On the other hand, Boulware and Deser explored that such nonlinear generalizations usually generate an equation of motion which has a higher derivative term yielding a ghost instability in the theory which is known as Boulware-Deser (BD) ghost [13]. However, these problems, arising in the construction of the massive gravity have been resolved in the last decade by introducing Stückelberg fields [20]. This allows a class of potential energies depending on the gravitational metric and an internal Minkowski metric (reference metric). In Refs. [21,22], de Rham, Gabadadze and Tolley (dRGT) introduced a new version of massive gravity which is free of vDVZ discontinuity and BD ghost in arbitrarily dimensions [23]. Although the equations of motion have no higher derivative term in the dRGT massive gravity, finding exact solutions in this theory of gravity is difficult. However, black hole solutions in dRGT massive gravity have been obtained by some authors in Refs. [24–27]. In the astrophysics context, Katsuragawa *et al.* evaluated the neutron stars in this theory and showed that the massive gravity leads to small deviation from the GR [28]. Mass-radius ratio bounds for compact objects in this gravity have been obtained in Ref. [29]. M. Yamazaki *et al.* discussed the boundary conditions for the relativistic stars in this theory of gravity [30]. From a cosmological point of view, bounce and cyclic cosmology [31], cosmological behavior [32], and another properties have been studied by some authors in Refs. [33–35]. On the other hand, the constraints imposed by the Type Ia Supernovae (SNe Ia), gamma ray bursts (GRBs), baryon acoustic oscillations (BAOs), cosmic microwave background radiation (CMBR) on the massive gravity have been investigated in Refs. [36,37]. In Ref. [38], Panpanich and Burikham evaluated the effects of nonzero graviton mass on the rotation curves of the Milky Way, spiral galaxies, and low surface brightness galaxies. Rotation curves of the most galaxies can be fitted well by considering the graviton's mass in the range $m \sim 10^{-21}$–10^{-30} eV (see Ref. [38], for more details). Aoki and Mukohyama studied graviton mass as a candidate for dark matter [39]. Indeed, they showed that if LIGO detects gravitational waves generated by preheating after inflation then the massive graviton with the mass of $\sim$0.01 GeV is a candidate of the dark matter. Cosmological perturbations in massive gravity have been studied by some authors in Refs. [40–44], and they obtained some constraints on parameters of this theory by considering observational cosmological data.

It is notable that, modification in the introduced reference metric in dRGT theory leads to the possibility of introduction of different classes of dRGT like massive theories [45]. One of the theories was proposed by Vegh which has applications in gauge/gravity duality [46]. Indeed this theory is similar to dRGT massive gravity with a difference that its reference metric is a singular one. Graviton in this massive gravity may behave like a lattice and exhibits a Drude peak [46]. It was shown that for arbitrary singular metric, this theory of massive gravity is ghost-free and stable [47]. Black hole solutions in this gravity have been obtained in Refs. [48,49]. The existence of van der Waals like behavior in extended phase space for the obtained black holes has been studied in this massive gravity by some authors in Refs. [50–53]. It was pointed out that it is possible to have a heat engine for nonspherical black holes in massive gravity [54]. In addition, magnetic solutions in this dRGT like massive gravity have been addressed in Refs. [55,56]. From the perspective of astrophysical, the modified Tolman-Oppenheimer-Volkoff (TOV) equation by considering this theory of massive gravity was obtained in Ref. [57], and it was shown that the maximum mass of neutron stars can be more than three times the solar mass. The existence of a remnant for a black hole in this theory of massive gravity has been evaluated by Eslam Panah *et al.* in Ref. [58], where they showed that this remnant may help to ameliorate the information paradox.

As we know, the massive graviton leads to the modification of long-range gravitational force. Therefore, one may expect that the graviton mass could be comparable to the cosmological constant, which could illustrate the accelerated expansion of the Universe without introducing the cosmological constant (see Refs. [4,59–62], for more details). It is very interesting to apply the dRGT like theories of gravity (in this work we consider Vegh's approach of massive gravity) to astrophysical phenomena. It is notable that, construction the general framework which quantifies the deviations from the predictions of the GR in strong-gravity regime is very difficult (see Ref. [63], for more details). In addition, it is very important that if we could conclude that cosmological and astrophysical applications are compatible with observations in a specific theory of modified gravity. According to the above reasons, it is necessary to study the compact objects in the massive gravity, especially dRGT like massive gravity as astrophysical test of this gravity in strong-gravity regime.

On the other hand, in recent years, some peculiar type SNe Ia: e.g., SN 2006gz, SN 2007if, SN 2009dc, SN 2003fg, have been observed [64–67] with exceptionally higher luminosities. It has been suggested that the progenitor mass to explain such SNe Ia stands in the range 2.1–2.8 $M_{\odot}$ [68,69], which exceeds significantly the Chandrasekhar mass limit about 1.45 $M_{\odot}$. Some authors explained such over luminous SNe Ia by proposing the existence of super strong uniform magnetic fields [70], rotation white dwarfs [71], electrical charge distribution white dwarfs [72], and modification to GR in white dwarfs [73], to generate some super-Chandrasekhar white dwarfs. Briefly, for explaining these massive white dwarfs, we can consider two approaches. The first approach: we can improve the equation of state by adding magnetic field [70]. The second approach is related to modified TOV equations [71–74]. According to the mentioned reasons for modification of GR, in this work, we are going to investigate the influence of Vegh's massive gravity [46] on the properties and stability of the white dwarfs.

The article is organized as follows: after an introduction about Vegh's massive gravity, we will present the modified TOV in this theory of massive gravity. In Sec. III, we reintroduce the Chandrasekhar's equation of state as a suitable equation of state. Then, by considering the mentioned modified TOV, we will study the properties of white dwarfs. We will evaluate another quantities such as Schwarzschild radius, average density, the Kretschmann scalar, and dynamical stability of these white dwarfs. Then we compare the properties of massive neutron stars with super-Chandrasekhar white dwarfs in this gravity. Some closing remarks are given in the last section.

II. BASIC EQUATIONS

The action of dRGT like massive gravity is given by [46]

$$\mathcal{I} = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\left[\mathcal{R} + m^2\sum_i^4 c_i\mathcal{U}_i(g,f)\right] + I_{\mathrm{matter}}, \quad (1)$$

where R and m are the Ricci scalar and the graviton mass, respectively. $\kappa = \frac{8\pi G}{c^4}$, and also f and g are a fixed symmetric tensor and metric tensor, respectively. In the above relation, I_{matter} is related to the action of matter. In addition, c_i's are free parameters of this theory which are arbitrary constants. Their values can be determined according to theoretical or observational considerations [4,5,75,76]. Also, $\mathcal{U}_i$'s are symmetric polynomials of the eigenvalues of 4×4 matrix $\mathcal{K}^\mu_\nu = \sqrt{g^{\mu\alpha}f_{\alpha\nu}}$ (for 4-dimensional spacetime) where they can be written in the following forms

$$\mathcal{U}_1 = [\mathcal{K}], \qquad \mathcal{U}_2 = [\mathcal{K}]^2 - [\mathcal{K}^2],$$
$$\mathcal{U}_3 = [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3],$$
$$\mathcal{U}_4 = [\mathcal{K}]^4 - 6[\mathcal{K}^2][\mathcal{K}]^2 + 8[\mathcal{K}^3][\mathcal{K}] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4].$$

where the square root in $\mathcal{K}$ stands for matrix square root, i.e., $\mathcal{K}^\mu{}_\nu = (\sqrt{\mathcal{K}})^\mu{}_\lambda(\sqrt{\mathcal{K}})^\lambda{}_\nu$, and the rectangular bracket denotes the trace $[\mathcal{K}] = \mathcal{K}^\mu{}_\mu$.

Considering a spherical symmetric space-time in 4-dimensional as

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = H(r)dt^2 - \frac{dr^2}{S(r)} - r^2h_{ij}dx_idx_j,$$
$$i,j = 1,2. \quad (2)$$

where $H(r)$ and $S(r)$ are unknown metric functions, and $h_{ij}dx_idx_j = (d\theta^2 + \sin^2\theta d\varphi^2)$. By variation of Eq. (1) with respect to the metric tensor g^ν_μ, the equation of motion for massive gravity can be written as

$$G^\nu_\mu + m^2\chi^\nu_\mu = \frac{8\pi G}{c^4}T^\nu_\mu, \quad (3)$$

where G is the gravitational constant, and also, G^ν_μ and c are the Einstein tensor and the speed of light in vacuum, respectively. T^ν_μ denotes the energy-momentum tensor which comes from the variation of I_{matter} and $\chi_{\mu\nu}$ is the massive term with the following explicit form

$$\begin{aligned}\chi_{\mu\nu} = &-\frac{c_1}{2}(\mathcal{U}_1g_{\mu\nu} - \mathcal{K}_{\mu\nu}) - \frac{c_2}{2}(\mathcal{U}_2g_{\mu\nu} - 2\mathcal{U}_1\mathcal{K}_{\mu\nu} + 2\mathcal{K}^2_{\mu\nu})\\ &-\frac{c_3}{2}(\mathcal{U}_3g_{\mu\nu} - 3\mathcal{U}_2\mathcal{K}_{\mu\nu} + 6\mathcal{U}_1\mathcal{K}^2_{\mu\nu} - 6\mathcal{K}^3_{\mu\nu})\\ &-\frac{c_4}{2}(\mathcal{U}_4g_{\mu\nu} - 4\mathcal{U}_3\mathcal{K}_{\mu\nu} + 12\mathcal{U}_2\mathcal{K}^2_{\mu\nu}\\ &- 24\mathcal{U}_1\mathcal{K}^3_{\mu\nu} + 24\mathcal{K}^4_{\mu\nu}).\end{aligned} \quad (4)$$

Considering the white dwarf as a perfect fluid with the following energy-momentum tensor as

$$T^{\mu\nu} = (c^2\rho + P)U^\mu U^\nu - Pg^{\mu\nu}, \quad (5)$$

where P and ρ are the pressure and density of the fluid which are measured by the local observer, respectively, and U^μ is the fluid four-velocity. The nonzero components of the energy-momentum tensor for perfect fluid are

$$T^0_0 = c^2\rho, \qquad T^1_1 = T^2_2 = T^3_3 = -P. \quad (6)$$

In order to obtain exact static spherical black hole solutions, the appropriate ansatz for the reference metric was introduced in the form; $f_{\mu\nu} = \mathrm{diag}(0,0,C^2h_{ij})$, see [48,49], for more details. In this work, we intend to obtain static spherical solutions similar to static spherical black hole solutions of massive gravity. So, we consider the mentioned appropriate ansatz for the reference metric $f_{\mu\nu}$ in 4-dimensional spacetime, which is given as

$$f_{\mu\nu} = \mathrm{diag}(0,0,C^2,C^2\sin^2\theta), \quad (7)$$

where C is known as parameter of reference metric which is a positive constant. In other words, $f_{\mu\nu}$ only depends on the spatial components h_{ij} of the spacetime metric (2). Using the mentioned information and ansatz, we can extract the explicit functional forms of U_i's in the following forms

$$\mathcal{U}_1 = \frac{2C}{r}, \qquad \mathcal{U}_2 = \frac{2C^2}{r^2},$$
$$\mathcal{U}_i = 0, \qquad i > 2. \tag{8}$$

Considering the field equation (3), the static spherical metric (2), and the mentioned reference metric (7), we can obtain the metric function $S(r)$, in the following form [57]

$$S(r) = 1 - m^2 C\left(\frac{c_1 r}{2} + c_2 C\right) - \frac{2GM(r)}{c^2 r}, \tag{9}$$

in which $M(r) = \int 4\pi r^2 \rho(r) dr$. After some calculations, we can extract the modified TOV equation in Vegh's massive gravity as [57]

$$\frac{dP}{dr} = \frac{G(c^2 M(r) + 4\pi r^3 P) - \frac{m^2 r^2 c_1 c^4 C}{4}}{\left(\frac{m^2 c_1 c^2 r^2 C}{2} + 2GM(r) + c^2 r(m^2 c_2 C^2 - 1)\right) c^2 r}(c^2\rho + P). \tag{10}$$

Considering the obtained modified TOV equation in Vegh's massive gravity, we want to investigate the properties of white dwarfs in this theory of gravity in the next sections.

Before we continue our study about white dwarfs in Vegh's massive gravity, we want to give a brief dimensional analysis of the parameters of massive and reference metric. It is notable that all terms of the metric function must be dimensionless, i.e., $\frac{m^2 c_1 C}{2} r$, $m^2 c_2 C^2$, and $\frac{2GM(r)}{c^2 r}$ are dimensionless. Also, in dimensional analysis we know that $[M(r)] = [m] = M$ (Mass), and $[r] = L$ (Length). So, the dimensional interpretation of massive terms are

$$[C] = L, \quad \& \quad [c_i] = M^{-2}L^{-2}, \quad i = 1, 2.$$

According to this fact that the action of massive gravity (1) is dimensionless, we find that dimensional interpretations of $\mathcal{R}$ and all of terms $m^2 \sum_i^4 c_i \mathcal{U}_i(g, f)$ are L^{-2}. Remembering that $[m^2 c_i] = L^{-2}$, one can conclude that U_i's in Eq. (1), are dimensionless. As we know, the dimension of the cosmological constant (Λ) is L^{-2}, so $m^2 c_i$ terms may play the role of the pressure in the extended phase space (see Ref. [77], for more details).

III. EQUATION OF STATE

We use the Chandrasekhar's equation of state (EoS), which are constituted from electron degenerate matter,

$$k_F = \hbar(3\pi^2\rho/(m_p\mu_e))^{1/3} \tag{11}$$

and

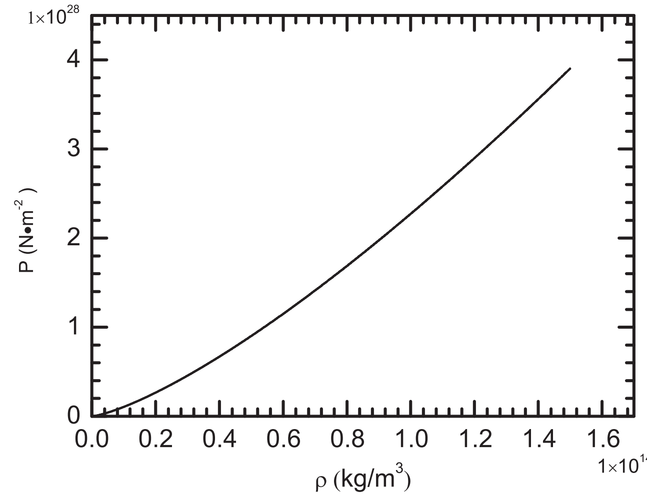

FIG. 1. Chandrasekhar's equation of state.

$$P = \frac{8\pi c}{3(2\pi\hbar)^3}\int_0^{k_F} \frac{k^2}{(k^2 + m_e^2 c^2)^{1/2}} k^2 dk, \tag{12}$$

where k is the momentum of electrons. m_p is the mass of proton. μ_e is the mean molecular weight per electron (we choose $\mu_e = 2$ for our work). $\hbar = h/2\pi$, where h is the Plank's constant. The Chandrasekhar's EoS of the electron degenerate matter was shown in Fig. 1.

The Chandrasekhar's EoS is one of famous EoSs for studying the structure of white dwarfs. In this regards, we review some properties of this EoS such as; energy conditions, stability, and Le Chatelier's principle.

The Chandrasekhar's EoS satisfies the energy conditions such as the null energy condition (NEC), weak energy condition (WEC), strong energy condition (SEC) and dominant energy condition (DEC) at the center of white dwarfs. These conditions are as

$$\text{NEC} \rightarrow P_c + \rho_c \geq 0, \tag{13}$$

$$\text{WEC} \rightarrow P_c + \rho_c \geq 0, \ \& \ \rho_c \geq 0, \tag{14}$$

$$\text{SEC} \rightarrow P_c + \rho_c \geq 0, \ \& \ 3P_c + \rho_c \geq 0, \tag{15}$$

$$\text{DEC} \rightarrow \rho_c > |P_c|, \tag{16}$$

where P_c and ρ_c are the pressure and density at the center of white dwarfs ($r = 0$), respectively. Using Fig. 1 and the mentioned conditions (13)–(16), our results are presented in Table I. According to Fig. 1 and Table I, we observe that all energy conditions are satisfied.

A. Stability

In order to evaluate the Chandrasekhar's EoS for a physically acceptable model, one expects that the velocity of sound ($v = \sqrt{\frac{dP}{d\rho}}$) be less than the light's velocity (c)

TABLE I. Energy conditions at the center of obtained white dwarfs of Chandrasekhar's EoS.

$\rho_c (10^{12} \frac{kg}{m^3})$	$P_c (10^{12} \frac{kg}{m^3})$	NEC	WEC	SEC	DEC
150.6590	4.0515	✓	✓	✓	✓

[78,79]. In other words, stability condition is in the form $0 \leq v^2 \leq c^2$. Therefore, by considering this stability condition and Fig. 1, and comparing them with diagrams related to speed of sound-density relationship in Fig. 2, it is evident that this EoS satisfies the inequality $0 \leq v^2 \leq c^2$.

B. Le Chatelier's principle

There is another important principle which is related to the matter of star and called Le Chatelier's principle. The matter of the star satisfies $dP/d\rho \geq 0$, which is a necessary condition of a stable body both as a whole and also with respect to the nonequilibrium elementary regions with spontaneous contraction or expansion (Le Chatelier's principle), see Ref. [80], for more details. Our calculations show that Le Chatelier's principle is established for the Chandrasekhar's EoS (see Fig. 2).

Our investigations indicated that the Chandrasekhar's EoS satisfies both energy and stability conditions, and also this EoS admits Le Chatelier's principle. Therefore, this EoS is a suitable EoS. It is notable that there are some realistic EoSs in order to have a good view of the behavior of white dwarfs. However, in this work we consider the Chandrasekhar's EoS, because we want to focus on the effects of modified gravity on the structure of white dwarfs.

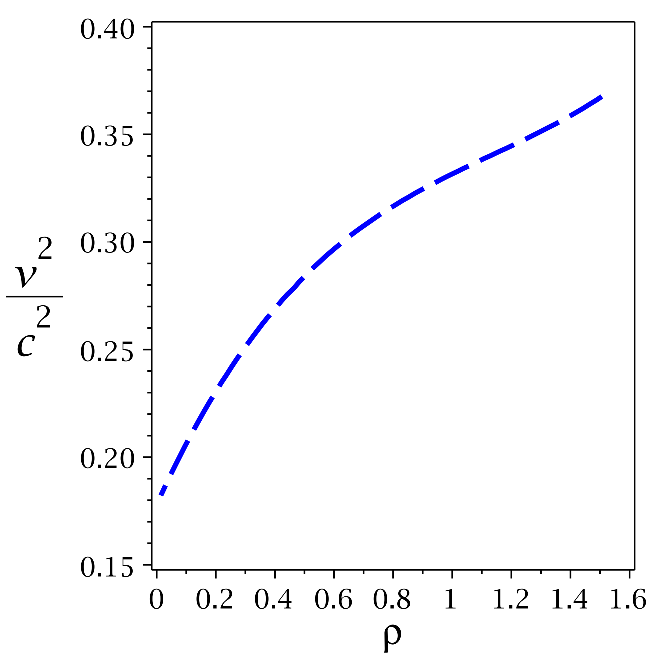

FIG. 2. Sound speed ($v^2/c^2 \times 10^{-18}$) vs density [$\rho \times 10^{14}$ (kg/m^3)].

IV. PROPERTIES OF WHITE DWARFS

Using the famous Chandrasekhar limit, the mass limit of the white dwarf is obtained in Newtonian, special relativity, GR, and nonrelativistic Newtonian theories in the range 1.41–1.45 $M_\odot$ [81]. On the other hand, the explosion of peculiar SN Ia provokes us to rethink the maximum mass of white dwarfs. Hence, the maximum mass of the white dwarf is still an open question. Here, we would like to see whether the maximum mass of the white dwarf in massive gravity and by employing the Chandrasekhar's EoS can be more than this limit (1.45 $M_\odot$). Then we want to study the effects of massive's parameter on properties of the white dwarfs, such as Schwarzschild radius, the Kretschmann scalar, and dynamical stability. It is notable that in this paper, we consider the graviton mass as 10^{-32} eV$/c^2 = 1.78 \times 10^{-65}$ g, which was extracted by A. F. Ali and S. Das in Ref. [82]. They have shown from theoretical considerations, that if the graviton has mass, its value will be about 10^{-32} eV$/c^2$, or 1.78×10^{-65} g. This estimate is consistent with those obtained from experiments, including the recent gravitational wave detection in the advanced LIGO/Virgo. Our results indicate that by considering the special values for the parameters of massive gravity, the maximum mass of white dwarf is an increasing (decreasing) function of C (m^2c_2), see Tables II and III. Our calculations show that the maximum mass of white dwarf in massive gravity can be more than Chandrasekhar limit ($M_{\mathrm{Max}} > 1.45\, M_\odot$). In other words, our results predict that the mass of white dwarfs in this gravity can be in the range upper than $2\, M_\odot$. Also, considering the values of $m^2c_2 \geq -10^{-3}$ and $C \leq 10^{-2}$, the maximum mass and radius of white dwarfs reduce to the obtained results of GR. It is notable that the variation of m^2c_1 has very interesting effect. In this case, by variation m^2c_1, the maximum mass and radius of white dwarfs are constant (see the Table IV).

In order to do more investigation, we plot the mass of white dwarf vs the central mass density ($M - \rho_c$), for different parameters of massive gravity and reference metric in left panels of Figs. 3 and 4. These figures show that, the maximum mass of white dwarf increases when m^2c_2 decreases or C increases. In addition, the variation of maximum mass vs radius ($M - R$) is also shown in right panels of Figs. 3 and 4.

Our calculations show that by considering fixed values for C and m^2c_1, in constant radius (for example $R = 5 \times 10^3$ km), by decreasing m^2c_2, the mass of white dwarfs increase (see right panel in Fig. 3). Also, there is the same behavior for C (see right panel in Fig. 4). In other words, in $R =$ constant, by varying the parameters of massive gravity and the reference metric, the mass of white dwarfs change. This result shows that the density within ofthe white dwarf depends on the parameters of this theory of gravity, so that it increases when the mass of the white dwarf increases.

Here we can ask this question: why does the maximum mass of white dwarfs increase by varying the parameters of

TABLE II. Structure properties of white dwarf in massive gravity for $C = 1$ and $m^2 c_1 = 1 \times 10^{-11}$.

$m^2 c_2$	$M_{\max}(M_\odot)$	R(km)	R_{Sch}(km)	$\bar{\rho}(10^{12}\ \mathrm{kg\,m^{-3}})$	$K(\mathrm{m^{-4}})$
-1×10^{-4}	1.41	871	4.17	1.01	6.37×10^{-32}
-1×10^{-3}	1.41	871	4.17	1.02	6.89×10^{-30}
-1×10^{-2}	1.43	875	4.19	1.02	6.82×10^{-28}
-1×10^{-1}	1.63	913	4.37	1.02	5.76×10^{-26}
-2×10^{-1}	1.86	954	4.57	1.02	1.93×10^{-25}
-4×10^{-1}	2.34	1030	4.93	1.02	5.68×10^{-25}
-6×10^{-1}	2.86	1101	5.27	1.02	9.80×10^{-25}
-8×10^{-1}	3.41	1168	4.79	1.02	1.38×10^{-24}

TABLE III. Structure properties of white dwarf in massive gravity for $m^2 c_1 = 10^{-11}$ and $m^2 c_2 = -2 \times 10^{-1}$.

C	$M_{\max}(M_\odot)$	R(km)	R_{Sch}(km)	$\bar{\rho}(10^{12}\ \mathrm{kg\,m^{-3}})$	$K(\mathrm{m^{-4}})$
0.01	1.41	870	4.17	1.02	2.78×10^{-33}
0.1	1.42	871	4.17	1.02	2.78×10^{-29}
0.5	1.52	892	4.27	1.02	1.58×10^{-26}
1.0	1.86	954	4.57	1.02	1.93×10^{-25}
1.5	2.46	1048	5.02	1.02	6.71×10^{-25}
2.0	3.41	1168	5.59	1.02	1.38×10^{-24}

this theory? The strength of gravity may change by varying the parameters of massive gravity and the reference metric. In other words, by increasing (C) or decreasing ($m^2 c_2$) parameters of this theory, the strength of gravity may decrease. As we know, there is a balance between the internal pressure (which origin it is electron degenerate) and gravitational force. Decreasing the strength of gravity, a star can bear more mass in order to keep this balance. Therefore, the maximum mass of white dwarf increases by increasing (C) or decreasing ($m^2 c_2$).

For completeness, in the following, we investigate other properties of white dwarf in massive gravity such as the Schwarzschild radius, average density, Kretschmann scalar, and dynamical stability.

A. Modified Schwarzschild radius

The Schwarzschild radius for this gravity is obtained as [57]

$$R_{\mathrm{Sch}} = \frac{c(1 - m^2 c_2 C^2)}{m^2 c c_1 C} - \frac{\sqrt{c^2 (m^2 c_2 C^2 - 1)^2 - 4 m^2 c_1 C G M}}{m^2 c c_1 C}. \tag{17}$$

The Schwarzschild radius of white dwarfs are obtained in Tables II and III. These results show that by increasing the maximum mass and radius of white dwarfs, the Schwarzschild radius increases and the obtained white dwarfs in massive gravity with mass more than the Chandrasekhar limit are out of the Schwarzschild radius (see Tables II and III). In other words, different parameters of massive gravity and the reference metric have different behavior on the Schwarzschild radius. For example, by considering the negative value of $m^2 c_2$ and increasing it, the Schwarzschild radius increases (see Table II). Also, by increasing C, the Schwarzschild radius increases (see Table III). On the other hand, by considering the positive (negative) values of $m^2 c_1$ and increasing (decreasing)

TABLE IV. Structure properties of white dwarf in massive gravity for $C = 1$ and $m^2 c_2 = -2 \times 10^{-1}$.

$m^2 c_1$	$M_{\max}(M_\odot)$	R(km)	R_{Sch}(km)	$\bar{\rho}(10^{12}\ \mathrm{kg\,m^{-3}})$	$K(\mathrm{m^{-4}})$
1×10^{-13}	1.86	953	4.56	1.02	1.94×10^{-25}
1×10^{-12}	1.86	953	4.56	1.02	1.94×10^{-25}
1×10^{-11}	1.86	954	4.57	1.02	1.93×10^{-25}
1×10^{-10}	1.86	956	4.58	1.02	1.91×10^{-25}
-1×10^{-13}	1.86	953	4.56	1.02	1.94×10^{-25}
-1×10^{-12}	1.86	953	4.56	1.02	1.94×10^{-25}
-1×10^{-11}	1.86	953	4.56	1.02	1.94×10^{-25}
-1×10^{-10}	1.85	950	4.55	1.02	1.96×10^{-25}

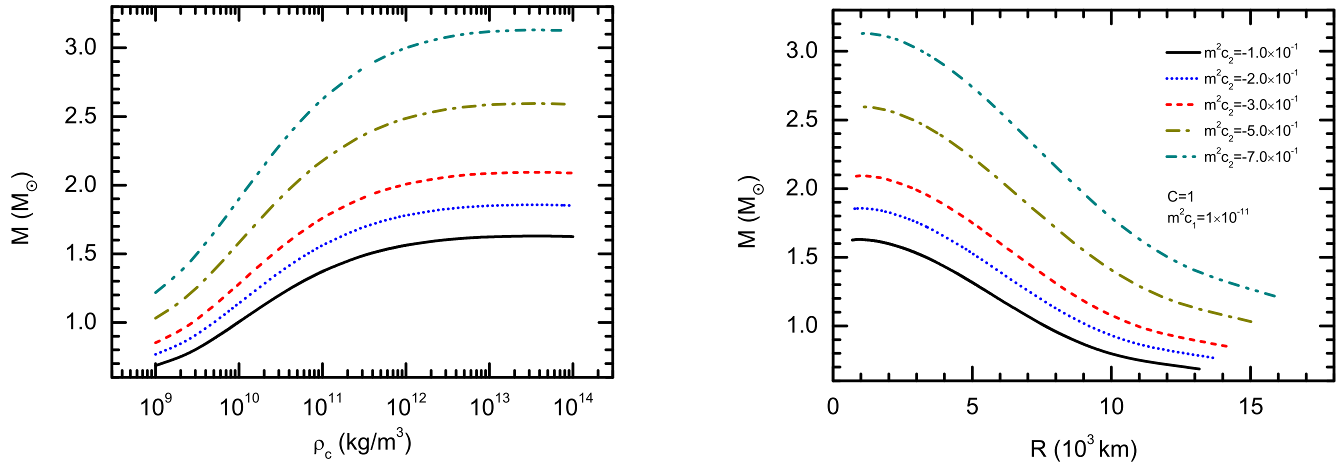

FIG. 3. Gravitational mass vs central density(radius) for $C = 1$ and $m^2c_1 = 1 \times 10^{-11}$. Left diagrams: gravitational mass vs central mass density for $m^2c_2 = -1.0 \times 10^{-1}$ (solid line), $m^2c_2 = -2.0 \times 10^{-1}$ (dotted line), $m^2c_2 = -3.0 \times 10^{-1}$ (dashed line), $m^2c_2 = -5.0 \times 10^{-1}$ (dashed-dotted line), and $m^2c_2 = -7.0 \times 10^{-1}$ (dashed-dotted-dotted line). Right diagrams: gravitational mass vs radius for $m^2c_2 = -1.0 \times 10^{-1}$ (solid line), $m^2c_2 = -2.0 \times 10^{-1}$ (dotted line), $m^2c_2 = -3.0 \times 10^{-1}$ (dashed line), $m^2c_2 = -5.0 \times 10^{-1}$ (dashed-dotted line), and $m^2c_2 = -7.0 \times 10^{-1}$ (dashed-dotted-dotted line).

m^2c_1, the Schwarzschild radius does not change (see Table IV).

B. Average density

We can evaluate the average density by using the obtained maximum mass and radius of white dwarfs in the massive gravity from the perspective of a distant observer (or an observer outside the neutron star). So the average density of the white dwarf is given

$$\bar{\rho} = \frac{3M}{4\pi R^3}, \tag{18}$$

where the results for variation of the massive parameters and the reference metric are presented in the Tables II–IV. There is an interesting result about the average density of the white dwarf in massive gravity from the perspective of a observer outside the white dwarf. By variations of the different parameters, the average density remains fixed (see Tables II–IV).

In order to investigate the strength of gravity, we study the Kretschmann scalar in the presence of nonzero graviton mass in the following subsection.

C. Kretschmann scalar

Spacetime curvature is a quantity that shows the strength of gravity. It is worth mentioning that in the Schwarzschild spacetime, the components of Ricci scalar (R) and the Ricci tensor ($R_{\mu\nu}$) are zero outside the star, and these quantities do

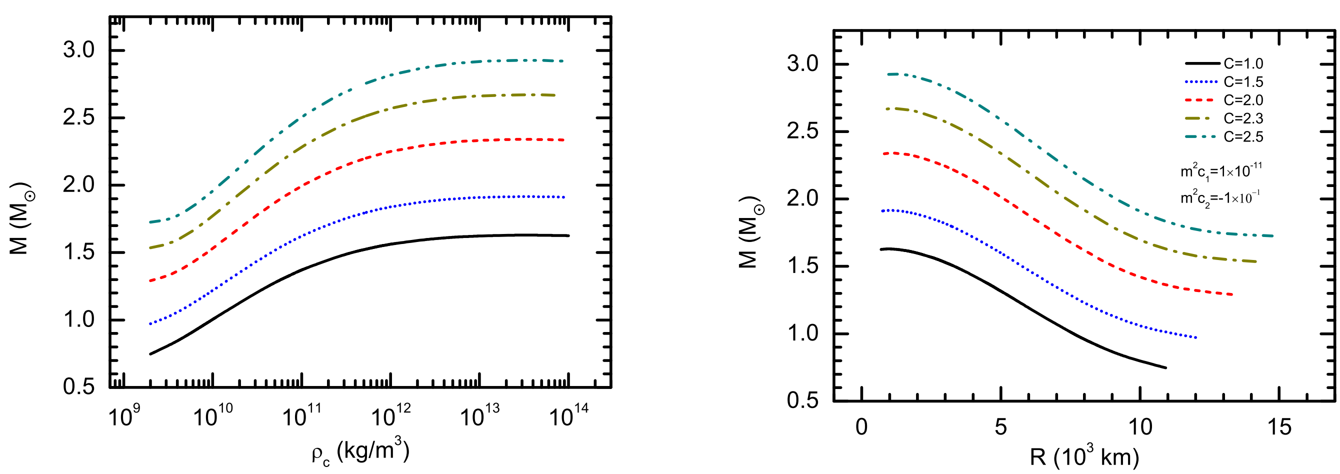

FIG. 4. Gravitational mass vs central density (radius) for $m^2c_1 = 1 \times 10^{-11}$ and $m^2c_2 = -1 \times 10^{-1}$. Left diagrams: gravitational mass vs central mass density for $C = 1.0$ (solid line), $C = 1.5$ (dotted line), $C = 2.0$ (dashed line), $C = 2.3$ (dashed-dotted line) and $C = 2.5$ (dashed-dotted-dotted line). Right diagrams: gravitational mass vs radius for $C = 1.0$ (solid line), $C = 1.5$ (dotted line), $C = 2.0$ (dashed line), $C = 2.3$ (dashed-dotted line) and $C = 2.5$ (dashed-dotted-dotted line).

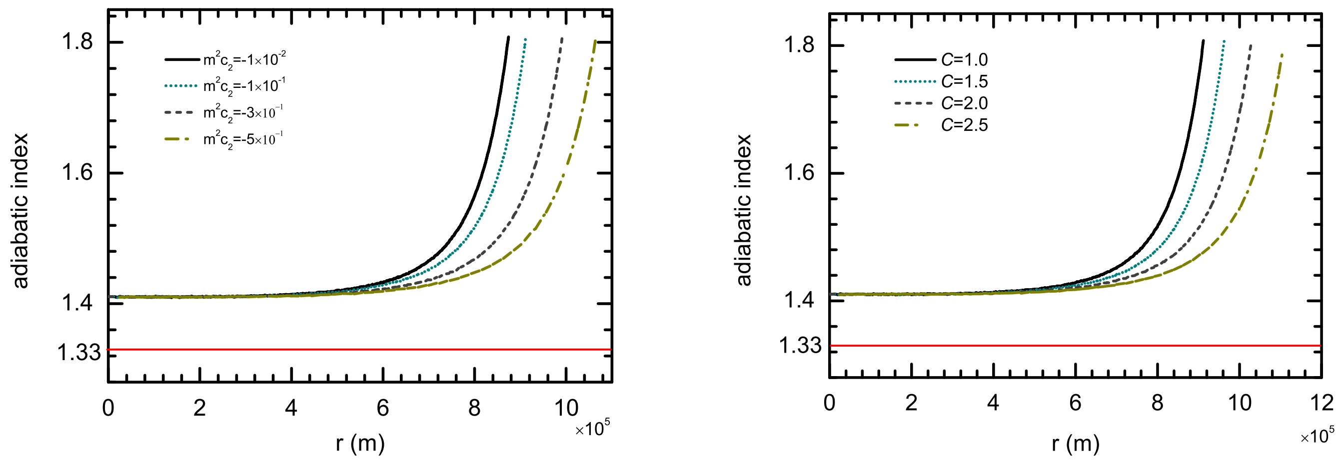

FIG. 5. Adiabatic index vs radius for $m^2c_1 = 1 \times 10^{-11}$. Left diagrams: for $C = 1$, $m^2c_2 = -1 \times 10^{-2}$ (continuous line), $m^2c_2 = -1 \times 10^{-1}$ (dotted line), $m^2c_2 = -3 \times 10^{-1}$ (dashed line), $m^2c_2 = -5 \times 10^{-1}$ (dashed-dotted line). Right diagrams: for $m^2c_2 = -1 \times 10^{-1}$, $C = 1.0$ (continuous line), $C = 1.5$ (dotted line), $C = 2.0$ (dashed line), $C = 2.5$ (dashed-dotted line).

not give us any information about the spacetime curvature (or the strength of gravity). In order to investigate the curvature of spacetime in more details, we use another quantity. The quantity is the Riemann tensor ($R_{\mu\nu\gamma\delta}$). It is notable that, the Riemann tensor may have more components, and, for simplicity, we can study the Kretschmann scalar for measurement of the curvature in a vacuum. After some calculations, we can obtain the curvature at the surface of a white dwarf in massive gravity as

$$\begin{aligned} K &= R_{\mu\nu\gamma\delta}R^{\mu\nu\gamma\delta} \\ &= \frac{2m^4c_1^2C^2}{R^2} + \frac{4m^4c_1c_2C^3}{R^3} + \frac{4m^4c_2^2C^4}{R^4} \\ &\quad + \frac{16m^2c_2C^2GM}{c^2R^5} + \frac{48G^2M^2}{c^4R^6}, \end{aligned} \tag{19}$$

where in the absence of graviton mass ($m = 0$), the above equation reduces to the Kretschmann scalar in Einstein gravity as $K = \frac{48G^2M^2}{c^4R^6}$, [83–85]. Our results show that by considering nonzero graviton mass, the strength of gravity from the perspective of a distant observer increases when the mass of white dwarfs increases (see Tables II and III).

D. Dynamical stability

Another important quantity is related to the dynamical stability of white dwarfs in massive gravity. Chandrasekhar introduced the dynamical stability of stellar model against the infinitesimal radial adiabatic perturbation in Ref. [86]. Some authors applied it to astrophysical cases [87–90]. The adiabatic index (γ) is defined in the following form

$$\gamma = \frac{\rho c^2 + P}{c^2 P}\frac{dP}{d\rho}. \tag{20}$$

We will encounter with the dynamical stability when γ is more than $4/3$ ($\gamma > 4/3 \simeq 1.33$), everywhere within the

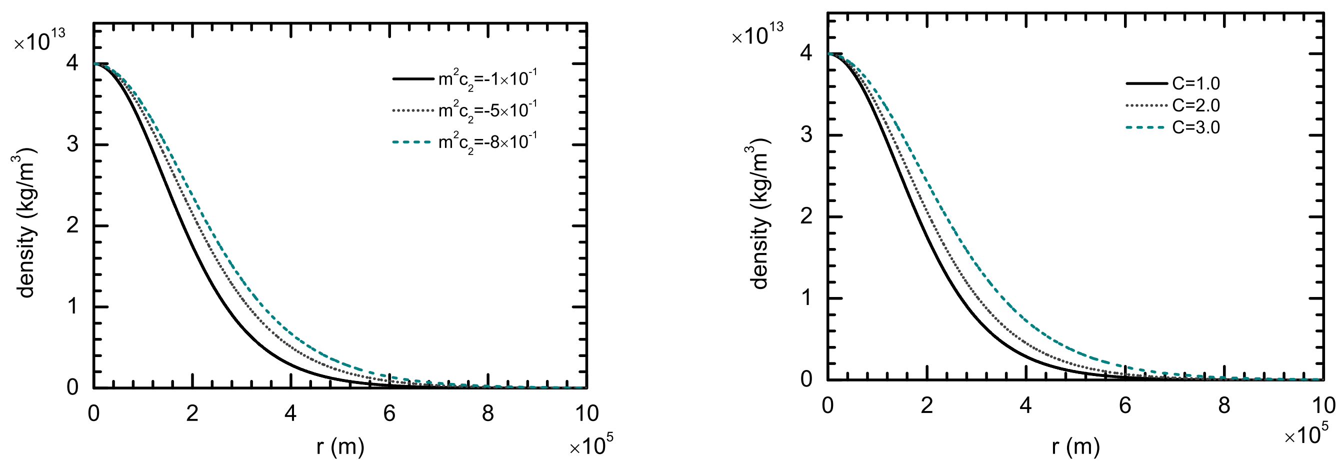

FIG. 6. Density vs radius for $m^2c_1 = 1 \times 10^{-11}$. Left diagrams: for $C = 1$, $m^2c_1 = -1 \times 10^{-1}$ (continuous line), $m^2c_1 = -5 \times 10^{-1}$ (dotted line), $m^2c_1 = -8 \times 10^{-1}$ (dashed line). Right diagrams: for $m^2c_2 = -1 \times 10^{-1}$, $C = 1.0$ (continuous line), $C = 2.0$ (dotted line), $C = 3.0$ (dashed line).

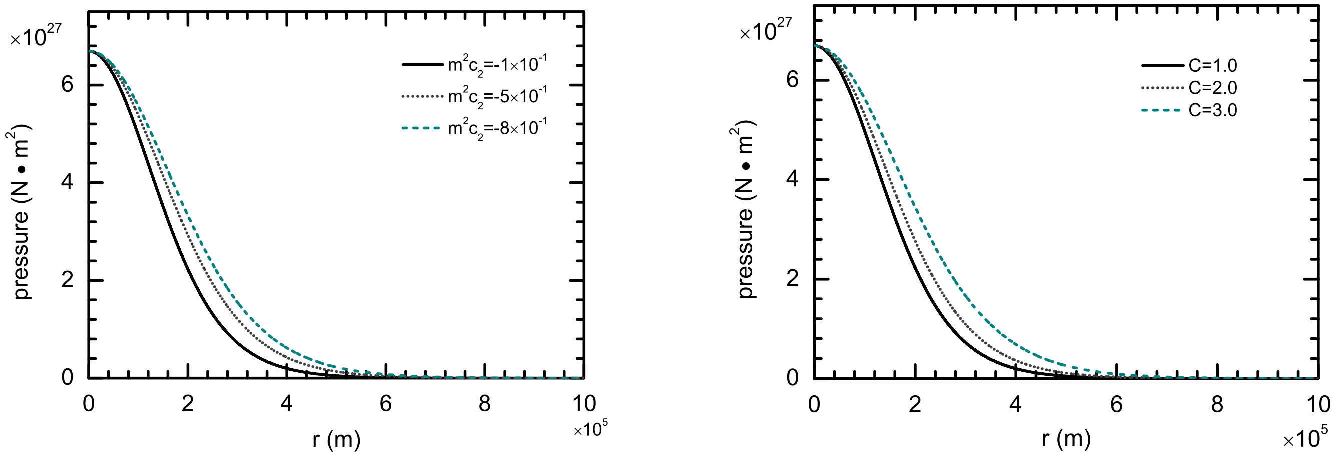

FIG. 7. Pressure vs radius for $m^2c_1 = 1 \times 10^{-11}$. Left diagrams: for $C = 1$, $m^2c_1 = -1 \times 10^{-1}$ (continuous line), $m^2c_1 = -5 \times 10^{-1}$ (dotted line), $m^2c_1 = -8 \times 10^{-1}$ (dashed line). Right diagrams: for $m^2c_2 = -1 \times 10^{-1}$, $C = 1.0$ (continuous line), $C = 2.0$ (dotted line), $C = 3.0$ (dashed line).

obtained white dwarfs. So, we plot two diagrams related to γ vs the radius for different values of m^2c_2 and C in Fig. 5. Our results show that, the super-Chandrasekhar white dwarfs or massive white dwarfs are stable against the radial adiabatic infinitesimal perturbations.

Here, we want to evaluate the behavior of density and pressure vs distance from the center to the surface of white dwarfs. For this goal, we plot them in Figs. 6 and 7. As one can see, the density and the pressure are maximum at the center and they decrease monotonically towards the boundary. These figures show that, in constant radius (for example $r = 4 \times 10^3$ km), by increasing C (or decreasing m^2c_2), the density and the pressure increase.

V. COMPARISON BETWEEN NEUTRON STARS AND WHITE DWARFS IN MASSIVE GRAVITY

In this section we want to compare the obtained neutron stars in Ref. [57] with the obtained white dwarfs in this paper.

Radius: the obtained radii of massive neutron stars were about 10 km ($R_{NS} \simeq 10$ km) [57], whereas the super-Chandrasekhar white dwarfs are about 1000 km, so the radii of these super-Chandrasekhar white dwarfs are one hundred times larger than radius of massive neutron stars ($R_{WD} \simeq 100 R_{NS}$).

Average density: the average density of massive neutron stars in massive gravity is $\bar{\rho}_{NS} \simeq 10^{15} g$ cm^{-3} [57], while the average density of the super-Chandrasekhar white dwarfs in this gravity is $\bar{\rho}_{WD} \simeq 10^9 g$ cm^{-3}. Therefore, the average density of massive white dwarfs is less than the average density of massive neutron stars, $\bar{\rho}_{WD} < \bar{\rho}_{NS}$ ($\bar{\rho}_{NS} \simeq 10^6 \bar{\rho}_{WD}$), as we expected.

Kretschmann scalar: in order to investigate the strength of gravity between massive neutron stars and massive white dwarfs, we compare the values of their Kretschmann scalars. Our calculations of the Kretschmann scalar for massive neutron stars are $K_{NS} \simeq 5.7 \times 10^{-16}$ m^{-4}, while for massive white dwarfs are $K_{WD} \simeq 6.7 \times 10^{-25}$ m^{-4}. Comparing these values show that, the strength of gravity of massive white dwarfs are less than the massive neutron stars, $K_{WD} < K_{NS}$ ($K_{NS} \simeq 10^9 K_{WD}$).

Dynamical stability: our results show that, both of them (the massive neutron stars and the super-Chandrasekhar white dwarfs) satisfy this condition. In other words, both of them are stable against the radial adiabatic infinitesimal perturbations.

VI. CLOSING REMARKS

As we mentioned before, in order to find the massive or super-Chandrasekhar white dwarfs, we can consider two approaches; (i) improve the EoS, and (ii) modified gravity. According to this fact that GR had some problems, and also the probability of existence massive graviton based on the recent observations by the advanced LIGO/Virgo [1,14] collaboration, which had put a tight bound on graviton's mass, and another theoretical and empirical limits on the mass of gravitons, in this work we considered the dRGT like massive gravity which is known as Vegh's massive gravity with a reference metric in the form of Eq. (7). Then we investigated the effects of this theory of gravity on the structure of white dwarfs. We employed the Chandrasekhar's EoS and considered the modified TOV equation in the presence of nonzero graviton mass. Our results showed that the maximum mass of white dwarfs in massive gravity can be more than Chandrasekhar limit ($M_{\text{Max}} > 1.45 M_{\odot}$), because the strength of gravity may change by varying the parameters of this gravity. Indeed, by increasing (C) or decreasing (m^2c_2) parameters of the reference metric and massive theory of gravity, respectively, the strength of gravity may decrease. As we know, there is a balance between the internal pressure and gravitational force. Decreasing the strength of gravity, a star can bear more mass in order to keep this balance. Therefore, the maximum mass of the white dwarf increased by increasing (C) or decreasing (m^2c_2). Then, we studied other properties of super-Chandrasekhar white dwarfs such as Schwarzschild radius, average density, and Kretschmann scalar in the presence of nonzero graviton mass. Next, we evaluated the dynamical stability in order to have physical super-Chandrasekhar white dwarfs. For having super-Chandrasekhar white dwarfs, we obtained some constraints on the parameters of massive gravity. Indeed, our results showed that the value of C has to be more than 0.1 ($C > 0.1$). The sign of c_2 was negative with the range $m^2c_2 < -1 \times 10^{-2}$. In addition, the sign of c_1 could be positive or negative, so that different values of this parameter did not affect the structure of white dwarfs (see Table IV).

Recently, in Ref. [76], was shown that there is a correspondence between the spherical black hole solutions in this theory of massive gravity with conformal gravity, when m^2c_1C and $m^2c_2C^2$ are in the ranges; (i) $m^2c_1C > 0$ and $-2 < m^2c_2C^2 < 0$, (ii) $m^2c_1C < 0$ and $m^2c_2C^2 < -2$, or (iii) $m^2c_1C < 0$ and $m^2c_2C^2 > 0$. The results obtained in this paper were consistent with case (i). Also, this range [case (i)] was valid for neutron stars in massive gravity [57].

Finally, in order to have a better view of these super-Chandrasekhar white dwarfs in massive gravity, we compared the obtained massive neutron stars in Ref. [57], with super-Chandrasekhar white dwarfs in the last section.

Briefly, we obtained the quite interesting results from massive gravity for the white dwarfs such as:

(I) Prediction of maximum mass for white dwarfs more than the Chandrasekhar limit ($M > 1.45\, M_\odot$), due to the existence of nonzero graviton mass. In other words, super-Chandrasekhar white dwarfs in massive gravity were acceptable.

(II) The super-Chandrasekhar white dwarfs in massive gravity are dynamically stable.

(III) Considering different values of parameters of massive gravity, the strength of gravity from the perspective of a distant observer by increasing the mass of white dwarf increased.

(IV) Density inside white dwarfs increased due to increasing the mass of white dwarf, which was one of the effects of massive gravity.

(V) Super-Chandrasekhar white dwarfs imposed some constraints on parameters of massive theory.

Finally, it is notable that rotating, slowly rotating and magnetized white dwarfs [91–99] in the context of massive gravity are interesting topics. In addition, it will be very interesting if we use another realistic equation of state in order to have a good view of the behavior of white dwarfs in massive gravity. We leave these issues for future works.

ACKNOWLEDGMENTS

We thank an anonymous referee for useful comments. We also acknowledge S. Panahiyan and S. H. Hendi for reading the manuscript. B. Eslam Panah thanks Shiraz University Research Council. The work of B. E. P. has been supported financially by Research Institute for Astronomy and Astrophysics of Maragha (RIAAM) under research Project No. 1/5750-49. The work of H. L. L. has been supported financially by the National Natural Science Foundation of China under No. 11803026.

[1] B. P. Abbott *et al.* (LIGO Scientific and Virgo Collaborations), Phys. Rev. Lett. **116**, 061102 (2016).
[2] K. Koyama, G. Niz, and G. Tasinato, Phys. Rev. Lett. **107**, 131101 (2011).
[3] A. H. Chamseddine and M. S. Volkov, Phys. Lett. B **704**, 652 (2011).
[4] G. D'Amico, C. de Rham, S. Dubovsky, G. Gabadadze, D. Pirtskhalava, and A. J. Tolley, Phys. Rev. D **84**, 124046 (2011).
[5] Y. Akrami, T. S. Koivisto, and M. Sandstad, J. High Energy Phys. 03 (2013) 099.
[6] K. Hinterbichler, Rev. Mod. Phys. **84**, 671 (2012).
[7] J. Zhang and S.-Y. Zhou, Phys. Rev. D **97**, 081501(R) (2018).
[8] M. Fierz and W. Pauli, Proc. R. Soc. A **173**, 211 (1939).
[9] P. Van Nieuwenhuizen, Nucl. Phys. **B60**, 478 (1973).
[10] H. van Dam and M. J. G. Veltman, Nucl. Phys. **B22**, 397 (1970).
[11] V. I. Zakharov, JETP Lett. **12**, 312 (1970).
[12] A. I. Vainshtein, Phys. Lett. **39B**, 393 (1972).
[13] D. G. Boulware and S. Deser, Phys. Rev. D **6**, 3368 (1972).
[14] B. P. Abbott *et al.* (LIGO Scientific and Virgo Collaborations), Phys. Rev. Lett. **116**, 221101 (2016).
[15] L. S. Finn and P. J. Sutton, Phys. Rev. D **65**, 044022 (2002).
[16] A. Gruzinov, New Astron. **10**, 311 (2005).
[17] A. S. Goldhaber and M. M. Nieto, Rev. Mod. Phys. **82**, 939 (2010).
[18] J. B. Jimenez, F. Piazza, and H. Velten, Phys. Rev. Lett. **116**, 061101 (2016).
[19] C. de Rham, J. T. Deskins, A. J. Tolley, and S.-Y. Zhou, Rev. Mod. Phys. **89**, 025004 (2017).

[20] N. Arkani-Hamed, H. Georgi, and M. D. Schwartz, Ann. Phys. (Berlin) **305**, 96 (2003).
[21] C. de Rham and G. Gabadadze, Phys. Rev. D **82**, 044020 (2010).
[22] C. de Rham, G. Gabadadze, and A. J. Tolley, Phys. Rev. Lett. **106**, 231101 (2011).
[23] S. F. Hassan and R. A. Rosen, Phys. Rev. Lett. **108**, 041101 (2012).
[24] Y. F. Cai, D. A. Easson, C. Gao, and E. N. Saridakis, Phys. Rev. D **87**, 064001 (2013).
[25] H. Kodama and I. Arraut, Prog. Theor. Exp. Phys. **2014**, 023E0 (2014).
[26] S. G. Ghosh, L. Tannukij, and P. Wongjun, Eur. Phys. J. C **76**, 119 (2016).
[27] D. C. Zou, R. Yue, and M. Zhang, Eur. Phys. J. C **77**, 256 (2017).
[28] T. Katsuragawa, S. Nojiri, S. D. Odintsov, and M. Yamazaki, Phys. Rev. D **93**, 124013 (2016).
[29] P. Kareeso, P. Burikham, and T. Harko, Eur. Phys. J. C **78**, 941 (2018).
[30] M. Yamazaki, T. Katsuragawa, S. D. Odintsov, and S. Nojiri, arXiv:1812.10239.
[31] Y. F. Cai, C. Gao, and E. N. Saridakis, J. Cosmol. Astropart. Phys. 10 (2012) 048.
[32] G. Leon, J. Saavedra, and E. N. Saridakis, Classical Quantum Gravity **30**, 135001 (2013).
[33] K. Hinterbichler, J. Stokes, and M. Trodden, Phys. Lett. B **725**, 1 (2013).
[34] M. Fasiello and A. J. Tolley, J. Cosmol. Astropart. Phys. 12 (2013) 002.
[35] K. Bamba, Md. Wali Hossain, R. Myrzakulov, S. Nojiri, and M. Sami, Phys. Rev. D **89**, 083518 (2014).
[36] V. F. Cardone, N. Radicella, and L. Parisi, Phys. Rev. D **85**, 124005 (2012).
[37] L. Heisenberg and A. Refregier, Phys. Lett. B **762**, 131 (2016).
[38] S. Panpanich and P. Burikham, Phys. Rev. D **98**, 064008 (2018).
[39] K. Aoki and S. Mukohyama, Phys. Rev. D **94**, 024001 (2016).
[40] M. Crisostomi, D. Comelli, and L. Pilo, J. High Energy Phys. 06 (2012) 085.
[41] G. D'Amico, Phys. Rev. D **86**, 124019 (2012).
[42] M. Fasiello and A. J. Tolley, J. Cosmol. Astropart. Phys. 11 (2012) 035.
[43] S. Nesseris and S. Tsujikawa, Phys. Rev. D **90**, 024070 (2014).
[44] A. E. Gümrükçüoğlu, L. Heisenberg, and S. Mukohyama, J. Cosmol. Astropart. Phys. 02 (2015) 022.
[45] C. de Rham, Living Rev. Relativity **17**, 7 (2014).
[46] D. Vegh, arXiv:1301.0537.
[47] H. Zhang and X. Z. Li, Phys. Rev. D **93**, 124039 (2016).
[48] R. G. Cai, Y. P. Hu, Q. Y. Pan, and Y. L. Zhang, Phys. Rev. D **91**, 024032 (2015).
[49] S. H. Hendi, B. Eslam Panah, and S. Panahiyan, J. High Energy Phys. 11 (2015) 157.
[50] J. Xu, L. M. Cao, and Y. P. Hu, Phys. Rev. D **91**, 124033 (2015).
[51] S. H. Hendi, B. Eslam Panah, and S. Panahiyan, Classical Quantum Gravity **33**, 235007 (2016).
[52] S. H. Hendi, R. B. Mann, S. Panahiyan, and B. Eslam Panah, Phys. Rev. D **95**, 021501(R) (2017).
[53] S. H. Hendi, B. Eslam Panah, and S. Panahiyan, Fortschr. Phys. **66**, 1800005 (2018).
[54] S. H. Hendi, B. Eslam Panah, S. Panahiyana, H. Liu, and X.-H. Meng, Phys. Lett. B **781**, 40 (2018).
[55] S. H. Hendi, B. Eslam Panah, S. Panahiyan, and M. Momennia, Phys. Lett. B **772**, 43 (2017).
[56] S. H. Hendi, B. Eslam Panah, S. Panahiyan, and M. Momennia, Phys. Lett. B **775**, 251 (2017).
[57] S. H. Hendi, G. H. Bordbar, B. Eslam Panah, and S. Panahiyan, J. Cosmol. Astropart. Phys. 07 (2017) 004.
[58] B. Eslam Panah, S. H. Hendi, and Y. C. Ong, arXiv:1808.07829.
[59] A. E. Gumrukcuoglu, C. Lin, and S. Mukohyama, J. Cosmol. Astropart. Phys. 11 (2011) 030.
[60] D. Comelli, M. Crisostomi, F. Nesti, and L. Pilo, J. High Energy Phys. 03 (2012) 067.
[61] D. Langlois and A. Naruko, Classical Quantum Gravity **29**, 202001 (2012).
[62] T. Kobayashi, M. Siino, M. Yamaguchi, and D. Yoshida, Phys. Rev. D **86**, 061505 (2012).
[63] D. Psaltis, Living Rev. Relativity **11**, 9 (2008).
[64] D. A. Howell *et al.*, Nature (London) **443**, 308 (2006).
[65] R. A. Scalzo *et al.*, Astrophys. J. **713**, 1073 (2010).
[66] M. Hicke, P. M. Garnavich, J. L. Prieto, S. Blondin, D. L. DePoy, R. P. Kirshner, and J. Parrent, Astrophys. J. **669**, L17 (2007).
[67] M. Yamanaka *et al.*, Astrophys. J. **707**, L118 (2009).
[68] J. M. Silverman, M. Ganeshalingam, W. Li, A. V. Filippenko, A. A. Miller, and D. Poznanski, Mon. Not. R. Astron. Soc. **410**, 585 (2011).
[69] S. Taubeberger *et al.*, Mon. Not. R. Astron. Soc. **412**, 2735 (2011).
[70] U. Das and B. Mukhopadhyay, Phys. Rev. Lett. **110**, 071102 (2013).
[71] K. Boshkayev, J. A. Rueda, R. Ruffini, and I. Siutsou, Astrophys. J. **762**, 117 (2013).
[72] H. Liu, X. Zhang, and D. Wen, Phys. Rev. D **89**, 104043 (2014).
[73] U. Das and B. Mukhopadhyay, J. Cosmol. Astropart. Phys. 05 (2015) 045.
[74] H. L. Liu and G. L. Lü, J. Cosmol. Astropart. Phys. 02 (2019) 040.
[75] L. Berezhiani, G. Chkareuli, C. de Rham, G. Gabadadze, and A. J. Tolley, Phys. Rev. D **85**, 044024 (2012).
[76] B. Eslam Panah and S. H. Hendi, Europhys. Lett. **125**, 60006 (2019).
[77] D. Kubiznak and R. B. Mann, J. High Energy Phys. 07 (2012) 033.
[78] L. Herrera, Phys. Lett. A **165**, 206 (1992).
[79] H. Abreu, H. Hernandez, and L. A. Nunes, Classical Quantum Gravity **24**, 4631 (2007).
[80] N. K. Glendenning, Phys. Rev. Lett. **85**, 1150 (2000).
[81] G. A. Carvalho, R. M. Marinho Jr, and M. Malheiro, Gen. Relativ. Gravit. **50**, 38 (2018).
[82] A. F. Ali and S. Das, Int. J. Mod. Phys. D **25**, 1644001 (2016).
[83] D. Psaltis, Living Rev. Relativity **11**, 9 (2008).

[84] K. Y. Eksi, C. Gungor, and M. M. Turkoglu, Phys. Rev. D **89**, 063003 (2014).
[85] B. Eslam Panah, G. H. Bordbar, S. H. Hendi, R. Ruffini, Z. Rezaei, and R. Moradi, Astrophys. J. **848**, 24 (2017).
[86] S. Chandrasekhar, Astrophys. J. **140**, 417 (1964).
[87] J. M. Bardeen, K. S. Thonre, and D. W. Meltzer, Astrophys. J. **145**, 505 (1966).
[88] H. Kuntsem, Mon. Not. R. Astron. Soc. **232**, 163 (1988).
[89] M. K. Mak and T. Harko, Eur. Phys. J. C **73**, 2585 (2013).
[90] M. Kalam, S. M. Hossein, and S. Molla, Res. Astron. Astrophys. **18**, 025 (2018).
[91] M. Livio and J. E. Pringle, Astrophys. J. **505**, 339 (1998).
[92] B. Franzon and S. Schramm, Phys. Rev. D **92**, 083006 (2015).
[93] S. Subramanian and B. Mukhopadhyay, Mon. Not. R. Astron. Soc. **454**, 752 (2015).
[94] D. M. Townsley, P. Arras, and L. Bildsten, arXiv:1601.02046.
[95] J. J. Hermes *et al.*, Astrophys. J. Suppl. Ser. **232**, 23 (2017).
[96] P. Ghosh and J. Craig Wheeler, Astrophys. J. **834**, 93 (2017).
[97] P. Neunteufel, S.-C. Yoon, and N. Langer, Astron. Astrophys. **602**, A55 (2017).
[98] D. Alvear Terrero, D. Manreza Paret, and A. Perez Martinez, Int. J. Mod. Phys. Conf. Ser. **45**, 1760025 (2017).
[99] D. García-Senz, R. M. Cabezón, and I. Domínguez, Astrophys. J. **862**, 27 (2018).