Isovector scalar $a_0(980)$ and $a_0(1450)$ resonances in the $B \to \psi(K\bar{K}, \pi\eta)$ decays

Zhou Rui,* Ya-Qian Li, and Jie Zhang

College of Sciences, North China University of Science and Technology, Tangshan 063009, China

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We present an analysis of two isovector scalar resonant contributions to the B decays into charmonia plus $K\bar{K}$ or $\pi\eta$ pair in the perturbative QCD approach. The Flatté model for the $a_0(980)$ resonance and the Breit Wigner formula for the $a_0(1450)$ resonance are adopted to parametrize the timelike form factors in the dimeson distribution amplitudes, which capture the important final state interactions in these processes. The predicted distribution in the K^+K^- invariant mass as well as its integrated branching ratio for the $a_0(980)$ resonance in the $B^0 \to J/\psi K^+K^-$ mode agree well with the current available experimental data. The obtained branching ratio of the quasi-two-body decay $B^0 \to J/\psi a_0(980)(\to \pi^0\eta)$ can reach the order of 10^{-6}, letting the corresponding measurement appear feasible. For the $a_0(1450)$ component, our results could be tested by further experiments in the LHCb and Belle II. We also discuss some theoretical uncertainties in detail in our calculation.

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I. INTRODUCTION

Many scalar mesons with quantum numbers $J^P = 0^+$ have been well established in the experiment [1]. Amongst them, two important low-lying scalar resonances, namely, isoscalar $f_0(980)$ and isovector $a_0(980)$, are of special interest. Their almost degenerate masses would lead to a mixing with each other through isospin violating effects [2–5]. As their masses proximity to the $K\bar{K}$ threshold, both can strong coupling to $K\bar{K}$. Besides, their main individual decay chain are $f_0(980) \to \pi\pi$ and $a_0(980) \to \pi\eta$, respectively. Up to now, several B decays involving scalar mesons have been observed, either with an $f_0(980)$ [6,7] or $a_0(980)$ [8] in the final state. Most recently, the BESIII Collaboration reports the first observation of $a_0(980)$ meson in the semileptonic decay $D^0 \to a_0(980)^- e^+ \nu_e$ [9], which provides one more arena in the investigation of the nature of the puzzling $a_0(980)$ states. Above the $a_0(980)$ mass, another important isovector scalar state, $a_0(1450)$, had been observed in $p\bar{p}$ annihilation experiments [10,11] and the three-body D decays [12,13]. Measurements of B decays into a scalar meson can provide valuable information on constraining any phenomenological models trying to understand the nature of scalar mesons. In the quark model scenario, the composition of $f_0(980)$ and $a_0(980)$ have turned out to be mysterious. Their intriguing internal structure allows tests of various hypotheses, such as quark-antiquark [14], tetraquarks [15], $K\bar{K}$ molecule [16], and hybrid states [17]. In contrast to the unclear assignment of $a_0(980)$, it is widely accepted that $a_0(1450)$ is the isovector scalar $q\bar{q}$ ground state [1]. In particular, the lattice QCD calculations support that the lowest isovector scalar $q\bar{q}$ state corresponds to $a_0(1450)$ rather than $a_0(980)$ [18–20]. For recent lattice QCD studies of light scalar mesons, refer to [21–23].

From the theoretical perspective, studies of the three-body decays of the B meson with final states including a J/ψ will help us to clarify the nature of the resonances involved. In Ref. [24], the $B_{(s)}$ decay into J/ψ plus $K\bar{K}$ or $\pi\eta$ pair are studied by the chiral unitary approach, where the $K\bar{K}$ and $\pi\eta$ mass distributions are calculated for the relevant processes. More general review about the use of the chiral unitary approach to study the final-state strong interactions in weak decays, one refer to [25] for details. It is found both $f_0(980)$ and $a_0(980)$ resonances contribute to the $B^0 \to J/\psi K^+K^-$, while only the $f_0(980)$ [$a_0(980)$] resonance influences the distribution in $B_s \to J/\psi K^+K^-$ ($B^0 \to J/\psi \pi^0\eta$). The obtained results compared reasonably well with present experimental information. In Ref. [26], the authors extract information on $\pi\eta$ scattering through the $B^0 \to J/\psi(\pi\eta, K\bar{K})$ decays by the dispersion theory. The dimeson scalar form factors are introduced to describe the S-wave decay amplitude for the considered processes. The predicted decay rates are of the same order of magnitude as those of $\pi\pi$ analogues. Experimentally,

*jindui1127@126.com

evidence of the $a_0(980)$ resonance is reported with statistical significance of 3.9 standard deviations in the $B^0 \to J/\psi K^+K^-$ decay by the LHCb Collaboration [27]. The product branching fraction of the $a_0(980)$ resonance mode is measured for the first time, yielding

$$\mathcal{B}(B^0 \to J/\psi a_0(980)(\to K^+K^-)) = (4.70 \pm 3.31 \pm 0.72) \times 10^{-7}, \tag{1}$$

where the first uncertainty is statistical and the second is systematic.

The perturbative QCD (pQCD) approach [28,29] is one of the recently developed theoretical tools based on QCD to deal with various exclusive processes [30]. In our previous papers [31,32], the S-wave $\pi\pi(K\pi)$ resonant contributions are studied in the $B \to J/\psi\pi\pi(K\pi)$ decays as well as the $\psi(2S)$ counterparts. The related scalar resonance candidates include $f_0(500)$, $f_0(980)$, $f_0(1500)$, $f_0(1790)$, $K_0^*(1430)$, and so on. More recently, we studied the P-wave resonances, such as $\rho(770)$, $\rho(1450)$, and $\rho(1700)$, in the $\pi^+\pi^-$ channel [33]. In the present paper, we mainly focus on the isovector scalar resonances $a_0(980)$ and $a_0(1450)$ in the $B \to \psi(K\bar{K}, \pi\eta)$ decays with charmonia $\psi = J/\psi, \psi(2S)$, while the corresponding B_s decay modes are forbidden because the $s\bar{s}$ pair that has $I = 0$ and does not allow the isovector resonance production upon hadronization. The subjects related to the crossed-channel such as ψP with $P = K$, π, η and other higher partial wave are beyond the scope of the present analysis.

As is well known, the QCD dynamics for the three-body B decays are much more complicated than those of two-body ones, and the energy release scale for the b quark mass may be too low to allow for a complete factorization in the central region of the Dalitz plot (DP) [34–36]. However, based on the experimental fact that the three-body decays of B and D mesons clearly receive important contributions from intermediate resonances [1], one can assume that two of the three final-state mesons form a collimated meson pair, which is interpreted as an intermediate quasi-two-body final state, and in this case the factorization can be applied. Within the quasi-two-body approximation, the dominant kinematic region is restricted to the edges of a Dalitz plot, where the three daughter mesons are quasialigned in the rest frame of the parent particle.

Taking the decay $B \to J/\psi K\bar{K}$ for example, the dominant contributions come from the kinematic region, where the two light kaon mesons move almost parallelly for producing a resonance. The final state interactions between the bachelor particle J/ψ and the kaon pair are expected to be suppressed in such conditions. The inherently nonperturbative dynamics associated with the kaon pair can be parametrized into the complex timelike form factors involved in the two-kaon distribution amplitudes (DAs). For the $K\bar{K}$ form factors, we adopt the form as a linear combination of the $a_0(980)$ and $a_0(1450)$ resonances, where the former is described by the popular Flatté mass shapes based on the coupled channels $\pi\eta$ and $K\bar{K}$ [37], while the latter refer to the relativistic Breit-Wigner (BW) form.[1] The complex coefficient for each resonance are extracted from the isobar model fit results for the $D^0 \to K_S^0 K^-\pi^+$ mode performed by the LHCb experiment [13]. Following the steps of Refs. [31,32,39], the decay amplitude for the decays under investigation can be conceptually written as

$$\mathcal{A} = \Phi_B \otimes H \otimes \Phi_{K\bar{K}} \otimes \Phi_\psi, \tag{2}$$

where Φ_B and Φ_ψ are the B meson and charmonium DAs, respectively. The two-kaon DA $\Phi_{K\bar{K}}$ absorbs the nonperturbative dynamics of the hadronization processes in the $K\bar{K}$ system. The hard kernel H, similar to the case of two-body decays, includes the leading-order contributions plus the vertex corrections. The symbol $\otimes$ denotes the convolution in parton momenta of all the perturbative and nonperturbative objects.

The paper is structured as follows. In Sec. II, the elementary kinematics, the distribution amplitudes of initial and final states, and the required isovector scalar form factors are described. In Sec. III, we present a discussion following the presentation of the significant results on the branching ratios. Finally, Sec. IV will be the conclusion of this work.

II. FRAMEWORK

We consider the decay $B \to J/\psi K\bar{K}$ as an illustration, where $K\bar{K}$ can be either a neutral or a charged kaon pair. In what follows, we will use the abbreviation a_0 to denote the $a_0(980)$ and $a_0(1450)$ for simplicity. It is convenient to work in the rest frame of the B meson. Its momentum p_B, along with the charmonium meson momentum p_3, the kaon pair momentum p and other quark momenta k_i in each meson, which are shown diagrammatically in Fig. 1(a), are chosen as [33]

$$\begin{aligned}
p_B &= \frac{M}{\sqrt{2}}(1,1,\mathbf{0}_T), \quad p_3 = \frac{M}{\sqrt{2}}(r^2,1-\eta,\mathbf{0}_T), \\
p &= \frac{M}{\sqrt{2}}(1-r^2,\eta,\mathbf{0}_T), \\
k_B &= \left(0,\frac{M}{\sqrt{2}}x_B,\mathbf{k}_{BT}\right), \quad k_3 = \left(\frac{M}{\sqrt{2}}r^2x_3,\frac{M}{\sqrt{2}}(1-\eta)x_3,\mathbf{k}_{3T}\right), \\
k &= \left(\frac{M}{\sqrt{2}}z(1-r^2),0,\mathbf{k}_T\right),
\end{aligned} \tag{3}$$

[1]Although the width of $a_0(1450)$ is somewhat large, the parametrizations of its resonant effect is still controversial. We further note that most resonances including $a_0(1450)$ are widely described using a relativistic BW parametrization by several collaborations in the Dalitz-plot analysis for the three-body B/D decays [12,13,38]. Hence, here we also model the $a_0(1450)$ by a simple BW line shape with an energy dependent width.

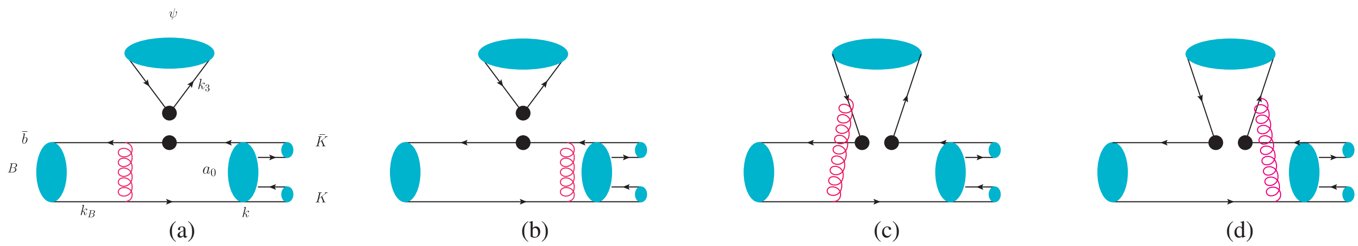

FIG. 1. The leading-order Feynman diagrams for the quasi-two-body decays $B \to \psi a_0(\to K\bar{K})$. (a,b) The factorizable diagrams, and (c,d) the nonfactorizable diagrams. a_0 is one of the isovector scalar intermediate states.

with the mass ratio $r = m/M$, and $m(M)$ is the mass of the charmonium (B) meson, the variable $\eta = \omega^2/(M^2 - m^2)$, and the invariant mass squared $\omega^2 = p^2$ for the kaon pair. The individual kaon momentum p_1 and p_2 in the $K\bar{K}$ pair are defined as

$$p_1 = \left(\zeta p^+, \eta(1-\zeta)p^+, \omega\sqrt{\zeta(1-\zeta)}, 0\right),$$
$$p_2 = \left((1-\zeta)p^+, \eta\zeta p^+, -\omega\sqrt{\zeta(1-\zeta)}, 0\right) \quad (4)$$

with ζ being the kaon momentum fraction. The momenta satisfy the momentum conservation $p = p_1 + p_2$. The three-momenta of the kaon and charmonium in the $K\bar{K}$ center of mass are given by

$$|\vec{p}_1| = \frac{\lambda^{1/2}(\omega^2, m_K^2, m_K^2)}{2\omega}, \quad |\vec{p}_3| = \frac{\lambda^{1/2}(M^2, m^2, \omega^2)}{2\omega}, \quad (5)$$

respectively, with m_K the kaon mass and the Källén function $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$. We do not spell out the kinematic relations for the $\pi\eta$ final state explicitly, inasmuch as they can be obtained from the above in a straightforward manner.

In the course of the PQCD calculations, the necessary inputs contain DAs of the initial and final states. The B meson can be treated as a heavy-light pseudoscalar system, the structure $\gamma_\mu\gamma_5$ and γ_5 components remain as leading contributions. Then, the B meson wave function with an intrinsic b (the conjugate space coordinate to k_T) dependence can be expressed by [40]

$$\Phi_B(x,b) = \frac{i}{\sqrt{2N_c}}[(\not{p}_B + M)\gamma_5\phi_B(x,b)], \quad (6)$$

with N_c the color factor. The DA $\phi_B(x,b)$ is adopted in the conventional form [40,41]

$$\phi_B(x,b) = Nx^2(1-x)^2\exp\left[-\frac{x^2M^2}{2\omega_b^2} - \frac{\omega_b^2 b^2}{2}\right], \quad (7)$$

with the shape parameter $\omega_b = 0.40 \pm 0.04$ GeV related to the factor N by the normalization

$$\int_0^1 \phi_B(x, b=0)dx = \frac{f_B}{2\sqrt{2N_c}}. \quad (8)$$

Since the concerned meson pair forms a spin-0 intermediate state, the final system only has a longitudinal component. For the charmonium states, the longitudinal polarized DAs are defined as [42,43]

$$\Phi_\psi^L = \frac{1}{\sqrt{2N_c}}[m\not{\epsilon}_L\phi^L(x,b) + \not{\epsilon}_L\not{p}_3\phi^t(x,b)], \quad (9)$$

with the longitudinal polarization vector $\epsilon_L = \frac{1}{\sqrt{2}r}(-r^2, 1-\eta, \mathbf{0}_T)$. The expressions of the $\phi^{L,t}$ are not shown here for the sake of brevity and can be found in Refs. [42,43].

The isovector scalar DAs are introduced in analogy with the case of two-pion ones [39,44], which are organized into

$$\Phi_{K\bar{K}(\pi\eta)}^{I=1} = \frac{1}{\sqrt{2N_c}}[\not{p}\phi_{v\mu=-}^{I=1}(z,\zeta,\omega^2) + \omega\phi_s^{I=1}(z,\zeta,\omega^2)$$
$$+ \omega(\not{n}\not{v} - 1)\phi_{t\mu=+}^{I=1}(z,\zeta,\omega^2)], \quad (10)$$

where $n = (1, 0, \mathbf{0}_T)$ and $v = (0, 1, \mathbf{0}_T)$ are two dimensionless vectors. For $I = 1$, $\phi_{v\mu=-}^{I=1}$ contributes at twist-2, while $\phi_s^{I=1}$ and $\phi_{t\mu=+}^{I=1}$ contribute at twist-3. It is worthwhile to mention that the concerned isovector scalar dimeson systems have similar asymptotic DAs as the ones for a light scalar meson [32,45], but we replace the scalar decay constants with the timelike form factor:

$$\phi_{v\mu=-}^{I=1}(z,\zeta,\omega^2) = \phi^0 = \frac{9}{\sqrt{2N_c}}F_s(\omega^2)B_1 z(1-z)(1-2z),$$
$$\phi_s^{I=1}(z,\zeta,\omega^2) = \phi^s = \frac{1}{2\sqrt{2N_c}}F_s(\omega^2),$$
$$\phi_{t\mu=+}^{I=1}(z,\zeta,\omega^2) = \phi^t = \frac{1}{2\sqrt{2N_c}}F_s(\omega^2)(1-2z), \quad (11)$$

which are the same as the two-pion one in [39], except for the different Gegenbauer moment B_1 due to the SU(3) breaking effects. Here we use $B_1 = 0.3$ for both $K\bar{K}$ and $\pi\eta$ pairs in the numerical analysis, which is determined from the data for the $B^0 \to J/\psi a_0(980)(\to K^+K^-)$ branching ratio [27].

As mentioned in the Introduction, the isovector scalar form factors for the concerned dimeson systems are given by the coherence summation of the two resonances $a_0(980)$ and $a_0(1450)$,

$$F^{I=1}_{K\bar{K}(\pi\eta)}(\omega^2) = \sum_{a_0} C_{a_0} M_{a_0}(\omega^2), \tag{12}$$

where $C_{a_0} = |C_{a_0}|e^{i\phi_{a_0}}$ is the corresponding complex amplitude for each intermediate state a_0. For the $K\bar{K}$ pair, the magnitude $|C_{a_0}|$ and phase ϕ_{a_0} can be obtained through a fit to the data, as done successfully in Ref. [13]. As mentioned in Ref. [13], the relevant parameters can be fixed in the isobar model fits in both the $D^0 \to K^0_S K^-\pi^+$ and $D^0 \to K^0_S K^+\pi^-$ modes, and two amplitude models have been constructed for each decay mode. Because the former has a higher signal yields and smaller mistag rate with respect to the latter, we prefer to the experimental solution

$$\begin{aligned} |C_{a_0(980)}| &= 1.07, \qquad \phi_{a_0(980)} = 82°, \\ |C_{a_0(1450)}| &= 0.43, \qquad \phi_{a_0(1450)} = -49°, \end{aligned} \tag{13}$$

which are taken from Table V of Ref. [13]. Since the experimental information on the $\pi\eta$ pair is not yet available, in this study, we roughly estimate their magnitudes $|C_{a_0}|$ by comparing the two form factors of the $K\bar{K}$ and $\pi\eta$ pairs. To achieve this, taking account of the charged dimeson pairs $\bar{K}^0K^+$ and $\eta\pi^+$, the relevant form factors $F_{K\bar{K}(\pi\eta)}(s)$, which enter the matrix elements for the transition from vacuum to the corresponding meson pairs via a $\bar{u}d$ source, are defined as [46,47]

$$\begin{aligned} \langle \bar{K}^0K^+|\bar{u}d|0\rangle &= B^0 F_{K\bar{K}}(\omega^2), \\ \langle \eta\pi^+|\bar{u}d|0\rangle &= B^0 F_{\pi\eta}(\omega^2), \end{aligned} \tag{14}$$

with the scale dependence factor B^0. Following the prescription in Refs. [48,49], by inserting a complete set of a_0 intermediate state into above matrix elements, we have

$$\begin{aligned} \langle \bar{K}^0K^+(\eta\pi^+)|\bar{u}d|0\rangle_{a_0} &\approx \langle \bar{K}^0K^+(\eta\pi^+)|a_0\rangle \frac{1}{BW_{a_0}} \langle a_0|\bar{u}d|0\rangle \\ &= \frac{g_{a_0KK}(g_{a_0\pi\eta})\bar{f}_{a_0}m_0}{BW_{a_0}}, \end{aligned} \tag{15}$$

with BW_{a_0} the resonance propagator [50]. Hereafter, m_0 refers to the pole mass of the resonance. The scalar decay constant and the strong coupling constant are defined by [34,51]

$$\begin{aligned} \langle a_0|\bar{u}d|0\rangle &= \bar{f}_{a_0}m_0, \\ \langle \bar{K}^0K^+(\eta\pi^+)|a_0\rangle &= g_{a_0KK}(g_{a_0\pi\eta}). \end{aligned} \tag{16}$$

By equating Eqs. (14) and (15), we link $F_{K\bar{K}(\pi\eta)}(\omega^2)$ with the usual Breit-Wigner expression through

$$F_{K\bar{K}(\pi\eta)}(\omega^2) = C^{K\bar{K}(\pi\eta)}_{a_0} \frac{m_0^2}{BW_{a_0}}, \tag{17}$$

with

$$C^{K\bar{K}(\pi\eta)}_{a_0} = \frac{g_{a_0KK}(g_{a_0\pi\eta})\bar{f}_{a_0}}{B^0 m_0}. \tag{18}$$

The combinations of $C^{K\bar{K}}_{a_0}$ and $C^{\pi\eta}_{a_0}$ lead to the ratio

$$\frac{C^{\pi\eta}_{a_0}}{C^{K\bar{K}}_{a_0}} = \frac{g_{a_0\pi\eta}}{g_{a_0KK}}, \tag{19}$$

where the values of the relative coupling $\frac{g_{a_0\pi\eta}}{g_{a_0KK}}$ for $a_0(980)$ and $a_0(1450)$ are taken from the Crystal Barrel experiment [52]. It can be seen the coefficients C_{a_0} have reflected the strength of the resonances a_0 decaying to the corresponding dimeson pair. We then can estimate the modules of the C_{a_0} for the $\pi\eta$ system by using Eq. (19), but keep their phases the same as in Eq. (13).

The partial amplitude M_R[2] appearing in Eq. (12) are chosen depends on the resonances in question. The $a_0(980)$ is a well established resonance but its shape is not well described by a simple Breit-Wigner formula because of the vicinity of the $K\bar{K}$ threshold. We follow the widely accepted prescription proposed by Flatté [37], based on the coupled channels $\pi\eta$ and $K\bar{K}$. The Flatté mass shapes are parametrized as

$$M_{a_0(980)}(\omega^2) = \frac{m_0^2}{m_0^2 - \omega^2 - i(g^2_{\pi\eta}\rho_{\pi\eta} + g^2_{KK}\rho_{KK})}, \tag{20}$$

with the nominal $a_0(980)$ mass $m_0 = 0.925$ GeV [13]. Note that the coupling constants $g_{KK}(g_{\pi\eta})$ in Eq. (20) are related to those in Eq. (16) through the relation $g_{KK}(g_{\pi\eta}) = g_{a_0KK}(g_{a_0\pi\eta})/(4\sqrt{\pi})$ according to the different definitions between Ref. [34] and Ref. [52]. In this study, we employ parameters $g_{\pi\eta} = 0.324$ GeV and $g^2_{K\bar{K}}/g^2_{\pi\eta} = 1.03$ from the Crystal Barrel experiment [52]. The ρ factors are given by the Lorentz-invariant phase space

[2]Here, we omit the relevant Blatt-Weisskopf centrifugal barrier factors and the angular distribution factors since in the scalar resonance case their values are equal to 1 [13,53,54].

$$\rho_{\pi\eta} = \sqrt{\left[1-\left(\frac{m_\eta - m_\pi}{\omega}\right)^2\right]\left[1-\left(\frac{m_\eta + m_\pi}{\omega}\right)^2\right]},$$
$$\rho_{K\bar{K}} = \frac{1}{2}\sqrt{1-\frac{4m_{K^\pm}^2}{\omega^2}} + \frac{1}{2}\sqrt{1-\frac{4m_{K^0}^2}{\omega^2}}. \tag{21}$$

The partial amplitude $M_{a_0 1450}(\omega^2)$ picks up the conventional Breit-Wigner model,

$$M_{a_0(1450)}(\omega^2) = \frac{m_0^2}{m_0^2 - \omega^2 - i m_0 \Gamma(\omega)}, \tag{22}$$

where $\Gamma(\omega)$ is its energy dependent width that is parametrized as in the case of a scalar resonance

$$\Gamma(\omega) = \Gamma_0 \frac{|\vec{p}_1|}{|\vec{p}_{10}|}\frac{m_0}{\omega}, \tag{23}$$

with $m_0 = 1.458$ GeV and $\Gamma_0 = 0.282$ GeV for the $a_0(1450)$ resonance [13]. The symbol $|\vec{p}_{10}|$ is used to indicate value of $|\vec{p}_1|$ at the resonance peak mass.

III. NUMERICAL RESULTS

The differential branching ratio for the considered decays is explicitly written as

$$\frac{d\mathcal{B}}{d\omega} = \frac{\tau\omega|\vec{p}_1||\vec{p}_3|}{32\pi^3 M^3}|\mathcal{A}|^2, \tag{24}$$

with τ the B meson lifetime. The resulting decay amplitudes $\mathcal{A}$ are equivalent to previous calculations in Ref. [39] by replacing the S-wave $\pi\pi$ form factor with the corresponding $K\bar{K}(\pi\eta)$ one in Eq. (12). To proceed with the numerical analysis, it is useful to summarize all of the input quantities entering the PQCD approach below:

(i) For the masses (in GeV) [1]: $M_B = 5.28$, $m_{J/\psi} = 3.097$, $m_{\psi(2S)} = 3.686$, $m_{K^\pm} = 0.494$, $m_{K^0} = 0.498$, $m_\eta = 0.548$, $m_{\pi^+} = 0.14$, $m_{\pi^0} = 0.135$, $m_b(\text{pole}) = 4.8$, $\overline{m_c}(\overline{m_c}) = 1.275$.

(ii) For the Wolfenstein parameters [1]: $\lambda = 0.22453$, $A = 0.836$, $\bar{\rho} = 0.122$, $\bar{\eta} = 0.355$.

(iii) For the decay constants (in GeV): $f_B = 0.19$ [1], $f_{J/\psi} = 0.405$ [42], $f_{\psi(2S)} = 0.296$ [43].

(iv) For the lifetimes (in ps) [1]: $\tau_{B_0} = 1.52$, $\tau_{B^+} = 1.638$.

The relevant parameters in the timelike form factors have been given in the previous section.

By using Eq. (24), integrating over the full invariant mass spectrum [$\omega_{\min} < \omega < M_B - m_\psi$ with $\omega_{\min} = 2m_{K^\pm}(m_{\pi^0} + m_\eta)$ for $K\bar{K}(\pi\eta)$ modes] separately for the individual resonant components, we derived the CP-averaged branching ratios for the neutral decay modes, which are summarized in Table I. The corresponding numbers for the charged decay modes can be obtained by multiplying the neutral branching ratios with a factor of $2\tau_{B^+}/\tau_{B^0}$ in the limit of isospin symmetry. For our results, we take into account the following theoretical uncertainties. The first uncertainty is from the shape parameter in the B meson wave function, $\omega_b = 0.40 \pm 0.04$. The second error originates from the Gegenbauer moment $B_1 = 0.3 \pm 0.1$ from the twist-2 DAs in Eq. (11). While the twist-3 DAs are taken as the asymptotic form for lack of better results from nonperturbative methods, this may also give large uncertainties. The third error is induced by the complex parameters C_{a_0} in Eq. (12). In the evaluation, we vary their magnitudes within a 10% range. The last one is caused by the variation of the hard scale from 0.75t to 1.25t, which characterizes the energy release in decay process. It is found that the main uncertainties of the concerned processes come from those nonperturbative parameters associated with the DAs of the B meson and dimeson pair. Their combined uncertainties can reach 50%. The uncertainties stemming from the Flatté parameters in Eq. (20) for the $a_0(980)$ channels are not included in Table I, whose effect on the branching ratio will be discussed in detail later. The errors from the uncertainty of the Cabibbo-Kobayashi-Maskawa matrix elements and the decay constants of charmonia are very small and have been neglected.

TABLE I. PQCD predictions for the concerned quasi-two-body decays involving the isovector scalar resonant a_0. The theoretical errors correspond to the uncertainties due to the shape parameters ω_b in the wave function of the B meson, the Gegenbauer moment B_1, the magnitude of the C_{a_0}, and the hard scale t, respectively.

Modes	$\mathcal{B}$
$B^0 \to J/\psi a_0(980)(\to K^+K^-)$	$(4.7^{+1.4+1.5+1.0+1.0}_{-0.8-1.0-0.9-0.4}) \times 10^{-7}$
$B^0 \to J/\psi a_0(980)(\to \pi^0\eta)$	$(6.0^{+1.6+1.9+1.3+1.1}_{-1.3-1.6-1.1-0.8}) \times 10^{-6}$
$B^0 \to J/\psi a_0(1450)(\to K^+K^-)$	$(6.8^{+3.8+0.7+1.4+0.4}_{-2.2-0.2-1.2-0.1}) \times 10^{-7}$
$B^0 \to J/\psi a_0(1450)(\to \pi^0\eta)$	$(1.1^{+0.5+0.1+0.2+0.0}_{-0.4-0.1-0.2-0.0}) \times 10^{-6}$
$B^0 \to \psi(2S) a_0(980)(\to K^+K^-)$	$(7.9^{+1.4+2.5+1.6+1.4}_{-1.3-1.7-1.5-0.8}) \times 10^{-8}$
$B^0 \to \psi(2S) a_0(980)(\to \pi^0\eta)$	$(1.5^{+0.3+0.4+0.3+0.2}_{-0.3-0.4-0.3-0.2}) \times 10^{-6}$
$B^0 \to \psi(2S) a_0(1450)(\to K^+K^-)$	$(6.1^{+4.5+0.9+1.3+0.4}_{-2.6-0.1-1.2-0.1}) \times 10^{-8}$
$B^0 \to \psi(2S) a_0(1450)(\to \pi^0\eta)$	$(1.2^{+0.6+0.1+0.3+0.0}_{-0.5-0.1-0.2-0.1}) \times 10^{-7}$

We notice that the branching ratios associated with $a_0(1450)$ modes are more sensitive to the shape parameter ω_b than the Gegenbauer moment B_1, whereas the situation is different for the corresponding processes of $a_0(980)$. It can be simply understood from the different twist contributions in the dimeson DAs. For $a_0(980)$ channels, the twist-2 and twist-3 contributions are of the same order, while for the $a_0(1450)$ case, the latter are more larger than the former. It is easy to observe that in Eq. (10), the twist-3 DAs always multiply by the invariant mass ω, and the larger pole mass induces larger contributions from twist-3 DAs. As the hard amplitude in Eq. (2) is convoluted with initial-state and final-state hadron DAs, the twist-3 contributions are concentrated in the endpoint region [41], which correspond to a small hard scale for the hard amplitude, such that the running coupling constant evaluated at that scale rise up rapidly. Therefore, the twist-3 contributions are more sensitive to the ω_b, which characterizes the shape of B meson DA. As stated above, the twist-3 DAs give the dominant contribution to the $a_0(1450)$ channels, thus their branching ratios depend heavily on ω_b and are less sensitive to the variation of the Gegenbauer moment B_1, which appears in the twist-2 DA.

It is well known that the $a_0(980)$ resonance mass always near the $K\bar{K}$ thresholds. As a result the predicted branching ratio of the $B^0 \to \psi a_0(980)(\to K\bar{K})$ decay is very sensitive to the choice of the resonance mass. It was pointed out in Refs. [55,56] that when we use the Flatté parametrization for the $a_0(980)$ resonance, there is a strong correlation between its mass and coupling constants. Therefore, here we do not take into account their individual uncertainty, but check the sensitivity of our results to the choice of these Flatté parameters. Actually, several phenomenological models [14,15,17] and experimental measurements [56–60] determined the relevant Flatté parameters (mass and couplings) as listed in Table II. Some coupling constants are converted into the numbers according to the definition in Ref. [52]. The first four parameter sets are taken from phenomenological models, while the remainder come from the experimental fitting. With each parameter set, we obtain the corresponding branching ratio shown in the last column of Table II. One can see that the parameters are quite model-dependent and suffers sizable uncertainty, which leads to the yielding branching ratios lie in a wide range $(2.8 \sim 18) \times 10^{-7}$. We expect that the new and improved data would help in constraining the relevant parameters and our theoretical understanding of the properties of the $a_0(980)$ resonances.

Now we turn to estimate the isospin breaking effect between the two physics final states K^+K^- and $K^0\bar{K}^0$ in the isovector a_0 channels. Since both the charged and neutral kaons decay channel open near the $a_0(980)$ resonance mass, the 8 MeV gap between the K^+K^- and $K^0\bar{K}^0$ thresholds make the latter mode suffer a further suppression from the phase space. Hence, the isospin breaking effect may be non-negligible in the $a_0(980)$ channels. In principle, as mentioned before, the nearly degenerate masses between the $a_0(980)$ and $f_0(980)$ resonances would lead to an admixture of them, which also cause to an important isospin-violating effects. However, it is not the theme of the present work. Here, we roughly estimate the isospin-violating effect from the kaon mass differences, but assume isospin symmetry for their coupling constants. To be more specific, we calculated the corresponding $K^0\bar{K}^0$ modes by using the same input parameters as the K^+K^- ones except for distinguishing the charged and neutral kaon masses, and found numerically that

$$\begin{aligned}
\mathcal{B}(B^0 \to J/\psi a_0(980)(\to K^0\bar{K}^0)) &= 4.3 \times 10^{-7}, \\
\mathcal{B}(B^0 \to \psi(2S) a_0(980)(\to K^0\bar{K}^0)) &= 7.1 \times 10^{-8}, \\
\mathcal{B}(B^0 \to J/\psi a_0(1450)(\to K^0\bar{K}^0)) &= 6.7 \times 10^{-7}, \\
\mathcal{B}(B^0 \to \psi(2S) a_0(1450)(\to K^0\bar{K}^0)) &= 5.8 \times 10^{-8}, \qquad (25)
\end{aligned}$$

which are typical smaller than the corresponding numbers for the charged kaon channels in Table I. For the $a_0(980)$ channels, the isospin breaking effect can reach roughly 10 percents even though the $a_0(980) - f_0(980)$ mixing is not included. For the case of $a_0(1450)$, the isospin breaking effect are rather small as expected since its resonance mass is far away from the two-kaon thresholds.

Let us also compare our results to those obtained in other methods. As stated before, the decay under investigation have been discussed in the chiral unitary approach [24] and the dispersion theory [26]. The authors of Ref. [24] subtract a smooth but large background from the differential decay to get the $a_0(980)$ contribution, and estimate the value $\mathcal{B}(B^0 \to J/\psi a_0(980)(\to \pi^0\eta)) = (2.2 \pm 0.2) \times 10^{-6}$, which is smaller than our numbers in Table I by a factor of 3. Nevertheless, a phenomenological estimate for the branching ratio of $B^0 \to J/\psi \pi^0\eta$ in the mass range above threshold up to 1.1 GeV in [26] gave a range $(6.0 \sim 6.4) \times 10^{-6}$ with the input phase $\delta_{12} = 90°$. For the sake of comparison, we derive a central value of 5.7×10^{-6} within the same energy region $[m_\pi + m_\eta, 1.1\ \mathrm{GeV}]$. This is consistent with their theoretical estimates within errors.

On the experimental side, the decay $B^0 \to J/\psi K^+K^-$ is first observed by the LHCb Collaboration. The relevant amplitude analysis is performed to separate resonant and nonresonant contributions in the K^+K^- spectrum. There is 3.9σ evidence for the $a_0(980)$ resonance with a product branching fraction of

$$\mathcal{B}(B^0 \to J/\psi a_0(980)(\to K^+K^-)) = (4.70 \pm 3.31 \pm 0.72) \times 10^{-7}, \qquad (26)$$

TABLE II. Masses and coupling constants of the $a_0(980)$ resonance in the Flatté parametrization determined from various theoretical models and experimental data. The last column correspond to the calculated branching ratios in the PQCD approach by using the corresponding parameters.

Model or experiment	$m_{a_0(980)}$ (MeV)	$g_{\pi\eta}$ (MeV)	g_{KK} (MeV)	$\mathcal{B}(B^0 \to J/\psi a_0(980)$ $(\to K^+K^-))$
$q\bar{q}$ model [14]	983	287	179	1.7×10^{-6}
$qq\bar{q}q$ model [14]	983	645	757	2.8×10^{-7}
$K\bar{K}$ model [15]	980	245	386	1.5×10^{-6}
$q\bar{q}g$ model [17]	980	355	278	1.2×10^{-6}
CB [56]	987.4	405	415	9.2×10^{-7}
SND [57]	995	439	592	6.6×10^{-7}
CLEO [58]	998	600	396	5.4×10^{-7}
KLOE [59][a]	982.5	303	397	1.3×10^{-6}
E852 [60]	1001	348	235	1.8×10^{-6}

[a]We quote the fit result for the KL model.

and an upper limit on its branching fraction is set to be 9.0×10^{-7} at 90% confidence level. The measured central value is close to our result in Table I, whereas the statistical uncertainty is too large to make a definite conclusion. As indicated in Table II, there is a notable uncertainty due to the different solutions of the Flatté parameters, and some of the results exceed the experimental upper limit. We suggest the experimentalists carry out a more precise measurement on this channel to constrain the relevant parameters, which allow us to discriminate between different models and improve the approach. On the other hand, the experiment information on the $\pi\eta$ channels are still scarcer. As a cross-check to the dynamical calculations, using the PQCD predictions as given in Table I we can estimate the relative ratios $\mathcal{R}^{\psi}_{a_0}=\frac{\mathcal{B}(B^0\to\psi a_0(\to K^+K^-))}{\mathcal{B}(B^0\to\psi a_0(\to\pi^0\eta))}$ as below,

$$\mathcal{R}^{J/\psi}_{a_0(980)}=0.08^{+0.04}_{-0.00},\qquad \mathcal{R}^{\psi(2S)}_{a_0(980)}=0.05^{+0.01}_{-0.00},$$
$$\mathcal{R}^{J/\psi}_{a_0(1450)}=0.62^{+0.02}_{-0.04},\qquad \mathcal{R}^{\psi(2S)}_{a_0(1450)}=0.51^{+0.10}_{-0.05},\tag{27}$$

where all uncertainties are added in quadrature. Under the narrow width approximation, above ratios obeys a simple factorization relation

$$\mathcal{R}^{\psi}_{a_0}\approx\frac{\mathcal{B}(B^0\to\psi a_0)\mathcal{B}(a_0\to K^+K^-)}{\mathcal{B}(B^0\to\psi a_0)\mathcal{B}(a_0\to\pi^0\eta)}=\frac{\Gamma(a_0\to K^+K^-)}{\Gamma(a_0\to\pi^0\eta)},\tag{28}$$

which allows us to test the ratios in Eq. (27). The average values of the relative partial decay widths $\Gamma(a_0\to K\bar{K})/\Gamma(a_0\to\pi^0\eta)$ given by Particle Data Group (PDG) [1] for the resonances $a_0(980)$ and $a_0(1450)$ are 0.183 ± 0.024 and 0.88 ± 0.23, respectively. Recalling that the isospin relation $\Gamma(a_0\to K^+K^-)=\Gamma(a_0\to K\bar{K})/2$, our calculations are in accordance with the data within errors.

The differential decay branching ratios versus the invariant mass ω are plotted in Fig. 2. Note that the $J/\psi-\psi(2S)$ mass difference causes significant differences in the range spanned in the respective decay modes. The blue solid and red dashed curves represent the contributions from the resonances $a_0(980)$ and $a_0(1450)$, respectively. The different shapes between the two resonances are mainly governed by the corresponding partial amplitude M_{a_0} and complex parameters C_{a_0} in Eq. (12). One can see in Fig. 2(a) and (c) that a clear narrow peak near the K^+K^- threshold for the $a_0(980)$ resonance, which makes

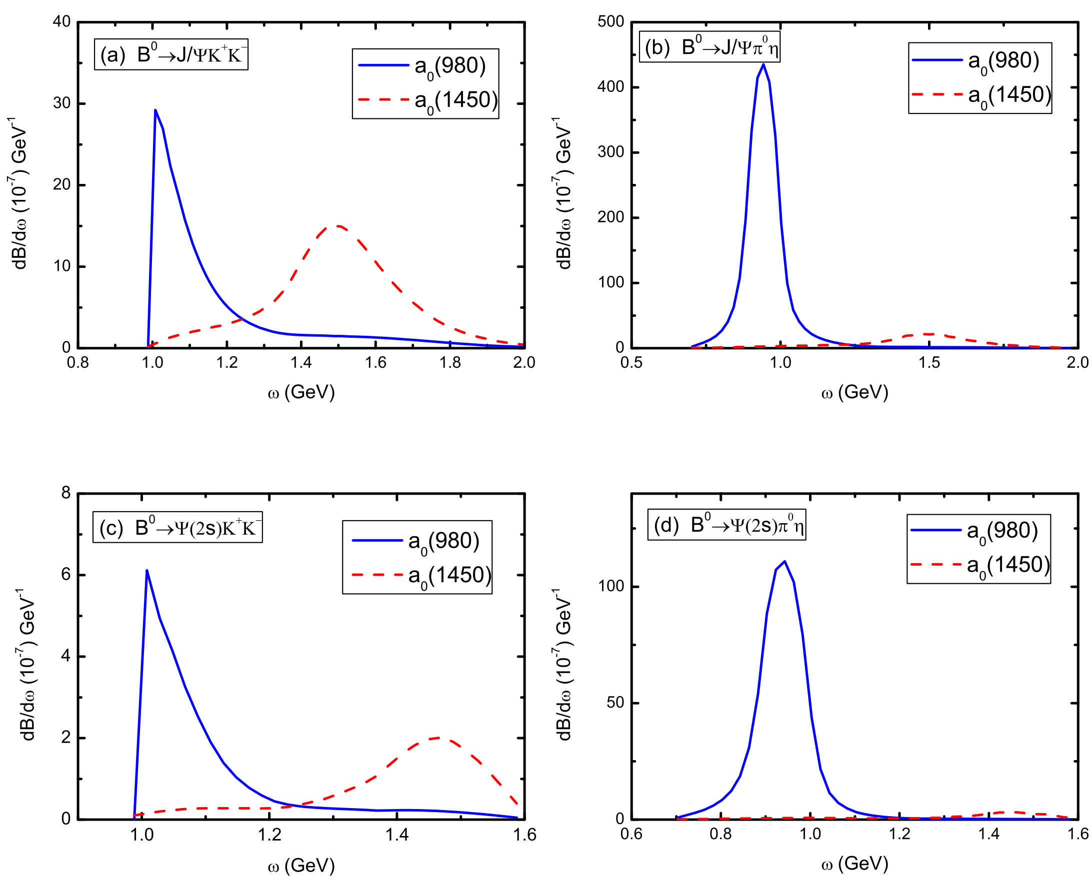

FIG. 2. Isovector scalar resonance contributions to the differential branching fractions of the modes (a) $B^0\to J/\psi K^+K^-$, (b) $B^0\to J/\psi\pi^0\eta$, (c) $B^0\to\psi(2S)K^+K^-$, and (d) $B^0\to\psi(2S)\pi^0\eta$. The blue solid lines corresponds to the resonant $a_0(980)$ contributions, while the red dashed to the $a_0(1450)$.

its distribution, is suppressed by the phase-space as mentioned before. The $a_0(1450)$ resonance peak has smaller strength than the $a_0(980)$ one, but its broader width compensate the integrated strength over the full K^+K^- invariant mass. Therefore, the contributions from the two resonances are of comparable size for the $K\bar{K}$ modes [see Table I]. In fact, the Crystal Barrel experiment [52] found the $a_0(1450)$ component is larger than that of $a_0(980)$ resonance in the process of $p\bar{p}$ annihilation into the $K\bar{K}\pi$ final state. In contrast to the $K\bar{K}$ channels, the $\pi^0\eta$ threshold far below the two resonance poles, and the strength of the $\pi^0\eta$ distribution is typical larger than the one for $K\bar{K}$, which enhanced its branching ratio accordingly. Just as expected, without the additional suppression from the phase space we observe an appreciable strength for $a_0(980)$ excitation and a less strong, but clearly visible excitation for the $a_0(1450)$ in Fig. 2(b) and 2(d). The obtained distribution for the $a_0(980)$ resonance contribution to the $B^0 \to J/\psi K^+K^-$ decay agrees fairly well with the LHCb data shown in Fig. 15 of Ref. [27], while other predictions could be tested by future experimental measurements.

IV. CONCLUSION

In this work we discuss the isovector scalar resonance contributions to the three-body $B^0 \to \psi(K\bar{K}, \pi\eta)$ decays under the quasi-two-body approximation based on the PQCD framework by introducing the corresponding dimeson DAs. The involved timelike form factors are parametrized as a linear combination of two components $a_0(980)$ and $a_0(1450)$, which can be described by the Flatté line shape and Breit-Wigner form, respectively. The predicted K^+K^- invariant mass distribution as well as its integrated branching ratio for the $a_0(980)$ resonance in the $B^0 \to J/\psi K^+K^-$ decay are in agreement with the findings by the LHCb Collaboration. It is found that the $a_0(1450)$ contribution is comparable with the $a_0(980)$ one for the $K\bar{K}$ modes, while fall short by a large factor for the $\pi\eta$ sector. In both resonances, the strength of the $\pi\eta$ invariant mass distribution are typical larger than the $K\bar{K}$ one in the channels with the same bachelor charmonia in the final state. The obtained branching ratios of the $B^0 \to \psi a_0(980)(\to\pi\eta)$ decays can reach the order of 10^{-6}, which would be straightforward for experimental observations.

We estimate the isospin breaking effect, which originates from the different thresholds of charged and neutral kaons, between the two physics final states K^+K^- and $K^0\bar{K}^0$ in the $a_0(980)$ and $a_0(1450)$ channels. For the former, the isospin breaking effect can reach roughly 10% even without the $a_0 - f_0$ mixing, while for the latter, the isospin breaking effect are negligible since its resonance mass is far away from the two-kaon thresholds.

We have discussed theoretical uncertainties arising from the nonperturbative parameters in the initial and final states DAs, and hard scale. The nonperturbative parameters contribute the main uncertainties in our approach, while the hard scale dependent uncertainty is less than 20% due to the inclusion of the vertex corrections. In addition, the $a_0(980)$ resonance contributions are largely dependence on the Flatté parameters, which should be constrained in the future.

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[1] M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018).
[2] J.-J. Wu, Q. Zhao, and B. S. Zou, Phys. Rev. D **75**, 114012 (2007).
[3] C. Hanhart, B. Kubis, and J. R. Pelaez, Phys. Rev. D **76**, 074028 (2007).
[4] T. Sekihara and S. Kumano, Phys. Rev. D **92**, 034010 (2015).
[5] W. Wang, Phys. Lett. B **759**, 501 (2016).
[6] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. D **72**, 072003 (2005); **74**, 099903(E) (2006).
[7] A. Garmash *et al.* (Belle Collaboration), Phys. Rev. Lett. **96**, 251803 (2006); Phys. Rev. D **75**, 012006 (2007).
[8] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. D **70**, 111102(R) (2004); **75**, 111102(R) (2007); **77**, 011101(R) (2008).
[9] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **121**, 081802 (2018).
[10] C. Amsler *et al.* (Crystal Barrel Collaboration), Phys. Lett. B **333**, 277 (1994).
[11] C. Amsler *et al.* (Crystal Barrel Collaboration), Phys. Lett. B **355**, 425 (1995).
[12] P. Rubin *et al.* (CLEO Collaboration), Phys. Rev. D **78**, 072003 (2008).
[13] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. D **93**, 052018 (2016).

[14] N. N. Achasov and V. N. Ivanchenko, Nucl. Phys. **B315**, 465 (1989).
[15] J. Weinstein and N. Isgur, Phys. Rev. D **27**, 588 (1983).
[16] J. Weinstein and N. Isgur, Phys. Rev. D **41**, 2236 (1990).
[17] S. Ishida *et al.*, in *Proceeding of the 6th International Conference on Hadron Spectroscopy, Manchester, UK, 1995* (World Scientific, Singapore, 1996), p. 574.
[18] S. Prelovsek, C. Dawson, T. Izubuchi, K. Orginos, and A. Soni, Phys. Rev. D **70**, 094503 (2004).
[19] T. Burch, C. Gattringer, L. Y. Glozman, C. Hagen, C. B. Lang, and A. Schafer, Phys. Rev. D **73**, 094505 (2006).
[20] N. Mathur, A. Alexandru, Y. Chen, S. J. Dong, T. Draper, I. Horváth, F. X. Lee, K. F. Liu, S. Tamhankar, and J. B. Zhang, Phys. Rev. D **76**, 114505 (2007).
[21] J. J. Dudek, R. G. Edwards, and D. J. Wilson (Hadron Spectrum Collaboration), Phys. Rev. D **93**, 094506 (2016).
[22] R. A. Briceño, J. J. Dudek, R. G. Edwards, and D. J. Wilson (Hadron Spectrum Collaboration), Phys. Rev. D **97**, 054513 (2018).
[23] C. Alexandrou, J. Berlin, M. Dalla Brida, J. Finkenrath, T. Leontiou, and M. Wagner, Phys. Rev. D **97**, 034506 (2018).
[24] W.-H. Liang, J.-J. Xie, and E. Oset, Eur. Phys. J. C **75**, 609 (2015).
[25] E. Oset *et al.*, Int. J. Mod. Phys. E **25**, 1630001 (2016).
[26] M. Albaladejo, J. T. Daub, C. Hanhart, B. Kubis, and B. Moussallam, J. High Energy Phys. 04 (2017) 010.
[27] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. D **88**, 072005 (2013).
[28] H. N. Li and H. L. Yu, Phys. Rev. Lett. **74**, 4388 (1995).
[29] H. N. Li, Phys. Lett. B **348**, 597 (1995).
[30] C. D. Lü, K. Ukai, and M. Z. Yang, Phys. Rev. D **63**, 074009 (2001); A. Ali, G. Kramer, Y. Li, C. D. Lü, Y. L. Shen, W. Wang, and Y. M. Wang, Phys. Rev. D **76**, 074018 (2007); W. Wang, Phys. Rev. D **83**, 014008 (2011); Q. Qin, Z. T. Zou, X. Yu, H. N. Li, and C. D. Lü, Phys. Lett. B **732**, 36 (2014); S. Cheng and Q. Qin, Phys. Rev. D **99**, 016019 (2019); C. D. Lü, W. Wang, Y. Xing, and Q.-A. Zhang, Phys. Rev. D **97**, 114016 (2018).
[31] Z. Rui, Y. Li, and W. F. Wang, Eur. Phys. J. C **77**, 199 (2017).
[32] Z. Rui and W. F. Wang, Phys. Rev. D **97**, 033006 (2018).
[33] Z. Rui, Y. Li, and H. N. Li, Phys. Rev. D **98**, 113003 (2018).
[34] H. Y. Cheng and C. K. Chua, Phys. Rev. D **88**, 114014 (2013).
[35] H. Y. Cheng, C. K. Chua, and Z. Q. Zhang, Phys. Rev. D **94**, 094015 (2016).
[36] S. Kränkl, T. Mannel, and J. Virto, Nucl. Phys. **B899**, 247 (2015).
[37] S. M. Flatté, Phys. Lett. **63B**, 224 (1976).
[38] J. P. Lees *et al.* (*BABAR* Collaboration), Phys. Rev. D **85**, 112010 (2012).
[39] W. F. Wang, H. N. Li, W. Wang, and C. D. Lü, Phys. Rev. D **91**, 094024 (2015).
[40] H. N. Li, Prog. Part. Nucl. Phys. **51**, 85 (2003), and references therein.
[41] T. Kurimoto, H. N. Li, and A. I. Sanda, Phys. Rev. D **65**, 014007 (2001).
[42] Z. Rui and Z. T. Zou, Phys. Rev. D **90**, 114030 (2014).
[43] Z. Rui, W. F. Wang, G. X. Wang, L. H. Song, and C. D. Lü, Eur. Phys. J. C **75**, 293 (2015).
[44] C. H. Chen and H. N. Li, Phys. Lett. B **561**, 258 (2003).
[45] U.-G. Meißner and W. Wang, Phys. Lett. B **730**, 336 (2014).
[46] D. Boito, J.-P. Dedonder, B. El-Bennich, R. Escribano, R. Kamiński, L. Leśniak, and B. Loiseau, Phys. Rev. D **96**, 113003 (2017).
[47] M. Albaladejo and B. Moussallam, Eur. Phys. J. C **75**, 488 (2015).
[48] D. R. Boito and R. Escribano, Phys. Rev. D **80**, 054007 (2009).
[49] W.-F. Wang, Phys. Lett. B **788**, 468 (2019).
[50] J. H. A. Nogueira *et al.*, arXiv:1605.03889.
[51] H.-Y. Cheng, C.-K. Chua, and K.-C. Yang, Phys. Rev. D **73**, 014017 (2006).
[52] A. Abele *et al.*, Phys. Rev. D **57**, 3860 (1998).
[53] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (Wiley/Springer-Verlag, New York, 1952).
[54] D. Asner, Phys. Lett. B **592**, 1 (2004).
[55] D. V. Bugg, J. Phys. G **35**, 075005 (2008).
[56] D. V. Bugg, Phys. Rev. D **78**, 074023 (2008).
[57] M. N. Achasov *et al.*, Phys. Lett. B **479**, 53 (2000).
[58] G. S. Adams *et al.* (CLEO Collaboration), Phys. Rev. D **84**, 112009 (2011).
[59] F. Ambrosino *et al.* (KLOE Collaboration), Phys. Lett. B **681**, 5 (2009).
[60] S. Teige *et al.* (E852 Collaboration), Phys. Rev. D **59**, 012001 (1998).