Test of higher-derivative gravitational relativistic models with the gravitational inverse-square law experiments

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The theory of general relativity has an ultraviolet(UV) problem that can be ameliorated by gravity with higher derivatives. The four-derivative gravity as an effective gravitational theory at UV scalars has two Yukawa-like corrections with parameters λ_0 and λ_2. By the analysis of experiments of HUST-2015, Newman's group, lake experiment, and Cassini spacecraft, we obtain the strong-bound regions for these parameters at submillimeter-to-millimeter, centimeter-to-meter, tens-of-meter, and solar-system scales, in which the properties of potential are clearly shown. Recently, the ghostfree and singularityfree gravity, a more potential modified gravity theory, introduces the novel conception of nonlocality. We test the scale of gravitational nonlocality $\lambda_m < 2.7 \times 10^{-5}$ m by the torsion pendulum experiment HUST-2015. The predicted decaying spatial oscillations are visible meaning possible violation at shorter ranges. Our result provides useful information for gravitational interaction at microscopic ranges.

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I. INTRODUCTION

Extended theories of gravity are alternative theories of gravitational interaction based on corrections and enlargements of the Einstein theory. These theories aim from one side to extend the positive results of general relativity and, on the other hand, to cure its shortcomings [1,2]. In order to address theoretical and experimental problems that have emerged in cosmology, astrophysics and high energy physics, these theories have been proposed to explain deviations at ultraviolet (UV) and infrared (IR) scales [3–5]. The problems presented at infrared scales are posed by the unexpected discovery of the accelerated expansion of the Universe that is measured from Ia supernovae and the cosmic microwave background (CMB) [6–8]. It is important to modify gravity at cosmological distances. Theoretical efforts in this area have recently gained attention and offered rich spectrum of new ideas that may be tested by experiments [9]. For problems at ultraviolet scales, it is unclear what the correct ultraviolet completion of Einstein theory is. Theoretical works, such as quantum gravity and the fifth force, predict new interactions in the short range motivating many groups around world to perform the high-precision test of gravitational inverse-square law. Experimental results and on-going searches [10] for a possible violation of the gravitational ISL in the ranges from micrometer to meter have been reported by Newman's group [11,12], Long's group [13], Adelberger's group [14], etc. All experimental efforts help to understand the gravitational interaction at ultraviolet scales.

Extended theories of gravity have become a sort of paradigm in the search of non-Einstein theory, which consists of searching for the deviations from special and general relativity. The paradigm consists of adding acceptable terms, such as higher-order curvature invariants and minimally or nonminimally coupled scalar fields. On the deviations at the ultraviolet (UV), four-derivative gravity is renormalizable by introducing such terms proportional to R^2 and $R^{\mu\nu}R_{\mu\nu}$ in the gravitational Lagrangian [15–18], but it inevitably suffers from unphysical ghost states [19,20]. This ghost makes it seem unlikely that four-derivative gravity has a place in an ultimate theory. However, as is pointed in Ref. [21], four-derivative model is still reasonable to represent an effective gravitational theory at UV scalars. Parametrized four-derivative gravity is relative to two parameters m_0 and m_2, whose values may reveal the microscopic properties of gravity. The experiments of gravity allow us to obtain bounds on these parameter. Since the large scale of astronomical tests, they are usually useless for testing the four-derivative gravity. Comparing the astronomical tests, laboratory experiments on the validity of Newtonian inverse square law are more useful to give better bounds.

String theory (ST) is a popular candidate in formulating a consistent quantum theory of gravity, whereas it

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approaches problem rather grandly because of its intention to unify gravity with the fundamental forces of nature. Loop quantum gravity (LQG), another popular candidate, aims to quantize the gravitational field. A common thread running through these theory is the presence of nonlocality in which gravitational interaction occurs not at a specific spatial point but over a region of space.

Recently, similar to the novel approach in ST and LQG, the ghostfree and singularityfree theories also propose the conception of nonlocality to address the issue of ghost. By introducing infinite derivatives to modify the gravitation propagator, ghostfree and singularityfree theories of gravity can soften the quantum UV divergences on one hand and make theory ghostfree on the other hand [22–24]. Furthermore, these infinite derivatives can remove the cosmological big bang singularity in a linearity limit, and remove the black hole type singularity in the static, linearity limit [25]. In order to avoid having to introduce new poles, such infinite derivatives are introduced in the form of an exponential of an entire function [26,27]. Actually, these infinite derivatives render the gravitational interactions nonlocal with a new scale in four dimensions $m \leq m_p \sim 2.4 \times 10^{18}$ GeV. It may be a novel connection between general relativity and quantum theory. This nonlocality can be tested by current gravitational experiments measuring the Newtonian potential. The experiments of measuring Newtonian potential are promising to expose the microscopic properties of gravity. So we wish to put a bound on the scale of nonlocality from the torsion pendulum experiments at millimeter ranges.

II. FOUR-DERIVATIVE GRAVITY

We will start our discussion with a modified Einstein-Hilbert action that includes both the action $\int(-g)^{1/2}Rd^4x$ and the higher-order terms in the curvature invariants. The theories of gravity including higher-order curvature invariant terms R^2 and $R^{\mu\nu}R_{\mu\nu}$ in the gravitational action permit us to renormalize the divergences in quantum corrections to the interactions of matter fields. Furthermore, from the conceptual point of view, there is no *a priori* reason to restrict the gravitational Lagrangian to a linear function of Ricci scalar and the minimal couple with matter. For example, when superstrings or supergravity take effective actions into account, higher-order terms in the curvature invariants or nonminimally coupled terms are present. Considering the higher-order terms in the curvature invariants, we write gravitational action as the form

$$\mathcal{S} = \int d^4x\sqrt{-g}(\chi R + f(A,B) + \mathcal{L}_{\mathrm{m}}) \tag{1}$$

where $\chi^{-1} = 16\pi G$, g is the determinant of metric tensor $g_{\mu\nu}$, $f(A,B)$ is an unspecified function of $A = R^2$ and $B = R^{\mu\nu}R_{\mu\nu}$ that describes deviations from the general relativity, and the $\mathcal{L}_{\mathrm{m}}$ is the Lagrangian density for matter. R and $R_{\mu\nu}$ are the Ricci scalar and Ricci tensor, respectively. It is worth noting that $\int dx^4\sqrt{-g}R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$ is not needed in the action because the Gauss-Bonnet relation $\sqrt{-g}(R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta} - 4R^{\mu\nu}R_{\mu\nu} + R^2) = \mathrm{divs}$ is topologically equivalent to flat space [28].

Varying the action with respect to the metric, we obtain the field equations

$$\begin{aligned}\chi\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) - \frac{1}{2}g_{\mu\nu}f + 2f_{,A}RR_{\mu\nu} + 2f_{,B}R^{\sigma}_{\mu}R_{\sigma\nu}\\ + \Box(f_{,B}R_{\mu\nu}) + g_{\mu\nu}\nabla_\alpha\nabla_\beta(f_{,B}R^{\alpha\beta}) - 2\nabla_\alpha\nabla_\beta[f_{,B}R^{\alpha}_{(\mu}\delta^{\beta}_{\nu)}]\\ + 2g_{\mu\nu}\Box(f_{,A}R) - 2\nabla_\mu\nabla_\nu(f_{,A}R) = \frac{1}{2}T_{\mu\nu},\end{aligned} \tag{2}$$

where $\Box$ is the d'Alembert operator, $f_{,A}$ and $f_{,B}$ are given by $\partial f/\partial A$ and $\partial f/\partial B$, respectively. The Einstein field equation will recover from this equation in the case of $f(A,B) = 0$ and $f_{,A/B} = 0$. The trace of the field equations (2) is

$$\begin{aligned}-\chi R + (2f_{,B}R^{\mu\nu}R_{\mu\nu} + 2\nabla_\mu\nabla_\nu(f_{,B}R^{\mu\nu}) + \Box(f_{,B}R)\\ + 2f_{,A}R^2 - 2f + 6\Box(f_{,A}R)) = \frac{T}{2}\end{aligned} \tag{3}$$

where $T = T^{\mu}_{\mu}$ is the trace of energy-momentum tensor. A special model can be determined by the special function $f(A,B)$, such as $f(R)$ gravity and four-derivative gravity.

In the case of weak field and slow motion, we consider the field equations. By introducing the quantities

$$\begin{aligned}m_{ab}^2 &= -\frac{\chi}{(6f_{,A} + 2f_{,B})},\\ m_b^2 &= \frac{\chi}{f_{,B}},\end{aligned} \tag{4}$$

and $m_{ab}^2, m_b^2 > 0$, the gravitational potential of a pointlike source is given by

$$V(r) = -\frac{GM}{r}\left(1 + \frac{1}{3}e^{-\sqrt{m_{ab}^2}r} - \frac{4}{3}e^{-\sqrt{m_b^2}r}\right). \tag{5}$$

If the matter source has a distribution function $\rho(\mathbf{x}')$, we have the gravitational potential

$$\begin{aligned}V(\mathbf{x}) = -\int d^3\mathbf{x}'\frac{G\rho(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}\\ \times\left(1 + \frac{1}{3}e^{-\sqrt{m_{ab}^2}|\mathbf{x}-\mathbf{x}'|} - \frac{4}{3}e^{-\sqrt{m_b^2}|\mathbf{x}-\mathbf{x}'|}\right),\end{aligned} \tag{6}$$

the last two terms are the corrections to the Newtonian gravitational potential. The Yukawa-like correction with m_{ab}^2 implies a stronger attracted force, while the correction with m_b^2 results in a repulsive force. In fact, these

Yukawa-like corrections imply the massive excitations, and a more massive field implies a shorter action range. When the m_{ab}^2, m_b^2 are large enough, these Yukawa-like corrections would not affect the macroscopical motions. If we choose the derivatives of $f(A,B)$ satisfying the condition $f_{,A} = f_{,B} = 0$, then we have m_{ab}^2, $m_b^2 \to \infty$ and the gravitational potential recovers the classical form.

A further step is to analyze specified models from the function $f(A,B)$. In higher-derivative formalism, we consider the simplest case where the function $f(A,B)$ is given by the function $f(A,B) = -\beta R^2 + \alpha R^{\mu\nu} R_{\mu\nu}$. Then the action takes the form

$$\mathcal{S} = \int d^4x \sqrt{-g}(\chi R - \beta R^2 + \alpha R^{\mu\nu} R_{\mu\nu} + \mathcal{L}_{\mathrm{m}}), \quad (7)$$

where α and β are dimensionless parameters, which may be limited to satisfy the various observables and experiments.

The modified Einstein equations obtained by varying the action (7) respect to the metric $g^{\mu\nu}$ are

$$\begin{aligned}\chi G_{\mu\nu} - \beta\left[2R\left(R_{\mu\nu} - \frac{1}{4}g_{\mu\nu}R\right) + 2(g_{\mu\nu}\Box R - \nabla_\mu\nabla_\nu R)\right] \\ + \alpha\left[2\left(R_\mu^\sigma R_{\sigma\nu} - \frac{1}{4}g_{\mu\nu}R^{\mu\nu}R_{\mu\nu}\right) + \Box R_{\mu\nu} \right. \\ \left. + g_{\mu\nu}\nabla_\alpha\nabla_\beta R^{\alpha\beta} - 2\nabla_\alpha\nabla_\beta[R^\alpha_{(\mu}\delta^\beta_{\nu)}]\right] = \frac{1}{2}T_{\mu\nu}. \quad (8)\end{aligned}$$

We consider the weak gravitational field of a point mass where $T_{\mu\nu} = \mathrm{diag}(M\delta(r), 0, 0, 0)$. The trace of this field equation is

$$\alpha(\Box R + 2\nabla_\mu\nabla_\nu R^{\mu\nu}) - 6\beta\Box R - \chi R = \frac{1}{2}T. \quad (9)$$

The parameters α and β in the field equations (7) may be limited by gravitational experiments. The field equations provide the solution in terms of the gravitational potential. In the weak field limit, the gravitational potential takes the form

$$V(r) = -\frac{GM}{r}\left(1 + \frac{1}{3}e^{-r/\lambda_0} - \frac{4}{3}e^{-r/\lambda_2}\right), \quad (10)$$

with $\lambda_0 = 1/m_0 = \sqrt{(3\beta - \alpha)32\pi G}$ and $\lambda_2 = 1/m_2 = \sqrt{16\pi G\alpha}$. The additional terms in action result in field equations containing four derivatives. In addition to the usual massless excitations of field, there are massive spin-two m_2 and massive scalar m_0 excitations for general amounts of two new terms, which results in the Yukawa-like potentials in the linearized solutions of the field equations. Although the linearized field energy of massive scalar and massless spin-two excitations is positive definite, the massive spin-two excitation has negative energy. Classically, the negative energy leads to the breakdown of causality, which poses the obstacle to physical conceptions. This is a characteristic of higher-derivative gravity models, and also makes it seem unlikely that higher-derivative models will find a place in an ultimate theory. However, if the massive fields are so large that it is only important on the distance scales near the Planck length ($\sim 10^{-33}$ cm), the breakdown of causality may only occur on microscopic scales. Therefore, it is not impossible that a higher-derivative gravity model could represent an effective gravitational theory at microscopic scalars. Giving the limitations of these parameters is significant for the developing of effective gravitational theories.

The limits of the two massive excitations for an extended source tend to a large but finite value. The corresponding parameters λ_0 and λ_2 can be bounded at a little but nonzero value. As mentioned in Ref. [29], λ_2 can be bounded independently of λ_0 using the gravitational wave (GW) events. With more and more GW events detected by LIGO and Virgo [30,31], λ_2 may be independently bounded in a higher level. At the same time, space-based GW missions are developed to detect GW in the low-frequency regime ($10^{-4} - 1$ Hz), such as LISA [32] and TianQin [33], which would provide more tests for four-derivative gravity. Generally, the experiments of gravity allow us to derive bounds on λ_0 and λ_2. Since the Yukawa-like corrections are exponential, the tests are sensitive to the scales of gravity experiments. However, astronomical tests are usually useless and laboratory experiments on the validity of Newtonian inverse square law seem much more promising. As an example of astronomical tests, considering the motion of Mercury where the orbital precession is known to about one part in 10^9, it can give a low bound on the masses $\sim 10^{-11}$ cm^{-1}. For ISL experiments, the interesting measurements of Long report a repulsive interaction deviation from $1/r^2$ that require a mass $\sim 1 \times 10^{-4}$ cm^{-1}, which corresponds to the distances $\sim 10^2$ m. However, the result is still along way from the microscopic domain. With some high-precision Cavendish-type experiments appear recently, we may improve the bounds of four-derivatives gravity using the pendulum type experiments.

Then, we consider the time and space components of metric that may be approximately written as [34]:

$$g_{00} = 1 - \frac{2GM}{c^2 r}\left(1 + \frac{1}{3}e^{-r/\lambda_0} - \frac{4}{3}e^{-r/\lambda_2}\right), \quad (11)$$

$$g_{ij} = -\delta_{ij} - \frac{2GM}{c^2 r}\left(1 - \frac{1}{3}e^{-r/\lambda_0} - \frac{2}{3}e^{-r/\lambda_2}\right)\delta_{ij}. \quad (12)$$

The time and space components of metric allow us to write post-Newtonian parameter γ as:

$$\gamma = \frac{h_{ij}|\delta_{ij}}{h_{00}} = \frac{3 - e^{-r/\lambda_0} - 2e^{-r/\lambda_2}}{3 + e^{-r/\lambda_0} - 4e^{-r/\lambda_2}}, \quad (13)$$

where we adopt the expansions $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. If the potential satisfies relation $\lambda_0 = \lambda_2$, we recover the general relativity limitation of $\gamma = 1$. On the other hand, we also can obtain the general relativity limitation in the case of λ_0, $\lambda_2 \to \infty$. The infinity case is neglected in our discussion because it cannot provide any explanation for UV problem. Although astronomical observables and experiments in Solar System can provide a effective method to bound the post-Newtonian parameter γ, they are not very useful for λ_0 and λ_2.

In $f(R)$ formalism, we also consider the simplest case where $f(A,B) = aR^2$. The action then takes the form

$$\mathcal{S} = \int d^4x\sqrt{-g}(\chi R + aR^2 + \mathcal{L}_{\mathrm{m}}). \tag{14}$$

Varying the $f(R)$ action (14) with respect to the metric leads to the field equations

$$\chi\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) + a\left[2R\left(R_{\mu\nu} - \frac{1}{4}g_{\mu\nu}R\right) + 2(g_{\mu\nu}\Box R - \nabla_\mu\nabla_\nu R)\right] = \frac{1}{2}T_{\mu\nu}. \tag{15}$$

The trace of this field equations is

$$6a\Box R - \chi R = \frac{1}{2}T. \tag{16}$$

The gravitational potential is

$$V(r) = -\frac{GM}{r}\left(1 + \frac{1}{3}e^{-r/\lambda}\right), \tag{17}$$

where parameter is $\lambda = (96\pi Ga)^{1/2}$. The potential has same form with Yukawa potential. Their difference is that the strength α of any new interaction with a length scale of λ is a constant in $f(R)$ gravity. Theoretically, the small value of a means very small λ, which leads to visible deviations from the ISL at UV scales and recover inverse-square form at IR scales. Thus from time and space components of metric, we obtain the post-Newtonian parameter

$$\gamma = \frac{3 - e^{-r/\lambda}}{3 + e^{-r/\lambda}}. \tag{18}$$

When $\lambda \to \infty$, we obtain a result $\gamma \to 1/2$ extremely deviating from general relativity; while $\lambda \to 0$, we recover the result $\gamma \to 1$, which has yielded the value $\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$ by the time delay of radar signals between the Earth and the Cassini spacecraft [35]. The result of the $f(R)$ model is identical with the result of a massive Brans-Dicke theories with $\omega = 0$. From the measurements of γ and β in the solar system, some works have discussed restrictions for massive Brans-Dicke theories [36,37], in which results could give corresponding restrictions for $f(R)$ model.

III. GHOSTFREE AND SINGULARITYFREE GRAVITY

As mentioned before, four-derivative gravity can ameliorate the UV behavior and is renormalizable, but it introduces a ghost term in the spin-2 excitations [28]. Higher derivative gravity theories are generally better behaved in the UV and make theory avoid singularity [38]. In order to make theory generally covariant and ghostfree at perturbative level, ghostfree and singularityfree gravity is introduced, which introduces infinite derivatives to soften UV divergences at the quantum level and cure instabilities of fourth order gravity [39]. By introducing the infinite derivatives in the action, the theory becomes nonlocal, meanwhile it does not introduce any new pole. This nonlocality would introduce a scale of gravitational nonlocality λ_{m}. In this theory, it predict the modified gravitational potential as the form [39,40]

$$V(r) = -\frac{GM}{r}f(\lambda_{\mathrm{m}}, r) \tag{19}$$

where

$$f(r) = \frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{dk}{k}e^{\tau(\lambda_{\mathrm{m}},k)}\sin(kr). \tag{20}$$

A typical form of τ may be taken as

$$\tau = -k^{2n}\lambda_{\mathrm{m}}^{2n}. \tag{21}$$

As shown in Ref. [40], in the case of large n, $f(\lambda_{\mathrm{m}}, r)$ can be well fit by the function

$$f(\lambda_{\mathrm{m}}, r) = a_1\frac{r}{\lambda_{\mathrm{m}}}, \qquad 0 < r/\lambda_{\mathrm{m}} < 1,$$
$$f(\lambda_{\mathrm{m}}, r) = 1 + a_2\frac{\lambda_{\mathrm{m}}}{r}\cos(r/\lambda_{\mathrm{m}} + \theta), \qquad r/\lambda_{\mathrm{m}} > 1, \tag{22}$$

where $a_1 = 0.544$, $a_2 = 0.572$, and $\theta = 0.885\pi$. It is clear that for $r \to 0$ the effective gravitational potential tends to a constant. This potential is finite when $r \approx 0$. At distances above scale of gravitational nonlocality λ_{m}, it recover the $1/r$ fall of Newtonian potential. The decaying oscillations with distances lead to that this oscillation cannot be measured at large distances. However, the oscillations are significant in the UV. Torsion pendulum experiments at submillimeter range or smaller range have a great potential to search the oscillatory potential. The fitted functions may be used as an effective potential in microscopical scale. From tests of the inverse square law, the result of Adelberger *et al.* obtains the scale of nonlocality $m \geq 0.004$ eV at submillimeter range [14,39]. However, we know very little about the gravitational interaction below the scale of submillimeter ranges. Analyzing more torsion pendulum experiments is indispensable for the ghostfree and singularityfree gravity.

IV. THE TEST OF RELATIVITY MODELS WITH THE DATA OF GRAVITATIONAL INVERSE-SQUARE LAW EXPERIMENTS

A. The test of four-derivative gravity

Before the discussion about four-derivative gravity, we consider the Yukawa gravitational potential given by the form as

$$V(\mathrm{r}) = -\frac{GM}{r}(1 + \alpha e^{-r/\lambda}), \tag{23}$$

where the α is the scale factor and λ the action range. Clearly, the tests of Yukawa potential are sensitive to the scales of the experiments. It also is the character of the four-derivative gravity. The gravitational inverse-square law experiments may give a better bounds for parameters λ_0 and λ_2 at ranges from micrometer to meter, such as Newman's experiment for mass separations from 2 to 105 cm [12] and HUST-2015 at submillimeter ranges [41]. However, the astrometric observables and space missions may bound the parameters at Solar System or bigger scales.

In a test of Yukawa-type gravitational potential model at submillimeter ranges, HUST-2015 experiment successes in limiting the Yukawa model parameters. The basic characteristics for that experiment are dual-modulation (density modulation and gravitational calibration modulation) and dual-compensation (test mass and source mass compensation), and the detail schematic drawing of the experimental setup information can be found in Fig. 1 of Ref. [41]. The aim of this null experiment is to measure signal at $8w$, and this aim signal should be zero in theory under experimental precision. The idiographic method of extracting signal is shown in Ref. [41], finally, the constraints on parameters are improved by up to a factor of 2 at the length scale $\lambda \sim 160\ \mu$m with the precision of 2×10^{-17} Nm at the 2σ level. Based on this experimental precision, we hope to use this typical experiment in our lab to give the limit of parameters for four-derivative gravity.

Comparing to the Yukawa-type gravitational potential model in Eq. (23), the modified gravitational potential in Eq. (6) has two action ranges λ_0 and λ_2, however the corresponding scale factors are constant $1/3$ and $-4/3$ in four-derivative theory, respectively. Therefore, we need to give the limit of this theory coefficients from the relation of two λ terms. Since the parameter α can be obtained by the measured signal (torque $\Delta\tau$) with the variable λ in Yukawa-type potential model experiment, meanwhile, the torque $\Delta\tau(\lambda_0)$ and $\Delta\tau(\lambda_2)$ of four-derivative theory can also be obtained by our lab experiment. The corresponding relation

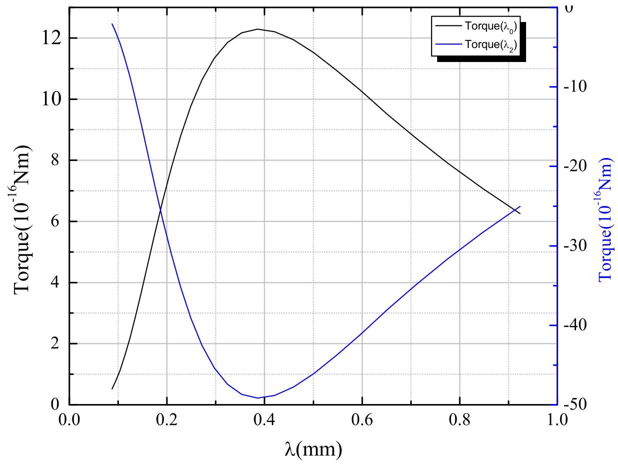

FIG. 1. The variation of torque with range λ. The dark line and blue line stand for the violating torque variations of λ_0 and λ_2, respectively.

between torque and range action is shown in Fig. 1, in which the dark line and blue line stand for the violating torque variations of λ_0 and λ_2, respectively. The correction term of λ_2 is more sensitive to the variation of distance than λ_0's. This characteristic implies that experimental test is able to give a stronger bound for λ_2 term. The zonal bounds for λ_2 in Fig. 2 have demonstrated it. Based on the similar analyses of Yukawa model and the geometry parameter information of HUST-2015, we try to bound the parameters for four-derivative gravity.

Figure 2 shows allowed regions for the parameters λ_0 and λ_2 at submillimeter and millimeter ranges. The upper boundary of green-zonal region and lower boundary of purple-zonal region are obtained from the positive torque 2×10^{-17} Nm. Inversely, the lower boundary of green-

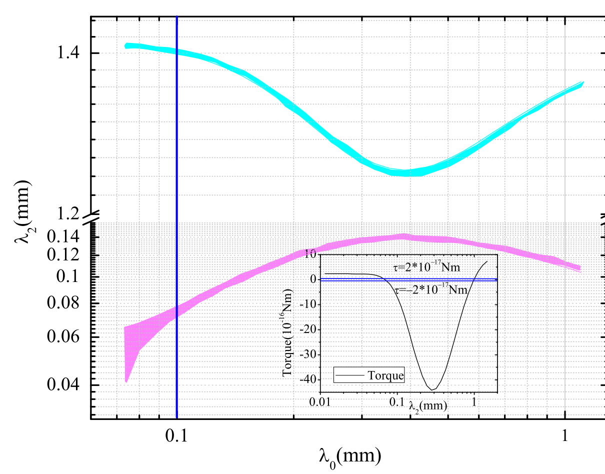

FIG. 2. The allowed regions for the parameters λ_0 and λ_2 at submillimeter and millimeter ranges. For 0.07 mm $\leq \lambda_0 \leq 1$ mm, the parameter λ_2 has two thin-allowed regions (the purple zonal region and green zonal region) that give the strong limitation for λ_2. The subsidiary figure represents the blue line $\lambda_0 = 0.1$ mm.

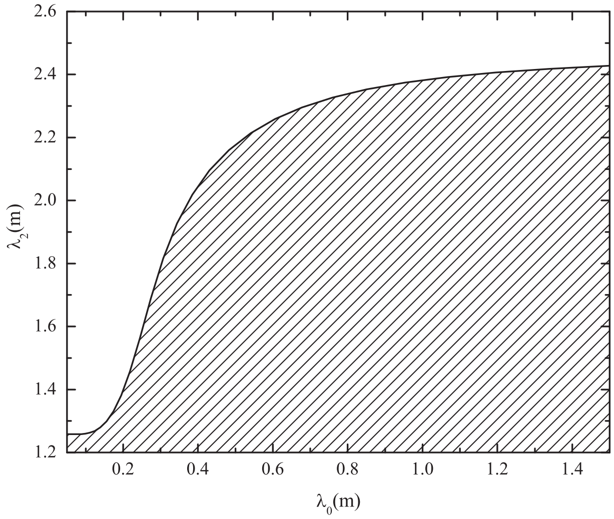

FIG. 3. The allowed regions for the parameters λ_0 and λ_2 at meter ranges. Shaded region is allowed for parameters λ_0 and λ_2 by the Newman-group experiment [12].

zonal region and upper boundary of purple-zonal region represent the result of the negative torque -2×10^{-17} Nm. Four boundaries give two thin-zonal regions (purple zones and green zones) with value range of λ_0 from 0.07 mm to 1 mm in which all values of λ_2 are allowed. It demonstrates that for four-derivative gravity the uncolored regions are excluded at a 95% confidence level. The two thin-allowed regions give a strong bound for four-derivative gravity in submillimater range where parameter λ_2 is allowed to have a very small value range. The great constraint at this region is expected. Since HUST-2015 is a submillimeter-range experiment, it has a great potential to constrain $(\lambda_0 - \lambda_2)$ parameter space in the millimeter range and submillimeter range. The subsidiary figure shows calculated torque of four-derivative gravity with parameter $\lambda_0 = 0.1$ mm for HUST-2015. In the ranges of $\lambda_2 \ll 0.1$ mm and $\lambda_2 \gg 1$ mm, the value of torque tends to the constant. When given the torques $\pm 2 \times 10^{-17}$, the four points of intersection coincide with the line $\lambda_0 = 0.1$ mm.

The deviation of four-derivative gravity from inverse-square law $1/r^2$ is sensitive to the scales of experiments. It means that the various inverse-square law experiments have prospect on giving the tests at corresponding regions. The Newman group's experiment testing gravitational inverse-square law for mass separations from 2 to 105 cm [12] obtained a value for the parameter $\epsilon = (1 \pm 7) \times 10^{-5}$ [ϵ is defined by $r \frac{d}{dr} \ln G(r)$], which may provide a reliable allowed region for four-derivative-gravity parameters. The shadow region in Fig. 3 is allowed for the parameters λ_0 and λ_2 at centimeter-to-meter range. The gently variations of λ_2 at centimeter and meter ranges mainly come from the experimental separations design of near mass (2 cm) and far mass (105 cm), which is consistent with the characteristic of sensitivities of λ_0 and λ_2. Then, the steep

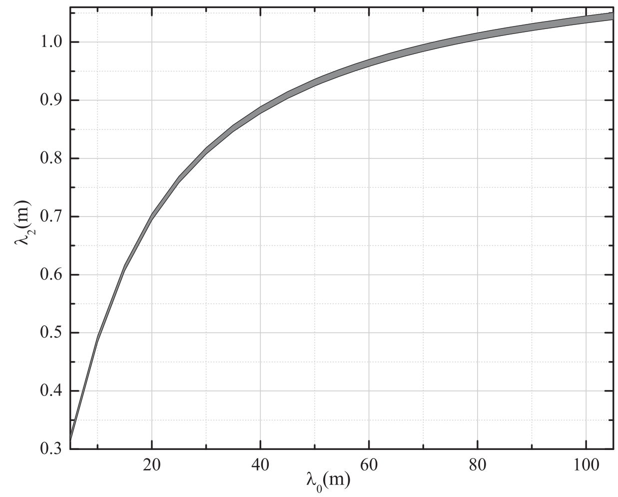

FIG. 4. The allowed regions for the parameters λ_0 and λ_2 at tens-of-meter ranges. The gray region is allowed region for parameters λ_0 and λ_2 by the lake experiment [42].

variation at range from 20 cm to 80 cm is understandable because of the absent configuration for mass separations with several centimeters.

For a longer-scale test, we take a lake experiment into account [42]. In this experiment, the accuracy of large G is given by using the gravimeter data at the two water levels beside the reservoir, in which the effective distances from the gravimeters is in the ranges 26–94 m. Meanwhile, it is also proved that there is no evidence for any "fifth force" deviation from Newtonian gravity. Using the basic information of this experiment, an estimate for constraint of parameters in the four-derivative gravity could be obtained. Figure 4 shows the lake-experiment constraint for four-derivative gravity in the tens-of-meter range. The sensitivity property of four-derivative gravity likewise is clearly

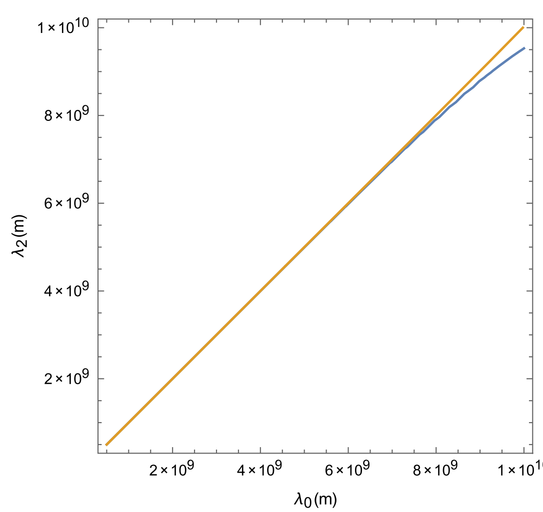

FIG. 5. The result of Cassini tracking experiment. The region between blue and orange lines is allowed at solar system scales.

visible from the curves where λ_0 variations from 10 to 100 m correspond to λ_2 variations 0.5–1.0 m.

At the solar system scales, we consider the most precise value for γ obtained from the time delay of radar signals between Earth and the Cassini spacecraft [35]. The radio signals passed near the Sun with a minimum impact 1.6 solar radii parameter (about 1.1×10^9 m) giving the result $\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$. The bound for four-derivative gravity is shown in Fig. 5 in which the region between blue and orange lines gives the allowed region at the ranges from 5×10^8 to 10^{10} m. It demonstrates that λ_0 and λ_2 almost have same values at given region, which accords with the general relativity limitation in the case of $\lambda_0 = \lambda_2$. Clearly, several-order improvement of γ almost does not narrow allowed regions, so the accuracy improvement of γ is not very efficient to bound the parameter of four-derivative gravity. At the IR scales, various-scales experiments or observations are useful for testing this gravity theory that may give a test of the approximative relationship $\lambda_0 = \lambda_2$ at corresponding ranges. The lunar laser ranging and orbit in galaxy could obtain same result at their scales. This is the characteristic for testing four-derivative gravity at IR scales.

B. The test of ghostfree and singularityfree gravity

The ghostfree and singularityfree theories behave well at the quantum level by introducing novel gravitational nonlocality. The modified gravitational potential introduces a possibility of decaying spatial oscillations on scales close to the scale of nonlocality. The predicted form is necessary to test by gravitational experiments. As mentioned before, torsion pendulum experiments can potentially test the scale of gravitational nonlocality, which means that we can now use the experiment of HUST-2015 to put the bound on gravitational nonlocality λ_m. Based on the detailed knowledge of parameters of experimental apparatus, we applied potential of ghostfree and singularityfree gravity (19) to calculate the torque. For the large n, Eq. (22) is convenient for calculating the torque τ. Adopting the same method (see Ref. [41]) of HUST-2015, we finally give torque curve in the parameter space (τ, λ_m) in Fig. 6.

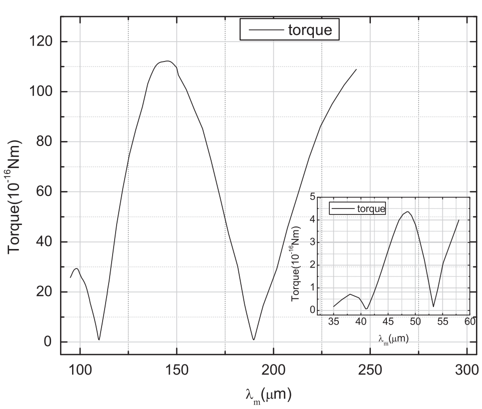

FIG. 6. The calculated torque curve of the ghostfree and singularityfree gravity in the λ_m regions 85 μm to 250 μm and 35 μm to 57 μm. The properties of torque curve coincide with the properties of function $\cos(1/x)$.

Figure 6 shows the calculated torque curves of ghostfree and singularityfree gravity for HUST-2015 in the λ_m ranges of 85 μm to 250 μm and 35 μm to 57 μm in which two curves have same variation tendency. From this figure, the torque has an enhanced-oscillation form with the λ_m parameter space. If the nonlocality λ_m is large enough, torsion pendulum experiments will reveal a measurable torque deviating from Newtonian results. This is why torsion pendulum experiments have a great potential to measure oscillating torque. Moreover, the torque curves have another two properties: the regions in the vicinity of (neighborhood of) the curve's extreme point near zero are sharper than that away from zero, and the wavelength of oscillation is smaller in smaller scale. These properties are understandable since the modified gravitational potential has the similar correction term of function $\cos x^{-1}$ (x is a variable). From the result of HUST-2015, constraint on the Yukawa interaction is set by the in-phase signal of the 295 μm separation experiment as 2.0×10^{-17} Nm. The ghostfree and singularityfree potential of a particular value scale of nonlocality leads to a larger divergence from GR than Yukawa potential of the same value λ, so we can constrain the scale of nonlocality on a lower bound. The further calculations, we obtain bound $\lambda_m < 2.7 \times 10^{-5}$ m at a 95% confidence level for the scale of gravitational nonlocality, which is better than previous constraint 0.004 eV (corresponds to 5×10^{-5} m) [14,39].

In addition, the solar system experiments are feasible for testing nonlocality λ_m. Comparing the gravitational ISL experiments, their limitations for nonlocality are weak in which the result is much bigger than the scale of meter. Since the visible-violation prediction of gravitational ISL occurs at the nearby scale of nonlocality, the short-range ISL experiments have a better possibility to detect deviations signal. For better determination of gravitational nonlocality, the gravitational experiments testing ISL at shorter separation with more precise accuracy are required that would help to understand the fundamental nature of gravity.

V. CONCLUSION

To summarize, we give an analysis of data from the gravitational inverse-square-law experiments to test the higher-derivative gravitational relativistic models. The ghostfree and singularityfree gravity and four-derivative gravity not only soften UV behavior of GR, but also recover the GR results in the IR limit. Their modified gravitational potentials may be tested by measuring the Newtonian potential in near-future experiments. As shown in Figs. 2–5, we give the allowed regions of parameters λ_0 and λ_2 at different ranges from the experiments of HUST-2015, Newman's group, lake experiment, and Cassini spacecraft. These zonal regions in Fig. 2 establish a strong bound for the four-derivative gravity at millimeter and submillimeter ranges. The shadow in Fig. 3 shows the allowed region of two parameters at range from centimeter to meter. As a test at medium range, Fig. 4 gives the constraint with λ_0 range from 5 to 105 m. All three results imply the characteristic that Yukawa-like correction of λ_2 is more sensitive to the variation of distance than that of λ_0. The Cassini spacecraft experiment gives the approximative relationship $\lambda_0 = \lambda_2$ at solar-system scales according with general relativity. Considering UV problems, the gravitational inverse-square-law experiments at short ranges have better potential to search microcosmic behaviors of four-derivative gravity.

Meanwhile using data of HUST-2015, we test the scale of gravitational nonlocality $\lambda_{\mathrm{m}} < 2.7 \times 10^{-5}$ m for the ghostfree and singularityfree gravity. The oscillation at UV scales is clear and visible in Fig. 6, which reveals the explanation that an enough small nonlocality leads to the zero results in detecting short-range ISL violation. Although we know very little about the gravitational interaction below the scale of 10^{-5} m, these results provide useful information for modified gravitational theory. The future experiments of measuring Newtonian potential have a great potential to expose the microscopic properties of gravity.

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