$\rho\rho$ scattering revisited with coupled channels of pseudoscalar mesons

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The $\rho\rho$ scattering has been studied by two groups which both claimed a dynamical generated scalar meson, most likely to be $f_0(1370)$. Here we investigate the influence of coupled channels of pseudoscalar mesons, i.e., $\pi\pi$ and $\bar{K}K$, on this dynamical generated scalar state. With the coupled-channel effect included, the pole and partial decay widths are found to be more close to PDG values for $f_0(1500)$.

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I. INTRODUCTION

The chiral unitary approach, which has made much progress in the study of pseudoscalar meson-meson [1] and meson-baryon [2,3] interactions, has been used to study the interaction of vector mesons among themselves. The first such study of the S-wave $\rho\rho$ interactions found that the $f_0(1370)$ and the $f_2(1270)$ could be dynamically generated [4]. The work found that the strength of the attractive interaction in the tensor channel is much stronger than that in the scalar channel, hence leads to a tighter bound tensor state than the corresponding scalar one.

The work [4] is based on the assumption that the three momenta of the ρ is negligibly small compared to its large mass. This assumption was questioned by a recent work [5], which pointed out the importance of relativistic effect for energies around $f_2(1270)$ well below the $\rho\rho$ threshold. The N/D method [6–10] was used to get the partial-wave amplitudes, which results in a pole for the scalar state similar to Ref. [4] but no pole for any tensor state in contradiction with Ref. [4]. However, this conclusion was not agreed upon by Ref. [11] in which the nonrelativistic assumption was dropped by evaluating exactly the loops with full relativistic ρ propagators in solving the B-S equation for $\rho\rho$ scattering. Both the scalar state and tensor state associated with $f_0(1370)$ and $f_2(1270)$, respectively, were found in consistence with the conclusion of Ref. [4].

From the studies of the above two groups, obviously, for the energies around $f_2(1270)$ well below $\rho\rho$ threshold, there is strong model dependence for the interaction of two far off-mass-shell ρ mesons. For the scalar state closer to the $\rho\rho$ threshold, the two groups got similar results rather model independently. In this paper, we shall study the influence of coupled channels of pseudoscalar mesons, i.e., $\pi\pi$ and $\bar{K}K$, on this dynamical generated scalar state. In the $\rho\rho-K\bar{K}$ coupling we consider the case of K and K^* exchange, while in the $\rho\rho-\pi\pi$ coupling we consider the case of π and ω exchange.

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II. FORMALISM

A. $\rho\rho\to\pi\pi$ with π exchange

We investigate the coupled-channel effect based on a chiral covariant framework [5]. We follow the formalism of the hidden gauge interaction which provides the $\rho\pi\pi$ coupling by means of the Lagrangian [12,13]

$$\mathcal{L}_{VPP} = -ig\langle V^\mu[P,\partial_\mu P]\rangle, \tag{1}$$

where the symbol $\langle\ldots\rangle$ stands for the trace in the $SU(3)$ space with the coupling constant $g = M_V/2f_\pi$ with M_V the mass of the vector meson and $f_\pi = 93$ MeV the pion decay constant. The matrices V_μ and P are given by

$$V_\mu = \begin{pmatrix} \frac{\rho_0}{\sqrt{2}}+\frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho_0}{\sqrt{2}}+\frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}_\mu,$$

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}}+\frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}}+\frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix}. \tag{2}$$

To get the three different isospin amplitudes for $\rho\rho\to\pi\pi$ we need the knowledge of the transitions $\rho^+(p_1)\rho^-(p_2)\to\pi^+(p_3)\pi^-(p_4)$, $\rho^+(p_1)\rho^-(p_2)\to\pi^0(p_3)\pi^0(p_4)$, etc.

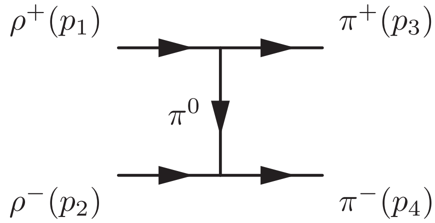

FIG. 1. π-exchange diagram for $\rho^+\rho^- \to \pi^+\pi^-$.

Starting with the Lagrangian in Eq. (1) we can immediately obtain the amplitude $A_t(p_1, p_2, p_3, p_4)$ of $\rho^+(p_1)\rho^-(p_2) \to \pi^+(p_3)\pi^-(p_4)$ corresponding to Fig. 1 as

$$A_t(p_1, p_2, p_3, p_4) = \frac{-8g^2}{(p_1 - p_3)^2 - m_\pi^2}\epsilon_1 \cdot p_3 \epsilon_2 \cdot p_4. \quad (3)$$

In this equation, the ϵ_i corresponds to the polarization vector of the ith ρ. Each polarization vector is characterized by its three-momentum $\mathbf{p}_i$ and third component of the spin σ_i. Explicit expressions of these polarization vectors are given by [5]

$$\epsilon(\mathbf{p}, 0) = \begin{pmatrix} \gamma\beta\cos\theta \\ \frac{1}{2}(\gamma - 1)\sin 2\theta\cos\phi \\ \frac{1}{2}(\gamma - 1)\sin 2\theta\sin\phi \\ \frac{1}{2}(1 + \gamma + (\gamma - 1)\cos 2\theta) \end{pmatrix},$$

$$\epsilon(\mathbf{p}, \pm 1) = \mp\frac{1}{\sqrt{2}}\begin{pmatrix} \gamma\beta e^{\pm i\phi}\sin\theta \\ 1 + (\gamma - 1)e^{\pm i\phi}\sin^2\theta\cos\phi \\ \pm i + (\gamma - 1)e^{\pm i\phi}\sin^2\theta\sin\phi \\ \frac{1}{2}(\gamma - 1)e^{\pm i\phi}\sin 2\theta \end{pmatrix}, \quad (4)$$

where $\beta = |\mathbf{p}|/E_p$, $\gamma = 1/\sqrt{1 - \beta^2}$, and θ and ϕ are the polar angle and azimuthal angle of $\mathbf{p}$, respectively. The u-channel π-exchange amplitude A_u can be obtained from the expression of A_t by exchanging $p_3 \leftrightarrow p_4$. In this way,

$$A_u(p_1, p_2, p_3, p_4) = A_t(p_1, p_2, p_4, p_3). \quad (5)$$

And now we write the tree-level amplitude for $\rho\rho \to \pi\pi$ with the π exchange

$$\begin{aligned} &\rho^+(p_1)\rho^-(p_2) \to \pi^+(p_3)\pi^-(p_4)\colon A_t, \\ &\rho^+(p_1)\rho^-(p_2) \to \pi^0(p_3)\pi^0(p_4)\colon A_t + A_u, \\ &\rho^0(p_1)\rho^0(p_2) \to \pi^+(p_3)\pi^-(p_4)\colon A_t + A_u, \\ &\rho^0(p_1)\rho^0(p_2) \to \pi^0(p_3)\pi^0(p_4)\colon 0. \end{aligned} \quad (6)$$

In order to obtain the S-wave amplitude in the isospin $I = 0$ channel we need the isospin eigenstates. We have

$$\begin{aligned} |\rho\rho, I = 0\rangle &= -\frac{1}{\sqrt{3}}|\rho^+(p_1)\rho^-(p_2) + \rho^-(p_1)\rho^+(p_2) \\ &\quad + \rho^0(p_1)\rho^0(p_2)\rangle, \\ |\pi\pi, I = 0\rangle &= -\frac{1}{\sqrt{3}}|\pi^+(p_1)\pi^-(p_2) + \pi^-(p_1)\pi^+(p_2) \\ &\quad + \pi^0(p_1)\pi^0(p_2)\rangle, \end{aligned} \quad (7)$$

where we have used the convention $|\rho^+\rangle = -|1, 1\rangle$ and $|\pi^+\rangle = -|1, 1\rangle$ of the isospin. By taking into account Eq. (7) and the amplitudes in Eq. (6) we can now write the isospin $I = 0$ amplitude for $\rho\rho \to \pi\pi$

$$T_\pi^{(0)} = 16g^2\left(\frac{\epsilon_1 \cdot p_3 \epsilon_2 \cdot p_4}{m_\pi^2 - t} + \frac{\epsilon_1 \cdot p_4 \epsilon_2 \cdot p_3}{m_\pi^2 - u}\right), \quad (8)$$

where $t = (p_1 - p_3)^2$ and $u = (p_1 - p_4)^2$.

B. $\rho\rho \to \pi\pi$ with ω exchange

One needs the $\rho\omega\pi$ coupling which is provided within the framework [14] of the hidden gauge formalism by means of the Lagrangian

$$\mathcal{L}_{VVP} = \frac{G'}{\sqrt{2}}\epsilon^{\mu\nu\alpha\beta}\langle\partial_\mu V_\nu \partial_\alpha V_\beta P\rangle \quad (9)$$

with

$$G' = \frac{3g'^2}{4\pi^2 f_\pi} \qquad g' = -\frac{G_V m_\rho}{\sqrt{2}f_\pi^2}, \quad (10)$$

where $G_V = 55$ MeV and $f_\pi = 93$ MeV. At this point we can write down the amplitude of $\rho^+(p_1)\rho^-(p_2) \to \pi^+(p_3)\pi^-(p_4)$ with the ω exchange corresponding to Fig. 2 as in the π-exchange case

$$\begin{aligned} B_t = \frac{-G'^2}{(p_1 - p_3)^2 - m_\omega^2}(&p_3 \cdot p_4 p_1 \cdot \epsilon_2 p_2 \cdot \epsilon_1 \\ &+ p_1 \cdot p_2 p_4 \cdot \epsilon_1 p_3 \cdot \epsilon_2 + p_1 \cdot p_4 p_2 \cdot p_3 \epsilon_1 \cdot \epsilon_2 \\ &- p_2 \cdot p_3 p_1 \cdot \epsilon_2 p_4 \cdot \epsilon_1 - p_1 \cdot p_4 p_2 \cdot \epsilon_1 p_3 \cdot \epsilon_2 \\ &- p_1 \cdot p_2 p_3 \cdot p_4 \epsilon_1 \cdot \epsilon_2). \end{aligned} \quad (11)$$

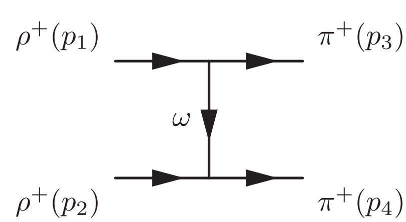

FIG. 2. ω-exchange diagram for $\rho^+\rho^- \to \pi^+\pi^-$.

And the u-channel ω-exchange amplitude $B_u(p_1,p_2,p_3,p_4)$ can be obtained from the expression of B_t as the case in the π exchange by exchanging $p_3 \leftrightarrow p_4$, thus

$$B_u(p_1,p_2,p_3,p_4) = B_t(p_1,p_2,p_4,p_3). \tag{12}$$

Next we write the tree-level amplitude for $\rho\rho \to \pi\pi$ with the ω exchange

$$\begin{aligned}
&\rho^+(p_1)\rho^-(p_2) \to \pi^+(p_3)\pi^-(p_4):\ B_t,\\
&\rho^+(p_1)\rho^-(p_2) \to \pi^0(p_3)\pi^0(p_4):\ 0,\\
&\rho^0(p_1)\rho^0(p_2) \to \pi^+(p_3)\pi^-(p_4):\ 0,\\
&\rho^0(p_1)\rho^0(p_2) \to \pi^0(p_3)\pi^0(p_4):\ B_t + B_u.
\end{aligned} \tag{13}$$

Then, using Eq. (7) we can get the $I = 0$ amplitude

$$\begin{aligned}
T_\omega^{(0)} = \frac{-G'^2}{(p_1-p_3)^2 - m_\omega^2}&(p_3\cdot p_4 p_1\cdot\epsilon_2 p_2\cdot\epsilon_1\\
&+ p_1\cdot p_2 p_4\cdot\epsilon_1 p_3\cdot\epsilon_2 + p_1\cdot p_4 p_2\cdot p_3 \epsilon_1\cdot\epsilon_2\\
&- p_2\cdot p_3 p_1\cdot\epsilon_2 p_4\cdot\epsilon_1 - p_1\cdot p_4 p_2\cdot\epsilon_1 p_3\cdot\epsilon_2\\
&- p_1\cdot p_2 p_3\cdot p_4 \epsilon_1\cdot\epsilon_2) + (p_3 \leftrightarrow p_4).
\end{aligned} \tag{14}$$

C. $\rho\rho \to K\bar{K}$ with K exchange

The ρKK coupling is provided in the same Lagrangian in Eq. (1), so we can immediately write down the amplitude of $\rho^+(p_1)\rho^-(p_2) \to K^+(p_3)K^-(p_4)$ with the K exchange corresponding to Fig. 3

$$C_t(p_1,p_2,p_3,p_4) = -4g^2 \frac{\epsilon_1\cdot p_3 \epsilon_2\cdot p_4}{(p_1-p_3)^2 - m_K^2}, \tag{15}$$

and the u channel

$$C_u(p_1,p_2,p_3,p_4) = C_t(p_1,p_2,p_4,p_3). \tag{16}$$

Then we can obtain the tree-level amplitudes for $\rho\rho \to K\bar{K}$ with the K exchange as the following:

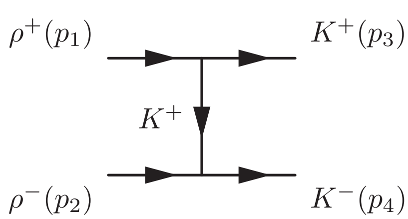

FIG. 3. K-exchange amplitude for $\rho^+(p_1)\rho^-(p_2) \to K^+(p_3)K^-(p_4)$.

$$\begin{aligned}
&\rho^+(p_1)\rho^-(p_2) \to K^+(p_3)K^-(p_4): C_t,\\
&\rho^+(p_1)\rho^-(p_2) \to K^0(p_3)\bar{K}^0(p_4): C_u,\\
&\rho^0(p_1)\rho^0(p_2) \to K^+(p_3)K^-(p_4): \frac{C_t + C_u}{2},\\
&\rho^0(p_1)\rho^0(p_2) \to K^0(p_3)\bar{K}^0(p_4): \frac{C_t + C_u}{2}.
\end{aligned} \tag{17}$$

Similar to Eq. (7) we need the isospin $I = 0$ eigenstate for $|K\bar{K}\rangle$. We have

$$|K\bar{K}\rangle = -\frac{1}{\sqrt{2}}|K^+(p_1)K^-(p_2) + K^0(p_1)\bar{K}^0(p_2)\rangle, \tag{18}$$

where we use the convention $|K^+\rangle = -|\frac{1}{2},\frac{1}{2}\rangle$ of the isospin. By using the isospin wave functions we can obtain for $I = 0$

$$T_K^{(0)} = 2\sqrt{6}g^2\left(\frac{\epsilon_1\cdot p_3\epsilon_2\cdot p_4}{m_K^2 - t} + \frac{\epsilon_1\cdot p_4\epsilon_2\cdot p_3}{m_K^2 - u}\right) \tag{19}$$

with t and u, the usual Mandelstam variable. We can see that Eq. (19) is similar to Eq. (8). The former can be obtained from the latter just by substituting $16 \to 2\sqrt{6}$ and $m_\pi \to m_K$.

D. $\rho\rho \to K\bar{K}$ with K^* exchange

As for the ρKK^* coupling, we use the Lagrangian in Eq. (9). Then we get the amplitude for $\rho^+(p_1)\rho^-(p_2) \to K^+(p_3)K^-(p_4)$ with the K^* exchange corresponding to Fig. 4 as

$$\begin{aligned}
D_t = \frac{-G'^2/2}{(p_1-p_3)^2 - m_{K^*}^2}&(p_3\cdot p_4 p_1\cdot\epsilon_2 p_2\cdot\epsilon_1\\
&+ p_1\cdot p_2 p_4\cdot\epsilon_1 p_3\cdot\epsilon_2 + p_1\cdot p_4 p_2\cdot p_3\epsilon_1\cdot\epsilon_2\\
&- p_2\cdot p_3 p_1\cdot\epsilon_2 p_4\cdot\epsilon_1 - p_1\cdot p_4 p_2\cdot\epsilon_1 p_3\cdot\epsilon_2\\
&- p_1\cdot p_2 p_3\cdot p_4\epsilon_1\cdot\epsilon_2),
\end{aligned} \tag{20}$$

and the u channel

$$D_u(p_1,p_2,p_3,p_4) = D_t(p_1,p_2,p_4,p_3). \tag{21}$$

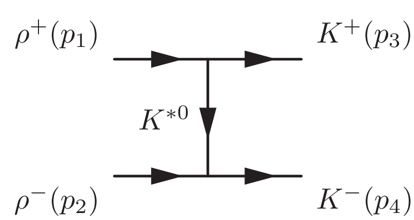

FIG. 4. K^*-exchange diagram for $\rho^+\rho^- \to K^+K^-$.

Next we list the tree-level amplitudes for $\rho\rho \to K\bar{K}$ with the K^*-exchange as the following:

$$\begin{aligned}
&\rho^+(p_1)\rho^-(p_2) \to K^+(p_3)K^-(p_4):\ D_t,\\
&\rho^+(p_1)\rho^-(p_2) \to K^0(p_3)\bar{K}^0(p_4):\ D_u,\\
&\rho^0(p_1)\rho^0(p_2) \to K^+(p_3)K^-(p_4):\ \frac{D_t+D_u}{2},\\
&\rho^0(p_1)\rho^0(p_2) \to K^0(p_3)\bar{K}^0(p_4):\ \frac{D_t+D_u}{2}.
\end{aligned} \tag{22}$$

Using Eqs. (7) and (18) we obtain the $I=0$ amplitude

$$\begin{aligned}
T_{K^*}^{(0)} = \frac{-\sqrt{6}G'^2/4}{(p_1-p_3)^2-m_{K^*}^2}&(p_3\cdot p_4 p_1\cdot\epsilon_2 p_2\cdot\epsilon_1\\
&+ p_1\cdot p_2 p_4\cdot\epsilon_1 p_3\cdot\epsilon_2 + p_1\cdot p_4 p_2\cdot p_3 \epsilon_1\cdot\epsilon_2\\
&- p_2\cdot p_3 p_1\cdot\epsilon_2 p_4\cdot\epsilon_1 - p_1\cdot p_4 p_2\cdot\epsilon_1 p_3\cdot\epsilon_2\\
&- p_1\cdot p_2 p_3\cdot p_4 \epsilon_1\cdot\epsilon_2) + (p_3 \leftrightarrow p_4),
\end{aligned} \tag{23}$$

which can be obtained from Eq. (14) by substituting $1 \to \sqrt{6}/4$ and $m_\omega \to m_{K^*}$.

E. Partial-wave decomposition

In term of these amplitudes with isospin $I=0$, we can calculate the partial-wave amplitudes in the ℓSJI basis [5], denoted as $T^{(JI)}_{\ell S;\bar{\ell}\bar{S}}(s)$ for the transition $(\bar{\ell}\bar{S}JI) \to (\ell SJI)$

$$\begin{aligned}
T^{(JI)}_{\ell S;\bar{\ell}\bar{S}}(s) = &\frac{Y^0_{\bar{\ell}}(\hat{\mathbf{z}})}{\sqrt{2}^N(2J+1)}\\
&\times \sum_{\substack{\sigma_1,\sigma_2,\bar{\sigma}_1\\ \bar{\sigma},m}} \int \mathrm{d}\hat{\mathbf{p}}'' Y^m_\ell(\hat{\mathbf{p}}'')^*(\sigma_1\sigma_2 M|s_1 s_2 S)\\
&\times (mM\bar{M}|\ell SJ)(\bar{\sigma}_1\bar{\sigma}_2\bar{M}|\bar{s}_1\bar{s}_2\bar{S})\\
&\times (0\bar{M}\bar{M}|\bar{\ell}\bar{S}J) T^{(I)}(p_1,p_2,p_3,p_4)
\end{aligned} \tag{24}$$

with $M=\sigma_1+\sigma_2$ and $\bar{M}=\bar{\sigma}_1+\bar{\sigma}_2$. And N accounts for identical particles, for example,

$$\begin{aligned}
N&=2 \quad \text{for } \rho\rho\to\pi\pi,\\
N&=1 \quad \text{for } \rho\rho\to K\bar{K}.
\end{aligned} \tag{25}$$

By using Eq. (24) we can calculate the partial-wave projected tree-level amplitudes of Eqs. (8), (14), (19), and (23) with quantum numbers I, ℓ, $S = 0, 0, 0$. We denote $T^{(00)}_{00;00}$ by V for simplicity and we have

for $\rho\rho\to\pi\pi$ with the π exchange

$$V_\pi = \frac{2g^2}{\sqrt{3}}\left(\frac{2(m_\rho^2-4m_\pi^2)}{\sqrt{s-4m_\pi^2}\sqrt{s-4m_\rho^2}}\ln\frac{s-2m_\rho^2+\sqrt{s-4m_\pi^2}\sqrt{s-4m_\rho^2}}{s-2m_\rho^2-\sqrt{s-4m_\pi^2}\sqrt{s-4m_\rho^2}}+\frac{s}{m_\rho^2}+2\right), \tag{26}$$

for $\rho\rho\to\pi\pi$ with the ω exchange

$$V_\omega = \frac{G'^2 s}{2\sqrt{3}}\left(\frac{m_\omega^2}{\sqrt{s-4m_\pi^2}\sqrt{s-4m_\rho^2}}\ln\frac{s+2m_\omega^2-2m_\pi^2-2m_\rho^2+\sqrt{s-4m_\pi^2}\sqrt{s-4m_\rho^2}}{s+2m_\omega^2-2m_\pi^2-2m_\rho^2-\sqrt{s-4m_\pi^2}\sqrt{s-4m_\rho^2}}-1\right), \tag{27}$$

for $\rho\rho\to K\bar{K}$ with the K exchange

$$V_K = \frac{g^2}{2}\left(\frac{2(m_\rho^2-4m_K^2)}{\sqrt{s-4m_K^2}\sqrt{s-4m_\rho^2}}\ln\frac{s-2m_\rho^2+\sqrt{s-4m_K^2}\sqrt{s-4m_\rho^2}}{s-2m_\rho^2-\sqrt{s-4m_K^2}\sqrt{s-4m_\rho^2}}+\frac{s}{m_\rho^2}+2\right), \tag{28}$$

for $\rho\rho\to K\bar{K}$ with the K^* exchange

$$V_{K^*} = \frac{G'^2 s}{4}\left(\frac{m_{K^*}^2}{\sqrt{s-4m_K^2}\sqrt{s-4m_\rho^2}}\ln\frac{s+2m_{K^*}^2-2m_K^2-2m_\rho^2+\sqrt{s-4m_K^2}\sqrt{s-4m_\rho^2}}{s+2m_{K^*}^2-2m_K^2-2m_\rho^2-\sqrt{s-4m_K^2}\sqrt{s-4m_\rho^2}}-1\right). \tag{29}$$

III. RESULTS AND DISCUSSION

We label the three channels, $\rho\rho$, $K\bar{K}$, and $\pi\pi$ as 1, 2, and 3, respectively. With the channel transition amplitudes V_π, V_ω, V_K, and V_{K^*} given in last section, we calculate the full amplitude and its pole positions by using the Bethe-Salpeter equation in its on-shell factorized form [4,5]

$$T = \frac{V}{1 - VG}. \tag{30}$$

G is a diagonal matrix made up by the two-point loop function [4,5]

$$G_{jj}(s) = i\int \frac{d^4q}{(2\pi)^4}\frac{1}{(q^2 - m_j^2)((P-q)^2 - m_j^2)} \tag{31}$$

with P the total four-momentum of the meson-meson systems and q the loop momentum. The channel is labeled by the subindex j. By using dimensional regularization, the integration can be recast as

$$G_{jj}(s) = \frac{1}{(4\pi)^2}\left(a(\mu) + \log\frac{m_j^2}{\mu^2} + \sigma\log\frac{\sigma+1}{\sigma-1}\right) \tag{32}$$

with

$$\sigma = \sqrt{1 - \frac{4m_j^2}{s}} \tag{33}$$

or using a momentum cutoff $q_{\max}$ as

$$G_{jj}(s) = \frac{1}{2\pi^2}\int_0^{q_{\max}} \mathrm{d}q \frac{q^2}{w(s - 4w^2 + i\epsilon)} \tag{34}$$

where $w = \sqrt{q^2 + m_j^2}$. The integral can be done algebraically

$$G_{jj}(s) = \frac{1}{(4\pi)^2}\left\{\sigma\log\frac{\sigma\sqrt{1+\frac{m_j^2}{q_{\max}^2}}+1}{\sigma\sqrt{1+\frac{m_j^2}{q_{\max}^2}}-1} - 2\log\left[\frac{q_{\max}}{m_j}\left(1+\sqrt{1+\frac{m_j^2}{q_{\max}^2}}\right)\right]\right\}. \tag{35}$$

Typical values of the cutoff $q_{\max}$ are around 1 GeV. $G_{jj}(s)$ has a right-hand cut above the threshold $2m_j$. In order to make an analytical extrapolation to the second Riemann sheet we make use of the continuity property

$$G_{jj}^{(2)}(\sqrt{s} + i\epsilon) = G_{jj}(\sqrt{s} - i\epsilon) \tag{36}$$

where the index (2) indicates the second Riemann sheet of G_{jj}. Then

$$\begin{aligned} G_{jj}^{(2)}(\sqrt{s}+i\epsilon) &= G_{jj}(\sqrt{s}-i\epsilon) \\ &= G_{jj}(\sqrt{s}+i\epsilon) - 2i\mathrm{Im}G_{jj}(\sqrt{s}+i\epsilon) \\ &= G_{jj}(\sqrt{s}+i\epsilon) + \frac{i}{4\pi}\frac{|\mathbf{p}|}{\sqrt{s}}. \end{aligned} \tag{37}$$

Other potentials of coupled channels like $\pi\pi - K\bar{K}$ can be found in [1]. Our results are shown in Table 1 for various $q_{\max}$ values. For comparison, the results for the $\rho\rho$ single channel without considering the coupled-channel effects as in Ref. [5] are shown in the second row. The $3\sim6$ rows show the results including one coupled channel with the exchanged meson listed in the first column. For example the π denotes the $\rho\rho - \pi\pi$ channel with the π exchange and so on. The seventh row gives the results including all three coupled channels of $\rho\rho$, $\pi\pi$, and $\bar{K}K$.

The above results show that the influence of the vector meson ω and K^* exchanges is very small; the largest influence comes from the $\rho\rho - \pi\pi$ channel coupling by the pion exchange, which shifts up the pole mass and results in a sizable $\pi\pi$ decay width, comparable with relevant PDG values for $f_0(1500)$ [15]. For the $\rho\rho - K\bar{K}$ coupled-channel case we can see that the width is consistent with $f_0(1500)$ decaying into $K\bar{K}$ in PDG, which is about

TABLE I. Pole position for coupled channels.

$q_{\max}$(GeV)	0.875	1.0	1.2	1.4
$\rho\rho$ only	1494.8	1467.2	1427.3	1395.0
π	$1530.0 - 4.9i$	$1519.5 - 8.4i$	$1501.5 - 12.3i$	$1488.6 - 14.6i$
ω	$1492.2 - 0.7i$	$1466.5 - 1.0i$	$1428.1 - 1.1i$	$1400.0 - 1.1i$
K	$1497.8 - 3.3i$	$1473.9 - 4.1i$	$1437.2 - 4.4i$	$1410.0 - 4.2i$
K^*	$1489.6 - 0.5i$	$1463.3 - 0.5i$	$1424.5 - 0.4i$	$1396.1 - 0.3i$
3 channels	$1529.8 - 4.9i$	$1519.0 - 8.6i$	$1500.9 - 13.5i$	$1488.4 - 16.7i$

8.9 MeV. When taking into account all the three channels, the pole position is close to the results by considering only the pion exchange contribution. With $q_{\max} = 1.4$ GeV, the pole mass and partial decay widths to $\pi\pi$ and $\bar{K}K$ are roughly consistent with PDG values for $f_0(1500)$. The largest decay channel should be 4π either through $\rho\rho$ directly or by its cross talk with $\sigma\sigma$. Note that due to the binding energy of the molecule as well as the kinetic energy of ρ inside the molecule, the 4π decay width through the decay of ρ inside the $\rho\rho$ molecule can be smaller than the decay width of a single free ρ meson. A similar effect was pointed out by Refs. [16,17] in their studies of $d^*(2380)$ as a $\Delta\Delta$ molecule which gets a decay width smaller than the decay width of a single free Δ state. This kind of effect was also observed by the study of other hadronic molecules [18,19].

In summary, the $\rho\rho$ scattering is revisited by including its coupled channels of pseudoscalar mesons, i.e., $\pi\pi$ and $\bar{K}K$. It is found that the coupled-channel effect is important and shifts up the pole mass of the dynamically generated scalar state significantly. It makes the state to be more consistent with $f_0(1500)$ rather than $f_0(1370)$ as favored by the previous studies [4,5] without including these coupled channels. This leads to a nicely consistent picture with a recent dispersive study [20] where a new parametrization for the scalar pion form factors is derived by fitting it to LHCb data on $\bar{B}_s^0 \to J/\psi\pi\pi$ and finding an $f_0(1500)$ at mass 1465 ± 18 MeV coupling strongly to $\rho\rho$ (or $\sigma\sigma$). The $\rho\rho$ scattering has been extended to the S-wave interactions for the whole vector-meson nonet by two groups [21,22]. Both propose $f_0(1710)$ to be the $K^*\bar{K}^*$ dynamically generated state. We expect similar significant coupled-channel effects there. By including its coupled channels of pseudoscalar mesons, the $K^*\bar{K}^*$ dynamically generated state could be $f_0(1790)$ suggested by the BES data [23–25] instead of $f_0(1710)$. The $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ have been studied before in the quarkonia-glueball mixing picture in Refs. [26–28], trying to pin down partial contributions of glueball, nonstrange, and strange quarkonia in these scalar mesons. With the new configuration of meson-meson dynamically generated states, the structure of these scalars should be richer than previous assumptions and deserve further exploration by expanding the configuration space.

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