Radiative corrections of order $O(\alpha E_e/m_N)$ to Sirlin's radiative corrections of order $O(\alpha/\pi)$ to the neutron lifetime

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We calculate the radiative corrections of order $O(\alpha E_e/m_N)$ as next-to-leading order corrections in the large nucleon mass expansion to Sirlin's radiative corrections of order $O(\alpha/\pi)$ to the neutron lifetime. The calculation is carried out within a quantum field theoretic model of strong low-energy pion-nucleon interactions described by the linear σ model (LσM) with chiral $SU(2)\times SU(2)$ symmetry and electroweak hadron-hadron, hadron-lepton and lepton-lepton interactions for the electron-lepton family with $SU(2)_L \times U(1)_Y$ symmetry of the standard electroweak model (SEM). Such a quantum field theoretic model is some kind a hadronized version of the Standard Model. From a gauge invariant set of the Feynman diagrams with one-photon exchanges we reproduce Sirlin's radiative corrections of order $O(\alpha/\pi)$, calculated to leading order in the large nucleon mass expansion, and calculate next-to-leading corrections of order $O(\alpha E_e/m_N)$. This confirms Sirlin's confidence level of the radiative corrections $O(\alpha E_e/m_N)$. The contributions of the LσM are taken in the limit of the infinite mass of the scalar isoscalar σ meson. In such a limit the LσM reproduces the results of the current algebra [S. Weinberg, Phys. Rev. Lett. **18**, 188 (1967)] in the form of effective chiral Lagrangians of pion-nucleon interactions with nonlinear realization of chiral $SU(2)\times SU(2)$ symmetry. In such a limit the LσM is also equivalent to Gasser-Leutwyler's chiral quantum field theory or chiral perturbation theory with chiral $SU(2)\times SU(2)$ symmetry and the exponential parametrization of a pion-field [(G. Ecker, Prog. Part. Nucl. Phys. **35**, 1 (1995)].

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I. INTRODUCTION

Nowadays the structure of the neutron in the β^- decay [1,2] is investigated at the level of 10^{-3} related to the radiative corrections of order $O(\alpha/\pi)$, where α is the fine-structure constant [3], and corrections of order $O(E_e/m_N)$, caused by the weak magnetism and proton recoil, where E_e and m_N are the electron energy and the nucleon mass [4–9]. The contributions of radiative corrections of order $O(\alpha/\pi)$ has a long history [10–38]. The contemporary shape of radiative corrections to the neutron lifetime has been calculated by Sirlin [14] in the approximation of the one-photon exchange and to leading order in the large nucleon mass expansion. The contributions to the radiative corrections of the neutron lifetime, which caused electroweak boson exchanges and QCD corrections, have been calculated by Marciano and Sirlin [24,33] and Czarnecki *et al.* [32]. Recently the result obtained in [33] has been improved by Seng *et al.* [36]. In turn, the contemporary shape of the radiative corrections to the correlation coefficients of the electron-antineutrino 3-momentum correlations and correlations between neutron spin and the electron 3-momentum has been calculated by Shann [17]. Recently radiative corrections of order $O(\alpha/\pi)$ to leading order in the large nucleon mass expansion have been calculated to the correlation coefficients of the neutron β^- decays with polarized neutron and electron and unpolarized proton, and polarized electron and unpolarized neutron and proton [7–9]. For the first time the contributions of the weak magnetism and proton recoil of order $O(E_e/m_N)$ to the neutron lifetime and correlation coefficients of the neutron β^- decay with polarized neutron and unpolarized electron and proton have been calculated by Bilen'kii *et al.* [39] and then by Wilkinson [40]. To the correlation coefficients of the neutron β^- decays with polarized neutron and electron and unpolarized proton, and with polarized electron and unpolarized neutron and proton have been calculated in [7–9]. At the level of 10^{-3} the neutron as well as the proton has been treated as a

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structureless particle. The contributions of strong low-energy interactions to the β^- decay of the structureless neutron with a structureless decay proton are described by the axial coupling constant g_A, and the isovector anomalous magnetic moment of the nucleon $\kappa = \kappa_p - \kappa_n$, where κ_p and κ_n are anomalous magnetic moments of the proton and neutron, respectively, measured in the nuclear magneton [3]. We would like to remind the reader that the axial coupling constant g_A appears in the standard $V - A$ theory of weak interactions [41–43] as a trace of strong low-energy interactions in the matrix element of the hadronic $n \to p$ transition after renormalization of the matrix element of the axial-vector hadronic current [44]. As has been shown by Sirlin [14,21] the radiative corrections of order $O(\alpha/\pi)$, calculated to leading order in the large nucleon mass expansion, are independent of the axial coupling constant g_A. In turn, the corrections of order $O(E_e/m_N)$, caused by the weak magnetism and proton recoil, depend strongly on the axial coupling constant g_A and the isovector anomalous magnetic moment of the nucleon κ [39,40] (see also [4,5,7,8]). The neutron lifetime $\tau_n = 879.6(1.1)$ s, calculated in [5] at the axial coupling constant $g_A = 1.2750(9)$ [1] (see also [45–48]), agrees well with the neutron lifetime $\tau_n = 879.6(6)$ s, averaged over the experimental values of the six bottle experiments [49–54] included in the Particle Date Group (PDG) [3]. The values of the neutron lifetime $\tau_n = 879.6(1.1)$ s and axial coupling constant $g_A = 1.2750(9)$ agree also well with (i) the values $\tau_n^{(\text{favoured})} = 879.6(4)$ s and $g_A^{(\text{favoured})} = 1.2755(11)$, which have been recommended by Czarnecki *et al.* [55] as *favoured* by a global analysis of the experimental data on the neutron lifetime and the electron asymmetry of the neutron β^- decay with a polarized neutron and unpolarized proton and electron, and (ii) recent experimental value $g_A = 1.27641(45)_{\text{stat}}(33)_{\text{sys}}$ [56].

For the first time deviations of the nucleon from a structureless pointlike particle in the neutron β^- decay have been taken into account by Wilkinson [40]. As has been shown in [8] these corrections are of order 10^{-5}. The problem of nontrivial influence of hadronic structure of the nucleon, caused by strong low-energy interactions, on gauge properties of radiative corrections of order $O(\alpha^2/\pi^2)$ has been pointed out in [57] within the standard $V - A$ effective theory of weak interactions. As has been found in [57] the interactions of real and virtual photons with hadronic structure of the neutron and proton should provide not only gauge invariance of radiative corrections of order $O(\alpha^2/\pi^2)$ but also nontrivial dependence of these corrections on the electron E_e and photon ω energies. This agrees well with Weinberg's assertion that strong low-energy interactions play an important role in weak decays [58]. Hence, according to Weinberg [58], contributions of strong low-energy interactions beyond the axial coupling constant g_A seem to be in principle important for the gauge invariant description of radiative corrections to neutron β^- decays to all orders in the fine-structure constant expansion. However, as has been shown by Sirlin [14,16,21] the contribution of strong low-energy interactions to the radiative corrections of order $O(\alpha/\pi)$ to the neutron lifetime, calculated to leading order in the large nucleon mass expansion, is a constant independent of the electron energy. Because of such a property of strong low-energy interactions their contributions to neutron β^- decays have been left at the level of the axial coupling constant g_A and screened in the radiative corrections [14–35] (see also [4–8]). As has been shown in [5] the contributions of the weak magnetism and proton recoil of order $O(E_e/m_N)$ to the neutron lifetime are much smaller than the contributions of the radiative corrections. An enhancement of the radiative corrections with respect to the corrections from the weak magnetism and proton recoil is caused also by the contributions of the electroweak-boson exchanges. The necessity to take into account contributions of electroweak-boson exchanges [59] for the calculation of radiative corrections of order $O(\alpha/\pi)$ has been pointed out by Sirlin [18,20,21,23]. The analysis of electroweak-boson exchanges and QCD corrections has been continued by Marciano and Sirlin [24,33], Degrassi and Sirlin [25], Czarnecki, Marciano and Sirlin [32], and Sirlin and Ferroglia [35]. As has been shown by Czarnecki *et al.* [32] the contributions of electroweak-boson exchanges change crucially the value of the radiative corrections of order $O(\alpha/\pi)$. Indeed, the radiative corrections to the neutron lifetime, averaged over the electron-energy spectrum, are equal to $\langle(\alpha/\pi)g_n(E_e)\rangle = 0.015056$ and $\langle(\alpha/\pi)g_n(E_e)\rangle = 0.0390(8)$ without and with the contributions of the electroweak-boson exchanges and QCD corrections, respectively [32], where the function $g_n(E_e)$ describes the radiative corrections to the neutron lifetime in notation [5,7]. In Fig. 1 the function $g_n(E_e)$ is plotted without (golden line) and with (blue line) the contributions of the electroweak-boson and QCD corrections. It is important to emphasize that the contribution of QCD corrections, caused by the quark structure of the neutron and proton and gluon exchanges, is by 2 orders of magnitude smaller than the contribution of the electroweak-boson exchanges [32].

For the correct gauge invariant calculation of radiative corrections of order $O(\alpha^2/\pi^2)$ and as well as $O(\alpha E_e/m_N)$ to the rate of the neutron radiative β^- decay $n \to p + e^- + \bar{\nu}_e + \gamma$ within the standard $V - A$ effective theory of weak interactions, an appearance of nontrivial contributions of strong low-energy interactions dependent on the energies of decay particles has been pointed out in [57]. The problem of gauge invariant and infrared stable nontrivial contributions of strong low-energy interactions to the radiative corrections to neutron β^- decays is closely related to the analysis of corrections of order 10^{-5}, calculated in the SM [7,57,60–62].

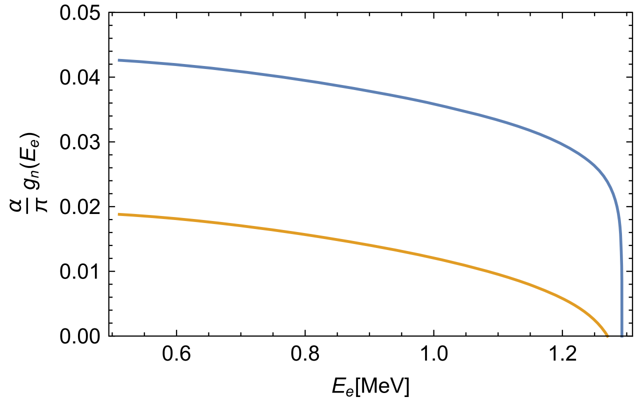

FIG. 1. Radiative corrections $(\alpha/\pi)g_n(E_e)$ to the neutron lifetime in the electron energy region $m_e \le E_e < E_0$. Blue and golden curves show the behavior of the function $(\alpha/\pi)g_n(E_e)$ with and without contributions of electroweak-boson exchanges and QCD corrections, respectively, where QCD corrections make up of about 1.7% of the contributions of electroweak-boson exchanges, which make up of about 60% of the total radiative corrections to the neutron lifetime averaged over the electron-energy spectrum [32].

A contemporary level of relative accuracy of about 10^{-4} or even better of the experimental analysis of the neutron β^- decay [63] for searches of contributions of interactions beyond the SM and contributions of second class currents [64–76] (see also [4,5,8,9]) demands the SM theoretical background of order 10^{-5}, since "discovery" experiments with the required 5σ sensitivity will require experimental uncertainties of a few parts in 10^{-5} [7–9]. The calculation of the radiative corrections of order $O(\alpha E_e/m_N) \sim 10^{-5}$ as next-to-leading order corrections in the large nucleon mass expansion to Sirlin's corrections of order $O(\alpha/\pi) \sim 10^{-3}$, which we carry out in this paper, is the first step to the calculation of the complete set of the SM corrections of order 10^{-5} to the neutron lifetime and correlation coefficients of the neutron β^- decays with different polarization states of the neutron and massive decay fermions.

The paper is organized as follows. In Sec. II we discuss briefly a low-energy hadronization of the Standard Model (SM), where strong low-energy pion-nucleon interactions are described at the hadronic level by the linear σ model (LσM) with linear realization of chiral $SU(2) \times SU(2)$ symmetry. In Sec. III we outline the structure and properties of the LσM with chiral $SU(2) \times SU(2)$ symmetry. In Sec. IV we demonstrate an equivalence at the Lagrangian level between the LσM, taken in the limit of the infinite mass of the scalar isoscalar σ-meson $m_\sigma \to \infty$, and chiral quantum field theories with nonlinear realization of chiral $SU(2) \times SU(2)$ symmetry by Weinberg and by Gasser and Leutwyler. In Sec. V we propose a quantum field theoretic model of strong low-energy and electroweak interactions with electroweak $SU(2)_L \times U(1)_Y$ symmetry as a hadronized version of the SM at low energies. Having switched off the electroweak coupling constants this model reduces to the LσM with chiral $SU(2) \times SU(2)$ symmetry. In Sec. VI we calculate the matrix element of the hadronic $n \to p$ transition in the neutron β^- decay in the tree approximation for electroweak interactions and to one-hadron-loop approximation for strong low-energy interactions in the quantum field theoretic model proposed in Sec. V and described by the Lagrangian Eq. (44). We show that the quantum field theoretic model, described by the Lagrangian Eq. (44), reproduces well the standard Lorentz structure of the matrix element of the hadronic $n \to p$ transition with the axial coupling constant $g_A \neq 1$, the isovector anomalous nucleon magnetic moment κ and the one-pion-pole contribution. The latter is important for gauge invariance of the matrix element of the hadronic $n \to p$ transition in the chiral limit $m_\pi \to 0$, where m_π is a pion-meson mass. Such a gauge invariance or an independence of a longitudinal part of the propagator of the electroweak W^- boson is required by conservation of the axial-vector hadronic current in the chiral limit $m_\pi \to 0$ [42]. Section VII is devoted to the analysis of the calculation of the radiative corrections of order $O(\alpha E_e/m_N)$ to Sirlin's radiative corrections of order $O(\alpha/\pi)$ to the neutron lifetime. We point out that for the calculation of the radiative corrections of order $O(\alpha E_e/m_N)$ as next-to-leading order corrections to Sirlin's corrections of order $O(\alpha/\pi)$ calculated to leading order in the large nucleon mass expansion, it is enough to analyze the contribution of the Feynman diagrams with one-virtual-photon exchanges in Fig. 6. Such a set of the Feynman diagrams is gauge invariant, i.e. independent of a gauge parameter ξ of a longitudinal part of the photon propagator. Then, to leading order in the large mass of the electroweak W^- boson exchanges the Feynman diagrams in Fig. 6 reduce to the Feynman diagrams, used by Sirlin for the calculation of the radiative corrections of order $O(\alpha/\pi)$ to the neutron lifetime [14]. The calculation of the contributions of hadronic structure of the nucleon to the radiative corrections of order $O(\alpha/\pi)$ and $O(\alpha E_e/m_N)$, caused by one-virtual-photon exchanges and demanding the analysis of two-loop Feynman diagrams, and the contributions of the Feynman diagrams with electroweak W and Z boson in the one-electroweak-loop approximation goes beyond the scope of this paper. We are planning to carry out these calculations in our forthcoming publications. In Sec. VIII we discuss the obtained results and perspectives of application of the quantum field theoretic model of strong low-energy and electroweak interactions, described by the Lagrangian Eq. (44), to the analysis of neutron lifetime and correlation coefficients of the neutron β^- decays with different polarization states of the neutron and massive decay fermions.

The Supplemental Material [77] including Appendices A, B, C and D, where we give detailed calculations of the matrix element of the hadronic $n \to p$ transition and radiative corrections of order $O(\alpha/\pi)$ and $O(\alpha E_e/m_N)$ to the amplitude of the neutron β^- decay, respectively, and discuss gauge properties of the amplitude of the neutron radiative β^- decay. In Appendix A we give the calculation of the matrix element of the hadronic $n \to p$ transition to one-hadron-loop approximation in the quantum field theoretic model described by the Lagrangian Eq. (44). In Appendices B and C we give the analysis of gauge properties of the Feynman diagrams in Fig. 6 and the calculation of these diagrams in details. We show that the Feynman diagrams in Fig. 6 are gauge invariant, i.e. independent of a gauge parameter ξ of the photon propagator, and renormalizable. In Appendix D we discuss gauge properties of the amplitude of the neutron radiative β^- decay taken to leading order in the large mass of the electroweak W^--boson expansion.

II. LOW-ENERGY DYNAMICS OF THE STANDARD MODEL

The Standard Model of particle physics is a quantum field theory based on the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry group which describes strong, weak and electromagnetic (or electroweak) interactions among fundamental particles, which are (i) eight gluons (g), mediating strong interactions between quarks with six flavors ($q = u, d, s, c, b, t$) and three color degrees of freedom each, electroweak bosons ($W^{\pm}, Z$) and photon (γ), mediating weak and electromagnetic interactions between quarks and three lepton families (ℓ^-, ν_ℓ) for $\ell = e, \mu, \tau$ or electron, muon and tauon and electron-, muon-, and tauon-neutrinos, and a Higgs boson (H) with mass $M_{\rm H} = 125$ GeV coupled to quarks, leptons, electroweak bosons, photon and gluons [3,78,79]. The part of the SM invariant under $SU(3)_C$ gauge symmetry or quantum chromodynamics (QCD) [3,78,79], describing strong interactions, was mainly formulated in [80–87]. In turn, the standard electroweak model (SEM) or the part of the SM invariant under $SU(2)_L \times U(1)_Y$ gauge symmetry has been formulated in [88–94]. Renormalizability of the SM, including a renormalizability of non-Abelian massless and massive Yang-Mills theories, has been proved in [95–101]. The number of colored quarks and lepton families is constrained by a requirement of renormalizability of the SEM to all orders of perturbation theory, violation of which can occur because of Adler-Bell-Jackiw anomalies [102–106]. Following Bijnens [78] the SM Lagrangian we write as follows:

$$\begin{aligned}\mathcal{L}_{\rm SM} &= \mathcal{L}_{\rm Higgs}(\phi) + \mathcal{L}_{\rm gauge}(g, W, Z, \gamma) + \sum_{q=u,d,s,c,b,t} \bar{\psi}_q i\gamma^\mu D_\mu \psi_q + \sum_{\ell=e,\mu,\tau} \bar{\psi}_\ell i\gamma^\mu D_\mu \psi_\ell + \sum_{\nu_\ell=\nu_e,\nu_\mu,\nu_\tau} \bar{\psi}_{\nu_\ell} i\gamma^\mu D_\mu \psi_{\nu_\ell} \\ &\quad - \sum_{q=u,c,t} g_{qq}(\bar{\Psi}_{qL}\psi_{qR}\phi^c + \phi^{c\dagger}\bar{\psi}_{qR}\Psi_{qL}) - \sum_{(qq')=(ud),(cs),(tb)} g_{qq'}(\bar{\Psi}_{qL}\psi_{q'R}\phi + \phi^\dagger\bar{\psi}_{q'R}\Psi_{qL}) \\ &\quad - \sum_{\ell=e,\mu,\tau} g_\ell(\bar{\Psi}_{\ell L}\psi_{\ell R}\phi + \phi^\dagger\bar{\psi}_{\ell R}\Psi_{\ell L}), \qquad (1)\end{aligned}$$

where $\mathcal{L}_{\rm Higgs}(\phi)$ is the Lagrangian of the Higgs field ϕ, $\mathcal{L}_{\rm gauge}(g, W, Z, \gamma)$ is the Lagrangian of the kinetic terms of gauge bosons, the third and fourth terms in $\mathcal{L}_{\rm SM}$ are the kinetic terms and interactions of quarks and leptons with gauge bosons and the last three terms in $\mathcal{L}_{\rm SM}$ define Yukawa interactions of quarks and leptons with the Higgs field. In the phase of the spontaneously broken $SU(2)_L \times U(1)_Y$ symmetry these interactions produce masses of charged fermions. Then, $\Psi_{uL}, \Psi_{cL}, \Psi_{tL}$ are quark left-handed doublets with components $(P_L\psi_u, P_L\psi_d)$, $(P_L\psi_c, P_L\psi_s)$ and $(P_L\psi_t, P_L\psi_b)$, respectively, and $\Psi_{\ell L}$ are the lepton left-handed $SU(2)_L \times U(1)_Y$ doublets with components $(P_L\psi_\ell, P_L\psi_{\nu_\ell})$ for $\ell = e, \mu$ and τ, respectively, $\psi_{qR} = P_R\psi_q$ and $\psi_{\ell R} = P_R\psi_\ell$ are the right-handed quark and charged lepton $SU(2)_L \times U(1)_Y$ singlets, where $P_{L,R} = (1 \mp \gamma^5)/2$ are the projection operators $P_L^2 = P_L$, $P_R^2 = P_R$ and $P_L P_R = P_R P_L = 0$. For $g_{qq} = g_{qq'} = 0$ the Lagrangian $\mathcal{L}_{\rm SM}$ is invariant under chiral $SU(N_f) \times SU(N_f)$ transformations of the quark fields [78,79], where N_f is the number of quark fields.

Having integrated over gluon and quark degrees of freedom we arrive at the effective Lagrangian for hadrons coupled to electroweak bosons (W, Z), photons (γ) and the Higgs field (H), and leptons. The strong low-energy interactions are described by the effective Lagrangian. After the integration over the fields of baryons with masses larger than $m_B > 1$ GeV and of mesons with masses larger than the π-meson mass $m_M > m_\pi = 0.14$ GeV we arrive at the effective quantum field theory for pions and nucleons pions, described by the chiral perturbation theory (ChPT) with a nonlinear realization of chiral $SU(2) \times SU(2)$ symmetry [107–121], based on the quantum field theory of chiral dynamics developed by Weinberg [122–124] and the general theory of phenomenological or effective chiral Lagrangians [125–127], which reproduce fully (see Dashen and Weinstein [128]) the results of the current algebra with partially conserved axial-vector hadronic current (PCAC) [129–131] (see also [43,44]) on all possible soft-pion theorems related to multipion production [132]. As has been shown by Weinberg [122] the effective chiral Lagrangians with nonlinear realization of chiral $SU(2) \times SU(2)$ symmetry can be derived from the linear σ model with linear realization of chiral $SU(2) \times SU(2)$ symmetry [133] in the limit of the infinite mass of the scalar σ meson. Then, by applying pion-field redefinition one may arrive at any form of an effective chiral Lagrangian with nonlinear realization of chiral $SU(2) \times SU(2)$ symmetry [123]. Such an effective theory can be generalized by a series of higher order terms of covariant derivatives of the pion-field producing perturbative corrections to the current algebra results, i.e. chiral perturbation theory [124]. A consistent realization of this idea has been carried out by Gasser and Leutwyler [107] (see also [108–121] and many other papers). The LσM with a linear realization of chiral $SU(2) \times SU(2)$ symmetry attracts strong attention by the following properties: (i) spontaneously broken chiral $SU(2) \times SU(2)$ symmetry, (ii) the partially conserved axial-vector hadronic current (PCAC) and the Goldberger-Treiman relation [134] at the quantum field theoretic level and (iii) renormalizability [135–140].

The analysis of the contributions of hadronic structure of the nucleon or strong low-energy interactions to the neutron β^- decay within the LσM has been carried out in [60–62] in the standard $V - A$ effective theory of weak interactions [41–43]. As has been shown in [60] (see also [141]) the contributions of the LσM, calculated to one-hadron-loop approximation, reproduce well the Lorentz structure of the matrix element $\langle p(\vec{k}_p, \sigma_p)|J^{(+)}_\mu(0)|n(\vec{k}_n, \sigma_n)\rangle$ of the hadronic $n \to p$ transition, where $J^{(+)}_\mu(0) = V^{(+)}_\mu(0) - A^{(+)}_\mu(0)$ is the charged weak hadronic current [41–43]. According to the analysis of contributions of hadronic structure of the nucleon to the radiative corrections of the neutron lifetime, described by QED and the LσM in the standard $V - A$ effective theory of weak interactions, the radiative corrections to order $O(\alpha/\pi)$ are gauge invariant with contributions of strong low-energy interactions described by the axial coupling constant g_A to leading order in the large nucleon mass m_N expansion only. This agrees well with the analysis of radiative corrections carried out by Sirlin [14,21]. In other words in such an approximation the neutron and proton can be treated as pointlike particles. Nontrivial contributions of hadronic structure of the nucleon to the radiative corrections can appear only to order $O(\alpha E_e/m_N)$ [60]. However, these contributions are gauge noninvariant and dependent on the ultraviolet cutoff, which cannot be removed by renormalization. As has been pointed out in [60–62] the problem of an appearance of gauge noninvariant contributions and contributions, violating renormalizability of the amplitude of the neutron β^- decays, to order $O(\alpha E_e/m_N)$ and even smaller, can be explained as follows. Indeed, the effective $V - A$ vertex of weak interactions is not the vertex of the combined quantum field theory including the LσM and QED. This implies that correct gauge invariant contributions to the amplitude of the neutron β^- decays can be obtained in any loop approximation and without violation of renormalizability only in the hadronized version of the SEM with renormalizable quantum field theory of strong low-energy interactions. In such a combined quantum field theory the vertex of the effective $V - A$ weak interactions is defined by the electroweak W^--boson exchange. This should result in a gauge invariant set of Feynman diagrams including electroweak bosons and photons coupled to leptons, nucleon and hadrons from hadronic structure of the nucleon, described by a renormalizable quantum field theory of strong low-energy interactions. Since the effective chiral Lagrangians with nonlinear realization of chiral $SU(2) \times SU(2)$ symmetry can be derived from the LσM in the limit of the infinite mass of the scalar isoscalar σ meson [122] (see also [127]) and by redefinition of hadronic quantum fields [123], for the description of strong low-energy interactions of the nucleon and pions we choose the LσM in the infinite limit of the scalar isoscalar σ-meson mass. Because of the equivalence theorem [142–145] such redefinitions of hadronic quantum fields do not affect observable quantities, defined by matrix elements of the $\mathbb{S}$-matrix on mass shell of interacting particles.

III. LINEAR σ MODEL (LσM) WITH CHIRAL $SU(2) \times SU(2)$ SYMMETRY [62]

A. Chirally symmetric phase of the LσM

The LσM with linear realization of chiral $SU(2) \times SU(2)$ symmetry describes strong low-energy nucleon-nucleon, pion-nucleon and pion-pion interactions with a mediation of the scalar isoscalar σ meson [133]. In the chirally symmetric phase the Lagrangian of the LσM is given by [44]

$$\mathcal{L}_{\mathrm{L}\sigma\mathrm{M}} = \bar{\psi}_N(i\gamma^\mu\partial_\mu - g_{\pi N}(\tau_0\sigma + i\vec{\tau}\cdot\vec{\pi}))\psi_N + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}) + \frac{1}{2}\mu^2(\sigma^2 + \vec{\pi}^2) - \frac{1}{4}\gamma(\sigma^2 + \vec{\pi}^2)^2, \tag{2}$$

where ψ_N is the isospin doublet of the nucleon field operator with components (ψ_p, ψ_n), where ψ_p and ψ_n are the proton and neutron field operators, respectively, σ and $\vec{\pi} = (\pi^+, \pi^0, \pi^-)$ are the scalar isospin-scalar (isoscalar) σ- and pseudoscalar isospin-vector (isovector) pion-meson field operators, μ^2, γ and $g_{\pi N}$ are input parameters of the LσM, $\vec{\tau} = (\tau_1, \tau_2, \tau_3)$ are the isospin 2×2 Pauli matrices and τ_0 is the isospin 2×2 unit matrix.

Under isospin-vector and isospin-axial-vector (or chiral) infinitesimal transformations with parameters $\vec{\alpha}_V$ and $\vec{\alpha}_A$, respectively, the nucleon and meson fields transform as follows:

$$\psi_N \xrightarrow{\vec{\alpha}_V} \psi'_N = \left(1 + i\frac{1}{2}\vec{\alpha}_V\cdot\vec{\tau}\right)\psi_N,$$
$$\bar{\psi}_N \xrightarrow{\vec{\alpha}_V} \bar{\psi}'_N = \bar{\psi}_N\left(1 - i\frac{1}{2}\vec{\alpha}_V\cdot\vec{\tau}\right),$$
$$\sigma \xrightarrow{\vec{\alpha}_V} \sigma' = \sigma, \qquad \vec{\pi} \xrightarrow{\vec{\alpha}_V} \vec{\pi}' = \vec{\pi} - \vec{\alpha}_V\times\vec{\pi},$$
$$\psi_N \xrightarrow{\vec{\alpha}_A} \psi'_N = \left(1 + i\frac{1}{2}\gamma^5\vec{\alpha}_A\cdot\vec{\tau}\right)\psi_N,$$
$$\bar{\psi}_N \xrightarrow{\vec{\alpha}_A} \bar{\psi}'_N = \bar{\psi}_N\left(1 + i\frac{1}{2}\gamma^5\vec{\alpha}_A\cdot\vec{\tau}\right),$$
$$\sigma \xrightarrow{\vec{\alpha}_A} \sigma' = \sigma + \vec{\alpha}_A\cdot\vec{\pi}, \qquad \vec{\pi} \xrightarrow{\vec{\alpha}_A} \vec{\pi}' = \vec{\pi} - \vec{\alpha}_A\sigma. \quad (3)$$

The Lagrangian Eq. (2) is invariant under global transformations Eq. (3). Under local transformations Eq. (3) the Lagrangian Eq. (2) acquires the following corrections:

$$\delta\mathcal{L}_{\mathrm{L}\sigma\mathrm{M}} = -\partial^\mu\vec{\alpha}_V\cdot\left(\bar{\psi}_N\gamma_\mu\frac{1}{2}\vec{\tau}\psi_N + \vec{\pi}\times\partial_\mu\vec{\pi}\right) - \partial^\mu\vec{\alpha}_A\cdot\left(\bar{\psi}_N\gamma_\mu\gamma^5\frac{1}{2}\vec{\tau}\psi_N + (\sigma\partial_\mu\vec{\pi} - \vec{\pi}\partial_\mu\sigma)\right), \quad (4)$$

which allow one to define the vector and axial-vector hadronic currents [129]

$$\vec{V}_\mu = -\frac{\delta\mathcal{L}_{\mathrm{L}\sigma\mathrm{M}}}{\delta\partial^\mu\vec{\alpha}_V} = \bar{\psi}_N\gamma_\mu\frac{1}{2}\vec{\tau}\psi_N + \vec{\pi}\times\partial_\mu\vec{\pi},$$
$$\vec{A}_\mu = -\frac{\delta\mathcal{L}_{\mathrm{L}\sigma\mathrm{M}}}{\delta\partial^\mu\vec{\alpha}_A} = \bar{\psi}_N\gamma_\mu\gamma^5\frac{1}{2}\vec{\tau}\psi_N + (\sigma\partial_\mu\vec{\pi} - \vec{\pi}\partial_\mu\sigma). \quad (5)$$

Using the equations of motion for the nucleon, scalar and pseudoscalar fields one may show that in the chirally symmetric phase the divergences of the vector and axial-vector hadronic currents vanish $\partial^\mu\vec{V}_\mu = \partial^\mu\vec{A}_\mu = 0$. This means that in the chirally symmetric phase the vector and axial-vector hadronic currents are locally conserved.

B. Phase of spontaneously broken chiral symmetry

We would like to notice that the nucleon, scalar and pseudoscalar fields in Eq. (2) are unphysical. Indeed, the nucleon is massless and the mass term of the scalar and pseudoscalar fields enters with incorrect sign. Hence, physical hadronic states can appear in the LσM only in the phase of spontaneously broken chiral symmetry [133]. In the LσM the phase of spontaneously broken chiral $SU(2)\times SU(2)$ symmetry can be described by the Lagrangian [44]

$$\mathcal{L}_{\mathrm{L}\sigma\mathrm{M}} = \bar{\psi}_N(i\gamma^\mu\partial_\mu - g_{\pi N}(\tau_0\sigma + i\gamma^5\vec{\tau}\cdot\vec{\pi}))\psi_N + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}) + \frac{1}{2}\mu^2(\sigma^2 + \vec{\pi}^2) - \frac{1}{4}\gamma(\sigma^2 + \vec{\pi}^2)^2 + a\sigma, \quad (6)$$

where the last term $a\sigma$ is noninvariant under chiral transformations Eq. (3).

The phase of spontaneously broken chiral symmetry characterizes by a nonvanishing vacuum expectation value of the σ-field $\langle\sigma\rangle = b \neq 0$. The transition to the fields of physical hadronic states goes through the change of the σ-field $\sigma \to \sigma + b$, where in the right-hand side the σ field possesses a vanishing vacuum expectation value. After such a change of the σ field the dynamics of physical hadronic states is described by the Lagrangian

$$\mathcal{L}_{\mathrm{L}\sigma\mathrm{M}} = \bar{\psi}_N(i\gamma^\mu\partial_\mu - m_N - g_{\pi N}(\tau_0\sigma + i\gamma^5\vec{\tau}\cdot\vec{\pi}))\psi_N + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) + \frac{1}{2}(\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} - m_\pi^2\vec{\pi}^2) - \gamma b\sigma(\sigma^2 + \vec{\pi}^2) - \frac{1}{4}\gamma(\sigma^2 + \vec{\pi}^2)^2, \quad (7)$$

where the masses of physical hadrons and coupling constants are determined by

$$m_N = g_{\pi N}b, \qquad m_\sigma^2 = 3\gamma b^2 - \mu^2,$$
$$m_\pi^2 = \gamma b^2 - \mu^2, \qquad a = m_\pi^2 b, \quad (8)$$

where $b = f_\pi$ with f_π is the π-meson leptonic coupling constant [44], and $\gamma = (m_\sigma^2 - m_\pi^2)/2f_\pi^2$. In the phase of spontaneously broken chiral symmetry the vector and axial-vector hadronic currents are equal to

$$\vec{V}_\mu = \bar{\psi}_N\gamma_\mu\frac{1}{2}\vec{\tau}\psi_N + \vec{\pi}\times\partial_\mu\vec{\pi},$$
$$\vec{A}_\mu = \bar{\psi}_N\gamma_\mu\gamma^5\frac{1}{2}\vec{\tau}\psi_N + (\sigma\partial_\mu\vec{\pi} - \vec{\pi}\partial_\mu\sigma) + b\partial_\mu\vec{\pi}. \quad (9)$$

Using the equations of motion for the nucleon, scalar and pseudoscalar fields one may show that the divergences of the vector and axial vector hadronic currents are given by $\partial^\mu\vec{V}_\mu = 0$ and $\partial^\mu\vec{A}_\mu = -m_\pi^2 f_\pi\vec{\pi}$. This result agrees well with that by Adler and Dashen [129] (see Eq. (1.49) of Ref. [129]). Thus, the LσM reproduces well the hypothesis of partial conservation of the axial-vector hadronic current (the PCAC hypothesis) at the quantum field theoretic level [133]. Unlike the axial-vector hadronic current the vector hadronic current is locally conserved even in the phase of spontaneously broken chiral symmetry. Conservation of the vector hadronic current in the LσM can be violated only by isospin symmetry breaking.

The mass of the scalar isoscalar σ-meson $m_\sigma = \sqrt{2 f_\pi^2 \gamma + m_\pi^2}$ is practically arbitrary because of an arbitrariness of the coupling constant γ. Following Weinberg [89] one may take the limit $m_\sigma \to \infty$ corresponding to the limit $\gamma \to \infty$ at $\sqrt{\mu^2/\gamma} =$ fixed. As has been pointed out by Weinberg [89] (see also [127]), in the limit $m_\sigma \to \infty$ (or $\gamma \to \infty$) the LσM reproduces the results of the current algebra [129], and it is equivalent to chiral quantum field theories of strong low-energy pion-nucleon interactions with nonlinear realizations of chiral $SU(2) \times SU(2)$ symmetry.

For massless pions $m_\pi = 0$ or $a = 0$ the vacuum expectation value of the σ field is equal to $\langle \sigma \rangle = \sqrt{\mu^2/\gamma} = f_\pi$. In this case the mass of the σ meson is $m_\sigma = \sqrt{3\gamma} f_\pi$.

C. Renormalization of the LσM

For the discussion of the renormalization procedure in the LσM we rewrite the Lagrangian Eq. (7) as follows [136–138]:

$$\begin{aligned}\mathcal{L}^{(0)}_{\mathrm{L}\sigma\mathrm{M}} &= \bar{\psi}^{(0)}_N (i\gamma^\mu \partial_\mu - m^{(0)}_N - g^{(0)}_{\pi N}(\tau_0 \sigma^{(0)} + i\gamma^5 \vec{\tau}\cdot\vec{\pi}^{(0)}))\psi^{(0)}_N + \frac{1}{2}(\partial_\mu \sigma^{(0)} \partial^\mu \sigma^{(0)} - m^{(0)2}_\sigma \sigma^{(0)2}) \\ &\quad + \frac{1}{2}(\partial_\mu \vec{\pi}^{(0)} \cdot \partial^\mu \vec{\pi}^{(0)} - m^{(0)2}_\pi \vec{\pi}^{(0)2}) + \gamma^{(0)} f^{(0)}_\pi \sigma^{(0)}(\sigma^{(0)2} + \vec{\pi}^{(0)2}) - \frac{1}{4}\gamma^{(0)}(\sigma^{(0)2} + \vec{\pi}^{(0)2})^2, \end{aligned} \tag{10}$$

where $\psi^{(0)}_N$, $\sigma^{(0)}$ and $\vec{\pi}^{(0)}$ are *bare* hadronic fields, $m^{(0)}_N$, $m^{(0)}_\sigma$, $m^{(0)}_\pi$ and $\gamma^{(0)}$, $f^{(0)}_\pi$ are *bare* hadronic masses and coupling constants, respectively. After the calculation of hadron-loop contributions the dynamics of physical fields is described by the Lagrangian

$$\begin{aligned}\mathcal{L}^{(r)}_{\mathrm{L}\sigma\mathrm{M}} &= \bar{\psi}^{(r)}_N (i\gamma^\mu \partial_\mu - m^{(r)}_N - g^{(r)}_{\pi N}(\tau_0 \sigma^{(r)} + i\gamma^5 \vec{\tau}\cdot\vec{\pi}^{(r)}))\psi^{(r)}_N + \frac{1}{2}(\partial_\mu \sigma^{(r)} \partial^\mu \sigma^{(r)} - m^{(r)2}_\sigma (\sigma^{(r)})^2) \\ &\quad + \frac{1}{2}(\partial_\mu \vec{\pi}^{(r)} \cdot \partial^\mu \vec{\pi}^{(r)} - m^{(r)2}_\pi (\vec{\pi}^{(r)})^2) + \gamma^{(r)} f^{(r)}_\pi \sigma^{(r)}((\sigma^{(r)})^2 + (\vec{\pi}^{(r)})^2) - \frac{1}{4}\gamma^{(r)}((\sigma^{(r)})^2 + (\vec{\pi}^{(r)})^2)^2 + \mathcal{L}^{(\mathrm{CT})}_{\mathrm{L}\sigma\mathrm{M}}, \end{aligned} \tag{11}$$

where the Lagrangian $\mathcal{L}^{(\mathrm{CT})}_{\mathrm{L}\sigma\mathrm{M}}$ is given by

$$\begin{aligned}\mathcal{L}^{(\mathrm{CT})}_{\mathrm{L}\sigma\mathrm{M}} &= (Z_N - 1)\bar{\psi}^{(r)}_N (i\gamma^\mu \partial_\mu - m^{(r)}_N)\psi^{(r)}_N - Z_N \delta m^{(r)}_N \bar{\psi}^{(r)}_N \psi^{(r)}_N - (Z_{MN} - 1) g^{(r)}_{\pi N} \bar{\psi}^{(r)}_N (\tau_0 \sigma^{(r)} + i\gamma^5 \vec{\tau}\cdot\vec{\pi}^{(r)})\psi^{(r)}_N \\ &\quad + (Z_M - 1)\frac{1}{2}(\partial_\mu \sigma^{(r)} \partial^\mu \sigma^{(r)} - m^{(r)2}_\sigma (\sigma^{(r)})^2) - Z_M \delta m^{(r)2}_\sigma (\sigma^{(r)})^2 + (Z_M - 1)\frac{1}{2}(\partial_\mu \vec{\pi}^{(r)} \cdot \partial^\mu \vec{\pi}^{(r)} - m^{(r)2}_\pi (\vec{\pi}^{(r)})^2) \\ &\quad - Z_M \delta m^{(r)2}_\pi (\vec{\pi}^{(r)})^2 + (Z_{3M} - 1)\gamma^{(r)} f^{(r)}_\pi \sigma^{(r)}((\sigma^{(r)})^2 + (\vec{\pi}^{(r)})^2) - (Z_{4M} - 1)\frac{1}{4}\gamma^{(r)}((\sigma^{(r)})^2 + (\vec{\pi}^{(r)})^2)^2. \end{aligned} \tag{12}$$

Here Z_N, Z_M and $\delta m^{(r)}_N$, $\delta m^{(r)2}_\sigma$, $\delta m^{(r)2}_\pi$ are renormalization constants of wave functions and masses of the nucleon, scalar and pseudoscalar fields, respectively. Then, Z_{MN}, Z_{3M} and Z_{4M} are renormalization constants of the corresponding vertices of meson-nucleon and meson-meson field interactions. The abbreviation "CT" means "counterterms." If the fields, masses, coupling constants and renormalization constants satisfy the relations

$$\begin{aligned} &\psi^{(0)}_N = \sqrt{Z_N}\psi^{(r)}_N, \qquad \sigma^{(0)} = \sqrt{Z_M}\sigma^{(r)}, \qquad \vec{\pi}^{(0)} = \sqrt{Z_M}\vec{\pi}^{(r)}, \\ &m^{(0)}_N = m^{(r)}_N + \delta m^{(r)}_N, \qquad m^{(0)2}_\sigma = m^{(r)2}_\sigma + \delta m^{(r)2}_\sigma, \qquad m^{(0)2}_\pi = m^{(r)2}_\pi + \delta m^{(r)2}_\pi, \\ &g^{(0)}_{\pi N} = Z_{MN} Z^{-1}_N Z^{-1/2}_M g^{(r)}_{\pi N}, \qquad f^{(0)}_\pi = Z_{3M} Z^{-1}_{4M} Z^{1/2}_M f^{(r)}_\pi, \qquad \gamma^{(0)} = Z_{4M} Z^{-2}_M \gamma^{(r)}, \\ &Z_{3M} = Z_{4M}, \end{aligned} \tag{13}$$

the Lagrangian Eq. (11) reduces to the Lagrangian Eq. (10). The relation $Z_{3M} = Z_{4M}$ implies that the pion decay constant $f^{(r)}_\pi$ is renormalized only by renormalization of the wave function of the $\vec{\pi}$ meson, i.e. $f^{(0)}_\pi = Z^{1/2}_M f^{(r)}_\pi$.

IV. EQUIVALENCE OF THE LσM TO QUANTUM FIELD THEORIES OF STRONG LOW-ENERGY PION-NUCLEON INTERACTIONS WITH NONLINEAR REALIZATION OF CHIRAL $SU(2)\times SU(2)$ SYMMETRY

In this section we discuss an equivalence of the LσM with a linear realization of chiral $SU(2)\times SU(2)$ symmetry to quantum field theories with nonlinear realizations of chiral $SU(2)\times SU(2)$ symmetry or chiral perturbation theory. For this aim we follow Ecker [112]. We introduce the fields

$$U=\frac{1}{f_\pi}(\tau_0\sigma+i\vec{\tau}\cdot\vec{\pi}),\qquad U^\dagger=\frac{1}{f_\pi}(\tau_0\sigma-i\vec{\tau}\cdot\vec{\pi}) \tag{14}$$

and rewrite the Lagrangian $\mathcal{L}_{\mathrm{L}\sigma\mathrm{M}}$ in Eq. (6) as follows:

$$\begin{aligned}\mathcal{L}_{\mathrm{L}\sigma\mathrm{M}}=&\bar{\psi}_N(i\gamma^\mu\partial_\mu-g_{\pi N}(\tau_0\sigma+i\gamma^5\vec{\tau}\cdot\vec{\pi}))\psi_N\\&+\frac{f_\pi^2}{4}\langle\partial_\mu U^\dagger\partial^\mu U\rangle+\frac{1}{4}m_\pi^2f_\pi^2\langle(U+U^\dagger)\rangle\\&+\frac{1}{4}m_\pi^2f_\pi^2\langle(1-U^\dagger U)\rangle-\frac{f_\pi^4}{8}\gamma\langle(1-U^\dagger U)^2\rangle,\end{aligned} \tag{15}$$

where $\langle\ldots\rangle$ is a trace over isospin matrices [112]. Taking the limit $\gamma\to\infty$ corresponding to the infinite limit of the scalar isoscalar σ-meson mass $m_\sigma\to\infty$, we get $U^\dagger U=1$. This allows us to transcribe Eq. (15) into the form

$$\begin{aligned}\mathcal{L}_{\mathrm{ChPT}}=&\bar{\psi}_N(i\gamma^\mu\partial_\mu-g_{\pi N}(\tau_0\sigma+i\gamma^5\vec{\tau}\cdot\vec{\pi}))\psi_N\\&+\frac{f_\pi^2}{4}\langle\partial_\mu U^\dagger\partial^\mu U\rangle+\frac{1}{4}m_\pi^2f_\pi^2\langle(U+U^\dagger)\rangle.\end{aligned} \tag{16}$$

From the condition $U^\dagger U=1$ we obtain $\sigma^2+\vec{\pi}^2=f_\pi^2$ [123]. Following again Ecker [112] we rewrite Eq. (16) as follows:

$$\begin{aligned}\mathcal{L}_{\mathrm{ChPT}}=&\bar{\psi}_{NL}i\gamma^\mu\partial_\mu\psi_{NL}+\bar{\psi}_{NR}i\gamma^\mu\partial_\mu\psi_{NR}\\&-m_N(\bar{\psi}_{NL}U\psi_{NR}+\bar{\psi}_{NR}U^\dagger\psi_{NL})\\&+\frac{f_\pi^2}{4}\langle\partial_\mu U^\dagger\partial^\mu U\rangle\rangle+\frac{1}{4}m_\pi^2f_\pi^2\langle(U+U^\dagger)\rangle,\end{aligned} \tag{17}$$

where $\psi_{NL}=P_L\psi_N$ and $\psi_{\mathrm{NR}}=P_R\psi_N$ are the left- and right-handed nucleon fields, respectively, $g_{\pi N}=m_N/f_\pi$ is the Goldberger-Treiman (GT) relation with the axial coupling constant $g_A=1$ [134]. Then, we make unitary transformations [112]

$$\begin{aligned}\psi_{NL}&=u\psi'_{NL},\qquad \psi_{NR}=u^\dagger\psi'_{NR},\\ \bar{\psi}_{NL}&=\bar{\psi}'_{NL}u^\dagger,\qquad \bar{\psi}_{NR}=\bar{\psi}'_{NR}u.\end{aligned} \tag{18}$$

Plugging Eq. (18) into Eq. (16) we arrive at the Lagrangian

$$\begin{aligned}\mathcal{L}_{\mathrm{ChPT}}=&\bar{\psi}'_{NL}i\gamma^\mu(\partial_\mu+u^\dagger\partial_\mu u)\psi'_{NL}\\&+\bar{\psi}'_{NR}i\gamma^\mu(\partial_\mu+u\partial_\mu u^\dagger)\psi'_{NR}\\&-m_N(\bar{\psi}'_{NL}u^\dagger Uu^\dagger\psi'_{NR}+\bar{\psi}'_{NR}uU^\dagger u\psi'_{NL})\\&+\frac{f_\pi^2}{4}\langle\partial_\mu U^\dagger\partial^\mu U\rangle\rangle+\frac{1}{4}m_\pi^2f_\pi^2\langle(U+U^\dagger)\rangle.\end{aligned} \tag{19}$$

Setting $u^\dagger Uu^\dagger=uU^\dagger u=1$ that gives $U=u^2$ we transcribe Eq. (19) into the form

$$\begin{aligned}\mathcal{L}_{\mathrm{ChPT}}=&\bar{\psi}'_{NL}\bigg(i\gamma^\mu\partial_\mu+i\gamma^\mu\frac{1}{2}[u^\dagger,\partial_\mu u]\\&-i\gamma^\mu\gamma^5\frac{1}{2}\{u^\dagger,\partial_\mu u\}-m_N\bigg)\psi'\\&+\frac{f_\pi^2}{4}\langle\partial_\mu U^\dagger\partial^\mu U\rangle\rangle+\frac{1}{4}m_\pi^2f_\pi^2\langle(U+U^\dagger)\rangle,\end{aligned} \tag{20}$$

where we have used the relation $u\partial_\mu u^\dagger=-\partial_\mu uu^\dagger$ [117] and denoted $[u^\dagger,\partial_\mu u]=u^\dagger\partial_\mu u-\partial_\mu uu^\dagger$ and $\{u^\dagger,\partial_\mu u\}=u^\dagger\partial_\mu u+\partial_\mu uu^\dagger=u^\dagger\partial_\mu Uu^\dagger$. The Lagrangian Eq. (20) can be written also in the following form [108,109,111,112,117]:

$$\begin{aligned}\mathcal{L}_{\mathrm{ChPT}}=&\bar{\psi}'_{NL}\bigg(i\gamma^\mu D_\mu-i\gamma^\mu\gamma^5\frac{1}{2}\{u^\dagger,\partial_\mu u\}-m_N\bigg)\psi'_N\\&+\frac{f_\pi^2}{4}\langle\partial_\mu U^\dagger\partial^\mu U\rangle\rangle+\frac{1}{4}m_\pi^2f_\pi^2\langle(U+U^\dagger)\rangle,\end{aligned} \tag{21}$$

where $D_\mu=\partial_\mu+\Gamma_\mu$ is the covariant derivative and $\Gamma_\mu=(1/2)[u^\dagger,\partial_\mu u]$ has a meaning of an affine connection [112,117].

A. Quantum field theory of strong low-energy pion-nucleon interactions with nonlinear chiral $SU(2)\times SU(2)$ symmetry in Weinberg's parametrization

In Weinberg's parametrization $u=(1+i\vec{\tau}\cdot\vec{\xi})/\sqrt{1+\vec{\xi}^2}$, where $\vec{\xi}=\vec{\pi}'/2f_\pi$ [89], the effective chiral Lagrangian Eq. (21) takes the form

$$\begin{aligned}\mathcal{L}_{\mathrm{ChPT}}=&\bar{\psi}'_{NL}\bigg(i\gamma^\mu\partial_\mu-m_N-\gamma^\mu\frac{1}{4f_\pi^2}\frac{\vec{\tau}\cdot(\vec{\pi}'\times\partial_\mu\vec{\pi}')}{1+\vec{\pi}'^2/4f_\pi^2}\\&+\gamma^\mu\gamma^5\frac{1}{2f_\pi}\frac{\vec{\tau}\cdot\partial_\mu\vec{\pi}'}{1+\vec{\pi}'^2/4f_\pi^2}\bigg)\psi'_N\\&+\frac{1}{2}\frac{\partial_\mu\vec{\pi}'\cdot\partial^\mu\vec{\pi}'-m_\pi^2\vec{\pi}'^2}{1+\vec{\pi}'^2/4f_\pi^2}\end{aligned} \tag{22}$$

and describes the quantum field theory of strong low-energy pion-nucleon interactions with nonlinear realization of chiral

$SU(2)\times SU(2)$ symmetry in Weinberg's chiral perturbation theory [89,123,124]. A deviation of the axial coupling constant from unity $g_A > 1$ can be obtained in the hadron-loop approximation.

B. Quantum field theory of strong low-energy pion-nucleon interactions with nonlinear chiral $SU(2)\times SU(2)$ symmetry in Gasser-Leutwyler's parametrization

In the exponential or Gasser-Leutwyler's parametrization $u = e^{i\vec{\tau}\cdot\vec{\xi}}$, where $\vec{\xi} = \vec{\pi}'/2f_\pi$, the effective chiral Lagrangian Eq. (21) retains its form with $U = u^2 = e^{i\vec{\tau}\cdot\vec{\pi}'/f_\pi}$ [107–121]

$$\mathcal{L}_{\rm ChPT} = \bar{\psi}'_{NL}\left(i\gamma^\mu D_\mu - m_N - i\gamma^\mu\gamma^5\frac{1}{2}u^\dagger\partial_\mu U u^\dagger\right)\psi'_N + \frac{f^2_\pi}{4}\langle\partial_\mu U^\dagger\partial^\mu U\rangle\rangle + \frac{1}{4}m^2_\pi f^2_\pi\langle(U+U^\dagger)\rangle, \tag{23}$$

and describes the quantum field theory of strong low-energy pion-nucleon interactions with nonlinear realization of chiral $SU(2)\times SU(2)$ symmetry in Gasser-Leutwyler's chiral perturbation theory [107–121]. The deviation of the axial coupling constant from unity $g_A > 1$ can be obtained in the hadron-loop approximation [108–121].

C. Quantum field theoretic model of strong low-energy pion-nucleon interactions for the description of hadronic structure of the nucleon in neutron β^- decays

Since in the limit of infinite mass of the scalar isoscalar σ-meson $m_\sigma \to \infty$ (or in the limit $\gamma \to \infty$) the LσM with linear realization of chiral $SU(2)\times SU(2)$ symmetry is equivalent to chiral perturbation theory with nonlinear realization of chiral $SU(2)\times SU(2)$ symmetry in Weinberg's and Gasser-Leutwyler's parametrizations, we shall use the LσM for the description of contributions of hadronic structure of the nucleon to the neutron β^- decays. We shall calculate the corresponding Feynman diagrams for contributions of strong low-energy interactions to the amplitude of the neutron β^- decays. We take the contributions of these Feynman diagrams in the limit of the infinite scalar isoscalar σ-meson mass $m_\sigma \to \infty$ (or in the limit $\gamma \to \infty$). After renormalization the obtained expressions of the matrix elements of the $\mathbb{S}$-matrix for the amplitudes of the neutron β^- decays should be in agreement with such properties of the $\mathbb{S}$-matrix as analyticity, unitarity, cluster decomposition and symmetry. This should imply that because of equivalence of the LσM in the infinite limit of the scalar isoscalar σ-meson mass to quantum field theories of strong low-energy pion-nucleon interactions with nonlinear realization of chiral $SU(2)\times SU(2)$ symmetry, the contributions of these Feynman diagrams should be the same as the contributions of quantum field theories with nonlinear realization of chiral $SU(2)\times SU(2)$ symmetry and, correspondingly, current algebra. Such an assertion is based on Weinberg's "theorem" [124].

According to Weinberg [124], "The 'theorem' says that although individual quantum field theories have of course a good deal of content, quantum field theory itself has no content beyond analyticity, unitarity, cluster decomposition, and symmetry. This can be put more precisely in the context of perturbation theory: if one writes down the most general possible Lagrangian, including all terms consistent with assumed symmetry principles, and then calculates matrix elements with this Lagrangian to any given order of perturbation theory, the result will simply be the most general possible S-matrix consistent with analyticity, perturbative unitarity, cluster decomposition and the assumed symmetry principles. As I said, this has not been proved, but any counterexamples would be of great interest, and I do not know of any. With this 'theorem,' one can obtain and justify the results of current algebra simply by writing down the most general Lagrangian consistent with the assumed symmetry principles, and then deriving low energy theorems by a direct study of the Feynman graphs, without operator algebra. However, in order for this to be a derivation and not merely a mnemonic, it is necessary to include all possible terms in the Lagrangian, and take account of graphs of all orders in perturbation theory."

According to this theorem, one may expect that the contributions of strong low-energy interactions described by the LσM to the neutron β^- decays are at Sirlin's confidence level of the description of contributions of strong low-energy interactions to radiative corrections for the neutron lifetime.

V. QUANTUM FIELD THEORETIC MODEL OF STRONG LOW-ENERGY PION-NUCLEON AND ELECTROWEAK INTERACTIONS FOR THE DESCRIPTION OF NEUTRON β^- DECAYS

A. General properties of the Lagrangian for quantum field theoretic model of strong low-energy and weak interactions of pion-nucleon system coupled to electron and neutrino

For the analysis of neutron β^- decays within the quantum field theoretic model of strong low-energy and electroweak interactions of the pion-nucleon system coupled to electron and neutrino (antineutrino), we propose to rewrite the Lagrangian of the LσM in the $SU(2)\times SU(2)$ symmetric phase, given by Eq. (2), in terms of the field operators

$$
\begin{aligned}
&\Psi_{NL}=P_L\psi_N=P_L\begin{pmatrix}\psi_p\\ \psi_n\end{pmatrix},\quad \psi_{pR}=P_R\psi_p,\quad \psi_{nR}=P_R\psi_n,\\
&\Phi=\frac{1}{\sqrt{2}}\begin{pmatrix}\sigma+i\pi^3\\ i(\pi^1+i\pi^2)\end{pmatrix}=\frac{1}{\sqrt{2}}\begin{pmatrix}\sigma+i\pi^0\\ i\sqrt{2}\pi^-\end{pmatrix},\\
&\Phi^c=-i\tau_{2L}\Phi^*=\frac{1}{\sqrt{2}}\begin{pmatrix}i(\pi^1-i\pi^2)\\ \sigma-i\pi^3\end{pmatrix}=\frac{1}{\sqrt{2}}\begin{pmatrix}i\sqrt{2}\pi^+\\ \sigma-i\pi^0\end{pmatrix},
\end{aligned}
\tag{24}
$$

where τ_{2L} is the Pauli 2×2 matrix of the "weak isospin." The field operators Eq. (24) have the following properties under the $SU(2)_L\times U(1)_Y$ transformations:

$$
\begin{aligned}
\Psi_{NL}\xrightarrow{\vec{\alpha}_L,\alpha_Y}\Psi'_{NL}&=\Big(1+i\frac{1}{2}\vec{\tau}_L\cdot\vec{\alpha}_L+i\frac{1}{2}Y\alpha_Y\Big)\Psi_{NL},\\
\psi_{pR}\xrightarrow{\vec{\alpha}_L,\alpha_Y}\psi'_{pR}&=\Big(1+i\frac{1}{2}Y\alpha_Y\Big)\psi_{pR},\\
\psi_{nR}\xrightarrow{\vec{\alpha}_L,\alpha_Y}\psi'_{nR}&=\Big(1+i\frac{1}{2}Y\alpha_Y\Big)\psi_{nR},\\
\Phi\xrightarrow{\vec{\alpha}_L,\alpha_Y}\Phi'&=\Big(1+i\frac{1}{2}\vec{\tau}_L\cdot\vec{\alpha}_L+i\frac{1}{2}Y\alpha_Y\Big)\Phi,\\
\Phi^c\xrightarrow{\vec{\alpha}_L,\alpha_Y}\Phi^{c\prime}&=\Big(1+i\frac{1}{2}\vec{\tau}_L\cdot\vec{\alpha}_L+i\frac{1}{2}Y\alpha_Y\Big)\Phi^c,
\end{aligned}
\tag{25}
$$

where $\vec{I}_L=\frac{1}{2}\vec{\tau}_L$ and Y are operators of the "weak isospin" and "weak hypercharge," respectively, $\vec{\alpha}_L$ and α_Y are infinitesimal parameters of the $SU(2)_L$ and $U(1)_Y$ gauge group transformations, respectively. The operators of the third component I_{3L} of the weak isospin $\vec{I}_L$ and the weak hypercharge Y are related by $Q=I_{3L}+Y/2$ [59,89] (see also [3]), where Q is the operator of electric charge, measured in e, which is the proton electric charge. The eigenvalues of the third component of the weak isospin and weak hypercharge are $((I_{3L})_{pL},Y_{pL})=(+1/2,+1)$, $((I_{3L})_{nL},Y_{nL})=(-1/2,+1)$, $((I_{3L})_{pR},Y_{pR})=(0,+2)$, $((I_{3L})_{nR},Y_{nR})=(0,0)$, $((I_{3L})_{\sigma+i\pi^0},Y_\Phi)=(+1/2,-1)$, $((I_{3L})_{\pi^-},Y_\Phi)=(-1/2,-1)$, $((I_{3L})_{\pi^+},Y_{\Phi^c})=(+1/2,+1)$ and $((I_{3L})_{\sigma-i\pi^0},Y_{\Phi^c})=(-1/2,+1$, respectively. In terms of the field operators Eq. (24) the Lagrangian Eq. (2) takes the form

$$
\begin{aligned}
\mathcal{L}_{\rm L\sigma M}=&\bar{\Psi}_{NL}i\gamma^\mu\partial_\mu\Psi_{NL}+\bar{\psi}_{pR}i\gamma^\mu\partial_\mu\psi_{pR}+\bar{\psi}_{nR}i\gamma^\mu\partial_\mu\psi_{nR}\\
&-\sqrt{2}g_{\pi N}(\bar{\Psi}_{NL}\Phi\psi_{pR}+\bar{\psi}_{pR}\Phi^\dagger\Psi_{NL})\\
&-\sqrt{2}g_{\pi N}(\bar{\Psi}_{NL}\Phi^c\psi_{nR}+\bar{\psi}_{nR}\Phi^{c\dagger}\Psi_{NL})\\
&+\partial_\mu\Phi^\dagger\partial^\mu\Phi+\mu^2\Phi^\dagger\Phi-\frac{1}{2}\gamma(\Phi^\dagger\Phi)^2.
\end{aligned}
\tag{26}
$$

The Lagrangian Eq. (26) is invariant under global $SU(2)_L\times U(1)_Y$ transformations Eq. (25). Invariance under local $SU(2)_L\times U(1)_Y$ transformations can be reached by the inclusion of the interactions with gauge boson fields $\vec{W}_\mu$ and B_μ [59,89]. This gives

$$
\begin{aligned}
\mathcal{L}_{\rm L\sigma M}=&\bar{\Psi}_{NL}\Big(i\gamma^\mu\partial_\mu+ig\frac{1}{2}\vec{\tau}_L\cdot\vec{W}_\mu+ig'\frac{1}{2}B_\mu\Big)\Psi_{NL}+\bar{\psi}_{pR}(i\gamma^\mu\partial_\mu+ig'B_\mu)\psi_{pR}+\bar{\psi}_{nR}i\gamma^\mu\partial_\mu\psi_{nR}\\
&-g_{\pi N}(\bar{\Psi}_{NL}\Phi\psi_{pR}+\bar{\psi}_{pR}\Phi^\dagger\Psi_{NL})-g_{\pi N}(\bar{\Psi}_{NL}\Phi^c\psi_{nR}+\bar{\psi}_{nR}\Phi^{c\dagger}\Psi_{NL})\\
&+\Big(\partial_\mu\Phi^\dagger-ig\frac{1}{2}\Phi^\dagger\vec{\tau}_L\cdot\vec{W}_\mu+ig'\frac{1}{2}\Phi^\dagger B_\mu\Big)\Big(\partial^\mu\Phi+ig\frac{1}{2}\vec{\tau}_L\cdot\vec{W}_\mu\Phi-ig'\frac{1}{2}B_\mu\Phi\Big)\\
&+\mu^2\Phi^\dagger\Phi-\frac{1}{2}\gamma(\Phi^\dagger\Phi)^2,
\end{aligned}
\tag{27}
$$

where g and g' are the electroweak coupling constants [59,89]. The gauge boson fields $\vec{W}_\mu$ and B_μ have the following transformation properties under the $SU(2)_L\times U(1)_Y$ local transformations:

$$
\begin{aligned}
\vec{W}_\mu\xrightarrow{\vec{\alpha}_L,\alpha_Y}\vec{W}'_\mu&=\vec{W}_\mu+\vec{W}_\mu\times\vec{\alpha}_L-\frac{1}{g}\partial_\mu\vec{\alpha}_L,\\
B_\mu\xrightarrow{\vec{\alpha}_L,\alpha_Y}B'_\mu&=B_\mu-\frac{1}{g'}\partial_\mu\alpha_Y.
\end{aligned}
\tag{28}
$$

Having added to the Lagrangian Eq. (27) the kinetic terms of the electroweak gauge boson fields, the interactions of the electroweak gauge boson fields with the electron and neutrino fields $(\Psi_{\ell L},\psi_{eR})$ and the Higgs field ϕ [89] we arrive at the Lagrangian

$$\begin{aligned}\mathcal{L}_{\mathrm{L}\sigma\mathrm{M+SEM}} = {}& \bar{\Psi}_{NL}\left(i\gamma^{\mu}\partial_{\mu} + ig\frac{1}{2}\vec{\tau}_{L}\cdot\vec{W}_{\mu} + ig'\frac{1}{2}B_{\mu}\right)\Psi_{NL} + \bar{\psi}_{pR}(i\gamma^{\mu}\partial_{\mu} + ig'B_{\mu})\psi_{pR} + \bar{\psi}_{nR}i\gamma^{\mu}\partial_{\mu}\psi_{nR}\\ & - \sqrt{2}g_{\pi N}(\bar{\Psi}_{NL}\Phi\psi_{pR} + \bar{\psi}_{pR}\Phi^{\dagger}\Psi_{NL}) - \sqrt{2}g_{\pi N}(\bar{\Psi}_{NL}\Phi^{c}\psi_{nR} + \bar{\psi}_{nR}\Phi^{c\dagger}\Psi_{NL})\\ & + \left(\partial_{\mu}\Phi^{\dagger} - ig\frac{1}{2}\Phi^{\dagger}\vec{\tau}_{L}\cdot\vec{W}_{\mu} + ig'\frac{1}{2}\Phi^{\dagger}B_{\mu}\right)\left(\partial^{\mu}\Phi + ig\frac{1}{2}\vec{\tau}_{L}\cdot\vec{W}_{\mu}\Phi - ig'\frac{1}{2}B_{\mu}\Phi\right)\\ & + \mu^{2}\Phi^{\dagger}\Phi - \frac{1}{2}\gamma(\Phi^{\dagger}\Phi)^{2} - \frac{1}{4}\vec{W}_{\mu\nu}\cdot\vec{W}^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \bar{\Psi}_{\ell L}i\gamma^{\mu}\left(\partial_{\mu} + ig\frac{1}{2}\vec{\tau}\cdot\vec{W}_{\mu} - ig'\frac{1}{2}B_{\mu}\right)\Psi_{\ell L}\\ & + \bar{\psi}_{eR}i\gamma^{\mu}(\partial_{\mu} - ig'B_{\mu})\psi_{eR} - \sqrt{2}g_{e}(\bar{\Psi}_{\ell L}\psi_{eR}\phi + \phi^{\dagger}\bar{\psi}_{eR}\Psi_{\ell L}) + \left(\partial_{\mu}\phi^{\dagger} - ig\frac{1}{2}\phi^{\dagger}\vec{\tau}\cdot\vec{W}_{\mu} - ig'\frac{1}{2}\phi^{\dagger}B_{\mu}\right)\\ & \times\left(\partial^{\mu}\phi + ig\frac{1}{2}\vec{\tau}\cdot\vec{W}^{\mu}\phi + ig'\frac{1}{2}B^{\mu}\phi\right) + \tilde{\mu}^{2}\phi^{\dagger}\phi - \tilde{\lambda}(\phi^{\dagger}\phi)^{2}\end{aligned} \quad (29)$$

of the quantum field theoretic model of strong low-energy and electroweak interactions, which we apply to the analysis of hadronic structure of the nucleon in the neutron β^- decays, where $\vec{W}_{\mu\nu}$ and $B_{\mu\nu}$ are the operators of the field strength tensors of the gauge boson $\vec{W}_{\mu}$ and B_{μ} fields

$$\begin{aligned}\vec{W}_{\mu\nu} &= \partial_{\mu}\vec{W}_{\nu} - \partial_{\nu}\vec{W}_{\mu} - g\vec{W}_{\mu}\times\vec{W}_{\nu},\\ B_{\mu\nu} &= \partial_{\mu}B_{\nu} - \partial_{\nu}B_{\mu}\end{aligned} \quad (30)$$

and the operators of the lepton and Higgs fields are defined by

$$\Psi_{\ell L} = P_{L}\begin{pmatrix}\psi_{\nu_e}\\ \psi_{e}\end{pmatrix},\quad \psi_{eR} = P_{R}\psi_{e},\quad \phi = \begin{pmatrix}\phi^{+}\\ \phi^{0}\end{pmatrix}, \quad (31)$$

having the following properties under the $SU(2)_L\times U(1)_Y$ transformations:

$$\begin{aligned}\vec{W}_{\mu\nu}&\xrightarrow{\vec{\alpha}_L,\alpha_Y}\vec{W}'_{\mu\nu} = \vec{W}_{\mu\nu} + \vec{W}_{\mu\nu}\times\vec{\alpha}_{L},\\ B_{\mu\nu}&\xrightarrow{\vec{\alpha}_L,\alpha_Y}B'_{\mu\nu} = B_{\mu\nu},\\ \Psi_{eL}&\xrightarrow{\vec{\alpha}_L,\alpha_Y}\Psi'_{NL} = \left(1 + i\frac{1}{2}\vec{\tau}\cdot\vec{\alpha}_{L} + i\frac{1}{2}Y\alpha_{Y}\right)\Psi_{eL},\\ \psi_{eR}&\xrightarrow{\vec{\alpha}_L,\alpha_Y}\psi'_{eR} = \left(1 + i\frac{1}{2}Y\alpha_{Y}\right)\psi_{eR},\\ \phi&\xrightarrow{\vec{\alpha}_L,\alpha_Y}\phi' = \left(1 + i\frac{1}{2}\vec{\tau}_{L}\cdot\vec{\alpha}_{L} + i\frac{1}{2}Y\alpha_{Y}\right)\phi.\end{aligned} \quad (32)$$

The eigenvalues of the third component of the "weak isospin" and "weak hypercharge" are $((I_{3L})_{eL}, Y_{eL}) = (-1/2,-1)$, $((I_{3L})_{\nu_e L}, Y_{\nu_e L}) = (+1/2,-1)$, $((I_{3L})_{pR}, Y_{pR}) = (0,+2)$, $((I_{3L})_{eR}, Y_{eR}) = (0,-2)$, $((I_{3L})_{\phi^+}, Y_{\phi}) = (+1/2,+1)$ and $((I_{3L})_{\pi^0}, Y_{\phi}) = (-1/2,+1)$, respectively. For the derivation of the Lagrangians Eqs. (27) and (29) we have used the following standard definitions of the covariant derivatives of the left-handed fermions and the Higgs field $D_{L\mu}$ and the right-handed fermions $D_{R\mu}$ defined by [89]

$$\begin{aligned}D_{L\mu} &= \partial_{\mu} + ig\frac{1}{2}\vec{\tau}_{L}\cdot\vec{W}_{\mu} + ig'\frac{1}{2}YB_{\mu},\\ D_{R\mu} &= \partial_{\mu} + ig'\frac{1}{2}YB_{\mu},\end{aligned} \quad (33)$$

where Y is the operator of the weak hypercharge [89]. In the physical phase or in the phase of spontaneously broken $SU(2)_L\times U(1)_Y$ symmetry reduced to $SU(2)_L\times U(1)_Y\to U(1)_{\mathrm{em}}$, where $U(1)_{\mathrm{em}}$ is a gauge group of electromagnetic interactions, the components of the Higgs field ϕ are equal to $\phi^+ = \phi^- = 0$ and $\phi^0 = \phi^{0*} = (v+H)/\sqrt{2}$, respectively, where v is the vacuum expectation value $\langle\phi^0\rangle = \langle\phi^{0*}\rangle = v$ and H is the observable scalar Higgs field with mass $M_H = 125$ GeV [3]. In turn in the physical phase the hadronic fields Φ and Φ^c are defined by

$$\begin{aligned}\Phi &= \frac{1}{\sqrt{2}}\begin{pmatrix}\sigma + i\pi^{0}\\ i\sqrt{2}\pi^{-}\end{pmatrix}\to\frac{1}{\sqrt{2}}\begin{pmatrix}f_{\pi} + \sigma + i\pi^{0}\\ i\sqrt{2}\pi^{-}\end{pmatrix},\\ \Phi^{c} &= \frac{1}{\sqrt{2}}\begin{pmatrix}i\sqrt{2}\pi^{+}\\ \sigma - i\pi^{0}\end{pmatrix}\to\frac{1}{\sqrt{2}}\begin{pmatrix}i\sqrt{2}\pi^{+}\\ f_{\pi} + \sigma - i\pi^{0}\end{pmatrix},\end{aligned} \quad (34)$$

where the transition to the fields of physical hadronic states goes through the change of the σ field $\sigma\to f_{\pi} + \sigma$ with a vanishing vacuum expectation value $\langle\sigma\rangle = 0$ of the σ field on the right-hand side. In terms of the fields of the physical states the Lagrangian Eq. (29) takes the form

$$
\begin{aligned}
\mathcal{L}_{\mathrm{L}\sigma\mathrm{M+SEM}} &= \bar{\psi}_p(i\gamma^\mu\partial_\mu - m_N)\psi_p + \bar{\psi}_n(i\gamma^\mu\partial_\mu - m_N)\psi_n + \partial_\mu\pi^+\partial^\mu\pi^- + \frac{1}{2}\partial_\mu\pi^0\partial^\mu\pi^0 + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2)\\
&- \sqrt{2}g_{\pi N}\bar{\psi}_p i\gamma^5\psi_n\pi^+ - \sqrt{2}g_{\pi N}\bar{\psi}_n i\gamma^5\psi_p\pi^- - g_{\pi N}(\bar{\psi}_p i\gamma^5\psi_p - \bar{\psi}_n i\gamma^5\psi_n)\pi^0 - g_{\pi N}(\bar{\psi}_p\psi_p + \bar{\psi}_n\psi_n)\sigma\\
&- \gamma f_\pi\sigma(\sigma^2 + 2\pi^+\pi^- + (\pi^0)^2) - \frac{1}{4}\gamma(\sigma^2 + 2\pi^+\pi^- + (\pi^0)^2)^2 - \frac{1}{2}W^+_{\mu\nu}W^{-\mu\nu} + M_W^2 W^+_\mu W^{-\mu} - \frac{1}{4}Z_{\mu\nu}Z^{\mu\nu}\\
&+ \frac{1}{2}M_Z^2 Z_\mu Z^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 + \frac{1}{2}\partial_\mu H\partial^\mu H - \frac{1}{2}M_H^2 H^2 + \bar{\psi}_e(i\gamma^\mu\partial_\mu - m_e)\psi_e + \bar{\psi}_{\nu L} i\gamma^\mu\partial_\mu\psi_{\nu L}\\
&- \frac{g}{2\sqrt{2}}(\bar{\psi}_p\gamma^\mu(1-\gamma^5)\psi_n + i\sqrt{2}(\pi^0\partial_\mu\pi^- - \partial_\mu\pi^0\pi^-) - \sqrt{2}(\sigma\partial_\mu\pi^- - \partial_\mu\sigma\pi^-) - \sqrt{2}f_\pi\partial_\mu\pi^-)W^+_\mu\\
&- \frac{g}{2\sqrt{2}}(\bar{\psi}_n\gamma^\mu(1-\gamma^5)\psi_p + i\sqrt{2}(\pi^+\partial_\mu\pi^0 - \partial_\mu\pi^+\pi^0) - \sqrt{2}(\sigma\partial_\mu\pi^+ - \partial_\mu\sigma\pi^+) - \sqrt{2}f_\pi\partial_\mu\pi^+)W^-_\mu\\
&- \frac{g}{2\cos\theta_W}\Big(\frac{1}{2}\bar{\psi}_p\gamma^\mu(1 - 4\sin^2\theta_W - \gamma^5)\psi_p - \frac{1}{2}\bar{\psi}_n\gamma^\mu(1-\gamma^5)\psi_n + i(1 - 2\sin^2\theta_W)(\pi^+\partial_\mu\pi^- - \partial_\mu\pi^+\pi^-)\\
&- (\sigma\partial_\mu\pi^0 - \partial_\mu\sigma\pi^0) - f_\pi\partial_\mu\pi^0\Big)Z_\mu - e(\bar{\psi}_p\gamma^\mu\psi_p + i(\pi^-\partial_\mu\pi^+ - \partial_\mu\pi^-\pi^+))A_\mu - \frac{g}{2\sqrt{2}}\bar{\psi}_e\gamma^\mu(1-\gamma^5)\psi_{\nu L}W^-_\mu\\
&- \frac{g}{2\sqrt{2}}\bar{\psi}_{\nu L}\gamma^\mu(1-\gamma^5)\psi_e W^+_\mu + \frac{g}{4\cos\theta_W}\bar{\psi}_e\gamma^\mu(1 - 4\sin^2\theta_W - \gamma^5)\psi_e Z_\mu - \frac{g}{4\cos\theta_W}\bar{\psi}_{\nu L}\gamma^\mu(1-\gamma^5)\psi_{\nu L}Z_\mu\\
&+ e\bar{\psi}_e\gamma^\mu\psi_e A_\mu + \frac{1}{2}(egA_\mu + g^2\cos\theta_W\tan^2\theta_W Z_\mu)(i(f_\pi + \sigma)(\pi^+W^{-\mu} - \pi^-W^{+\mu}) - \pi^0(\pi^+W^{-\mu} + \pi^-W^{+\mu}))\\
&+ \frac{1}{4}g^2(2f_\pi\sigma^2 + (\pi^0)^2 + 2\pi^+\pi^-)W^+_\mu W^{-\mu} + \frac{1}{8}\frac{g^2}{\cos^2\theta_W}(2f_\pi\sigma^2 + (\pi^0)^2)Z_\mu Z^\mu + \pi^+\pi^-\Big(eA_\mu + \frac{g}{2\cos\theta_W}\\
&\times(1 - 2\sin^2\theta_W)Z_\mu\Big)\Big(eA^\mu + \frac{g}{2\cos\theta_W}(1 - 2\sin^2\theta_W)Z^\mu\Big) + i\frac{1}{2}eW^-_{\mu\nu}(W^{+\mu}A^\nu - A^\mu W^{+\nu}) + i\frac{1}{2}g\cos\theta_W\\
&\times W^-_{\mu\nu}(W^{+\mu}Z^\nu - Z^\mu W^{+\nu}) + i\frac{1}{2}eW^+_{\mu\nu}(A^\mu W^{-\nu} - W^{-\mu}A^\nu) + i\frac{1}{2}g\cos\theta_W W^+_{\mu\nu}(Z^\mu W^{-\nu}W^{-\mu}Z^\nu)\\
&+ \frac{1}{2}e^2(W^+_\mu A_\nu - A_\mu W^+_\nu)(A^\mu W^{-\nu} - W^{-\mu}A^\nu) + \frac{1}{2}g^2\cos^2\theta_W(W^+_\mu Z_\nu - Z_\mu W^+_\nu)(Z^\mu W^{-\nu} - W^{-\mu}Z^\nu)\\
&+ \frac{1}{2}eg\cos\theta_W(W^+_\mu A_\nu - A_\mu W^+_\nu)(Z^\mu W^{-\nu} - W^{-\mu}Z^\nu) + \frac{1}{2}eg\cos\theta_W(W^+_\mu Z_\nu - Z_\mu W^+_\nu)(A^\mu W^{-\nu} - W^{-\mu}A^\nu)\\
&+ \frac{1}{2}ieF_{\mu\nu}(W^{-\mu}W^{+\nu} - W^{+\mu}W^{-\nu}) + \frac{1}{2}ig\cos\theta_W Z_{\mu\nu}(W^{-\mu}W^{+\nu} - W^{+\mu}W^{-\nu}) + \frac{1}{4}g^2(W^-_\mu W^+_\nu - W^+_\mu W^-_\nu)\\
&\times(W^{-\mu}W^{+\nu} - W^{+\mu}W^{-\nu}) + \frac{M_W^2}{v}W^+_\mu W^{-\mu}H + \frac{1}{4}\frac{M_W^2}{v^2}W^+_\mu W^{-\mu}H^2 + \frac{1}{2}\frac{M_Z^2}{v}Z_\mu Z^\mu H + \frac{1}{8}\frac{M_Z^2}{v^2}Z_\mu Z^\mu H^2\\
&- \frac{m_e}{v}\bar{\psi}_e\psi_e H - \frac{1}{2}\frac{M_H^2}{v}H^3 - \frac{1}{8}\frac{M_H^2}{v^2}H^4,
\end{aligned}
\tag{35}
$$

where θ_W is the Weinberg angle defined by $\tan\theta_W = g'/g$ [3,89], the field operators $W^\pm = (W^1 \mp iW^2)/\sqrt{2}$ of the $W^\pm$ boson, $Z_\mu = W^3_\mu\cos\theta_W - B_\mu\sin\theta_W$ of the Z boson and $A_\mu = B_\mu\cos\theta_W + W^3_\mu\sin\theta_W$ of the electromagnetic fields, respectively, $e = g\sin\theta_W$ is the proton electric charge. Then, we have denoted $X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$ for $X = W^\pm$, Z and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field strength tensor. The term $(1/2\xi)(\partial_\mu A^\mu)^2$ fixes a gauge of the electromagnetic field, where ξ is a gauge parameter [146]. The massive fields of the $W^\pm$- and Z-electroweak bosons are defined in the physical gauge with masses equal to

$$
M_W^2 = \frac{1}{4}g^2(v^2 + f_\pi^2), \qquad M_Z^2 = \frac{M_W^2}{\cos^2\theta_W} \tag{36}
$$

with the hadronic contribution defined by the term proportional to f_π^2. The vacuum expectation values v and f_π of the Higgs and σ-meson fields are equal to $v = \sqrt{\tilde{\mu}^2/\tilde{\lambda}}$ and $f_\pi = \sqrt{\mu^2/\gamma}$, respectively. The masses of the hadrons, electron and Higgs boson are given by

$$
\begin{aligned}
&m_N = g_{\pi N}f_\pi, \qquad m_\pi^2 = 0, \qquad m_\sigma^2 = 2f_\pi^2\gamma,\\
&m_e = g_e v, \qquad M_H^2 = 2v^2\tilde{\lambda}.
\end{aligned}
\tag{37}
$$

The Lagrangian Eq. (35) as well as the Lagrangian Eq. (29) is invariant under gauge $SU(2)_L \times U(1)_Y$ transformations Eqs. (25), (28) and (32). Such an invariance is being retained as long as pions $\vec{\pi} = (\pi^{\pm}, \pi^0)$ are massless.

The term violating chiral $SU(2) \times SU(2)$ invariance and providing a nonvanishing pion mass is equal to $\delta\mathcal{L}_{\mathrm{L}\sigma\mathrm{M}} = m_\pi^2 f_\pi \sigma$. This leads to the hadronic masses

$$m_N = g_{\pi N} f_\pi, \qquad m_\pi^2 = f_\pi^2 \gamma - \mu^2, \qquad m_\sigma^2 = 2 f_\pi^2 \gamma + m_\pi^2 \tag{38}$$

and $f_\pi = \sqrt{(\mu^2 + m_\pi^2)/\gamma} > \sqrt{\mu^2/\gamma}$. Since the σ field is a component of the $SU(2)_L \times U(1)_Y$ doublet $\sigma = (\Phi_{+1/2} + \Phi^c_{-1/2})/\sqrt{2}$, where $\Phi_{+1/2}$ and $\Phi^c_{-1/2}$ are the *up* and *down* components of the $SU(2)_L \times U(1)_Y$ doublets Φ and Φ^c, respectively, the term $\delta\mathcal{L}_{\mathrm{L}\sigma\mathrm{M}} \to \delta\mathcal{L}_{\mathrm{L}\sigma\mathrm{M+SEM}} = m_\pi^2 f_\pi \sigma = m_\pi^2 f_\pi (\Phi_{+1/2} + \Phi^c_{-1/2})/\sqrt{2}$ violates also invariance under $SU(2)_L \times U(1)_Y$ transformations. Restoration of invariance under $SU(2)_L \times U(1)_Y$ transformations can be reached following Weinberg [59] and introducing the interaction

$$\delta\mathcal{L}_{\mathrm{L}\sigma\mathrm{M+SEM}} = \frac{m_\pi^2}{\sqrt{2}} \frac{f_\pi}{v} (\Phi^{c\dagger}\phi + \phi^\dagger \Phi^c). \tag{39}$$

This allows us to deal with the term $\delta\mathcal{L}_{\mathrm{L}\sigma\mathrm{M}} = m_\pi^2 f_\pi \sigma$ in the form invariant under $SU(2)_L \times U(1)_Y$ transformations. In the phase of spontaneously broken $SU(2)_L \times U(1)_Y$ symmetry the interaction Eq. (39) acquires a form

$$\delta\mathcal{L}_{\mathrm{L}\sigma\mathrm{M+SEM}} = m_\pi^2 f_\pi \sigma \left(1 + \frac{H}{v}\right). \tag{40}$$

In the chirally broken phase, when $\sigma \to f_\pi + \sigma$, the contribution of the interaction Eq. (40) to the Lagrangian $\mathcal{L}_{\mathrm{L}\sigma\mathrm{M+SEM}}$ in Eq. (35) is given by

$$\delta\mathcal{L}_{\mathrm{L}\sigma\mathrm{M+SEM}} \to -m_\pi^2 \pi^+\pi^- - \frac{1}{2} m_\pi^2 (\pi^0)^2 + \frac{m_\pi^2 f_\pi}{v} \sigma H. \tag{41}$$

The terms linear in σ and H, which appear in the $SU(2)_L \times U(1)_Y$ symmetry broken phase, lead to a redefinition of the vacuum expectation value v of the Higgs field only. A relative correction $\delta v/v_0 = f_\pi^2 m_\pi^2 / v_0^2 M_H^2$ to the standard value $v_0 = \sqrt{\tilde{\mu}^2/\tilde{\lambda}} = 246$ GeV [3] is of about 10^{-13}, calculated for the Higgs-boson mass $M_H = 125$ GeV, $f_\pi = 92.4$ MeV and $m_\pi = 140$ MeV [3]. We would like to accentuate that the interaction Eq. (41) amends only invariance under global $SU(2)_L \times U(1)_Y$ transformations but not gauge ones. Indeed, a nonvanishing pion mass leads to nonconservation (or partial conservation) of the axial-vector hadronic current, violating invariance under $SU(2)_L \times U(1)_Y$ gauge transformations. Below we show this by example of the neutron β^- decays.

Together with the contribution of the interaction Eq. (39), taken in the physical phase given by Eq. (41), the quantum field theoretic model of strong low-energy pion-nucleon and electroweak hadron-hadron, hadron-lepton and lepton-lepton interactions, where leptons are an electron e^- and neutrino ν_e, is described by the Lagrangian

$$\begin{aligned}
\mathcal{L}_{\mathrm{L}\sigma\mathrm{M+SEM}} &= \bar{\psi}_p (i\gamma^\mu \partial_\mu - m_N)\psi_p + \bar{\psi}_n (i\gamma^\mu \partial_\mu - m_N)\psi_n + (\partial_\mu \pi^+ \partial^\mu \pi^- - m_\pi^2)\pi^+\pi^- \\
&+ \frac{1}{2}(\partial_\mu \pi^0 \partial^\mu \pi^0 - m_\pi^2 (\pi^0)^2) + \frac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - \sqrt{2} g_{\pi N} \bar{\psi}_p i\gamma^5 \psi_n \pi^+ - \sqrt{2} g_{\pi N} \bar{\psi}_n i\gamma^5 \psi_p \pi^- \\
&- g_{\pi N}(\bar{\psi}_p i\gamma^5 \psi_p - \bar{\psi}_n i\gamma^5 \psi_n)\pi^0 - g_{\pi N}(\bar{\psi}_p \psi_p + \bar{\psi}_n \psi_n)\sigma - \gamma f_\pi \sigma(\sigma^2 + 2\pi^+\pi^- + (\pi^0)^2) \\
&- \frac{1}{4}\gamma(\sigma^2 + 2\pi^+\pi^- + (\pi^0)^2)^2 - \frac{1}{2} W^+_{\mu\nu} W^{-\mu\nu} + M_W^2 W^+_\mu W^{-\mu} - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\
&- \frac{1}{2\xi}(\partial_\mu A^\mu)^2 + \frac{1}{2}\partial_\mu H \partial^\mu H - \frac{1}{2} M_H^2 H^2 + \bar{\psi}_e (i\gamma^\mu \partial_\mu - m_e)\psi_e + \bar{\psi}_{\nu L} i\gamma^\mu \partial_\mu \psi_{\nu L} \\
&- \frac{g}{2\sqrt{2}}(\bar{\psi}_p \gamma^\mu (1 - \gamma^5)\psi_n + i\sqrt{2}(\pi^0 \partial_\mu \pi^- - \partial_\mu \pi^0 \pi^-) - \sqrt{2}(\sigma \partial_\mu \pi^- - \partial_\mu \sigma \pi^-) - \sqrt{2} f_\pi \partial_\mu \pi^-) W^+_\mu \\
&- \frac{g}{2\sqrt{2}}(\bar{\psi}_n \gamma^\mu (1 - \gamma^5)\psi_p + i\sqrt{2}(\pi^+ \partial_\mu \pi^0 - \partial_\mu \pi^+ \pi^0) - \sqrt{2}(\sigma \partial_\mu \pi^+ - \partial_\mu \sigma \pi^+) - \sqrt{2} f_\pi \partial_\mu \pi^+) W^-_\mu \\
&- \frac{g}{2\cos\theta_W}\bigg(\frac{1}{2}\bar{\psi}_p \gamma^\mu (1 - 4\sin^2\theta_W - \gamma^5)\psi_p - \frac{1}{2}\bar{\psi}_n \gamma^\mu (1 - \gamma^5)\psi_n + i(1 - 2\sin^2\theta_W)(\pi^+ \partial_\mu \pi^- - \partial_\mu \pi^+ \pi^-) \\
&- (\sigma \partial_\mu \pi^0 - \partial_\mu \sigma \pi^0) - f_\pi \partial_\mu \pi^0\bigg) Z_\mu - e(\bar{\psi}_p \gamma^\mu \psi_p + i(\pi^- \partial_\mu \pi^+ - \partial_\mu \pi^- \pi^+))A_\mu - \frac{g}{2\sqrt{2}}\bar{\psi}_e \gamma^\mu (1 - \gamma^5)\psi_{\nu L} W^-_\mu
\end{aligned}$$

$$
\begin{aligned}
&-\frac{g}{2\sqrt{2}}\bar{\psi}_{\nu L}\gamma^{\mu}(1-\gamma^5)\psi_e W^+_{\mu} + \frac{g}{4\cos\theta_W}\bar{\psi}_e\gamma^{\mu}(1-4\sin^2\theta_W-\gamma^5)\psi_e Z_{\mu} - \frac{g}{4\cos\theta_W}\bar{\psi}_{\nu L}\gamma^{\mu}(1-\gamma^5)\psi_{\nu L}Z_{\mu}\\
&+ e\bar{\psi}_e\gamma^{\mu}\psi_e A_{\mu} + \frac{1}{2}(egA_{\mu} + g^2\cos\theta_W\tan^2\theta_W Z_{\mu})(i(f_{\pi}+\sigma)(\pi^+W^{-\mu} - \pi^-W^{+\mu}) - \pi^0(\pi^+W^{-\mu} + \pi^-W^{+\mu}))\\
&+ \frac{1}{4}g^2(2f_{\pi}\sigma + \sigma^2 + (\pi^0)^2 + 2\pi^+\pi^-)W^+_{\mu}W^{-\mu} + \frac{1}{8}\frac{g^2}{\cos^2\theta_W}(2f_{\pi}\sigma + \sigma^2 + (\pi^0)^2)Z_{\mu}Z^{\mu} + \pi^+\pi^-\bigg(eA_{\mu} + \frac{g}{2\cos\theta_W}\\
&\times(1-2\sin^2\theta_W)Z_{\mu}\bigg)\bigg(eA^{\mu} + \frac{g}{2\cos\theta_W}(1-2\sin^2\theta_W)Z^{\mu}\bigg) + i\frac{1}{2}eW^-_{\mu\nu}(W^{+\mu}A^{\nu} - A^{\mu}W^{+\nu}) + i\frac{1}{2}g\cos\theta_W\\
&\times W^-_{\mu\nu}(W^{+\mu}Z^{\nu} - Z^{\mu}W^{+\nu}) + i\frac{1}{2}eW^+_{\mu\nu}(A^{\mu}W^{-\nu} - W^{-\mu}A^{\nu}) + i\frac{1}{2}g\cos\theta_W W^+_{\mu\nu}(Z^{\mu}W^{-\nu} - W^{-\mu}Z^{\nu})\\
&+ \frac{1}{2}e^2(W^+_{\mu}A_{\nu} - A_{\mu}W^+_{\nu})(A^{\mu}W^{-\nu} - W^{-\mu}A^{\nu}) + \frac{1}{2}g^2\cos^2\theta_W(W^+_{\mu}Z_{\nu} - Z_{\mu}W^+_{\nu})(Z^{\mu}W^{-\nu} - W^{-\mu}Z^{\nu})\\
&+ \frac{1}{2}eg\cos\theta_W(W^+_{\mu}A_{\nu} - A_{\mu}W^+_{\nu})(Z^{\mu}W^{-\nu} - W^{-\mu}Z^{\nu}) + \frac{1}{2}eg\cos\theta_W(W^+_{\mu}Z_{\nu} - Z_{\mu}W^+_{\nu})(A^{\mu}W^{-\nu} - W^{-\mu}A^{\nu})\\
&+ \frac{1}{2}ieF_{\mu\nu}(W^{-\mu}W^{+\nu} - W^{+\mu}W^{-\nu}) + \frac{1}{2}ig\cos\theta_W Z_{\mu\nu}(W^{-\mu}W^{+\nu} - W^{+\mu}W^{-\nu}) + \frac{1}{4}g^2(W^-_{\mu}W^+_{\nu} - W^+_{\mu}W^-_{\nu})\\
&\times(W^{-\mu}W^{+\nu} - W^{+\mu}W^{-\nu}) + \frac{M^2_W}{v}W^+_{\mu}W^{-\mu}H + \frac{1}{4}\frac{M^2_W}{v^2}W^+_{\mu}W^{-\mu}H^2 + \frac{1}{2}\frac{M^2_Z}{v}Z_{\mu}Z^{\mu}H + \frac{1}{8}\frac{M^2_Z}{v^2}Z_{\mu}Z^{\mu}H^2\\
&- \frac{m_e}{v}\bar{\psi}_e\psi_e H - \frac{1}{2}\frac{M^2_H}{v}H^3 - \frac{1}{8}\frac{M^2_H}{v^2}H^4 + \frac{m^2_{\pi}f_{\pi}}{v}\sigma H. \qquad (42)
\end{aligned}
$$

We would like to emphasize that the $W^{\pm}$ bosons couple to the $V-A$ hadronic currents, providing in the tree approximation a standard $V-A$ effective low-energy interaction for the description of the neutron β^- decays [41,42]. The vector and axial-vector hadronic currents have baryonic and mesonic parts in agreement with Eq. (9), which are necessary for conservation of vector and partial conservation of axial-vector hadronic currents [41,42,60,141]. A partial conservation of the axial-vector hadronic current assumes a proportionality of the divergence of the axial-vector hadronic current to the squared pion mass [129]. In the chiral limit, i.e. in the limit of zero pion mass $m_{\pi}\to 0$, the axial-vector hadronic current is conserved [42]. An influence of partial conservation of the axial-vector hadronic current on gauge invariance of radiative corrections, caused by hadronic structure of the nucleon, we shall investigate below by example of radiative corrections of order $O(\alpha E_e/m_N)$ to the neutron lifetime.

B. Renormalization of the quantum field theory of strong low-energy and electroweak interactions described by the Lagrangian Eq. (42)

For the discussion of the renormalization procedure in the quantum field theoretic model LσM + SEM we rewrite the Lagrangian Eq. (42) as follows:

$$
\begin{aligned}
\mathcal{L}^{(0)}_{\mathrm{L}\sigma\mathrm{M+SEM}} &= \bar{\psi}^{(0)}_p(i\gamma^{\mu}\partial_{\mu} - m^{(0)}_N)\psi^{(0)}_p + \bar{\psi}^{(0)}_n(i\gamma^{\mu}\partial_{\mu} - m^{(0)}_N)\psi^{(0)}_n + (\partial_{\mu}\pi^{(0)+}\partial^{\mu}\pi^{(0)-} - m^{(0)2}_{\pi})\pi^{(0)+}\pi^{(0)-}\\
&+ \frac{1}{2}(\partial_{\mu}\pi^{(0)0}\partial^{\mu}\pi^{(0)0} - m^{(0)2}_{\pi}(\pi^{(0)0})^2) + \frac{1}{2}(\partial_{\mu}\sigma^{(0)}\partial^{\mu}\sigma^{(0)} - m^{(0)2}_{\sigma}(\sigma^{(0)})^2) - \sqrt{2}g^{(0)}_{\pi N}\bar{\psi}^{(0)}_p i\gamma^5\psi^{(0)}_n\pi^{(0)+}\\
&- \sqrt{2}g^{(0)}_{\pi N}\bar{\psi}^{(0)}_n i\gamma^5\psi_p\pi^{(0)-} - g^{(0)}_{\pi N}(\bar{\psi}^{(0)}_p i\gamma^5\psi^{(0)}_p - \bar{\psi}^{(0)}_n i\gamma^5\psi^{(0)}_n)\pi^{(0)0} - g^{(0)}_{\pi N}(\bar{\psi}^{(0)}_p\psi^{(0)}_p + \bar{\psi}^{(0)}_n\psi^{(0)}_n)\sigma^{(0)}\\
&- \gamma^{(0)}f^{(0)}_{\pi}\sigma^{(0)}((\sigma^{(0)})^2 + 2\pi^{(0)+}\pi^{(0)-} + (\pi^{(0)0})^2) - \frac{1}{4}\gamma^{(0)}((\sigma(0))^2 + 2\pi^{(0)+}\pi^{(0)-} + (\pi^{(0)0})^2)^2 - \frac{1}{2}W^{(0)+}_{\mu\nu}W^{(0)-\mu\nu}\\
&+ M^{(0)2}_W W^{(0)+}_{\mu}W^{(0)-\mu} - \frac{1}{4}Z^{(0)}_{\mu\nu}Z^{(0)\mu\nu} + \frac{1}{2}M^{(0)2}_Z Z^{(0)}_{\mu}Z^{(0)\mu} - \frac{1}{4}F^{(0)}_{\mu\nu}F^{(0)\mu\nu} - \frac{1}{2\xi^{(0)}}(\partial_{\mu}A^{(0)\mu})^2 + \frac{1}{2}\partial_{\mu}H^{(0)}\partial^{\mu}H^{(0)}\\
&- \frac{1}{2}M^{(0)2}_H(H^{(0)})^2 + \bar{\psi}^{(0)}_e(i\gamma^{\mu}\partial_{\mu} - m^{(0)}_e)\psi^{(0)}_e + \bar{\psi}^{(0)}_{\nu L}i\gamma^{\mu}\partial_{\mu}\psi^{(0)}_{\nu L} - \frac{g^{(0)}}{2\sqrt{2}}(\bar{\psi}^{(0)}_p\gamma^{\mu}(1-\gamma^5)\psi^{(0)}_n + i\sqrt{2}(\pi^{(0)0}\partial_{\mu}\pi^{(0)-}\\
&- \partial_{\mu}\pi^{(0)0}\pi^{(0)-}) - \sqrt{2}(\sigma^{(0)}\partial_{\mu}\pi^{(0)-} - \partial_{\mu}\sigma^{(0)}\pi^{(0)-}) - \sqrt{2}f^{(0)}_{\pi}\partial_{\mu}\pi^{(0)-})W^{(0)+}_{\mu} - \frac{g^{(0)}}{2\sqrt{2}}(\bar{\psi}^{(0)}_n\gamma^{\mu}(1-\gamma^5)\psi^{(0)}_p
\end{aligned}
$$

$$
\begin{aligned}
&+i\sqrt{2}(\pi^{(0)+}\partial_\mu\pi^{(0)0}-\partial_\mu\pi^{(0)+}\pi^{(0)0})-\sqrt{2}(\sigma^{(0)}\partial_\mu\pi^{(0)+}-\partial_\mu\sigma^{(0)}\pi^{(0)+})-\sqrt{2}f_\pi^{(0)}\partial_\mu\pi^{(0)+})W_\mu^{(0)-}\\
&-\frac{g^{(0)}}{2\cos\theta_W}\left(\frac{1}{2}\bar{\psi}_p^{(0)}\gamma^\mu(1-4\sin^2\theta_W-\gamma^5)\psi_p^{(0)}-\frac{1}{2}\bar{\psi}_n^{(0)}\gamma^\mu(1-\gamma^5)\psi_n^{(0)}+i(1-2\sin^2\theta_W)(\pi^{(0)+}\partial_\mu\pi^{(0)-}\right.\\
&\left.-\partial_\mu\pi^{(0)+}\pi^{(0)-})-(\sigma^{(0)}\partial_\mu\pi^{(0)0}-\partial_\mu\sigma^{(0)}\pi^{(0)0})-f_\pi^{(0)}\partial_\mu\pi^{(0)0}\right)Z_\mu^{(0)}-e^{(0)}(\bar{\psi}_p^{(0)}\gamma^\mu\psi_p^{(0)}+i(\pi^{(0)-}\partial_\mu\pi^{(0)+}\\
&-\partial_\mu\pi^{(0)-}\pi^{(0)+}))A_\mu^{(0)}-\frac{g^{(0)}}{2\sqrt{2}}\bar{\psi}_e^{(0)}\gamma^\mu(1-\gamma^5)\psi_{\nu L}^{(0)}W_\mu^{(0)-}-\frac{g^{(0)}}{2\sqrt{2}}\bar{\psi}_{\nu L}^{(0)}\gamma^\mu(1-\gamma^5)\psi_e^{(0)}W_\mu^{(0)+}+\frac{g^{(0)}}{4\cos\theta_W}\\
&\times\bar{\psi}_e^{(0)}\gamma^\mu(1-4\sin^2\theta_W-\gamma^5)\psi_e^{(0)}Z_\mu^{(0)}-\frac{g^{(0)}}{4\cos\theta_W}\bar{\psi}_{\nu L}^{(0)}\gamma^\mu(1-\gamma^5)\psi_{\nu L}^{(0)}Z_\mu^{(0)}+e^{(0)}\bar{\psi}_e^{(0)}\gamma^\mu\psi_e^{(0)}A_\mu^{(0)}\\
&+\frac{1}{2}(e^{(0)}g^{(0)}A_\mu^{(0)}+g^{(0)2}\cos\theta_W\tan^2\theta_W Z_\mu^{(0)})(i(f_\pi^{(0)}+\sigma^{(0)})(\pi^{(0)+}W^{(0)-\mu}-\pi^{(0)-}W^{(0)+\mu})\\
&-\pi^{(0)0}(\pi^{(0)+}W^{(0)-\mu}+\pi^{(0)-}W^{(0)+\mu}))+\frac{1}{4}g^{(0)2}(2f_\pi^{(0)}\sigma^{(0)}+(\sigma^{(0)})^2+(\pi^{(0)0})^2+2\pi^{(0)+}\pi^{(0)-})W_\mu^{(0)+}W^{(0)-\mu}\\
&+\frac{1}{8}\frac{g^{(0)2}}{\cos^2\theta_W}(2f_\pi^{(0)}\sigma^{(0)}+(\sigma^{(0)})^2+(\pi^{(0)0})^2)Z_\mu^{(0)}Z^{(0)\mu}+\pi^{(0)+}\pi^{(0)-}\left(e^{(0)}A_\mu^{(0)}+\frac{g^{(0)}}{2\cos\theta_W}(1-2\sin^2\theta_W)Z_\mu^{(0)}\right)\\
&\times\left(e^{(0)}A^{(0)\mu}+\frac{g^{(0)}}{2\cos\theta_W}(1-2\sin^2\theta_W)Z^{(0)\mu}\right)+i\frac{1}{2}e^{(0)}W_{\mu\nu}^{(0)-}(W^{(0)+\mu}A^{(0)\nu}-A^{(0)\mu}W^{(0)+\nu})+i\frac{1}{2}g^{(0)}\cos\theta_W\\
&\times W_{\mu\nu}^{(0)-}(W^{(0)+\mu}Z^{(0)\nu}-Z^{(0)\mu}W^{(0)+\nu})+i\frac{1}{2}e^{(0)}W_{\mu\nu}^{(0)+}(A^{(0)\mu}W^{(0)-\nu}-W^{(0)-\mu}A^{(0)\nu})+i\frac{1}{2}g^{(0)}\cos\theta_W W_{\mu\nu}^{(0)+}\\
&\times(Z^{(0)\mu}W^{(0)-\nu}-W^{(0)-\mu}Z^{(0)\nu})+\frac{1}{2}e^{(0)2}(W_\mu^{(0)+}A_\nu^{(0)}-A_\mu^{(0)}W_\nu^{(0)+})(A^{(0)\mu}W^{(0)-\nu}-W^{(0)-\mu}A^{(0)\nu})+\frac{1}{2}g^{(0)2}\\
&\times\cos^2\theta_W(W_\mu^{(0)+}Z_\nu^{(0)}-Z_\mu^{(0)}W_\nu^{(0)+})(Z^{(0)\mu}W^{(0)-\nu}-W^{(0)-\mu}Z^{(0)\nu})+\frac{1}{2}e^{(0)}g^{(0)}\cos\theta_W(W_\mu^{(0)+}A_\nu^{(0)}-A_\mu^{(0)}W_\nu^{(0)+})\\
&\times(Z^{(0)\mu}W^{(0)-\nu}-W^{(0)-\mu}Z^{(0)\nu})+\frac{1}{2}e^{(0)}g^{(0)}\cos\theta_W(W_\mu^{(0)+}Z_\nu^{(0)}-Z_\mu^{(0)}W_\nu^{(0)+})(A^{(0)\mu}W^{(0)-\nu}-W^{(0)-\mu}A^{(0)\nu})\\
&+\frac{1}{2}ie^{(0)}F_{\mu\nu}^{(0)}(W^{(0)-\mu}W^{(0)+\nu}-W^{(0)+\mu}W^{(0)-\nu})+\frac{1}{2}ig^{(0)}\cos\theta_W Z_{\mu\nu}^{(0)}(W^{(0)-\mu}W^{(0)+\nu}-W^{(0)+\mu}W^{(0)-\nu})\\
&+\frac{1}{4}g^{(0)2}(W_\mu^{(0)-}W_\nu^{(0)+}-W_\mu^{(0)+}W_\nu^{(0)-})(W^{(0)-\mu}W^{(0)+\nu}-W^{(0)+\mu}W^{(0)-\nu})+\frac{M_W^{(0)2}}{v^{(0)}}W_\mu^{(0)+}W^{(0)-\mu}H^{(0)}\\
&+\frac{1}{4}\frac{M_W^{(0)2}}{v^{(0)2}}W_\mu^{(0)+}W^{(0)-\mu}(H^{(0)})^2+\frac{1}{2}\frac{M_Z^{(0)2}}{v^{(0)}}Z_\mu^{(0)}Z^{(0)\mu}H^{(0)}+\frac{1}{8}\frac{M_Z^{(0)2}}{v^{(0)2}}Z_\mu^{(0)}Z^{(0)\mu}(H^{(0)})^2-\frac{m_e^{(0)}}{v^{(0)}}\bar{\psi}_e^{(0)}\psi_e^{(0)}H^{(0)}\\
&-\frac{1}{2}\frac{M_H^{(0)2}}{v^{(0)}}(H^{(0)})^3-\frac{1}{8}\frac{M_H^{(0)2}}{v^{(0)2}}(H^{(0)})^4+\frac{m_\pi^{(0)2}f_\pi^{(0)}}{v^{(0)}}\sigma^{(0)}H^{(0)},
\end{aligned}
\tag{43}
$$

where the subscript (0) denotes *bare* fields and their *bare* masses and coupling constants, respectively. After the calculation of loop-contributions the dynamics of strong low-energy and electroweak interactions of physical fields is described in the quantum field theoretic model LσM + SEM by the Lagrangian

$$\begin{aligned}
\mathcal{L}^{(r)}_{\mathrm{L}\sigma\mathrm{M+SEM}} &= \bar{\psi}^{(r)}_p(i\gamma^\mu\partial_\mu - m^{(r)}_N)\psi^{(r)}_p + \bar{\psi}^{(r)}_n(i\gamma^\mu\partial_\mu - m^{(r)}_N)\psi^{(r)}_n + (\partial_\mu\pi^{(r)+}\partial^\mu\pi^{(r)-} - m^{(r)2}_\pi)\pi^{(r)+}\pi^{(r)-} \\
&+ \frac{1}{2}(\partial_\mu\pi^{(r)0}\partial^\mu\pi^{(r)0} - m^{(r)2}_\pi(\pi^{(r)0})^2) + \frac{1}{2}(\partial_\mu\sigma^{(r)}\partial^\mu\sigma^{(r)} - m^{(r)2}_\sigma(\sigma^{(r)})^2) - \sqrt{2}g^{(r)}_{\pi N}\bar{\psi}^{(r)}_p i\gamma^5\psi^{(r)}_n\pi^{(r)+} \\
&- \sqrt{2}g^{(r)}_{\pi N}\bar{\psi}^{(r)}_n i\gamma^5\psi_p\pi^{(r)-} - g^{(r)}_{\pi N}(\bar{\psi}^{(r)}_p i\gamma^5\psi^{(r)}_p - \bar{\psi}^{(r)}_n i\gamma^5\psi^{(r)}_n)\pi^{(r)0} - g^{(r)}_{\pi N}(\bar{\psi}^{(r)}_p\psi^{(r)}_p + \bar{\psi}^{(r)}_n\psi^{(r)}_n)\sigma^{(r)} \\
&- \gamma^{(r)}f^{(r)}_\pi\sigma^{(r)}((\sigma^{(r)})^2 + 2\pi^{(r)+}\pi^{(r)-} + (\pi^{(r)0})^2) - \frac{1}{4}\gamma^{(r)}((\sigma(r))^2 + 2\pi^{(r)+}\pi^{(r)-} + (\pi^{(r)0})^2)^2 - \frac{1}{2}W^{(r)+}_{\mu\nu}W^{(r)-\mu\nu} \\
&+ M^{(r)2}_W W^{(r)+}_\mu W^{(r)-\mu} - \frac{1}{4}Z^{(r)}_{\mu\nu}Z^{(r)\mu\nu} + \frac{1}{2}M^{(r)2}_Z Z^{(r)}_\mu Z^{(r)\mu} - \frac{1}{4}F^{(r)}_{\mu\nu}F^{(r)\mu\nu} - \frac{1}{2\xi^{(r)}}(\partial_\mu A^{(r)\mu})^2 + \frac{1}{2}\partial_\mu H^{(r)}\partial^\mu H^{(r)} \\
&- \frac{1}{2}M^{(r)2}_H(H^{(r)})^2 + \bar{\psi}^{(r)}_e(i\gamma^\mu\partial_\mu - m^{(r)}_e)\psi^{(r)}_e + \bar{\psi}^{(r)}_{\nu L}i\gamma^\mu\partial_\mu\psi^{(r)}_{\nu L} - \frac{g^{(r)}}{2\sqrt{2}}(\bar{\psi}^{(r)}_p\gamma^\mu(1-\gamma^5)\psi^{(r)}_n + i\sqrt{2}(\pi^{(r)0}\partial_\mu\pi^{(r)-} \\
&- \partial_\mu\pi^{(r)0}\pi^{(r)-}) - \sqrt{2}(\sigma^{(r)}\partial_\mu\pi^{(r)-} - \partial_\mu\sigma^{(r)}\pi^{(r)-}) - \sqrt{2}f^{(r)}_\pi\partial_\mu\pi^{(r)-})W^{(r)+}_\mu - \frac{g^{(r)}}{2\sqrt{2}}(\bar{\psi}^{(r)}_n\gamma^\mu(1-\gamma^5)\psi^{(r)}_p \\
&+ i\sqrt{2}(\pi^{(r)+}\partial_\mu\pi^{(r)0} - \partial_\mu\pi^{(r)+}\pi^{(r)0}) - \sqrt{2}(\sigma^{(r)}\partial_\mu\pi^{(r)+} - \partial_\mu\sigma^{(r)}\pi^{(r)+}) - \sqrt{2}f^{(r)}_\pi\partial_\mu\pi^{(r)+})W^{(r)-}_\mu \\
&- \frac{g^{(r)}}{2\cos\theta_W}\bigg(\frac{1}{2}\bar{\psi}^{(r)}_p\gamma^\mu(1 - 4\sin^2\theta_W - \gamma^5)\psi^{(r)}_p - \frac{1}{2}\bar{\psi}^{(r)}_n\gamma^\mu(1-\gamma^5)\psi^{(r)}_n + i(1 - 2\sin^2\theta_W)(\pi^{(r)+}\partial_\mu\pi^{(r)-} \\
&- \partial_\mu\pi^{(r)+}\pi^{(r)-}) - (\sigma^{(r)}\partial_\mu\pi^{(r)0} - \partial_\mu\sigma^{(r)}\pi^{(r)0}) - f^{(r)}_\pi\partial_\mu\pi^{(r)0}\bigg)Z^{(r)}_\mu - e^{(r)}(\bar{\psi}^{(r)}_p\gamma^\mu\psi^{(r)}_p + i(\pi^{(r)-}\partial_\mu\pi^{(r)+} \\
&- \partial_\mu\pi^{(r)-}\pi^{(r)+}))A^{(r)}_\mu - \frac{g^{(r)}}{2\sqrt{2}}\bar{\psi}^{(r)}_e\gamma^\mu(1-\gamma^5)\psi^{(r)}_{\nu L}W^{(r)-}_\mu - \frac{g^{(r)}}{2\sqrt{2}}\bar{\psi}^{(r)}_{\nu L}\gamma^\mu(1-\gamma^5)\psi^{(r)}_e W^{(r)+}_\mu + \frac{g^{(r)}}{4\cos\theta_W} \\
&\times \bar{\psi}^{(r)}_e\gamma^\mu(1 - 4\sin^2\theta_W - \gamma^5)\psi^{(r)}_e Z^{(r)}_\mu - \frac{g^{(r)}}{4\cos\theta_W}\bar{\psi}^{(r)}_{\nu L}\gamma^\mu(1-\gamma^5)\psi^{(r)}_{\nu L}Z^{(r)}_\mu + e^{(r)}\bar{\psi}^{(r)}_e\gamma^\mu\psi^{(r)}_e A^{(r)}_\mu \\
&+ \frac{1}{2}(e^{(r)}g^{(r)}A^{(r)}_\mu + g^{(r)2}\cos\theta_W\tan^2\theta_W Z^{(r)}_\mu)(i(f^{(r)}_\pi + \sigma^{(r)})(\pi^{(r)+}W^{(r)-\mu} - \pi^{(r)-}W^{(r)+\mu}) \\
&- \pi^{(r)0}(\pi^{(r)+}W^{(r)-\mu} + \pi^{(r)-}W^{(r)+\mu})) + \frac{1}{4}g^{(r)2}(2f^{(r)}_\pi\sigma^{(r)} + (\sigma^{(r)})^2 + (\pi^{(r)0})^2 + 2\pi^{(r)+}\pi^{(r)-})W^{(r)+}_\mu W^{(r)-\mu} \\
&+ \frac{1}{8}\frac{g^{(r)2}}{\cos^2\theta_W}(2f^{(r)}_\pi\sigma^{(r)} + (\sigma^{(r)})^2 + (\pi^{(r)0})^2)Z^{(r)}_\mu Z^{(r)\mu} + \pi^{(r)+}\pi^{(r)-}\bigg(e^{(r)}A^{(r)}_\mu + \frac{g^{(r)}}{2\cos\theta_W}(1 - 2\sin^2\theta_W)Z^{(r)}_\mu\bigg) \\
&\times\bigg(e^{(r)}A^{(r)\mu} + \frac{g^{(r)}}{2\cos\theta_W}(1 - 2\sin^2\theta_W)Z^{(r)\mu}\bigg) + i\frac{1}{2}e^{(r)}W^{(r)-}_{\mu\nu}(W^{(r)+\mu}A^{(r)\nu} - A^{(r)\mu}W^{(r)+\nu}) + i\frac{1}{2}g^{(r)}\cos\theta_W \\
&\times W^{(r)-}_{\mu\nu}(W^{(r)+\mu}Z^{(r)\nu} - Z^{(r)\mu}W^{(r)+\nu}) + i\frac{1}{2}e^{(r)}W^{(r)+}_{\mu\nu}(A^{(r)\mu}W^{(r)-\nu} - W^{(r)-\mu}A^{(r)\nu}) + i\frac{1}{2}g^{(r)}\cos\theta_W W^{(r)+}_{\mu\nu} \\
&\times(Z^{(r)\mu}W^{(r)-\nu} - W^{(r)-\mu}Z^{(r)\nu}) + \frac{1}{2}e^{(r)2}(W^{(r)+}_\mu A^{(r)}_\nu - A^{(r)}_\mu W^{(r)+}_\nu)(A^{(r)\mu}W^{(r)-\nu} - W^{(r)-\mu}A^{(r)\nu}) + \frac{1}{2}g^{(r)2} \\
&\times\cos^2\theta_W(W^{(r)+}_\mu Z^{(r)}_\nu - Z^{(r)}_\mu W^{(r)+}_\nu)(Z^{(r)\mu}W^{(r)-\nu} - W^{(r)-\mu}Z^{(r)\nu}) + \frac{1}{2}e^{(r)}g^{(r)}\cos\theta_W(W^{(r)+}_\mu A^{(r)}_\nu - A^{(r)}_\mu W^{(r)+}_\nu) \\
&\times(Z^{(r)\mu}W^{(r)-\nu} - W^{(r)-\mu}Z^{(r)\nu}) + \frac{1}{2}e^{(r)}g^{(r)}\cos\theta_W(W^{(r)+}_\mu Z^{(r)}_\nu - Z^{(r)}_\mu W^{(r)+}_\nu)(A^{(r)\mu}W^{(r)-\nu} - W^{(r)-\mu}A^{(r)\nu}) \\
&+ \frac{1}{2}ie^{(r)}F^{(r)}_{\mu\nu}(W^{(r)-\mu}W^{(r)+\nu} - W^{(r)+\mu}W^{(r)-\nu}) + \frac{1}{2}ig^{(r)}\cos\theta_W Z^{(r)}_{\mu\nu}(W^{(r)-\mu}W^{(r)+\nu} - W^{(r)+\mu}W^{(r)-\nu}) \\
&+ \frac{1}{4}g^{(r)2}(W^{(r)-}_\mu W^{(r)+}_\nu - W^{(r)+}_\mu W^{(r)-}_\nu)(W^{(r)-\mu}W^{(r)+\nu} - W^{(r)+\mu}W^{(r)-\nu}) + \frac{M^{(r)2}_W}{v^{(r)}}W^{(r)+}_\mu W^{(r)-\mu}H^{(r)} \\
&+ \frac{1}{4}\frac{M^{(r)2}_W}{v^{(r)2}}W^{(r)+}_\mu W^{(r)-\mu}(H^{(r)})^2 + \frac{1}{2}\frac{M^{(r)2}_Z}{v^{(r)}}Z^{(r)}_\mu Z^{(r)\mu}H^{(r)} + \frac{1}{8}\frac{M^{(r)2}_Z}{v^{(r)2}}Z^{(r)}_\mu Z^{(r)\mu}(H^{(r)})^2 - \frac{m^{(r)}_e}{v^{(r)}}\bar{\psi}^{(r)}_e\psi^{(r)}_e H^{(r)} \\
&- \frac{1}{2}\frac{M^{(r)2}_H}{v^{(r)}}(H^{(r)})^3 - \frac{1}{8}\frac{M^{(r)2}_H}{v^{(r)2}}(H^{(r)})^4 + \frac{m^{(r)2}_\pi f^{(r)}_\pi}{v^{(r)}}\sigma^{(r)}H^{(r)} + \mathcal{L}^{(\mathrm{CT})}_{\mathrm{L}\sigma\mathrm{M+SEM}},
\end{aligned} \tag{44}$$

where the Lagrangian $\mathcal{L}^{(\mathrm{CT})}_{\mathrm{L}\sigma\mathrm{M+SEM}}$ contains the contributions of the counterterms. We define it following [60–62,147–157]

$$
\begin{aligned}
\mathcal{L}^{(\mathrm{CT})}_{\mathrm{L}\sigma\mathrm{M+SEM}} ={}& (Z_N\tilde{Z}_2^{(N)}Z_2^{(p)}-1)\bar{\psi}_p(i\gamma^\mu\partial_\mu - m_N)\psi_p + (Z_N\tilde{Z}_2^{(N)}-1)\bar{\psi}_n(i\gamma^\mu\partial_\mu - m_N)\psi_n - Z_N\tilde{Z}_2^{(N)}Z_2^{(p)}\delta m_N^{(r)} \\
&\times\bar{\psi}_p^{(r)}\psi_p^{(r)} - Z_N\tilde{Z}_2^{(N)}\delta m_N^{(r)}\bar{\psi}_n^{(r)}\psi_n^{(r)} + (Z_M Z_2^{(\pi)}\tilde{Z}_2^{(M)}-1)(\partial_\mu\pi^{(r)+}\partial^\mu\pi^{(r)-} - (m_\pi^{(r)})^2\pi^{(r)+}\pi^{(r)-}) - Z_M Z_2^{(\pi)}\tilde{Z}_2^{(M)} \\
&\times\delta m_\pi^{(r)2}\pi^{(r)+}\pi^{(r)-} + \frac{1}{2}(Z_M\tilde{Z}_2^{(M)}-1)(\partial_\mu\pi^{(r)0}\partial^\mu\pi^{(r)0} - (m_\pi^{(r)})^2(\pi^{(r)0})^2) - \frac{1}{2}Z_M\tilde{Z}_2^{(M)}\delta m_\pi^{(r)2}(\pi^{(r)0})^2 \\
&+\frac{1}{2}(Z_M\tilde{Z}_2^{(M)}-1)(\partial_\mu\sigma^{(r)}\partial^\mu\sigma^{(r)} - (m_\sigma^{(r)})^2(\sigma^{(r)})^2) - \frac{1}{2}Z_M\tilde{Z}_2^{(M)}\delta m_\sigma^{(r)2}(\sigma^{(r)})^2 - (Z_{MN}\tilde{Z}_2^{(N)}\sqrt{Z_2^{(p)}Z_2^{(\pi)}\tilde{Z}_2^{(M)}}-1) \\
&\times\sqrt{2}g_{\pi N}^{(r)}\bar{\psi}_p^{(r)}i\gamma^5\psi_n^{(r)}\pi^{(r)+} - (Z_{MN}\tilde{Z}_2^{(N)}\sqrt{Z_2^{(p)}Z_2^{(\pi)}\tilde{Z}_2^{(M)}}-1)\sqrt{2}g_{\pi N}^{(r)}\bar{\psi}_n^{(r)}i\gamma^5\psi_p\pi^{(r)-} - (Z_{MN}\tilde{Z}_2^{(N)}Z_2^{(p)}\sqrt{\tilde{Z}_2^{(M)}}-1) \\
&\times g_{\pi N}^{(r)}\bar{\psi}_p^{(r)}i\gamma^5\psi_p^{(r)}\pi^{(r)0} + (Z_{MN}\tilde{Z}_2^{(N)}\sqrt{\tilde{Z}_2^{(M)}}-1)g_{\pi N}^{(r)}\bar{\psi}_n^{(r)}i\gamma^5\psi_n^{(r)}\pi^{(r)0} - (Z_{MN}\tilde{Z}_2^{(N)}Z_2^{(p)}\sqrt{\tilde{Z}_2^{(M)}}-1)g_{\pi N}^{(r)}\bar{\psi}_p^{(r)}\psi_p^{(r)}\sigma^{(r)} \\
&-(Z_{MN}\tilde{Z}_2^{(N)}\sqrt{\tilde{Z}_2^{(M)}}-1)g_{\pi N}^{(r)}\bar{\psi}_n^{(r)}\psi_n^{(r)}\sigma^{(r)} - (Z_{3M}(\tilde{Z}_2^{(M)})^{3/2}-1)\gamma^{(r)}f_\pi^{(r)}(\sigma^{(r)})^3 - (Z_{3M}(\tilde{Z}_2^{(M)})^{3/2}Z_2^{(\pi)}-1) \\
&\times 2\gamma^{(r)}f_\pi^{(r)}\sigma^{(r)}\pi^{(r)+}\pi^{(r)-} - (Z_{3M}(\tilde{Z}_2^{(\sigma)})^{3/2}-1)\gamma^{(r)}f_\pi^{(r)}\sigma^{(r)}(\pi^{(r)0})^2 - (Z_{4M}(\tilde{Z}_2^{(M)})^2-1)\frac{1}{4}\gamma^{(r)}(\sigma^{(r)})^4 \\
&-(Z_{4M}(\tilde{Z}_2^{(M)}Z_2^{(\pi)})^2-1)\gamma^{(r)}(\pi^{(r)+}\pi^{(r)-})^2 - (Z_{4M}(\tilde{Z}_2^{(M)})^2-1)\frac{1}{4}\gamma^{(r)}(\pi^{(r)0})^4 - (Z_{4M}(\tilde{Z}_2^{(M)})^2Z_2^{(\pi)}-1)\gamma^{(r)}(\sigma^{(r)})^2 \\
&\times\pi^{(r)+}\pi^{(r)-} - (Z_{4M}(\tilde{Z}_2^{(M)})^2-1)\frac{1}{2}\gamma^{(r)}(\sigma^{(r)})^2(\pi^{(r)0})^2 - (Z_{4M}(\tilde{Z}_2^{(M)})^2Z_2^{(\pi)}-1)\gamma^{(r)}\pi^{(r)+}\pi^{(r)-}(\pi^{(r)0})^2 \\
&+(Z_3^{(W)}-1)\left(-\frac{1}{2}W_{\mu\nu}^{(r)+}W^{(r)-\mu\nu} + M_W^{(r)2}W_\mu^{(r)+}W^{(r)-\mu}\right) + Z_3^{(W)}\delta M_W^{(r)2}W_\mu^{(r)+}W^{(r)-\mu} + (Z_3^{(Z)}-1)\left(-\frac{1}{4}Z_{\mu\nu}^{(r)}Z^{(r)\mu\nu}\right. \\
&\left.+\frac{1}{2}M_Z^{(r)2}Z_\mu^{(r)}Z^{(r)\mu}\right) + Z_3^{(Z)}\frac{1}{2}\delta M_Z^{(r)2}Z_\mu^{(r)}Z^{(r)\mu} - (Z_3^{(\gamma)}-1)\frac{1}{4}F_{\mu\nu}^{(r)}F^{(r)\mu\nu} - \frac{Z_3^{(\gamma)}-1}{Z_\xi}\frac{1}{2\xi^{(r)}}(\partial_\mu A^{(r)\mu})^2 + \frac{1}{2}(Z_2^{(H)}-1) \\
&\times(\partial_\mu H^{(r)}\partial^\mu H^{(r)} - M_H^{(r)2}(H^{(r)})^2) - \frac{1}{2}Z_2^{(H)}\delta M_H^{(r)2}(H^{(r)})^2 + (Z_2^{(e)}\tilde{Z}_2^{(e)}-1)\bar{\psi}_e^{(r)}(i\gamma^\mu\partial_\mu - m_e^{(r)})\psi_e^{(r)} - Z_2^{(e)}\tilde{Z}_2^{(e)} \\
&\times\delta m_e^{(r)}\bar{\psi}_e^{(r)}\psi_e^{(r)} + (\tilde{Z}_2^{(\ell)}-1)\bar{\psi}_{\nu L}^{(r)}i\gamma^\mu\partial_\mu\psi_{\nu L}^{(r)} - (\tilde{Z}_1^{(N)}Z_N\sqrt{Z_2^{(p)}}-1)\frac{g^{(r)}}{2\sqrt{2}}\bar{\psi}_p^{(r)}\gamma^\mu(1-\gamma^5)\psi_n^{(r)}W_\mu^{(r)+} - (\tilde{Z}_1^{(M)}Z_M \\
&\times\sqrt{Z_2^{(\pi)}}-1)\frac{g^{(r)}}{2\sqrt{2}}i\sqrt{2}(\pi^{(r)0}\partial_\mu\pi^{(r)-} - \partial_\mu\pi^{(r)0}\pi^{(r)-})W_\mu^{(r)+} - (\tilde{Z}_1^{(M)}Z_M\sqrt{Z_2^{(\pi)}}-1)\frac{g^{(r)}}{2\sqrt{2}}\sqrt{2}(\sigma^{(r)}\partial_\mu\pi^{(r)-} - \partial_\mu\sigma^{(r)} \\
&\times\pi^{(r)-})W_\mu^{(r)+} - (\tilde{Z}_1^{(M)}Z_M\sqrt{Z_2^{(\pi)}/\tilde{Z}_2^{(M)}}-1)\frac{g^{(r)}}{2\sqrt{2}}\sqrt{2}f_\pi^{(r)}\partial_\mu\pi^{(r)-}W_\mu^{(r)+} - (\tilde{Z}_1^{(N)}Z_N\sqrt{Z_2^{(p)}}-1)\frac{g^{(r)}}{2\sqrt{2}}\bar{\psi}_n^{(r)}\gamma^\mu(1-\gamma^5) \\
&\times\psi_p^{(r)}W_\mu^{(r)-} - (\tilde{Z}_1^{(M)}Z_M\sqrt{Z_2^{(\pi)}}-1)\frac{g^{(r)}}{2\sqrt{2}}i\sqrt{2}(\pi^{(r)+}\partial_\mu\pi^{(r)0} - \partial_\mu\pi^{(r)+}\pi^{(r)0})W_\mu^{(r)-} - (\tilde{Z}_1^{(M)}Z_M\sqrt{Z_2^{(\pi)}}-1)\frac{g^{(r)}}{2\sqrt{2}} \\
&\times\sqrt{2}(\sigma^{(r)}\partial_\mu\pi^{(r)+} - \partial_\mu\sigma^{(r)}\pi^{(r)+})W_\mu^{(r)-} - (\tilde{Z}_1^{(M)}Z_M\sqrt{Z_2^{(\pi)}/\tilde{Z}_2^{(M)}}-1)\sqrt{2}f_\pi^{(r)}\partial_\mu\pi^{(r)+}W_\mu^{(r)-} - (\tilde{Z}_1^{(N)}Z_N Z_2^{(p)}-1) \\
&\times\frac{g^{(r)}}{2\cos\theta_W}\left(\frac{1}{2}\bar{\psi}_p^{(r)}\gamma^\mu(1-4\sin^2\theta_W-\gamma^5)\psi_p^{(r)}Z_\mu^{(r)} + (\tilde{Z}_1^{(N)}Z_N-1)\frac{g^{(r)}}{2\cos\theta_W}\frac{1}{2}\bar{\psi}_n^{(r)}\gamma^\mu(1-\gamma^5)\psi_n^{(r)}Z_\mu^{(r)} - (\tilde{Z}_1^{(M)}Z_M\right. \\
&\times Z_2^{(\pi)}-1)\frac{g^{(r)}}{2\cos\theta_W}(1-2\sin^2\theta_W)i(\pi^{(r)+}\partial_\mu\pi^{(r)-} - \partial_\mu\pi^{(r)+}\pi^{(r)-})Z_\mu^{(r)} + (\tilde{Z}_1^{(M)}Z_M-1)\frac{g^{(r)}}{2\cos\theta_W}(\sigma^{(r)}\partial_\mu\pi^{(r)0} \\
&-\partial_\mu\sigma^{(r)}\pi^{(r)0})Z_\mu^{(r)} + (\tilde{Z}_1^{(M)}Z_M(\tilde{Z}_2^{(M)})^{-1/2}-1)\frac{g^{(r)}}{2\cos\theta_W}f_\pi^{(r)}\partial_\mu\pi^{(r)0}Z_\mu^{(r)} - (Z_1^{(p)}Z_N\tilde{Z}_2^{(N)}-1)e^{(r)}\bar{\psi}_p^{(r)}\gamma^\mu\psi_p^{(r)}A_\mu^{(r)} \\
&-(Z_1^{(\pi)}Z_M\tilde{Z}_2^{(M)}-1)e^{(r)}i(\pi^{(r)-}\partial_\mu\pi^{(r)+} - \partial_\mu\pi^{(r)-}\pi^{(r)+})A_\mu^{(r)} - (\tilde{Z}_1^{(\ell)}\sqrt{Z_2^{(e)}}-1)\frac{g^{(r)}}{2\sqrt{2}}\bar{\psi}_e^{(r)}\gamma^\mu(1-\gamma^5)\psi_{\nu L}^{(r)}W_\mu^{(r)-} \\
&-(\tilde{Z}_1^{(\ell)}\sqrt{Z_2^{(e)}}-1)\frac{g^{(r)}}{2\sqrt{2}}\bar{\psi}_{\nu L}^{(r)}\gamma^\mu(1-\gamma^5)\psi_e^{(r)}W_\mu^{(r)+} + (\tilde{Z}_1^{(\ell)}Z_2^{(e)}\sqrt{Z_3^{(Z)}/Z_3^{(W)}}-1)\frac{g^{(r)}}{4\cos\theta_W}\bar{\psi}_e^{(r)}\gamma^\mu(1-4\sin^2\theta_W-\gamma^5)
\end{aligned}
$$

$$
\times\psi_e^{(r)}Z_\mu^{(r)} - (\tilde{Z}_1^{(\ell)}\sqrt{Z_3^{(Z)}/Z_3^{(W)}} - 1)\frac{g^{(r)}}{4\cos\theta_W}\bar{\psi}_{\nu L}^{(r)}\gamma^\mu(1-\gamma^5)\psi_{\nu L}^{(r)}Z_\mu^{(r)} + (Z_1^{(e)}\tilde{Z}_2^{(\ell)} - 1)e^{(r)}\bar{\psi}_e^{(r)}\gamma^\mu\psi_e^{(r)}A_\mu^{(r)}
$$

$$
+ (Z_1^{(\pi)}\tilde{Z}_1^{(M)}Z_M - 1)\frac{1}{2}if_\pi^{(r)}e^{(r)}g^{(r)}A_\mu^{(r)}(\pi^{(r)+}W^{(r)-\mu} - \pi^{(r)-}W^{(r)+\mu}) + (Z_1^{(\pi)}\tilde{Z}_1^{(M)}Z_M - 1)\frac{1}{2}e^{(r)}g^{(r)}A_\mu^{(r)}
$$

$$
\times(i\sigma^{(r)}(\pi^{(r)+}W^{(r)-\mu} - \pi^{(r)-}W^{(r)+\mu}) - \pi^{(r)0}(\pi^{(r)+}W^{(r)-\mu} + \pi^{(r)-}W^{(r)+\mu})) + ((\tilde{Z}_1^{(M)})^2 Z_M\sqrt{Z_2^{(\pi)}/Z_3^{(W)}}
$$

$$
\times(\tilde{Z}_2^{(M)})^{-3/2} - 1)\frac{1}{2}g^{(r)2}\cos\theta_W\tan^2\theta_W if_\pi^{(r)}Z_\mu^{(r)}(\pi^{(r)+}W^{(r)-\mu} - \pi^{(r)-}W^{(r)+\mu}) + ((\tilde{Z}_1^{(M)})^2 Z_M\sqrt{Z_2^{(\pi)}/Z_3^{(W)}}
$$

$$
\times(\tilde{Z}_2^{(M)})^{-1} - 1)\frac{1}{2}g^{(r)2}\cos\theta_W\tan^2\theta_W Z_\mu^{(r)}(i\sigma^{(r)}(\pi^{(r)+}W^{(r)-\mu} - \pi^{(r)-}W^{(r)+\mu}) - \pi^{(r)0}(\pi^{(r)+}W^{(r)-\mu}
$$

$$
+ \pi^{(r)-}W^{(r)+\mu})) + ((\tilde{Z}_1^{(M)})^2 Z_M(\tilde{Z}_2^{(M)})^{-1/2} - 1)\frac{1}{4}g^{(r)2}2f_\pi^{(r)}\sigma^{(r)}W_\mu^{(r)+}W^{(r)-\mu} + ((\tilde{Z}_1^{(M)})^2 Z_M - 1)\frac{1}{4}g^{(r)2}
$$

$$
\times((\sigma^{(r)})^2 + (\pi^{(r)0})^2)W_\mu^{(r)+}W^{(r)-\mu} + \left((\tilde{Z}_1^{(M)})^2 Z_M Z_2^{(\pi)} - 1)\frac{1}{4}g^{(r)2}2\pi^{(r)+}\pi^{(r)-}\right)W_\mu^{(r)+}W^{(r)-\mu}
$$

$$
+ ((\tilde{Z}_1^{(M)})^2 Z_M Z_3^{(Z)}(Z_3^{(W)})^{-1}(\tilde{Z}_2^{(M)})^{-1/2} - 1)\frac{1}{8}\frac{g^{(r)2}}{\cos^2\theta_W}2f_\pi^{(r)}\sigma^{(r)}Z_\mu^{(r)}Z^{(r)\mu} + ((\tilde{Z}_1^{(M)})^2 Z_M Z_3^{(Z)}(Z_3^{(W)})^{-1} - 1)
$$

$$
\times\frac{1}{8}\frac{g^{(r)2}}{\cos^2\theta_W}((\sigma^{(r)})^2 + (\pi^{(r)0})^2)Z_\mu^{(r)}Z^{(r)\mu} + ((Z_1^{(\pi)})^2(Z_2^{(\pi)})^{-1} - 1)e^{(r)2}\pi^{(r)+}\pi^{(r)-}A_\mu^{(r)}A^{(r)\mu} + (Z_1^{(\pi)}\tilde{Z}_1^{(M)}
$$

$$
\times\sqrt{Z_3^{(Z)}/Z_3^{(W)}} - 1)\frac{2e^{(r)}g^{(r)}}{2\cos\theta_W}(1 - 2\sin^2\theta_W)\pi^{(r)+}\pi^{(r)-}A_\mu^{(r)}Z^{(r)\mu} + ((Z_1^{(M)})^2 Z_M Z_2^{(\pi)}Z_3^{(Z)}/Z_3^{(W)} - 1)\frac{g^{(r)2}}{4\cos^2\theta_W}
$$

$$
\times(1 - 2\sin^2\theta_W)^2\pi^{(r)+}\pi^{(r)-}Z_\mu^{(r)}Z^{(r)\mu} + (Z_1^{(W)} - 1)i\frac{1}{2}e^{(r)}W_{\mu\nu}^{(r)-}(W^{(r)+\mu}A^{(r)\nu} - A^{(r)\mu}W^{(r)+\nu}) + (Z_1^{(W)}\sqrt{Z_3^{(Z)}}
$$

$$
\times(Z_3^{(W)})^{-1/2} - 1)i\frac{1}{2}g^{(r)}\cos\theta_W W_{\mu\nu}^{(r)-}(W^{(r)+\mu}Z^{(r)\nu} - Z^{(r)\mu}W^{(r)+\nu}) + (Z_3^{(W)} - 1)i\frac{1}{2}e^{(r)}W_{\mu\nu}^{(r)+}(A^{(r)\mu}W^{(r)-\nu}
$$

$$
- W^{(r)-\mu}A^{(r)\nu}) + (Z_1^{(W)}\sqrt{Z_3^{(Z)}/Z_3^{(W)}} - 1)i\frac{1}{2}g^{(r)}\cos\theta_W W_{\mu\nu}^{(r)+}(Z^{(r)\mu}W^{(r)-\nu} - W^{(r)-\mu}Z^{(r)\nu}) + (Z_3^{(W)} - 1)
$$

$$
\times\frac{1}{2}e^{(r)2}(W_\mu^{(r)+}A_\nu^{(r)} - A_\mu^{(r)}W_\nu^{(r)+})(A^{(r)\mu}W^{(r)-\nu} - W^{(r)-\mu}A^{(r)\nu}) + ((Z_1^{(W)})^2 Z_3^{(Z)}(Z_3^{(W)})^{-2} - 1)\frac{1}{2}g^{(r)2}\cos^2\theta_W
$$

$$
\times(W_\mu^{(r)+}Z_\nu^{(r)} - Z_\mu^{(r)}W_\nu^{(r)+})(Z^{(r)\mu}W^{(r)-\nu} - W^{(r)-\mu}Z^{(r)\nu}) + (Z_1^{(W)}\sqrt{Z_3^{(Z)}/Z_3^{(W)}} - 1)\frac{1}{2}e^{(r)}g^{(r)}\cos\theta_W(W_\mu^{(r)+}A_\nu^{(r)}
$$

$$
- A_\mu^{(r)}W_\nu^{(r)+})(Z^{(r)\mu}W^{(r)-\nu} - W^{(r)-\mu}Z^{(r)\nu}) + (Z_1^{(W)}\sqrt{Z_3^{(Z)}/Z_3^{(W)}} - 1)\frac{1}{2}e^{(r)}g^{(r)}\cos\theta_W(W_\mu^{(r)+}Z_\nu^{(r)} - Z_\mu^{(r)}W_\nu^{(r)+})
$$

$$
\times(A^{(r)\mu}W^{(r)-\nu} - W^{(r)-\mu}A^{(r)\nu}) + (Z_3^{(W)} - 1)\frac{1}{2}ie^{(r)}F_{\mu\nu}^{(r)}(W^{(r)-\mu}W^{(r)+\nu} - W^{(r)+\mu}W^{(r)-\nu}) + (Z_1^{(W)}\sqrt{Z_3^{(Z)}/Z_3^{(W)}}
$$

$$
- 1)\frac{1}{2}ig^{(r)}\cos\theta_W Z_{\mu\nu}^{(r)}(W^{(r)-\mu}W^{(r)+\nu} - W^{(r)+\mu}W^{(r)-\nu}) + ((Z_1^{(W)})^2\sqrt{Z_3^{(W)}} - 1)\frac{1}{4}g^{(r)2}(W_\mu^{(r)-}W_\nu^{(r)+} - W_\mu^{(r)+}
$$

$$
\times W_\nu^{(r)-})(W^{(r)-\mu}W^{(r)+\nu} - W^{(r)+\mu}W^{(r)-\nu}) + (Z_3^{(W)}\sqrt{Z_2^{(H)}}Z_v^{-1} - 1)\frac{M_W^{(r)2}}{v^{(r)}}W_\mu^{(r)+}W^{(r)-\mu}H^{(r)} + Z_3^{(W)}\sqrt{Z_2^{(H)}}Z_v^{-1}
$$

$$
\times\frac{\delta M_W^{(r)2}}{v^{(r)}}W_\mu^{(r)+}W^{(r)-\mu}H^{(r)} + (Z_3^{(W)}Z_2^{(H)}Z_v^{-2} - 1)\frac{1}{4}\frac{M_W^{(r)2}}{v^{(r)2}}W_\mu^{(r)+}W^{(r)-\mu}(H^{(r)})^2 + Z_3^{(W)}Z_2^{(H)}Z_v^{-2}\frac{1}{4}\frac{\delta M_W^{(r)2}}{v^{(r)2}}
$$

$$\times W^{(r)+}_\mu W^{(r)-\mu}(H^{(r)})^2 + \Big(Z^{(Z)}_3\sqrt{Z^{(H)}_2}Z^{-1}_v - 1\Big)\frac{1}{2}\frac{M^{(r)2}_Z}{v^{(r)}}Z^{(r)}_\mu Z^{(r)\mu}H^{(r)} + Z^{(Z)}_3\sqrt{Z^{(H)}_2}Z^{-1}_v\frac{1}{2}\frac{\delta M^{(r)2}_Z}{v^{(r)}}Z^{(r)}_\mu Z^{(r)\mu}H^{(r)}$$
$$+ \Big(Z^{(Z)}_3 Z^{(H)}_2 Z^{-2}_v - 1\Big)\frac{1}{8}\frac{M^{(r)2}_Z}{v^{(r)2}}Z^{(r)}_\mu Z^{(r)\mu}(H^{(r)})^2 + Z^{(Z)}_3 Z^{(H)}_2 Z^{-2}_v\frac{1}{8}\frac{\delta M^{(r)2}_Z}{v^{(r)2}}Z^{(r)}_\mu Z^{(r)\mu}(H^{(r)})^2 - \Big(Z^{(e)}_2\tilde{Z}^{(\ell)}_2\sqrt{Z^{(H)}_2} - 1\Big)$$
$$\times\frac{m^{(r)}_e}{v^{(r)}}\bar{\psi}^{(r)}_e\psi^{(r)}_e H^{(r)} - \Big((Z^{(H)}_2)^{3/2}Z^{-1}_v - 1\Big)\frac{1}{2}\frac{M^{(r)2}_H}{v^{(r)}}(H^{(r)})^3 - (Z^{(H)}_2)^{3/2}Z^{-1}_v\frac{1}{2}\frac{\delta M^{(r)2}_H}{v^{(r)}}(H^{(r)})^3 - \Big((Z^{(H)}_2)^2 Z^{-2}_v - 1\Big)$$
$$\times\frac{1}{8}\frac{M^{(r)2}_H}{v^{(r)2}}(H^{(r)})^4 - (Z^{(H)}_2)^2 Z^{-2}_v\frac{1}{8}\frac{\delta M^{(r)2}_H}{v^{(r)2}}(H^{(r)})^4 + \Big(Z_M\sqrt{Z^{(H)}_2}Z^{-1}_v - 1\Big)\frac{m^{(r)2}_\pi f^{(r)}_\pi}{v^{(r)}}\sigma^{(r)}H^{(r)} + Z_M\sqrt{Z^{(H)}_2}Z^{-1}_v$$
$$\times\frac{\delta m^{(r)2}_\pi f^{(r)}_\pi}{v^{(r)}}\sigma^{(r)}H^{(r)}, \tag{45}$$

where Z_{MN}, Z_N, Z_M, $Z^{(a)}_j$ and $\tilde{Z}^{(a')}_{j'}$ are renormalization constants of the field operators and vertices of strong and electroweak interactions. Then, Z_v is a renormalization constant of the vacuum expectation value $v^{(r)}$, and $\delta m^{(r)}_N$, $\delta m^{(r)2}_\pi$, $\delta m^{(r)2}_\sigma$, $\delta M^{(r)2}_W$ and so on are the counterterms of mass renormalization. Rescaling the field operators and the coupling constants

$$\psi^{(0)}_p = \sqrt{Z_N Z^{(p)}_2\tilde{Z}^{(N)}_2}\psi^{(r)}_p, \qquad \psi^{(0)}_n = \sqrt{Z_N\tilde{Z}^{(N)}_2}\psi^{(r)}_n, \qquad \pi^{(0)\pm} = \sqrt{Z_M Z^{(\pi)}_2\tilde{Z}^{(M)}_2}\pi^{(r)\pm}, \qquad \pi^{(0)0} = \sqrt{Z_M\tilde{Z}^{(M)}_2}\pi^{(r)0},$$
$$\sigma^{(0)} = \sqrt{Z_M\tilde{Z}^{(M)}_2}\sigma^{(r)}, \qquad A^{(0)}_\mu = \sqrt{Z^{(\gamma)}_3}A^{(r)}_\mu, \qquad W^{(0)\pm}_\mu = \sqrt{Z^{(W)}_3}W^{(r)\pm}_\mu, \qquad Z^{(0)}_\mu = \sqrt{Z^{(Z)}_3}Z^{(r)}_\mu, \qquad H^{(0)} = \sqrt{Z^{(H)}_2}H^{(r)},$$
$$\psi^{(0)}_e = \sqrt{Z^{(e)}_2\tilde{Z}^{(\ell)}_2}\psi^{(r)}_e, \qquad \psi^{(0)}_{\nu L} = \sqrt{\tilde{Z}^{(\ell)}_2}\psi^{(r)}_{\nu L},$$
$$g^{(0)}_{\pi N} = Z_{MN}Z^{-1}_N Z^{-1/2}_M g^{(r)}_{\pi N}, \qquad f^{(0)}_\pi = Z^{1/2}_M f^{(r)}_\pi, \qquad \gamma^{(0)} = Z_{3M}Z^{-2}_M\gamma^{(r)},$$
$$e^{(0)} = Z^{(p)}_1 Z^{(p)-1}_2 Z^{(\gamma)-1/2}_3 e^{(r)} = Z^{(\pi)}_1 Z^{(\pi)-1}_2 Z^{(\gamma)-1/2}_3 e^{(r)} = Z^{(e)}_1 Z^{(e)-1}_2 Z^{(\gamma)-1/2}_3 e^{(r)} = Z^{(\gamma)-1/2}_3 e^{(r)},$$
$$g^{(0)} = \tilde{Z}^{(N)}_1\tilde{Z}^{(N)-1}_2 Z^{(W)-1/2}_3 g^{(r)} = \tilde{Z}^{(M)}_1\tilde{Z}^{(M)-1}_2 Z^{(W)-1/2}_3 g^{(r)} = \tilde{Z}^{(\ell)}_1\tilde{Z}^{(\ell)-1}_2 Z^{(W)-1/2}_3 g^{(r)}$$
$$= Z^{(W)}_1 Z^{(W)-3/2}_3 g^{(r)}, \qquad v^{(0)} = Z_v v^{(r)}, \qquad \xi^{(0)} = Z_\xi\xi^{(r)}, \tag{46}$$

where we have set $Z_{4M} = Z_{3M}$ [see Eq. (13)], and using the relations

$$m^{(0)}_N = m^{(r)}_N + \delta m^{(r)}_N, \qquad m^{(0)2}_\pi = m^{(r)2}_\pi + \delta m^{(r)2}_\pi, \qquad m^{(0)2}_\sigma = m^{(r)2}_\sigma + \delta m^{(r)2}_\sigma,$$
$$M^{(0)2}_W = M^{(r)2}_W + \delta M^{(r)2}_W, \qquad M^{(0)2}_Z = M^{(r)2}_Z + \delta M^{(r)2}_Z, \qquad M^{(0)2}_H = M^{(r)2}_H + \delta M^{(r)2}_H,$$
$$m^{(0)}_e = m^{(r)}_e + \delta m^{(r)}_e \tag{47}$$

we transcribe the Lagrangian in Eq. (44) into the Lagrangian in Eq. (43).

VI. MATRIX ELEMENT OF THE HADRONIC $n \to p$ TRANSITION IN THE NEUTRON β^- DECAY $n \to p + e^- + \bar{\nu}_e$

The amplitude of the neutron β^- decay is defined by [60,141]

$$M(n \to p e^-\bar{\nu}_e) = \Big\langle \text{out}, \bar{\nu}_e\Big(\vec{k}_{\bar{\nu}}, +\frac{1}{2}\Big), e^-(\vec{k}_e, \sigma_e), p(\vec{k}_p, \sigma_p)\Big| n(\vec{k}_n, \sigma_n), \text{in}\Big\rangle, \tag{48}$$

where $\langle \text{out}, \chi(\vec{k}_\chi, \sigma_\chi)|$ and $|\text{in}, n(\vec{k}_n, \sigma_n)\rangle$ are the wave functions of the free antineutrino, electron and proton ($\chi = \bar{\nu}_e, e^-, p$) in the final state (i.e. out-state at $t \to +\infty$) and the free neutron in the initial state (i.e. in-state at $t \to -\infty$) [146]. Using the relation $\langle \text{out}, \prod_\chi \chi(\vec{k}_\chi, \sigma_\chi)| = \langle \text{in}, \prod_\chi \chi(\vec{k}_\chi, \sigma_\chi)|\mathbb{S}$, where $\mathbb{S}$ is the S-matrix, we rewrite the matrix element Eq. (48) as follows:

$$M(n \to p e^- \bar{\nu}_e) = \left\langle \mathrm{in}, \bar{\nu}_e\left(\vec{k}_{\bar{\nu}}, +\frac{1}{2}\right), e^-(\vec{k}_e, \sigma_e), p(\vec{k}_p, \sigma_p)|\mathbb{S}|n(\vec{k}_n, \sigma_n), \mathrm{in}\right\rangle. \quad (49)$$

The corresponding S-matrix is determined by [60,141,146]

$$\mathbb{S} = \mathrm{T}e^{i\int d^4x \mathcal{L}_{\mathrm{L}\sigma\mathrm{M+SEM}}(x)}, \quad (50)$$

where T is a time-ordering operator and $\mathcal{L}_{\mathrm{L}\sigma\mathrm{M+SEM}}$ is given by Eq. (44). Plugging Eq. (50) into Eq. (49) we get [60,141]

$$M(n \to p e^- \bar{\nu}_e) = \left\langle \mathrm{in}, \bar{\nu}_e\left(\vec{k}_{\bar{\nu}}, +\frac{1}{2}\right), e^-(\vec{k}_e, \sigma_e), p(\vec{k}_p, \sigma_p)\right| \times \mathrm{T}e^{i\int d^4x \mathcal{L}_{\mathrm{L}\sigma\mathrm{M+SEM}}(x)}\left|n(\vec{k}_n, \sigma_n), \mathrm{in}\right\rangle. \quad (51)$$

The wave functions of fermions we determine in terms of the operators of creation (annihilation)

$$|n(\vec{k}_n, \sigma_n), \mathrm{in}\rangle = a^{\dagger}_{n,\mathrm{in}}(\vec{k}_n, \sigma_n)|0\rangle,$$
$$\left\langle \mathrm{in}, \bar{\nu}_e\left(\vec{k}_{\bar{\nu}}, +\frac{1}{2}\right), e^-(\vec{k}_e, \sigma_e), p(\vec{k}_p, \sigma_p)\right. = \langle 0|b_{\bar{\nu}_e,\mathrm{in}}\left(\vec{k}_{\bar{\nu}}, +\frac{1}{2}\right)a_{e,\mathrm{in}}(\vec{k}_e, \sigma_e)a_{p,\mathrm{in}}(\vec{k}_p, \sigma_p). \quad (52)$$

The operators of creation (annihilation) obey standard anticommutation relations [141,146].

A. Neutron beta decay in the tree approximation for strong low-energy and electroweak interactions described by the Lagrangian Eq. (44)

In the tree approximation for the electroweak W^--boson exchange and strong low-energy interactions, the amplitude of neutron β^- decay $n \to p + e^- + \bar{\nu}_e$ is defined by the Feynman diagrams in Fig. 2

$$M(n \to p e^- \bar{\nu}_e) = G_V \langle p(\vec{k}_p, \sigma_p)|J^+_\mu(0)|n(\vec{k}_n, \sigma_n)\rangle_{\mathrm{Fig.\,2}} \times \frac{M^2_W}{M^2_W - q^2 - i0}\left(-\eta^{\mu\nu} + \frac{q^\mu q^\nu}{M^2_W}\right) \times \left[\bar{u}_e(\vec{k}_e, \sigma_e)\gamma_\nu(1-\gamma^5)v_{\bar{\nu}}\left(\vec{k}_{\bar{\nu}}, +\frac{1}{2}\right)\right], \quad (53)$$

where $G_V = g^2/8M^2_W$, $J^+_\mu(0) = V^+_\mu(0) - A^+_\mu(0)$ is the $V - A$ charged hadronic current [41,42], appearing naturally in our model caused by the electroweak W^--boson exchanges [see Eqs. (43) and (44)], where the vector and axial-vector current possess both baryonic and mesonic parts [see Eq. (9)]. Then, $\bar{u}_e$ and v_ν are Dirac wave functions of the free electron and electron antineutrino, respectively,

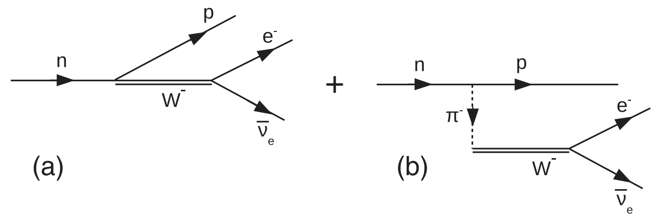

FIG. 2. The Feynman diagrams, defining the amplitude of the neutron β^- decay $n \to p + e^- + \bar{\nu}_e$ in the tree approximation in the quantum field theoretic model of strong low-energy and electroweak interactions described by the Lagrangian Eq. (44).

a momentum transferred of the decay is equal to $q = k_p - k_n = -k_e - k_\nu$. Then, since strong low-energy interactions give the contributions to the matrix element of the charged hadronic current only, we have denoted $\langle \mathrm{in}, p(\vec{k}_p,\sigma_p)|\mathrm{T}(e^{i\int d^4x \mathcal{L}_{\mathrm{L}\sigma\mathrm{M}}(x)} J^+_\mu(0))|n(\vec{k}_n,\sigma_n),\mathrm{in}\rangle = \langle p(\vec{k}_p,\sigma_p)|J^+_\mu(0)|n(\vec{k}_n,\sigma_n)\rangle$. This matrix element describes the hadronic $n \to p$ transition in the neutron β^- decay [60,141,158]. The matrix element of the hadronic $V - A$ current $\langle p(\vec{k}_p, \sigma_p)|J^+_\mu(0)|n(\vec{k}_n, \sigma_n)\rangle$, calculated in the tree approximation (see Fig. 2), is equal to (see also [60,141])

$$\langle p(\vec{k}_p, \sigma_p)|J^+_\mu(0)|n(\vec{k}_n, \sigma_n)\rangle_{\mathrm{Fig.\,2}} = \bar{u}_p(\vec{k}_p, \sigma_p)\left(\gamma_\mu(1-\gamma^5) - \frac{2g_{\pi N}f_\pi}{m^2_\pi - q^2}q_\mu\gamma^5\right)u_n(\vec{k}_n, \sigma_n), \quad (54)$$

where $\bar{u}_p$ and u_n are the Dirac wave functions of the free proton and neutron. The matrix element of the divergence of the charged hadronic current $\partial^\mu J^+_\mu$ is equal to

$$\langle p(\vec{k}_p, \sigma_p)|\partial^\mu J^+_\mu(0)|n(\vec{k}_n, \sigma_n)\rangle_{\mathrm{Fig.\,2}} = i\bar{u}_p(\vec{k}_p, \sigma_p)\left((-2m_N + 2g_{\pi N}f_\pi)\gamma^5 - 2g_{\pi N}f_\pi\frac{m^2_\pi}{m^2_\pi - q^2}\gamma^5\right) \times u_n(\vec{k}_n, \sigma_n). \quad (55)$$

Because of the Goldberger-Treiman (GT) relation $g_{\pi N} = m_N/f_\pi$ [134] (see also [42,133,135,139]), which appears naturally in the LσM [see Eq. (8)] at $b = f_\pi$ with the axial coupling constant g_A equal to $g_A = 1$, we get

$$\langle p(\vec{k}_p, \sigma_p)|\partial^\mu J^+_\mu(0)|n(\vec{k}_n, \sigma_n)\rangle_{\mathrm{Fig.\,2}} = -2g_{\pi N}f_\pi\frac{m^2_\pi}{m^2_\pi - q^2}\bar{u}_p(\vec{k}_p, \sigma_p)i\gamma^5 u_n(\vec{k}_n, \sigma_n). \quad (56)$$

Because of conservation of the charged hadronic vector current $\partial^\mu V^+_\mu = 0$ [41,60,141,158] leading to

$$\langle p(\vec{k}_p, \sigma_p)|\partial^\mu V^+_\mu(0)|n(\vec{k}_n, \sigma_n)\rangle_{\mathrm{Fig.\,2}} = iq^\mu\langle p(\vec{k}_p, \sigma_p)|V^+_\mu(0)|n(\vec{k}_n, \sigma_n)\rangle_{\mathrm{Fig.\,2}} = 0, \quad (57)$$

the right-hand side of Eq. (56) is fully defined by the divergence of the charged hadronic axial-vector current

$$\langle p(\vec{k}_p,\sigma_p)|\partial^\mu A^+_\mu(0)|n(\vec{k}_n,\sigma_n)\rangle_{\rm Fig.\,2} = 2g_{\pi N}f_\pi \frac{m^2_\pi}{m^2_\pi - q^2}\bar{u}_p(\vec{k}_p,\sigma_p)i\gamma^5 u_n(\vec{k}_n,\sigma_n). \quad (58)$$

Such a matrix element is caused by the partial conservation of the axial-vector hadronic current (PCAC) $\partial^\mu A^+_\mu = -\sqrt{2}m^2_\pi f_\pi \pi^+$ [129,133]. Plugging the GT relation $g_{\pi N} = m_N/f_\pi$ into Eq. (37) we arrive at the matrix element of the charged $V - A$ hadronic current, calculated in the tree approximation in the LσM + SEM (see also [141])

$$\langle p(\vec{k}_p,\sigma_p)|J^+_\mu(0)|n(\vec{k}_n,\sigma_n)\rangle_{\rm Fig.\,1} = \bar{u}_p(\vec{k}_p,\sigma_p)\left(\gamma_\mu(1-\gamma^5) - \frac{2m_N}{m^2_\pi - q^2}q_\mu\gamma^5\right)u_n(\vec{k}_n,\sigma_n). \quad (59)$$

The matrix element of the charged hadronic current Eq. (59) has the standard Lorentz structure with the vector, axial-vector and pseudoscalar form factors equal to unity [42,43,158] (see also [141]). The amplitude of the neutron β^- decay in the tree approximation is equal to

$$M(n\to pe^-\bar{\nu}_e) = G_V\bar{u}_p(\vec{k}_p,\sigma_p)\left(\gamma_\mu(1-\gamma^5) - \frac{2m_N}{m^2_\pi - q^2}q_\mu\gamma^5\right) \times u_n(\vec{k}_n,\sigma_n)\frac{M^2_W}{M^2_W - q^2 - i0}\left(-\eta^{\mu\nu} + \frac{q^\mu q^\nu}{M^2_W}\right) \times \left[\bar{u}_e(\vec{k}_e,\sigma_e)\gamma_\nu(1-\gamma^5)v_{\bar{\nu}}\left(\vec{k}_{\bar{\nu}},+\frac{1}{2}\right)\right], \quad (60)$$

As a consequence of the PCAC the longitudinal part of the electroweak W^--boson propagator, proportional to $q^\mu q^\nu/M^2_W$, does not vanish. This violates gauge invariance, as we have pointed out above. The contribution of such a violation of gauge invariance to the amplitude of the neutron β^- decay is of order $O(2m_N m_e/M^2_W) \sim 1.5\times 10^{-7}$. This is 2 orders of magnitude smaller than the corrections of order $O(\alpha E_e/m_N) \sim 10^{-5}$, which we are searching for. In the chiral limit $m_\pi \to 0$, that is in the limit of a vanishing pion mass, the right-hand-side of Eq. (58) and, correspondingly, Eq. (56) vanish that leads to local conservation of the charged axial-vector hadronic current $\partial^\mu A^+_\mu = 0$, providing gauge invariance of the amplitude of the neutron β^- decay, i.e. independence of the longitudinal part of the electroweak W^--boson propagator.

B. Neutron beta decay in the tree approximation for electroweak interactions and to one-hadron-loop approximation for strong low-energy interactions described by the Lagrangian Eq. (44)

The amplitude of the neutron β^- decay in the tree approximation for the electroweak W^--boson exchange and to one-hadron-loop approximation can be taken in the following form [60]:

$$M(n\to pe^-\bar{\nu}_e) = G_V\langle p(\vec{k}_p,\sigma_p)|J^+_\mu(0)|n(\vec{k}_n,\sigma_n)\rangle_{\rm Fig.\,2+\ldots+Fig.\,5} \times \frac{M^2_W}{M^2_W - q^2 - i0}\left(-\eta^{\mu\nu} + \frac{q^\mu q^\nu}{M^2_W}\right) \times \left[\bar{u}_e(\vec{k}_e,\sigma_e)\gamma_\nu(1-\gamma^5)v_{\bar{\nu}}\left(\vec{k}_{\bar{\nu}},+\frac{1}{2}\right)\right]. \quad (61)$$

For the calculation of the one-hadron-loop corrections we shall use the normal ordered form of the Lagrangians Eqs. (44) and (45), respectively [159]. This allows us to avoid the tadpole contributions. Using the normal ordered form of the Lagrangians Eqs. (44) and (45) the Feynman diagrams, defining the one-hadron-loop contributions to the amplitude of the neutron β^- decay, are shown in Figs. 3–5, respectively. The Feynman diagrams in Figs. 3 and 4 define the contributions of the self-energy

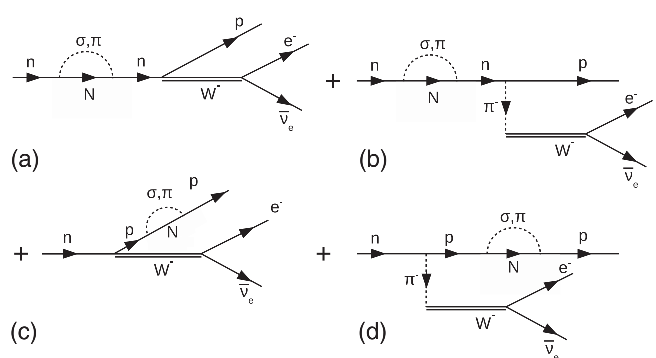

FIG. 3. Feynman diagrams, describing the contributions to the amplitude of the neutron β^- decay of the self-energy corrections to the neutron and proton states in the one-hadron-loop approximation in the LσM and SEM described by the Lagrangian Eq. (44).

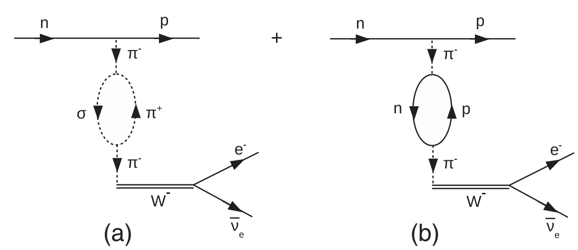

FIG. 4. Feynman diagrams, describing self-energy corrections to the π^--meson state in the one-hadron-loop approximation in the LσM and SEM described by the Lagrangian Eq. (44).

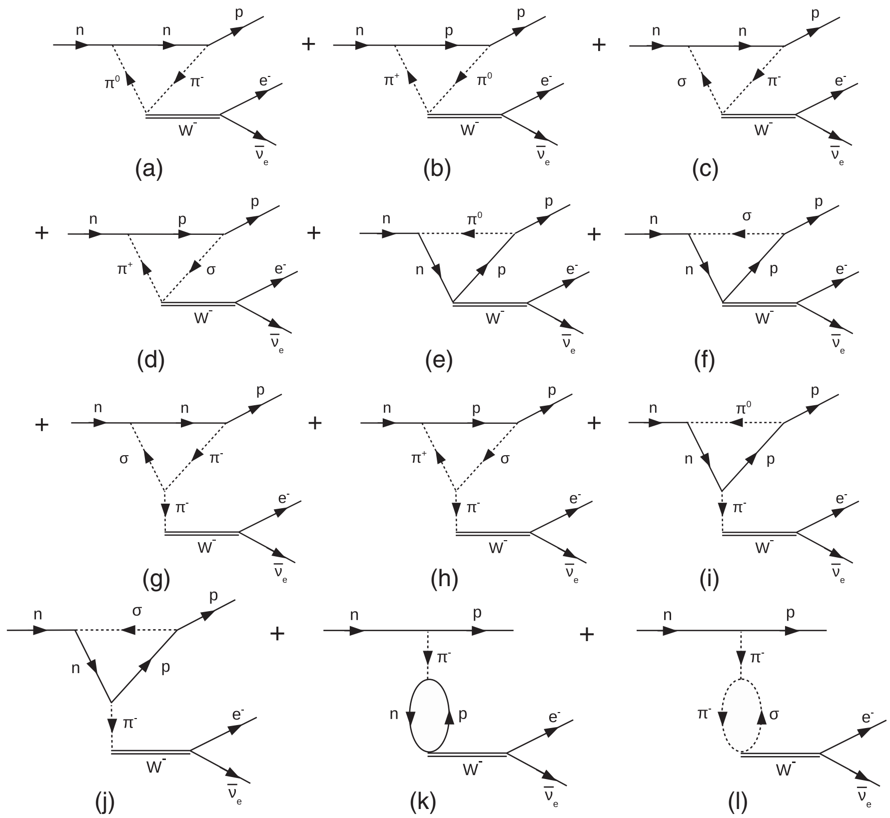

FIG. 5. Feynman diagrams, describing the contributions of the hadronic structure of the neutron and proton, and the π^- meson to the amplitude of the neutron β^- decay in the one-hadron-loop approximation in the LσM and SEM described by the Lagrangian Eq. (44).

corrections, caused by strong low-energy interactions to the neutron and proton and π^- states, respectively. It is obvious that after normalization the contributions of these diagrams to matrix element of the hadronic $n \to p$ transition vanish [57,60]. The nontrivial structure of the matrix element of the hadronic $n \to p$ transition is caused by the contributions of the Feynman diagrams in Fig. 5 [60].

The matrix element of the hadronic $n \to p$ transition, calculated in the one-hadron-loop approximation, is equal to (see Appendix A of Supplemental Material [77])

$$\begin{aligned}\langle p(\vec{k}_p,\sigma_p)|J^+_\mu(0)|n(\vec{k}_n,\sigma_n)\rangle_{\text{Fig. 2}+\ldots+\text{Fig. 5}} = \bar{u}_p(\vec{k}_p,\sigma_p)\bigg\{&\bigg[1+(\tilde{Z}^{(N)}_1-1)+(Z_N-1)+\frac{g^2_{\pi N}}{8\pi^2}\bigg(\ell n\frac{\Lambda^2}{m^2_N}-\frac{1}{4}\ell n\frac{m^2_\sigma}{m^2_N}\bigg)\bigg]\gamma_\mu\\ &-\bigg[1+(\tilde{Z}^{(N)}_1-1)+(Z_N-1)+\frac{g^2_{\pi N}}{8\pi^2}\bigg(\frac{5}{4}\ell n\frac{m^2_\sigma}{m^2_N}-\ell n\frac{\Lambda^2}{m^2_N}\bigg)\bigg]\gamma_\mu\gamma^5\\ &+\frac{5g^2_{\pi N}}{16\pi^2}\frac{i\sigma_{\mu\nu}q^\nu}{2m_N}-\frac{2m_N q_\mu}{m^2_\pi-q^2-i0}\gamma^5\bigg[1+(Z_{MN}-1)+(\tilde{Z}^{(N)}_2-1)+(\tilde{Z}^{(M)}_1-1)\\ &+(Z_M-1)+\frac{g^2_{\pi N}}{8\pi^2}\bigg(\ell n\frac{m^2_\sigma}{m^2_N}+\ell n\frac{\Lambda^2}{m^2_N}\bigg)\bigg]\bigg\}u_n(\vec{k}_n,\sigma_n). \end{aligned} \tag{62}$$

Using Eq. (A6) in Supplemental Material [77], we arrive at the matrix element of the hadronic $n \to p$ transition, calculated to the one-hadron-loop approximation. We get

$$\langle p(\vec{k}_p,\sigma_p)|J^+_\mu(0)|n(\vec{k}_n,\sigma_n)\rangle_{\text{Fig. 2+...+Fig. 5}} = \bar{u}_p(\vec{k}_p,\sigma_p)\Big\{[1+(\tilde{Z}^{(N)}_1-1)-(\tilde{Z}^{(N)}_2-1)]\gamma_\mu - \Big[1+(\tilde{Z}^{(N)}_1-1)-(\tilde{Z}^{(N)}_2-1) + \frac{g^2_{\pi N}}{8\pi^2}\Big(\frac{3}{2}\ell n\frac{m^2_\sigma}{m^2_N}-2\ell n\frac{\Lambda^2}{m^2_N}\Big)\Big]\gamma_\mu\gamma^5 + \frac{5g^2_{\pi N}}{16\pi^2}\frac{i\sigma_{\mu\nu}q^\nu}{2m_N} - \frac{2m_N q_\mu}{m^2_\pi-q^2-i0}\gamma^5\Big[1+\Big((Z_{MN}-1)-(Z_N-1)-\frac{Z_M-1}{2}\Big)+(\tilde{Z}^{(M)}_1-1) - \frac{3}{2}(\tilde{Z}^{(M)}_2-1)-\frac{g^2_{\pi N}}{8\pi^2}\Big(3\ell n\frac{\Lambda^2}{m^2_N}-\frac{5}{4}\ell n\frac{m^2_\sigma}{m^2_N}\Big)\Big]\Big\}u_n(\vec{k}_n,\sigma_n). \tag{63}$$

Because of gauge invariance and Ward identities $\tilde{Z}^{(N)}_2=\tilde{Z}^{(N)}_1$ and $\tilde{Z}^{(M)}_2=\tilde{Z}^{(M)}_1$ we transcribe the right-hand side of Eq. (63) into the form

$$\langle p(\vec{k}_p,\sigma_p)|J^+_\mu(0)|n(\vec{k}_n,\sigma_n)\rangle_{\text{Fig. 2+...+Fig. 5}} = \bar{u}_p(\vec{k}_p,\sigma_p)\Big\{\gamma_\mu - \Big[1+\frac{g^2_{\pi N}}{8\pi^2}\Big(\frac{3}{2}\ell n\frac{m^2_\sigma}{m^2_N}-2\ell n\frac{\Lambda^2}{m^2_N}\Big)\Big]\gamma_\mu\gamma^5 + \frac{5g^2_{\pi N}}{16\pi^2}\frac{i\sigma_{\mu\nu}q^\nu}{2m_N} - \frac{2m_N q_\mu}{m^2_\pi-q^2-i0}\gamma^5\Big[1+\Big((Z_{MN}-1)-(Z_N-1)-\frac{Z_M-1}{2}\Big) - \frac{1}{2}(\tilde{Z}^{(M)}_2-1)-\frac{g^2_{\pi N}}{8\pi^2}\Big(3\ell n\frac{\Lambda^2}{m^2_N}-\frac{5}{4}\ell n\frac{m^2_\sigma}{m^2_N}\Big)\Big]\Big\}u_n(\vec{k}_n,\sigma_n). \tag{64}$$

Since the counterterm $Z_{MN}Z^{-1}_N Z^{-1/2}_M$ renormalizes the pion-nucleon coupling constant $g_{\pi N}$, we set

$$(Z_{MN}-1)-(Z_N-1)-\frac{Z_M-1}{2} = g_A - 1, \tag{65}$$

where $g_A \neq 1$ is the axial coupling constant, defining a finite nontrivial renormalization of the pion-nucleon coupling constant $g_{\pi N}$ [44]. Setting then the relation

$$\tilde{Z}^{(M)}_2 - 1 = -\frac{g^2_{\pi N}}{8\pi^2}\Big(3\ell n\frac{\Lambda^2}{m^2_N}-\frac{5}{4}\ell n\frac{m^2_\sigma}{m^2_N}\Big) \tag{66}$$

we arrive at the following matrix element of the hadronic $n\to p$ transition

$$\langle p(\vec{k}_p,\sigma_p)|J^+_\mu(0)|n(\vec{k}_n,\sigma_n)\rangle_{\text{Fig. 2+...+Fig. 5}} = \bar{u}_p(\vec{k}_p,\sigma_p)\Big\{\gamma_\mu - \Big[1+\frac{g^2_{\pi N}}{8\pi^2}\Big(\frac{3}{2}\ell n\frac{m^2_\sigma}{m^2_N}-2\ell n\frac{\Lambda^2}{m^2_N}\Big)\Big]\gamma_\mu\gamma^5 + \frac{5g^2_{\pi N}}{16\pi^2}\frac{i\sigma_{\mu\nu}q^\nu}{2m_N} - \frac{2m_N g_A q_\mu}{m^2_\pi-q^2-i0}\gamma^5\Big\}u_n(\vec{k}_n,\sigma_n). \tag{67}$$

In the chiral limit $m_\pi \to 0$ the matrix element Eq. (67) should obey the requirement of conservation of the axial-vector hadronic current [42], i.e.

$$q^\mu \lim_{m_\pi\to 0}\langle p(\vec{k}_p,\sigma_p)|J^+_\mu(0)|n(\vec{k}_n,\sigma_n)\rangle_{\text{Fig. 2+...+Fig. 5}} = 0. \tag{68}$$

This allows us to impose the following relation:

$$g_A = 1+\frac{g^2_{\pi N}}{8\pi^2}\Big(\frac{3}{2}\ell n\frac{m^2_\sigma}{m^2_N}-2\ell n\frac{\Lambda^2}{m^2_N}\Big). \tag{69}$$

The axial coupling constant g_A defines a finite renormalization of the axial-vector hadronic current [44]. This means that the right-hand side of Eq. (69) should be finite even in the limit $\Lambda \to \infty$ and $m_\sigma \to \infty$. This can be reached if $m_\sigma^2 = (\Lambda^2/m_N^2)^{4/3}M^2$, where M is a finite scale parameter. The fact that the mass of the σ meson tends to infinity faster than the ultraviolet cutoff does not contradict our analysis of the equivalence of the LσM to the chiral quantum field theories with nonlinear realizations of chiral $SU(2) \times SU(2)$ symmetry (see Sec. IV). As a result, the matrix element of the hadronic $n \to p$ transition takes the standard form [158]

$$\langle p(\vec{k}_p, \sigma_p)|J^+_\mu(0)|n(\vec{k}_n, \sigma_n)\rangle_{\text{Fig. 2}+\ldots+\text{Fig. 5}} = \bar{u}_p(\vec{k}_p, \sigma_p)\left\{\gamma_\mu(1 - g_A\gamma^5) + \frac{\kappa}{2m_N} i\sigma_{\mu\nu}q^\nu - \frac{2m_N g_A q_\mu}{m_\pi^2 - q^2 - i0}\gamma^5\right\}u_n(\vec{k}_n, \sigma_n), \quad (70)$$

where $\kappa = 5g_{\pi N}^2/16\pi^2$ is the isovector anomalous n magnetic moment of the nucleon defining the intensity of the so-called weak magnetism [43]. The experimental value of the isovector anomalous magnetic moment of the nucleon is equal to $\kappa = \kappa_p - \kappa_n = 3.70589$ with $\kappa_p = 1.7928473$ and $\kappa_n = -1.9130427$, where κ_p and κ_n are anomalous magnetic moments of the proton and neutron, respectively [3]. Setting $\kappa = 3.70589$ one may estimate the value of the pion-nucleon coupling constant $g_{\pi N} = \sqrt{\kappa 16\pi^2/5} = 10.82$. This defines the leptonic decay (or the PCAC) constant of pion $f_\pi = 86.8$ MeV at $m_N = (m_p + m_n)/2 = 939$ MeV [3], which agrees well with the definition of a *bare* leptonic decay constant of a pion [117]. In our approach a *bare* leptonic decay constant of pion $f_\pi = 86.8$ MeV deviates from the observable value of the pion-leptonic constant is equal to $f_\pi^{(\text{obs})} = 92.4$ MeV [3] by about 6%.

Thus, we have shown that the matrix element of the hadronic $n \to p$ transition, calculated to one-hadron-loop approximation in the quantum field theoretic model of strong low-energy and electroweak interactions described by the Lagrangian Eq. (44), possesses a standard Lorentz structure, where contributions of strong low-energy interactions are defined by the axial coupling constant $g_A \neq 1$, the isovector anomalous magnetic moment of the nucleon κ and the one-pion-pole exchange. In the chiral limit the matrix element of the hadronic $n \to p$ transition provides independence of the amplitude of the neutron β^- decay of the longitudinal part of the electroweak W^--boson propagator. This agrees well with a requirement of conservation of the axial-vector hadronic current in the chiral limit [42].

Using the experimental value of the axial coupling constant $g_A^{(\text{exp})} = 1.27641(45)_{\text{stat}}(33)_{\text{syst}}$, measured recently by the spectrometer PERKEO III [56], we estimate the value of the scale parameter $M \simeq 1$ GeV, agreeing well with a scale $\Lambda_\chi \sim 1$ GeV of spontaneous breakdown of chiral symmetry [117].

VII. RADIATIVE ONE-LOOP ELECTROMAGNETIC CORRECTIONS TO THE NEUTRON β^- DECAY $n \to p + e^- + \bar{\nu}_e$

In this section we proceed to the calculation of the radiative corrections of order $O(\alpha/\pi)$ and its next-to-leading order corrections of order $O(\alpha E_e/m_N)$ to the neutron β^- decay, caused by one-virtual-photon exchanges. For this aim we start with the analysis of the radiative electromagnetic corrections to the amplitude of the neutron β^- decay taken in the tree approximation for strong low-energy interactions at $g_A = 1$ and described by the Feynman diagrams in Fig. 6. We show that the set of Feynman diagrams in Fig. 6 is gauge invariant, i.e. independent of a gauge parameter ξ of the photon propagator. Then, we show that gauge properties of the Feynman diagrams in Fig. 6 are not changed even for $g_A \neq 1$ and calculate these diagrams setting $g_A \neq 1$. This allows us to take contributions of strong low-energy interactions at Sirlin's confidence level [14,21].

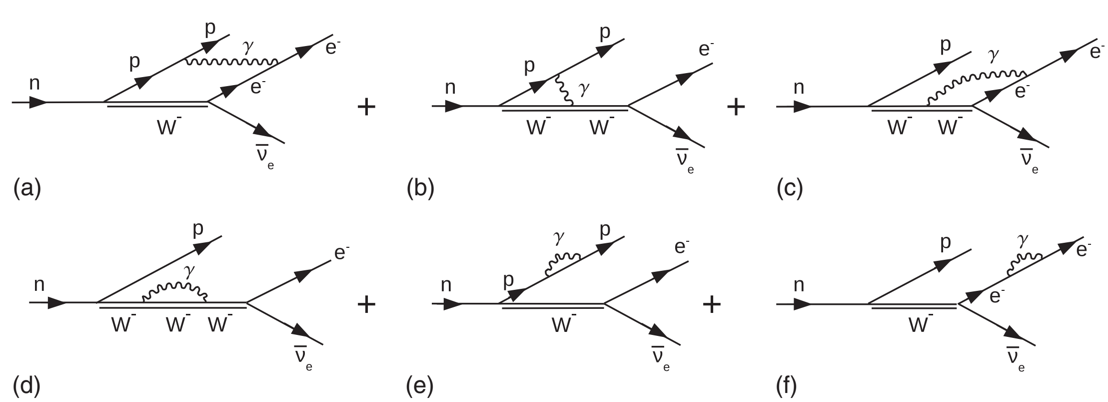

FIG. 6. Feynman diagrams, describing the one-photon-loop radiative corrections to the part of the amplitude of the neutron β^- decay, described by the Feynman diagram in Fig. 2(a).

A. Amplitude of the neutron β^- decay in the tree approximation for strong low-energy interactions and to one-loop approximation for electromagnetic interactions described by the Lagrangian Eq. (44)

The amplitude of the neutron β^- decay, calculated in the tree approximation for strong low-energy and electroweak interactions is described by the Feynman diagrams in Fig. 2. The radiative corrections to this part of the amplitude of the neutron β^- decay, caused by one-virtual-photon exchanges and described by the Lagrangian Eq. (44), are defined by the Feynman diagrams in Fig. 6. We do not analyze the radiative corrections to the one-pion-pole exchange, since as has been shown in [62] the radiative corrections to the one-pion-pole exchange are of order 10^{-9} and can be neglected in comparison with the radiative corrections of order 10^{-5}, which we are searching for in this paper. The details of the calculation of the Feynman diagrams in Fig. 6 one may find in Appendices B and C of the Supplemental Material [77], where we show that the Feynman diagrams in Fig. 6 are gauge invariant and do not depend on a gauge parameter ξ of the photon propagator. We show also that gauge invariance of the Feynman diagrams in Fig. 6 retains even for the axial coupling constant $g_A \neq 1$. As has been shown in Sec. VI [see Eq. (70)], the axial coupling constant $g_A \neq$ in the amplitude of the neutron β^- decay appears as a contribution of one-hadron-loop diagrams, caused by strong low-energy interactions described by the Lagrangian Eq. (44). Gauge invariance of the Feynman diagrams in Fig. 6 for $g_A \neq 1$ allows us to take into account partly the contributions of strong low-energy interactions to the one-hadron-loop approximation. As we have shown in Appendices B and C of the Supplemental Material [77] the amplitude of the neutron β^- decay with radiative corrections of order $O(\alpha/\pi)$ and $O(\alpha E_e/m_N)$ after renormalization takes the form

$$\begin{aligned} M(n \to p e^- \bar{\nu}_e) = -2m_N G_V\Big\{&\Big[1 + \frac{\alpha}{2\pi}\Big(f_{\beta^-_c}(E_e,\mu) + \frac{E_e}{m_N} f_V(E_e)\Big)\Big][\varphi^\dagger_p \varphi_n][\bar{u}_e \gamma^0(1-\gamma^5)v_{\bar{\nu}}] \\ &+ g_A\Big[1 + \frac{\alpha}{2\pi}\Big(f_{\beta^-_c}(E_e,\mu) + \frac{E_e}{m_N} f_A(E_e) + \frac{5}{2}\frac{m_N^2}{M_W^2}\ell n\frac{M_W^2}{m_N^2}\Big)\Big][\varphi^\dagger_p \vec{\sigma}\varphi_n]\cdot[\bar{u}_e\vec{\gamma}(1-\gamma^5)v_{\bar{\nu}}] \\ &+ \frac{\alpha}{2\pi}\Big[[\varphi^\dagger_p\varphi_n][\bar{u}_e(1-\gamma^5)v_{\bar{\nu}}]\Big(-\frac{\sqrt{1-\beta^2}}{2\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big) + \frac{E_e}{m_N}f_S(E_e)\Big) + g_A[\varphi^\dagger_p\vec{\sigma}\varphi_n]\cdot[\bar{u}_e\gamma^0\vec{\gamma}(1-\gamma^5)v_{\bar{\nu}}] \\ &\times\Big(-\frac{\sqrt{1-\beta^2}}{2\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big) + \frac{E_e}{m_N}f_T(E_e)\Big) + \Big(\Big[\varphi^\dagger_p\frac{\vec{k}_e\cdot\vec{\sigma}}{E_e}\varphi_n\Big]\frac{E_e}{m_N}g_S(E_e) + \Big[\varphi^\dagger_p\frac{\vec{k}_{\bar{\nu}}\cdot\vec{\sigma}}{E_e}\varphi_n\Big]\frac{E_e}{m_N}h_S(E_e)\Big) \\ &\times[\bar{u}_e(1-\gamma^5)v_{\bar{\nu}}] + \Big[\varphi^\dagger_p\frac{\vec{k}_e\cdot\vec{\sigma}}{E_e}\varphi_n\Big][\bar{u}_e\gamma^0(1-\gamma^5)v_{\bar{\nu}}]\frac{E_e}{m_N}g_V(E_e) \\ &+ \Big[\varphi^\dagger_p\frac{(\vec{k}_e\cdot\vec{\sigma})\vec{\sigma}}{E_e}\varphi_n\Big]\cdot[\bar{u}_e\vec{\gamma}(1-\gamma^5)v_{\bar{\nu}}]\frac{E_e}{m_N}h_A(E_e)\Big]\Big\}, \end{aligned} \tag{71}$$

where the functions $f_{\beta^-_c}(E_e,\mu)$, $f_V(E_e)$, $f_A(E_e)$, $f_S(E_e)$ and so on are calculated in Appendixes B and C of the Supplemental Material [77] and are given in Eq. (C4). The function $f_{\beta^-_c}(E_e,\mu)$, where μ is an infinitesimal photon mass, realizing relativistic covariant infrared regularization of the radiative corrections caused by one-photon loop exchanges [14], has been calculated by Sirlin [14] (the details of the calculation one may find in [5]). This function together with the terms [see Eq. (C5) in Supplemental Material [77]] which survive to leading order in the large nucleon mass expansion, define the famous Sirlin's function $\bar{g}(E_e)$ [14], describing radiative corrections to the neutron lifetime. The functions $f_V(E_e)$, $f_A(E_e)$, $f_S(E_e)$, $f_T(E_e)$, $g_S(E_e)$, $h_S(E_e)$, $g_V(E_e)$ and $h_A(E_e)$ [see Eq. (C4) in Supplemental Material [77]] are related to the radiative corrections of order $O(\alpha E_e/m_N)$. The term $(\alpha/\pi)(5m_N^2/2M_W^2)\ell n(M_W^2/m_N^2) \sim 10^{-5}$, calculated at $m_N = 0.939$ GeV and $M_W = 80.379$ GeV [3], is the rest of the contributions of the virtual electroweak W^--boson exchanges (see Feynman diagrams in Fig. 6) after renormalization (see Appendixes B and C).

B. Rate of the neutron β^- decay $n \to p + e^- + \bar{\nu}_e$ described by the amplitude Eq. (71). Corrections of order $O(\alpha E_e/m_N)$ to Sirlin's function

First, following [5] we calculate the electron energy and angular distribution of the neutron β^- decay $n \to p + e^- + \bar{\nu}_e$ with unpolarized massive fermions, described in the amplitude Eq. (71). We get

$$\frac{d^5\lambda_{\beta^-_c}(E_e,\vec{k}_e,\vec{k}_{\bar{\nu}},\mu)}{dE_e d\Omega_e d\Omega_{\bar{\nu}}} = (1+3g_A^2)\frac{|G_V|^2}{32\pi^5}\Big\{1+\frac{\alpha}{\pi}\Big(f_{\beta^-_c}(E_e,\mu)-\frac{1-\beta^2}{2\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big)\Big)+\frac{1-g_A^2}{1+3g_A^2}\Big(1+\frac{\alpha}{\pi}f_{\beta^-_c}(E_e,\mu)\Big)\frac{\vec{k}_e\cdot\vec{k}_{\bar{\nu}}}{E_eE_{\bar{\nu}}}$$
$$+\frac{\alpha}{\pi}\Big[\frac{1}{1+3g_A^2}\frac{E_e}{m_N}\Big(f_V(E_e)+\sqrt{1-\beta^2}f_S(E_e)+\beta^2h_A(E_e)+g_A\beta^2g_V(E_e)+g_A\frac{E_0-E_e}{E_e}\sqrt{1-\beta^2}h_S(E_e)\Big)$$
$$+\frac{3g_A^2}{1+3g_A^2}\frac{E_e}{m_N}\Big(f_A(E_e)+\sqrt{1-\beta^2}f_T(E_e)\Big)+\frac{3g_A^2}{1+3g_A^2}\frac{5}{2}\frac{m_N^2}{M_W^2}\ell n\frac{M_W^2}{m_N^2}\Big]$$
$$+\frac{\alpha}{\pi}\Big[\frac{1}{1+3g_A^2}\frac{E_e}{m_N}(f_V(E_e)+h_A(E_e)+g_A\sqrt{1-\beta^2}g_S(E_e)+g_Ag_V(E_e))-\frac{g_A^2}{1+3g_A^2}\frac{E_e}{m_N}f_A(E_e)$$
$$-\frac{g_A^2}{1+3g_A^2}\frac{5}{2}\frac{m_N^2}{M_W^2}\ell n\frac{M_W^2}{m_N^2}\Big]\frac{\vec{k}_e\cdot\vec{k}_{\bar{\nu}}}{E_eE_{\bar{\nu}}}\Big\}\sqrt{E_e^2-m_e^2}E_eF(E_e,Z=1), \quad (72)$$

where $d\Omega_e$ and $d\Omega_{\bar{\nu}}$ are infinitesimal solid angles in the directions of the electron and antineutrino 3-momenta, $F(E_e, Z=1)$ is the well-known relativistic Fermi function, describing the electron-proton Coulomb final-state interaction (see, for example, [40]). The rate $\lambda_{\beta^-_c}(\mu)$ of the neutron β^- decay $n\to p+e^-+\bar{\nu}_e$ is defined by the integral

$$\lambda_{\beta^-_c}(\mu) = (1+3g_A^2)\frac{|G_V|^2}{2\pi^3}\int_{m_e}^{E_0}\Big\{1+\frac{\alpha}{\pi}\Big(f_{\beta^-_c}(E_e,\mu)-\frac{1-\beta^2}{2\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big)\Big)+\frac{\alpha}{\pi}\Big[\frac{1}{1+3g_A^2}\frac{E_e}{m_N}\Big(f_V(E_e)$$
$$+\sqrt{1-\beta^2}f_S(E_e)+\beta^2h_A(E_e)+g_A\beta^2g_V(E_e)+g_A\frac{E_0-E_e}{E_e}\sqrt{1-\beta^2}h_S(E_e)\Big)+\frac{3g_A^2}{1+3g_A^2}\frac{E_e}{m_N}(f_A(E_e)$$
$$+\sqrt{1-\beta^2}f_T(E_e))+\frac{3g_A^2}{1+3g_A^2}\frac{5}{2}\frac{m_N^2}{M_W^2}\ell n\frac{M_W^2}{m_N^2}\Big]\Big\}\sqrt{E_e^2-m_e^2}E_eF(E_e,Z=1)dE_e, \quad (73)$$

where $E_0 = (m_n^2-m_p^2+m_e^2)/2m_n = 1.2927$ MeV is the end-point energy of the electron-energy spectrum [5]. In the integrand the first term of order $O(\alpha/\pi)$ reproduces fully Sirlin's result [14], calculated to leading order in the large nucleon mass expansion.

For the cancellation of the infrared divergence in the rate $\lambda_{\beta^-_c}(\mu)$ of the neuron β^- decay $n\to p+e^-+\bar{\nu}_e$ and to calculate the total rate of the neutron β^- decays we have to take into account the rate $\lambda_{\beta^-_c\gamma}(\mu)$ of the neutron radiative β^- decay $n\to p+e^-+\bar{\nu}_e+\gamma$, where γ is a real photon [10–15]. The Feynman diagrams, describing the amplitude of the neutron radiative β^- decay in the tree approximation for electroweak, electromagnetic and strong low-energy interactions, are shown in Fig. 7. The Feynman diagrams are drawn to leading order in the large mass M_W of the electroweak W^--boson expansion at the neglect of the

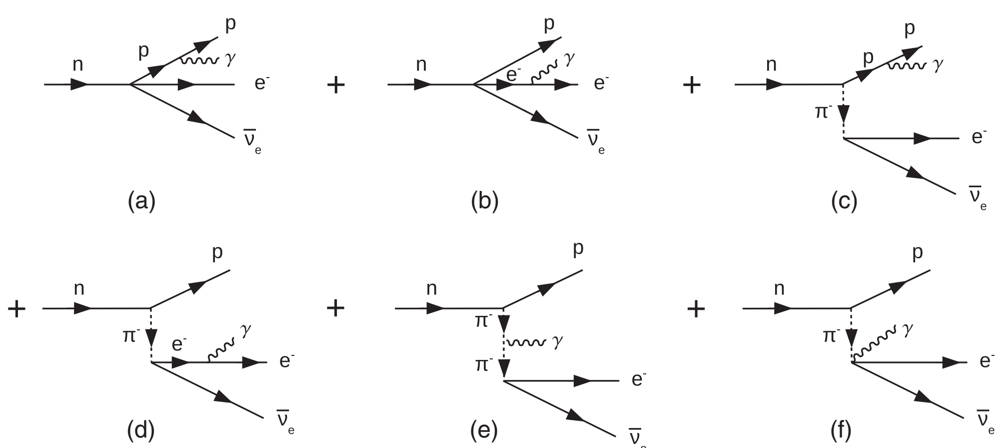

FIG. 7. Feynman diagrams, defining in the tree approximation for the electroweak, electromagnetic and strong low-energy interactions the amplitude of the neutron radiative β^- decay calculated with the Lagrangian Eq. (44). The Feynman diagrams are drawn to leading order in the large mass M_W of the electroweak W^--boson expansion at the neglect of the Feynman diagram with the vertex $W^-W^-\gamma$, the contribution of which is suppressed by the factor $q\cdot k/M_W^2$, where k is a 4-momentum of a real photon.

Feynman diagram with the vertex $W^-W^-\gamma$, the contribution of which is suppressed by the factor $q\cdot k/M_W^2$, where k is a 4-momentum of a real photon. The Feynman diagrams in Figs. 7(c)–7(f) are caused by the mesonic part of the charged hadronic axial-vector current. The calculation of the Feynman diagrams in Fig. 7 has been carried out in [60] (see also Appendix D of the Supplemental Material [77]). For the calculation of the neutron lifetime τ_n, related to the rate $\tau_n = 1/\lambda_n$, where $\lambda_n = \lambda_{\beta^-_c}(\mu) + \lambda_{\beta^-_c\gamma}(\mu)$, we may use $\lambda_{\beta^-_c\gamma}(\mu)$, calculated to leading order in the large nucleon mass expansion. Using the results, obtained in Appendix B of Ref. [5] (see also Appendix D in the Supplemental Material [77]), we get the rate $\lambda_{\beta^-_c\gamma}(\mu)$ of the neutron radiative β^- decay

$$\begin{aligned}\lambda_{\beta^-_c\gamma}(\mu) = {}& (1+3g_A^2)\frac{\alpha|G_V|^2}{\pi\,2\pi^3}\int_{m_e}^{E_0}\Big\{\Big[2\ell n\Big(\frac{2(E_0-E_e)}{\mu}\Big)\\ &-3+\frac{2}{3}\frac{E_0-E_e}{E_e}\Big(1+\frac{1}{8}\frac{E_0-E_e}{E_e}\Big)\Big]\Big[\frac{1}{2\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big)-1\Big]\\ &+1+\frac{1}{12}\frac{(E_0-E_e)^2}{E_e^2}+\frac{1}{2\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big)\\ &-\frac{1}{4\beta}\ell n^2\Big(\frac{1+\beta}{1-\beta}\Big)-\frac{1}{\beta}\mathrm{Li}_2\Big(\frac{2\beta}{1+\beta}\Big)\Big\}\\ &\times\sqrt{E_e^2-m_e^2}\,E_e F(E_e,Z=1)\,dE_e \qquad (74)\end{aligned}$$

and the total rate $\lambda_n = \lambda_{\beta^-_c}(\mu) + \lambda_{\beta^-_c\gamma}(\mu)$ of the neutron β^- decay

$$\begin{aligned}\lambda_n = {}&(1+3g_A^2)\frac{|G_V|^2}{2\pi^3}\int_{m_e}^{E_0}\Big(1+\frac{\alpha}{\pi}\bar{g}_n(E_e)+\frac{\alpha}{\pi}\frac{E_e}{m_N}\bar{h}_n(E_e)\Big)\\ &\times\sqrt{E_e^2-m_e^2}\,E_e F(E_e,Z=1)\,dE_e, \qquad (75)\end{aligned}$$

where the function $\bar{g}_n(E_e)$ is Sirlin's function equal to [14] (see also Appendix D of Ref. [5])

$$\begin{aligned}\bar{g}_n(E_e) = {}&\frac{3}{4}\ell n\Big(\frac{m_N^2}{m_e}\Big)-\frac{3}{8}+2\Big[\frac{1}{2\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big)-1\Big]\\ &\times\Big[\ell n\Big(\frac{2(E_0-E_e)}{m_e}\Big)-\frac{3}{2}+\frac{1}{3}\frac{E_0-E_e}{E_e}\Big]\\ &-\frac{2}{\beta}\mathrm{Li}_2\Big(\frac{2\beta}{1+\beta}\Big)+\frac{1}{2\beta}\ell n\Big(\frac{1+\beta}{1-\beta}\Big)\\ &\times\Big[(1+\beta^2)+\frac{1}{12}\frac{(E_0-E_e)^2}{E_e^2}-\ell n\Big(\frac{1+\beta}{1-\beta}\Big)\Big], \qquad (76)\end{aligned}$$

and the function $\bar{h}_n(E_e)$, defining gauge invariant radiative corrections of order $O(\alpha E_e/m_N)$ to Sirlin's function $\bar{g}_n(E_e)$, is given by

$$\begin{aligned}\bar{h}_n(E_e) = {}&\frac{1}{1+3g_A^2}\Big(f_V(E_e)+\sqrt{1-\beta^2}f_S(E_e)+\beta^2 h_A(E_e)\\ &+g_A\beta^2 g_V(E_e)+g_A\frac{E_0-E_e}{E_e}\sqrt{1-\beta^2}h_S(E_e)\Big)\\ &+\frac{3g_A^2}{1+3g_A^2}\big(f_A(E_e)+\sqrt{1-\beta^2}f_T(E_e)\big)\\ &+\frac{3g_A^2}{1+3g_A^2}\frac{5}{2}\frac{m_N}{E_e}\frac{m_N^2}{M_W^2}\ell n\frac{M_W^2}{m_N^2}, \qquad (77)\end{aligned}$$

where the functions $f_V(E_e)$, $f_V(E_e)$, $f_S(E_e)$, $f_T(E_e)$, $g_S(E_e)$, $g_V(E_e)$, $h_S(E_e)$ and $h_A(E_e)$ are adduced in Eq. (C4) of Appendix C in the Supplemental Material [77]. Thus, we have reproduced fully Sirlin's radiative corrections of order $O(\alpha/\pi)\sim 10^{-3}$ to the neutron lifetime, calculated to leading order in the large nucleon mass expansion, and obtained radiative corrections of order $O(\alpha E_e/m_N)\sim 10^{-5}$ or corrections to Sirlin's function $\bar{g}_n(E_e)$ in the gauge invariant and renormalizable quantum field theoretic model of strong low-energy and electroweak interactions, described by the Lagrangians Eqs. (44) and (45). In Fig. 8 we plot the function $(\alpha/\pi)(E_e/m_N)\bar{h}_n(E_e)$, where (i) the black curve is defined by the contributions of all three terms in Eq. (77), (ii) the red curve is given by the contribution of only the first term, (iii) the blue curve is defined by the contributions of the last three terms and (iv) the green line determines the contribution of the last term, caused by the contribution of the electroweak W^--boson exchanges. The function $(\alpha/\pi)(E_e/m_N)\bar{h}_n(E_e)$ depends strongly on the axial coupling constant g_A. The curves in Fig. 8 are calculated at $g_A = 1.2764$ [56],

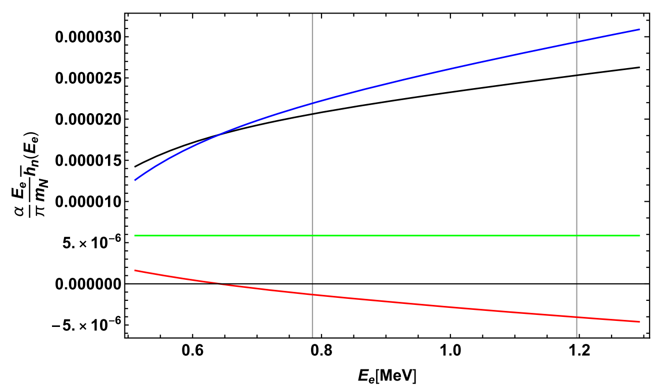

FIG. 8. Radiative corrections of order $O(\alpha E_e/m_N)$, which are described by the function $\bar{h}_n(E_e)$ defining next-to-leading order corrections in the large nucleon mass expansion to Sirlin's function $\bar{g}_n(E_e)$, calculated to leading order in the large nucleon mass expansion [see Eq. (77)], where (i) black, (ii) red, (iii) blue and (iv) green curves are defined by the contributions of (i) all three terms, (ii) of the first term, (iii) of the last two terms and (iv) of the last term of Eq. (77), respectively. The radiative corrections $O(\alpha E_e/m_N)$ are calculated in the electron-energy region $m_e < E_e < E_0$.

$m_e = 0.511$ MeV, $m_N = (m_n + m_p)/2 = 939$ MeV and $M_W = 80379$ MeV, respectively [3], in the electron-energy region $m_e < E_e < E_0$.

VIII. DISCUSSION

We have calculated radiative corrections of order $O(\alpha E_e/m_N) \sim 10^{-5}$ as next-to-leading order corrections in the large nucleon mass expansion to Sirlin's radiative corrections of order $O(\alpha/\pi)$, calculated to leading order in the large nucleon mass expansion to the neutron lifetime [14]. For the extension of Sirlin's result on the contributions of order $O(\alpha E_e/m_N)$ we have followed the assertion pointed out in [60–62] that for the analysis of corrections of order $O(\alpha E_e/m_N)$ as next-to-leading order corrections in the large nucleon mass expansion to Sirlin's radiative corrections of order $O(\alpha/\pi)$ one has to deal with a combined quantum field theoretic model at the hadronic level for strong low-energy pion-nucleon interactions and electroweak interactions of the standard electroweak model with $SU(2)_L \times U(1)_Y$ symmetry. Thus, for the calculation of radiative corrections of order $O(\alpha E_e/m_N) \sim 10^{-5}$ as next-to-leading order corrections in the large nucleon mass expansion to Sirlin's radiative corrections of order $O(\alpha/\pi)$ we have proposed a gauge invariant quantum field theoretic model of strong low-energy pion-nucleon interactions and electroweak pion-nucleon-lepton interactions with electroweak $SU(2)_L \times U(1)_Y$ gauge symmetry, described by the Lagrangian Eq. (44). In the limit of vanishing electroweak coupling constants such a quantum field theoretic model reduces to the linear σ model of strong low-energy pion-nucleon interactions with chiral $SU(2) \times SU(2)$ symmetry, which is treated as a hadronized version of low-energy QCD. The latter is justified by an equivalence of the LσM with a linear realization of chiral $SU(2) \times SU(2)$ symmetry to Gasser-Leutwyler's chiral perturbation theory with nonlinear realization of chiral $SU(2) \times SU(2)$ symmetry in the limit of the infinite mass $m_\sigma \to \infty$ of the scalar isoscalar σ meson (see Sec. IV and [112]). We have shown that the quantum field theoretic model of strong low-energy and electroweak interactions, described by the Lagrangian Eq. (44), reproduces well in the tree approximation for electroweak W^--boson exchanges and to one-hadron-loop approximation, calculated in the limit of the infinitely heavy scalar isoscalar σ meson, a correct Lorentz structure of the matrix element of the hadronic $n \to p$ transition in the amplitude of the neutron β^- decay. The contributions of strong low-energy interactions are presented in the matrix element of the hadronic $n \to p$ transition in terms of the axial coupling constant $g_A \neq 1$, the anomalous isovector magnetic moment of the nucleon κ and the one-pion-pole exchange. In the chiral limit $m_\pi \to 0$ such a matrix element does not depend on a longitudinal part of the electroweak W^--boson propagator. This agrees well with the analysis of weak decays within effective standard $V - A$ theory of weak interactions, carried out by Feynman and Gell-Mann [41] and Nambu [42] (see also [43,141,158]).

In the quantum field theoretic model, described by the Lagrangian Eq. (44), the radiative corrections of order $O(\alpha/\pi)$ are defined by the one-photon-loop Feynman diagrams in the tree approximation for strong low-energy hadronic interactions and by two-loop Feynman diagrams with one-virtual-photon and -hadron exchanges. After renormalization these Feynman diagrams define the radiative corrections of order $O(\alpha/\pi)$ to the neutron β^- decay with the traces of strong low-energy hadronic interactions in terms of the axial coupling constant $g_A \neq 1$ and the contributions of hadronic structure of the nucleon, which do not reduce to the axial coupling constant.

As a first step towards a calculation of radiative corrections of order $O(\alpha/\pi)$ valid to any order in the large nucleon mass expansion and an understanding of gauge properties of these corrections in dependence of one-virtual-photon exchanges with hadronic structure of the neutron and proton, we have investigated the contributions of one-photon-loop Feynman diagrams in the tree approximation for strong low-energy hadronic interactions, which are shown in Fig. 6. To leading order in the large electroweak W^--boson exchanges these diagrams reduce to the set of one-photon-loop Feynman diagrams, defined by Figs. 6(a), 6(e) and 6(f), with pointlike neutron and proton, defined within the standard $V - A$ effective theory of weak interactions and quantum electrodynamics (QED). Such a reduced set of Feynman diagrams has been investigated by Sirlin [14] to leading order in the large nucleon mass expansion for the calculation of the radiative corrections to the neutron lifetime, defined by the function $(\alpha/\pi)\bar{g}_n(E_e)$. As has been pointed out by Sirlin [14], these Feynman diagrams with one-virtual photon coupled to pointlike proton and electron is not gauge invariant, and for a gauge invariant set of Feynman diagrams defining observable radiative corrections of order $O(\alpha/\pi)$ one has to take into account Feynman diagrams of one-virtual-photon exchanges with hadronic structure of the neutron and proton. Keeping only the leading order contributions in the large nucleon mass expansion Sirlin obtained that the observable radiative corrections of order $O(\alpha/\pi)$ to the neutron lifetime do not depend on the axial coupling constant $g_A \neq 1$ and the contributions of hadronic structure of the nucleon coupled to one-virtual photon, responsible for gauge invariance of radiative corrections of order $O(\alpha/\pi)$, do not depend on the electron energy E_e [14,21]. Thus, the analysis of the Feynman diagrams in Fig. 6, taken in the tree approximation for strong low-energy interactions, only for the first step in the calculation of radiative corrections of order $O(\alpha/\pi)$ should shed light on the influence of the hadronic structure of the nucleon on gauge invariance of radiative corrections of order $O(\alpha/\pi)$ valid to any order in the large nucleon mass expansion.

The analytical expressions for these Feynman diagrams, adduced in Appendix B of the Supplemental Material [77], can be obtained by using the Lagrangian Eq. (44) with the axial coupling constant g_A equal to $g_A = 1$. As we have shown in Appendix B of the Supplemental Material [77] these Feynman diagrams are gauge invariant and do not depend on a gauge parameter ξ of the longitudinal part of the photon propagator. Having noticed that such an independence of a gauge parameter ξ retains also for $g_A \neq 1$, we have calculated the contributions of the Feynman diagrams in Fig. 6 at $g_A \neq 1$. This has allowed us to take into account partly the contributions of strong low-energy interactions in terms of the axial coupling constant and to deal with gauge invariant radiative contributions of order $O(\alpha/\pi)$ valid to any order in the large nucleon mass expansion at Sirlin's confidence level [14]. The latter is very important for the calculation of next-to-leading corrections in the large nucleon mass expansion to Sirlin's radiative corrections, calculated to leading order in the large nucleon mass expansion. After renormalization of the one-photon-loop contributions we have obtained the radiative corrections to the amplitude of the neutron β^- decay of order $O(\alpha/\pi) \sim 10^{-3}$, agreeing fully with Sirlin's result [14] (see also Appendices C and D of Ref. [5]) calculated to leading order in the large nucleon mass expansion, and have taken into account next-to-leading order corrections in the large nucleon mass expansion of order $O(\alpha E_e/m_N) \sim 10^{-5}$ [see Eq. (70)]. The amplitude of the neutron β^- decay, given by Eq. (70) and supplemented by next-to-leading order $1/m_N$ proton recoil corrections and contributions of the weak magnetism, might be used for the analysis of the neutron lifetime and correlation coefficients of the neutron β^- decays with different polarization states of the neutron and massive decay fermions to order 10^{-5}. We are planning to carry out such an analysis in our forthcoming publications.

The $O(\alpha E_e/m_N)$ corrections, defined by the function $\bar{h}_n(E_e e)$ [see Eq. (77)], to Sirlin's function $\bar{g}_n(E_e)$ is plotted in Fig. 8 in the electron-energy region $m_e < E_e < E_0$. The order of the $O(\alpha E_e/m_N)$ corrections is of about 10^{-5}. Unlike Sirlin's corrections $(\alpha/\pi)\bar{g}_n(E_e)$ of order $O(\alpha/\pi) \sim 10^{-3}$, which do not depend on the axial coupling constant g_A, the corrections of order $O(\alpha E_e/m_N) \sim 10^{-5}$ depend strongly on the axial coupling constant or on strong low-energy interactions. It is important to emphasize that the term $(\alpha/\pi)(5/2)(m_N^2/M_W^2)\ell n(M_W^2/m_N^2) \simeq 5 \times 10^{-6}$ does not depend on the electron energy E_e. Such a contribution comes from the Feynman diagrams in Figs. 6(b) and 6(c), which are important for gauge invariance of the one-photon-loop exchanges, and agrees well with Sirlin's assertion that the contribution of Feynman diagrams restoring gauge invariance of the Feynman diagrams with one-virtual photon exchanges, when a virtual photon emitted by the proton is hooked by the electron and self-energy proton and electron Feynman diagrams, do not depend on the electron energy. So one may assert that the radiative corrections of order $O(\alpha E_e/m_N) \sim 10^{-5}$ calculated as next-to-leading order corrections to Sirlin's radiative corrections of order $O(\alpha/\pi)$, are defined at Sirlin's confidence level of radiative corrections of order $O(\alpha/\pi)$. In addition the calculation of the radiative corrections of order $O(\alpha/\pi)$ being valid to any order in the large nucleon mass expansion and defined by a gauge invariant set of Feynman diagrams in Fig. 6 testifies that the contributions of one-virtual photon interactions with hadronic structure of the nucleon should be described by a set of Feynman diagrams, which are self-gauge invariant. The shape of radiative corrections of order $O(\alpha/\pi)$ as functions of the electron energy E_e, caused by one-virtual photon coupled to hadronic structure of the nucleon, is to some extent model-dependent and can be calculated within the quantum field theoretic model of strong low-energy and electroweak interactions defined by the Lagrangian Eq. (44). We would like also to notice that the radiative corrections of order $O(\alpha E_e/m_N)$ to the amplitude of the neutron β^- decay Eq. (71) can be also used for the calculation of radiative corrections of order 10^{-5} to the proton recoil distribution of the neutron β^- decay [26–31].

Thus, concluding our discussion of the radiative corrections of order $O(\alpha E_e/m_N)$ as next-to-leading order corrections in the large nucleon mass expansion to Sirlin's radiative corrections of order $O(\alpha/\pi)$, calculated to leading order in the large nucleon mass expansion, we may argue that there are else three problems, the analysis of which goes beyond the scope of this paper. They are (i) the radiative corrections to two-loop approximation with one-hadron- and one-photon-loop exchanges, the contributions of which do not reduce to the axial coupling constant, (ii) the contribution of the electroweak W- and Z-boson exchanges to one-virtual electroweak boson approximation and (iii) the radiative corrections to two-loop approximation with one-hadron- and one-electroweak-boson-loop exchanges.

The contributions of the electroweak W- and Z-boson exchanges are defined by more than 24 Feynman diagrams with intermediate W- and Z-boson virtual exchanges. Practically, they have been calculated by Sirlin with coworkers (see, for example, [32]). We have to show that such a result can be obtained in the quantum field theoretic model of strong low-energy and electroweak interactions, described by the Lagrangian Eq. (44). According to [32], the contribution of the electroweak W- and Z-boson exchanges do not depend on the electron energy E_e. Our analysis of the Feynman diagrams in Appendices B and C of the Supplemental Material [77], where we have shown that the contributions of Feynman diagrams with virtual electroweak W^--boson exchanges do not depend on the electron energy E_e, agrees well with independence of the electron energy the corrections caused by the electroweak

W- and Z-boson exchanges. So that the contribution of the Feynman diagrams with W- and Z-boson virtual exchanges should not add any corrections of order $O(\alpha E_e/m_N)$. As next-to-leading order corrections to the result obtained in [32] we may expect the corrections of order $(m_N^2/M_X^2)\ell n(M_X^2/m_N^2) \sim 10^{-5}$ for $X = W$ or Z, respectively. We are planning to take into account the contributions of the virtual electroweak W- and Z-boson exchanges in our forthcoming publication.

Then, the contributions of the two-loop Feynman diagrams with virtual hadron and photon exchanges, which cannot be reduced to the contribution of the axial coupling constant, and electroweak W- and Z-boson exchanges are defined by a huge number of Feynman diagrams which could in principle give some nontrivial but finite independent of the electron energy E_e contributions to the radiative corrections of order $O(\alpha/\pi)$ and, correspondingly, to next-to-leading order corrections in the large nucleon mass expansion. However, according to Sirlin's analysis [14,21], the corrections of hadronic structure of the nucleon to order $O(\alpha/\pi)$ and taken to leading in the large nucleon mass expansion do not depend on the electron energy E_e and can be removed by renormalization of the Fermi and axial coupling constants. Nevertheless, we are planning to carry out the investigation of the problem of contributions of hadronic structure of the nucleon coupled to one-virtual photon and virtual electroweak bosons, which cannot be reduced to the axial coupling constant g_A, in our forthcoming publications.

Finally we would like to notice that in the quantum field theoretic model of strong low-energy and electroweak interactions strong low-energy interactions are described by the LσM with a linear realization of chiral $SU(2) \times SU(2)$ symmetry. We calculate the contributions of strong low-energy interactions in the limit of the infinite mass of the scalar isoscalar σ meson. In such a limit the LσM is equivalent to the quantum field theory with a nonlinear realization of chiral $SU(2) \times SU(2)$ symmetry. In the exponential parametrization of the pion field the LσM reduces to ChPT (see [112]), which is accepted as a low-energy QCD [113,117]). Since the use of the LσM as a quantum field theoretic model of strong low-energy pion-nucleon interactions makes to some extent the results of our analysis of contributions of strong low-energy interactions to the neutron β^- decays model dependent, we are planning to reformulate the quantum field theoretic model of strong low-energy and electroweak interactions, presented by the Lagrangian Eq. (29) and, correspondingly, Eq. (44) with the sector of strong low-energy pion-nucleon interactions, described by ChPT with a nonlinear realization of chiral $SU(2) \times SU(2)$ symmetry. However, first of all we would like to investigate the problems mentioned above for subsequent investigations of an influence of strong low-energy interactions on gauge properties and renormalizability of radiative corrections to the neutron β^- decays in the quantum field theoretic model of strong low-energy and electroweak interactions with the sector of strong low-energy interactions described by the LσM. According to Weinberg's "theorem" [124] (see also subsection C in Sec. III), because of an equivalence of the LσM to the ChPT [112] (see also Sec. IV) the results obtained in the LσM and in the ChPT as quantum field theoretic models of strong low-energy interactions in the neutron β^- decays should be in principle the same. A comparison of the results, obtained within these to quantum field theoretic models of strong low-energy hadronic interactions with chiral $SU(2) \times SU(2)$ symmetry in the neutron β^- decays, should be to some extent a good verification of Weinberg's theorem, which is required by Weinberg [124].

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