Three body open flavor decays of higher charmonium and bottomonium

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In the present work, we study the Okubo-Zweig-Iizuka-allowed three body open flavor decay properties of higher vector charmonium and bottomonium states with an extended quark pair creation model. For the bottomonium system, we get that (i) the $BB\pi$ and $B^*B^*\pi$ partial decay widths of the $\Upsilon(10860)$ state are consistent with the experiment, and the $BB^*\pi$ partial decay width of the $\Upsilon(10860)$ state is smaller but very close to Belle's experiment. Meanwhile, (ii) the $BB^*\pi$ and $B^*B^*\pi$ decay widths of $\Upsilon(11020)$ can reach 2–3 MeV. In addition, (iii) for most of the higher vector charmonium states, the partial decay widths of the $DD^*\pi$ and $D^*D^*\pi$ modes can reach up to several MeV, which may be observed in future experiments.

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I. INTRODUCTION

In 2003, the Belle Collaboration reported the first observation of the charmoniumlike state $X(3872)$ in exclusive $B^\pm \to K^\pm\pi^+\pi^-J/\psi$ decays [1]. This state was later confirmed by the CDF [2], D0 [3], *BABAR* [4], LHCb [5], CMS [6], and BESIII [7] Collaborations. Its quantum number is determined to be $I^GJ^{PC} = 0^+1^{++}$ by the LHCb Collaboration [5]. Following the discovery of $X(3872)$, a large number of charmoniumlike states have been observed over the last decades, such as $X(3940)$ [8], $X(4140)$ [9], $X(4160)$ [10], $\psi(4260)$ [11], $\psi(4360)$ [12], $\psi(4660)$ [13], and so on. These states have attracted lots of attention from theorists. Various hadron configurations including molecular state [14,15], hybrid meson [16,17], tetraquark [18,19], etc. have been proposed to explain their nature. A detailed review can be found in Ref. [20] and references therein.

Since the charmoniumlike states with normal quantum numbers have similar masses compared to the normal charmonium, in order to understand the nature of the exotic states, it is necessary to have a better understanding of the normal charmonium spectroscopy. In Ref. [21], Li *et al.* investigated the spectrum of higher charmonium with screened potential, and found that the vector states $\psi(4008)$, $\psi(4260)$, $\psi(4320/4360)$, and $\psi(4660)$ might be assigned as the $\psi(3S)$, $\psi(4S)$, $\psi(3D)$, and $\psi(6S)$ states, respectively, while $X(3940)$ and $X(4160)$ might be the $\eta_c(3S)$ and $\chi_{c0}(3P)$ states. However, according to the constituent quark model description by Segovia *et al.* [22], the mass of $\psi(4040)$, $\psi(4160)$, $X(4360)$, $\psi(4415)$, $\psi(4630)$, and $\psi(4660)$ are compatible with the $\psi(3S)$, $\psi(2D)$, $\psi(4S)$, $\psi(3D)$, $\psi(5S)$, and $\psi(4D)$ states. Among the charmonium or charmoniumlike states, the 1^{--} states are of special interest because they can be easily produced in the e^+e^- annihilation. In Table I, we have listed the predicted masses of the vector charmonium states from various models.

In addition, the decay properties of charmonium play a pivotal role in revealing the nature of charmonium. From Table I, we see that the masses of these states are well above the allowed two body open-charm decay threshold; thus the decay widths mainly come from the strong decays. A widely used framework for the strong decay is the quark pair creation ($^3P_0$) model. In this model, the $c\bar{c}$ pair in the initial charmonium regroups with a $q\bar{q}$ pair created from the vacuum, which carries the vacuum quantum number $J^{PC} = 0^{++}$, and then decays into the outgoing open-charm mesons. About forty years ago, Le Yaouanc *et al.* [23,24] used this model to study the open-charm strong decays of $\psi(4040)$ and $\psi(4415)$. In 2005, Barnes *et al.* performed a systematic study of the higher charmonium states just above 4.4 GeV, with the charmonium masses calculated in the Godfrey-Isgur (GI) model and a nonrelativistic potential model [25]. In 2012, Segovia *et al.* [22] studied the strong decays of the vector charmonium states. For $\psi(3770)$, $\psi(4040)$, $\psi(4160)$, and $\psi(4360)$ with $\psi(1D)$,

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TABLE I. The predicted charmonium masses from various models (in units of MeV).

State	QM [38]	QM [39]	QM [22]	SSE/EA [40]	NR/GI [25]	SP [21]	LP/SP [41]
$\psi(3^3S_1)$	4225	4100	4097	4078/4096	4072/4100	4022	4078/4030
$\psi(4^3S_1)$	4625	4450	4389	4398/4426	4406/4450	4273	4412/4281
$\psi(5^3S_1)$	⋯	⋯	4614	4642/4672	⋯	4463	4711/4472
$\psi(6^3S_1)$	⋯	⋯	⋯	4804/4828	⋯	4608	⋯
$\psi(2^3S_1)$	4230	4190	4153	4156/4165	4142/4194	4089	4144/4095
$\psi(3^3S_1)$	⋯	4520	4426	4464/4477	⋯	4317	4478/4336
$\psi(4^3S_1)$	⋯	⋯	4641	4690/4707	⋯	⋯	⋯
$\psi(5^3S_1)$	⋯	⋯	⋯	4840/4855	⋯	⋯	⋯

$\psi(3S)$, $\psi(2D)$, and $\psi(4S)$ assignments, the calculated widths are compatible with the experimental values. While for the $\psi(4415)$, $X(4640)$, and $\psi(4660)$ states, the difference between theoretical and experimental values of the total widths is larger. Recently, Gui *et al.* [26] studied the open-charm strong decays of higher charmonium states up to the $6P$ multiplet with their wave functions of charmonium states calculated in the linear potential and screened potential quark models. Moreover, the $^3P_0$ model has also been used to study the strong decays of bottomonium states [27–29].

Besides the two body decays, three body open flavor decay is also important access to dig into the properties of charmonium and bottomonium. In 2008, the Belle Collaboration [30] first measured the exclusive cross section for $e^+e^- \to D^0D^-\pi^+$ over the center-of-mass energy range (4.0–5.0) GeV with the initial-state radiation (ISR) method and observed the decay $\psi(4415) \to D^0D^-\pi^+$. A detailed study found that the decay is dominated by $\psi(4415) \to D\bar{D}_2^*(2460)$ and

$$\frac{\mathcal{B}(\psi(4415) \to D^0D^-\pi^+_{\text{nonresonant}})}{\mathcal{B}(\psi(4415) \to D\bar{D}_2^*(2460) \to D^0D^-\pi^+)} < 0.22 \quad (1)$$

at 90% C.L. In 2009, they further measured the cross section of the $e^+e^- \to D^0D^{*-}\pi^+ + \text{c.c.}$ process and found no evidence of $\psi(4260)$, $\psi(4360)$, $\psi(4415)$, $\psi(4630)$, or $\psi(4660)$ with limited statistics [31]. Recently, the BESIII Collaboration found two resonances in the $e^+e^- \to D^0D^{*-}\pi^+$ process [32–34]. The lower mass one is in good agreement with the $Y(4220)$, and the other one might be $\psi(4415)$. For the bottomonium state, the Belle Collaboration also measured $\Upsilon(10860)$ decays into B mesons [35]. The measured fractions are

$$f(B\bar{B}\pi) = (0.0 \pm 1.1 \pm 0.3)\%, \quad (2)$$

$$f(B\bar{B}^*\pi + B^*\bar{B}\pi) = (7.3^{+2.3}_{-2.1} \pm 0.8)\%, \quad (3)$$

$$f(B^*\bar{B}^*\pi) = (1.0^{+1.4}_{-1.3} \pm 0.4)\%. \quad (4)$$

The measured three-body fractions are significantly larger than the older predictions [36].

In Ref. [37], we extended the $^3P_0$ model to study the $\psi(4660) \to \Lambda_c\bar{\Lambda}_c$ process with two $q\bar{q}$ pairs created from the vacuum. In this paper, we follow the extended $^3P_0$ model to study the three body open flavor decays of higher charmonium and bottomonium states through a different rearrangement (see Fig. 1). In the framework of the extended $^3P_0$ model, we find that (i) the $BB\pi$ and $B^*B^*\pi$ partial decay widths of the $\Upsilon(10860)$ state are consistent with the experiment. (For simplicity, we abbreviate the $B\bar{B}\pi$, $B\bar{B}^*\pi + B^*\bar{B}\pi$, and $B^*\bar{B}^*\pi$ to $BB\pi$, $BB^*\pi$, and $B^*B^*\pi$, respectively. A similar abbreviation is also used for the charmonium decays.) The $BB^*\pi$ partial decay width of the $\Upsilon(10860)$ state is smaller but very close to Belle's experiment. (ii) The partial decay widths of the $DD^*\pi$ and $D^*D^*\pi$ modes can reach up to several MeV for the higher vector charmonium states. The three body open charm decay channels may be observed in the near future.

For the singly heavy hadrons that contain one heavy quark, the heavy quark symmetry plays a very important role in the discussion of their properties such as masses and decay widths. The theoretical framework is the well-known heavy quark effective theory (HQET). With the help of HQET, the strong interaction between the heavy hadron multiplets may be described by one or two coupling constants, which can greatly simplify the discussion. However, if there exist two heavy quarks, like the heavy quarkonium that we study here, the traditional heavy quark expansion does not work anymore because they may have different velocities. In the present work, the strong decay widths for the heavy quarkonium states are restricted by

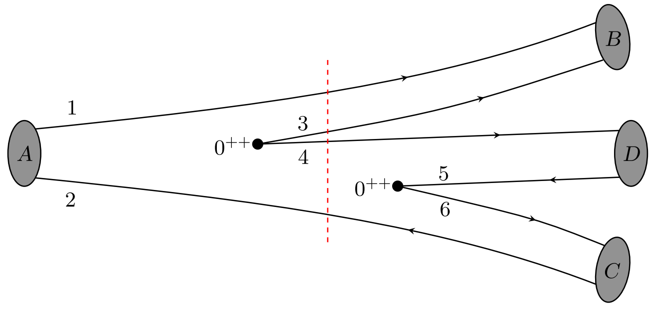

FIG. 1. The quarkonium (A) decays into three mesons (B+C+D). The intermediate state is marked by a red dashed line.

many factors, such as the Clebsch-Gordan series, masses of the states, and so on.

This paper is organized as follows. In Sec. II the $^3P_0$ model and its extension are briefly introduced. The numerical results are presented and discussed in Sec. III. Finally, a quick summary is given in Sec. IV.

II. THE $^3P_0$ MODEL

The $^3P_0$ model is widely used to calculate the Okubo-Zweig-Iizuka (OZI) allowed strong decays. It was first proposed by Micu [42] to study the strong decay properties of the *P*-wave mesons. Le Yaouanc *et al.* of the Orsay group further developed this model, and used it to study the open flavor strong decays of mesons [23,24,43] and baryons [44,45]. Since then, this model has been widely used in the study of baryon strong decays [25,28,46–51]. In the $^3P_0$ model, a light $q\bar{q}$ pair is created with the vacuum quantum number $J^{PC} = 0^{++}$ (hence "the $^3P_0$ model"), and then rearranged with the quarks within the initial meson to produce two final mesons. The decay matrix element can be described by the interaction Hamiltonian [25,48,49]

$$H_{q\bar{q}} = \gamma \sum_f 2m_f \int \mathrm{d}^3x \bar{\psi}_f \psi_f, \tag{5}$$

where m_f is the constituent quark mass of light flavor f, and ψ_f is a Dirac field of quark. γ is a dimensionless constant standing for the $q\bar{q}$ pair creation strength, which can be extracted by fitting to data.

In Ref. [37], we extended the $^3P_0$ model to study the $\psi(4660) \to \Lambda_c \bar{\Lambda}_c$ decay process, which requires two light $q\bar{q}$ pairs to be created. Here, we go a further step to study the higher heavy quarkonium decaying into two heavy mesons plus a light meson, as shown in Fig. 1. The corresponding helicity amplitude $\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}M_{J_D}}$ is

$$\begin{aligned}\delta^3(\mathbf{p}_A - \mathbf{p}_B - \mathbf{p}_C - \mathbf{p}_D)\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}M_{J_D}} \\ = \sum_k \frac{\langle BCD|H_{q\bar{q}}|k\rangle\langle k|H_{q\bar{q}}|A\rangle}{E_k - E_A},\end{aligned} \tag{6}$$

where $\mathbf{p}_i$'s are the momenta of the hadrons; $|k\rangle$ stands for the intermediate state; E_A and E_k are energies of the initial and intermediate states, respectively. We first invoke the quark-hadron duality [52] to simplify the calculation of the rhs of the Eq. (6) at the quark level. More precisely, we take $E_k - E_A$ as a constant, namely $E_k - E_A \equiv 2m_q$ since the intermediate state differs from the initial state by a created $q\bar{q}$ pair at the quark level [37] (see Fig. 1). Under the above approximation, we can rewrite the Eq. (6) as

$$\begin{aligned}\delta^3(\mathbf{p}_A - \mathbf{p}_B - \mathbf{p}_C - \mathbf{p}_D)\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}M_{J_D}} \\ = \frac{\langle BCD|H_{q\bar{q}}H_{q\bar{q}}|A\rangle}{2m_q}.\end{aligned} \tag{7}$$

The corresponding transition operator in the nonrelativistic limit reads [37]

$$\begin{aligned}T = {}& \frac{9\gamma^2}{2m_q}\sum_{m,m'}\langle 1m; 1-m|00\rangle\langle 1m'; 1-m'|00\rangle \\ & \times \int \mathrm{d}^3\mathbf{p}_3\mathrm{d}^3\mathbf{p}_4\mathrm{d}^3\mathbf{p}_5\mathrm{d}^3\mathbf{p}_6\delta^3(\mathbf{p}_3+\mathbf{p}_4)\delta^3(\mathbf{p}_5+\mathbf{p}_6) \\ & \times \varphi_0^{34}\omega_0^{34}\chi_{1,-m}^{34}\mathcal{Y}_1^m\left(\frac{\mathbf{p}_3-\mathbf{p}_4}{2}\right)a_{3i}^\dagger b_{4j}^\dagger \\ & \times \varphi_0^{56}\omega_0^{56}\chi_{1,-m'}^{56}\mathcal{Y}_1^{m'}\left(\frac{\mathbf{p}_5-\mathbf{p}_6}{2}\right)a_{5i'}^\dagger b_{6j'}^\dagger,\end{aligned} \tag{8}$$

where $\mathbf{p}_i$ is the momentum of the ith quark created from vacuum. $\varphi_0 = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$ and $\omega_0 = \delta_{ij}$ stand for the flavor and color singlets, respectively. The solid harmonic polynomial $\mathcal{Y}_1^{m(m')}(\mathbf{p}) \equiv |\mathbf{p}|\mathcal{Y}_1^{m(m')}(\Omega_\mathbf{p})$ corresponds to the *P*-wave $q\bar{q}$ pair, and $\chi_{1,-m(m')}$ is the spin triplet state for the created $q\bar{q}$ pair. $a_i^\dagger b_j^\dagger$ is the creation operator denoting the $q\bar{q}$ pair creation in the vacuum.

We use the mock state [53] to define the meson (A)

$$\begin{aligned}& |A(N_A{}^{2S_A+1}L_A J_A M_{J_A})(\mathbf{p}_A)\rangle \\ & \quad = \sqrt{2E_A}\varphi_A^{12}\omega_A^{12}\sum_{M_{L_A},M_{S_A}}\langle L_A M_{L_A}; S_A M_{S_A}|J_A M_{J_A}\rangle \\ & \qquad \times \int \mathrm{d}^3\mathbf{p}_1\mathrm{d}^3\mathbf{p}_2\delta^3(\mathbf{p}_1+\mathbf{p}_2-\mathbf{p}_A) \\ & \qquad \times \Psi_{N_A L_A M_{L_A}}(\mathbf{p}_1,\mathbf{p}_2)\chi_{S_A M_{S_A}}^{12}|q_1(\mathbf{p}_1)q_2(\mathbf{p}_2)\rangle.\end{aligned} \tag{9}$$

Here the $\mathbf{p}_i$ ($i = 1$, 2) is the momentum of quarks in meson A. Then the helicity amplitude in the center-of-mass frame can be written as

$$\begin{aligned}\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}M_{J_D}}(A \to BCD) = {}& \frac{\gamma^2}{2m_q}\sqrt{16E_A E_B E_C E_D} \times \sum_{mm'}\sum_{M_{L_{A,B,C,D}},M_{S_{A,B,C,D}}}\langle 1m; 1-m|00\rangle\langle 1m'; 1-m'|00\rangle \\ & \times \langle L_A M_{L_A} S_A M_{S_A}|J_A M_{J_A}\rangle\langle L_B M_{L_B} S_B M_{S_B}|J_B M_{J_B}\rangle\langle L_C M_{L_C} S_C M_{S_C}|J_C M_{J_C}\rangle \\ & \times \langle L_D M_{L_D} S_D M_{S_D}|J_D M_{J_D}\rangle\langle \chi_{S_B M_{S_B}}^{13}\chi_{S_C M_{S_C}}^{26}\chi_{S_D M_{S_D}}^{45}|\chi_{S_A M_{S_A}}^{12}\chi_{1,-m}^{34}\chi_{1,-m'}^{56}\rangle \\ & \times \langle \varphi_B^{13}\varphi_C^{26}\varphi_D^{45}|\varphi_A^{12}\varphi_0^{34}\varphi_0^{56}\rangle \times I_{M_{L_B}M_{L_C}M_{L_D}}^{M_{L_A}mm'}(\mathbf{p}),\end{aligned} \tag{10}$$

where the factor $(-3)^2$ has been canceled by the color factor

$$\langle \omega_B^{13}\omega_C^{26}\omega_D^{45}|\omega_A^{12}\omega_0^{34}\omega_0^{56}\rangle = \frac{1}{9} \tag{11}$$

and $I^{M_{L_A},mm'}_{M_{L_B},M_{L_C},M_{L_D}}(\mathbf{p})$ is the momentum-space integration and more detailed calculations are shown in the Appendix. Finally, the decay width Γ reads

$$\Gamma = \int_0^\infty dE_B dE_C \frac{\pi^3}{M_A}\frac{1}{2J_A+1} \times \sum_{M_{J_{A,B,C,D}}} |\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}M_{J_D}}|^2. \tag{12}$$

Following the literature in this field [23,25,28,47–51], we adopt the simple harmonic oscillator (SHO) wave function to describe the momentum-space wave function of the meson

$$\psi^{\mathrm{SHO}}_{nlm}(\mathbf{p}) = \frac{(-1)^n(-i)^l}{\beta^{3/2}}\sqrt{\frac{2n!}{\Gamma(n+l+3/2)}}\left(\frac{p}{\beta}\right)^l \times \exp\left(-\frac{\mathbf{p}^2}{2\beta^2}\right)L_n^{l+1/2}\left(\frac{p^2}{\beta^2}\right)\mathcal{Y}_l^m(\Omega_{\mathbf{p}}), \tag{13}$$

where $L_n^{l+1/2}(p^2/\beta^2)$ is an associated Laguerre polynomial.

In the present work, we set $m_u = m_d = 220$ MeV, $m_s = 419$ MeV, $m_c = 1628$ MeV, and $m_b = 4977$ MeV for the constituent quark masses [39]. The masses of final state mesons are listed in Table II. For simplicity, we ignore the isospin breaking and obtain the meson masses by taking their isospin averages.

The harmonic oscillator strength β of light mesons takes the average value 400 MeV [37,51]. The parameter β's of heavy-light mesons (see Table II) are taken from Refs. [28,51], which are obtained by the relation

$$\int d^3\mathbf{p}|\psi^{\mathrm{SHO}}_{nlm}(\mathbf{p})|^2p^2 = \int d^3\mathbf{p}|\Phi(\mathbf{p})|^2p^2, \tag{14}$$

where the lhs is the root-mean-square momentum of the SHO wave function, and the rhs is the root-mean-square momentum calculated through the GI model [28,51]. We use $\beta = 500$ MeV for charmonium [25,54]. In Ref. [28], Godfrey *et al.* showed that the parameter β's are 638, 600, and 578 MeV for $\Upsilon(4^3S_1)$, $\Upsilon(5^3S_1)$, and $\Upsilon(6^3S_1)$, respectively; thus we adopt the average value as 600 MeV for bottomonium states in this work.

For the $q\bar{q}$ pair creation strength, we use $\gamma(c\bar{c}) = 6.95$ for charmonium decays, which is $\sqrt{96\pi}$ times of in Refs. [25,47] due to a different definition. However, in Ref. [28] it is found that this value underestimated the two body strong decay widths of bottomonium, and the fitting of the open bottom decays of the Υ sector gives $\gamma(b\bar{b}) = 10.42$. Here we adopt the same value of γ for Υ sector as in Ref. [28]. The uncertainty of γ is about 30% [28,49,51,56], and the partial decay width is proportional to γ^4. Thus the uncertainty of our results may be quite large.

TABLE II. Masses and harmonic oscillator strength β's of final state mesons used in the decays (in units of MeV).

Meson	State	Mass [55]	β [28,51]
π	$^1S_0$	138.0	400
ρ	$^3S_1$	775.3	400
ω	$^3S_1$	782.6	400
η	$^1S_0$	547.9	400
D	$^1S_0$	1867.2	600
D^*	$^3S_1$	2008.6	520
D_s	$^1S_0$	1968.3	650
D_s^*	$^3S_1$	2112.2	560
B	$^1S_0$	5279.5	580
B^*	$^3S_1$	5324.6	540
B_s	$^1S_0$	5366.9	640
B_s^*	$^3S_1$	5415.4	600

III. NUMERICAL RESULTS

A. Charmonium

There are six charmoniumlike states above the $DD\pi$ threshold, $\psi(4040)$, $\psi(4160)$, $\psi(4260)$, $\psi(4360)$, $\psi(4415)$, and $\psi(4660)$ with $J^{PC} = 1^{--}$. These states are of special interest since they can be easily produced from the e^+e^- annihilation. Note that the $\psi(4260)$ is usually not considered to be a conventional charmonium state [26], so we do not discuss it in the following.

1. ψ(4360)

The state $\psi(4360)$ was first observed by the *BABAR* Collaboration in the $e^+e^- \to \gamma_{\mathrm{ISR}}\pi^+\pi^-\psi(2S)$ process [12]. Later, the Belle Collaboration confirmed this state in the same process with a statistical significance of more than 8σ [13]. The average values of mass and width listed in PDG are $M = 4368 \pm 13$ MeV and $\Gamma_{\mathrm{tot.}} = 96 \pm 7$ MeV [55]. An interesting feature is that only the $\psi(2S)\pi^+\pi^-$ [and possibly $\psi_2(3823)\pi^+\pi^-$] decay mode(s) was observed, while the open charm decay modes are still missing [12,13,57].

$\psi(4360)$ was interpreted to be a 3^3D_1 state in the nonrelativistic screened potential model [21]. Ding *et al.* also interpreted $\psi(4360)$ as a 3^3D_1 charmonium by evaluating its e^+e^- leptonic widths, E1 transitions, M1 transitions, and the open flavor strong decays in the flux tube model. However, the possibility of the 4^3S_1 assignment cannot be ruled out [22]. As the possible assignments of $\psi(4360)$, it is crucial to study the decay properties of the $\psi(4^3S_1)$ and $\psi(3^3D_1)$. The theoretical predictions are listed in Table III.

TABLE III. The partial decay widths (in MeV) of the vector charmonium with a mass of 4368 MeV.

State	$\psi(4^3S_1)$	$\psi(3^3D_1)$
$\Gamma_{DD\pi}$	0.27	0.14
$\Gamma_{DD^*\pi}$	1.40	1.21
$\Gamma_{D^*D^*\pi}$	0.60	0.25
$\Gamma_{DD\eta}$	0.6 keV	0.3 keV

From Table III, the dominant three body decay mode for both $\psi(4^3S_1)$ and $\psi(3^3D_1)$ is $D\bar{D}^*\pi$ with a mass of $M = 4368$ MeV, and the predicted partial decay widths are

$$\Gamma[\psi(4^3S_1) \to DD^*\pi] \sim 1.40 \text{ MeV}, \tag{15}$$

and

$$\Gamma[\psi(3^3D_1) \to DD^*\pi] \sim 1.21 \text{ MeV}. \tag{16}$$

Combing the measured width of $\psi(4360)$, we further get the branching ratios

$$\mathcal{B}[\psi(4^3S_1) \to DD^*\pi] \sim 1.5\%, \tag{17}$$

$$\mathcal{B}[\psi(3^3D_1) \to DD^*\pi] \sim 1.3\%. \tag{18}$$

The sizeable branching ratios indicates that this state has a good potential to be observed in the $DD^*\pi$ decay channel if it indeed turns out to be either the state $\psi(4^3S_1)$ or $\psi(3^3D_1)$.

Meanwhile, the partial decay widths of $DD\pi$ and $D^*D^*\pi$ are sizable for the two assignments. If $\psi(4360)$ is the 4^3S_1 state, we predict

$$\Gamma(DD\pi):\Gamma(DD^*\pi):\Gamma(D^*D^*\pi) \sim 1.0:5.1:2.2, \tag{19}$$

while the 3^3D_1 assignment gives

$$\Gamma(DD\pi):\Gamma(DD^*\pi):\Gamma(D^*D^*\pi) \sim 1.0:8.1:1.7. \tag{20}$$

The $DD\eta$ decay mode is also available kinetically. However, our calculation shows that its width $[\mathcal{O}(0.1 \text{ keV})]$ is too small to be observed because of its tiny phase space.

Unfortunately the three body decay properties of the two assignments $\psi(4^3S_1)$ and $\psi(3^3D_1)$ are very similar, which cannot be used to distinguish these two states in future experiments.

2. $\psi(4415)$

The $\psi(4415)$ state was discovered by SLAC and LBL in e^+e^- annihilation [58]. Later, it was confirmed by the DASP Collaboration [59]. Its mass and width are (4421 ± 4) and (62 ± 20) MeV [55], respectively. This state is the unique vector charmonium with experimental data of three body decays. The present study of the state $\psi(4415)$ can not only provide an important test of our model but also let us obtain more information about the nature of $\psi(4415)$.

In Ref. [24], Le Yaouanc *et al.* used the $^3P_0$ model to calculate its open flavor decay and assigned it to be the 4^3S_1 state. Later, Barnes *et al.* confirmed this assignment by comparing the mass spectrum from GI model calculation. They calculated all ten open-charmed decay widths of $\psi(4415)$ using the $^3P_0$ model, and found that the total widths and the decay patterns were consistent with experiments [25]. Moreover, they predicted that DD_1 and DD_2^* were the major decay modes of $\psi(4415)$, and the latter prediction was confirmed by Belle Collaboration [30]. Thus it is essential to study the three body decay properties of $\psi(4^3S_1)$.

Fixing the mass of $\psi(4^3S_1)$ at $M = 4421$ MeV, we calculated its partial decay widths and listed them in Table IV. According to our calculation, its three body strong decay is governed by the $DD^*\pi$ channel with the branching ratio

$$\mathcal{B}[\psi(4^3S_1) \to DD^*\pi] \sim 3.2\%, \tag{21}$$

which is less than the upper limit ($< 11\%$) listed in PDG [55].

The role of the $D^*D^*\pi$ channel is also important in the decays. The predicted partial width ratio between $D^*D^*\pi$ and $DD^*\pi$ is

$$\frac{\Gamma[\psi(4^3S_1) \to D^*D^*\pi]}{\Gamma[\psi(4^3S_1) \to DD^*\pi]} \sim 0.5. \tag{22}$$

Meanwhile, the partial decay width of the $DD\pi$ mode is predicted to be

$$\Gamma[\psi(4^3S_1) \to DD\pi] \sim 0.38 \text{ MeV} \tag{23}$$

with the branching ratio

$$\mathcal{B}[\psi(4^3S_1) \to DD\pi] \sim 0.6\%. \tag{24}$$

This value is also less than the upper limit (2.2%) obtained by the Belle Collaboration [30]. To further confirm the

TABLE IV. The partial decay widths (in MeV) of the vector charmonium with a mass of 4421 MeV.

State	$\psi(4^3S_1)$	$\psi(5^3S_1)$	$\psi(3^3D_1)$
$\Gamma_{DD\pi}$	0.38	0.11	0.21
$\Gamma_{DD^*\pi}$	2.01	0.96	1.84
$\Gamma_{D^*D^*\pi}$	1.07	0.59	0.52
$\Gamma_{DD\eta}$	5.4 keV	1.7 keV	2.9 keV

nature of $\psi(4415)$ and test our results, more precise experimental data are badly needed.

Besides the 4^3S_1 assignment, there are other interpretations of $\psi(4415)$. In Ref. [21], Li *et al.* proposed that the mass of $\psi(4415)$ was compatible with 5^3S_1 rather than 4^3S_1 in the nonrelativistic screened potential model. However, Segovia *et al.* suggested that $\psi(4415)$ could be a 3^3D_1 state [22]. In the present work, we also calculate the partial decay widths of $\psi(5^3S_1)$ and $\psi(3^3D_1)$ with the mass of 4421 MeV. The predictions are collected in Table IV.

Fixing the masses at $M = 4421$ MeV, the dominant three body decay mode of $\psi(5^3S_1)$ and $\psi(3^3D_1)$ is $DD^*\pi$. The partial decay widths are

$$\Gamma[\psi(5^3S_1) \to DD^*\pi] \sim 0.96 \text{ MeV} \tag{25}$$

and

$$\Gamma[\psi(3^3D_1) \to DD^*\pi] \sim 1.84 \text{ MeV}. \tag{26}$$

The above decay widths are large enough to be observed in future experiments. Moreover,

$$\frac{\Gamma[\psi(3^3D_1) \to DD^*\pi]}{\Gamma[\psi(5^3S_1) \to DD^*\pi]} \sim 1.9, \tag{27}$$

which indicates that the $DD^*\pi$ branching ratio of $\psi(3^3D_1)$ is larger that of $\psi(5^3S_1)$.

In addition, considering the uncertainty of the predicted masses in various models, we plot the partial decay widths of the 4^3S_1, 5^3S_1, and 3^3D_1 $c\bar{c}$ states as the functions of the mass in the range of $M = (4300 - 4700)$ MeV in Figs. 2–4.

3. ψ(**4660**)

In 2007, the Belle Collaboration reported an enhancement $\psi(4660)$ when they measured the cross section of the $e^+e^- \to \pi^+\pi^-\psi(2S)$ process [13]. Later, the *BABAR* Collaboration confirmed the existence of the $\psi(4660)$ state in the same process [60]. The mass and width of the $\psi(4660)$ are (4643 ± 9) and (72 ± 11) MeV, respectively. According to various quark model calculations, there are six excited vector charmonium states around 4.6 GeV (see Table I), namely $\psi(4S)$, $\psi(5S)$, $\psi(6S)$, $\psi(3D)$, $\psi(4D)$, and $\psi(5D)$. In Ref. [37], we have studied the $\Lambda_c\bar{\Lambda}_c$ partial decay widths of these states. Here we discuss their three body decays.

According to the quark model calculation, the mass of $\psi(5^3S_1)$ is very close to $\psi(4660)$. Ding *et al.* also suggested that the $\psi(4660)$ is a 5^3S_1 charmonium after studying its e^+e^- leptonic widths, E1 transitions, M1 transitions, and the open flavor strong decays in the flux-tube model [61].

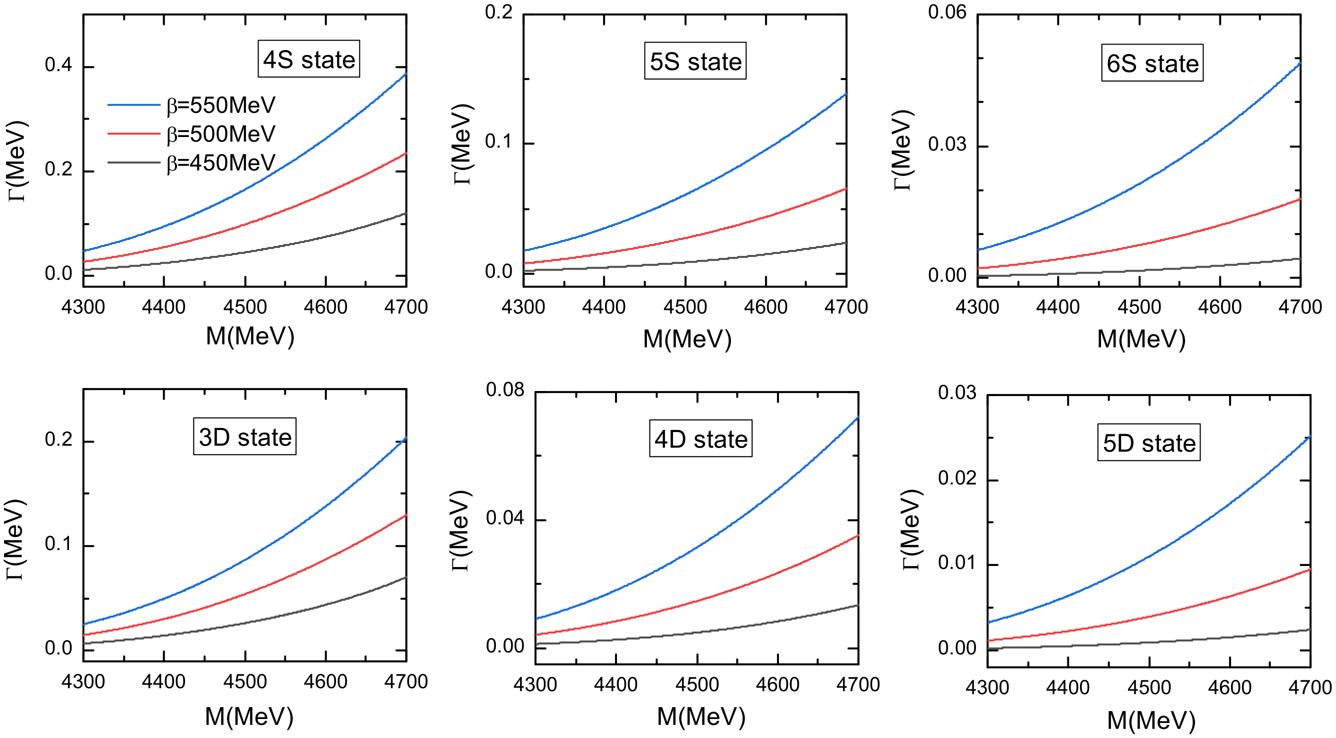

FIG. 2. The variation of the $D^+D^-\pi^0$ partial decay width with the mass of the D-wave vector charmonium. Note that $\Gamma_{D^+D^-\pi^0} = \frac{1}{6}\Gamma_{DD\pi}$ since we have ignored the isospin breaking. The blue, red, and black lines correspond to the predictions with different values of the harmonic oscillator strength $\beta = 450, 500,$ and 550 MeV, respectively.

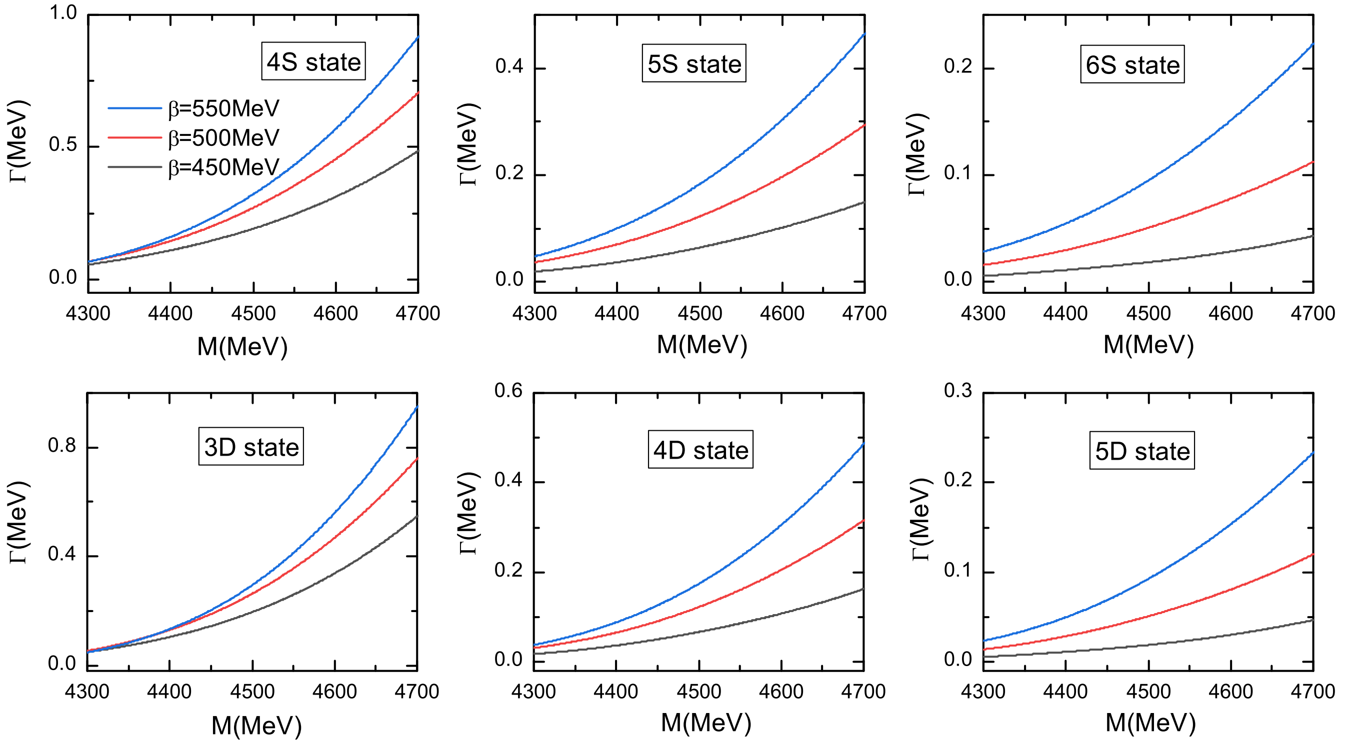

FIG. 3. The variation of the $D^+D^{*-}\pi^0$ partial decay width with the mass of the D-wave vector charmonium. Note that $\Gamma_{D^+D^{*-}\pi^0} = \frac{1}{12}\Gamma_{DD^*\pi}$ since we have ignored the isospin breaking. The blue, red, and black lines correspond to the predictions with different values of the harmonic oscillator strength $\beta = 450, 500$, and 550 MeV, respectively.

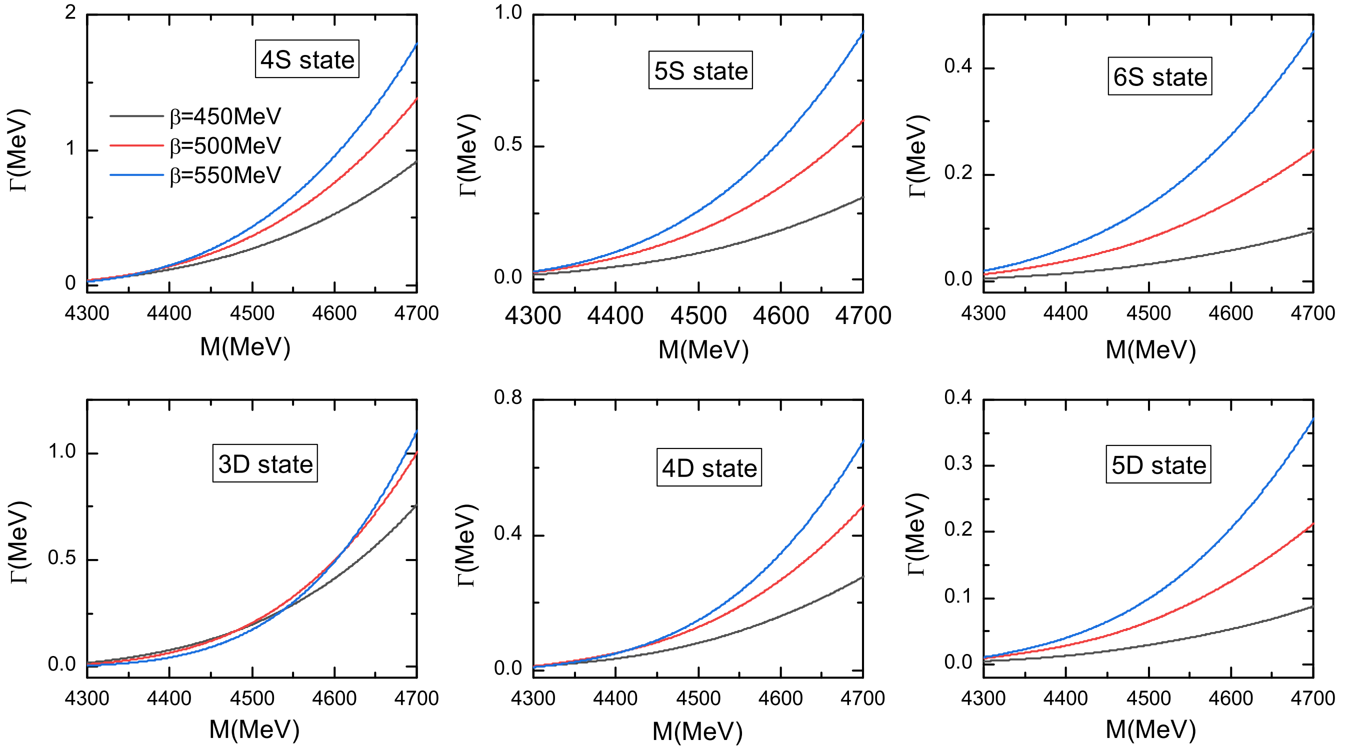

FIG. 4. The variation of the $D^{*+}D^{*-}\pi^0$ partial decay width with the mass of the D-wave vector charmonium. Note that $\Gamma_{D^{*+}D^{*-}\pi^0} = \frac{1}{6}\Gamma_{D^*D^*\pi}$ since we have ignored the isospin breaking. The blue, red, and black lines correspond to the predictions with different values of the harmonic oscillator strength $\beta = 450, 500$, and 550 MeV, respectively.

TABLE V. The partial decay width (MeV) of the vector charmonium with a mass of 4643 MeV.

State	$\psi(4^3S_1)$	$\psi(5^3S_1)$	$\psi(6^3S_1)$	$\psi(3^3D_1)$	$\psi(4^3D_1)$	$\psi(5^3D_1)$
$\Gamma_{DD\pi}$	1.14	0.31	0.09	0.63	0.17	0.05
$\Gamma_{DD^*\pi}$	6.65	2.83	1.10	6.99	2.99	1.16
$\Gamma_{D^*D^*\pi}$	5.97	2.68	1.13	4.12	2.11	0.96
$\Gamma_{DD\rho}$	0.85	0.41	0.16	1.86	0.64	0.22
$\Gamma_{DD\omega}$	0.24	0.12	0.05	0.59	0.20	0.07
$\Gamma_{DD\eta}$	53.2 keV	15.3 keV	4.2 keV	29.1 keV	8.2 keV	2.2 keV
$\Gamma_{DD^*\eta}$	0.25	0.12	0.05	0.20	0.11	0.05
$\Gamma_{D^*D^*\eta}$	58.2 keV	38.7 keV	19.1 keV	8.5 keV	9.9 keV	7.5 keV
$\Gamma_{D_sD_s\eta}$	3.0 keV	0.8 keV	0.2 keV	1.6 keV	0.4 keV	0.1 keV
$\Gamma_{D_sD_s^*\eta}$	1.9 keV	1.3 keV	0.6 keV	12 eV	11 eV	7 eV

As a possible assignment, we first study the decay property of the $\psi(5^3S_1)$ and list the corresponding results in Table V.

From the table, we find that the partial decay widths of $DD^*\pi$ and $D^*D^*\pi$ modes are quite large, which read

$$\mathcal{B}[\psi(5^3S_1) \to DD^*\pi] \sim 3.9\% \tag{28}$$

and

$$\mathcal{B}[\psi(5^3S_1) \to D^*D^*\pi] \sim 3.7\%, \tag{29}$$

respectively. The values are large enough to be observed in experiment. Meanwhile, the partial decay width of $\Gamma[\psi(5^3S_1) \to DD\pi]$ is considerable. The partial decay width ratio is

$$\frac{\Gamma[\psi(5^3S_1) \to DD\pi]}{\Gamma[\psi(5^3S_1) \to DD^*\pi]} \sim 0.1. \tag{30}$$

Besides $\psi(5^3S_1)$, the possibility that $\psi(4660)$ is a $\psi(4^3S_1)$ or $\psi(6^3S_1)$ state cannot be excluded completely. Thus we also calculate the partial decay widths of the $\psi(4^3S_1)$ and $\psi(6^3S_1)$ states. Similarly, we fix the mass of $\psi(4^3S_1)$ and $\psi(6^3S_1)$ at $M = 4643$ MeV, and collect their partial decay widths in Table V.

As listed in Table V, we obtain that the partial decay widths of $DD^*\pi$ and $D^*D^*\pi$ for $\psi(4^3S_1)$ are the largest compared to those for $\psi(5^3S_1)$ and $\psi(6^3S_1)$. The predicted branching ratios are

$$\begin{aligned} &\mathcal{B}[\psi(4^3S_1) \to DD^*\pi] \sim 9.2\%, \\ &\mathcal{B}[\psi(4^3S_1) \to D^*D^*\pi] \sim 8.3\%. \end{aligned} \tag{31}$$

However, the $\psi(6^3S_1)$ state gives the smallest branching ratios, which are

$$\begin{aligned} &\mathcal{B}[\psi(6^3S_1) \to DD^*\pi] \sim 1.5\%, \\ &\mathcal{B}[\psi(6^3S_1) \to D^*D^*\pi] \sim 1.6\%. \end{aligned} \tag{32}$$

Furthermore, we study the decay properties of the $\psi(3^3D_1)$, $\psi(4^3D_1)$, and $\psi(5^3D_1)$ states, and list their decay properties in Table V as well. Combined with the total width of $\psi(4660)$, we obtain the branching ratios of the $\psi(3^3D_1)$, $\psi(4^3D_1)$, and $\psi(5^3D_1)$ states as follows:

$$\mathcal{B}[\psi(3^3D_1) \to DD\pi] \sim 0.6\%, \tag{33}$$

$$\mathcal{B}[\psi(3^3D_1) \to DD^*\pi] \sim 9.7\%, \tag{34}$$

$$\mathcal{B}[\psi(3^3D_1) \to D^*D^*\pi] \sim 5.7\%; \tag{35}$$

$$\mathcal{B}[\psi(4^3D_1) \to DD\pi] \sim 0.2\%, \tag{36}$$

$$\mathcal{B}[\psi(4^3D_1) \to DD^*\pi] \sim 4.2\%, \tag{37}$$

$$\mathcal{B}[\psi(4^3D_1) \to D^*D^*\pi] \sim 2.9\%; \tag{38}$$

$$\mathcal{B}[\psi(5^3D_1) \to DD\pi] \sim 0.06\%, \tag{39}$$

$$\mathcal{B}[\psi(5^3D_1) \to DD^*\pi] \sim 1.6\%, \tag{40}$$

$$\mathcal{B}[\psi(5^3D_1) \to D^*D^*\pi] \sim 1.3\%. \tag{41}$$

These branching ratios are comparable to those of the S-wave states. If $\psi(4660)$ is a D-wave state, it is possible to be observed in the $DD^*\pi$ and $D^*D^*\pi$ channels as well.

In addition to the $DD\pi$, $DD^*\pi$, and $D^*D^*\pi$ channels, $\psi(4660)$ can also decay into $DD\rho$, $DD\omega$, $DD\eta$, $DD^*\eta$, $D^*D^*\eta$, $D_sD_s\eta$, and $D_sD_s^*\eta$ channels. In the same way, we fix the mass of the states $\psi(4S)$, $\psi(5S)$, $\psi(6S)$, $\psi(3D)$, $\psi(4D)$, and $\psi(5D)$ at $M = 4643$ MeV, and calculate their widths of decaying into these channels. The results are collected in Table V. The partial decay widths of these channels are relatively smaller. Among them, the partial decay widths of the $DD\rho$, $DD\omega$, and $DD^*\eta$ modes are around several tenths of MeV. If $\psi(4660)$ is one of the above states, it is still possible to observe these channels.

The mass spectrum predicted by various quark models bears a large uncertainty, and may have an effect on the partial decay widths. To investigate this effect, we vary the mass of the states $\psi(4S)$, $\psi(5S)$, $\psi(6S)$, $\psi(3D)$, $\psi(4D)$, and $\psi(5D)$ from 4300 to 4700 MeV, and calculate their corresponding decay widths. Here, we just plot the results for the $DD\pi$, $DD^*\pi$, and $D^*D^*\pi$ channels in Figs. 2–4, and omit the theoretical predictions of other channels since their decay widths are relatively smaller.

4. $\psi(4040)$ and $\psi(4160)$

There are two remaining states with $J^{PC}=1^{--}$, namely the $\psi(4040)$ and $\psi(4160)$ state. The $\psi(4040)$ state is commonly believed to be the 3^3S_1 $c\bar{c}$ state [25]. Its average mass and width are $M=4039\pm1$ MeV and $\Gamma_{\rm tot}=80\pm10$ MeV [55], respectively. This is the first 1^{--} charmonium above the $D\bar{D}\pi$ threshold. The mass of $\psi(4160)$ is 4191 ± 5 MeV [55], which is about 150 MeV heavier than that of $\psi(4040)$. According to the mass predictions in the quark model [25], this state is suggested to be the 2^3D_1 $c\bar{c}$ state. Its two body open charm decays have been studied by many authors, which also support this assignment [23,25,26].

We present our results in Table VI. According to our calculation, the partial decay width is

$$\Gamma[\psi(4040)\to DD\pi]\sim 20.9\ \text{keV}, \tag{42}$$

which is quite small compared to the total decay width of $\psi(4040)$. More precisely, the branching ratio is

$$\mathcal{B}[\psi(4040)\to DD\pi]\sim 2.6\times10^{-4}. \tag{43}$$

This ratio is smaller than that of the hidden charm decay modes of $\psi(4040)$ by 1 order. Because of the narrow partial decay width, the $DD\pi$ decay mode might be not easy to be observed.

The $DD^*\pi$ mode is also available for $\psi(4040)$. Since the $DD^*\pi$ mode has little phase space, the partial width of $\psi(4040)$ decaying into $DD^*\pi$ is about one magnitude smaller than the $DD\pi$ partial width. The partial decay width ratio is

$$\frac{\Gamma[\psi(4040)\to DD^*\pi]}{\Gamma[\psi(4040)\to DD\pi]}\sim 0.2. \tag{44}$$

TABLE VI. The $D^{(*)}D^{(*)}\pi$ partial decay width (in keV) for the two established charmonium states $\psi(4040)$ and $\psi(4160)$.

Meson	State	Mode	$\Gamma_{\rm th}^{\beta=450\ \rm MeV}$	$\Gamma_{\rm th}^{\beta=500\ \rm MeV}$	$\Gamma_{\rm th}^{\beta=550\ \rm MeV}$
$\psi(4040)$	3^3S_1	$D\bar{D}\pi$	14.7	20.9	24.0
		$DD^*\pi$	15.1	3.2	0.5
$\psi(4160)$	2^3D_1	$DD\pi$	80.4	119.8	142.8
		$DD^*\pi$	188.3	132.6	134.3
		$D^*D^*\pi$	0.4	0.1	0.2

We also analyze the three body decay properties of $\psi(4160)$ as the 2^3D_1 $c\bar{c}$ state, and collect its partial strong decay widths in Table VI. We obtain the partial decay widths

$$\Gamma[\psi(4160)\to DD\pi]\sim 119.8\ \text{keV} \tag{45}$$

and

$$\Gamma[\psi(4160)\to DD^*\pi]\sim 132.6\ \text{keV}. \tag{46}$$

The values are much bigger than the corresponding one of the $\psi(4040)$. These widths seem not large compared to its total width ($\Gamma_{\rm tot}=70\pm10$ MeV), but it is enough to be observed in those decay channels in experiments. Moreover, the branching ratios are predicted to be

$$\mathcal{B}[\psi(4160)\to DD\pi]\sim 1.7\times10^{-3} \tag{47}$$

and

$$\mathcal{B}[\psi(4160)\to DD^*\pi]\sim 1.9\times10^{-3}, \tag{48}$$

which are comparable to the upper limit of hidden charm decays of $\psi(4160)$. The partial decay width of $D^*D^*\pi$ mode is

$$\Gamma[\psi(4160)\to D^*D^*\pi]\sim 0.1\ \text{keV}. \tag{49}$$

This value is small and hard to be searched for at present.

The results of $\psi(4040)$ and $\psi(4160)$ may have large uncertainties due to their lower masses. At the hadron level, the allowed intermediate states with the spin parity $J^{PC}=1^{--}$ are $D\bar{D}_1$, $D^*\bar{D}_1$, $D^*\bar{D}_0$, $D^*\bar{D}_2$, $J/\psi f_0(500)$, $h_c(1P)\pi$, $h_c(1P)\eta$, $\chi_{c0}(1P)\omega(782)$, $\chi_{c2}(1P)\omega(782)$ and tetraquark states [18,62–64] and so on. Their masses are about $\sim(4.0-4.1)$ GeV. Thus for the higher mass states, such as $\psi(4660)$, $\psi(4415)$, and $\psi(4360)$, taking E_k-E_A as a constant is a reasonable assumption both at quark and hadron levels. However, for the lower mass states like $\psi(4040)$ and $\psi(4160)$, the E_k-E_A's are quite small and are sensitive to the masses of intermediate state. In this case, taking E_k-E_A as a constant will introduce a large uncertainty in our calculation.

B. Bottomonim

For the bottomonium system, there are three $b\bar{b}$ states above the open bottom threshold, namely $\Upsilon(4S)$, $\Upsilon(10860)$, and $\Upsilon(11020)$. A number of studies are available on the study of their strong decays with the $^3P_0$ model [27,28,65] and other models [36,66]. Most of them focus on the two body strong decays. However, the

$\Upsilon(10860)$ and $\Upsilon(11020)$ states can also decay into two bottomed mesons plus a π meson. Furthermore, these channels for the $\Upsilon(10860)$ state have recently been observed by the Belle Collaboration [35]. We investigate the three body decays of $\Upsilon(10860)$ and $\Upsilon(11020)$ with the extended ${}^3P_0$ model.

The $\Upsilon(10860)$ and $\Upsilon(11020)$ were discovered by the CLEO Collaboration in the e^+e^- annihilation [67]. Their masses and widths are [55]

$$m_{\Upsilon(10860)} = 10889.9^{+3.2}_{-2.6}\ \text{MeV}, \tag{50}$$

$$\Gamma_{\Upsilon(10860)} = 51^{+6}_{-7}\ \text{MeV}, \tag{51}$$

$$m_{\Upsilon(11020)} = 10992.9^{+10.0}_{-3.1}\ \text{MeV}, \tag{52}$$

$$\Gamma_{\Upsilon(11020)} = 49^{+9}_{-15}\ \text{MeV}. \tag{53}$$

They are usually assigned to be the 5^3S_1 and 6^3S_1 $b\bar{b}$ states in the quark model. We discuss the three body decays of the $\Upsilon(10860)$ and $\Upsilon(11020)$ states with this assignment.

The partial decay widths of the $\Upsilon(10860)$ state are listed in Table VII. According to our calculation, we obtain

$$\Gamma[\Upsilon(10860) \to BB\pi] \sim 0.20\ \text{MeV}, \tag{54}$$

$$\Gamma[\Upsilon(10860) \to BB^*\pi] \sim 1.22\ \text{MeV}, \tag{55}$$

$$\Gamma[\Upsilon(10860) \to B^*B^*\pi] \sim 0.61\ \text{MeV}. \tag{56}$$

The $BB^*\pi$ decay width is the largest one. Combining with the total width of $\Upsilon(10860)$, we obtain the branching ratios as follows:

$$\mathcal{B}[\Upsilon(10860) \to BB\pi] \sim 0.4\%, \tag{57}$$

$$\mathcal{B}[\Upsilon(10860) \to BB^*\pi] \sim 2.4\%, \tag{58}$$

$$\mathcal{B}[\Upsilon(10860) \to B^*B^*\pi] \sim 1.2\%. \tag{59}$$

The predicted branching ratios of the $BB\pi$ and $B^*B^*\pi$ decay modes are within the ranges of experimental values measured by the Belle Collaboration [55]. For the $BB^*\pi$ decay mode, our result is slightly smaller than the experiment data.

The partial decay widths of $\Upsilon(11020)$ state are also presented in Table VII. According to our calculations, we obtain the corresponding branching ratios

$$\mathcal{B}[\Upsilon(11020) \to BB\pi] \sim 0.7\%, \tag{60}$$

$$\mathcal{B}[\Upsilon(11020) \to BB^*\pi] \sim 6.5\%, \tag{61}$$

$$\mathcal{B}[\Upsilon(11020) \to B^*B^*\pi] \sim 5.5\%. \tag{62}$$

The branching ratios of the $BB^*\pi$ and $B^*B^*\pi$ channels are quite large. Thus these two channels may be observed by the Belle II Collaboration in the near future.

C. The effect of β

We have investigated the three body open flavor decays of five charmoniumlike states with various assignments and two bottomonium states. In this work, we carried out the calculation by fixing the harmonic oscillator parameter β to be 500 MeV (600 MeV) for charmonium (bottomonium) states. However, the parameter β of the initial states is not determined precisely, which may bring in uncertainty to our results. To estimate this effect, we carry out the preceding calculation by varying the parameter β of the initial states by 50 MeV. We investigate the decay properties with two different β values, $\beta = 450$ MeV and $\beta = 550$ MeV for charmonium states and $\beta = 550$ MeV and $\beta = 650$ MeV for bottomonium states. The numerical results are presented in Table VI, Figs. 2–4 for charmonium states, and Table VII for bottomonium states.

For the charmonium states $\psi(4360)$, $\psi(4415)$, and $\psi(4660)$ and bottomonium states $\Upsilon(10860)$ and $\Upsilon(11020)$, we notice that within a reasonable range of the parameter β, our main predictions and conclusions hold. However, for the $\psi(4040)$ and $\psi(4160)$ states, the decay widths are quite sensitive to β. Particularly, for $\psi(4040)$ decaying into $DD^*\pi$, when β changes by 50 MeV, the width varies by a factor of 5. As pointed out in Sec. III A 4, the $\psi(4040)$ and $\psi(4160)$ states are close to the mass of

TABLE VII. The $B^{(*)}\bar{B}^{(*)}\pi$ partial decay widths of the vector bottomonium (in units of MeV). $\mathcal{B}_{\text{exp}}$ represents the branching ratio for each corresponding channel.

			$\beta = 550$ MeV		$\beta = 600$ MeV		$\beta = 650$ MeV			
Meson	State	Mode	Γ_{th}	$\mathcal{B}_i$	Γ_{th}	$\mathcal{B}_i$	Γ_{th}	$\mathcal{B}_i$	[36]	$\mathcal{B}_{\text{exp}}$ [55]
$\Upsilon(10860)$	5^3S_1	$BB\pi$	0.12	0.2%	0.20	0.4%	0.28	0.5%		$(0.0 \pm 1.2)\%$
		$BB^*\pi$	1.36	2.7%	1.22	2.4%	0.94	1.8%	(23–30) keV	$(7.3 \pm 2.3)\%$
		$B^*B^*\pi$	0.68	1.3%	0.61	1.2%	0.47	0.9%	(5–6.6) keV	$(1.0 \pm 1.4)\%$
$\Upsilon(11020)$	6^3S_1	$BB\pi$	0.17	0.3%	0.34	0.7%	0.55	1.1%		
		$BB^*\pi$	2.50	5.1%	3.17	6.5%	3.41	7.0%		
		$B^*B^*\pi$	2.12	4.3%	2.69	5.5%	2.97	6.1%		

intermediate states, and taking $E_k - E_A$ as a constant introduces a large uncertainty in our calculation.

IV. CONCLUSIONS

In the present work, we have investigated the OZI-allowed three body open flavor decays of excited vector charmoniumlike states and bottomonium states in the framework of the extended $^3P_0$ model. In spite of the large uncertainty caused by the parameter γ, it is the first attempt along this direction in literatures to study this type of decay modes by considering the creation of two light $q\bar{q}$ pairs from vacuum. Our main results are summarized as follows.

For the well-established states $\psi(4040)$ and $\psi(4160)$, we estimate their three body open flavor decay properties with the assignments $\psi(3^3S_1)$ and $\psi(2^3D_1)$, respectively. The partial decay widths of $\psi(4040)$ should be fairly small (about several tens keV), and those of $\psi(4160)$ are a little larger, which are about 0.1 MeV for the $DD\pi$ and $DD^*\pi$ modes.

We also discuss the decay properties of $\psi(4360)$ as a candidate of $\psi(4^3S_1)$ or $\psi(3^3D_1)$. From our calculation, the partial decay width of $DD^*\pi$ mode can reach up to 1 MeV in both cases. Thus if $\psi(4360)$ is one of these states, it may be observed in the $DD^*\pi$ channel.

With the $\psi(4^3S_1)$ assignment, the $DD^*\pi$ and $D^*D^*\pi$ decay widths of $\psi(4415)$ are larger than 1 MeV. Meanwhile, the $DD\pi$ decay mode is sizable with a width of ∼0.38 MeV. Our predictions for the branching ratios of the $DD\pi$ and $DD^*\pi$ channels are within the upper limits measured by the Belle Collaboration [30,31]. However, assigning $\psi(4415)$ to be the $\psi(5^3S_1)$ or $\psi(3^3D_1)$ state, we obtain similar decay properties. Thus, to further determine the inner structure of $\psi(4415)$, more precise experimental data are needed.

We calculated the three body open flavor decay widths of $\psi(4660)$ with various assignments, $\psi(4S, 5S, 6S)$ and $\psi(3D, 4D, 5D)$. In both cases, its three body decays are dominated by the $DD^*\pi$ and $D^*D^*\pi$ channels, and the partial decay widths can reach up to several MeV. Meanwhile, we notice that the $DD\rho$ and $DD\omega$ decay widths of the D-wave states are larger than those of the S-wave states. If $\psi(4660)$ turns out to be $\psi(3D)$, its $DD\rho$ decay width even reaches up to 1.86 MeV.

We have also investigated the three body open flavor decays of $\Upsilon(10860)$ and $\Upsilon(11020)$. The branching ratios of $\Upsilon(10860)$ decaying into $BB\pi$ and $B^*B^*\pi$ are consistent with the experimental data, while the $BB^*\pi$ braching ratio is smaller but very close to the Belle measurement. For $\Upsilon(11020)$, the $BB\pi$, $BB^*\pi$, and $B^*B^*\pi$ decay widths are 0.34, 3.17, and 2.69 MeV, respectively. Hopefully the $BB^*\pi$ and $B^*B^*\pi$ decay modes of the $\Upsilon(11020)$ state will be observed by the Belle II Collaboration in the very near future.

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APPENDIX: THE MOMENTUM-SPACE INTEGRATION

The momentum-space integration $I^{M_{L_A},mm'}_{M_{L_B},M_{L_C},M_{L_D}}(\mathbf{p})$ reads

$$\begin{aligned} I^{M_{L_A},mm'}_{M_{L_B},M_{L_C},M_{L_D}}(\mathbf{p}) = \int d^3\mathbf{p}_1 & \mathcal{Y}_1^m(\mathbf{P}_B - \mathbf{p}_1)\mathcal{Y}_1^{m'}(-\mathbf{P}_C - \mathbf{p}_1) \\ & \times \psi^*_{n_B L_B M_{L_B}}(\mathbf{p}_1 - \kappa_1 \mathbf{P}_B) \\ & \times \psi^*_{n_C L_C M_{L_C}}(-\mathbf{p}_1 - \kappa_2 \mathbf{P}_C) \\ & \times \psi^*_{n_D L_D M_{L_D}}(\mathbf{p}_1 - \kappa_{32}\mathbf{P}_B + \kappa_{31}\mathbf{P}_C) \\ & \times \psi_{n_A L_A M_{L_A}}(\mathbf{p}_1), \end{aligned} \tag{A1}$$

where

$$\kappa_1 = \frac{m_1}{m_1 + m_3}, \tag{A2}$$

$$\kappa_2 = \frac{m_2}{m_2 + m_6}, \tag{A3}$$

$$\kappa_{31} = \frac{m_4}{m_4 + m_5}, \tag{A4}$$

$$\kappa_{32} = \frac{m_5}{m_4 + m_5}. \tag{A5}$$

In our calculation, only the S-wave states are considered for the final states. Thus we can rewrite the momentum-space integration $I^{M_{L_A},mm'}_{M_{L_B},M_{L_C},M_{L_D}}(\mathbf{p})$ as $\Pi(M_{L_A}, m, m')$.

For the decay of the 1S state,

$$\begin{aligned} \Pi(0, m, m') = & \left(\frac{1}{\pi\beta^2}\right)^{3/4}\left(\frac{1}{\pi\beta_B^2}\right)^{3/4}\left(\frac{1}{\pi\beta_C^2}\right)^{3/4}\left(\frac{1}{\pi\beta_D^2}\right)^{3/4} \\ & \times \exp\left[f(\mathbf{P}_B, \mathbf{P}_C)\right] \times \left(\frac{\pi}{\lambda_1}\right)^{3/2} \\ & \times \left[\frac{3(-1)^m}{8\pi\lambda_1}\delta_{m',-m} + \mathcal{Y}_1^m(-(\eta - 1)\mathbf{P}_B + \varpi\mathbf{P}_C)\right. \\ & \left. \times \mathcal{Y}_1^{m'}(-\eta\mathbf{P}_B + (\varpi - 1)\mathbf{P}_C)\right]. \end{aligned} \tag{A6}$$

For the decay of the 1D state

$$\begin{aligned}\Pi(M_{L_A},m,m') ={}& -\left(\frac{16}{15\sqrt{\pi}}\right)^{1/2}\frac{1}{\beta^{7/2}}\cdot\left(\frac{1}{\pi\beta_B^2}\right)^{3/4}\left(\frac{1}{\pi\beta_C^2}\right)^{3/4}\left(\frac{1}{\pi\beta_D^2}\right)^{3/4}\times\exp\left[f(\mathbf{P}_B,\mathbf{P}_C)\right]\left(\frac{\pi}{\lambda_1}\right)^{3/2}\\ &\times\left\{\frac{15}{16\pi\lambda_1^2}\cdot k^{(112)}_{m,m',M_{L_A}}\delta_{m+m',-M_{L_A}}+\frac{3}{8\pi\lambda_1}\cdot\left[(-1)^m\delta_{m',-m}\mathcal{Y}_2^{M_{L_A}}(\eta\mathbf{P}_B-\varpi\mathbf{P}_C)\right.\right.\\ &-(-1)^m\sqrt{\frac{40\pi}{3}}\mathcal{Y}_1^{m'}(-\eta\mathbf{P}_B+(\varpi-1)\mathbf{P}_C)\times\mathcal{Y}_1^{M_{L_A}+m}(\eta\mathbf{P}_B-\varpi\mathbf{P}_C)\times\langle 1-m;1M_{L_A}+m|2M_{L_A}\rangle\\ &\left.-(-1)^{m'}\sqrt{\frac{40\pi}{3}}\mathcal{Y}_1^{m}(-(\eta-1)\mathbf{P}_B+\varpi\mathbf{P}_C)\times\mathcal{Y}_1^{M_{L_A}+m'}(\eta\mathbf{P}_B-\varpi\mathbf{P}_C)\times\langle 1-m';1M_{L_A}+m'|2M_{L_A}\rangle\right]\\ &\left.+\mathcal{Y}_1^{m}(-(\eta-1)\mathbf{P}_B+\varpi\mathbf{P}_C)\times\mathcal{Y}_1^{m'}(-\eta\mathbf{P}_B+(\varpi-1)\mathbf{P}_C)\times\mathcal{Y}_2^{M_{L_A}}(\eta\mathbf{P}_B-\varpi\mathbf{P}_C)\right\}.\end{aligned}\tag{A7}$$

Here,

$$\lambda_1=\frac{1}{2\alpha^2}+\frac{1}{2\alpha_B^2}+\frac{1}{2\alpha_C^2}+\frac{1}{2\alpha_D^2},\tag{A8}$$

$$\lambda_2=\frac{\kappa_1}{\alpha_B^2}+\frac{\kappa_{32}}{\alpha_D^2},\tag{A9}$$

$$\lambda_3=\frac{\kappa_2}{\alpha_C^2}+\frac{\kappa_{31}}{\alpha_D^2};\tag{A10}$$

$$\begin{aligned}f(\mathbf{P}_B,\mathbf{P}_C)={}&\frac{(\lambda_2\mathbf{P}_B-\lambda_3\mathbf{P}_C)^2}{4\lambda_1}-\frac{\kappa_1^2\mathbf{P}_B^2}{2\alpha_B^2}\\ &-\frac{\kappa_2^2\mathbf{P}_C^2}{2\alpha_C^2}-\frac{(\kappa_{32}\mathbf{P}_B-\kappa_{31}\mathbf{P}_C)^2}{2\alpha_D^2},\end{aligned}\tag{A11}$$

$$\eta=\frac{\lambda_2}{2\lambda_1},\tag{A12}$$

$$\varpi=\frac{\lambda_3}{2\lambda_1}.\tag{A13}$$

When $m+m'+M_{L_A}=0$, $k^{(112)}_{m,m',M_{L_A}}$'s are nonvanishing and take the following values,

$$k^{(112)}_{\pm1,\pm1,\mp2}=\sqrt{\frac{3}{10\pi}},\tag{A14}$$

$$k^{(112)}_{\pm1,0,\mp1}=k^{(112)}_{0,\pm1,\mp1}=-\sqrt{\frac{3}{20\pi}},\tag{A15}$$

$$k^{(112)}_{\pm1,\mp1,0}=\frac{1}{\sqrt{20\pi}},\tag{A16}$$

$$k^{(112)}_{0,0,0}=\frac{1}{\sqrt{5\pi}}.\tag{A17}$$

Applying the above momentum space integrations to Eq. (10), we can calculate the $1S$ and $1D$ amplitudes. Amplitudes of the radially and orbitally excited states can be obtained by following recursion relations [37,68],

$$\mathcal{M}_{3S}=\sqrt{\frac{2}{15}}\left(\alpha^2\frac{\partial^2}{\partial\alpha^2}+\alpha\frac{\partial}{\partial\alpha}+\frac{3}{2}\right)\mathcal{M}_{1S},\tag{A18}$$

$$\mathcal{M}_{4S}=\frac{2}{3\sqrt{35}}\left(\alpha^3\frac{\partial^3}{\partial\alpha^3}+3\alpha^2\frac{\partial^2}{\partial\alpha^2}+\frac{15}{2}\alpha\frac{\partial}{\partial\alpha}\right)\mathcal{M}_{1S},\tag{A19}$$

$$\begin{aligned}\mathcal{M}_{5S}={}&\frac{2}{9\sqrt{70}}\left(\alpha^4\frac{\partial^4}{\partial\alpha^4}+6\alpha^3\frac{\partial^3}{\partial\alpha^3}+24\alpha^2\frac{\partial^2}{\partial\alpha^2}\right.\\ &\left.+18\alpha\frac{\partial}{\partial\alpha}+\frac{63}{4}\right)\mathcal{M}_{1S},\end{aligned}\tag{A20}$$

$$\begin{aligned}\mathcal{M}_{6S}={}&\frac{2}{45\sqrt{77}}\left(\alpha^5\frac{\partial^5}{\partial\alpha^5}+10\alpha^4\frac{\partial^4}{\partial\alpha^4}+60\alpha^3\frac{\partial^3}{\partial\alpha^3}\right.\\ &\left.+120\alpha^2\frac{\partial^2}{\partial\alpha^2}+\frac{675}{4}\alpha\frac{\partial}{\partial\alpha}\right)\mathcal{M}_{1S};\end{aligned}\tag{A21}$$

$$\mathcal{M}_{2D}=\sqrt{\frac{2}{7}}\alpha\frac{\partial}{\partial\alpha}\mathcal{M}_{1D},\tag{A22}$$

$$\mathcal{M}_{3D}=\frac{1}{3}\sqrt{\frac{2}{7}}\left(\alpha^2\frac{\partial^2}{\partial\alpha^2}+\alpha\frac{\partial}{\partial\alpha}+\frac{7}{2}\right)\mathcal{M}_{1D},\tag{A23}$$

$$\mathcal{M}_{4D}=\frac{2}{3\sqrt{231}}\left(\alpha^3\frac{\partial^3}{\partial\alpha^3}+3\alpha^2\frac{\partial^2}{\partial\alpha^2}+\frac{27}{2}\alpha\frac{\partial}{\partial\alpha}\right)\times\mathcal{M}_{1D},\tag{A24}$$

$$\begin{aligned}\mathcal{M}_{5D}={}&\frac{1}{3}\sqrt{\frac{2}{3003}}\left(\alpha^4\frac{\partial^4}{\partial\alpha^4}+6\alpha^3\frac{\partial^3}{\partial\alpha^3}+36\alpha^2\frac{\partial^2}{\partial\alpha^2}\right.\\ &\left.+30\alpha\frac{\partial}{\partial\alpha}+\frac{231}{4}\right)\mathcal{M}_{1D}.\end{aligned}\tag{A25}$$

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