Where is the stable pentaquark?

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(Received 5 December 2018; published 16 May 2019)

We systematically analyze the flavor color spin structure of the pentaquark $q^4\bar{Q}$ system in a constituent quark model based on the chromomagnetic interaction in both the SU(3) flavor symmetric and SU(3) flavor broken case with and without charm quarks. We show that the originally proposed pentaquark state $\bar{Q}sqqq$ by Gignoux *et al.* and by Lipkin indeed belongs to the most stable pentaquark configuration, but that when charm quark mass correction based on recent experiments are taken into account, a doubly charmed antistrange pentaquark configuration P_{cc} $(udcc\bar{s})$ could be the most attractive flavor exotic configuration that could be stable and realistically searched for at present through the $P_{cc} \to \Lambda_c K^+ K^- \pi^+$ final states. The proposed final state is just reconstructing K^+ instead of π^+ in the measurement of $\Xi_{cc}^{++} \to \Lambda_c K^- \pi^+ \pi^+$ reported by the LHCb collaboration and hence measurable immediately.

DOI: 10.1103/PhysRevD.99.094023

I. INTRODUCTION

The possible existence of mutiquark hadrons beyond the ordinary hadrons was first discussed for the tetraquark states in Refs. [1,2] and for the H-dibaryon in Ref. [3]. Later, possible stable pentaquark configurations $\bar{Q}sqqq$ were proposed in Ref. [4] and in Ref. [5]. The long experimental search for the H-dibaryon has not been successful so far but is still planned at JPARC [6]. The search by Fermilab E791 [7] for the proposed pentaquark state also failed to find any significant signal for the exotic configurations.

On the other hand, starting from the $X(3872)$ [8], possible exotic meson configurations XYZ and the pentaquark P_c [9] were recently found. These states are not flavor exotic but are known to contain $\bar{c}c$ quarks. Heavy quarks were for many years considered to be stable color sources that would allow for a stable multiquark configuration that does not fall into usual hadrons. In particular, with the recent experimental confirmation of the doubly charmed baryon [10–12], there is new excitement in the physics of exotics in general and in hiterto unobserved flavor exotic states with more than one heavy quarks [13–17].

In this work, we systematically analyze the color flavor spin structure of the pentaquark configuration within a constituent quark model based on chromomagnetic interaction. We show that the originally proposed pentaquark state $\bar{Q}sqqq$ indeed belongs to the most stable pentaquark configuration, but that when charm quark mass correction based on recent experiments is taken into account, a doubly charmed antistrange pentaquark configuration P_{cc} $(udcc\bar{s})$ could be the most attractive flavor exotic configuration that could be stable and realistically searched for at present.

It is useful to view the classification in terms of $\mathrm{SU}(4)_F$, which deserves a full analysis in our approach in the future. It can be shown that the most stable configuration that we find belongs to a **140** multiplet in such a scheme [18]. Here, we are restricting our discussion to categorizing the four quark structure in terms of the light flavors. This is so because the color spin interaction effects are suppressed when a heavy quark is involved. Hence, for a first order estimate, our analysis should be a valid starting point.

II. SYSTEMATIC ANALYSIS OF $q^4\bar{Q}$

We first discuss the classification of the flavor, color and spin wave function for the ground state of the pentaquark composed of q^4 light quarks and one heavy antiquark Q ($\bar{c}$ or $\bar{b}$) assuming that the spatial parts of the wave function for all quarks are in the s-wave. We categorize them into the flavor states in $\mathrm{SU}(3)_F$, and then examine the color ⊗ spin states.

The flavor states for q^4 can be decomposed into the direct sum of the irreducible representation of $\mathrm{SU}(3)_F$ as follows:

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TABLE I. The $\mathrm{SU}(6)_{CS}$ representations containing the $[1_C, S]$ multiplet.

$\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_S$ state	$\mathrm{SU}(6)_{CS}$ representation
$[1_C, 1/2]$	$[2^4 1]$, $[32^2 1^2]$, $[21]$, $[421^3]$
$[1_C, 3/2]$	$[1^3]$, $[32^2 1^2]$, $[3^2 1^3]$, $[421^3]$
$[1_C, 5/2]$	$[32^2 1^2]$

$$\mathbf{3}_F \otimes \mathbf{3}_F \otimes \mathbf{3}_F \otimes \mathbf{3}_F = \mathbf{15} \oplus 3 \times \mathbf{15}' \oplus 3 \times \mathbf{3} \oplus 2 \times \bar{\mathbf{6}}. \quad (1)$$

Here, the $\mathrm{SU}(3)_F$ multiplets can be represented by the corresponding Young diagrams; **15** multiplet corresponds to Young diagram [4], $\mathbf{15}'$ multiplet to Young diagram [31], **3** multiplet to Young diagram $[21^2]$, and $\bar{\mathbf{6}}$ to Young diagram $[2^2]$.

The 7776 dimensional color $\otimes$ spin states of $q^4\bar{Q}$ can be classified as the direct sum of the irreducible representations of $\mathrm{SU}(6)_{CS}$ as follows:

$$\begin{aligned}
&(\mathbf{6}_{CS} \otimes \mathbf{6}_{CS} \otimes \mathbf{6}_{CS} \otimes \mathbf{6}_{CS} \otimes \bar{\mathbf{6}}_{CS})_{[7776]} \\
&\quad = ([51^4] \oplus [3])_{[4]} \oplus 2([3^2 1^3] \oplus [21])_{[2^2]} \\
&\qquad \oplus 3([421^3] \oplus [3] \oplus [21])_{[31]} \oplus ([2^4 1] \oplus [1^3])_{[1^4]} \\
&\qquad \oplus 3([32^2 1^2] \oplus [21] \oplus [1^3])_{[21^2]}. \qquad (2)
\end{aligned}$$

The direct sum in the right-hand side has been divided by large brackets according to the $\mathrm{SU}(6)_{CS}$ representation for the light quark sector q^4, with the subscript indicating the corresponding Young diagram.

The $\mathrm{SU}(6)_{CS}$ representation is made up of the sum of $\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_S$ multiplets, represented here by [color state, spin state]. To select out physical states, among all the $\mathrm{SU}(6)_{CS}$ representations in Eq. (2) we have identified the $\mathrm{SU}(6)_{CS}$ representation that contains color singlet states 1_C with certain spin $S = 1/2, 3/2, 5/2$. The second column in Table I shows the $\mathrm{SU}(6)_{CS}$ representation that contains the allowed color singlet states with the possible spin states, denoted by $[1_C, S]$ in the first column.

Therefore, since the $SU(6)_{CS}$ representation of $q^4\bar{Q}$ as well as those of q^4 are given in Eq. (2), we can construct the flavor $\otimes$ color $\otimes$ spin states with color singlet, by using the fully antisymmetric property together with the conjugate relation between the flavor in Eq. (1) and the $SU(6)_{CS}$ representation in Eq. (2) among the four light quarks. Such a combination will finally determine the allowed flavor and spin content of the pentaquarks in the flavor SU(3) symmetric limit.

III. COLOR SPIN INTERACTION FOR THE PENTAQUARK SYSTEM

In the constituent quark model based on the color spin interaction, the stability of a pentaquark depends critically on the expectation value of the interaction. Therefore, we derive the following elegant formula of the chromomagnetic interaction relevant for the pentaquark configuration, which is similar to that of a tetraquark in Ref. [2], by introducing the quadratic Casimir operator of $\mathrm{SU}(6)_{CS}$, which is denoted by C^6:

$$\begin{aligned}
-\sum_{i<j}^{5} \lambda_i^c \lambda_j^c \vec{\sigma}_i \cdot \vec{\sigma}_j &= 4C_5^6 - 8C_4^6 - 2C_5^3 + 4C_4^3 \\
&\quad - \frac{4}{3}(\vec{S}\cdot\vec{S})_5 + \frac{8}{3}(\vec{S}\cdot\vec{S})_4 + 24I. \qquad (3)
\end{aligned}$$

Here, C_5^6 (C_4^6) is the quadratic Casimir operator of $SU(6)_{CS}$ for the pentaquark (the four light quarks), C_5^3 (C_4^3) is the quadratic Casimir operator of $SU(3)_C$ for the pentaquark (the four light quarks), $(\vec{S}\cdot\vec{S})_5$ $[(\vec{S}\cdot\vec{S})_4]$ the spin operator for the pentaquark (the four light quarks), and I the identity operator.

A. Spin = 3/2

Let us discuss in detail the flavor $\mathbf{15}'$ case with $S = 3/2$. Here, there are two flavor $\otimes$ color $\otimes$ spin states that are orthonormal to each other. There are two methods to obtain these states.

In one approach based on the coupling scheme, the first (second) state comes from the coupling scheme of the color $\otimes$ spin state in which the spin among the four quarks is one (two), as given in Eq. (26) [Eq. (32)] in [19]. The fully antisymmetric orthonormal flavor $\otimes$ color $\otimes$ spin states for $S = 3/2$ among the four quarks can be obtained by multiplying the color $\otimes$ spin state by their conjugate flavor $\mathbf{15}'$ state represented by Young diagram [31]. The fully antisymmetric orthonormal flavor $\otimes$ color $\otimes$ spin states for $S = 3/2$ among the four quarks are given by

$$\begin{aligned}
|\psi_1\rangle = \frac{1}{\sqrt{3}}\Bigg(& \ytableaushort{123,4}_F \otimes \ytableaushort{14,2,3}_{CS^1} - \ytableaushort{124,3}_F \otimes \ytableaushort{13,2,4}_{CS^1} \\
& + \ytableaushort{134,2}_F \otimes \ytableaushort{12,3,4}_{CS^1} \Bigg), \\
|\psi_2\rangle = \frac{1}{\sqrt{3}}\Bigg(& \ytableaushort{123,4}_F \otimes \ytableaushort{14,2,3}_{CS^2} - \ytableaushort{124,3}_F \otimes \ytableaushort{13,2,4}_{CS^2} \\
& + \ytableaushort{134,2}_F \otimes \ytableaushort{12,3,4}_{CS^2} \Bigg),
\end{aligned} \qquad (4)$$

where the superscript of S denotes the spin state among the four quark, and subscript F stands for flavor state, which is represented by Young-Yammanuochi bases of Young diagram [31]. Here, it should be noted that the Young-Yammanuochi bases in both $|\psi_1\rangle$ and $|\psi_2\rangle$ in Eq. (4) belong to the multiplets of $SU(6)_{CS}$ for four quarks corresponding to Young diagram $[21^2]$. This means that these states are the eigenstates of the quadratic Casimir operator of $SU(6)_{CS}$ for four quarks, C_4^6, with $26/3$ being their eigenvalue.

In the other approach, the two states can be directly obtained from Eq. (2). As we can see in Eq. (2) and Table I, both the $[32^21^2]$ and the $[1^3]$ $SU(6)_{CS}$ representations have the state of the color singlet and $S=3/2$. Also, these states involve the $[21^2]$ multiplets in the $SU(6)_{CS}$ representation among the four quarks, which are conjugate to the flavor $\mathbf{15}'$ states, so that the two fully antisymmetric orthonormal flavor ⊗ color ⊗ spin states can be constructed from the $SU(6)_{CS}$ representation $[32^21^2]$ and $[1^3]$,

$$
\begin{aligned}
|\Psi_1\rangle = \frac{1}{\sqrt{3}}\Bigg(& \begin{ytableau}1&2&3\\4\end{ytableau}_F \otimes \Bigg|\begin{ytableau}1&4\\2\\3\end{ytableau}, [32^21^2]\Bigg\rangle_{CS^{3/2}} - \begin{ytableau}1&2&4\\3\end{ytableau}_F \\
& \otimes \Bigg|\begin{ytableau}1&3\\2\\4\end{ytableau}, [32^21^2]\Bigg\rangle_{CS^{3/2}} + \begin{ytableau}1&3&4\\2\end{ytableau}_F \otimes \Bigg|\begin{ytableau}1&2\\3\\4\end{ytableau}, [32^21^2]\Bigg\rangle_{CS^{3/2}}\Bigg),
\end{aligned}
$$

$$
\begin{aligned}
|\Psi_2\rangle = \frac{1}{\sqrt{3}}\Bigg(& \begin{ytableau}1&2&3\\4\end{ytableau}_F \otimes \Bigg|\begin{ytableau}1&4\\2\\3\end{ytableau}, [1^3]\Bigg\rangle_{CS^{3/2}} - \begin{ytableau}1&2&4\\3\end{ytableau}_F \\
& \otimes \Bigg|\begin{ytableau}1&3\\2\\4\end{ytableau}, [1^3]\Bigg\rangle_{CS^{3/2}} + \begin{ytableau}1&3&4\\2\end{ytableau}_F \otimes \Bigg|\begin{ytableau}1&2\\3\\4\end{ytableau}, [1^3]\Bigg\rangle_{CS^{3/2}}\Bigg),
\end{aligned}
\tag{5}
$$

where the superscript of S indicates the total spin of $[32^21^2]([1^3])$ of $SU(6)_{CS}$ representation. Unlike Eq. (4), we did not add subscripts in the CS part of the Young-Yamanouchi bases in Eq. (5) as the total spins for the four quarks are not determined. It should be noted that $|\Psi_1\rangle$ ($|\Psi_2\rangle$) in Eq. (5) not only shows the symmetry property with respect to the four quarks, but also the explicit dependence on the multiplet $[32^21^2]([1^3])$ of $SU(6)_{CS}$ representation for the pentaquark. This means that the color spin parts of $|\Psi_1\rangle$ ($|\Psi_2\rangle$) in Eq. (5) are the eigenstates of the quadratic Casimir operator, C_5^6, with $49/4$ $(21/4)$ being its eigenvalue.

From the $SU(6)_{CS}$ representation point of view, we can infer that the linear sum of two fully antisymmetric flavor ⊗ color ⊗ spin states coming from the coupling scheme must belong to either the $[32^21^2]$ state or $[1^3]$ state. We find that the coefficients of the linear sum can be calculated from the condition that these are the eigenstates of the Casimir operator of $\mathrm{SU}(6)_{CS}$, given by

$$
\begin{aligned}
|\Psi_1\rangle &= \frac{\sqrt{5}}{\sqrt{7}}|\psi_1\rangle + \frac{\sqrt{2}}{\sqrt{7}}|\psi_2\rangle, \\
|\Psi_2\rangle &= -\frac{\sqrt{2}}{\sqrt{7}}|\psi_1\rangle + \frac{\sqrt{5}}{\sqrt{7}}|\psi_2\rangle.
\end{aligned}
\tag{6}
$$

The emphasis here is that $|\Psi_1\rangle$ and $|\Psi_2\rangle$ are both eigenstates of the quadratic Casimir operator C_5^6 of the $SU(6)_{CS}$ because $|\Psi_1\rangle$ and $|\Psi_2\rangle$ are themselves the eigenstates. The eigenvalue can be calculated using the following formula:

$$
\begin{aligned}
C_5^6 = & -\frac{1}{4}\sum_{i=1}^{4}\lambda_i^c\lambda_5^c\vec{\sigma}_i\cdot\vec{\sigma}_5 + C_4^6 + \frac{1}{2}C_5^3 - \frac{1}{2}C_4^3 \\
& + \frac{1}{3}(\vec{S}\cdot\vec{S})_5 - \frac{1}{3}(\vec{S}\cdot\vec{S})_4 + 2I.
\end{aligned}
\tag{7}
$$

We can verify from Eqs. (6) and (7) that the following eigenvalue equation holds for the Casimir operator C_5^6 of the $SU(6)_{CS}$:

$$
\begin{aligned}
C_5^6\Bigg|\begin{ytableau}1&2\\3\\4\end{ytableau}, [32^21^2]\Bigg\rangle_{CS^{3/2}} &= 49/4\Bigg|\begin{ytableau}1&2\\3\\4\end{ytableau}, [32^21^2]\Bigg\rangle_{CS^{3/2}}, \\
C_5^6\Bigg|\begin{ytableau}1&3\\2\\4\end{ytableau}, [32^21^2]\Bigg\rangle_{CS^{3/2}} &= 49/4\Bigg|\begin{ytableau}1&3\\2\\4\end{ytableau}, [32^21^2]\Bigg\rangle_{CS^{3/2}}, \\
C_5^6\Bigg|\begin{ytableau}1&4\\2\\3\end{ytableau}, [32^21^2]\Bigg\rangle_{CS^{3/2}} &= 49/4\Bigg|\begin{ytableau}1&4\\2\\3\end{ytableau}, [32^21^2]\Bigg\rangle_{CS^{3/2}}, \\
C_5^6\Bigg|\begin{ytableau}1&2\\3\\4\end{ytableau}, [1^3]\Bigg\rangle_{CS^{3/2}} &= 21/4\Bigg|\begin{ytableau}1&2\\3\\4\end{ytableau}, [1^3]\Bigg\rangle_{CS^{3/2}}, \\
C_5^6\Bigg|\begin{ytableau}1&3\\2\\4\end{ytableau}, [1^3]\Bigg\rangle_{CS^{3/2}} &= 21/4\Bigg|\begin{ytableau}1&3\\2\\4\end{ytableau}, [1^3]\Bigg\rangle_{CS^{3/2}}, \\
C_5^6\Bigg|\begin{ytableau}1&4\\2\\3\end{ytableau}, [1^3]\Bigg\rangle_{CS^{3/2}} &= 21/4\Bigg|\begin{ytableau}1&4\\2\\3\end{ytableau}, [1^3]\Bigg\rangle_{CS^{3/2}}.
\end{aligned}
\tag{8}
$$

From Eq. (6), the multiplets of $[32^21^2]([1^3])$ of $SU(6)_{CS}$ representation in Eq. (5) become the eigenstates of the quadratic Casimir operator of $SU(6)_{CS}$ for four quarks, C_4^6, having $26/3$ as its eigenvalue, as follows:

$$
\begin{aligned}
C_4^6\Bigg|\begin{ytableau}1&2\\3\\4\end{ytableau}, [32^21^2]\Bigg\rangle_{CS^{3/2}} &= 26/3\Bigg|\begin{ytableau}1&2\\3\\4\end{ytableau}, [32^21^2]\Bigg\rangle_{CS^{3/2}}, \\
C_4^6\Bigg|\begin{ytableau}1&3\\2\\4\end{ytableau}, [32^21^2]\Bigg\rangle_{CS^{3/2}} &= 26/3\Bigg|\begin{ytableau}1&3\\2\\4\end{ytableau}, [32^21^2]\Bigg\rangle_{CS^{3/2}}, \\
C_4^6\Bigg|\begin{ytableau}1&4\\2\\3\end{ytableau}, [32^21^2]\Bigg\rangle_{CS^{3/2}} &= 26/3\Bigg|\begin{ytableau}1&4\\2\\3\end{ytableau}, [32^21^2]\Bigg\rangle_{CS^{3/2}}.
\end{aligned}
\tag{9}
$$

Also, this eigenvalue equation of C_4^6 is true of the Young-Yamanouchi bases in $[1^3]$ multiplets.

Therefore, we can calculate the matrix element of Eq. (3) in terms of $|\Psi_1\rangle$ and $|\Psi_2\rangle$ given in Eq. (5), by using Eqs. (9), (6), and (8).

Following the same procedure, one can construct the flavor ⊗ color ⊗ spin states for the remaining flavor cases for $S = 3/2$, which satisfy the antisymmetry property among four quarks. The number of independent color spin states can be obtained by noting that for the pentaquark, there are three independent color singlet states. Hence multiplying this to the spin degeneracy gives the total number of independent states for a given spin state. From the result, it is found that there are all together 12 color ⊗ spin states that are both color singlet and $S = 3/2$. These are expressed by the Young-Yamanouchi bases of the $SU(6)_{CS}$ representation among the four quarks, together with the $SU(6)_{CS}$ Young diagram for the full $q^4\bar{Q}$ pentaquark state,

$$\begin{aligned}
&\left|\begin{array}{|c|c|c|}\hline 1&2&3\\\hline 4\\\cline{1-1}\end{array}, [421^3]\right\rangle_{CS^{3/2}}, \quad \left|\begin{array}{|c|c|c|}\hline 1&2&4\\\hline 3\\\cline{1-1}\end{array}, [421^3]\right\rangle_{CS^{3/2}},\\
&\left|\begin{array}{|c|c|c|}\hline 1&3&4\\\hline 2\\\cline{1-1}\end{array}, [421^3]\right\rangle_{CS^{3/2}}, \quad \left|\begin{array}{|c|c|}\hline 1&2\\\hline 3\\\cline{1-1} 4\\\cline{1-1}\end{array}, [32^21^2]\right\rangle_{CS^{3/2}},\\
&\left|\begin{array}{|c|c|}\hline 1&3\\\hline 2\\\cline{1-1} 4\\\cline{1-1}\end{array}, [32^21^2]\right\rangle_{CS^{3/2}}, \quad \left|\begin{array}{|c|c|}\hline 1&4\\\hline 2\\\cline{1-1} 3\\\cline{1-1}\end{array}, [32^21^2]\right\rangle_{CS^{3/2}},\\
&\left|\begin{array}{|c|c|}\hline 1&2\\\hline 3\\\cline{1-1} 4\\\cline{1-1}\end{array}, [1^3]\right\rangle_{CS^{3/2}}, \quad \left|\begin{array}{|c|c|}\hline 1&3\\\hline 2\\\cline{1-1} 4\\\cline{1-1}\end{array}, [1^3]\right\rangle_{CS^{3/2}},\\
&\left|\begin{array}{|c|c|}\hline 1&4\\\hline 2\\\cline{1-1} 3\\\cline{1-1}\end{array}, [1^3]\right\rangle_{CS^{3/2}}, \quad \left|\begin{array}{|c|c|}\hline 1&2\\\hline 3&4\\\hline\end{array}, [3^21^3]\right\rangle_{CS^{3/2}},\\
&\left|\begin{array}{|c|c|}\hline 1&3\\\hline 2&4\\\hline\end{array}, [3^21^3]\right\rangle_{CS^{3/2}}, \quad \left|\begin{array}{|c|}\hline 1\\\hline 2\\\hline 3\\\hline 4\\\hline\end{array}, [1^3]\right\rangle_{CS^{3/2}}.
\end{aligned} \tag{10}$$

B. Other spin states

In analogy to the $S = 3/2$ case, we can apply the same procedure to the $S = 1/2$ case. In this case, there are all together 15 color ⊗ spin states that are both color singlet and $S = 1/2$, and that are expressed by the Young-Yamanouchi bases of the $SU(6)_{CS}$ representation among the four quarks, like Eq. (10),

$$\begin{aligned}
&\left|\begin{array}{|c|c|c|}\hline 1&2&3\\\hline 4\\\cline{1-1}\end{array}, [421^3]\right\rangle_{CS^{1/2}}, \quad \left|\begin{array}{|c|c|c|}\hline 1&2&4\\\hline 3\\\cline{1-1}\end{array}, [421^3]\right\rangle_{CS^{1/2}},\\
&\left|\begin{array}{|c|c|c|}\hline 1&3&4\\\hline 2\\\cline{1-1}\end{array}, [421^3]\right\rangle_{CS^{1/2}}, \quad \left|\begin{array}{|c|c|}\hline 1&2\\\hline 3\\\cline{1-1} 4\\\cline{1-1}\end{array}, [32^21^2]\right\rangle_{CS^{1/2}},\\
&\left|\begin{array}{|c|c|}\hline 1&3\\\hline 2\\\cline{1-1} 4\\\cline{1-1}\end{array}, [32^21^2]\right\rangle_{CS^{1/2}}, \quad \left|\begin{array}{|c|c|}\hline 1&4\\\hline 2\\\cline{1-1} 3\\\cline{1-1}\end{array}, [32^21^2]\right\rangle_{CS^{1/2}},\\
&\left|\begin{array}{|c|c|}\hline 1&2\\\hline 3\\\cline{1-1} 4\\\cline{1-1}\end{array}, [21]\right\rangle_{CS^{1/2}}, \quad \left|\begin{array}{|c|c|}\hline 1&3\\\hline 2\\\cline{1-1} 4\\\cline{1-1}\end{array}, [21]\right\rangle_{CS^{1/2}},\\
&\left|\begin{array}{|c|c|}\hline 1&4\\\hline 2\\\cline{1-1} 3\\\cline{1-1}\end{array}, [21]\right\rangle_{CS^{1/2}}, \quad \left|\begin{array}{|c|c|}\hline 1&2\\\hline 3&4\\\hline\end{array}, [21]\right\rangle_{CS^{1/2}},\\
&\left|\begin{array}{|c|c|}\hline 1&3\\\hline 2&4\\\hline\end{array}, [21]\right\rangle_{CS^{1/2}}, \quad \left|\begin{array}{|c|c|c|}\hline 1&2&3\\\hline 4\\\cline{1-1}\end{array}, [21]\right\rangle_{CS^{1/2}},\\
&\left|\begin{array}{|c|c|c|}\hline 1&2&4\\\hline 3\\\cline{1-1}\end{array}, [21]\right\rangle_{CS^{1/2}}, \quad \left|\begin{array}{|c|c|c|}\hline 1&3&4\\\hline 2\\\cline{1-1}\end{array}, [21]\right\rangle_{CS^{1/2}},\\
&\left|\begin{array}{|c|}\hline 1\\\hline 2\\\hline 3\\\hline 4\\\hline\end{array}, [2^41]\right\rangle_{CS^{1/2}}.
\end{aligned} \tag{11}$$

Finally, in the $S = 5/2$ case, there exists only one color ⊗ spin state coming from the $[32^21^2]$ representation in Table I,

$$\begin{aligned}
&\left|\begin{array}{|c|c|}\hline 1&2\\\hline 3\\\cline{1-1} 4\\\cline{1-1}\end{array}, [32^21^2]\right\rangle_{CS^{1/2}}, \quad \left|\begin{array}{|c|c|}\hline 1&3\\\hline 2\\\cline{1-1} 4\\\cline{1-1}\end{array}, [32^21^2]\right\rangle_{CS^{1/2}},\\
&\left|\begin{array}{|c|c|}\hline 1&4\\\hline 2\\\cline{1-1} 3\\\cline{1-1}\end{array}, [32^21^2]\right\rangle_{CS^{1/2}}.
\end{aligned} \tag{12}$$

It is straightforward to calculate the expectation value of $\lambda_1^c\lambda_2^c\vec{\sigma}_1\cdot\vec{\sigma}_2$ and $\lambda_4^c\lambda_5^c\vec{\sigma}_4\cdot\vec{\sigma}_5$ with respect to the 12 color ⊗ spin states for $S = 3/2$ given in Eq. (10) in a 12 by 12 matrix from. Then, by applying the permutation operator on either $\lambda_1^c\lambda_2^c\vec{\sigma}_1\cdot\vec{\sigma}_2$ or $\lambda_4^c\lambda_5^c\vec{\sigma}_4\cdot\vec{\sigma}_5$, one can obtain all the expectation values in the 12 by 12 matrix from. Here, we use the formula given by

$$(ij)\lambda_1^c\lambda_i^c\vec{\sigma}_1\cdot\vec{\sigma}_i(ij) = \lambda_1^c\lambda_j^c\vec{\sigma}_1\cdot\vec{\sigma}_j, \tag{13}$$

where (ij) is a permutation operator of finite group, S_4, which acts on the color ⊗ spin states in Eq. (10) as well as on the Young-Yamanouchi states corresponding to the Young diagram of q^4. The (ij) operator can be expressed

TABLE II. The expectation value of $-\sum_{i<j}^{5}\frac{1}{m_i m_j}\lambda_i^c\lambda_j^c\vec{\sigma}_i\cdot\vec{\sigma}_j$ of $q^4\bar{Q}$ in $SU(3)_F$ limit, which means $m_4=m_3=m_2=m_1$. The eigenvalue indicates the value of Eq. (3) when $m_5=m_1$.

(F,S)	$-\langle\sum_{i<j}^{5}\frac{1}{m_i m_j}\lambda_i^c\lambda_j^c\vec{\sigma}_i\cdot\vec{\sigma}_j\rangle$
$(\mathbf{15},1/2)$	$\frac{56}{3m_1{}^2}+\frac{32}{3m_1m_5}$
$SU(6)_{CS}$	$[2^4 1]$
Eigenvalue	$\frac{88}{3}$
$(\mathbf{15},3/2)$	$\frac{56}{3m_1{}^2}-\frac{16}{3m_1m_5}$
$SU(6)_{CS}$	$[1^3]$
Eigenvalue	$\frac{40}{3}$
$(\mathbf{15}',1/2)$	$\begin{pmatrix}\frac{4}{3m_1{}^2}-\frac{20}{3m_1m_5} & \frac{4}{3m_1{}^2}+\frac{4}{3m_1m_5} \\ \frac{4}{3m_1{}^2}+\frac{4}{3m_1m_5} & \frac{4}{3m_1{}^2}+\frac{28}{3m_1m_5}\end{pmatrix}$
$SU(6)_{CS}$	$[21]$, $[32^2 1^2]$
Eigenvalue	$-\frac{8}{3}(\sqrt{10}-1)$, $\frac{8}{3}(\sqrt{10}+1)$
$(\mathbf{15}',3/2)$	$\begin{pmatrix}\frac{88}{21m_1{}^2}+\frac{172}{21m_1m_5} & \frac{16\sqrt{10}}{21m_1{}^2}+\frac{16\sqrt{10}}{21m_1m_5} \\ \frac{16\sqrt{10}}{21m_1{}^2}+\frac{16\sqrt{10}}{21m_1m_5} & \frac{136}{21m_1{}^2}-\frac{368}{21m_1m_5}\end{pmatrix}$
$SU(6)_{CS}$	$[32^2 1^2]$, $[1^3]$
Eigenvalue	-12, $\frac{40}{3}$
$(\mathbf{15}',5/2)$	$\frac{8}{m_1{}^2}+\frac{16}{3m_1m_5}$
$SU(6)_{CS}$	$[32^2 1^2]$
Eigenvalue	$\frac{40}{3}$
$(\mathbf{3},1/2)$	$\begin{pmatrix}-\frac{14}{m_1{}^2}-\frac{22}{m_1m_5} & -\frac{2}{\sqrt{3}m_1{}^2}-\frac{2}{\sqrt{3}m_1m_5} \\ -\frac{2}{\sqrt{3}m_1{}^2}-\frac{2}{\sqrt{3}m_1m_5} & -\frac{46}{3m_1{}^2}+\frac{26}{3m_1m_5}\end{pmatrix}$
$SU(6)_{CS}$	$[21]$ $[421^3]$
Eigenvalue	$-\frac{8}{3}(\sqrt{31}+8)$, $\frac{8}{3}(\sqrt{31}-8)$
$(\mathbf{3},3/2)$	$-\frac{40}{3m_1{}^2}+\frac{20}{3m_1m_5}$
$SU(6)_{CS}$	$[421^3]$
Eigenvalue	$-\frac{20}{3}$
$(\bar{\mathbf{6}},1/2)$	$-\frac{16}{3m_1{}^2}-\frac{40}{3m_1m_5}$
$SU(6)_{CS}$	$[21]$
Eigenvalue	$-\frac{56}{3}$
$(\bar{\mathbf{6}},3/2)$	$-\frac{16}{3m_1{}^2}+\frac{20}{3m_1m_5}$
$SU(6)_{CS}$	$[3^2 1^3]$
Eigenvalue	$\frac{4}{3}$

using the generator of $SU(3)_C$ and $SU(2)_S$, given by $(1/3I+1/2\lambda_i^c\lambda_j^c)\times(1/2I+1/2\vec{\sigma}_i\cdot\vec{\sigma}_j)$ [20] acting on the color and spin space. Furthermore, it is also block diagonal in both spaces in a 12 by 12 matrix form. In the same way, we can calculate the expectation value of $\lambda_i^c\lambda_j^c\vec{\sigma}_i\cdot\vec{\sigma}_j$ with respect to the 15 color $\otimes$ spin states for $S=1/2$ given in Eq. (11) in a 15 by 15 matrix from.

By using the flavor $\otimes$ color $\otimes$ spin states for $S=1/2$, $S=3/2$, and $S=5/2$, the expectation values of Eq. (3) can be calculated, as given in Table II. In Table II, below each matrix element, we also show the relevant $SU(6)_{CS}$ representations for the pentaquark state as well as the eigenvalue of Eq. (3). As can be seen in the table, the most attractive channel is given by the 2×2 matrix valued $(F,S)=(\mathbf{3},1/2)$ state. Upon diagonalizing the matrix in the $m_5\to\infty$ one finds the eigenvalues $(-16, -40/3)$, where the lowest one corresponds to the most attractive pentaquark state discussed in Refs. [4,5]. It should be noted that the factor -16 in this case can also be naively obtained by assuming two diquarks (ud, us) in the $udus\bar{c}$

pentaquark. However, as noted from the case of H dibaryon, SU(3) breaking effects together with the additional attraction from the strong charm quarks are important to the realistic estimate of the stability: The color spin interaction from the $m_{J/\psi} - m_{\eta_c}$ is much stronger than from naively scaling the color spin splitting in the light quark sector by the charm quark mass [14].

IV. PENTAQUARK BINDING IN THE $SU(3)_F$ BROKEN CASE

To analyze the stability of the pentaquarks against the lowest threshold, we introduce a simplified form for the matrix element of the hyperfine potential term, where we approximate the spatial overlap factors by constants that depend only on the constituent quark masses of the two quarks involved,

$$H_{hyp} = -\sum_{i<j}^{5} C_{m_i m_j} \lambda_i^c \lambda_j^c \vec{\sigma}_i \cdot \vec{\sigma}_j. \tag{14}$$

We then assume that the difference between the pentaquark energy and the lowest threshold baryon meson states arises only from the hyperfine energy difference [21]. This is because other potential terms are linear in the number of quarks involved so that assuming that all hadrons occupy the same size, the differences of their contribution to the pentaquark and baryon meson cancel. For the color-color two body force, one notes

$$\sum \lambda_j^c \lambda_i^c = -\frac{1}{2} N \lambda_q^2, \tag{15}$$

where $N = N_q + N_{\bar{q}}$ is the sum of quark and antiquarks in the configuration and $\lambda_q^2 = \frac{16}{3}$ for both the quark and antiquark. In a full constituent quark model calculation, the spatial size will be different for all quark pairs involved. However, we believe that the dominant repulsion and attraction at short distance will be dominated by the color spin interaction as this force will be proportional to the sizes of the wave function while the others are to the differences in the sizes. In fact, it was shown that the short-range nuclear force in different channels can be well understood in terms of Pauli principle and color spin interaction along the line of arguments given in the present work [22].

To evaluate the binding energy of the pentaquark in terms of Eq. (14), we extract the $C_{m_i m_j}$ values from the relevant mass differences between baryons and between mesons when involving one antiquark. The relations are given by

$$\begin{aligned} \Delta - N &= 16 C_{uu}, \\ \Sigma^* - \Sigma + \Xi^* - \Xi &= 32 C_{us}, \\ \Omega_c^* - \Omega_c &= 16 C_{sc}, \qquad \Sigma_c^* - \Sigma_c = 16 C_{uc}, \\ 2\Omega + \Delta - (2\Xi^* + \Xi) &= 8 C_{ss} + 8 C_{uu}. \end{aligned} \tag{16}$$

TABLE III. The value of $C_{m_i m_j}$ (unit MeV).

C_{uu}	C_{us}	C_{ss}	C_{sc}	C_{uc}	$C_{u\bar{s}}$
18.25	12.87	6.55	4.43	4.12	18.65
$C_{s\bar{s}}$	$C_{s\bar{c}}$	$C_{u\bar{c}}$	$C_{c\bar{c}}$	$C_{u\bar{b}}$	$C_{s\bar{b}}$
13.49	6.75	6.65	5.29	2.15	2.25

We show the value of $C_{m_i m_j}$ in Table III, and for C_{cc} we take it to be $1/2 C_{c\bar{c}}$. It is important to extract the numbers from experimental mass difference rather than assessing the mass dependence through the naive relation $C_{m_i m_j} \propto \frac{1}{m_i m_j}$. For example, the mass difference between the J/ψ and η_c, which is around 113 MeV and comes dominantly from the color spin interaction, is much larger than that between ρ and π, which is around 640 MeV, multiplied by the ratios in the constituent quark masses $m_u^2/m_c^2 \sim 1/25$, which gives a mass difference of 26 MeV only. Therefore, although Table II is used to highlight the coefficients multiplying each mass dependent term, in actual calculation, the mass effect in the color spin interaction should be extracted from a more realistic calculation taking into account the wave function or from the observed mass difference originating from the color spin interaction, which is the approach taken in this work.

V. ISOSPIN BASIS

We now investigate the stability of the pentaquark with respect to isospin (I) and spin (S), and allow the antiquark of the pentaquark to be either $\bar{s}$, $\bar{c}$ or $\bar{b}$. When considering the $SU(3)_F$ broken case, one can use the Young-Yamanuchi basis for q^4 in the $SU(6)_{CS}$ representation for the pentaquark given in Eqs. (10)–(12) together with the permutation operators to find the flavor ⊗ color ⊗ spin states suitable for a certain symmetry, which is allowed by the Pauli principle. Since there are several flavor ⊗ color ⊗ spin states possible, we denote the number of those possible states by the multiplicity as given in Tables IV and V. By using those flavor ⊗ color ⊗ spin states, the value of Eq. (14) for the pentaquark is obtained from diagonalizing the matrix element of the hyperfine potential energy. Then, we calculate the binding energy, denoted by ΔE, by taking the difference of the hyperfine potential energy given in Eq. (14) between the pentaquark and its lowest threshold, which is given in Tables IV and V for each pentaquark.

We need to characterize isospin states of q^4 in order to classify the pentaquark with respect to I. As can be seen in [23], the iospin states to q^4 can be decomposed in the following way: $I = 0$ with Young diagram $[2^2]$ consisting of $uudd$ component, $I = 1$ with Young diagram [31] consisting of $uuud$ component, and $I = 2$ with Young diagram [4] consisting of $uuuu$.

TABLE IV. The quark configurations of $I = 1/2$ and $I = 3/2$ pentaquark states with their lowest threshold hadron components and their binding energies, ΔE, in units of MeV. The numbers in the brackets show the multiplicities of the flavor ⊗ color ⊗ spin states.

	$I=1/2$					
	$S=1/2$		$S=3/2$		$S=5/2$	
Quark Config.	ΔE	State	ΔE	State	ΔE	State
$uuds\bar{b}$	−77	$NB_s(5)$	−45	$NB_s^*(4)$	−4	$\Sigma^* B^*(1)$
$uuds\bar{c}$	−99	$ND_s(5)$	−39	$ND_s^*(4)$	−4	$\Sigma^* D^*(1)$
$uudc\bar{s}$	17	$\Lambda_c K(5)$	−88	$\Sigma_c^* K(4)$	−1	$\Sigma_c^* K^*(1)$
$uudc\bar{c}$	−34	$N\eta_c(5)$	−15	$NJ/\psi(4)$	−9	$\Sigma_c^* D^*(1)$
$sssu\bar{c}$	133	$\Xi D_s(3)$	−17	$\Xi D_s^*(3)$	−34	$\Xi^* D_s^*(1)$
$sssu\bar{b}$	87	$\Xi B_s(3)$	73	$\Xi B_s^*(3)$	−34	$\Xi^* B_s^*(1)$
	$I=3/2$					
	$S=1/2$		$S=3/2$		$S=5/2$	
Quark Config.	ΔE	State	ΔE	State	ΔE	State
$uuus\bar{c}$	214	$\Sigma D(3)$	−42	$\Delta D_s(3)$	0	$\Delta D_s^*(1)$
$uuus\bar{b}$	170	$\Sigma B(3)$	142	$\Sigma B^*(3)$	0	$\Delta B_s^*(1)$
$uuuc\bar{s}$	274	$\Sigma_c K(3)$	186	$\Sigma_c^* K(3)$	0	$\Delta D_s^*(1)$
$uuuc\bar{c}$	191	$\Sigma_c D(3)$	−20	$\Delta\eta_c(3)$	0	$\Delta J/\psi(1)$

TABLE V. The quark configurations of $I = 0$ and $I = 1$ pentaquark states with the lowest threshold hadron components and their binding energies, ΔE, in units of MeV. The numbers in the brackets show the multiplicities of the flavor ⊗ color ⊗ spin states.

	$I=0$						$I=1$					
	$S=1/2$		$S=3/2$		$S=5/2$		$S=1/2$		$S=3/2$		$S=5/2$	
Quark Config.	ΔE	State	ΔE	State	ΔE	State	ΔE	State	ΔE	State	ΔE	State
$udsc\bar{c}$	−124	$\Lambda\eta_c(7)$	−43	$\Lambda J/\psi(5)$	−12	$\Xi_c^* D^*(1)$	−46	$\Sigma\eta_c(8)$	−31	$\Sigma J/\psi(7)$	−44	$\Sigma^* J/\psi(2)$
$udss\bar{c}$	−117	$\Lambda D_s(4)$	−62	$\Lambda D_s^*(3)$	−7	$\Xi^* D^*(1)$	54	$\Sigma D_s(4)$	1	$\Sigma D_s^*(4)$	−17	$\Sigma^* D_s^*(1)$
$udcc\bar{s}$	−135	$\Xi_{cc} K(4)$	−94	$\Xi_{cc}^* K(3)$	−3	$\Xi_{cc}^* K^*(1)$	133	$\Xi_{cc} K(4)$	85	$\Xi_{cc}^* K(4)$	6	$\Xi_{cc}^* K^*(1)$
$udcc\bar{c}$	−38	$\Lambda_c\eta_c(4)$	−43	$\Lambda_c J/\psi(3)$	−17	$\Xi_{cc}^* D^*(1)$	14	$\Sigma_c\eta_c(4)$	−31	$\Sigma_c^*\eta_c(4)$	0	$\Sigma_c^* J/\psi(1)$
$udss\bar{b}$	−92	$\Lambda B_s(4)$	−67	$\Lambda B_s^*(3)$	−7	$\Xi^* B^*(1)$	24	$\Sigma B_s(4)$	20	$\Sigma B_s^*(4)$	−17	$\Sigma^* B_s^*(1)$
$uudd\bar{s}$	98	$NK(1)$	74	$NK^*(1)$								
$uudd\bar{c}$	66	$ND(1)$	58	$ND^*(1)$								
$uudd\bar{b}$	54	$NB(1)$	52	$NB^*(1)$								
$uuud\bar{s}$							337	$NK(2)$	−74	$\Delta K(2)$	0	$\Delta K^*(1)$
$uuud\bar{c}$							223	$ND(2)$	79	$ND^*(2)$	0	$\Delta D^*(1)$
$uuud\bar{b}$							175	$NB(2)$	172	$NB^*(2)$	0	$\Delta B^*(1)$

VI. RESULT AND SUMMARY

The result for the binding energy defined as the difference between the hyperfine interaction of the pentaquark against its lowest threshold values is given in Tables IV and V. As can be seen in Tables IV and V, it is found that the most attractive pentaquark states are those with $(I,S) = (0, 1/2)$, apart from $udcc\bar{c}$, as well as the $uuds\bar{c}$ with $(I,S) = (1/2, 1/2)$. To understand the reason why these particles could be bound states, we need to analyze the expectation matrix value of Eq. (14) in terms of a dominant color ⊗ spin state among the possible states. For these states, the dominant color ⊗ spin state comes from the $SU(6)_{CS}$ representation [21] having the Young diagram [31] for the four quarks,[1] for which the expectation value of Eq. (3) is −36, as can be seen in $(F = \mathbf{3}, S = 1/2)$ sector of Table II when $m_1 = m_5$. In fact, the $SU(6)_{CS}$ representation [21] state with $S = 1/2$ gives the most attractive contribution to the expectation value of Eq. (3) than any other state, and both the $I = 0$ and $I = 1/2$ comes from the breaking of the flavor **3** state of this representation.

In Table VI, we show the expectation value of Eq. (14) in terms of only a color ⊗ spin state coming from the $SU(6)_{CS}$ representation [21] as well as the corresponding binding

[1]This state corresponds to the most stable $P_{\bar{c}s}$ state discussed in Ref. [5].

TABLE VI. The expectation value of Eq. (14) coming from the dominant color ⊗ spin state for stable pentaquark candidate states (unit MeV).

$I = 0,\ S = 1/2$	$udcc\bar{s}$ $(\Delta E = -131)$
H_{hyp}	$11/4C_{cc}$-$11/2C_{c\bar{s}}$-$25/2C_{uc}$ -$33/2C_{u\bar{s}}$-$17/4C_{uu}$
$\Xi_{cc}K$	$8/3C_{cc}$-$32/3C_{uc}$-$16C_{u\bar{s}}$
$I = 0,\ S = 1/2$	$udsc\bar{c}$ $(\Delta E = -122)$
H_{hyp}	-$22/3C_{s\bar{c}}$-$44/3C_{u\bar{c}}$-$28/3C_{us}$-$14/3C_{uu}$
$\Lambda\eta_c$	-$8C_{uu}$-$16C_{c\bar{c}}$
$I = 0,\ S = 1/2$	$udss\bar{c}$ $(\Delta E = -113)$
H_{hyp}	$11/4C_{ss}$-$11/2C_{s\bar{c}}$-$25/2C_{us}$ -$33/2C_{u\bar{c}}$-$17/4C_{uu}$
ΛD_s	-$8C_{uu}$-$16C_{s\bar{c}}$
$I = 1/2,\ S = 1/2$	$uuds\bar{c}$ $(\Delta E = -92)$
H_{hyp}	-$33/4C_{s\bar{c}}$-$55/4C_{u\bar{c}}$-$21/2C_{us}$-$7/2C_{uu}$
PD_s	-$8C_{uu}$-$16C_{s\bar{c}}$

energy against its threshold represented in the third row for each state. It should be noted that H_{hyp} for each state reduces to $-36C_{m_im_j}$ when the $C_{m_im_j}$'s are taken to be a quark mass independent constant. It should be noted that all these possible stable states are related to the attractive pentaquark states discussed in Ref. [4,5] in the flavor SU(3) symmetric limit. However, it should also be pointed out that when the charm quark is also included, together with its hyperfine contribution, it is the $P_{cc}(udcc\bar{s})$ pentaquark configuration that is most attractive. This state has also been discussed recently in Ref. [24]. The next attractive state is $udsc\bar{c}$, which could also be stable.

It should be noted that for the pentaquark to be stable against strong decay, the respective attraction obtained in Table V should be large enough to overcome the additional kinetic energy needed to bring the pentaquark into a compact configuration. This additional repulsive energy can be estimated in a constituent quark model by comparing the total kinetic energy of a separated baryon meson configuration to that of the corresponding compact pentaquark configuration. The energy is related to $p_{\rm rel}^2/2\mu$, where $p_{\rm rel}$ is the relative momentum and thus the inverse size of the compact configuration while μ is the reduced mass of the lowest threshold baryon meson system [19]. For the $P_{cc}(udcc\bar{s})$ state, this is typically of order 100 MeV. Hence, the proposed pentaquark state could be a weakly stable pentaquark or a resonance state slightly above the lowest threshold, which is $\Xi_{cc} + K$ for this state.

The $P_{cc}(udcc\bar{s})$ could also be easily observed with the present detection limits. Noting that Ξ_{cc} has been recently discovered, one can just add an additional kaon to look for this possible resonance state. If the state is strongly bound, one could look at the $P_{cc}(udcc\bar{s}) \to \Lambda_c K^+ K^- \pi^+$ decay or any hadronic decay mode similar to those of $\Lambda_c D_s^+$. The proposed final state is just reconstructing K^+ instead of π^+ in the measurement of $\Xi_{cc}^{++} \to \Lambda_c K^- \pi^+ \pi^+$ reported in Ref. [10] and hence measurable immediately. Such a measurement would be the first confirmation of a flavor exotic pentaquark state.

ACKNOWLEDGMENTS

This work was supported by the Korea National Research Foundation under the Grants No. 2016R1D1A1B03930089 and No. 2017R1D1A1B03028419(NRF), and by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIP) (Grant No. 2016R1C1B1016270).

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