Class $\mathcal{S}$ anomalies from M-theory inflow

Ibrahima Bah,[1] Federico Bonetti,[1] Ruben Minasian,[2] and Emily Nardoni[3]

[1]*Department of Physics and Astronomy, Johns Hopkins University, 3400 North Charles Street, Baltimore, Maryland 21218, USA*

[2]*Institut de Physique Théorique, Université Paris Saclay, CNRS, CEA, F-91191 Gif-sur-Yvette, France*

[3]*Mani L. Bhaumik Institute for Theoretical Physics, Department of Physics and Astronomy, University of California, Los Angeles, California 90095, USA*

(Received 15 February 2019; published 30 April 2019)

We present a first principles derivation of the anomaly polynomials of 4d $\mathcal{N} = 2$ class $\mathcal{S}$ theories of type A_{N-1} with arbitrary regular punctures, using anomaly inflow in the corresponding M-theory setup with N M5-branes wrapping a punctured Riemann surface. The labeling of punctures in our approach follows entirely from the analysis of the 11d geometry and G_4 flux. We highlight the applications of the inflow method to the AdS/CFT correspondence.

DOI: 10.1103/PhysRevD.99.086020

I. INTRODUCTION

't Hooft anomalies are measures of degrees of freedom of quantum systems that are preserved under renormalization group flow. Thus, anomalies provide powerful tools for exploring phases and nonperturbative regimes of quantum theories.

In the last ten years, a new approach to studying quantum field theories (QFTs) has emerged with the discovery of $\mathcal{N} = 2$ class $\mathcal{S}$ superconformal field theories (SCFTs) [1,2], where a large class of 4d $\mathcal{N} = 2$ SCFTs are geometrically defined from reductions of 6d (2,0) SCFTs on punctured Riemann surfaces. A choice of 6d SCFT and boundary data at the punctures completely specifies a 4d SCFT and its various protected sectors. A typical theory in this class is non-Lagrangian and strongly coupled, and yet it can be analyzed from the geometric construction. The approach of the class $\mathcal{S}$ program has been generalized and adopted for studying SCFTs in different dimensions with varying amount of supersymmetry. The geometrization program has become a standard tool in the study of QFTs.

A key feature of the class $\mathcal{S}$ program is the richness of the variety of punctures on the Riemann surface. The anomalies of $\mathcal{N} = 2$ class $\mathcal{S}$ SCFTs in the presence of regular punctures have been indirectly obtained from field theoretic arguments [3–5]. However, a direct derivation of the anomalies from the geometric definition of class $\mathcal{S}$ SCFTs is lacking. In this paper we use anomaly inflow in M-theory to provide a first principles derivation, building on [6]. Our procedure can be generalized to obtain the anomalies of other classes of SCFTs with geometric descriptions. Further, our prescription suggests a method for extracting the exact anomalies of a holographic SCFT from its gravity dual.

The 't Hooft anomalies of a d-dimensional QFT are neatly encoded in the $(d+2)$-form anomaly polynomial. In this paper we derive the anomaly polynomials of 4d $\mathcal{N} = 2$ class $\mathcal{S}$ SCFTs with regular punctures engineered from the 6d (2,0) A_{N-1} SCFTs. First, we describe the relevant geometric setup from a stack of N M5-branes in M-theory, and the inflow procedure. Then we provide a novel description of the boundary data at punctures in terms of the four-form flux of M-theory. Finally, we compute the anomaly polynomial and discuss its implications for holography. A companion paper [7] to this letter contains more complete derivations and a broader study of the results and their implications.

II. SETUP AND INFLOW

A 4d $\mathcal{N} = 2$ class $\mathcal{S}$ theory of type A_{N-1} is engineered in M-theory by taking the low-energy limit of a configuration with N coincident M5-branes wrapping a punctured Riemann surface. Let W_6 denote the 6d world volume of the M5-brane stack inside the ambient 11d space M_{11}. The normal bundle to W_6, denoted NW_6, encodes the five transverse directions to the stack and generically has structure group $SO(5)$. We study the case $W_6 = M_4 \times \Sigma_{g,n}$, where M_4 is external spacetime and $\Sigma_{g,n}$ is a Riemann surface of genus g with n punctures.

We are interested in setups that preserve 4d $\mathcal{N} = 2$ supersymmetry (for $M_4 = \mathbb{R}^{1,3}$). In this case, the structure group of NW_6 reduces from $SO(5)$ to $SO(2) \times SO(3)$, and

Published by the American Physical Society under the terms of the Creative Commons Attribution 4.0 International license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

correspondingly NW_6 decomposes as $NW_6 = N_{SO(3)} \oplus N_{SO(2)}$. The (universal cover) of $SO(2) \times SO(3)$ is identified with the $U(1)_r \times SU(2)_R$ R-symmetry of the 4d field theory. In summary, the tangent bundle to 11d spacetime restricted on W_6 decomposes as

$$TM_{11}|_{W_6} = TM_4 \oplus T\Sigma_{g,n} \oplus N_{SO(2)} \oplus N_{SO(3)}. \quad (1)$$

The total space of the $N_{SO(2)}$ fibration over $\Sigma_{g,n}$ is the cotangent bundle $T^*\Sigma_{g,n}$, and is hyper-Kähler. The twisting of $N_{SO(2)}$ over $\Sigma_{g,n}$ implements a partial topological twist of the 6d (2,0) A_{N-1} theory living on the stack. If $\hat{n}$ denotes the Chern root of $N_{SO(2)}$, then

$$\hat{n} = -\hat{t} + 2c_1^r, \qquad \int_{\Sigma_{g,n}} \hat{t} = \chi(\Sigma_{g,n}), \quad (2)$$

where c_1^r is the first Chern class of $U(1)_r$, $\hat{t}$ is the Chern root of $T\Sigma_{g,n}$, and $\chi(\Sigma_{g,n}) = 2(1-g) - n$ is the Euler characteristic of the punctured Riemann surface. In order to specify the 4d theory, we must supplement each puncture with appropriate data, encoding the boundary conditions for the 6d theory. The puncture data is determined by the branching pattern of the M5-branes which governs the flavor symmetry of the 4d theory.

From the point of view of M-theory, the combined system of the M5-brane stack and the 11d bulk enjoys a nonanomalous diffeomorphism invariance. The total system is free from local anomalies in 11d due to a cancellation between the anomaly generated by the chiral massless degrees of freedom localized on W_6, and anomaly inflow from the bulk.

The anomaly inflow from the bulk amounts to a classical anomalous variation of the M-theory effective action under 11d diffeomorphisms, due to the presence of the M5-brane stack. The latter acts as a magnetic source for the M-theory four-form G_4 with delta-function support on W_6, $dG_4 = 2\pi N \delta_{W_6}$. In order to analyze anomaly inflow in the supergravity approximation we must smooth out the delta-function singularity [8,9]. This is achieved by cutting out a small tubular neighborhood of the M5-brane stack. As a result, we are now considering M-theory on a manifold with a boundary $M_{10} = \partial M_{11}$, which is diffeomorphic to an S^4 bundle over W_6. The information about the original delta-function source is translated into a smoothed-out $\tilde{G}_4$ flux,

$$\frac{\tilde{G}_4}{2\pi} = \frac{dC_3}{2\pi} - df \wedge E_3^{(0)} - f E_4, \qquad \int_{S^4} E_4 = N. \quad (3)$$

The quantity f is a bump function that depends only on the radial distance away from the M5-brane stack, smoothly interpolating between -1 at the boundary M_{10} and 0 away from it. The four-form E_4 is globally-defined, closed, invariant under the action of the structure group of NW_6, and can be written locally as $E_4 = dE_3^{(0)}$. The integral of E_4 over the S^4 surrounding the stack measures the total magnetic charge N of the M5-branes.

The anomalous variation of the M-theory effective action is expressed as an integral over M_{10} and is conveniently formulated in the framework of descent, $\delta S = 2\pi \int_{M_{10}} \mathcal{I}_{10}^{(1)}$, $d\mathcal{I}_{10}^{(1)} = \delta \mathcal{I}_{11}^{(0)}$, $d\mathcal{I}_{11}^{(0)} = \mathcal{I}_{12}$. The formal quantity $\mathcal{I}_{12}$ is a twelve-form characteristic class constructed from E_4 and given by

$$\mathcal{I}_{12} = -\frac{1}{6}(E_4)^3 - E_4 I_8. \quad (4)$$

On the right-hand side we suppressed wedge products for brevity, and we introduced the eight-form class I_8, which is defined in terms of the Pontryagin classes of TM_{11} as

$$I_8 = \frac{1}{192}[p_1(TM_{11})^2 - 4p_2(TM_{11})]. \quad (5)$$

The inflow contribution to the anomaly polynomial of the 4d CFT is extracted by integrating $\mathcal{I}_{12}$ over the total space of the S^4 bundle over $\Sigma_{g,n}$, denoted M_6,

$$\mathcal{I}_6^{\rm inf} = \int_{M_6} \mathcal{I}_{12}, \qquad S^4 \hookrightarrow M_6 \to \Sigma_{g,n}. \quad (6)$$

Anomaly cancellation requires $\mathcal{I}_6^{\rm inf}$ to cancel against the CFT anomaly, up to decoupling modes, $\mathcal{I}_6^{\rm inf} + \mathcal{I}_6^{\rm CFT} + \mathcal{I}_6^{\rm decoup} = 0$.

To compute the integral in (6), we excise small disks around each puncture on $\Sigma_{g,n}$, together with the S^4 fibers on top of them. We thus obtain a space $\tilde{M}_6$, which is an S^4 fibration over a smooth Riemann surface with n boundaries. We replace the excised portions of M_6 with suitable local geometries X_6^α, with $\alpha = 1, \ldots, n$, glued smoothly to $\tilde{M}_6$. This decomposition of M_6 translates to

$$\begin{aligned} \mathcal{I}_6^{\rm inf} &= \int_{\tilde{M}_6} \mathcal{I}_{12} + \sum_{\alpha=1}^n \int_{X_6^\alpha} \mathcal{I}_{12} \\ &\equiv \mathcal{I}_6^{\rm inf}(\Sigma_{g,n}) + \sum_{\alpha=1}^n \mathcal{I}_6^{\rm inf}(P_\alpha), \end{aligned} \quad (7)$$

where P_α denotes the $\alpha^{\rm th}$ puncture on $\Sigma_{g,n}$. We refer to $\mathcal{I}_6^{\rm inf}(\Sigma_{g,n})$ as the bulk contribution to $\mathcal{I}_6^{\rm inf}$.

Each geometry X_6^α is locally $S_\Omega^2 \times X_4^\alpha$, where the S_Ω^2 encodes the angular directions of $N_{SO(3)}$, while X_4^α comprises the directions of the excised disk, together with the fibers of $N_{SO(2)}$ on top of it. More precisely, X_4^α is the local space that models $T^*\Sigma_{g,n}$ in the vicinity of the puncture P_α. Thus, the possible choices of X_4^α in M-theory encode the puncture data. The space X_4^α admits a $U(1)$ isometry, which is identified with the $U(1)$ action on $N_{SO(2)}$ in the bulk of $T^*\Sigma_{g,n}$.

III. BULK CONTRIBUTION TO INFLOW

To write E_4, we realize S^4 as an $S^1_\phi \times S^2_\Omega$ fibration over an interval with coordinate $\mu \in [0,1]$, with S^1_ϕ, S^2_Ω associated to $N_{SO(2)}$, $N_{SO(3)}$, respectively, see (1). At $\mu=0$, S^2_Ω shrinks, while at $\mu=1$, S^1_ϕ shrinks. The $N_{SO(2)}$ bundle is captured by $D\phi = d\phi - \mathcal{A}$, where $\mathcal{A}$ is a connection with field strength $d\mathcal{A} = 2\pi\hat{n}$, see (2). Using this notation, the general E_4 reads

$$E_4 = N\left[d\gamma \wedge \frac{D\phi}{2\pi} - \gamma\hat{n}\right] \wedge e_2^\Omega. \tag{8}$$

The function γ depends on μ only, satisfies $\gamma(0)=0$, $\gamma(1)=1$, and has no zeros within the interval (0,1), but is otherwise arbitrary. The two-form e_2^Ω is the closed, $SO(3)$-invariant completion of the volume form on S^2_Ω, normalized to integrate to 1. The overall normalization in (8) is fixed by (3).

The class I_8 on $\tilde{M}_6$ is obtained via the decomposition of $p_1(TM_{11})$, $p_2(TM_{11})$ under (1), using standard formulas for Pontryagin classes of direct sums of vector bundles. Notice that $p_1(T\Sigma_{g,n}) = \hat{t}^2$, $p_1(N_{SO(2)}) = \hat{n}^2$, while $p_1(N_{SO(3)}) = -4c_2^R$, where c_2^R is the second Chern class of $SU(2)_R$. The only terms in I_8 that can contribute to the integral over $\tilde{M}_6$ are those linear in $\hat{t}$,

$$I_8 = \frac{1}{48}\hat{t}c_1^r[4(c_1^r)^2 + 4c_2^R - p_1(TM_4)] + \cdots \tag{9}$$

We are now in a position to compute the integral of $\mathcal{I}_{12}$ over $\tilde{M}_6$. To this end, it is useful to recall the Bott-Cattaneo formula [10] $\int_{S^2_\Omega}(e_2^\Omega)^3 = -c_2^R$. The result reads

$$\mathcal{I}_6^{\rm inf}(\Sigma_{g,n}) = \frac{1}{2}N\chi(\Sigma_{g,n})\left[\frac{(c_1^r)^3}{3} - \frac{c_1^r p_1(TM_4)}{12}\right] - \frac{1}{6}(4N^3 - N)\chi(\Sigma_{g,n})c_1^r c_2^R. \tag{10}$$

The quantity $\mathcal{I}_6^{\rm inf}(\Sigma_{g,n})$ coincides with the dimensional reduction along $\Sigma_{g,n}$ of the inflow eight-form anomaly polynomial for a stack of M5-branes [6].

IV. PUNCTURE GEOMETRY AND FLUX

To introduce the αth puncture, we excise a portion of M_6 of the form $D_\alpha \times S^4$, where D_α is a small disk centered at P_α with polar angle β. We replace $D_\alpha \times S^4$ with a space X_6^α, which admits an $SO(3)\times U(1)^2$ isometry inherited from $S^2_\Omega \times S^1_\phi \subset S^4$ and $S^1_\beta \subset D_\alpha$.

The space X_6^α is given as a fibration of S^2_Ω over a 4d space X_4^α, which is modeled by an S^1_β fibration over $\mathbb{R}^3$. We use cylindrical coordinates (ρ,η,χ) on $\mathbb{R}^3$, with η the axial coordinate, ρ the radial coordinate, and χ the azimuthal angle, related to ϕ,β by $\chi = \phi+\beta$. The circle S^1_χ shrinks along the η axis *in the base space* $\mathbb{R}^3$, while S^2_Ω shrinks at $\eta=0$.

The S^1_β fibration admits monopole sources located along the η axis at $\eta=\eta_a$, $a=1,\ldots,p$, at which S^1_β shrinks. The space X_4^α corresponds to a small region that surrounds the interval $[0,\eta_p]$ on the η axis. The S^1_β fibration is captured by

$$D\beta = d\beta - Ld\chi, \qquad S^1_\beta \hookrightarrow X_4^\alpha \to \mathbb{R}^3. \tag{11}$$

L is a function of ρ,η that approaches a piecewise constant function of η for $\rho\to 0$. Denote the piecewise constant values of L by

$$L = \ell_a \quad \text{for } \eta_{a-1} < \eta < \eta_a; \qquad \ell_{p+1} = 0. \tag{12}$$

The charge k_a of each monopole is measured by

$$\int_{S^2_a}\frac{dD\beta}{2\pi} \equiv k_a = \ell_a - \ell_{a+1} \in \mathbb{Z}, \tag{13}$$

for S^2_a the 2-sphere surrounding the monopole in $\mathbb{R}^3$.

Since the space X_4^α is a local model for $T^*\Sigma_{g,n}$ in the neighborhood of the puncture P_α, its geometry is constrained. In particular, $k_a>0$ for all a, so that the ℓ_a are a sequence of decreasing integers. Furthermore, the local geometry near each monopole is an ALF hyper-Kähler space, modeled by a single-center Taub-NUT space with charge k_a, denoted TN_{k_a}. This space has an $\mathbb{R}^4/\mathbb{Z}_{k_a}$ orbifold singularity which can be resolved to yield a smooth hyper-Kähler space $\widetilde{\mathrm{TN}}_{k_a}$.

Now we discuss E_4 in the geometry X_6^α. The most general form of E_4 compatible with the symmetries is

$$E_4 = d(YD\chi - W\widetilde{D\beta}) \wedge e_2^\Omega + E_4^{\rm fl}, \quad D\chi \equiv d\chi - \mathcal{A}, \tag{14}$$

where the gauging of χ with the connection $\mathcal{A}$ is inherited from ϕ, $\widetilde{D\beta}$ denotes $D\beta$ as in (11) with $d\chi \to D\chi$, and $E_4^{\rm fl}$ is a flavor contribution discussed below. The field strength $d\mathcal{A}$ in the puncture region only receives contributions from the term $2c_1^r$ in (2). The quantities Y, W are functions of ρ,η and are constrained by flux quantization of E_4. They vanish at $\eta=0$, where S^2_Ω shrinks.

We start by defining the relevant cycles. There is a four-cycle $\mathcal{B}_a$ for $a=1,\ldots,p$, consisting of the interval $[\eta_{a-1},\eta_a]$ at $\rho=0$, S^1_β, and S^2_Ω. For $a\geq 2$, S^1_β shrinks at the endpoints of $[\eta_{a-1},\eta_a]$ and thus we also have a two-cycle $\mathcal{S}_a$, depicted in Fig. 1.

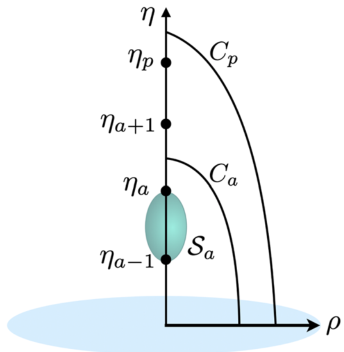

FIG. 1. A generic profile of monopoles. The C_a arcs form part of the four-cycle $\mathcal{C}_a$. The bubble denotes the two-cycle $\mathcal{S}_a$, which is part of the four-cycle $\mathcal{B}_a$.

Next, consider the arc C_a connecting a point on the ρ axis to a point within the (η_a, η_{a+1}) interval, with $a = 1, \ldots, p-1$, as depicted in Fig. 1. The arc C_a, together with S^2_Ω and the combination of S^1_χ and S^1_β that shrinks along (η_a, η_{a+1}), gives the four-cycle $\mathcal{C}_a$. The arc C_p in Fig. 1, combined with S^1_ϕ and S^2_Ω, gives a four-cycle $\mathcal{C}_p$ that is equivalent to the bulk S^4.

Supersymmetry requires the flux of E_4 through the $\mathcal{C}_a$ and $\mathcal{B}_a$ cycles to respectively carry the same sign. We choose the orientations such that $\int_{\mathcal{B}_a} E_4$ and $\int_{\mathcal{C}_a} E_4$ are positive, and we find

$$\int_{\mathcal{B}_a} E_4 = W(0, \eta_a) - W(0, \eta_{a-1}) \equiv w_a - w_{a-1}, \tag{15}$$

such that $w_0 = 0$ and $\{w_a\}_{a=1}^p$ is an increasing sequence of positive integers.

The flux $\int_{\mathcal{C}_a} E_4$ equals Y evaluated at the endpoint of the C_a arc on the η axis. Since the endpoint can be freely moved within (η_a, η_{a+1}), Y is piecewise constant along the η axis, and takes non-negative integer values,

$$Y(0,\eta) = y_a \in \mathbb{Z}_{\geq 0} \quad \text{for } \eta_a < \eta < \eta_{a+1}. \tag{16}$$

Although Y is discontinuous along the η axis, E_4 must be continuous. This condition gives $y_a - y_{a-1} = w_a k_a$,

$$y_a = \sum_{b=1}^{a} w_b k_b, \qquad N = \sum_{a=1}^{p} w_a k_a, \tag{17}$$

where $y_0 = 0$ and we used $\mathcal{C}_p \cong S^4$. Continuity of E_4 thus implies the partition of N labeling a regular puncture.

For each nontrivial two-cycle in X_6^α, we can turn on an additional contribution to E_4 of the form $\omega \wedge F$, for ω the Poincaré dual of the two-cycle and F the field strength of a background $U(1)$ connection on M_4. One such two-cycle is $\mathcal{S}_a$ depicted in Fig. 1, with Poincaré dual denoted ω_a. Additional two-cycles are introduced upon resolving the orbifold singularities at the monopoles. The resolved space $\widetilde{\mathrm{TN}}_{k_a}$ admits $k_a - 1$ two-cycles, with Poincaré duals $\{\hat{\omega}_{a,I}\}_{I=1}^{k_a-1}$. Their intersection pairings give the Cartan matrix $C^{\mathfrak{su}(k_a)}$ of $\mathfrak{su}(k_a)$,

$$\int_{\widetilde{\mathrm{TN}}_{k_a}} \hat{\omega}_{a,I} \wedge \hat{\omega}_{a,J} = -C^{\mathfrak{su}(k_a)}_{IJ}. \tag{18}$$

The flavor terms in E_4 are thus

$$E_4^{\mathrm{fl}} = \sum_{a=2}^{p} \omega_a \wedge \frac{F_a}{2\pi} + \sum_{a=1}^{p} \sum_{I=1}^{k_a-1} \hat{\omega}_{a,I} \wedge \frac{\hat{F}_{a,I}}{2\pi}, \tag{19}$$

where F_a and $\hat{F}_{a,I}$ are 4d field strengths. Equation (19) only captures the Cartan subgroup of the full 4d flavor group $G_F = S[\prod_{a=1}^p U(k_a)]$.

The class I_8 in the puncture geometry is computed using the decomposition $TM_{11} = TM_4 \oplus N_{SO(3)} \oplus TX_4^\alpha$. The Pontryagin classes of TX_4^α are given in terms of the Chern roots λ_1, λ_2 as $p_1(TX_4^\alpha) = \lambda_1^2 + \lambda_2^2$, $p_2(TX_4^\alpha) = \lambda_1^2\lambda_2^2$. To account for the gauging of the angle χ in (14), the Chern roots are shifted by c_1^r,

$$\lambda_1 \to \lambda_1 + c_1^r, \qquad \lambda_2 \to \lambda_2 + c_1^r. \tag{20}$$

The relevant terms of I_8 are

$$I_8 = \frac{1}{96}[4(c_1^r)^2 + 4c_2^R - p_1(TM_4)]p_1(TX_4^\alpha) + \cdots \tag{21}$$

where $p_1(TX_4^\alpha)$ is understood before the shift (20). The total $p_1(TX_4^\alpha)$ decomposes into a sum of $p_1(\widetilde{\mathrm{TN}}_{k_a})$ terms, which satisfy $\int_{\widetilde{\mathrm{TN}}_{k_a}} p_1(\widetilde{\mathrm{TN}}_{k_a}) = 2k_a$ [11].

V. CFT COMPARISON

We now have the necessary components to compute $\mathcal{I}_6^{\mathrm{inf}}(P_\alpha) = \int_{X_6^\alpha} \mathcal{I}_{12}$ in (7). We use the standard parametrization of $\mathcal{I}_6$ for 4d $\mathcal{N} = 2$ SCFTs

$$\begin{aligned} \mathcal{I}_6 = (n_v - n_h)\left[\frac{(c_1^r)^3}{3} - \frac{c_1^r p_1(TM_4)}{12}\right] \\ - n_v c_1^r c_2^R + \sum_G k_G c_1^r c_2(G), \end{aligned} \tag{22}$$

where n_v and n_h are the effective numbers of vector multiplets and hypermultiplets respectively, and k_G is the flavor central charge of a factor G of the 4d flavor group.

A direct computation of the integrals yields

$$(n_v - n_h)^{\mathrm{inf}}(P_\alpha) = \frac{1}{2} \sum_{a=1}^{p} N_a k_a, \tag{23}$$

$$n_v^{\rm inf}(P_\alpha) = \sum_{a=1}^{p} \left[\frac{2}{3} \ell_a^2 (w_a^3 - w_{a-1}^3) - \frac{1}{6} N_a k_a + \ell_a (N_a - w_a \ell_a)(w_a^2 - w_{a-1}^2) \right], \qquad (24)$$

$$k_{SU(k_a)}^{\rm inf} = -2N_a, \qquad N_a \equiv \sum_{b=1}^{a} (w_b - w_{b-1}) \ell_b. \qquad (25)$$

Note that there is an enhancement of the $k_a - 1$ Cartan components to the second Chern class of the full non-Abelian $SU(k_a)$ factor in G_F.

The partition of N in (17) defines a Young diagram with rows $\{\tilde{\ell}_i\}_{i=1}^{w_p}$, where $\tilde{\ell}_i = \ell_a$ for $w_{a-1} + 1 \leq i \leq w_a$. We define $\tilde{k}_i = \tilde{\ell}_i - \tilde{\ell}_{i+1}$ and $\tilde{N}_i = \sum_{j=1}^{i} \tilde{\ell}_j$. It follows that (23)–(24) are equivalently written as

$$(n_v - n_h)^{\rm inf}(P_\alpha) = \frac{1}{2} \sum_{i=1}^{w_p} \tilde{N}_i \tilde{k}_i, \qquad (26)$$

$$n_v^{\rm inf}(P_\alpha) = \sum_{i=1}^{w_p} (N^2 - \tilde{N}_i^2) + \frac{1}{2} N^2. \qquad (27)$$

We can also read off $n_{v,h}^{\rm inf}(\Sigma_{g,n})$ from (10),

$$(n_v - n_h)^{\rm inf}(\Sigma_{g,n}) = \frac{1}{2} N \chi(\Sigma_{g,n}), \qquad (28)$$

$$n_h^{\rm inf}(\Sigma_{g,n}) = \frac{1}{6}(4N^3 - N)\chi(\Sigma_{g,n}). \qquad (29)$$

According to (7), the total $n_v^{\rm inf}$, $n_h^{\rm inf}$ are

$$n_{v,h}^{\rm inf} = n_{v,h}^{\rm inf}(\Sigma_{g,n}) + \sum_{\alpha=1}^{n} n_{v,h}^{\rm inf}(P_\alpha). \qquad (30)$$

These quantities can now be compared to the known CFT answers [4], as presented in [6]. We find

$$n_v^{\rm inf} + n_v^{\rm CFT} = \frac{1}{2}\chi(\Sigma_{g,0}), \qquad n_h^{\rm inf} + n_h^{\rm CFT} = 0, \qquad (31)$$

$$k_{SU(k_a)}^{\rm inf} + k_{SU(k_a)}^{\rm CFT} = 0. \qquad (32)$$

The inflow and CFT contributions cancel, up to minus the anomaly of a free 6d (2, 0) tensor multiplet reduced on a genus-g Riemann surface $\Sigma_{g,0}$ *with no punctures*. We identify this free tensor multiplet with the center-of-mass mode of the M5-brane stack. Our results show that this mode is insensitive to the presence of punctures.

VI. CONCLUSION AND APPLICATIONS TO HOLOGRAPHY

In this paper we provided a first principles derivation of the anomaly polynomials of 4d $\mathcal{N}=2$ A_{N-1} class $\mathcal{S}$ theories with arbitrary regular punctures, using anomaly inflow in the corresponding M-theory setup with N M5-branes wrapping a punctured Riemann surface.

In our approach, the puncture data are entirely specified by the topological properties of the 11d geometry and G_4 flux in the vicinity of the puncture. Remarkably, the anomaly inflow cancels exactly the known anomalies of the 4d SCFTs, up to the contribution of the center-of-mass free tensor multiplet on the M5-brane stack.

Our method for analyzing $\mathcal{N}=2$ regular punctures is generalizable to irregular punctures and setups with less supersymmetry. Many interesting QFTs can be realized via branes probing geometries in string theory and M-theory. In such cases, inflow can be a robust tool to compute anomalies, and therefore provides a handle on nonperturbative aspects of these QFTs.

We conclude with a discussion of applications to holography. An important motivation for our analysis of the local puncture geometry and E_4 flux comes from the holographic M-theory duals of $\mathcal{N}=2$ and $\mathcal{N}=1$ class $\mathcal{S}$ theories with punctures [3,12]. In particular, the fibration in (11) is related to and inspired by the Bäcklund transform of [3]. The solutions are warped products of AdS_5 with an internal space $M_6^{\rm hol}$ with four-form flux $G_4^{\rm hol}$.

We observe that the topological properties of $M_6^{\rm hol}$ in [3] are the same as those of M_6 in (6). Furthermore,

$$\frac{G_4^{\rm hol}}{2\pi} = \bar{E}_4 \quad \text{in cohomology}, \qquad (33)$$

where $\bar{E}_4$ is E_4 with all 4d connections turned off and $G_4^{\rm hol}$ is the four-form flux of [3]. In the bulk of $\Sigma_{g,n}$ $\bar{E}_4 = S^4$, but $\bar{E}_4$ is nontrivial in the puncture geometry and encodes the puncture labelling.

Kaluza-Klein reduction of 11d supergravity on $M_6^{\rm hol}$ yields a 5d gauged supergravity model with an AdS_5 vacuum. The full reduction ansatz requires a $G_4^{\rm hol}$ that captures the fluctuations of the AdS_5 gauge fields beyond the linearized level. E_4 is a natural candidate for constructing such an ansatz [9].

In the solutions of [3] the *classical* objects $M_6^{\rm hol}$, $G_4^{\rm hol}$ provide the *exact* topological data of M_6, $\bar{E}_4$ to all orders in N. This data determines the E_4 and I_8 needed to carry out the inflow procedure, which (subtracting the $\mathcal{O}(1)$ contribution of decoupling modes) yields the exact anomaly coefficients of the dual SCFT. This route to the exact a and c central charges bypasses a computation with the AdS_5 effective action, which would require a detailed knowledge of higher-derivative corrections.

An interesting question is whether (33) extends to more general AdS_5 solutions in M-theory, with varying amount of supersymmetry. If so, we may use inflow and classical data of the supergravity solution to access exact anomaly coefficients, providing a systematic way to compute quantum corrections in AdS_5.

ACKNOWLEDGMENTS

We are grateful to J. Distler, T. Dumitrescu, S. Giacomelli, K. Intriligator, J. Kaplan, C. Lawrie, G. Moore, S. Schäfer-Nameki, J. Song, Y. Tachikawa, A. Tomasiello, and Y. Wang for interesting conversations and correspondence. The work of E. N. is supported in part by DOE Grant No. DE-SC0009919, and a UC President's Dissertation Fellowship. The work of I. B. and F. B. is supported in part by NSF Grant No. PHY-1820784. Part of this work was performed at the Aspen Center for Physics, which is supported by NSF Grant No. PHY-1607611.

[1] D. Gaiotto, J. High Energy Phys. 08 (2012) 034.
[2] D. Gaiotto, G. W. Moore, and A. Neitzke, arXiv:0907.3987.
[3] D. Gaiotto and J. Maldacena, J. High Energy Phys. 10 (2012) 189.
[4] O. Chacaltana, J. Distler, and Y. Tachikawa, Int. J. Mod. Phys. A **28**, 1340006 (2013).
[5] Y. Tachikawa, Prog. Theor. Exp. Phys. **2015**, 11B102 (2015).
[6] I. Bah and E. Nardoni, J. High Energy Phys. 03 (2019) 024.
[7] I. Bah, F. Bonetti, R. Minasian, and E. Nardoni (to be published).
[8] D. Freed, J. A. Harvey, R. Minasian, and G. W. Moore, Adv. Theor. Math. Phys. **2**, 601 (1998).
[9] J. A. Harvey, R. Minasian, and G. W. Moore, J. High Energy Phys. 09 (1998) 004.
[10] R. Bott and A. S. Cattaneo, J. Diff. Geom. **48**, 91 (1998).
[11] G. W. Gibbons, C. N. Pope, and H. Romer, Nucl. Phys. **B157**, 377 (1979).
[12] I. Bah, J. High Energy Phys. 09 (2015) 163.