Muon anomalies and the $SU(5)$ Yukawa relations

A. E. Cárcamo Hernández[1,*] and Stephen F. King[2,†]

[1]*Universidad Técnica Federico Santa María and Centro Científico-Tecnológico de Valparaíso, Casilla 110-V Valparaíso, Chile*

[2]*School of Physics and Astronomy, University of Southampton, SO17 1BJ Southampton, United Kingdom*

(Received 8 January 2019; published 3 May 2019)

We show that, within the framework of $SU(5)$ grand unified theories (GUTs), multiple vectorlike families at the GUT scale which transform under a gauged $U(1)'$ (under which the three chiral families are neutral) can result in a single vectorlike family at low energies which can induce nonuniversal and flavorful Z' couplings, which can account for the B physics anomalies in $R_{K^{(*)}}$. In such theories, we show that the same muon couplings which explain $R_{K^{(*)}}$ also correct the Yukawa relation $Y_e = Y_d^T$ in the muon sector without the need for higher Higgs representations. To illustrate the mechanism, we construct a concrete model based on $SU(5) \times A_4 \times Z_3 \times Z_7$ with two vectorlike families at the GUT scale, and two right-handed neutrinos, leading to a successful fit to quark and lepton (including neutrino) masses, mixing angles, and CP phases, where the constraints from lepton-flavor violation require Y_e to be diagonal.

DOI: 10.1103/PhysRevD.99.095003

I. INTRODUCTION

Most Z' models [1] have universal couplings to the three families of quarks and leptons. The reason for this is both theoretical and phenomenological. First, many theoretical models naturally predict universal Z' couplings. Second, from a phenomenological point of view, having universal couplings avoids dangerous flavor changing neutral currents (FCNCs) mediated by tree-level Z' exchange. The most sensitive processes involve the first two families, such as $K_0 - \bar{K}_0$ mixing, $\mu - e$ conversion in muonic atoms, and so on, leading to stringent bounds on the Z' mass and couplings [1].

Recently, the phenomenological motivation for considering nonuniversal Z' models has increased due to mounting evidence for semileptonic B decays which violate $\mu - e$ universality at rates which exceed those predicted by the Standard Model (SM) [2–4]. In particular, the LHCb Collaboration and other experiments have reported a number of anomalies in $B \to K^{(*)} l^+ l^-$ decays such as the R_K [5] and R_{K^*} [6] ratios of $\mu^+\mu^-$ to e^+e^- final states, which are observed to be about 70% of their expected values with a 4σ deviation from the SM, and the P_5' angular variable, not to mention the $B \to \phi\mu^+\mu^-$ mass distribution in $m_{\mu^+\mu^-}$.

*antonio.carcamo@usm.cl
†king@soton.ac.uk

Following the recent measurement of R_{K^*} [6], a number of phenomenological analyses of these data (see, e.g., [7–12]) favor a new physics operator of the $C_{9\mu}^{NP} = -C_{10\mu}^{NP}$ form [13,14],

$$-\frac{1}{(31.5\ \text{TeV})^2}\bar{b}_L\gamma^\mu s_L\bar{\mu}_L\gamma_\mu\mu_L \qquad (1)$$

or of the $C_{9\mu}^{NP}$ form,

$$-\frac{1}{(31.5\ \text{TeV})^2}\bar{b}_L\gamma^\mu s_L\bar{\mu}\gamma_\mu\mu \qquad (2)$$

or some linear combination of these two operators. Other solutions different than $C_{9\mu}^{NP} = -C_{10\mu}^{NP}$ allowing for a successful explanation of the R_{K^*} anomalies are studied in detail in Ref. [15]. However the solution $C_{9\mu}^{NP} = -C_{10\mu}^{NP}$ can provide a simultaneous explanation of the R_{K^*} and R_{D^*} anomalies [16].

In a flavorful Z' model, the new physics operator in Eq. (1) will arise from the tree-level Z' exchange, where the Z' must dominantly couple to $\mu\mu$ over ee, and must also have the quark flavor changing coupling $b_L s_L$ which must dominate over $b_R s_R$. The coefficient of the tree-level Z' exchange operator is therefore of the form,

$$\frac{C_{b_L s_L} C_{\mu_L\mu_L}}{M_Z'^2} \approx -\frac{1}{(31.5\ \text{TeV})^2}. \qquad (3)$$

In realistic models, the product of the Z' couplings $C_{b_L s_L} C_{\mu_L\mu_L}$ is much smaller than unity since the constraint from the B_s mass difference will imply that $\frac{|C_{b_L s_L}|}{|C_{\mu_L \mu_L}|} \lesssim \frac{1}{50}$, so if $C_{\mu_L \mu_L} \lesssim 1$ then $C_{b_L s_L} \lesssim 1/50$ which implies that $M'_Z \lesssim 5$ TeV, making the Z' possibly observable at the LHC, depending on its coupling to light quarks. Studies of lepton-flavor violating (LFV) B decays in generic Z' models before the R_{K^*} measurement but compatible with it are provided in Ref. [17]. In addition, two and three Higgs doublet models with a nonuniversal $U(1)'$ gauge symmetry have been used as the first explanations for the R_K and R_{K^*} anomalies [18]. An alternative explanation of the R_K and R_{K^*} anomalies in the framework of a two Higgs doublet model with two scalar singlets and nonuniversal $U(1)'$ gauge symmetry is provided in Ref. [19]. Another explanation for the R_K and R_{K^*} anomalies is an extended inert doublet model having an extra nonuniversal $U(1)'$ gauge symmetry, where the SM fermion mass hierarchy is generated from sequential loop suppression [20,21]. Furthermore, the R_K and R_{K^*} anomalies can be explained in an aligned two Higgs doublet model with right-handed Majorana neutrinos mediating linear and inverse scale seesaw mechanisms to generate light active neutrino masses [22]. Apart from these explanations, the R_K and R_{K^*} anomalies can also be explained in models with extended $SU(3)_C \times SU(3)_L \times U(1)'$ symmetry, with nonminimal particle content, as done in Ref. [23]. Finally, a vector leptoquark in the Standard Model representation $(3,1)_{2/3}$ arising from a Pati-Salam-like theory has been shown for the first time to provide a good fit to the R_{K^*} anomalies [24].

In a recent paper, we showed how to obtain a flavorful Z' suitable for explaining R_{K^*} by adding a fourth vectorlike family with nonuniversal $U(1)'$ charges [25]. The idea is that the Z' couples universally to the three chiral families, which then mix with the nonuniversal fourth family to induce effective nonuniversal couplings in the physical light mixed quarks and leptons. Such a mechanism has wide applicability; for example, it was recently discussed in the context of F-theory models with nonuniversal gauginos [26]. Two explicit examples were discussed in [25]: an $SO(10) \to SU(5) \times U(1)_X$ model, where we identified $U(1)' \equiv U(1)_X$, which, however, was subsequently shown to be not consistent with both explaining R_{K^*} and respecting the B_s mass difference [27], and a fermiophobic model where the $U(1)'$ charges are not carried by the three chiral families, only by a fourth vectorlike family. The fermiophobic looks more promising, since, with suitable couplings, it can overcome all the phenomenological flavor changing and collider constraints, and can, in addition, also provide an explanation for dark matter, as recently discussed [28].

On the other hand, the existing pattern of SM fermion masses is extended over a range of 5 orders of magnitude in the quark sector and a much wider range of about 12 orders of magnitude, when neutrinos are included. Unlike in the quark sector where the mixing angles are very small, two of the three leptonic mixing angles, i.e., the atmospheric θ_{23} and the solar θ_{12}, are large, while the reactor angle θ_{13} is comparatively small. This suggests a different kind of underlying physics for the neutrino sector than what should be responsible for the observed hierarchy of quark masses and mixing angles. That flavor puzzle of the SM indicates that new physics has to be advocated to explain the observed SM fermion mass and mixing pattern. That SM "flavor puzzle" motivates us to build models with additional scalars and fermions in their particle spectrum and with an extended gauge group, supplemented by discrete flavor symmetries, which are usually spontaneously broken, in order to generate the observed pattern of SM fermion masses and mixing angles. Recent reviews of discrete flavor groups can be found in Refs. [29–33]. Several discrete groups such as S_3 [34–62], A_4 [63–105], S_4 [106–125], D_4 [126–134], Q_6 [135–145], T_7 [146–155], T_{13} [156–159], T' [160–168], $\Delta(27)$ [169–194], $\Delta(54)$ [195], $\Delta(96)$ [196–198], $\Delta(6N^2)$ [199–201], and A_5 [202–213] have been implemented in extensions of the SM to provide a nice description of the observed pattern of fermion masses and mixing angles.

In this paper we focus on an $SU(5) \times U(1)'$ model with a vectorlike fourth family where the three chiral families do not couple to the $U(1)'$, but the fourth vectorlike family has arbitrary $U(1)'$ charges for the different multiplets, which mix with the three families, thereby inducing effective nonuniversal couplings for the light physical mixed quarks and leptons. The particular scheme we consider involves induced Z' couplings to third family left-handed quark doublets and second family left-handed lepton doublets, similar to the model discussed recently in [28]. However, in addition, we also allow induced Z' couplings to the right-handed muon, in order to provide nonuniversality for both left-handed and right-handed muons, and hence give corrections to the physical muon Yukawa coupling. We show that such an $SU(5)$ model with the vector sector can account for the muon anomalies $R_{K^{(*)}}$ and correct the Yukawa relation $Y_e \neq Y_d^T$ without the need for higher Higgs representations. The same applies to flavored grand unified theories (GUTs) such as $SU(5) \times A_4$ with a vector sector. In addition, we study the implications of a A_4 flavored $SU(5) \times U(1)'$ GUT with five generations of fermions on SM fermion masses and mixings. To successfully describe the observed pattern of SM fermion masses and mixing angles, we supplement the A_4 family symmetry of that model by the $Z_3 \times Z_7$ discrete group and we extend the particle content of our model by adding two right-handed Majorana neutrinos and several $SU(5)$ singlet scalar fields. The discrete $A_4 \times Z_3 \times Z_7$ discrete group is needed in order to reproduce the specific patterns of mass matrices in the quark and lepton sectors, consistent with the low energy SM fermion flavor data. The two right-handed Majorana neutrinos are required for the implementation of the type-I seesaw mechanism at tree level to generate the masses for the light active neutrinos as pointed out for the first time in

Refs. [214,215]. In this framework, the active neutrinos acquire small masses scaled by the inverse of the large type-I seesaw mediators, thus providing a natural explanation for the smallness of neutrino masses.

The layout of the remainder of the paper is as follows. In Sec. II we describe a two Higgs doublet model with four generations of fermions, several scalar singlets, and an extra $U(1)'$ gauge symmetry under which the SM fermions are neutral and the fourth generation of fermions is charged. In Sec. III we present the $SU(5)\times U(1)'$ GUT theory with five generations of fermions in the $\bar{\mathbf{5}}$ and $\mathbf{10}$ irreps of $SU(5)$. In Sec. IV we outline the A_4 flavored $SU(5)\times U(1)'$ GUT theory with five generations of fermions and we discuss its implications on SM fermion masses and mixings. Finally, we conclude in Sec. V. Appendix A provides a brief description of the A_4 discrete group.

II. STANDARD MODEL WITH A VECTOR SECTOR

In this section we analyze the model defined in Table I. The three chiral families and the Higgs doublets do not carry any $U(1)'$ charges. We allow the vectorlike family to carry arbitrary $U(1)'$ charges. The scalars ϕ couple the vectorlike family to the three chiral families.

A. Higgs Yukawa couplings

The Higgs Yukawa couplings of the first three chiral families ψ_i are

$$\mathcal{L}^{Yuk} = y^u_{ij} H_u \bar{Q}_{Li} u_{Rj} + y^d_{ij} H_d \bar{Q}_{Li} d_{Rj} + y^e_{ij} H_d \bar{L}_{Li} e_{Rj} + \text{H.c.} \tag{4}$$

where $i, j = 1, \ldots, 3$.

TABLE I. The general framework considered in this paper.

Field	Representation/Charge			
	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)'$
Q_{Li}	**3**	**2**	1/6	0
u_{Ri}	**3**	**1**	2/3	0
d_{Ri}	**3**	**1**	−1/3	0
L_{Li}	**1**	**2**	−1/2	0
e_{Ri}	**1**	**1**	−1	0
ν_{Ri}	**1**	**1**	0	0
H_u	**1**	**2**	−1/2	0
H_d	**1**	**2**	1/2	0
Q_{L4},$\tilde{Q}_{R4}$	**3**	**2**	1/6	q_{Q_4}
u_{R4},$\tilde{u}_{L4}$	**3**	**1**	2/3	q_{u_4}
d_{R4},$\tilde{d}_{L4}$	**3**	**1**	−1/3	q_{d_4}
L_{L4},$\tilde{L}_{R4}$	**1**	**2**	−1/2	q_{L_4}
e_{R4},$\tilde{e}_{L4}$	**1**	**1**	−1	q_{e_4}
$\phi_{Q,u,d,L,e}$	**1**	**1**	0	$q_{\phi_{Q,u,d,L,e}}$

In addition we allow the possibility of the fourth vectorlike family Higgs Yukawa couplings,

$$\mathcal{L}^{Yuk}_4 = y^u_4 H_u \bar{Q}_{L4} u_{R4} + y^d_4 H_d \bar{Q}_{L4} d_{R4} + y^e_4 H_d \bar{L}_{L4} e_{R4} + \text{H.c.} \tag{5}$$

although the existence of these couplings will depend on the choice of the $U(1)'$ charges for the vectorlike family, and some or all of these couplings could be zero.

B. Heavy masses

In this subsection we ignore the Higgs Yukawa couplings (which give electroweak scale masses) and consider only the heavy mass Lagrangian (which gives multi-TeV masses).

The vectorlike family can mix with the three chiral families via the ϕ scalars, and also can have explicit masses, leading to the heavy Lagrangian,

$$\begin{aligned}\mathcal{L}^{\text{heavy}} = {} & x^Q_i \phi_Q \bar{Q}_{Li} \tilde{Q}_{R4} + x^u_i \phi_u \bar{\tilde{u}}_{L4} u_{Ri} + x^d_i \phi_d \bar{\tilde{d}}_{L4} d_{Ri} \\ & + x^L_i \phi_L \bar{L}_{Li} \tilde{L}_{R4} + x^e_i \phi_e \bar{\tilde{e}}_{L4} e_{Ri} + M^Q_4 \bar{Q}_{L4} \tilde{Q}_{R4} \\ & + M^u_4 \bar{\tilde{u}}_{L4} u_{R4} + M^d_4 \bar{\tilde{d}}_{L4} d_{R4} + M^L_4 \bar{L}_{L4} \tilde{L}_{R4} \\ & + M^e_4 \bar{\tilde{e}}_{L4} e_{R4} + \text{H.c.}\end{aligned} \tag{6}$$

After the singlet fields ϕ develop vacuum expectation values (VEVs), the $U(1)'$ gauge symmetry is broken and yields a massive Z' gauge boson whose mass is of order of the largest VEV of the ϕ fields. Then we may define new mass parameters $M^Q_i = x^Q_i \langle \phi_Q \rangle$, and similarly for the other mass parameters, give

$$\begin{aligned}\mathcal{L}^{\text{heavy}} = {} & M^Q_\alpha \bar{Q}_{L\alpha} \tilde{Q}_{R4} + M^u_\alpha \bar{\tilde{u}}_{L4} u_{R\alpha} + M^d_\alpha \bar{\tilde{d}}_{L4} d_{R\alpha} \\ & + M^L_\alpha \bar{L}_{L\alpha} \tilde{L}_{R4} + M^e_\alpha \bar{\tilde{e}}_{L4} e_{R\alpha} + \text{H.c.}\end{aligned} \tag{7}$$

where $\alpha = 1, \ldots, 4$ in a compact notation.

All these mass terms are heavy, of order a few TeV, and our first task is to identify the heavy mass states and integrate them out. Actually only one linear combination of the four "normal chirality" states will get heavy, while the other three orthogonal linear combinations will remain massless (ignoring the Higgs Yukawa couplings). We will identify the three physical massless families with the quarks and leptons of the Standard Model.

C. Diagonalizing the heavy masses

We now focus on $\mathcal{L}^{\text{heavy}}$ (ignoring the Higgs Yukawa Lagrangian) and show how the heavy masses may be diagonalized, denoting the fields in this basis by primes. The goal is to identify the light states of the low energy effective SM below the few TeV scale, after the heavy states have been integrated out.

In the primed basis, the fourth family is massive (before electroweak symmetry breaking),

$$\begin{aligned}\mathcal{L}^{\text{mass}} = \tilde{M}_4^Q \bar{Q}'_{L4}\tilde{Q}_{R4} + \tilde{M}_4^u \bar{\tilde{u}}_{L4} u'_{R4} + \tilde{M}_4^d \bar{\tilde{d}}_{L4} d'_{R4} \\ + \tilde{M}_4^L \bar{L}'_{L4}\tilde{L}_{R4} + \tilde{M}_4^e \bar{\tilde{e}}_{L4} e'_{R4} + \text{H.c.}\end{aligned} \tag{8}$$

The first three families in the primed basis have zero mass (before electroweak symmetry breaking), and are identified as the quarks and leptons of the SM.

The fields in the primed basis and the original basis are related by unitary 4×4 mixing matrices,

$$\begin{aligned}&Q'_L = V_{Q_L} Q_L, \qquad u'_R = V_{u_R} u_R, \qquad d'_R = V_{d_R} d_R, \\ &L'_L = V_{L_L} L_L, \qquad e'_R = V_{e_R} e_R.\end{aligned} \tag{9}$$

In our scheme we will consider only the nonzero mixing angles to be $\theta_{34}^{Q_L}$, in order to generate the Z' coupling to the third family quark doublet including b'_L, and also $\theta_{24}^{L_L}$ and $\theta_{24}^{e_R}$ to generate the Z' coupling to the second family lepton doublet including μ'_L and also μ'_R, in the primed basis. This is very similar to the model in [28], where the nonzero angles $\theta_{34}^{Q_L}$ and $\theta_{24}^{L_L}$ were considered, and whose main focus was on the phenomenological viability of the model including dark matter. The model considered here includes, in addition, the nonzero angle $\theta_{24}^{e_R}$ which generates an additional Z' coupling to μ'_R, which is important for the main focus of the present paper, namely, the effect of the model on the $SU(5)$ Yukawa relations.

To summarize, in this paper we consider

$$V_{Q_L} = V_{34}^{Q_L} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_{34}^{Q_L} & s_{34}^{Q_L} \\ 0 & 0 & -s_{34}^{Q_L} & c_{34}^{Q_L} \end{pmatrix}, \tag{10}$$

$$V_{L_L} = V_{24}^{L_L} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{24}^{L_L} & 0 & s_{24}^{L_L} \\ 0 & 0 & 1 & 0 \\ 0 & -s_{24}^{L_L} & 0 & c_{24}^{L_L} \end{pmatrix}, \tag{11}$$

$$V_{e_R} = V_{24}^{e_R} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{24}^{e_R} & 0 & s_{24}^{e_R} \\ 0 & 0 & 1 & 0 \\ 0 & -s_{24}^{e_R} & 0 & c_{24}^{e_R} \end{pmatrix}, \tag{12}$$

denoting $c = \cos\theta$ and $s = \sin\theta$.

D. The Lagrangian in the primed basis

1. Yukawa couplings in the primed basis

In the original basis, the Yukawa couplings in Eq. (4) may be written in terms of the three chiral families ψ_i plus the same chirality fourth family ψ_4 in a 4×4 matrix notation as

$$\mathcal{L}^{Yuk} = H_u \bar{Q}_L \tilde{y}^u u_R + H_d \bar{Q}_L \tilde{y}^d d_R + H_d \bar{L}_L \tilde{y}^e e_R + \text{H.c.} \tag{13}$$

where $\tilde{y}^u$, $\tilde{y}^d$, $\tilde{y}^e$ are 4×4 matrices consisting of the original 3×3 matrices, y^u, y^d, y^e, but augmented by a fourth row and column, as follows:

$$\tilde{y}^e = \begin{pmatrix} y_{11}^e & y_{12}^e & y_{13}^e & y_{14}^e \\ y_{21}^e & y_{22}^e & y_{23}^e & y_{24}^e \\ y_{31}^e & y_{32}^e & y_{33}^e & y_{34}^e \\ y_{41}^e & y_{42}^e & y_{43}^e & y_{44}^e \end{pmatrix}. \tag{14}$$

In the primed basis in Eq. (9), where only the fourth components of the fermions are very heavy, the Yukawa couplings become

$$\mathcal{L}^{Yuk} = H_u \bar{Q}'_L \tilde{y}'^u u'_R + H_d \bar{Q}'_L \tilde{y}'^d d'_R + H_d \bar{L}'_L \tilde{y}'^e e'_R + \text{H.c.} \tag{15}$$

where

$$\tilde{y}'^u = V_{Q_L}\tilde{y}^u V_{u_R}^\dagger, \quad \tilde{y}'^d = V_{Q_L}\tilde{y}^d V_{d_R}^\dagger, \quad \tilde{y}'^e = V_{L_L}\tilde{y}^e V_{e_R}^\dagger. \tag{16}$$

In the primed basis it is trivial to integrate out the heavy family by simply removing the fourth rows and columns of the primed Yukawa matrices in Eq. (16), to leave the upper 3×3 blocks, which describe the three massless families, in the low energy effective theory involving the massless fermions ψ'_i,

$$\begin{aligned}\mathcal{L}^{Yuk}_{\text{light}} = y'^u_{ij} H_u \bar{Q}'_{Li} u'_{Rj} + y'^d_{ij} H_d \bar{Q}'_{Li} d'_{Rj} \\ + y'^e_{ij} H_d \bar{L}'_{Li} e'_{Rj} + \text{H.c.}\end{aligned} \tag{17}$$

where

$$\begin{aligned}&y'^u_{ij} = (V_{Q_L}\tilde{y}^u V_{u_R}^\dagger)_{ij}, \qquad y'^d_{ij} = (V_{Q_L}\tilde{y}^d V_{d_R}^\dagger)_{ij}, \\ &y'^e_{ij} = (V_{L_L}\tilde{y}^e V_{e_R}^\dagger)_{ij}\end{aligned} \tag{18}$$

and $i, j = 1, \ldots, 3$. The physical three family quark and lepton masses in the low energy effective theory should be calculated using the 3×3 Yukawa matrices in Eq. (18).

For example, from Eqs. (11), (12), (14), and (16) we see that, if y_{44} is large, then this mixing may enhance significantly y'^e_{22} compared to its original value y^e_{22},

$$\tilde{y}'^{e} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{24}^{L_L} & 0 & s_{24}^{L_L} \\ 0 & 0 & 1 & 0 \\ 0 & -s_{24}^{L_L} & 0 & c_{24}^{L_L} \end{pmatrix} \begin{pmatrix} y_{11}^{e} & y_{12}^{e} & y_{13}^{e} & y_{14}^{e} \\ y_{21}^{e} & y_{22}^{e} & y_{23}^{e} & y_{24}^{e} \\ y_{31}^{e} & y_{32}^{e} & y_{33}^{e} & y_{34}^{e} \\ y_{41}^{e} & y_{42}^{e} & y_{43}^{e} & y_{44}^{e} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{24}^{e_R} & 0 & -s_{24}^{e_R} \\ 0 & 0 & 1 & 0 \\ 0 & s_{24}^{e_R} & 0 & c_{24}^{e_R} \end{pmatrix}$$
$$= \begin{pmatrix} y_{11}^{e} & y_{12}^{e} & y_{13}^{e} & y'^{e}_{14} \\ y_{21}^{e} & y'^{e}_{22} & y_{23}^{e} & y'^{e}_{24} \\ y_{31}^{e} & y_{32}^{e} & y_{33}^{e} & y'^{e}_{34} \\ y'^{e}_{44} & y'^{e}_{44} & y'^{e}_{44} & y'^{e}_{44} \end{pmatrix} \tag{19}$$

where the 22 element of the 3×3 light physical Yukawa matrix gets modified as follows:

$$y'^{e}_{22} = c_{24}^{L_L} c_{24}^{e_R} y_{22}^{e} + c_{24}^{L_L} s_{24}^{e_R} y_{24}^{e} + s_{24}^{L_L} c_{24}^{e_R} y_{42}^{e} + s_{24}^{L_L} s_{24}^{e_R} y_{44}^{e} \approx y_{22}^{e} + \theta_{24}^{e_R} y_{24}^{e} + \theta_{24}^{L_L} y_{42}^{e} + \theta_{24}^{L_L} \theta_{24}^{e_R} y_{44}^{e} \tag{20}$$

where the approximation is for small angles. This may be a rather large correction if $y_{44}^{e} \gg y_{22}^{e}$ or $y_{24}^{e} \gg y_{22}^{e}$ or $y_{42}^{e} \gg y_{22}^{e}$ even for small angle rotations. Such an enhancement is not present for y'^{d}_{22}, due to the assumed zero angles $\theta_{24}^{Q_L} = \theta_{24}^{d_R} = 0$. Therefore any relation between y_{22}^{e} and y_{22}^{d} will not be respected by the physical couplings y'^{e}_{22} and y'^{d}_{22}, after the mixing with the vectorlike family has been taken into account.

By a similar argument, turning on the mixing angles $\theta_{14}^{L_L}$, $\theta_{14}^{e_R}$ would lead to

$$y'^{e}_{11} \approx y_{11}^{e} + \theta_{14}^{e_R} y_{14}^{e} + \theta_{14}^{L_L} y_{41}^{e} + \theta_{14}^{L_L} \theta_{14}^{e_R} y_{44}^{e}, \tag{21}$$

where these mixing angles $\theta_{14}^{L_L}$, $\theta_{14}^{e_R}$ could be much smaller than $\theta_{24}^{Q_L}$, $\theta_{24}^{d_R}$ and still give a significant correction, since the 11 element of the charged lepton matrix is more sensitive to such corrections than the 22 element (since the electron mass is much smaller than the muon mass).

2. Z' gauge couplings in the primed basis

There is a Glashow-Iliopoulos-Maiani mechanism in the electroweak sector leading to no FCNCs. However, in the physics of Z' gauge bosons, the $U(1)'$ charges depend on the family index α. This leads to nonuniversality and possibly FCNCs due to the Z' gauge boson exchange, as we discuss. After $U(1)'$ breaking, we have a massive Z' gauge boson with diagonal gauge couplings to the four families of quarks and leptons, in the original basis,

$$\mathcal{L}_{Z'}^{\text{gauge}} = g' Z'_{\mu} (\bar{Q}_L D_Q \gamma^{\mu} Q_L + \bar{u}_R D_u \gamma^{\mu} u_R + \bar{d}_R D_d \gamma^{\mu} d_R + \bar{L}_L D_L \gamma^{\mu} L_L + \bar{e}_R D_e \gamma^{\mu} e_R) \tag{22}$$

where only the fourth family has nonzero charges,

$$D_Q = \text{diag}(0,0,0,q_{Q4}), \qquad D_u = \text{diag}(0,0,0,q_{u4}), \qquad D_d = \text{diag}(0,0,0,q_{d4}),$$
$$D_L = \text{diag}(0,0,0,q_{L4}), \qquad D_e = \text{diag}(0,0,0,q_{e4}). \tag{23}$$

In the diagonal heavy mass (primed) basis, given by the unitary transformations in Eq. (9), the Z' couplings to the four families of quarks and leptons in Eq. (22) become

$$\mathcal{L}_{Z'}^{\text{gauge}} = g' Z'_{\mu} (\bar{Q}'_L D'_Q \gamma^{\mu} Q'_L + \bar{u}'_R D'_u \gamma^{\mu} u'_R + \bar{d}'_R D'_d \gamma^{\mu} d'_R + \bar{L}'_L D'_L \gamma^{\mu} L'_L + \bar{e}'_R D'_e \gamma^{\mu} e'_R) \tag{24}$$

where

$$D'_Q = V_{Q_L} D_Q V_{Q_L}^{\dagger}, \qquad D'_u = V_{u_R} D_u V_{u_R}^{\dagger}, \qquad D'_d = V_{d_R} D_d V_{d_R}^{\dagger},$$
$$D'_L = V_{L_L} D_L V_{L_L}^{\dagger}, \qquad D'_e = V_{e_R} D_e V_{e_R}^{\dagger}. \tag{25}$$

In the low energy effective theory, after decoupling the fourth heavy family, Eq. (24) gives the Z' couplings to the three massless families of quarks and leptons,

$$\mathcal{L}_{Z'}^{\text{gauge}} = g' Z'_{\mu} (\bar{Q}'_L \tilde{D}'_Q \gamma^{\mu} Q'_L + \bar{u}'_R \tilde{D}'_u \gamma^{\mu} u'_R + \bar{d}'_R \tilde{D}'_d \gamma^{\mu} d'_R + \bar{L}'_L \tilde{D}'_L \gamma^{\mu} L'_L + \bar{e}'_R \tilde{D}'_e \gamma^{\mu} e'_R) \tag{26}$$

where the 3×3 matrices $\tilde{D}'$ are given by

$$(\tilde{D}'_Q)_{ij} = (V_{Q_L} D_Q V^\dagger_{Q_L})_{ij}, \qquad (\tilde{D}'_u)_{ij} = (V_{u_R} D_u V^\dagger_{u_R})_{ij}, \qquad (\tilde{D}'_d)_{ij} = (V_{d_R} D_d V^\dagger_{d_R})_{ij},$$
$$(\tilde{D}'_L)_{ij} = (V_{L_L} D_L V^\dagger_{L_L})_{ij}, \qquad (\tilde{D}'_e)_{ij} = (V_{e_R} D_e V^\dagger_{e_R})_{ij}, \tag{27}$$

where $i, j = 1, \ldots, 3$.

Without the fourth family, mixing all these Z' couplings would be zero, since the three original chiral families have zero $U(1)'$ charges. However, with Eqs. (10)–(12), this mixing induces Z' couplings to the third family left-handed quarks and to the muons, as we discuss in the next subsection.

E. Phenomenology

The example we consider is one in which the quarks and leptons start out not coupling to the Z' at all, as in fermiophobic models. We show that such fermiophobic Z' models may be converted to flavorful Z' models via mixing with fourth and fifth vectorlike families with Z' couplings. We consider both fourth and fifth vectorlike families of charged fermions to account for the R_K and R_{K^*} anomalies and at the same time to allow embedding the model in a $SU(5)$ GUT theory in such a way that the mixings between the heavy and light states will yield a realistic SM quark mass spectrum at low energies without adding a scalar field in the 45 irrep representation of $SU(5)$ as we will shown in detail in Sec. IV. Without the inclusion of the fifth fermion family it will not be possible to embed our model in a $SU(5)$ GUT theory consistent with the low energy SM fermion flavor data and at the same time allowing for an explanation of the R_K and R_{K^*} anomalies, without invoking **45** irrep scalar of $SU(5)$. We start by considering the following scenario where the mixing matrices for the fermionic fields Q_L, L_L, and e_R are

$$V_{Q_L} = V^{Q_L}_{35} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & c^{Q_L}_{35} & 0 & s^{Q_L}_{35} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -s^{Q_L}_{35} & 0 & c^{Q_L}_{35} \end{pmatrix},$$
$$V_{L_L} = V^L_{24} V^L_{14} V^L_{15} = \begin{pmatrix} c^L_{14}c^L_{15} & 0 & 0 & s^L_{14} & c^L_{14}s^L_{15} \\ -c^L_{15}s^L_{14}s^L_{24} & c^L_{24} & 0 & c^L_{14}s^L_{24} & -s^L_{14}s^L_{15}s^L_{24} \\ 0 & 0 & 1 & 0 & 0 \\ -c^L_{15}c^L_{24}s^L_{14} & -s^L_{24} & 0 & c^L_{14}c^L_{24} & -c^L_{24}s^L_{14}s^L_{15} \\ -s^L_{15} & 0 & 0 & 0 & c^L_{15} \end{pmatrix},$$
$$V_{e_R} = V^{e_R}_{24} V^{e_R}_{14} V^{e_R}_{15} = \begin{pmatrix} c^{e_R}_{14}c^{e_R}_{15} & 0 & 0 & s^{e_R}_{14} & c^{e_R}_{14}s^{e_R}_{15} \\ -c^{e_R}_{15}s^{e_R}_{14}s^{e_R}_{24} & c^{e_R}_{24} & 0 & c^{e_R}_{14}s^{e_R}_{24} & -s^{e_R}_{14}s^{e_R}_{15}s^{e_R}_{24} \\ 0 & 0 & 1 & 0 & 0 \\ -c^{e_R}_{15}c^{e_R}_{24}s^{e_R}_{14} & -s^{e_R}_{24} & 0 & c^{e_R}_{14}c^{e_R}_{24} & -c^{e_R}_{24}s^{e_R}_{14}s^{e_R}_{15} \\ -s^{e_R}_{15} & 0 & 0 & 0 & c^{e_R}_{15} \end{pmatrix}. \tag{28}$$

In addition we consider that only the fourth and fifth families have nonvanishing charges:

$$D_Q = \mathrm{diag}(0,0,0,q_{Q4},q_{Q5}), \qquad D_L = \mathrm{diag}(0,0,0,q_{L4},q_{L5}), \qquad D_e = \mathrm{diag}(0,0,0,q_{e4},q_{e5}). \tag{29}$$

Then, by replacing in Eq. (27) we find the following relations:

$$\tilde{D}'_Q = q_{Q5}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (s^Q_{35})^2 \end{pmatrix},$$
$$\tilde{D}'_L = \begin{pmatrix} q_{L_4}(s^L_{14})^2 + q_{L_5}(s^L_{15})^2(c^L_{14})^2 & c^L_{14}s^L_{14}s^L_{24}[q_{L_4} - q_{L_5}(s^L_{15})^2] & 0 \\ c^L_{14}s^L_{14}s^L_{24}[q_{L_4} - q_{L_5}(s^L_{15})^2] & q_{L_4}(s^L_{24})^2(c^L_{14})^2 + q_{L_5}(s^L_{15})^2(s^L_{14})^2(s^L_{24})^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
$$\tilde{D}'_e = \begin{pmatrix} q_{e_4}(s^e_{14})^2 + q_{e_5}(s^e_{15})^2(c^e_{14})^2 & c^e_{14}s^e_{14}s^e_{24}[q_{e_4} - q_{e_5}(s^e_{15})^2] & 0 \\ c^e_{14}s^e_{14}s^e_{24}[q_{e_4} - q_{e_5}(s^e_{15})^2] & q_{e_4}(s^e_{24})^2(c^e_{14})^2 + q_{e_5}(s^e_{15})^2(s^e_{14})^2(s^e_{24})^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (30)$$

so that the Z' couplings from Eq. (26) become

$$\begin{aligned} \mathcal{L}^{\text{gauge}}_{Z'} &= g' Z'_\lambda \{q_{Q5}(s^Q_{35})^2 \bar{Q}'_{L3}\gamma^\lambda Q'_{L3} + [q_{L_4}(s^L_{14})^2 + q_{L_5}(s^L_{15})^2(c^L_{14})^2]\bar{L}'_{L1}\gamma^\lambda L'_{L1}\} \\ &+ g' Z'_\lambda [q_{e_4}(s^e_{14})^2 + q_{e_5}(s^e_{15})^2(c^e_{14})^2]\bar{e}'_{R1}\gamma^\lambda e'_{R1} \\ &+ g' Z'_\lambda [q_{L_4}(s^L_{24})^2(c^L_{14})^2 + q_{L_5}(s^L_{15})^2(s^L_{14})^2(s^L_{24})^2]\bar{L}'_{L2}\gamma^\lambda L'_{L2} \\ &+ g' Z'_\lambda [q_{e_4}(s^e_{24})^2(c^e_{14})^2 + q_{e_5}(s^e_{15})^2(s^e_{14})^2(s^e_{24})^2]\bar{e}'_{R2}\gamma^\lambda e'_{R2} \\ &+ g' Z'_\lambda c^L_{14}s^L_{14}s^L_{24}[q_{L_4} - q_{L_5}(s^L_{15})^2](\bar{L}'_{L1}\gamma^\lambda L'_{L2} + \bar{L}'_{L2}\gamma^\lambda L'_{L1}) \\ &+ g' Z'_\lambda c^e_{14}s^e_{14}s^e_{24}[q_{e_4} - q_{e_5}(s^e_{15})^2](\bar{e}'_{R1}\gamma^\lambda e'_{R2} + \bar{e}'_{R2}\gamma^\lambda e'_{1R}) \end{aligned} \quad (31)$$

where the Z' couples only to the third family left-handed quark doublets $Q'_{L3} = (t'_L, b'_L)$ and the muons $L'_{L2} = (\nu'_{\mu L}, \mu'_L)$ and $e'_{R2} = \mu'_R$, where the primes indicate that these are the states before the Yukawa matrices are diagonalized.

Ignoring any charged lepton mixing amongst the three light families (to start with), this will lead the couplings,

$$\begin{aligned} \mathcal{L}^{\text{gauge}}_{Z'} &= Z'_\lambda (C_{b_L s_L}\bar{b}_L\gamma^\lambda s_L + C_{\mu_L\mu_L}\bar{\mu}_L\gamma^\lambda \mu_L + C_{\mu_R\mu_R}\bar{\mu}_R\gamma^\lambda\mu_R + C_{e_L e_L}\bar{e}_L\gamma^\lambda e_L \\ &+ C_{e_R e_R}\bar{e}_R\gamma^\lambda e_R + C_{\mu_L e_L}(\bar{\mu}_L\gamma^\lambda e_L + \bar{e}_L\gamma^\lambda\mu_L) + C_{\mu_R e_R})(\bar{\mu}_R\gamma^\lambda e_R + \bar{e}_R\gamma^\lambda\mu_R) + \ldots) \end{aligned} \quad (32)$$

with the different couplings of the Z' gauge bosons with the charged leptonic fields appearing in Eq. (33) given by

$$\begin{aligned} C_{b_L s_L} &\equiv g' q_{Q5}(s^Q_{35})^2 (V'^\dagger_{dL})_{32}, \qquad C_{\mu_L\mu_L} \equiv g'[q_{L_4}(s^L_{24})^2(c^L_{14})^2 + q_{L_5}(s^L_{15})^2(s^L_{14})^2(s^L_{24})^2] \\ C_{\mu_R\mu_R} &\equiv g'[q_{e_4}(s^e_{24})^2(c^e_{14})^2 + q_{e_5}(s^e_{15})^2(s^e_{14})^2(s^e_{24})^2], \\ C_{e_L e_L} &\equiv g'[q_{L_4}(s^L_{14})^2 + q_{L_5}(s^L_{15})^2(c^L_{14})^2], \qquad C_{e_R e_R} \equiv g'[q_{e_4}(s^e_{14})^2 + q_{e_5}(s^e_{15})^2(c^e_{14})^2] \\ C_{\mu_L e_L} &\equiv g'\{c^L_{14}s^L_{14}s^L_{24}[q_{L_4} - q_{L_5}(s^L_{15})^2] + [q_{L_4}(s^L_{24})^2(c^L_{14})^2 + q_{L_5}(s^L_{15})^2(s^L_{14})^2(s^L_{24})^2]s^L_{12}\}, \\ C_{\mu_R e_R} &\equiv g'\{c^e_{14}s^e_{14}s^e_{24}[q_{e_4} - q_{e_5}(s^e_{15})^2] + [q_{e_4}(s^e_{24})^2(c^e_{14})^2 + q_{e_5}(s^e_{15})^2(s^e_{14})^2(s^e_{24})^2]s^e_{12}\}, \end{aligned} \quad (33)$$

where the mixing parameters $s^{L,e}_{12}$ appear after expressing the leptonic fields in the interaction basis in terms of the leptonic fields in the mass eigenstates, considering, for the sake of simplicity, only the mixing in the 1-2 plane. In addition, we have expanded the quark primed fields in terms of mass eigenstates as follows:

$$b'_L = (V'^\dagger_{dL})_{31} d_L + (V'^\dagger_{dL})_{32} s_L + (V'^\dagger_{dL})_{33} b_L \quad (34)$$

and assumed from the hierarchy of the Cabibbo-Kobayashi-Maskawa matrix that

$$|(V'^\dagger_{dL})_{31}|^2 \ll |(V'^\dagger_{dL})_{32}|^2 \ll (V'^\dagger_{dL})^2_{33} \approx 1. \quad (35)$$

Then the Z' exchange generates the effective operators, as in Eq. (1), where the operator corresponds to $C^{NP}_{9\mu} = -C^{NP}_{10\mu}$. For the sake of simplicity, we ignore the contribution of the right-handed muon operator and we neglect the contribution arising from the mixing between the first and fourth generations of charged leptons, i.e., we set $\theta^{(L,R)}_{14} = 0$. Let us note that we are considering a scenario where the fifth family of vectorlike fermions only couples with the third generation of SM quarks as well as with the first generation of charged leptons, whereas the fourth family will only couple with the second generation of SM charged leptons; thus, we are assuming that only

$\theta_{35}^{Q_L}$, $\theta_{24}^{L_L}$, $\theta_{24}^{e_R}$, $\theta_{15}^{L_L}$, $\theta_{15}^{e_R}$ are nonzero with all other mixing angles being zero (see Sec. IV for a justification of those assumptions in terms of symmetries).

To explain the R_K and R_{K^*} anomalies, we require the coefficient to have the correct sign and magnitude, as discussed in Eq. (3), leading to

$$|C_{b_L s_L} C_{\mu_L \mu_L}| \approx 10^{-3} \left(\frac{M'_Z}{1\ \mathrm{TeV}}\right)^2. \tag{36}$$

There are important flavor violating processes such as $B_s - \bar{B}_s$ mixing which can rule out models, due to the Z' coupling to bs. As discussed for example in [27], this leads to the constraint,

$$|C_{b_L s_L}|^2 \lesssim 2 \times 10^{-5} \left(\frac{M'_Z}{1\ \mathrm{TeV}}\right)^2. \tag{37}$$

From Eqs. (36) and (37) we find the constraint,

$$\frac{|C_{b_L s_L}|}{|C_{\mu_L \mu_L}|} \lesssim \frac{1}{50}. \tag{38}$$

From Eq. (33), this implies

$$\frac{|q_{Q5}(s_{35}^Q)^2 (V'^{\dagger}_{dL})_{32}|}{|q_{L4}(s_{24}^L)^2|} \lesssim \frac{1}{50}. \tag{39}$$

This is easily satisfied, since for example if $(V'^{\dagger}_{dL})_{32} \sim V_{ts} \sim \lambda^2 \sim (1/5)^2 \sim 1/25$ then this by itself is almost sufficient to satisfy the constraint.

For example, if we saturate the bound in Eq. (37), then Eq. (36) implies

$$|C_{\mu_L \mu_L}| = g' q_{L4} (s_{24}^L)^2 \approx 0.22 \left(\frac{M'_Z}{1\ \mathrm{TeV}}\right). \tag{40}$$

This shows that the mixing angle θ_{24}^L cannot be too small. Note that the LHC limits on the Z' mass are very weak since it does not couple to light quarks at leading order, and its coupling to strange quarks is suppressed by a factor of $(V'^{\dagger}_{dL})_{32}^2$.

For a more detailed discussion of the phenomenological constraints on this particular model arising from both flavor violating processes such as $B_s - \bar{B}_s$ mixing and LHC limits on the Z' mass, see [28]. Furthermore, note that the model has very small FCNC in the Z couplings as explained in Ref. [25]. In addition, the loop effects of fermions charged under both the SM and extra $U(1)'$ groups will generate a small $Z - Z'$ mixing of the order of $\frac{M_T^2 (s_{35}^Q)^2}{16\pi^2}$, with M_T being the mass of the fifth family of quarks. Considering $M_T \approx M_{Z'}$, the $Z - Z'$ mixing angle will be of the order of 6×10^{-3}, thus leading to suppressed FCNC in the Z couplings.

There are other important constraints due to LFV processes such as $\mu \to eee$ as recently discussed for example in [27].[1] However, as discussed there, violations of lepton universality do not always lead to lepton-flavor violation: it depends on the mixing angles $\theta_{12}^{L,R}$ arising from the left-handed (L) and right-handed (R) rotations which diagonalize the charged lepton Yukawa matrix. This leads to a $Z'\mu e$ flavor changing coupling suppressed by $\theta_{12}^{L,R}$ and a $Z'ee$ flavor conserving coupling to electrons suppressed by $(\theta_{12}^{L,R})^2$. We may estimate the branching ratios for $\mu \to eee$ by taking the ratio of the Z' exchange diagram squared to the W exchange diagram squared,

$$\mathrm{Br}(\mu_L \to e_L e_L e_L) \approx (C_{\mu_L \mu_L})^4 (\theta_{12}^L)^6 \left(\frac{M_W}{M_{Z'}}\right)^4 \tag{41}$$

$$\mathrm{Br}(\mu_R \to e_R e_R e_R) \approx (C_{\mu_R \mu_R})^4 (\theta_{12}^R)^6 \left(\frac{M_W}{M_{Z'}}\right)^4. \tag{42}$$

For typical charged lepton mixing angles such as $\theta_{12}^{L,R} \sim \lambda/3 \sim 0.07$, the coefficient in Eq. (40) will lead to branching ratios such as

$$\mathrm{Br}(\mu_L \to e_L e_L e_L) \approx (0.22)^4 (0.07)^6 (0.08)^4 \approx 10^{-14} \tag{43}$$

below the current experimental limit of $\mathrm{Br}(\mu \to eee) \lesssim 10^{-12}$ but within the range of future experiments.

Although the above constraints may be satisfied, our current framework can lead to the LFV decay $\mu \to e\gamma$, which is only induced by the $\theta_{14}^{L,R}$ mixing angles in the case of a diagonal SM charged lepton mass matrix, as shown in Appendix B. Thus, to avoid all LFV decays and at the same time to generate the correct value of the electron mass, we need to also suppress the $\theta_{14}^{L,R}$ mixing angles while at the same time correcting the charged lepton masses. This can be achieved by adding a fifth vectorlike family as discussed in the next section.

Finally, we remark that the models discussed in this paper will be supersymmetric (SUSY). It is well known that SUSY must be broken in realistic models, leading to additional sources of flavor violation coming from the SUSY breaking sector via SUSY loop contributions. These have been recently studied for a class of SUSY $SU(5) \times A_4$ models [104] which includes the type of model described in Sec. IV. Interestingly, according to the model independent analysis based on the region of SUSY parameter space consistent with smuon assisted dark matter [104], the most constraining SUSY loop induced flavor observables are also $\mu \to eee$ and $\mu \to e\gamma$, which are the same modes as discussed above. Such lepton-flavor violating decays could

[1]We do not consider $\mu - e$ conversion since the Z' does not couple to light quarks at leading order.

therefore be mediated by either SUSY loops or by a Z' exchange in this model.

III. $SU(5)$ WITH A VECTOR SECTOR

We now suppose that the SM with a vector sector considered in the previous subsection descends from a supersymmetric $SU(5)$ GUT. The three chiral families result from three families of F_i transforming as $\bar{\mathbf{5}}$, and T_i transforming as $\mathbf{10}$, which all carry zero $U(1)'$ charges. The Higgs H_u and H_d arise from $\mathbf{5}$ and $\bar{\mathbf{5}}$ representations, after doublet-triplet splitting (which we do not address). This results in the $SU(5)$ Yukawa relation, $Y_e = Y_d^T$ in the usual way.

Now we consider adding the previous vector sector to the $SU(5)$ GUT. In order to violate the $SU(5)$ relation $Y_e = Y_d^T$ we will suppose that the fourth vectorlike family at low energies results from multiple $\bar{\mathbf{5}} + \mathbf{5}$ and $\mathbf{10} + \overline{\mathbf{10}}$ at the GUT scale, where each pair has equal and opposite $U(1)'$ charges, but which differ each from another pair. Similar arguments apply for the origin of the fifth family. At low energies below the GUT scale, only the matter content of two vectorlike families survives with various $U(1)'$ charges, similarly as in Table I, with the remaining components of the multiple $\bar{\mathbf{5}} + \mathbf{5}$ and $\mathbf{10} + \overline{\mathbf{10}}$ states having GUT scale masses. Below the GUT scale, the model in Table II leads to the SM plus vector sector in Table I. Thus, the $SU(5)$ plus vector sector can explain the muon anomalies exactly like we discussed in the previous section (see in particular Sec. II E).

We now focus on the $SU(5)$ Yukawa relation, $Y_d = Y_e^T$ and show that it is violated by the $SU(5)$ plus mixing with the vector sector. At the GUT scale, we identify $Y_e = y_{ij}^e$ and $Y_d = y_{ij}^d$ in Eq. (4).

The Yukawa terms in $SU(5)$ may be written as

$$y_{ij}^u H_{\mathbf{5}} T_i T_j + y_{ij}^\nu H_{\mathbf{5}} F_i \nu_j^c + y_{ij}^d H_{\bar{\mathbf{5}}} T_i F_j. \tag{44}$$

TABLE II. The $SU(5)$ model considered in this paper. The indices $i = 1, 2, 3$ while $a = 4, 5, \ldots$.

	Representation/Charge	
Field	$SU(5)$	$U(1)'$
F_i	$\bar{\mathbf{5}}$	0
T_i	$\mathbf{10}$	0
H_u	$\mathbf{5}$	0
H_d	$\bar{\mathbf{5}}$	0
F_a	$\bar{\mathbf{5}}$	q_{Fa}
$\bar{F}_a$	$\mathbf{5}$	$-q_{Fa}$
T_a	$\mathbf{10}$	q_{Ta}
$\bar{T}_a$	$\overline{\mathbf{10}}$	$-q_{Ta}$
ϕ_{Fa}	$\mathbf{1}$	q_{Fa}
ϕ_{Ta}	$\mathbf{1}$	q_{Ta}

These give SM Yukawa terms,

$$y_{ij}^u H_u Q_i u_j^c + y_{ij}^\nu H_u L_i \nu_j^c + y_{ij}^d (H_d Q_i d_j^c + H_d e_i^c L_j). \tag{45}$$

From this equation we identify the charged lepton Yukawa matrix as

$$Y_e = Y_d^T, \tag{46}$$

at the GUT scale. This means that after RG effects are considered we have at low energy,

$$Y_e \approx \frac{1}{3} Y_d^T, \tag{47}$$

where QCD corrections lead to an overall scaling factor of about 3 for the quark Yukawa couplings as compared to those of the leptons. This implies that

$$y_\tau = \frac{1}{3} y_b, \qquad y_\mu = \frac{1}{3} y_s, \qquad y_e = \frac{1}{3} y_d. \tag{48}$$

Though successful for the third family, this fails for the first and second families.

Georgi and Jarlskog [216] proposed that the (2,2) matrix entry of the Yukawa matrices may be given by

$$y_{22}^d H_{\overline{\mathbf{45}}} T_2 F_2, \tag{49}$$

involving a Higgs field $H_{\overline{\mathbf{45}}}$, where H_d is the light linear combination of the electroweak doublets contained in $H_{\bar{\mathbf{5}}}$ and $H_{\overline{\mathbf{45}}}$. This term reduces to

$$y_{22}^d (H_d Q_2 d_2^c - 3 H_d e_2^c L_2), \tag{50}$$

where the factor of -3 is a Clebsch-Gordan coefficient. Assuming a zero Yukawa element (texture) in the (1,1) position, and symmetric and hierarchical Yukawa matrices, this leads to the relations at low energy,

$$y_\tau = \frac{1}{3} y_b, \qquad y_\mu = y_s, \qquad y_e = \frac{1}{9} y_d, \tag{51}$$

which are approximately consistent with the low energy masses.

In our approach we do not wish to consider such large Higgs representations to modify the Yukawa matrices at the GUT scale. Instead we note that these are not the physical Yukawa matrices due to mixing with the fourth family. By following our discussion given in Sec. II D 1 we find that the mixing with the fourth family may enhance y'^e_{22} compared to its original value y_{22}^e,

$$\begin{aligned} y'^e_{22} = {} & y_{22}^e \cos\theta_{24}^{L_L} \cos\theta_{24}^{e_R} + y_{24}^e \cos\theta_{24}^{L_L} \sin\theta_{24}^{e_R} \\ & + y_{42}^e \sin\theta_{24}^{L_L} \cos\theta_{24}^{e_R} + y_{44}^e \sin\theta_{24}^{L_L} \sin\theta_{24}^{e_R} \equiv f y_{22}^e, \end{aligned} \tag{52}$$

which may be a rather large correction if $y^e_{41} \gg y^e_{22}$, even for small angle rotations. We can easily achieve an enhancement by a factor of 3, or indeed any other factor f. Such an enhancement is not present in y'^d_{22} due to our choice of zero mixing angles $\theta^{Q_L}_{24} = \theta^{d_R}_{24} = 0$.

Assuming as before, a zero Yukawa element (texture) in the (1,1) position, and symmetric and hierarchical Yukawa matrices, Eq. (52) leads to the relations at low energy,

$$y_\tau = \frac{1}{3} y_b, \qquad y_\mu = \frac{f}{3} y_s, \qquad y_e = \frac{1}{3f} y_d. \tag{53}$$

These relations are approximately consistent with the low energy masses for $f \approx 2$–3.

It is worth noting that the requirement for enhancing y'^e_{22} but not y'^d_{22} relies on the assumption that $\theta^{L_L}_{24} \neq 0$ or $\theta^{e_R}_{24} \neq 0$ but $\theta^{Q_L}_{24} \theta^{d_R}_{24} = 0$. If we had assumed that the vectorlike family originated from a single $\bar{\mathbf{5}} + \mathbf{5}$ and $\mathbf{10} + \overline{\mathbf{10}}$ representation, denoted as $F_4 + \bar{F}_4$ and $T_4 + \bar{T}_4$, then this would constrain the choice of charges for the vectorlike fourth family to be $\pm q_{F4}$ for the states L_{L4} and d_{R4}, together with $\pm q_{T4}$ for the states Q_{L4}, u_{R4}, and e_{R4}, and their vector partners. In particular, the vectorlike family in Table I would have constrained charges $q_{L_4} = -q_{d4}$ and also $q_{Q_4} = -q_{u4} = -q_{e4}$. This would eventually have led to the constraint on the fourth family mixing that $V_{L_L} = V^\dagger_{d_R}$. Similarly it would have implied that $V_{Q_L} = V^\dagger_{u_R} = V^\dagger_{e_R}$. These relations would imply from Eq. (16) that the $SU(5)$ relation at low energy would be preserved, $Y'_e \approx \frac{1}{3} Y'^T_d$. Furthermore, for enhancing y'^e_{11}, we require $\theta^{L_L}_{15} \neq 0$ or $\theta^{e_R}_{15} \neq 0$ but $\theta^{Q_L}_{15} \theta^{d_R}_{15} = 0$.

In summary, we need $\theta^{L_L}_{24} \neq 0$ or $\theta^{e_R}_{24} \neq 0$ and $\theta^{L_L}_{15} \neq 0$ or $\theta^{e_R}_{15} \neq 0$ but $\theta^{Q_L}_{24} \theta^{d_R}_{24} = 0$ and $\theta^{Q_L}_{15} \theta^{d_R}_{15} = 0$. This can be done if the fourth and fifth vectorlike families at low energies result from multiple $\bar{\mathbf{5}} + \mathbf{5}$ and $\mathbf{10} + \overline{\mathbf{10}}$ at the GUT scale, where each pair has equal and opposite $U(1)'$ charges, but which differ each from another pair, as assumed in Table II. Assuming this, then we have shown that the $SU(5)$ theory can account for the muon anomalies $R_{K^{(*)}}$ and obtain $Y_e \neq \frac{1}{3} Y^T_d$ without the need for higher Higgs representations.

The above discussion assumes that there is a zero Yukawa element (texture) in the (1,1) position, with a symmetric and hierarchical charged lepton Yukawa matrix. If, on the other hand, we would assume that the charged lepton Yukawa matrix is diagonal, then we would need to assume corrections as in both Eqs. (20) and (21) in order to account for the correct low energy mass relations in Eq. (51). We will see an example of such a model in the next section.

IV. $SU(5) \times A_4$ WITH A VECTOR SECTOR

In this section we will extend the particle content of our supersymmetric model by adding fourth and fifth generations of fermions in the $\bar{\mathbf{5}}$ and $\mathbf{10}$ irreps of $SU(5)$, two right-handed Majorana neutrinos, i.e., ν_{1R}, ν_{2R} and several $SU(5)$ singlet scalar fields. In addition, we will implement the A_4 family symmetry, which will be supplemented by the $Z_3 \times Z_7$ discrete group. These modifications in our simplified version of our model are done in order to get viable and predictive textures for the fermion sector, which will allow us to successfully describe the current pattern of SM fermion masses and mixing angles, as we will show later in this section.

The particle content of the model and the field assignments under the $SU(5) \times U(1)' \times A_4 \times Z_3 \times Z_7$ group are shown in Table III. Let us note, that we use the A_4 family symmetry, since A_4 is the smallest discrete group having a three-dimensional irreducible representation and three different one-dimensional irreducible representations, which

TABLE III. The $SU(5) \times A_4$ model considered in this paper. Notice that we included the field $H^{(3)}_d$ with the same quantum numbers of $H^{(2)}_d$ in order to fulfill the anomaly cancellation condition without introducing extra mixing terms between the light and heavy vectorlike fermions.

	Representation/Charge				
Field	$SU(5)$	$U(1)'$	A_4	Z_3	Z_7
F	$\bar{\mathbf{5}}$	0	**3**	0	0
T_1	**10**	0	**1**	2	3
T_2	**10**	0	**1**	1	2
T_3	**10**	0	**1**	0	0
F_4	$\bar{\mathbf{5}}$	q_{F_4}	**1**	−1	−2
$\bar{F}_4$	**5**	$-q_{F_4}$	**1**	1	2
F_5	$\bar{\mathbf{5}}$	q_{F_5}	**1**	−2	−3
$\bar{F}_5$	**5**	$-q_{F_5}$	**1**	2	3
T_4	**10**	q_{T_4}	**1**	1	2
T_5	**10**	q_{T_4}	**1**	0	0
$\bar{T}_4$	$\overline{\mathbf{10}}$	$-q_{T_4}$	**1**	−1	−2
$\bar{T}_5$	$\overline{\mathbf{10}}$	$-q_{T_5}$	**1**	0	0
ν_{1R}	**1**	0	**1**	0	−3
ν_{2R}	**1**	0	**1**	0	0
$H^{(1)}_u$	**5**	0	**1**	2	0
$H^{(2)}_u$	**5**	0	**1**	1	0
$H^{(3)}_u$	**5**	0	**1**	0	0
$H^{(1)}_d$	$\bar{\mathbf{5}}$	0	**1**	0	0
$H^{(2)}_d$	$\bar{\mathbf{5}}$	$-q_{F_4}$	**1**	0	5
$H^{(3)}_d$	$\bar{\mathbf{5}}$	$-q_{F_4}$	**1**	0	5
ϕ_{F_4}	**1**	q_{F_4}	**3**	−1	−2
ϕ_{F_5}	**1**	q_{F_5}	**3**	−2	−3
ϕ_T	**1**	q_{T_5}	**1**	0	0
σ	**1**	0	**1**	0	−1
ξ_e	**1**	0	**3**	−2	−3
ξ_μ	**1**	0	**3**	−1	−2
ξ_τ	**1**	0	**3**	0	0
η_1	**1**	0	**3**	0	3
η_2	**1**	0	**3**	0	0

allows us to naturally accommodate the three fermion families. Specifically, we grouped the three generations of SM fermionic $\bar{\mathbf{5}}_i \approx F_i$ ($i = 1, 2, 3$) irreps of $SU(5)$ in an A_4 triplet, whereas the three generations of SM fermionic $\mathbf{10}_i \sim T_i$ ($i = 1, 2, 3$) irreps of $SU(5)$ are assigned into A_4 trivial singlets. The exotic fermionic fields are also assigned into A_4 trivial singlets. As a consequence of the aforementioned fermion assignments under the $A_4 \times Z_3 \times Z_7$ discrete group, three A_4 triplets, $SU(5)$ scalar singlets are needed to provide the masses for the SM down type quarks and charged leptons. In addition, we need two extra A_4 scalar triplets to generate a viable and predictive light active neutrino mass matrix as well as three A_4 triplets, and $SU(5)$ scalar quintuplets, with different Z_3 charges, are required to generate the SM up type quark masses and quark mixing parameters. Thus, in view of the above, the $SU(5)$ singlet scalar fields neutral under $U(1)'$ are accommodated into five A_4 triplets, i.e., ξ_e, ξ_μ, ξ_τ, η_1, η_2, and one A_4 trivial singlet, i.e., σ. Out of the A_4 scalar triplets, only η_1 and η_2 will participate in the neutrino Yukawa interactions, whereas the remaining A_4 triplets will appear in the charged lepton and down type quark Yukawa terms. That separation of the A_4 scalar triplets, resulting from the $Z_3 \times Z_7$ discrete symmetry, allows us to treat the neutrino and the charged fermion sectors independently.

In addition, the Z_3 symmetry allows us to have a SM charged lepton mass matrix diagonal, which is crucial to completely suppress the lepton-flavor violating decays. The Z_7 symmetry give rises to the hierarchical structure of the charged fermion mass matrices that yields the observed pattern of charged fermion masses and quark mixing angles. Furthermore, we introduce two right-handed Majorana neutrinos, i.e., ν_{1R}, ν_{2R}, in order to implement a realistic type-I seesaw mechanism at tree level for the generation of the light active neutrino masses. Having only one right-handed Majorana neutrino would lead to two massless active neutrinos, which is obviously in contradiction with the experimental data on neutrino oscillations. On the other hand, in order to get predictive SM fermion mass matrices consistent with low energy fermion flavor data, we assume the following VEV pattern for the A_4 triplet $SU(5)$ singlet scalars:

$$
\begin{aligned}
&\langle\xi_e\rangle = v_\xi^{(e)}(1,0,0), \qquad \langle\xi_\mu\rangle = v_\xi^{(\mu)}(0,1,0), \qquad \langle\xi_\tau\rangle = v_\xi^{(\tau)}(0,0,1),\\
&\langle\eta_1\rangle = v_{\eta_1}(0,1,1), \qquad \langle\eta_2\rangle = v_{\eta_2}e^{i\frac{\phi_\nu}{2}}(1,3,1), \qquad \langle\phi_{F_4}\rangle = v_{\phi_{F_4}}(0,1,0),\\
&\langle\phi_{F_5}\rangle = v_{\phi_{F_5}}(1,0,0),
\end{aligned} \tag{54}
$$

where the complex phases ϕ_ν are introduced in the VEV pattern of the A_4 triplet scalar η_2 in order to successfully reproduce the experimental values of the leptonic mixing angles. Since the breaking of the $A_4 \times Z_3 \times Z_7$ discrete group generates the hierarchy among charged fermion masses and quark mixing angles and in order to relate the quark masses with the quark mixing parameters, we set the VEVs of the $SU(5)$ singlet scalars σ, ξ_e, ξ_μ, ξ_τ, η_s ($s = 1, 2$), ϕ_{F_4}, and ϕ_{F_5} with respect to the Wolfenstein parameter $\lambda = 0.225$ and the model cutoff Λ, as follows:

$$
v_{\phi_{F_4}} \sim v_{\phi_{F_5}} \ll v_\xi^{(e)} \sim \lambda^7\Lambda \ll v_\xi^{(\mu)} \sim \lambda^5\Lambda \ll v_\xi^{(\tau)} \sim \lambda^3\Lambda < v_\sigma \sim v_{\eta_s} \sim \lambda\Lambda, \tag{55}
$$

where $s = 1, 2$. The aforementioned VEV patterns are consistent with the scalar potential minimization equations for a large region parameter space. In particular, the VEV pattern of the A_4 scalar triplets η_1 and η_2 that participate in the neutrino Yukawa interactions have been derived for the first time in Ref. [74] in the framework of an A_4 flavor model. Assuming that the scale of breaking of the discrete symmetries is of the order of the GUT scale $\Lambda_{\mathrm{GUT}} \approx 10^{16}$ GeV, from Eq. (55) we find for the model cutoff the estimate $\Lambda \approx 4.4 \times 10^{16}$ GeV.

With the above particle content, the following Yukawa terms invariant under the group $SU(5) \times U(1)' \times A_4 \times Z_3 \times Z_7$ arise:

$$
\begin{aligned}
-\mathcal{L}_{\mathrm{Y}} = {}& y_{11}^{(u)}T_1T_1H_u^{(1)}\frac{\sigma^6}{\Lambda^6} + y_{12}^{(u)}T_1T_2H_u^{(3)}\frac{\sigma^5}{\Lambda^5} + y_{22}^{(u)}T_2T_2H_u^{(2)}\frac{\sigma^4}{\Lambda^4} + y_{13}^{(u)}T_1T_3H_u^{(2)}\frac{\sigma^3}{\Lambda^3} + y_{23}^{(u)}T_2T_3H_u^{(1)}\frac{\sigma^2}{\Lambda^2} + y_{33}^{(u)}T_3T_3H_u^{(3)}\\
&+ y_{11}^{(d)}T_1FH_d^{(1)}\frac{\xi_e}{\Lambda} + y_{22}^{(d)}T_2FH_d^{(1)}\frac{\xi_\mu}{\Lambda} + y_{33}^{(d)}T_3FH_d^{(1)}\frac{\xi_\tau}{\Lambda} + y_{24}^{(F)}T_2F_4H_d^{(2)}\frac{\sigma^5}{\Lambda^5} + z_{24}^{(F)}T_2F_4H_d^{(3)}\frac{\sigma^5}{\Lambda^5} + x_{24}^{(F)}\bar{F}_4F\phi_{F_4}\\
&+ x_{15}^{(F)}\bar{F}_5F\phi_{F_5} + x_{35}^{(T)}\bar{T}_5T_3\phi_T^* + \sum_{a=4}^{5}M_{F_a}\bar{F}_aF_a + \sum_{a=4}^{5}M_{T_a}\bar{T}_aT_a + x_{45}^{(F)}\bar{F}_4F_5\frac{\sigma^2\phi_{F_4}\phi_{F_5}^*}{\Lambda^3} + x_{54}^{(F)}\bar{F}_5F_4\frac{\sigma^2\phi_{F_5}\phi_{F_4}^*}{\Lambda^3}\\
&+ \sum_{s=1}^{2}y_s^{(\nu)}FH_u^{(3)}\nu_{sR}\frac{\eta_s}{\Lambda} + x_1^{(\nu)}\nu_{1R}\overline{\nu_{1R}^C}\sigma + M^{(\nu)}\nu_{2R}\overline{\nu_{2R}^C},
\end{aligned} \tag{56}
$$

where the Yukawa couplings are $\mathcal{O}(1)$ dimensionless parameters, assumed to be real for the sake of simplicity, whereas M_{F_a}, M_{T_a} $(a = 4, 5)$ and $M^{(\nu)}$ are dimensionful parameters.

On the other hand, it is worth mentioning that the lightest of the physical neutral scalar states of $H_u^{(1)}$, $H_u^{(2)}$, $H_u^{(3)}$, $H_d^{(1)}$, $H_d^{(2)}$, and $H_d^{(3)}$ is the SM-like 125 GeV Higgs discovered at the LHC. As clearly seen from Eq. (56), the top quark mass mainly arises from $H_u^{(3)}$. Consequently, the dominant contribution to the SM-like 125 GeV Higgs mainly arises from the CP even neutral state of the $SU(2)$ doublet part of $H_u^{(3)}$. In addition, let us note that the scalar potential of our model has many free parameters, which allows us freedom to assume that the remaining scalars are heavy and outside the LHC reach. In addition, the loop effects of the heavy scalars contributing to precision observables can be suppressed by making an appropriate choice of the free parameters in the scalar potential. These adjustments do not affect the physical observables in the quark and lepton sectors, which are determined mainly by the Yukawa couplings.

From the Yukawa interactions given above, it follows that the SM mass matrices for quarks and charged leptons are given by

$$M_U = \begin{pmatrix} a_{11}^{(u)}\lambda^6 & a_{12}^{(u)}\lambda^5 & a_{13}^{(u)}\lambda^3 \\ a_{12}^{(u)}\lambda^5 & a_{22}^{(u)}\lambda^4 & a_{23}^{(u)}\lambda^2 \\ a_{13}^{(u)}\lambda^3 & a_{23}^{(u)}\lambda^2 & a_{33}^{(u)} \end{pmatrix}\frac{v}{\sqrt{2}},$$

$$M_D = \begin{pmatrix} a_{11}^{(d)}\lambda^7 & 0 & 0 \\ 0 & a_{22}^{(d)}\lambda^5 & 0 \\ 0 & 0 & a_{33}^{(d)}\lambda^3 \end{pmatrix}\frac{v}{\sqrt{2}},$$

$$M_l = \begin{pmatrix} a_{11}^{(l)}\lambda^7 & 0 & 0 \\ 0 & a_{22}^{(l)}\lambda^5 & 0 \\ 0 & 0 & a_{33}^{(l)}\lambda^3 \end{pmatrix}\frac{v}{\sqrt{2}},$$

$$a_{ij}^{(l)} \approx \frac{\kappa}{3}[1 + \delta_{i2}\delta_{j2}(f_2 - 1) + \delta_{i1}\delta_{j1}(f_1 - 1)]a_{ji}^{(d)}, \qquad (57)$$

where $v = 246$ GeV is the electroweak symmetry breaking scale, the factor of 3 includes the QCD corrections, the κ parameter is introduced to account for the threshold corrections to the down type quarks and charged lepton mass matrices [217], and the factors f_1 and f_2 consider the effects of the mixings with the fourth and fifth families, respectively, of charged leptons as in Eqs. (20) and (21). Let us note that we have assumed, as follows from an extension of our discussion given in Sec. II D 1, with appropriate modifications of Eqs. (21) and (20), that the factors f_1 and f_2 are given by

$$f_1 \approx \cos\theta_{15}^L, \qquad \tan\theta_{15}^L \approx -\frac{x_{15}^{(F)} v_{\phi_{F_5}}}{M_{F_5}}, \qquad (58)$$

$$f_2 \approx \cos\theta_{24}^L + y_{24}^{(F)}\frac{\sqrt{2}v_{H_d^{(2)}}}{v}\sin\theta_{24}^L, \quad \tan\theta_{24}^L \approx -\frac{x_{24}^{(F)} v_{\phi_{F_4}}}{M_{F_4}}. \qquad (59)$$

Then, considering $M_{F_4} \sim M_{F_5} \sim v_{\phi_{F_4}} \sim v_{\phi_{F_5}} \sim \mathcal{O}(1)$ TeV and $x_{15}^{(F)} \sim x_{24}^{(F)} \sim \mathcal{O}(1)$, we find that factors f_1 and f_2 will be of order unity, which is crucial to generate the right values of the electron and muon masses without spoiling our predictions for the SM down type quark mass spectrum.

The mechanism described above works because the fifth generation of vectorlike leptons only mixes with the first family of charged leptons. Thus, as a result of this mixing, the 11 entry of the charged lepton mass matrix will receive a correction proportional to $\sin\theta_{15}^{L_L}\sin\theta_{15}^{e_R}$ instead of the quantity $\theta_{14}^{L_L}\theta_{14}^{e_R}$ shown in Eq. (21), thus yielding the right value of the electron mass (without spoiling the predictions of the down quark mass) and at the same time preventing the $\mu \to e\gamma$ decay. Thus, the present flavor model has the features $\theta_{14}^{L,R} = \theta_{25}^{L,R} = 0$, $\theta_{15}^R \approx 0$, $\theta_{24}^R \approx 0$ and $\theta_{15}^L \neq 0$, and $\theta_{24}^L \neq 0$. In this model, due to the discrete symmetry assignments, the mass matrices for SM down type quarks and charged leptons are diagonal and the right values of the electron and muon masses arise from the θ_{15}^L and θ_{24}^L mixing angles, respectively, and the mixing between the fourth and fifth generation of vectorlike leptons is very tiny, thus allowing us to have a realistic SM fermion mass spectrum and strongly suppressing the $\mu \to e\gamma$ rate.

Additionally, as seen from the Yukawa terms given in Eq. (56), considering $v_{\phi_{F_4}} \approx v_{\phi_{F_5}} \approx \mathcal{O}(1)$ TeV and assuming that the scale of breaking of the discrete symmetries is of the order of the GUT scale $\Lambda_{\mathrm{GUT}} \approx 10^{16}$ GeV, we find that for dimensionless coupling of order unity, the mass mixing term between the fourth and the fifth generations of charged fermions is of the order of 10^{-10} GeV. Considering fourth and the fifth generations of charged leptons contained in the 5, $\bar{5}$ $SU(5)$ representations have masses around $\mathcal{O}(1)$ TeV, we find a mixing angle between these fermions to be $\theta_{45} \approx 10^{-13}$, which implies that branching fractions for the charged lepton-flavor violating decays induced by this mixing will be very tiny and well below their corresponding experimentally upper bound. Furthermore, as seen from Eq. (57) and Yukawa terms $x_{24}^{(F)}\bar{F}_4 F\phi_{F_4}$, $x_{15}^{(F)}\bar{F}_5 F\phi_{F_5}$, $y_{24}^{(F)}T_2F_4H_d^{(2)}\frac{\sigma^5}{\Lambda^5}$, and $z_{24}^{(F)}T_2F_4H_d^{(3)}\frac{\sigma^5}{\Lambda^5}$ shown in Eq. (56), the SM charged lepton mass matrix is diagonal and $\theta_{24}^L \neq 0$, $\theta_{15}^L \neq 0$, respectively, whereas $\theta_{14}^{L,R} = \theta_{25}^{L,R} = 0$, $\theta_{15}^R \approx 0$, $\theta_{24}^R \approx 0$, thus preventing contributions to the $\mu \to e\gamma$ decay rate arising from these mixing angles, as follows from Appendix B. Besides that, it is worth mentioning that we are considering incomplete $SU(5)$ multiplets for the fourth and fifth generations of fermions, which can be justified by assuming that the exotic down type quark fields contained in the 5 and $\bar{5}$ irreps of $SU(5)$, F_4, F_5, $\bar{F}_4$, $\bar{F}_5$ as well as the charged exotic leptons and down type quarks included in the 10, $\overline{10}$ irreps of $SU(5)T_4$, T_5, $\bar{T}_4$, $\bar{T}_5$, have masses much larger than the TeV scale, whereas the remaining fermions inside these representations do acquire TeV scale masses. That assumption will guarantee that $\theta^Q_{24} = \theta^d_{24} = \theta^Q_{15} = \theta^d_{15} = \theta^d_{35} = \theta^e_{35} = 0$, $\theta^e_{15} \approx 0$, $\theta^e_{24} \approx 0$ despite the fact that $\theta^L_{24} \neq 0$, $\theta^L_{15} \neq 0$, and $\theta^u_{35} \neq 0$.

Since we assume that the dimensionless Yukawa couplings appearing in Eq. (56) are roughly of the same order of magnitude and we consider the VEVs $v_{H_u^{(1)}}$, $v_{H_u^{(2)}}$, $v_{H_u^{(3)}}$, $v_{H_d^{(1)}}$, and $v_{H_d^{(2)}}$ of the order of the electroweak scale $v \simeq 246$ GeV, the hierarchy of charged fermion masses and quark mixing matrix elements arises from the breaking of the $A_4 \times Z_3 \times Z_7$ symmetry. Let us note that despite the fact that the running of Yukawa couplings from the GUT scale up to the electroweak scale is not explicitly included in our calculations, our effective Yukawa couplings can accommodate for the renormalization groups effects, since these effective Yukawa couplings depend not only on the Yukawa couplings but also on the VEVs of the scalar fields participating in the Yukawa interactions and those VEVs can be adjusted to account for these effects. This freedom in adjusting the VEVs of the scalars fields participating in the Yukawa interactions is due to the large number of parameters in the scalar potential. Furthermore, we recall that we adjust the corresponding effective Yukawa couplings instead of the Yukawa couplings to fit the physical observables in the quark and lepton sector to their experimental values at the M_Z scale.

TABLE IV. Model and experimental values of the charged fermion masses and CKM parameters.

Observable	Model value	Experimental value
m_e(MeV)	0.487	0.487
m_μ(MeV)	102.8	102.8 ± 0.0003
m_τ(GeV)	1.75	1.75 ± 0.0003
m_u(MeV)	1.45	$1.45^{+0.56}_{-0.45}$
m_c(MeV)	635	635 ± 86
m_t(GeV)	172.1	$172.1 \pm 0.6 \pm 0.9$
m_d(MeV)	2.9	$2.9^{+0.5}_{-0.4}$
m_s(MeV)	57.7	$57.7^{+16.8}_{-15.7}$
m_b(GeV)	2.82	$2.82^{+0.09}_{-0.04}$
$\sin\theta^{(q)}_{12}$	0.225	0.225
$\sin\theta^{(q)}_{23}$	0.0414	0.0414
$\sin\theta^{(q)}_{13}$	0.00355	0.00357
J	2.99×10^{-5}	$2.96^{+0.20}_{-0.16} \times 10^{-5}$

The charged lepton and quark masses [218,219], the quark mixing angles, and the Jarskog invariant [220] can be well reproduced in terms of natural parameters of order one, as shown in Table IV, starting from the following benchmark point:

$$\begin{aligned}
&a^{(u)}_{11} \simeq 1.884 + 0.387i, \qquad a^{(u)}_{12} \simeq -1.933 - 0.211i, \qquad a^{(u)}_{22} \simeq 1.974 - 0.023i,\\
&a^{(u)}_{33} \simeq 0.989, \qquad a^{(u)}_{13} \simeq 0.691 + 0.277i, \qquad a^{(u)}_{23} \simeq -0.788 + 0.014i,\\
&a^{(l)}_{11} \simeq 0.095, \qquad a^{(l)}_{22} \simeq 1.016, \qquad a^{(l)}_{33} \simeq 0.879,\\
&\kappa \simeq 1.862, \qquad f_1 \simeq -0.729, \qquad f_2 \simeq 1.871.
\end{aligned} \tag{60}$$

In Table V we show the model and experimental values for the physical observables of the quark sector. We use the M_Z-scale experimental values of the quark masses given by Ref. [218] (which are similar to those in [219]). The experimental values of the CKM parameters are taken from Ref. [220]. As indicated by Table IV, the obtained quark masses, quark mixing angles, and CP violating phase are consistent with the low energy quark flavor data. As shown from Table IV, the obtained values for the SM down type quark masses are inside the 1σ experimentally allowed range. In addition, our obtained values for the SM up type quark masses are inside the 1σ experimentally allowed range, as indicated in Table IV.

On the other hand, from the neutrino Yukawa interactions, we find that the Dirac and Majorna neutrino mass matrices are given by

$$m_{\nu D} = \begin{pmatrix} 0 & b \\ a & 3b \\ a & b \end{pmatrix}, \qquad M_R = \begin{pmatrix} M_{\rm atm} & 0 \\ 0 & M_{\rm sol} \end{pmatrix},$$
$$b = |b| e^{i\frac{\phi_\nu}{2}}. \tag{61}$$

Since the right-handed Majorana neutrinos ν_{1R} and ν_{2R} acquire very large masses, the light active neutrino masses are generated via the tree-level type-I seesaw mechanism and thus the light neutrino mass matrix takes the following form:

TABLE V. Model and experimental values of the light active neutrino masses, leptonic mixing angles and CP violating phase for the scenario of normal neutrino mass hierarchy. The difference $\alpha_3-\alpha_2$ between the Majorana phases predicted by the model is also shown. The experimental values are taken from Refs. [221,222].

Observable	Model	bpf $\pm 1\sigma$ [221]	bpf $\pm 1\sigma$ [222]	3σ Range [221]	3σ Range [222]
Δm_{21}^2 $[10^{-5}\ \mathrm{eV}^2]$	7.38	$7.55^{+0.20}_{-0.16}$	$7.40^{+0.21}_{-0.20}$	7.05–8.14	6.80–80.2
Δm_{31}^2 $[10^{-3}\ \mathrm{eV}^2]$	2.48	2.50 ± 0.03	$2.494^{+0.033}_{-0.031}$	2.41–2.60	2.399–2.593
$\theta_{12}^{(l)}(^\circ)$	34.32	$34.5^{+1.2}_{-1.0}$	$36.62^{+0.78}_{-0.76}$	31.5–38.0	31.42–36.05
$\theta_{13}^{(l)}(^\circ)$	8.67	$8.45^{+0.16}_{-0.14}$	8.54 ± 0.15	8.0–8.9	8.09–8.98
$\theta_{23}^{(l)}(^\circ)$	45.77	$47.9^{+1.0}_{-1.7}$	$47.2^{+1.9}_{-3.9}$	41.8–50.7	40.3–51.5
$\delta_{CP}^{(l)}(^\circ)$	−86.67	-142^{+38}_{-27}	-108^{+43}_{-31}	157–349	144–374
$(\alpha_3-\alpha_2)(^\circ)$	−71.90	…	…	…	…

$$m_\nu = m_{\nu D} M_R^{-1} m_{\nu D}^T = m_{\nu a}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_{\nu b} e^{i\phi_\nu}\begin{pmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{pmatrix}, \tag{62}$$

where $m_{\nu a}$ and $m_{\nu b}$ are given by

$$m_{\nu a} = \frac{a^2}{M_{\mathrm{atm}}}, \qquad m_{\nu b} = \frac{b^2}{M_{\mathrm{sol}}}. \tag{63}$$

The neutrino mass squared splittings, light active neutrino masses, leptonic mixing angles, and CP violating phase for the scenario of the normal neutrino mass hierarchy can be very well reproduced, as shown in Table V, for the following benchmark point:

$$m_{\nu a} \simeq 26.57\ \mathrm{meV}, \qquad m_{\nu b} \simeq 2.684\ \mathrm{meV}, \qquad \phi_\nu = 120^\circ. \tag{64}$$

In addition, we find that the light active neutrino masses are

$$m_1 = 0, \qquad m_2 = 8.59\ \mathrm{meV} \qquad m_3 = 49.81\ \mathrm{meV}. \tag{65}$$

From Table V, it follows that the neutrino mass squared splittings, i.e., Δm_{21}^2 and Δm_{31}^2, the leptonic mixing angles $\theta_{12}^{(l)}, \theta_{23}^{(l)}, \theta_{13}^{(l)}$, and the Dirac leptonic CP violating phase are consistent with neutrino oscillation experimental data for the scenario of normal neutrino mass hierarchy. Let us note that, for the inverted neutrino mass hierarchy, the obtained leptonic mixing parameters are very much outside the 3σ experimentally allowed range. Consequently, our model is only viable for the scenario of the normal neutrino mass hierarchy.

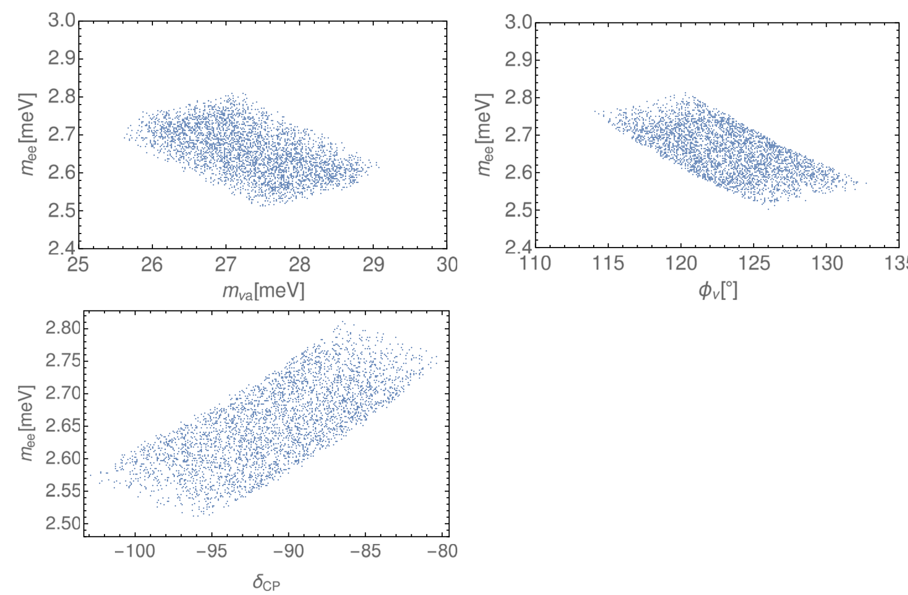

FIG. 1. Effective Majorana neutrino mass parameters as functions of the $m_{\nu a}$, ϕ_ν parameters and leptonic Dirac CP violating phase δ_{CP}.

Another important observable, worth determining in this model, is the effective Majorana neutrino mass parameter of the neutrinoless double beta decay, which gives us information on the Majorana nature of neutrinos. The amplitude for this process is directly proportional to the effective Majorana mass parameter, which is defined as

$$m_{ee} = \left|\sum_j U_{ek}^2 m_{\nu_k}\right| = |m_{\nu_1} c_{12}^2 c_{13}^2 + m_{\nu_2} s_{12}^2 c_{13}^2 e^{i\alpha_{21}} + m_{\nu_3} s_{13}^2 e^{i(\alpha_{31} - 2\delta_{CP}^{(l)})}|, \tag{66}$$

where U_{ej} and m_{ν_k} are the Pontecorvo-Maki-Nakagawa leptonic mixing matrix elements and the neutrino Majorana masses, respectively. Furthermore, $s_{ij} = \sin\theta_{ij}^{(l)}$, $c_{ij} = \cos\theta_{ij}^{(l)}$, $\alpha_{ij} = \alpha_i - \alpha_j$, being α_i the Majorana phases, with $i \neq j$ and $i,\ j = 1,\ 2,\ 3$. Note that since $m_{\nu_1} = 0$ in our model, then m_{ee} only depends on the relative phase $\alpha_{32} - 2\delta_{CP}^{(l)}$ where $\alpha_{32} = \alpha_3 - \alpha_2$.

Figure 1 shows the effective Majorana neutrino mass parameter as functions of the $m_{\nu a}$, ϕ_ν and δ_{CP} parameters (here δ_{CP} is the leptonic Dirac CP violating phase). To obtain the plots of Fig. 1, the parameters $m_{\nu a}$, ϕ_ν, and δ_{CP} were randomly generated in a range of values where the neutrino mass squared splittings and leptonic mixing parameters are inside the 3σ experimentally allowed range. As indicated by Fig. 1, our model predicts teh effective Majorana neutrino mass parameter in the range $2.5\ \text{meV} \lesssim m_{ee} \lesssim 2.8$ meV, for the scenario of the normal neutrino mass hierarchy.

Our obtained range of values for the effective Majorana neutrino mass parameter is beyond the reach of the present and forthcoming $0\nu\beta\beta$-decay experiments. The current most stringent experimental upper limit on the effective Majorana neutrino mass parameter $m_{ee} \leq 160$ meV is set by $T_{1/2}^{0\nu\beta\beta}(^{136}\text{Xe}) \geq 1.1 \times 10^{26}$ yr at 90% C.L. from the KamLAND-Zen experiment [223].

V. CONCLUSION

In this paper we have shown that $SU(5)$ GUTs with multiple vectorlike families at the GUT scale which transform under a gauged $U(1)'$ (under which the three chiral families are neutral) can result from two vectorlike families at low energies which can induce nonuniversal and flavorful Z' couplings, which can account for the B physics anomalies in $R_{K^{(*)}}$. In such theories, we have shown that the same physics which explains $R_{K^{(*)}}$ also corrects the Yukawa relation $Y_e = Y_d^T$ in the muon sector without the need for higher Higgs representations.

To illustrate the mechanism, we have constructed a concrete model based on $SU(5) \times A_4 \times Z_3 \times Z_7$ with two vectorlike families at the GUT scale, and two right-handed neutrinos, leading to successful fit to quark and lepton (including neutrino) masses, mixing angles, and CP phases, where the constraints from lepton-flavor violation require Y_e to be diagonal. This particular model predicts normal neutrino mass ordering with the inverted ordering disfavored by our fit, and an effective Majorana neutrino mass parameter in the range $2.5\ \text{meV} \lesssim m_{ee} \lesssim 2.8$ meV, for the scenario of the normal neutrino mass hierarchy.

In conclusion, we have shown that the idea of a flavorful Z' arising from mixing with vectorlike families can be extended to $SU(5)$ GUTs. In such theories, we have shown that the physics responsible for explaining the B physics anomalies in $R_{K^{(*)}}$ as a result of modified couplings in the muon sector can also lead to violation of the $SU(5)$ Yukawa relations $Y_e = Y_d^T$ in the muon sector without the need for higher Higgs representations.

ACKNOWLEDGMENTS

S. F. K. acknowledges the STFC Consolidated Grant No. ST/L000296/1 and the European Union's Horizon 2020 Research and Innovation programme under Marie Skłodowska-Curie Grant Agreements Elusives ITN No. 674896 and InvisiblesPlus RISE No. 690575 and would like to thank A. E. C. H. and the Universidad Técnica Federico Santa María for their hospitality. A. E. C. H. has been supported by Chilean grants Fondecyt No. 1170803 and CONICYT PIA/Basal FB0821. S. F. K. thanks Universidad Técnica Federico Santa María for hospitality, where this work was started. The visit of S. F. K. to Universidad Técnica Federico Santa María was supported by Chilean Grant Fondecyt No. 1170803.

APPENDIX A: THE PRODUCT RULES FOR A_4

The A_4 group, which is the group of even permutations of four elements, is the smallest discrete group having one three-dimensional representation, i.e., **3** as well as three inequivalent one-dimensional representations, i.e., **1**, **1′** and **1″**, satisfying the following product rules:

$$\mathbf{3} \otimes \mathbf{3} = \mathbf{3}_s \oplus \mathbf{3}_a \oplus \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}'',$$
$$\mathbf{1} \otimes \mathbf{1} = \mathbf{1}, \quad \mathbf{1}' \otimes \mathbf{1}'' = \mathbf{1}, \quad \mathbf{1}' \otimes \mathbf{1}' = \mathbf{1}'', \quad \mathbf{1}'' \otimes \mathbf{1}'' = \mathbf{1}'. \tag{A1}$$

Considering (x_1, y_1, z_1) and (x_2, y_2, z_2) as the basis vectors for two A_4 triplets **3**, the following relations are fulfilled:

$$\begin{aligned}
(\mathbf{3}\otimes\mathbf{3})_{\mathbf{1}} &= x_1y_1 + x_2y_2 + x_3y_3,\\
(\mathbf{3}\otimes\mathbf{3})_{\mathbf{1}'} &= x_1y_1 + \omega x_2y_2 + \omega^2 x_3y_3,\\
(\mathbf{3}\otimes\mathbf{3})_{\mathbf{1}''} &= x_1y_1 + \omega^2 x_2y_2 + \omega x_3y_3\\
(\mathbf{3}\otimes\mathbf{3})_{\mathbf{3}_s} &= (x_2y_3 + x_3y_2, x_3y_1 + x_1y_3, x_1y_2 + x_2y_1),\\
(\mathbf{3}\otimes\mathbf{3})_{\mathbf{3}_a} &= (x_2y_3 - x_3y_2, x_3y_1 - x_1y_3, x_1y_2 - x_2y_1),
\end{aligned} \tag{A2}$$

where $\omega = e^{i\frac{2\pi}{3}}$. The representation **1** is trivial, while the nontrivial **1**$'$ and **1**$''$ are complex conjugate to each other. Some reviews of discrete symmetries in particle physics are found in Refs. [29–33].

APPENDIX B: BRANCHING RATIO OF $\mu \to e\gamma$

The branching ratio of the $\mu \to e\gamma$ decay in our model, for the scenario where the charged lepton masses are much smaller than the Z' mass, is given by [224–226]

$$\mathrm{Br}(\mu \to e\gamma) = \frac{m_\mu^3}{2304\pi^4\Gamma_\mu M_{Z'}^4}[|3C_{e_RE_R}C_{\mu_LE_L}m_E + C_{e_R\mu_R}(3C_{\mu_L\mu_L} - C_{\mu_R\mu_R})m_\mu|^2 + |3C_{e_LE_L}C_{\mu_RE_R}m_E + C_{e_L\mu_L}(3C_{\mu_R\mu_R} - C_{\mu_L\mu_L})m_\mu|^2] \tag{B1}$$

where

$$\begin{aligned} C_{\mu_LE_L} &= g'q_{L4}\sin\theta_{24}^L, \qquad C_{\mu_RE_R} = g'q_{e4}\sin\theta_{24}^R \\ C_{e_LE_L} &= \sin\theta_{12}^L C_{\mu_LE_L} = g'q_{L4}\sin\theta_{12}^L\sin\theta_{24}^L, \\ C_{e_RE_R} &= \sin\theta_{12}^R C_{\mu_RE_R} = g'q_{e4}\sin\theta_{12}^R\sin\theta_{24}^R, \\ C_{e_L\mu_L} &= g'q_{L4}(\sin\theta_{12}^L\sin^2\theta_{24}^L\cos^2\theta_{24}^L + \sin\theta_{14}^L\sin\theta_{24}^L\cos\theta_{14}^L), \\ C_{e_R\mu_R} &= g'q_{e4}(\sin\theta_{12}^R\sin^2\theta_{24}^R\cos^2\theta_{24}^R + \sin\theta_{14}^R\sin\theta_{24}^R\cos\theta_{14}^R), \end{aligned} \tag{B2}$$

where $\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} = 3\times 10^{-19}$ GeV is the total muon decay width. The generalization to the fifth generation of fermions is straightforward and is made by replacing $\theta_{n4}^{L,R}$ by $\theta_{n5}^{L,R}$ $(n = 1, 2)$. Note that the branching ratio becomes zero for a diagonal SM charged lepton mass matrix provided that $\theta_{14}^L = \theta_{14}^R = \theta_{25}^L = \theta_{25}^R = 0$, which is the case of our flavor model described in Sec. IV.

[1] P. Langacker, Rev. Mod. Phys. **81**, 1199 (2009).
[2] S. Descotes-Genon, J. Matias, and J. Virto, Phys. Rev. D **88**, 074002 (2013).
[3] W. Altmannshofer and D. M. Straub, Eur. Phys. J. C **73**, 2646 (2013).
[4] D. Ghosh, M. Nardecchia, and S. A. Renner, J. High Energy Phys. 12 (2014) 131.
[5] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. Lett. **113**, 151601 (2014).
[6] S. Bifani *et al.* (LHCb Collaboration), Search for new physics with $b \to s\ell^+\ell^-$ decays at LHCb, CERN Seminar, 2017, https://cds.cern.ch/record/2260258.
[7] G. Hiller and I. Nisandzic, Phys. Rev. D **96**, 035003 (2017).
[8] M. Ciuchini, A. M. Coutinho, M. Fedele, E. Franco, A. Paul, L. Silvestrini, and M. Valli, Eur. Phys. J. C **77**, 688 (2017).
[9] L. S. Geng, B. Grinstein, S. Jäger, J. Martin Camalich, X. L. Ren, and R. X. Shi, Phys. Rev. D **96**, 093006 (2017).
[10] B. Capdevila, A. Crivellin, S. Descotes-Genon, J. Matias, and J. Virto, J. High Energy Phys. 01 (2018) 093.
[11] D. Ghosh, Eur. Phys. J. C **77**, 694 (2017).
[12] D. Bardhan, P. Byakti, and D. Ghosh, Phys. Lett. B **773**, 505 (2017).
[13] S. L. Glashow, D. Guadagnoli, and K. Lane, Phys. Rev. Lett. **114**, 091801 (2015).
[14] G. D'Amico, M. Nardecchia, P. Panci, F. Sannino, A. Strumia, R. Torre, and A. Urbano, J. High Energy Phys. 09 (2017) 010.
[15] S. Descotes-Genon, L. Hofer, J. Matias, and J. Virto, J. High Energy Phys. 06 (2016) 092.
[16] L. Calibbi, A. Crivellin, and T. Ota, Phys. Rev. Lett. **115**, 181801 (2015).
[17] A. Crivellin, L. Hofer, J. Matias, U. Nierste, S. Pokorski, and J. Rosiek, Phys. Rev. D **92**, 054013 (2015).
[18] A. Crivellin, G. D'Ambrosio, and J. Heeck, Phys. Rev. D **91**, 075006 (2015).
[19] C. Bonilla, T. Modak, R. Srivastava, and J. W. F. Valle, Phys. Rev. D **98**, 095002 (2018).
[20] A. E. Cárcamo Hernández, S. Kovalenko, R. Pasechnik, and I. Schmidt, arXiv:1901.02764.
[21] A. E. Cárcamo Hernández, S. Kovalenko, R. Pasechnik, and I. Schmidt, arXiv:1901.09552.
[22] L. Delle Rose, S. Khalil, S. J. D. King, and S. Moretti, arXiv:1903.11146.
[23] S. Descotes-Genon, M. Moscati, and G. Ricciardi, Phys. Rev. D **98**, 115030 (2018).
[24] N. Assad, B. Fornal, and B. Grinstein, Phys. Lett. B **777**, 324 (2018).
[25] S. F. King, J. High Energy Phys. 08 (2017) 019.
[26] M. C. Romao, S. F. King, and G. K. Leontaris, Phys. Lett. B **782**, 353 (2018).

[27] S. Antusch, C. Hohl, S. F. King, and V. Susic, Nucl. Phys. **B934**, 578 (2018).
[28] A. Falkowski, S. F. King, E. Perdomo, and M. Pierre, J. High Energy Phys. 08 (2018) 061.
[29] H. Ishimori, T. Kobayashi, H. Ohki, Y. Shimizu, H. Okada, and M. Tanimoto, Prog. Theor. Phys. Suppl. **183**, 1 (2010).
[30] G. Altarelli and F. Feruglio, Rev. Mod. Phys. **82**, 2701 (2010).
[31] S. F. King and C. Luhn, Rep. Prog. Phys. **76**, 056201 (2013).
[32] S. F. King, A. Merle, S. Morisi, Y. Shimizu, and M. Tanimoto, New J. Phys. **16**, 045018 (2014).
[33] S. F. King, Prog. Part. Nucl. Phys. **94**, 217 (2017).
[34] J. M. Gerard, Z. Phys. C **18**, 145 (1983).
[35] J. Kubo, A. Mondragon, M. Mondragon, and E. Rodriguez-Jauregui, Prog. Theor. Phys. **109**, 795 (2003); **114**, 287(E) (2005).
[36] J. Kubo, Phys. Lett. B **578**, 156 (2004); **619**, 387(E) (2005).
[37] T. Kobayashi, J. Kubo, and H. Terao, Phys. Lett. B **568**, 83 (2003).
[38] S. L. Chen, M. Frigerio, and E. Ma, Phys. Rev. D **70**, 073008 (2004); **70**, 079905(E) (2004).
[39] A. Mondragon, M. Mondragon, and E. Peinado, Phys. Rev. D **76**, 076003 (2007).
[40] A. Mondragon, M. Mondragon, and E. Peinado, Rev. Mex. Fis. **54**, 81 (2008) [Rev. Mex. Fis. Suppl. **54**, 0181 (2008)].
[41] G. Bhattacharyya, P. Leser, and H. Pas, Phys. Rev. D **83**, 011701 (2011).
[42] P. V. Dong, H. N. Long, C. H. Nam, and V. V. Vien, Phys. Rev. D **85**, 053001 (2012).
[43] A. G. Dias, A. C. B. Machado, and C. C. Nishi, Phys. Rev. D **86**, 093005 (2012).
[44] D. Meloni, J. High Energy Phys. 05 (2012) 124.
[45] F. Gonzalez Canales, A. Mondragon, and M. Mondragon, Fortschr. Phys. **61**, 546 (2013).
[46] F. González Canales, A. Mondragón, M. Mondragón, U. J. Saldaña Salazar, and L. Velasco-Sevilla, Phys. Rev. D **88**, 096004 (2013).
[47] E. Ma and B. Melic, Phys. Lett. B **725**, 402 (2013).
[48] Y. Kajiyama, H. Okada, and K. Yagyu, Nucl. Phys. **B887**, 358 (2014).
[49] A. E. Cárcamo Hernández, R. Martinez, and F. Ochoa, Eur. Phys. J. C **76**, 634 (2016).
[50] E. Ma and R. Srivastava, Phys. Lett. B **741**, 217 (2015).
[51] A. E. Cárcamo Hernández, R. Martinez, and J. Nisperuza, Eur. Phys. J. C **75**, 72 (2015).
[52] A. E. Cárcamo Hernández, E. Cataño Mur, and R. Martinez, Phys. Rev. D **90**, 073001 (2014).
[53] S. Gupta, C. S. Kim, and P. Sharma, Phys. Lett. B **740**, 353 (2015).
[54] A. E. Cárcamo Hernández, I. de Medeiros Varzielas, and E. Schumacher, Phys. Rev. D **93**, 016003 (2016).
[55] A. E. Cárcamo Hernández, I. de Medeiros Varzielas, and N. A. Neill, Phys. Rev. D **94**, 033011 (2016).
[56] A. E. Cárcamo Hernández, I. de Medeiros Varzielas, and E. Schumacher, arXiv:1601.00661.
[57] A. E. Cárcamo Hernández, Eur. Phys. J. C **76**, 503 (2016).
[58] A. E. Cárcamo Hernández, S. Kovalenko, and I. Schmidt, J. High Energy Phys. 02 (2017) 125.
[59] C. Arbeláez, A. E. Cárcamo Hernández, S. Kovalenko, and I. Schmidt, Eur. Phys. J. C **77**, 422 (2017).
[60] J. C. Gómez-Izquierdo, Eur. Phys. J. C **77**, 551 (2017).
[61] A. A. Cruz and M. Mondragón, arXiv:1701.07929.
[62] A. E. Cárcamo Hernández, J. Vignatti, and A. Zerwekh, arXiv:1807.05321.
[63] E. Ma and G. Rajasekaran, Phys. Rev. D **64**, 113012 (2001).
[64] I. de Medeiros Varzielas, S. F. King, and G. G. Ross, Phys. Lett. B **644**, 153 (2007).
[65] X. G. He, Y. Y. Keum, and R. R. Volkas, J. High Energy Phys. 04 (2006) 039.
[66] M. C. Chen and S. F. King, J. High Energy Phys. 06 (2009) 072.
[67] T. J. Burrows and S. F. King, Nucl. Phys. **B842**, 107 (2011).
[68] S. F. King and C. Luhn, J. High Energy Phys. 03 (2012) 036.
[69] S. Antusch, S. F. King, C. Luhn, and M. Spinrath, Nucl. Phys. **B856**, 328 (2012).
[70] Y. H. Ahn and S. K. Kang, Phys. Rev. D **86**, 093003 (2012).
[71] I. K. Cooper, S. F. King, and C. Luhn, J. High Energy Phys. 06 (2012) 130.
[72] N. Memenga, W. Rodejohann, and H. Zhang, Phys. Rev. D **87**, 053021 (2013).
[73] S. F. King, J. High Energy Phys. 01 (2014) 119.
[74] S. F. King, J. High Energy Phys. 07 (2013) 137.
[75] G. J. Ding, S. F. King, and A. J. Stuart, J. High Energy Phys. 12 (2013) 006.
[76] R. Gonzalez Felipe, H. Serodio, and J. P. Silva, Phys. Rev. D **88**, 015015 (2013).
[77] I. de Medeiros Varzielas and D. Pidt, J. High Energy Phys. 03 (2013) 065.
[78] H. Ishimori and E. Ma, Phys. Rev. D **86**, 045030 (2012).
[79] S. F. King, S. Morisi, E. Peinado, and J. W. F. Valle, Phys. Lett. B **724**, 68 (2013).
[80] S. Antusch, S. F. King, and M. Spinrath, Phys. Rev. D **87**, 096018 (2013).
[81] A. E. Carcamo Hernandez, I. de Medeiros Varzielas, S. G. Kovalenko, H. Päs, and I. Schmidt, Phys. Rev. D **88**, 076014 (2013).
[82] K. S. Babu, E. Ma, and J. W. F. Valle, Phys. Lett. B **552**, 207 (2003).
[83] G. Altarelli and F. Feruglio, Nucl. Phys. **B741**, 215 (2006).
[84] I. de Medeiros Varzielas, G. G. Ross, and M. Serna, Phys. Rev. D **80**, 073002 (2009).
[85] S. Gupta, A. S. Joshipura, and K. M. Patel, Phys. Rev. D **85**, 031903 (2012).
[86] S. Morisi, M. Nebot, K. M. Patel, E. Peinado, and J. W. F. Valle, Phys. Rev. D **88**, 036001 (2013).
[87] G. Altarelli and F. Feruglio, Nucl. Phys. **B720**, 64 (2005).
[88] A. Kadosh and E. Pallante, J. High Energy Phys. 08 (2010) 115.
[89] A. Kadosh, J. High Energy Phys. 06 (2013) 114.
[90] F. del Aguila, A. Carmona, and J. Santiago, J. High Energy Phys. 08 (2010) 127.
[91] M. D. Campos, A. E. Cárcamo Hernández, S. Kovalenko, I. Schmidt, and E. Schumacher, Phys. Rev. D **90**, 016006 (2014).

[92] V. V. Vien and H. N. Long, Int. J. Mod. Phys. A **30**, 1550117 (2015).
[93] S. F. King, J. High Energy Phys. 08 (2014) 130.
[94] A. S. Joshipura and K. M. Patel, Phys. Lett. B **749**, 159 (2015).
[95] A. E. Cárcamo Hernández and R. Martinez, Nucl. Phys. **B905**, 337 (2016).
[96] F. Björkeroth, F. J. de Anda, I. de Medeiros Varzielas, and S. F. King, J. High Energy Phys. 06 (2015) 141.
[97] P. Chattopadhyay and K. M. Patel, Nucl. Phys. **B921**, 487 (2017).
[98] A. E. Cárcamo Hernández and H. N. Long, J. Phys. G **45**, 045001 (2018).
[99] A. E. Cárcamo Hernández, S. Kovalenko, H. N. Long, and I. Schmidt, J. High Energy Phys. 07 (2018) 144.
[100] S. Centelles Chuliá, R. Srivastava, and J. W. F. Valle, Phys. Lett. B **773**, 26 (2017).
[101] F. Björkeroth, E. J. Chun, and S. F. King, Phys. Lett. B **777**, 428 (2018).
[102] A. S. Belyaev, S. F. King, and P. B. Schaefers, Phys. Rev. D **97**, 115002 (2018).
[103] S. F. King and Y. L. Zhou, J. High Energy Phys. 11 (2018) 173.
[104] J. Bernigaud, B. Herrmann, S. F. King, and S. J. Rowley, J. High Energy Phys. 03 (2019) 067.
[105] F. J. de Anda, S. F. King, and E. Perdomo, arXiv:1812.05620.
[106] K. M. Patel, Phys. Lett. B **695**, 225 (2011).
[107] S. Morisi, K. M. Patel, and E. Peinado, Phys. Rev. D **84**, 053002 (2011).
[108] A. E. Cárcamo Hernández, J. M. González, and U. J. Saldaña-Salazar, arXiv:1904.09993.
[109] R. N. Mohapatra and C. C. Nishi, Phys. Rev. D **86**, 073007 (2012).
[110] P. S. Bhupal Dev, B. Dutta, R. N. Mohapatra, and M. Severson, Phys. Rev. D **86**, 035002 (2012).
[111] C. Hagedorn, S. F. King, and C. Luhn, Phys. Lett. B **717**, 207 (2012).
[112] I. de Medeiros Varzielas and L. Lavoura, J. Phys. G **40**, 085002 (2013).
[113] G. J. Ding, S. F. King, C. Luhn, and A. J. Stuart, J. High Energy Phys. 05 (2013) 084.
[114] H. Ishimori, Y. Shimizu, M. Tanimoto, and A. Watanabe, Phys. Rev. D **83**, 033004 (2011).
[115] G. J. Ding and Y. L. Zhou, Nucl. Phys. **B876**, 418 (2013).
[116] C. Hagedorn and M. Serone, J. High Energy Phys. 10 (2011) 083.
[117] M. D. Campos, A. E. Cárcamo Hernández, H. Päs, and E. Schumacher, Phys. Rev. D **91**, 116011 (2015).
[118] P. V. Dong, H. N. Long, D. V. Soa, and V. V. Vien, Eur. Phys. J. C **71**, 1544 (2011).
[119] V. V. Vien, H. N. Long, and D. P. Khoi, Int. J. Mod. Phys. A **30**, 1550102 (2015).
[120] M. Dimou, S. F. King, and C. Luhn, Phys. Rev. D **93**, 075026 (2016).
[121] S. F. King and C. Luhn, J. High Energy Phys. 09 (2016) 023.
[122] F. Björkeroth, F. J. de Anda, S. F. King, and E. Perdomo, J. High Energy Phys. 10 (2017) 148.
[123] F. J. de Anda, S. F. King, and E. Perdomo, J. High Energy Phys. 12 (2017) 075.
[124] F. J. de Anda and S. F. King, J. High Energy Phys. 07 (2018) 057.
[125] A. E. Cárcamo Hernández and S. F. King, arXiv:1903.02565.
[126] P. H. Frampton and T. W. Kephart, Int. J. Mod. Phys. A **10**, 4689 (1995).
[127] W. Grimus and L. Lavoura, Phys. Lett. B **572**, 189 (2003).
[128] W. Grimus, A. S. Joshipura, S. Kaneko, L. Lavoura, and M. Tanimoto, J. High Energy Phys. 07 (2004) 078.
[129] M. Frigerio, S. Kaneko, E. Ma, and M. Tanimoto, Phys. Rev. D **71**, 011901 (2005).
[130] A. Adulpravitchai, A. Blum, and C. Hagedorn, J. High Energy Phys. 03 (2009) 046.
[131] H. Ishimori, T. Kobayashi, H. Ohki, Y. Omura, R. Takahashi, and M. Tanimoto, Phys. Lett. B **662**, 178 (2008).
[132] C. Hagedorn and R. Ziegler, Phys. Rev. D **82**, 053011 (2010).
[133] D. Meloni, S. Morisi, and E. Peinado, Phys. Lett. B **703**, 281 (2011).
[134] V. V. Vien and H. N. Long, Int. J. Mod. Phys. A **28**, 1350159 (2013).
[135] K. S. Babu and J. Kubo, Phys. Rev. D **71**, 056006 (2005).
[136] Y. Kajiyama, E. Itou, and J. Kubo, Nucl. Phys. **B743**, 74 (2006).
[137] Y. Kajiyama, J. High Energy Phys. 04 (2007) 007.
[138] N. Kifune, J. Kubo, and A. Lenz, Phys. Rev. D **77**, 076010 (2008).
[139] K. S. Babu and Y. Meng, Phys. Rev. D **80**, 075003 (2009).
[140] K. Kawashima, J. Kubo, and A. Lenz, Phys. Lett. B **681**, 60 (2009).
[141] Y. Kaburaki, K. Konya, J. Kubo, and A. Lenz, Phys. Rev. D **84**, 016007 (2011).
[142] K. S. Babu, K. Kawashima, and J. Kubo, Phys. Rev. D **83**, 095008 (2011).
[143] T. Araki and Y. F. Li, Phys. Rev. D **85**, 065016 (2012).
[144] J. C. Gómez-Izquierdo, F. González-Canales, and M. Mondragon, Eur. Phys. J. C **75**, 221 (2015).
[145] J. C. Gómez-Izquierdo, F. Gonzalez-Canales, and M. Mondragón, Int. J. Mod. Phys. A **32**, 1750171 (2017).
[146] C. Luhn, S. Nasri, and P. Ramond, Phys. Lett. B **652**, 27 (2007).
[147] C. Hagedorn, M. A. Schmidt, and A. Y. Smirnov, Phys. Rev. D **79**, 036002 (2009).
[148] Q. H. Cao, S. Khalil, E. Ma, and H. Okada, Phys. Rev. Lett. **106**, 131801 (2011).
[149] C. Luhn, K. M. Parattu, and A. Wingerter, J. High Energy Phys. 12 (2012) 096.
[150] Y. Kajiyama, H. Okada, and K. Yagyu, J. High Energy Phys. 10 (2013) 196.
[151] C. Bonilla, S. Morisi, E. Peinado, and J. W. F. Valle, Phys. Lett. B **742**, 99 (2015).
[152] V. V. Vien and H. N. Long, J. High Energy Phys. 04 (2014) 133.
[153] H. N. Long and V. V. Vien, Int. J. Mod. Phys. A **29**, 1450072 (2014).
[154] A. E. Cárcamo Hernández and R. Martinez, J. Phys. G **43**, 045003 (2016).
[155] C. Arbeláez, A. E. Cárcamo Hernández, S. Kovalenko, and I. Schmidt, Phys. Rev. D **92**, 115015 (2015).

[156] G. J. Ding, Nucl. Phys. **B853**, 635 (2011).
[157] C. Hartmann, Phys. Rev. D **85**, 013012 (2012).
[158] C. Hartmann and A. Zee, Nucl. Phys. **B853**, 105 (2011).
[159] Y. Kajiyama and H. Okada, Nucl. Phys. **B848**, 303 (2011).
[160] A. Aranda, C. D. Carone, and R. F. Lebed, Phys. Rev. D **62**, 016009 (2000).
[161] S. Sen, Phys. Rev. D **76**, 115020 (2007).
[162] A. Aranda, Phys. Rev. D **76**, 111301 (2007).
[163] M. C. Chen and K. T. Mahanthappa, Phys. Lett. B **652**, 34 (2007).
[164] P. H. Frampton, T. W. Kephart, and S. Matsuzaki, Phys. Rev. D **78**, 073004 (2008).
[165] D. A. Eby, P. H. Frampton, X. G. He, and T. W. Kephart, Phys. Rev. D **84**, 037302 (2011).
[166] P. H. Frampton, C. M. Ho, and T. W. Kephart, Phys. Rev. D **89**, 027701 (2014).
[167] M. C. Chen, J. Huang, K. T. Mahanthappa, and A. M. Wijangco, J. High Energy Phys. 10 (2013) 112.
[168] V. V. Vien, H. N. Long, and A. E. Cárcamo Hernández, Mod. Phys. Lett. A **34**, 1950005 (2019).
[169] G. C. Branco, J. M. Gerard, and W. Grimus, Phys. Lett. **136B**, 383 (1984).
[170] I. de Medeiros Varzielas, S. F. King, and G. G. Ross, Phys. Lett. B **648**, 201 (2007).
[171] E. Ma, Phys. Lett. B **660**, 505 (2008).
[172] F. Bazzocchi and I. de Medeiros Varzielas, Phys. Rev. D **79**, 093001 (2009).
[173] I. de Medeiros Varzielas, D. Emmanuel-Costa, and P. Leser, Phys. Lett. B **716**, 193 (2012).
[174] G. Bhattacharyya, I. de Medeiros Varzielas, and P. Leser, Phys. Rev. Lett. **109**, 241603 (2012).
[175] P. M. Ferreira, W. Grimus, L. Lavoura, and P. O. Ludl, J. High Energy Phys. 09 (2012) 128.
[176] E. Ma, Phys. Lett. B **723**, 161 (2013).
[177] C. C. Nishi, Phys. Rev. D **88**, 033010 (2013).
[178] I. de Medeiros Varzielas and D. Pidt, J. Phys. G **41**, 025004 (2014).
[179] A. Aranda, C. Bonilla, S. Morisi, E. Peinado, and J. W. F. Valle, Phys. Rev. D **89**, 033001 (2014).
[180] E. Ma and A. Natale, Phys. Lett. B **734**, 403 (2014).
[181] M. Abbas and S. Khalil, Phys. Rev. D **91**, 053003 (2015).
[182] M. Abbas, S. Khalil, A. Rashed, and A. Sil, Phys. Rev. D **93**, 013018 (2016).
[183] I. de Medeiros Varzielas, J. High Energy Phys. 08 (2015) 157.
[184] F. Björkeroth, F. J. de Anda, I. de Medeiros Varzielas, and S. F. King, Phys. Rev. D **94**, 016006 (2016).
[185] P. Chen, G. J. Ding, A. D. Rojas, C. A. Vaquera-Araujo, and J. W. F. Valle, J. High Energy Phys. 01 (2016) 007.
[186] V. V. Vien, A. E. Cárcamo Hernández, and H. N. Long, Nucl. Phys. **B913**, 792 (2016).
[187] A. E. Cárcamo Hernández, H. N. Long, and V. V. Vien, Eur. Phys. J. C **76**, 242 (2016).
[188] A. E. Cárcamo Hernández, S. Kovalenko, J. W. F. Valle, and C. A. Vaquera-Araujo, J. High Energy Phys. 07 (2017) 118.
[189] I. de Medeiros Varzielas, G. G. Ross, and J. Talbert, J. High Energy Phys. 03 (2018) 007.
[190] N. Bernal, A. E. Cárcamo Hernández, I. de Medeiros Varzielas, and S. Kovalenko, J. High Energy Phys. 05 (2018) 053.
[191] A. E. Cárcamo Hernández, H. N. Long, and V. V. Vien, Eur. Phys. J. C **78**, 804 (2018).
[192] I. De Medeiros Varzielas, M. L. López-Ibáñez, A. Melis, and O. Vives, J. High Energy Phys. 09 (2018) 047.
[193] A. E. Cárcamo Hernández, J. C. Gómez-Izquierdo, S. Kovalenko, and M. Mondragón, arXiv:1810.01764.
[194] A. E. Cárcamo Hernández, S. Kovalenko, J. W. F. Valle, and C. A. Vaquera-Araujo, J. High Energy Phys. 02 (2019) 065.
[195] B. Carballo-Perez, E. Peinado, and S. Ramos-Sanchez, J. High Energy Phys. 12 (2016) 131.
[196] S. F. King, C. Luhn, and A. J. Stuart, Nucl. Phys. **B867**, 203 (2013).
[197] S. F. King, T. Neder, and A. J. Stuart, Phys. Lett. B **726**, 312 (2013).
[198] G. J. Ding and S. F. King, Phys. Rev. D **89**, 093020 (2014).
[199] H. Ishimori and S. F. King, Phys. Lett. B **735**, 33 (2014).
[200] S. F. King and T. Neder, Phys. Lett. B **736**, 308 (2014).
[201] H. Ishimori, S. F. King, H. Okada, and M. Tanimoto, Phys. Lett. B **743**, 172 (2015).
[202] L. L. Everett and A. J. Stuart, Phys. Rev. D **79**, 085005 (2009).
[203] F. Feruglio and A. Paris, J. High Energy Phys. 03 (2011) 101.
[204] I. K. Cooper, S. F. King, and A. J. Stuart, Nucl. Phys. **B875**, 650 (2013).
[205] I. de Medeiros Varzielas and L. Lavoura, J. Phys. G **41**, 055005 (2014).
[206] J. Gehrlein, J. P. Oppermann, D. Schäfer, and M. Spinrath, Nucl. Phys. **B890**, 539 (2014).
[207] J. Gehrlein, S. T. Petcov, M. Spinrath, and X. Zhang, Nucl. Phys. **B896**, 311 (2015).
[208] A. Di Iura, C. Hagedorn, and D. Meloni, J. High Energy Phys. 08 (2015) 037.
[209] P. Ballett, S. Pascoli, and J. Turner, Phys. Rev. D **92**, 093008 (2015).
[210] J. Gehrlein, S. T. Petcov, M. Spinrath, and X. Zhang, Nucl. Phys. **B899**, 617 (2015).
[211] J. Turner, Phys. Rev. D **92**, 116007 (2015).
[212] C. C. Li and G. J. Ding, J. High Energy Phys. 05 (2015) 100.
[213] G. J. Ding, S. F. King, and C. C. Li, Nucl. Phys. **B925**, 470 (2017).
[214] S. F. King, Nucl. Phys. **B576**, 85 (2000).
[215] S. F. King, J. High Energy Phys. 09 (2002) 011.
[216] H. Georgi and C. Jarlskog, Phys. Lett. **86B**, 297 (1979).
[217] G. Ross and M. Serna, Phys. Lett. B **664**, 97 (2008).
[218] K. Bora, Horizons in biochemistry and biophysics **2**, 112 (2013).
[219] Z. z. Xing, H. Zhang, and S. Zhou, Phys. Rev. D **77**, 113016 (2008).
[220] C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C **40**, 100001 (2016).
[221] P. F. de Salas, D. V. Forero, C. A. Ternes, M. Tortola, and J. W. F. Valle, Phys. Lett. B **782**, 633 (2018).
[222] I. Esteban, M. C. Gonzalez-Garcia, M. Maltoni, I. Martinez-Soler, and T. Schwetz, J. High Energy Phys. 01 (2017) 087.

[223] A. Gando *et al.* (KamLAND-Zen Collaboration), Phys. Rev. Lett. **117**, 082503 (2016); **117**, 109903 (2016).
[224] C. W. Chiang, Y. F. Lin, and J. Tandean, J. High Energy Phys. 11 (2011) 083.
[225] S. Raby and A. Trautner, Phys. Rev. D **97**, 095006 (2018).
[226] M. Lindner, M. Platscher, and F. S. Queiroz, Phys. Rep. **731**, 1 (2018).