

## Higgs scalar potential in asymptotically safe quantum gravity

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The effect of gravitational fluctuations on the quantum effective potential for scalar fields is a key ingredient for predictions of the mass of the Higgs boson, understanding the gauge hierarchy problem, and a possible explanation of an—asymptotically—vanishing cosmological constant. We find that the quartic self-interaction of the Higgs scalar field is an irrelevant coupling at the asymptotically safe ultraviolet fixed point of quantum gravity. This renders the ratio between the masses of the Higgs boson and top quark predictable. If the flow of couplings below the Planck scale is approximated by the Standard Model, this prediction is consistent with the observed value. The quadratic term in the Higgs potential is irrelevant if the strength of gravity at short distances exceeds a bound that is determined here as a function of the particle content. In this event, a tiny value of the ratio between the Fermi scale and the Planck scale is predicted.

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### I. INTRODUCTION

The quantum effective potential for the Higgs field is the central quantity for understanding the electroweak symmetry breaking in the Standard Model of particle physics (SM). The vacuum expectation value of the Higgs field  $\phi_0$  is determined by the location of the minimum of the potential. It defines the Fermi scale. For given gauge couplings and Yukawa couplings, it sets the mass of  $W$  and  $Z$  bosons as well as of quarks and charged leptons. In turn, the vacuum expectation value depends on two renormalizable couplings, the mass parameter  $m_H^2$  and the quartic scalar coupling  $\lambda_H$ . The observable mass of Higgs boson obeys  $M_H = \sqrt{2\lambda_H}|\phi_0|$ .

The renormalizable couplings of the SM can be extrapolated to momenta much larger than the Fermi scale. In renormalization group (RG) improved perturbation theory, their running is computed with an expansion in loops. Let us now assume that the SM is part of an “effective low-energy theory” model for scales below some transition scale  $k_t$  where gravitational fluctuations decouple. Typically,  $k_t$  is close to the Planck mass. In the absence of gravitational fluctuations for momenta smaller than  $k_t$ , all couplings are small and in the perturbative regime. For a

given model, the “initial values” of  $m_H^2(k_t)$  and  $\lambda_H(k_t)$  can be extrapolated perturbatively to momenta of the order of the Fermi scale where they determine the observable quantities.

For possible predictions of the Fermi scale and the mass of the Higgs boson, the decisive question is the predictability of  $m_H^2(k_t)$  and  $\lambda_H(k_t)$ . For this issue, gravitational fluctuations become important. For the flow of couplings at momenta larger than  $k_t$ , the gravitational fluctuations strongly influence the running of  $m_H^2$  and  $\lambda_H$ . It has been argued that the gravitational fluctuations drive  $\lambda_H$  to a fixed-point value close to zero, such that  $\lambda_H(k_t)$  has a tiny value. The extrapolation to low momenta within the SM as effective low-energy theory has predicted [1] the mass of the Higgs particle in accordance with later observation.

We aim here for a systematic investigation of the effects of gravitational fluctuations on the shape of the effective scalar potential. Beyond the Higgs sector of the SM, this is relevant for other theories with scalars, such as grand unified theories. For cosmology, gravitational fluctuations play an important role for the shape of scalar potentials responsible for the inflationary epoch or dynamical dark energy. Since gravity is not perturbatively renormalizable, any investigation of the role of gravitational fluctuations at momentum scales larger than  $k_t$  has to employ some suitable nonperturbative method.

In the present work, we use the functional renormalization group (FRG) for the effective average action [2]. The FRG has proven to be a successful nonperturbative method for various systems in both condensed matter and

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elementary particle physics. A central object within its formulation is the scale-dependent quantum effective action or effective average action  $\Gamma_k$ , which includes all effects of quantum fluctuations with momenta larger than an IR cutoff  $k$ . The scale dependence of  $\Gamma_k$  obeys an exact flow equation [2].

The FRG is capable of understanding quantitatively asymptotically safe renormalizable quantum field theories, which is crucial for studying gravitational fluctuations near and beyond the Planck scale. For asymptotically safe theories, the interactions do not vanish at the UV fixed point, such that perturbative renormalizability is often not given. Well-studied examples for nonperturbative asymptotic safety are the Wilson-Fischer fixed point for three-dimensional scalar theories or four-dimensional theories with four-fermion interactions, for which FRG has proven quantitative reliability. Quantum gravity presumably belongs to this class of asymptotically safe theories. For the quantitative study of this work, we assume asymptotic safety [3,4] as a working hypothesis, leaving the fixed-point value of the dimensionless Planck mass as a not yet fully quantitatively determined parameter. This is sufficient to obtain rather robust results for the effect of gravitational fluctuations on the effective scalar potential.

The asymptotic-safety hypothesis for quantum gravity has found support by many investigations [4–12]. It is crucial for this scenario that the system has a nontrivial UV fixed point, the Reuter fixed point, at which the UV complete action is defined. Using the FRG, the existence of such a fixed point has been investigated in pure gravity as well as for gravity coupled to elementary particles. The methods of approximations to the functional flow equation include the background field approximation [4,13–39], the vertex expansion [40–53], the geometrical approach [54–56], and the bimetric method [57,58]. Also, a gauge invariant flow equation for quantum gravity has been proposed [59].

Quantum gravity coupled to elementary particles reveals a new predictive power for particle properties. This is connected to the number of relevant parameters at the fixed point, which may be smaller than the number of renormalizable couplings in the SM. This entails that certain relations among the SM couplings become, in principle, computable. Initial values of running couplings become fixed at the Planck scale if they correspond to irrelevant parameters at the fixed point. This allows a computation of observable quantities such as the Higgs-boson mass and the top-quark mass in the low-energy regime [1,60–63]. The fixed-point structure and the RG flow could also determine the potential of scalar fields of which the time evolution characterizes the history of our Universe [64–67].

In this paper, we investigate quantum gravity effects on the effective scalar potential in asymptotically safe gravity. Taking proper account of gauge symmetries, in our case diffeomorphism symmetry, is crucial for quantitative

reliability. For this purpose, we concentrate on a “physical gauge fixing,” which purely acts on the gauge modes among the metric fluctuations, leaving the physical fluctuations untouched [59]. We employ the physical metric decomposition [68], where the metric fluctuations are split into physical modes consisting of the traceless-transverse tensor (graviton) and a scalar, and the gauge modes, which comprise a transverse vector and a scalar. Employing the physical gauge fixing, the two-point function becomes block diagonal in the physical and gauge modes. A simple relation between ghost and gauge modes allows us to combine their contributions to a universal measure factor, which does not depend on the value of the scalar field [59].

Our paper can also be seen as a first application of the gauge invariant flow equation employing only one macroscopic metric field [59]. In fact, at the level of truncation employed here, there is no difference between the background formalism with physical gauge fixing and the gauge invariant flow equation. The proposed universal measure contribution [59] comes out directly in our approximation for the background formalism. The flow equation for the effective potential is the same for the truncated background formalism and the gauge invariant flow equation. The contribution to the gauge invariant flow equation from physical fluctuations involves formally nonlocal projections. This projection is implicitly performed in the background field formalism by the inversion of the second functional derivative of the effective action in the presence of the physical gauge fixing term. The relevant projected differential operators for the graviton and the physical scalar metric fluctuations are second-order differential operators. No nonlocality is encountered explicitly.

The propagator and interactions for the physical modes are derived here from a gauge invariant effective action. In the background formalism, this is an approximation, while for the gauge invariant flow equation, this is a genuine property. We also compute the flow of the mass term and quartic coupling by taking derivatives of the flow equation for the effective potential. In the truncated background formalism, this is an approximation, while for the gauge invariant flow equation, this is an exact property. Here, we do not enter into the discussion of whether the employed one-loop form of the gauge invariant flow equation is itself an approximation or whether it can be made exact by a suitable definition of the macroscopic field [59].

Our main results for the effects of gravitational fluctuations on the scalar effective potential are the following:

- (i) A UV fixed point for the cosmological constant (value of scalar potential at its minimum) exists, provided the dimensionless squared Planck mass  $\tilde{M}_p^2 = M_p^2/k^2$  is above a minimal value  $\tilde{M}_{p,c}^2$ . This value depends on the number  $N$  of particle degrees of freedom (d.o.f.), as shown in Fig. 1.
- (ii) The quartic scalar coupling  $\lambda_H$  of the Higgs boson is an irrelevant coupling at the UV fixed point. For

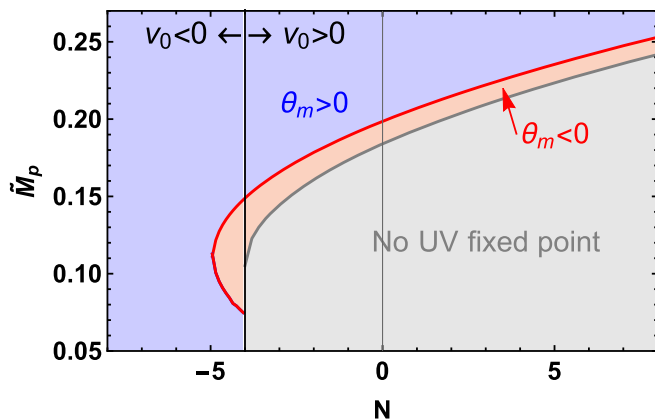


FIG. 1. Existence of the UV fixed point and sign of the critical exponent of the scalar mass term, as a function of the fixed-point value  $\tilde{M}_p$  for the running dimensionless Planck mass and the number of effective particle d.o.f.  $N = N_S + 2N_V - 2N_F$ . The grey line is the critical value of the Planck mass  $\tilde{M}_{p,c}$ . In the red-colored region, the critical exponent of the scalar mass term becomes negative, i.e., this coupling is irrelevant, while it is relevant in the blue-colored region. The quartic scalar coupling is irrelevant whenever a fixed point exists.

large  $k^2$ , the gravitational fluctuations drive it very close to zero, enforcing an initial value for the low energy effective theory  $\lambda_H(k_t) \approx 0$ . For a given low-energy theory at momentum scales below the Planck mass and a given observed mass of the top quark, the mass of the Higgs boson  $M_H$  becomes predictable. If the low-energy theory is the SM, the predicted value [1] is  $M_H = 126$  GeV with a few giga-electron-volts uncertainty. For a higher-loop computation and dependence on the top mass, see Refs. [69,70].

- (iii) For small enough  $\tilde{M}_p^2$  (red region in Fig. 1), the quadratic term in the scalar potential is also an irrelevant parameter. Then, the model is predicted to be located on the critical phase-transition surface of the vacuum electroweak phase transition, realizing self-organized criticality. The gauge hierarchy [71,72] of a tiny ratio between the Fermi scale and Planck scale could then be explained by the resurgence mechanism [73].

First indications that a quartic scalar coupling could be an irrelevant parameter can be found in Refs. [13,14,17,18,74,75]. At the time of the prediction [1] of the Higgs-boson mass, important uncertainties about the sign and magnitude of the anomalous dimension for the quartic coupling of the Higgs scalar persisted, however. Emphasis on the dominant role of the graviton fluctuations (“graviton approximation”) has shown [76] the positive sign of the anomalous dimension  $A$  and estimated its magnitude to be of the order 1. These are precisely the requirements for the prediction of the mass of the Higgs boson [1]. In the present paper, we confirm the graviton domination by an explicit computation of the contribution

of all other fluctuations, including the universal measure term for a physical gauge fixing. This allows for a quantitative comparison with the dominant graviton contribution. We also show that extensions of the truncation do not alter the main conclusion that quartic scalar couplings are irrelevant parameters at the UV fixed point for asymptotic safety.

The quantitative precision of the present approach allows for the first time an estimate under which circumstances the scalar mass term can be an irrelevant coupling [73]. Typically, this occurs for  $A > 2$ . The size of  $A$  depends strongly on the fixed-point value of the dimensionless Planck mass. This value depends, in turn, on the precise particle content of the model and requires a computation of the flow equation with similar detail as the one for the effective potential investigated here. Only once this task is accomplished, a definite statement on the predictive power of asymptotic safety for the gauge hierarchy will be possible.

This paper is organized as follows. In the next section, we present the flow equation for the scalar potential. The technical aspects are given in the Appendix. In Sec. III, we analyze the fixed-point structure and the critical exponent for the cosmological constant. Section IV investigates the critical exponents for the scalar mass term and the quartic interaction of the scalar field. We address here the predictive power of quantum gravity for properties of the Higgs scalar. Section V discusses the robustness of our results by extending the truncation and varying the cutoff function. Section VI is devoted to summarizing our results and to discussing their robustness and remaining quantitative uncertainty.

## II. FLOW OF THE SCALAR POTENTIAL

The flow equation for the effective scalar potential is extracted from the exact flow equation for the effective average action by taking space- and time-independent field values for the scalar field configuration. The flow is evaluated for a flat spacetime geometry that we take here to be Euclidean. The crucial quantity for the flow equation is the inverse propagator, which is given by the matrix of second functional derivatives  $\Gamma_k^{(2)}$  of the effective action, evaluated for the given scalar and metric fluctuation. Precision and robustness of results depend on the validity of the approximations used for  $\Gamma_k^{(2)}$ . Gravity is a local gauge theory, with gauge transformations associated to diffeomorphism or general coordinate transformations. It is a crucial issue to take the gauge symmetry properly into account. The fluctuations around any given metric configuration can be split into gauge fluctuations or gauge modes and physical fluctuations or modes. The gauge modes correspond to the infinitesimal changes of the given metric induced by an infinitesimal gauge transformation.

We follow the standard treatment of functional integrals for gauge theories, implementing gauge fixing and the

associated Faddeev-Popov determinant. We impose a particular physical gauge fixing [59]. A physical gauge fixing acts only on the gauge modes. Choosing a decomposition of the metric fluctuations into physical modes and gauge modes [68], the physical gauge-fixing term renders  $\Gamma_k^{(2)}$  effectively block diagonal, with separate blocks for the physical modes and the gauge modes. Furthermore, imposing the physical gauge constraint on the fields in the effective action leaves a gauge invariant effective action [59]. We can therefore employ an ansatz where the effective action consists of a gauge invariant part  $\bar{\Gamma}_k$  plus a gauge-fixing part. For the inverse propagator  $\Gamma_k^{(2)}$ , the block for the physical modes is given by the second functional derivative of  $\bar{\Gamma}_k$ . This is an important advantage, since gauge symmetry severely restricts the form of  $\bar{\Gamma}_k$ .

We find that the contribution of the gauge modes, together with the contribution from the Faddeev-Popov determinant or the corresponding ghosts, results in a simple universal contribution to the flow equation. This ‘‘measure contribution’’ depends on the metric but not on the values of scalar fields. For the flow of the effective potential, it only concerns an overall constant but not the field dependence.

What remains to be done is an effective approximation for the physical inverse propagator  $\bar{\Gamma}_k^{(2)}$ . This is done by making an ansatz for the gauge invariant effective action  $\bar{\Gamma}_k$ . We approximate the gravitational part of  $\bar{\Gamma}_k$  by the Einstein-Hilbert action, with coefficient of the curvature scalar given by the running or scale-dependent squared Planck mass  $M_p^2(k)$ . The cosmological constant is included as part of the effective scalar potential, namely, its value at the minimum. We discuss in the conclusions how this ansatz can also incorporate effects of higher-derivative invariants in  $\bar{\Gamma}_k$ , as  $R^2$  or  $R_{\mu\nu}R^{\mu\nu}$ . This is done by an adaptation of the definition of  $M_p^2(k)$ .

According to our assumption of asymptotic safety, the running Planck mass has to scale at and near the UV fixed point proportional to  $k$ ,

$$M_p^2(k) = \tilde{M}_{p*}^2 k^2. \quad (1)$$

The fixed-point value  $\tilde{M}_{p*}^2$  depends on the particular model. For the purpose of this paper, we treat it as an unknown parameter. Some of the predictions depend on the precise value of this parameter, while others such as the quartic coupling  $\lambda_H$  being an irrelevant parameter are independent of the precise value.

We first consider a single real scalar field  $\phi$  coupled to gravity. The detailed steps of the computation along the lines sketched above are displayed in Appendix. We obtain for the flow of the effective potential  $U(\rho)$  at fixed  $\rho = \phi^2/2$  a differential equation with a rather simple form,

$$\partial_t U = k \partial_k U = \tilde{\pi}_2 + \tilde{\pi}_0 + \tilde{\eta}. \quad (2)$$

Here,  $\tilde{\pi}_2$  is the contribution of the graviton fluctuations corresponding to the traceless-transverse metric fluctuations; the term  $\tilde{\pi}_0$  combines the physical scalar fluctuations, both from  $\phi$  and the physical scalar mode in the metric fluctuations. Finally,  $\tilde{\eta}$  is the measure contribution. Employing a Litim-type cutoff function [77], the terms are given by

$$\begin{aligned} \tilde{\pi}_2 &= \frac{5}{24\pi^2} \left(1 - \frac{\eta_g}{8}\right) \frac{k^4}{1-v}, \\ \tilde{\pi}_0 &= \frac{1}{24\pi^4} \left[ \left(1 - \frac{\eta_g}{8}\right) (1 + \tilde{U}' + 2\tilde{\rho}\tilde{U}'') \right. \\ &\quad \left. + \frac{3}{4} \left(1 - \frac{\eta_\phi}{6}\right) \left(1 - \frac{v}{4}\right) \right] \\ &\quad \times \frac{k^4}{[(1-v/4)(1 + \tilde{U}' + 2\tilde{\rho}\tilde{U}'') + 3\tilde{\rho}\tilde{U}'^2/\tilde{M}_p^2]}, \\ \tilde{\eta} &= -\frac{k^4}{8\pi^2}. \end{aligned} \quad (3)$$

Here, we have defined the dimensionless quantities,

$$\tilde{U}(\tilde{\rho}) = U(\rho)/k^4, \quad \tilde{\rho} = Z_\phi \rho/k^2, \quad (4)$$

with  $Z_\phi$  the coefficient of the scalar kinetic term and primes denoting derivatives with respect to  $\tilde{\rho}$ . The dimensionless ratio

$$v(\rho) = \frac{2U(\rho)}{M_p^2 k^2} = \frac{2\tilde{U}(\rho)}{\tilde{M}_p^2} \quad (5)$$

depends on  $\rho$ . The poles at  $v = 1$  and  $v = 4$  correspond to tachyonic instabilities in the graviton and the scalar mode of metric fluctuation propagator, respectively. They are not reached by the flow. We furthermore define  $\eta_g = -\partial_t \ln \tilde{M}_p^2 = 2 - \partial_t \ln M_p^2$  and the anomalous dimension of the scalar field,  $\eta_\phi = -\partial_t \ln Z_\phi$ .

We will see that at the fixed point the minimum of  $U(\rho)$  occurs for  $\rho = 0$ . In the vicinity of this point, we can neglect the term  $3\tilde{\rho}\tilde{U}'^2/\tilde{M}_p^2$  in the denominator of (3), such that also the effect of scalar fluctuations becomes block diagonal,

$$\tilde{\pi}_0 \simeq \tilde{\pi}_{0,g} + \tilde{\pi}_{0,\phi}, \quad (6)$$

with

$$\begin{aligned} \tilde{\pi}_{0,g} &= \frac{1}{24\pi^2} \left(1 - \frac{\eta_g}{8}\right) \frac{k^4}{1-v/4}, \\ \tilde{\pi}_{0,\phi} &= \frac{1}{32\pi^2} \left(1 - \frac{\eta_\phi}{6}\right) \frac{k^4}{1 + \tilde{U}' + 2\tilde{\rho}\tilde{U}''}. \end{aligned} \quad (7)$$



The first contribution  $\tilde{\pi}_{0,g}$  arises from the scalar mode in the metric, while the second term  $\tilde{\pi}_{0,\phi}$  is the standard flow contribution from scalar fields in flat space [2].

The flow equation (2) holds at constant  $\rho$ . For a discussion of a fixed point and the behavior close to the fixed point, we have to translate to the flow at constant  $\tilde{\rho}$ . Furthermore, we are interested in the flow of the dimensionless scalar potential  $\tilde{U}(\tilde{\rho})$ . Generalizing to  $N$  real scalars with  $O(N)$  symmetry and employing the approximation (6), we obtain the beta function or flow generator for  $\tilde{U}$  as

$$\begin{aligned} \partial_t \tilde{U} = & -4\tilde{U} + (2 + \eta_\phi)\tilde{\rho}\tilde{U}' \\ & + \frac{1}{24\pi^2} \left(1 - \frac{\eta_g}{8}\right) \left[\frac{5}{1-v} + \frac{1}{1-v/4}\right] + \frac{\Delta N - 4}{32\pi^2} \\ & + \frac{1}{32\pi^2} \left(1 - \frac{\eta_\phi}{6}\right) \left[\frac{1}{1 + \tilde{U}' + 2\tilde{\rho}\tilde{U}''} + \frac{N-1}{1 + \tilde{U}'}\right]. \end{aligned} \quad (8)$$

The dependence of the gravitational contributions on  $\tilde{\rho}$  arises through the quantity  $v = v(\tilde{\rho})$ . Also, the dimensionless Planck mass enters in (8) only through  $v$ . The first two terms on the right-hand side of (8) are the canonical scaling of the effective potential.

We have extended the scalar sector to  $N$  scalars with  $SO(N)$  symmetry. For scalar theories with  $SO(N)$  symmetry, the term proportional to  $N-1$  arises from the fluctuations in the Goldstone directions. For the Higgs doublet, one has  $N=4$ . Furthermore, we have included in (8) the contribution of fluctuations beyond the gravitational d.o.f. and the Higgs sector. For massless particles, as gauge bosons or chiral fermions, they contribute to  $\partial_t U$  a field-independent term  $\Delta N/(32\pi^2)$ , where

$$\Delta N = \Delta N_S + 2N_V - 2N_F, \quad (9)$$

with  $\Delta N_S$  the number of additional scalars,  $N_V$  the number of gauge bosons (with two physical d.o.f. each), and  $N_F$  the number of Weyl fermions. For the SM-matter content, this number is  $\Delta N_{\text{SM}} = -66$ ; for a grand unified gauge theory based on  $SO(10)$ , one has  $\Delta N_{\text{GUT}} = N_S + 10$  with  $N_S$  the total number of real scalars beyond a complex 10-representation.

Around the origin at  $\tilde{\rho} = 0$ , we expand

$$\tilde{U} = \tilde{V} + \tilde{m}_H^2 \tilde{\rho} + \frac{\tilde{\lambda}_H}{2} \tilde{\rho}^2 + \dots \quad (10)$$

Inserting into (8) yields the beta function for each coupling,

$$\partial_t \tilde{V} = -4\tilde{V} + \frac{1}{24\pi^2} \left(1 - \frac{\eta_g}{8}\right) \left[\frac{5}{1-v_0} + \frac{1}{1-v_0/4}\right] + \frac{N}{32\pi^2} \left(1 - \frac{\eta_\phi}{6}\right) \frac{1}{1 + \tilde{m}_H^2} + \frac{\Delta N - 4}{32\pi^2}, \quad (11)$$

$$\partial_t \tilde{m}_H^2 = (-2 + \eta_\phi)\tilde{m}_H^2 + \frac{\tilde{m}_H^2}{48\pi^2 \tilde{M}_p^2} \left(1 - \frac{\eta_g}{8}\right) \left[\frac{20}{(1-v_0)^2} + \frac{1}{(1-v_0/4)^2}\right] - \frac{(N+2)\tilde{\lambda}_H}{32\pi^2} \left(1 - \frac{\eta_\phi}{6}\right) \frac{1}{(1 + \tilde{m}_H^2)^2}, \quad (12)$$

$$\begin{aligned} \partial_t \tilde{\lambda}_H = & 2\eta_\phi \tilde{\lambda}_H + \frac{\tilde{\lambda}_H}{48\pi^2 \tilde{M}_p^2} \left(1 - \frac{\eta_g}{8}\right) \left[\frac{20}{(1-v_0)^2} + \frac{1}{(1-v_0/4)^2}\right] \\ & + \frac{\tilde{m}_H^4}{48\pi^2 \tilde{M}_p^4} \left(1 - \frac{\eta_g}{8}\right) \left[\frac{80}{(1-v_0)^3} + \frac{1}{(1-v_0/4)^3}\right] + \frac{(N+8)\tilde{\lambda}_H^2}{16\pi^2} \left(1 - \frac{\eta_\phi}{6}\right) \frac{1}{(1 + \tilde{m}_H^2)^3}. \end{aligned} \quad (13)$$

Here, we have defined the dimensionless renormalized parameters as  $\tilde{V} = U(\rho=0)/k^4$ ,  $\tilde{m}_H^2 = m_H^2/(Z_\phi k^2)$ ,  $\tilde{\lambda}_H = \lambda_H/Z_\phi^2$ , and  $v_0 = 2\tilde{V}/\tilde{M}_p^2$ . In general, Eqs. (11)–(13) receive contributions from the term  $3\tilde{\rho}\tilde{U}'^2/\tilde{M}_p^2$  in (3), neglected in (7) and (8). These contributions are proportional to higher orders of the coupling constants, e.g.,  $\tilde{m}_H^4$  or  $\tilde{m}_H^2 \tilde{\lambda}_H$ . If the fixed point of matter interactions occurs for  $\tilde{m}_{H^*}^2 = \tilde{\lambda}_{H^*} = 0$ , these terms do not contribute to the critical exponents defined below; see (16).

The effects of Yukawa couplings or gauge couplings to the Higgs sector correspond to the standard perturbative contributions to the beta functions. These effects are small and are not included in our discussion of the UV fixed point. In the present approximation, the additional particles only influence the flow of  $\tilde{V}$ , with no direct influence on (12) and (13).

We observe that for  $\tilde{m}_H^2 = 0$ , as appropriate for the UV fixed point, and  $\eta_\phi = 0$  the fluctuations of the Higgs scalar ( $N=4$ ) cancel the measure contribution,  $\tilde{\pi}_{0,\phi} + \tilde{\eta} = 0$ .

The two last terms in (11) can then be collected into  $\Delta N/(32\pi^2)$ .

Instead of the cosmological constant  $\tilde{V}$ , it is useful to introduce the beta function of the dimensionless quantity  $v_0 = 2\tilde{V}/\tilde{M}_p^2$ , which reads

$$\begin{aligned} \partial_t v_0 &= (-4 + \eta_g)v_0 \\ &+ \frac{1}{12\pi^2\tilde{M}_p^2} \left(1 - \frac{\eta_g}{8}\right) \left[\frac{5}{1-v_0} + \frac{1}{1-v_0/4}\right] \\ &+ \frac{N}{16\pi^2\tilde{M}_p^2} \left(1 - \frac{\eta_\phi}{6}\right) \frac{1}{1+\tilde{m}_H^2} + \frac{\Delta N - 4}{16\pi^2\tilde{M}_p^2}. \end{aligned} \quad (14)$$

Equations (12)–(14) constitute a system of three coupled nonlinear differential equations. They are solved numerically. We employ  $\eta_g = 0$  as appropriate for the UV fixed point and also neglect the presumably small scalar anomalous dimension  $\eta_\phi$ , which arises from the flow of the kinetic term for  $\phi$ . The result of the numerical solution is shown in Fig. 1. For the purpose of this figure, we define an effective  $N = \Delta N + 4$ . For  $\tilde{M}_{p^*}$  outside the grey region, we find indeed a UV fixed point. For  $N \geq -4$ , our assumption of asymptotic safety holds only if gravity is not too strong, such that  $\tilde{M}_{p^*}$  remains above the lower bound indicated by the grey line in Fig. 1.

### III. FIXED POINT AND CRITICAL EXPONENTS FOR THE SCALAR POTENTIAL

Let us now investigate the fixed-point structure and the critical exponents for the scalar potential  $U$ . To this end, we need the dimensionless Planck mass  $\tilde{M}_p$ , which enters directly in (12) and (13) and indirectly through  $v_0 = 2\tilde{V}/\tilde{M}_p^2$ . In order to close the system, the beta function for  $\tilde{M}_p$  would be needed. The latter depends on the particle content of the theory. Its computation is also influenced by the truncations of the system, and the choices of gauge parameters, and the regulator. We assume here only that a fixed point of the Planck mass exists and treat  $\tilde{M}_p$  as a free constant parameter. Since the Newton coupling is defined as  $G_N = 1/(8\pi M_p^2)$ , a small value of the Planck mass corresponds to a strong interaction of gravity. A constant  $\tilde{M}_p$  results in  $\eta_g = 0$ .

We also assume that the system has a Gaussian-matter fixed point, namely, that a nontrivial fixed point is present in the gravity sector, while gauge and Yukawa couplings in the matter sector vanish at the fixed point. We will discuss in Sec. VI the possibility that the matter interactions have a nontrivial fixed point and their effects on the critical exponents. We neglect the small [33] anomalous dimension  $\eta_\phi$ . With these assumptions, we obtain the fixed points and the critical exponents as functions of  $\tilde{M}_p$ .

### A. Critical exponents

Before discussing the structure of the beta functions, we briefly recall the definition of the critical exponents. We denote the renormalized couplings that span the theory space by  $g = \{g_1, \dots, g_i, \dots\}$ . The RG equations are generally given by

$$\partial_t \tilde{g}_i = \beta_i(\tilde{g}) = -d_i \tilde{g}_i + f_i(\tilde{g}), \quad (15)$$

where  $\tilde{g}_i = g_i k^{-d_i}$  is a dimensionless coupling and  $d_i$  is the mass dimension of  $g_i$ . The first term on the right-hand side reflects the canonical scaling, whereas the second one is the fluctuation contribution obtained from the flow equation. Suppose that there exists a nontrivial fixed point  $\tilde{g}_{i^*}$ . The critical exponents characterize the RG flow in the vicinity of the fixed point. We therefore linearly expand the RG equation (15)

$$\partial_t \tilde{g}_i = \sum_j \frac{\partial \beta_i}{\partial \tilde{g}_j} \Big|_{\tilde{g}=\tilde{g}_*} (\tilde{g}_j - \tilde{g}_{j^*}) = -T_{ij} (\tilde{g}_j - \tilde{g}_{j^*}). \quad (16)$$

The matrix  $T$  is the stability matrix, and its eigenvalues, denoted by  $\theta_l$ , are the critical exponents. The solution to (16) is

$$\tilde{g}_i = \tilde{g}_{i^*} + \sum_l C_l V_l^i \left(\frac{k}{\mu}\right)^{-\theta_l}, \quad (17)$$

where  $V_l^i$  is the matrix that diagonalizes the stability matrix and  $C_l$  are constant coefficients given at a reference scale  $\mu$ . Positive critical exponents correspond to relevant couplings, whereas the irrelevant couplings have negative critical exponent. As  $k$  is lowered, the irrelevant couplings flow toward their fixed-point values. Defining a theory at some UV fixed point, the irrelevant couplings take their fixed-point values, setting  $C_l = 0$  for all  $l$  with  $\theta_l < 0$  in (17). The coefficients  $C_l$  for the relevant parameters are the only free parameters of the theory.

Close to a fixed point with  $\tilde{m}_{H^*}^2 = 0$ ,  $\tilde{\lambda}_{H^*} = 0$ , we linearize in  $\tilde{m}_H^2$  and  $\tilde{\lambda}_H$ . Taking  $N = 4$ , the flow equations simplify to

$$\begin{aligned} \partial_t v_0 &= -4v_0 + \frac{\Delta N}{16\pi^2\tilde{M}_p^2} \\ &+ \frac{1}{12\pi^2\tilde{M}_p^2} \left[\frac{5}{1-v_0} + \frac{1}{1-v_0/4}\right] - \frac{\tilde{m}_H^2}{4\pi^2\tilde{M}_p^2}, \end{aligned} \quad (18)$$

$$\begin{aligned} \partial_t \tilde{m}_H^2 &= -2\tilde{m}_H^2 - \frac{3\tilde{\lambda}_H}{16\pi^2} \\ &+ \frac{\tilde{m}_H^2}{48\pi^2\tilde{M}_p^2} \left[\frac{20}{(1-v_0)^2} + \frac{1}{(1-v_0/4)^2}\right], \end{aligned} \quad (19)$$

$$\partial_t \tilde{\lambda}_H = \frac{\tilde{\lambda}_H}{48\pi^2 \tilde{M}_p^2} \left[ \frac{20}{(1-v_0)^2} + \frac{1}{(1-v_0/4)^2} \right]. \quad (20)$$

The stability matrix in the space of couplings  $v_0, \tilde{m}_H^2, \tilde{\lambda}_H$  follows by taking derivatives at the fixed-point values  $v_{0*}, \tilde{m}_{H*}^2 = \tilde{\lambda}_{H*} = 0$ ,

$$T = \begin{pmatrix} 4-A & \frac{1}{4\pi^2 \tilde{M}_p^2} & 0 \\ 0 & 2-A & \frac{3}{16\pi^2} \\ 0 & 0 & -A \end{pmatrix}. \quad (21)$$

The quantity

$$A = \frac{1}{48\pi^2 \tilde{M}_p^2} \left[ \frac{20}{(1-v_0)^2} + \frac{1}{(1-v_0/4)^2} \right] \quad (22)$$

depends only on  $\tilde{M}_p^2$  and  $v_0$ , not on  $\Delta N$ . The eigenvalues of  $T$  are simply the diagonal elements of the matrix (21). In a more complete setting with a beta function for  $\tilde{M}_p^2$  depending on  $v_0$  and  $\tilde{m}_H^2$ , the extended stability matrix involves mixing effects with the sector describing the flow of  $\tilde{M}_p^2$ , as well as possibly with other flowing parameters in the gravitational sector. These mixing effects are neglected in the present work. They concern only the critical exponent for  $v_0$ . Since the beta functions for  $\tilde{m}_H^2$  and  $\tilde{\lambda}_H$  vanish for  $\tilde{m}_H^2 = 0, \tilde{\lambda}_H = 0$ , the derivative of these functions with respect to  $\tilde{M}_p^2$  does not contribute at the fixed point. As a consequence, the critical exponents for  $\tilde{m}_H^2$  and  $\tilde{\lambda}_H$  are not affected by the mixing and remain to be given by  $2-A$  and  $-A$ .

These features allow for rather robust predictions of the critical exponents once the fixed-point values for  $\tilde{M}_p^2$  and  $v_0$  are known. One only needs the computation of  $A$  in (22). The first term in (22) is the graviton contribution from  $\tilde{\pi}_2$ . It typically exceeds the second term by more than a factor 20. This validates the graviton approximation for the computation of the critical exponents in Ref. [59].

## B. Cosmological constant

The value of the cosmological constant  $v_0$  near the fixed point has a substantial influence on the size of the gravitational fluctuations. We determine here the fixed-point value and the associated critical exponent.

### 1. Fixed point as a function of the Planck mass

We first look for a possible fixed point of the cosmological constant as a function of the Planck mass by setting the right-hand side of (18) to zero. We concentrate on  $N = 4$ , as appropriate for the Higgs doublet. The resulting quadratic equation for  $v_{0*}$  admits two solutions: One is a UV fixed point, and the other is an IR one. For instance, in the graviton approximation, namely, taking only the contribution of the traceless transverse mode  $\pi_2$  into account, one finds for  $\Delta N = 0$  [76]

$$v_{0*}^{(\text{UV})} = \frac{1}{2} \left( 1 - \sqrt{1 - \frac{5}{12\pi^2 \tilde{M}_{p*}^2}} \right), \quad (23)$$

$$v_{0*}^{(\text{IR})} = \frac{1}{2} \left( 1 + \sqrt{1 - \frac{5}{12\pi^2 \tilde{M}_{p*}^2}} \right). \quad (24)$$

These values obtain small corrections from the scalar mode in the metric. In Fig. 2, we plot the dependence of the fixed-point value of  $v_0$  on the Planck mass for  $N = 4$  and  $\Delta N = 0$ . We show both the full result (blue) and the graviton approximation (only  $\pi_2$ , red). We conclude that the graviton approximation is valid up to corrections approximately 10%.

In the limit  $\tilde{M}_{p*} \rightarrow \infty$ , the UV fixed point  $v_{0*}^{(\text{UV})}$  merges to the Gaussian fixed point, while the IR one converges to 1. In contrast, for decreasing  $\tilde{M}_{p*}$ , these two fixed points approach each other. They merge at a critical value  $\tilde{M}_{p,c} = 0.217$ . For  $\tilde{M}_p < \tilde{M}_{p,c}$ , no fixed point exists, and quantum gravity is not asymptotically safe. The critical value  $\tilde{M}_{p,c}$  depends on the particle content in the UV. We plot  $\tilde{M}_{p,c}$  as a function of  $N$  in Fig. 1. Here,  $N$  stands for  $4 + \Delta N$ . We include the range of negative  $N$ , as relevant for a sufficient number of fermions; see (9). Indeed, gauge bosons and scalars give a positive contribution to  $N$ , while for fermions, the contribution is negative. The value  $\tilde{M}_{p,c}^2$  is obtained by imposing at  $v_0$  the simultaneous requirements  $\partial_t v_0 = \beta_v = 0$  and  $\partial \beta_v / \partial v_0 = 0$ . For a given  $N$ , asymptotic safety is only realized for  $\tilde{M}_p > \tilde{M}_{p,c}(N)$ , as is visible in Fig. 1. According to (21) and (22), the value  $\tilde{M}_{p,c}$  corresponds to  $A = 4$ . In the vicinity of this critical value, both the mass term and the quartic scalar coupling are irrelevant couplings, with critical exponents  $-2$  and  $-4$ , respectively.

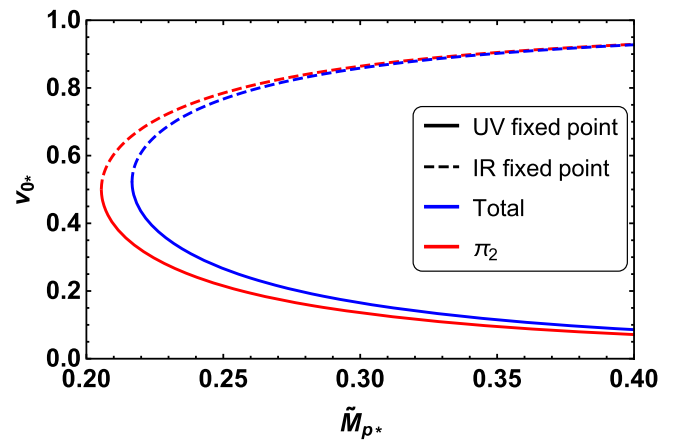


FIG. 2. The fixed-point value of the cosmological constant  $v_{0*}$  as a function of  $\tilde{M}_{p*}$  for  $N = 4$  and  $\Delta N = 0$ . The effects of the graviton fluctuations dominate, as shown by including only  $\pi_2$  in (2).

The IR-fixed point for  $v_0$  always exists for a given  $\tilde{M}_{p^*} > \tilde{M}_{p,c}$ , since the flow of  $v_0$  is always stopped before the pole of the beta function at  $v_0 = 1$  is reached. On the other hand,  $\tilde{M}_p$  depends on  $v_0$ . The function  $\tilde{M}_{p^*}(v_0)$  corresponds to a curve in Fig. 2 that is not computed in the present work. The UV fixed point corresponds to the intersection of this curve with the curve shown in Fig. 2. If there exist two intersection points, both with the solid and the dashed lines, both a UV and an IR-fixed point exist. If present, one could alternatively define the theory at the IR-fixed point. Since  $\partial\beta_v/\partial v_0 > 0$  at the IR-fixed point, one infers from (21) and (22) that  $A > 4$ . Thus, all three parameters  $v_0$ ,  $\tilde{m}_H^2$ , and  $\tilde{\lambda}_H$  are irrelevant couplings. In the present paper, we do not pursue this possible alternative and rather concentrate on the UV fixed point.

Once  $\tilde{M}_p^2$  increases as this coupling moves away from the UV fixed point, the IR-fixed point for  $v_0$  approaches the pole in the beta function. Indeed, the value  $v_0 = 1$  corresponds to a pole of the propagator of the graviton. The existence of the IR-fixed point close to  $v_0 = 1$  induces strong fluctuation effects of the graviton in the IR regime and could be a key point to resolve the cosmological constant problem [76,78].

For general  $\Delta N$ , we parametrize the ratio between the scalar and tensor gravitational contributions by

$$w_s(v) = \frac{\tilde{\pi}_{0,g}}{\tilde{\pi}_2} = \frac{1-v}{5(1-v/4)}. \quad (25)$$

The fixed points for  $v_0$  occur for

$$v_{0*} = \frac{1}{2} \left( 1 + z\Delta N \pm \sqrt{(1-z\Delta N)^2 - \frac{80z(1+w_s)}{3}} \right), \quad (26)$$

with

$$z = \frac{1}{64\pi^2 \tilde{M}_p^2}. \quad (27)$$

For large negative  $\Delta N$ , the UV fixed point occurs for a negative value,

$$v_{0*}^{(\text{UV})} \approx z\Delta N + \frac{20z(1+w_s)}{3(1-z\Delta N)}. \quad (28)$$

As long as  $z\Delta N$  remains small as compared to 1, one has  $w_s \approx 1/5$  and

$$v_{0*}^{(\text{UV})} \approx z(\Delta N + 8). \quad (29)$$

The IR-fixed point approaches 1, with  $w_s \rightarrow 0$ ,

$$v_{0*}^{(\text{IR})} \approx 1 - \frac{20z(1+w_s)}{3(1-z\Delta N)}. \quad (30)$$

The approximations (28)–(30) remain valid for positive  $\Delta N$  as long as  $z\Delta N \ll 1$ .

For the critical  $z_c$  at which the fixed point disappears, one has

$$v_{0c} = \frac{1}{2}(1+z_c\Delta N), \quad w_s(v_c) = \frac{2(1-z_c\Delta N)}{15(1+\frac{1}{6}(1-z_c\Delta N))}, \quad (31)$$

and therefore

$$(1-z_c\Delta N)^2 = \frac{80z_c}{3} + \frac{32z_c(1-z_c\Delta N)}{9(1+\frac{1}{6}(1-z_c\Delta N))}. \quad (32)$$

For large  $\Delta N$ , this results in a value of  $z_c\Delta N$  close to 1 such that the second term on the right-hand side of (32) can be neglected. Therefore the critical boundary for  $\tilde{M}_p^2$  linearly increases as a function of  $\Delta N$ ,

$$\tilde{M}_{p,c}^2 \approx \frac{1}{64\pi^2} \left( \Delta N + \frac{4\sqrt{15}}{3} \sqrt{\Delta N} \right). \quad (33)$$

The corresponding critical  $v_{0c}$  approaches 1,

$$v_{0c} = 1 - \frac{2\sqrt{15}}{3\sqrt{\Delta N}}, \quad (34)$$

such that the graviton contribution is enhanced. The graviton approximation becomes rather accurate for values of  $\tilde{M}_p^2$  and  $v_0$  in the vicinity of (33) and (34).

## 2. Critical exponent

With the value of the UV fixed point, we obtain the critical exponent of the cosmological constant or  $v_0$ ,

$$\begin{aligned} \theta_v &= - \left. \frac{\partial\beta_v}{\partial v_0} \right|_{\text{at FP}} = 4 - A \\ &= 4 - \frac{1}{12\pi^2 \tilde{M}_{p^*}^2} \left[ \frac{5}{(1-v_{0*})^2} + \frac{1}{4(1-v_{0*}/4)^2} \right] \\ &= \frac{4(1-2v_{0*})}{1-v_{0*}} \\ &\quad + \frac{1}{16\pi^2 \tilde{M}_{p^*}^2 (1-v_{0*})} \left( \Delta N + \frac{1}{(1-v_{0*}/4)^2} \right). \end{aligned} \quad (35)$$

Figure 3 displays the dependence of  $\theta_v$  on the fixed-point value of the Planck mass for  $N = 4$  and  $\Delta N = 0$ . For the limit  $\tilde{M}_{p^*} \rightarrow \infty$  (weak interaction), the critical exponent of the cosmological constant asymptotically converges to 4,



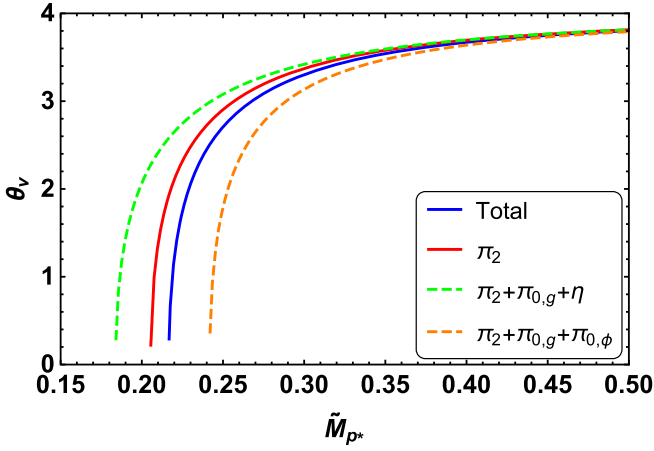


FIG. 3. The critical exponent of the cosmological constant  $\theta_v$  as a function of the fixed-point value of the Planck mass ( $N = 4$ ). The cosmological constant or, equivalently,  $v_0$  is a relevant parameter. The individual contributions of various fluctuations are visualized by including in (2) only the specified parts.

which is its canonical dimension. On the other hand, for  $\tilde{M}_p \rightarrow \tilde{M}_{p,c}$  the critical exponent approaches zero.

We display in Fig. 3 various approximations. Besides the total contribution (blue) and the graviton approximation (red), we show the total contributions of the gravitational d.o.f. without contributions of other particles, i.e.,  $N = 0$  and  $\Delta N = 0$ . This corresponds in (2) to  $\tilde{\pi}_2 + \tilde{\pi}_{0,g} + \tilde{\eta}$  (dashed green line). The total gravitational contribution is again well approximated by the graviton approximation. Finally, the green dashed line omits the measure contribution  $\tilde{\eta}$ .

#### IV. PREDICTIONS OF ASYMPTOTIC SAFETY FOR THE PROPERTIES OF THE HIGGS SCALAR

As mentioned in the Introduction, a UV fixed point for quantum gravity can predict those renormalizable couplings in the SM that correspond to irrelevant couplings at the fixed point. We find that the quartic coupling of the Higgs scalar is always irrelevant, and the scalar mass term is irrelevant for a certain range of the fixed-point value of the Planck mass.

##### A. Effective low-energy theory

So far, we have concentrated on the flow equations in the vicinity of the UV fixed point of asymptotically safe quantum gravity. This corresponds to a constant value of  $\tilde{M}_p^2$  in the flow equation (8). Let us now assume that the Planck mass corresponds to a relevant parameter in quantum gravity. At short distances, it scales according to the UV fixed point behavior,  $M_p^2(k^2) = \tilde{M}_{p*}^2 k^2$ . For small  $k$ , it deviates from this scaling behavior and takes a fixed value  $M^2$ . This results in the qualitative behavior

$$M_p^2(k) = \begin{cases} \tilde{M}_{p*}^2 k^2 & \text{for } k > k_t, \\ M^2 & \text{for } k < k_t. \end{cases} \quad (36)$$

The transition scale is found as

$$k_t^2 = \frac{M^2}{\tilde{M}_{p*}^2}. \quad (37)$$

A more complete treatment smoothens the transition. Details of the threshold behavior are not important for our purpose. One may use the simple form

$$M_p^2(k) = M^2 + \tilde{M}_{p*}^2 k^2. \quad (38)$$

For  $k$  below the transition scale  $k_t$ , the dimensionless coupling  $\tilde{M}_p^2$  increases rapidly,

$$\tilde{M}_p^2 = \frac{M_p^2}{k^2} = \tilde{M}_{p*}^2 \left[ \left( \frac{k_t}{k} \right)^2 + 1 \right]. \quad (39)$$

As a result, the gravitational contributions in (12) and (13)—not in (11)—become rapidly tiny and can be neglected. This leads to a simple picture. For  $k < k_t$ , the flow enters the regime of an effective low-energy theory for which the effect of gravitational fluctuations can be neglected, except for the cosmological constant  $\tilde{V}$ . This effective low-energy theory may be the SM or a possible extension of it.

To rather good accuracy, the flow of dimensionless couplings as  $\tilde{m}_H^2$  and  $\tilde{\lambda}_H$  can be divided into two regimes. For the UV regime  $k > k_t$ , it follows the flow in the vicinity of the UV fixed point. In contrast, for the IR regime  $k < k_t$ , the flow is given by the low-energy effective theory. In this picture the initial values of couplings for the IR flow, e.g., their values at  $k_t$ , are determined by their final values of the UV flow. In case of irrelevant couplings, the initial values for the IR flow are simply the UV fixed point values. They are therefore predicted. Following the IR flow from  $k_t$  in the vicinity of  $M$  down to observable energy scales leads then to predictions for observable quantities.

##### B. Scalar mass term

Next, we turn to the behavior of the scalar mass term in the vicinity of the UV fixed point. Depending on the value of  $\tilde{M}_{p*}^2$ , this can be a relevant or an irrelevant parameter. This issue has important consequences for the gauge hierarchy problem [71,72]. Therefore, we start from the discussion of this problem in the context of flow equations.

##### 1. Scalar mass flow and gauge hierarchy problem

Let us first discuss RG improved perturbation theory in the SM. The RG equation of the scalar mass term at one-loop level is given by

$$\partial_t \tilde{m}_H^2 = (-2 + \gamma_m) \tilde{m}_H^2, \quad (40)$$

where the first term on the right-hand side reflects the canonical scaling and the second one is the anomalous dimension, which reads

$$\gamma_m = \frac{1}{16\pi^2} \left( 2\lambda_H + 6y_t^2 - \frac{9}{2}g^2 - \frac{3}{2}g'^2 \right). \quad (41)$$

More precisely, the coupling  $\tilde{m}_H^2$  measures the distance from the critical surface of the (almost) second-order vacuum electroweak phase transition [79]. In (41),  $\lambda$ ,  $y_t$ ,  $g$ , and  $g'$  are the quartic coupling of the Higgs field, top-Yukawa coupling, SU(2) gauge coupling, and U(1) gauge coupling, respectively. Using the values of the couplings at the Fermi scale, the anomalous dimension  $\gamma_m \approx 0.027$  is much smaller than 2.

For the marginal couplings in (41), the scale dependence is logarithmic. Neglecting their runnings the solution of (40) reads

$$\tilde{m}_H^2 = \tilde{m}_0^2 \left( \frac{k}{M} \right)^{-2+\gamma_m}, \quad (42)$$

where  $\tilde{m}_0$  is the initial value of the scalar mass term at a reference scale  $M$ . For  $M$  being of the order of the Planck scale, one has to set a very tiny mass term  $\tilde{m}_0^2 \simeq 10^{-34} M^2$  at the Planck scale in order to obtain the Higgs mass  $\tilde{m}_H^2 = m_H^2 / \Lambda_{\text{EW}}^2 \simeq 1$  at the electroweak scale  $k = \Lambda_{\text{EW}} \simeq \mathcal{O}(10^2)$  GeV. This is the gauge hierarchy problem. It is directly related to the role of  $\tilde{m}_H^2 = m_H^2 / k^2$  being a relevant coupling for the (approximate) fixed point of the SM, with critical exponent  $\theta_m = 2 - \gamma_m$ .

A frequent discussion of the gauge hierarchy problem relies on the fact that the one-loop correction to the scalar mass involves a quadratic divergence as the UV cutoff is sent to infinity. In perturbation theory, the observed Higgs mass is given by the cancellation between the squared bare mass and the quadratic divergence. The quadratic divergence strongly depends on the cutoff scheme. It is not present for dimensional regularization, while the momentum cutoff regularization and the Pauli-Villars type cutoff yield different values, depending on the precise implementation.

In terms of the RG, the quadratic divergence indicates the position of the phase boundary in the space of bare couplings. This boundary or “critical surface” separates the symmetric and broken phases and corresponds to the massless (critical) situation [73,79–84]. The position of the phase boundary depends on the precise definition and choice of the bare couplings and on the precise regularization. It changes under a coordinate change in “theory space” if the latter is parametrized by the bare couplings. Different choices of the cutoff scheme also correspond to a coordinate transformation in theory space. In quantum field

theory, the precise choice of bare couplings is usually not of much interest.

On the other hand, the deviation from the phase boundary corresponds to a renormalized coupling. Its behavior is independent of the precise choice of microphysics as regularizations and the precise definition of bare couplings. This explains why the flow equation (19) only involves renormalized couplings, while no trace of the quadratic divergence appears. The vanishing of the right-hand side reflects the basic property of a second-order phase transition. No trajectory can cross the phase boundary. Couplings on the critical surface stay on the critical surface.

## 2. Quantum gravity effects

We next add the effects of the gravitational quantum fluctuations in the range  $k \gg M$ . The beta function of the scalar mass becomes

$$\beta_m = -(2 - \gamma_m - A) \tilde{m}_H^2. \quad (43)$$

Comparing with (40), we see that  $A$  corresponds to the gravitational contribution to the anomalous dimension. Neglecting for simplicity the small value of  $\gamma_m$  as compared to  $A$ , the critical exponent of the scalar mass parameter reads

$$\begin{aligned} \theta_m &= - \left. \frac{\partial \beta_m}{\partial \tilde{m}_H^2} \right|_{\text{at FP}} = 2 - A \\ &= 2 - \frac{1}{48\pi^2 \tilde{M}_{\text{p}^*}^2} \left[ \frac{20}{(1 - v_{0^*})^2} + \frac{1}{(1 - v_{0^*}/4)^2} \right]. \end{aligned} \quad (44)$$

An important observation is that the sign of  $A$  is positive. Gravitational fluctuations lower the value of the critical exponent  $\theta_m$ . As long as  $A$  stays smaller than 2, the scalar mass term  $\tilde{m}_H^2$  remains a relevant parameter,  $\theta_m > 0$ . In this case, the distance from the vacuum electroweak phase transition, as measured by the value of the Fermi constant, cannot be predicted. It is simply a free parameter specifying the theory. A dramatic change occurs for  $A > 2$ . In this event, the scalar mass term  $\tilde{m}_H^2$  turns out to be an irrelevant coupling. The flow trajectory is always toward the phase-transition surface—an example of “self-organized criticality.” If asymptotically safe gravity is realized in a model leading to  $A > 2$ , it predicts that  $\tilde{m}^2(k)$  vanishes for  $k = M$ . This explains the tiny value of the ratio  $\tilde{m}_H^2(k)k^2 = \tilde{m}_H^2$ , evaluated at  $k = M$ , which is required by the observed Fermi scale  $\tilde{m}_H^2(k = M) = 10^{-34}$ . It produces an even stronger gauge hierarchy, namely,  $\tilde{m}_H^2(k = M) = 0$ . If the vacuum electroweak phase transition would be an exact second-order phase transition, any model with  $A > 2$  would predict a vanishing Fermi scale.

The vacuum electroweak transition is not an exact second-order phase transition. This is due to the running

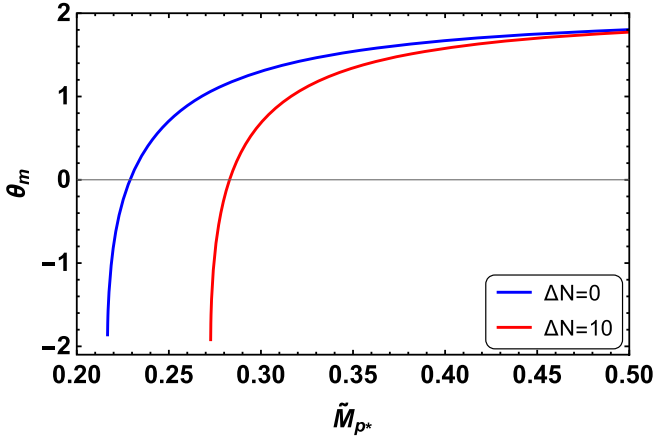


FIG. 4. The critical exponent of the scalar mass  $\theta_m$  as a function of the fixed-point value of the Planck mass in the cases  $\Delta N = 0$  and  $\Delta N = 10$ . The mass term becomes irrelevant in the region where  $\theta_m$  is negative (red-colored region in Fig. 1).

gauge and Yukawa couplings that prevent the realization of exact scale symmetry in the effective low-energy theory below the Planck mass. The dominant effect is believed to be due to the running strong gauge coupling. Chiral symmetry breaking induces a quark-antiquark condensate  $\langle \bar{q}q \rangle$ . For the light quarks, this condensate sets a scale of the order 100 MeV, implying a lower bound on the Fermi scale of the same order of magnitude. The detailed effects of the scale violation in the top-quark–Higgs-scalar sector are not known quantitatively. If they are not substantially larger than the effects of the light quark condensate, asymptotically safe quantum gravity coupled to the SM predicts a  $W$ -boson mass around 100 MeV in case of  $A > 2$ . This is not compatible with observation.

For  $A > 2$ , extensions would be needed, as discussed in the resurgence mechanism [73]. This may involve either new particles with masses near or below the Fermi scale or a more complicated UV fixed point structure for quantum gravity. We recall in this context that a definition of the theory at the IR-fixed point for  $v_0$  (which becomes then the effective UV fixed point) implies  $A > 4$ . The scalar mass term is always irrelevant for this setting.

In Fig. 4, we show the dependence of the critical exponent of the scalar mass term on the fixed-point value of the Planck mass  $\tilde{M}_p$  for  $N = 4$  and  $\Delta N = 0$ . It falls below zero ( $A > 2$ ) if the strength of gravity exceeds a certain bound or  $\tilde{M}_p$  becomes small enough. This behavior extends to other values of  $\Delta N$ , as shown in Fig. 1 where  $N$  stands for  $4 + \Delta N$ . The red region in this figure occurs for  $\theta_m < 0$ ,  $A > 2$ .

For large positive  $\Delta N$ , the physical scalar metric fluctuation can be neglected, yielding for the condition  $A = 2$  the relation

$$(1 - v_0)^2 = \frac{40z}{3}. \quad (45)$$

In this approximation, the fixed-point relation between  $z$  and  $1 - v_0$  inferred from (18) for  $\partial_t v_0 = 0$  reads

$$(1 - v_0) + z\Delta N - 1 + \frac{20z}{3(1 - v_0)} = 0. \quad (46)$$

With

$$(1 - v_0)^2 = (1 - z\Delta N)^2 - \frac{20z}{3} - \frac{1}{2}(1 - z\Delta N) \times \left[ 1 - z\Delta N - \sqrt{(1 - z\Delta N)^2 - \frac{80z}{3}} \right], \quad (47)$$

and

$$(1 - z\Delta N)^2 = 30z, \quad (48)$$

the line  $A = 2$  occurs for

$$\tilde{M}_{p*}^2 \approx \frac{1}{64\pi^2} (\Delta N + \sqrt{30\Delta N}). \quad (49)$$

For large  $\Delta N$ , this comes very close to the critical value (33) for which the UV fixed point disappears. Only a small region with negative critical exponent  $\theta_m$  remains.

### C. Quartic scalar coupling

We finally investigate the quantum gravity effects on the quartic scalar coupling. The critical exponent of the quartic scalar coupling is given by the beta function in linear order in  $\tilde{\lambda}_H$ ,  $\beta_\lambda = -\theta_\lambda \tilde{\lambda}_H$ ,

$$\theta_\lambda = -\left. \frac{\partial \beta_\lambda}{\partial \tilde{\lambda}_H} \right|_{\text{at FP}} = -A = -\frac{1}{48\pi^2 \tilde{M}_{p*}^2} \left[ \frac{20}{(1 - v_{0*})^2} + \frac{1}{(1 - v_{0*}/4)^2} \right]. \quad (50)$$

The dependence of the critical exponent on  $\tilde{M}_p$  is shown in Fig. 5.

The irrelevance of the quartic coupling at the fixed point  $\tilde{\lambda}_{H*} = 0$  means that the coupling constant vanishes for  $k$  above the Planck scale. In other words, the boundary condition of the RG equation of the quartic coupling is given by the fixed-point value  $\tilde{\lambda}_{H*} = 0$  at  $k_t$  close to the Planck scale. With  $\tilde{\lambda}_H(k_t) = 0$ , the flow of  $\tilde{\lambda}_H(k)$  can be followed in the effective low-energy theory for  $k < k_t$ . Assuming that this IR flow is well approximated by the SM, the mass of the Higgs boson can be predicted as a function of the top-quark Yukawa coupling. The result of this prediction [1] was  $m_H = 126$  GeV with only a few giga-electron-volts uncertainty. It agrees well with the observed value of the Higgs boson mass  $m_H = 125$  GeV.

In the flow equation for  $\tilde{\lambda}_H$ , we have neglected effects of Yukawa couplings and gauge couplings. They contribute to

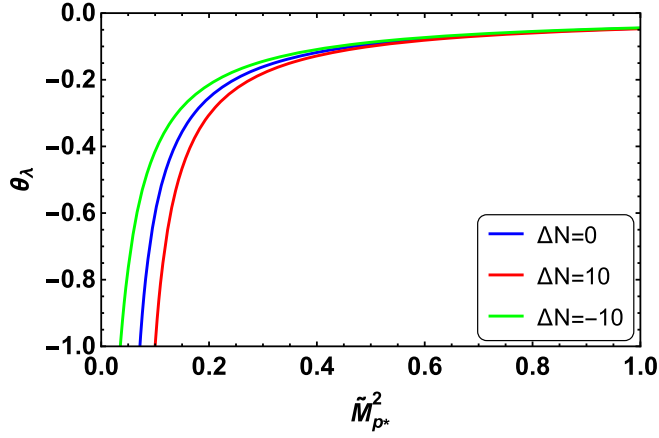


FIG. 5. The critical exponent of the quartic coupling  $\theta_\lambda$  as a function of the fixed-point value of the squared Planck mass in the cases  $\Delta N = 0$ ,  $\Delta N = 10$ , and  $\Delta N = -10$ .

the wave function renormalization, leading to a small anomalous dimension  $\eta_\phi$ . Yukawa and gauge couplings also shift the fixed point  $\tilde{\lambda}_{H^*}$  to a tiny nonzero value. For  $\lambda_{H^*} \neq 0$ , the term proportional to  $\lambda_H^2$  in  $\beta_\lambda$  contributes to the anomalous dimension, as seen from the last term in (13). This effectively enhances the anomalous dimension and makes the critical exponent  $\theta_\lambda$  more negative. A nonzero fixed-point value  $\tilde{\lambda}_{H^*}$  could also be induced by nonminimal scalar interactions involving higher derivatives that may be generated by gravitational fluctuations. In any case, even for nonzero  $\tilde{\lambda}_H$  at the UV fixed point, this value will be small. It therefore has only little impact on our results.

In summary, the negative sign of the critical exponent for the quartic scalar coupling seems to be a rather robust finding. For a given low-energy model, the mass of the Higgs scalar is predictable in asymptotically safe quantum gravity.

## V. ROBUSTNESS OF RESULTS

In view of the far-reaching consequences of our findings for the predictivity of quantum gravity for SM parameters, some tests of the robustness of these results seem appropriate. Possible errors are connected with a possibly insufficient truncation of the exact flow equation. Typical tests are the extension of the truncation and the sensitivity to the choice of the infrared cutoff function. Gauge dependence is a minor issue in our approach since we are bound to employ a physical gauge fixing that only acts on the gauge d.o.f. in the metric. Within this class of physical gauge fixings, the dependence on the precise gauge fixing is small [59,85]. At the present stage, the main uncertainty concerns the fixed-point value of the dimensionless Planck mass. We will see that truncation errors can typically be compensated by a change of this value. The error analysis will become more meaningful at a later stage when the fixed-point value for the Planck mass is also

calculated. Nevertheless, our two main statements seem rather robust:

- (i) The quartic scalar coupling is an irrelevant parameter at the UV fixed point.
- (ii) There exists a range of fixed-point values for the Planck mass for which the scalar mass term is also an irrelevant parameter.

## A. Extension of truncation

So far, we have analyzed the Einstein-Hilbert truncation for the gravity sector. Extensions of the truncation are possible in various directions. They are, however, restricted by diffeomorphism symmetry of the effective action. One possibility is the inclusion of higher-order curvature invariants. Only the quadratic invariants influence the propagator of the metric fluctuations in flat space, which is the only quantity involved for the computation of the flow of the effective scalar potential. From the quadratic curvature invariants, it is only the squared Weyl tensor that enters in the transverse traceless mode of the metric propagator, which constitutes the dominant graviton contribution. Denoting by  $D$  the coefficient of the squared Weyl tensor, the inverse graviton propagator becomes at the fixed point

$$G_g^{-1} = \frac{\tilde{M}_{\text{p}^*}^2}{4} (k^2 q^2 - k^4 v_{0^*}) + \frac{D_*}{2} q^4. \quad (51)$$

The flow is dominated by momenta  $q^2 \approx k^2$ , such that  $D$  becomes important only for

$$D_* \gtrsim \frac{\tilde{M}_{\text{p}^*}^2}{2} (1 - v_{0^*}). \quad (52)$$

A positive  $D_*$  lowers the anomalous dimension  $A$ , while a negative  $D_*$  enhances it. The sign of  $A$  is not changed as long as gravity is stable for positive  $G_g^{-1}$  in the Euclidean domain. The inclusion of  $D$  modifies the factor  $(1 - v_0)^{-2}$  in the first term of (22) to  $[(1 - v_0) + 2D/\tilde{M}_{\text{p}^*}^2]^{-2}$ . For large  $D$ , this replaces  $\tilde{M}_{\text{p}^*}^{-2}(1 - v_0)^{-2}$  by  $\tilde{M}_{\text{p}^*}^2/(4D^2)$ . The crucial property  $A > 0$  also holds within the perturbatively renormalizable quartic gravity [86,87].

For a second point, we investigate a possible dependence of the effective Planck mass on the scalar field, extending the truncation by a nonminimal interaction between the scalar field and the curvature,

$$\Gamma_k = -\frac{1}{2} \int_x \sqrt{g} (M_{\text{p}}^2 + \xi \rho) R. \quad (53)$$

We consider here a constant  $\xi$ , i.e., assumed to be at its fixed-point value. In the following, we derive the shift in the fixed-point values of  $m_H$  and  $\lambda_H$ , the changes in the stability matrix, as well as new contributions due to the mixing in the scalar sector. For a main result, we find that



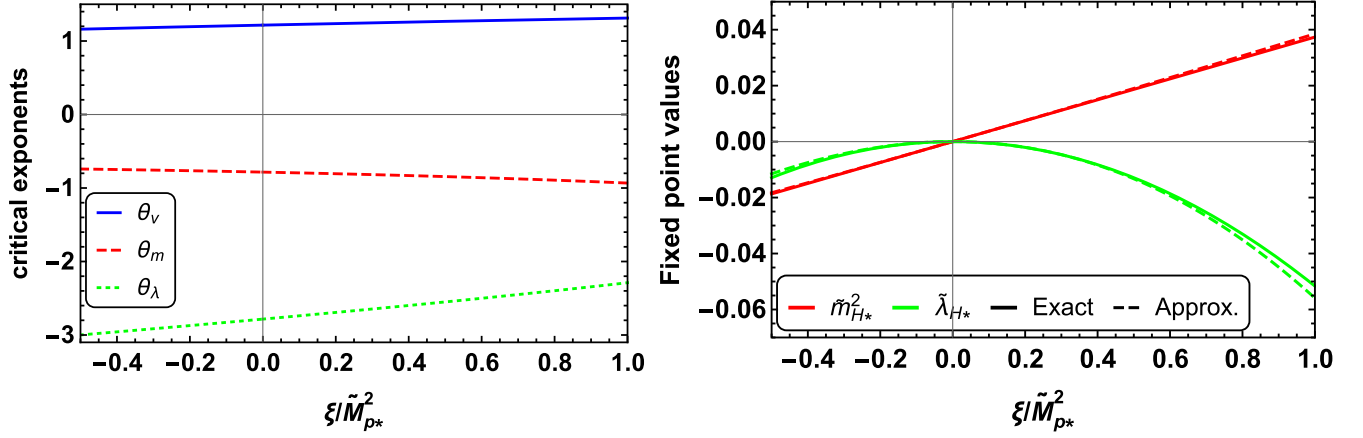


FIG. 6. Critical exponents (left) and fixed-point values (right) of the scalar mass term and quartic coupling as functions of the nonminimal coupling  $\xi/\tilde{M}_{p*}$ . For the dimensionless Planck mass, we take the value  $\tilde{M}_{p*} = 0.22$ . In the right-hand panel, we compare the fixed-point values for the numerical solutions including all terms discussed here (solid line) and the approximated ones given in (61) (dashed line). Importantly, the critical exponents, displayed in the left-hand panel, are almost independent of the nonminimal coupling  $\xi/\tilde{M}_{p*}$ .

the dependence of the critical exponents on the nonminimal coupling is small as displayed in Fig. 6.

The flow equation (3) for the effective potential remains valid if we replace  $\tilde{M}_p^2 \rightarrow \tilde{M}_p^2 + \xi\tilde{\rho}$ ,  $v = 2\tilde{U}/(\tilde{M}_p^2 + \xi\tilde{\rho})$ . The additional dependence of  $v$  on  $\tilde{\rho}$ ,

$$\frac{\partial v}{\partial \tilde{\rho}} = \frac{2}{\tilde{M}_p^2 + \xi\tilde{\rho}} \left( \tilde{U}' - \frac{\xi v}{2} \right), \quad (54)$$

modifies the flow equation for the mass term  $\tilde{m}_H^2 = \partial\tilde{U}/\partial\tilde{\rho}|_{\tilde{\rho}=0}$  and the quartic coupling  $\lambda_H = \partial^2\tilde{U}/\partial\tilde{\rho}^2|_{\tilde{\rho}=0}$  [88]. On the other hand, the flow equation for  $v_0 = 2\tilde{V}/\tilde{M}_p^2$  depends on  $\xi$  only indirectly through the dependence of the scalar contribution  $\pi_0$  on  $\tilde{m}_H^2$ .

We first neglect the mixing between the scalar modes, which will be added below. The flow equation (19) for  $\tilde{m}_H^2$  gets extended to

$$\partial_t \tilde{m}_H^2 = (-2 + A)\tilde{m}_H^2 - \frac{\xi A v_0}{2} - \frac{3\tilde{\lambda}_H}{16\pi^2(1 + \tilde{m}_H^2)^2}. \quad (55)$$

Similarly, the flow equation for  $\lambda_H$  (20) becomes

$$\begin{aligned} \partial_t \tilde{\lambda}_H &= A\tilde{\lambda}_H + \frac{2}{\tilde{M}_p^2} \frac{\partial A}{\partial v} \left( \tilde{m}_H^2 - \frac{\xi v_0}{2} \right)^2 \\ &\quad - \frac{2\xi A}{\tilde{M}_p^2} \left( \tilde{m}_H^2 - \frac{\xi v_0}{2} \right) + \frac{3\tilde{\lambda}_H^2}{4\pi^2(1 + \tilde{m}_H^2)^3}, \end{aligned} \quad (56)$$

where all quantities are evaluated at  $\tilde{\rho} = 0$  and we take  $N = 4$  for the number of scalars in (13).

The fixed-point value of  $\tilde{m}_H^2$  is now nonvanishing (assuming  $A \neq 2$ ),

$$\tilde{m}_{H*}^2 = \frac{1}{A-2} \left( \frac{\xi A v_{0*}}{2} + \frac{3\tilde{\lambda}_{H*}}{16\pi^2(1 + \tilde{m}_{H*}^2)^2} \right). \quad (57)$$

For positive  $\xi$  and positive or small  $\tilde{\lambda}_{H*}$ , the origin at  $\tilde{\rho} = 0$  is a local minimum only for  $A > 2$ . Similarly, the fixed-point value for  $\tilde{\lambda}_H$  is also nonzero. A negative value of  $\tilde{\lambda}_H$  is not an indication of instability. The Taylor expansion is not valid for larger values of  $\tilde{\rho}$  for which  $U$  would get negative in a quartic approximation. If the last term  $\sim \tilde{\lambda}_H^2$  in (56) can be neglected and if one can approximate  $\tilde{m}_{H*}^2 \ll 1$  in (57), one has

$$\tilde{\lambda}_{H*} = \frac{C}{A}, \quad (58)$$

with

$$C = \frac{8\tilde{m}_{H*}^4}{A v_{0*} \tilde{M}_{p*}^2} \left( A - 2 - \frac{\partial \ln A}{\partial \ln v} \right), \quad (59)$$

and

$$\tilde{A} = A + \frac{3\tilde{m}_{H*}^2}{4\pi^2 A v_{0*} \tilde{M}_{p*}^2} \left( 4 - A + 2 \frac{\partial \ln A}{\partial \ln v} \right). \quad (60)$$

This approximation breaks down as  $\tilde{A}$  comes close to zero. For the full fixed-point solution of Eqs. (55) and (56), both  $\tilde{m}_{H*}^2$  and  $\tilde{\lambda}_{H*}$  remain finite. For small  $\xi$ , one has  $\tilde{m}_{H*}^2 \sim \xi$ ,  $\tilde{\lambda}_{H*} \sim \xi^2$ , such that in lowest order in  $\xi$  one obtains

$$\begin{aligned}\tilde{m}_{H^*}^2 &= \frac{\xi A v_{0^*}}{2(A-2)}, \\ \tilde{\lambda}_{H^*} &= \frac{2\xi^2 v_{0^*} A - 2 - \frac{\partial \ln A}{\partial \ln v}}{\tilde{M}_{p^*}^2 (A-2)^2}.\end{aligned}\quad (61)$$

For the graviton approximation, one may use the simple relation

$$\frac{\partial \ln A}{\partial \ln v} = \frac{2v_0}{1-v_0}.\quad (62)$$

The stability matrix  $T$  receives additional off-diagonal entries for  $\tilde{m}_{H^*}^2 \neq 0$ ,  $\tilde{\lambda}_{H^*} \neq 0$ ,

$$T = \begin{pmatrix} 4-A & \frac{1}{4\pi^2 \tilde{M}_{p^*}^2 (1+\tilde{m}_{H^*}^2)^2} & 0 \\ B & 2-A - \frac{3\tilde{\lambda}_{H^*}}{8\pi^2 (1+\tilde{m}_{H^*}^2)^3} & \frac{3}{16\pi^2 (1+\tilde{m}_{H^*}^2)^2} \\ E & F & -A - \frac{3\tilde{\lambda}_{H^*}}{2\pi^2 (1+\tilde{m}_{H^*}^2)^3} \end{pmatrix},\quad (63)$$

where

$$\begin{aligned}B &= \frac{\xi A}{2} \left( 1 + \frac{\partial \ln A}{\partial \ln v} \right) - \frac{\partial A}{\partial v} \tilde{m}_{H^*}^2 \\ &= \frac{\xi A}{2} \left( 1 - \frac{2}{A-2} \frac{\partial \ln A}{\partial \ln v} \right),\end{aligned}\quad (64)$$

and

$$\begin{aligned}F &= \frac{1}{\tilde{M}_{p^*}^2} \left[ 2\xi A - 4 \frac{\partial A}{\partial v} \left( \tilde{m}_{H^*}^2 - \frac{\xi v_{0^*}}{2} \right) \right] + \frac{9\tilde{\lambda}_{H^*}^2}{4\pi^2 (1+\tilde{m}_{H^*}^2)^4} \\ &= \frac{2\xi A}{\tilde{M}_{p^*}^2} \left( 1 - \frac{2}{A-2} \frac{\partial \ln A}{\partial \ln v} \right).\end{aligned}\quad (65)$$

Here, the second lines in (64) and (65) use the approximation (61). For  $E$ , one obtains

$$\begin{aligned}E &= \frac{2}{\tilde{M}_{p^*}^2} \left[ 2\xi \frac{\partial A}{\partial v} \left( \tilde{m}_{H^*}^2 - \frac{\xi v_{0^*}}{2} \right) - \frac{\xi^2 A}{2} \right. \\ &\quad \left. - \frac{\partial^2 A}{\partial v^2} \left( \tilde{m}_{H^*}^2 - \frac{\xi v_{0^*}}{2} \right)^2 \right] - \frac{\partial A}{\partial v} \lambda_{H^*},\end{aligned}\quad (66)$$

which is of the order  $\xi^2$ . The corrections from  $\xi$  are small as long as

$$\begin{aligned}\frac{|B|}{4\pi^2 \tilde{M}_{p^*}^2} &< |(4-A)(2-A)|, \\ \frac{3|F|}{16\pi^2} &< |A(2-A)|.\end{aligned}\quad (67)$$

For an order of magnitude estimate, this holds for

$$|\xi| < |A-2| \pi^2 \tilde{M}_{p^*}^2.\quad (68)$$

Since for  $\xi \neq 0$  the scaling solutions for  $\tilde{m}_{H^*}^2$  and  $\tilde{\lambda}_{H^*}$  occur for nonzero values, there is also a contribution to the flow equation from the mixing in the scalar sector. This only concerns a subleading term. For small  $\tilde{\rho} \tilde{U}'^2 / (\tilde{M}_p^2 + \xi \tilde{\rho})$ , the mixing contributes an additional term to  $\tilde{\pi}_0$ ,

$$\Delta(\partial_t \tilde{U}) = \frac{\Delta \tilde{\pi}_0}{k^4} = -\tilde{\rho} \tilde{U}'^2 H,$$

with

$$H = \frac{1 + \tilde{U}' + 2\tilde{\rho} \tilde{U}'' + \frac{3}{4}(1 - \frac{v}{4})}{8\pi^2 (\tilde{M}_p^2 + \xi \tilde{\rho}) (1 - \frac{v}{4})^2 (1 + \tilde{U}' + 2\tilde{\rho} \tilde{U}'')^2},\quad (69)$$

where we take  $\eta_g = \eta_\phi = 0$ . This does not contribute to the flow of  $V$  or  $v_0$ , but the contribution to the flow of  $\tilde{m}_H^2$  and  $\tilde{\lambda}_H$  vanishes only for  $\tilde{m}_H^2 = 0$ ,  $\tilde{\lambda}_H = 0$ . One finds, with  $H_0 = H(\tilde{\rho} = 0)$ ,

$$\Delta(\partial_t \tilde{m}_H^2) = -H_0 \tilde{m}_H^4,\quad (70)$$

and

$$\Delta(\partial_t \tilde{\lambda}_H) = -2H_0 \tilde{m}_H^2 \tilde{\lambda}_H - \tilde{m}_H^4 \frac{\partial H}{\partial \tilde{\rho}} \Big|_{\tilde{\rho}=0}.\quad (71)$$

For small  $\xi$ , this shifts the fixed-point value  $\tilde{m}_{H^*}^2$  by a small amount approximately  $\xi^2$ , while the shift in  $\tilde{\lambda}_{H^*}$  is approximately  $\xi^3$ . In leading order in  $\xi$  these shifts can be neglected.

The dominant contribution of the mixing effect to the stability matrix  $T$  is a shift in the diagonal terms for  $\delta \tilde{m}_H^2$  and  $\delta \tilde{\lambda}_H$  (not for  $\delta v$ ),

$$A \rightarrow A - 2H_0 \tilde{m}_{H^*}^2.\quad (72)$$

As discussed before, it vanishes for  $\xi \rightarrow 0$ ,  $\tilde{m}_{H^*}^2 \rightarrow 0$ . With

$$H_0 = \frac{1 + \tilde{m}_{H^*}^2 + \frac{3}{4}(1 - \frac{v_{0^*}}{4})}{8\pi^2 \tilde{M}_{p^*}^2 (1 - \frac{v_{0^*}}{4})^2 (1 + \tilde{m}_{H^*}^2)^2},\quad (73)$$

we may neglect in leading order  $\tilde{m}_{H^*}^2$  in  $H_0$  and employ Eq. (61) for  $\tilde{m}_{H^*}^2$  in (72). We observe that  $H_0$  is positive such that  $A$  is enhanced for negative  $\tilde{m}_{H^*}^2$ . For  $A < 2$ , one has  $\tilde{m}_{H^*}^2 < 0$  such that the critical exponent for the scalar mass term moves closer to zero by the mixing effect.

The inclusion of the nonminimal coupling  $\xi$  further modifies the off-diagonal parts in the inverse propagator of the spin-0 metric fluctuation in (A24), such that

$$\frac{1}{2}U'\phi \rightarrow \frac{1}{2}(-\xi q^2 + U')\phi. \quad (74)$$

This effect induces a shift in the anomalous dimension, adding to (72) a further piece,

$$A \rightarrow A - 2H_0\tilde{m}_{H^*}^2 + \Delta A_i, \quad (75)$$

with

$$\begin{aligned} \Delta A_v &= 0, & \Delta A_m &= \frac{3\xi(\xi + 2)}{32\pi^2\tilde{M}_p^2(1 - v_0)^2}, \\ \Delta A_\lambda &= \frac{3\xi(3\xi + 2)}{16\pi^2\tilde{M}_p^2(1 - v_0)^2}. \end{aligned} \quad (76)$$

Even if the fixed-point values of  $\tilde{m}_{H^*}^2$  and  $\tilde{\lambda}_{H^*}$  are zero, this modification differs from zero as long as  $\xi \neq 0$ .

A characteristic quantity for the field dependence of the effective Planck mass is  $\xi/\tilde{M}_p^2$ . For  $\xi/\tilde{M}_p^2 \ll 1$ , the field dependence is weak, while for  $\xi/\tilde{M}_p^2 \gtrsim 1$ , the Planck mass varies very rapidly with the scalar field. Figure 6 displays the dependence of the critical exponents on  $\xi/\tilde{M}_p^2$  by evaluating the eigenvalues of the stability matrix (63) including the shift (75) in the diagonal elements. We use the value of the dimensionless Planck mass  $\tilde{M}_{p^*} = 0.22$  at which we have the anomalous dimension  $A \approx 2.78$  for vanishing  $\xi$ . The dominant effect arises from the nonzero values of  $\tilde{m}_{H^*}^2$  and  $\tilde{\lambda}_{H^*}$ . In a realistic setting, also gauge and Yukawa couplings influence these fixed-point values. If the difference of  $\tilde{m}_{H^*}$  and  $\tilde{\lambda}_{H^*}$  from zero can be neglected, the effect of  $\xi$  is much smaller, given by (76). For a main result, we find that the dependence of the critical exponents on  $\xi$  is small, supporting our conclusions from the previous sections.

## B. Regulator dependence

The functional flow equation is exact for an arbitrary choice of the regulator function  $\mathcal{R}_k$ . Provided that  $\mathcal{R}_k$  obeys the requirements for an efficient IR cutoff, with  $\partial_t \mathcal{R}_k$  decaying fast for high momenta, the results for observable quantities should not depend on the choice of  $\mathcal{R}_k$ . In any practical calculation, they do, however, and this is due to the choice of a truncation. This observation can be used for a test of validity of a given truncation. The dependence on  $\mathcal{R}_k$  should disappear for a ‘‘perfect truncation,’’ and any remaining dependence can be taken as some form of measure for the error induced by the truncation.

For a general cutoff function, the graviton contribution to the flow of the potential is given by the threshold function  $\ell_0^A$ ,

$$\tilde{\pi}_2 = \frac{5k^4}{16\pi^2} \ell_0^A(-v_0), \quad (77)$$

with  $\ell_0^A(\tilde{w})$  defined in terms of the propagator  $G(q^2)$  as

$$\begin{aligned} \ell_0^A(\tilde{w}) &= \frac{8\pi^2}{k^4} \int_q \partial_t \mathcal{R}_k(q^2) G(q^2) \\ &= \frac{1}{2} \int_0^\infty dx x (p(x) + \tilde{w})^{-1} f(x) r(x), \end{aligned} \quad (78)$$

with  $x = q^2/k^2$  and

$$\begin{aligned} p(x) &= x + r(x), & r_k(x) &= \frac{4\mathcal{R}_k}{M_p^2 k^2}, & \eta_M &= \frac{\partial_t M_p^2}{M_p^2}, \\ f_k(x) &= \frac{\partial_t \mathcal{R}_k}{\mathcal{R}_k} = 2 + \eta_M - 2 \frac{\partial \ln r}{\partial \ln x}. \end{aligned} \quad (79)$$

For the Litim cutoff, one takes

$$\mathcal{R}_k = \frac{M_p^2}{4} (k^2 - q^2) \theta(k^2 - q^2), \quad (80)$$

with

$$f_k(x)r_k(x) = 2 + \eta_M(1 - x) \quad (81)$$

and

$$p(x) = \begin{cases} 1 & \text{for } x < 1, \\ x & \text{for } x > 1. \end{cases} \quad (82)$$

This results in the threshold function

$$\ell_0^A(\tilde{w}) = \left( \frac{1}{2} + \frac{\eta_M}{12} \right) (1 + \tilde{w})^{-1}. \quad (83)$$

For  $\eta_M = 2$ , corresponding to  $\eta_g = 0$ , we recover (3).

For a general cutoff function, one replaces in the flow equation for  $\tilde{U}$

$$(1 - v_0)^{-1} \rightarrow \frac{3}{2} \ell_0^A(-v_0), \quad (84)$$

a similar replacement is performed for the scalar contributions. For the anomalous dimension  $A$ , one has to replace

$$(1 - v_0)^{-2} \rightarrow \frac{3}{2} \ell_1^A(-v_0), \quad (85)$$

with

$$\ell_1^A(\tilde{w}) = -\frac{\partial \ell_0^A(\tilde{w})}{\partial \tilde{w}}. \quad (86)$$

Since  $A$  is dominated by the graviton contribution, a good estimate of the effect of a general cutoff function is the multiplication of Eq. (22) by a factor  $f$ ,

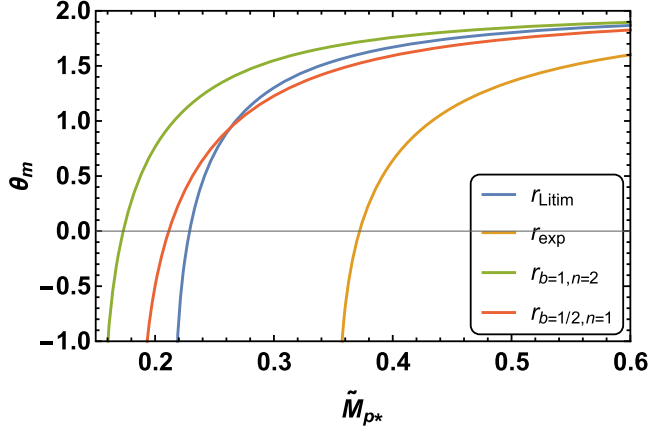


FIG. 7. Critical exponent of the scalar mass term as a function of the fixed-point value of the Planck mass for different regulators. The Litim regulator is depicted in blue, the exponential regulator is in yellow,  $r_{\text{int}}(x, b = 1, n = 2)$  is in green, and  $r_{\text{int}}(x, b = \frac{1}{2}, n = 1)$  is in red. The main effect is the expected multiplicative factor in the value of  $\tilde{M}_{p^*}$ .

$$f = \frac{3}{2} \ell_1^4 (-v_0)(1 - v_0)^2. \quad (87)$$

For the Litim cutoff,  $f = 1$ . Correspondingly, the value of  $\tilde{M}_{p^*}^2$  needed to realize a given  $A$  has to be multiplied by  $f$ . This feature is not surprising. A dominant effect of a change in the IR-cutoff function can be viewed as an effective rescaling of  $k$ , which may be absorbed by a multiplicative redefinition of  $k$ . The dimensionless ratio  $\tilde{M}_p^2 = M_p^2/k^2$  is directly affected by such a rescaling. This property demonstrates that our general results hold independently of the precise choice of the cutoff function.

Let us investigate the regulator dependence numerically. To this end, we use an exponential interpolating cutoff function for the explicit regulator comparison

$$r_{\text{int}}(x, b, n) = \frac{(1 - bx)x^{n-1}}{\exp(x^n) - 1}. \quad (88)$$

This cutoff function has the limits  $r_{\text{int}}(x, b = 1, n \rightarrow \infty) = r_{\text{Litim}}(x) = (x^{-1} - 1)\theta(1 - x)$  as well as  $r_{\text{int}}(x, b = 0, n = 1) = r_{\text{exp}}(x) = (\exp(x) - 1)^{-1}$ . We further use the regulator at the values  $r_{\text{int}}(x, b = 1, n = 2)$  as well as  $r_{\text{int}}(x, b = \frac{1}{2}, n = 1)$ . The results are displayed in Fig. 7. One can see that the fact that the critical exponent of the scalar mass  $\theta_m = 2 - A$  becomes negative for a certain value of the dimensionless Planck mass is not changed. The dominant effect is indeed a simple rescaling of  $\tilde{M}_{p^*}^2$ . For a given model and a given truncation,  $\tilde{M}_{p^*}^2$  will depend sensitively on the chosen cutoff function. For a valid truncation, this cutoff dependence should drop out in the final value or  $\theta_m$ . A computation of the flow equation for  $\tilde{M}_p^2$  will be needed for this check.

## VI. DISCUSSION

We have computed quantum gravity predictions for the mass and couplings of the Higgs scalar within the asymptotic safety scenario. We consider the SM of particle physics coupled to gravity, with possible extensions of the particle content. The value of the dimensionless flowing Planck mass at the fixed point,  $\tilde{M}_p^2 = M_p^2(k)/k^2$ , is influenced by the particle content of the model. We treat it here as an unknown parameter, to be determined for any given model.

Our main findings are the following:

- (i) The quartic self-coupling  $\tilde{\lambda}_H$  of the Higgs scalar is an irrelevant coupling. Its value at the Planck scale is predicted to be very close to zero. For a given low-energy model below the Planck scale, where gravitational contributions decouple, this initial value at the Planck scale is mapped by the renormalization flow to the value at the Fermi scale. The ratio between Higgs-boson mass and top-quark mass is therefore predicted. This prediction works well if the low-energy theory is the SM. The consistency of other low-energy models has to be tested.
- (ii) The dimensionless mass term  $\tilde{m}_H^2$  for the Higgs scalar can be a relevant or an irrelevant coupling, depending on the fixed-point value  $\tilde{M}_p^2$  and on the d.o.f., as shown in Fig. 1. If  $\tilde{m}_H^2$  is relevant, the value of the Fermi scale, or more precisely the ratio between Fermi scale and Planck scale  $M_W/M_p$ , cannot be predicted. For a relevant  $\tilde{m}_H^2$ , the gauge hierarchy is a free parameter. In contrast, if  $\tilde{m}_H^2$  is irrelevant, the model predicts that nature is located on the critical surface of the vacuum electroweak phase transition, with only a small deviation induced by running gauge and Yukawa couplings. Depending on the model, this may overpredict the gauge hierarchy to be  $10^{-40}$  instead of  $10^{-34}$ .
- (iii) For a given particle content, as expressed by  $N$ , we find a lower bound on the fixed-point value of  $\tilde{M}_p^2$  if  $N > -4$ .

These results extend to other quantum field theories with scalar fields coupled to gravity, such as extensions of the SM or grand unified theories. The gravitational contribution to the critical exponents is universal. Quartic scalar couplings are irrelevant parameters at the UV fixed point.

Our findings are of high relevance for the interplay between quantum gravity and particle physics. The validity of approximations and the robustness of results should therefore be critically questioned. First of all, quantum gravity contributions to the effective scalar potential can be performed in flat space, allowing for the full use of Euclidean  $\text{SO}(4)$  symmetry or Lorentz symmetry for Minkowski space. Second, our split of gravitational fluctuations into physical modes and gauge modes, together with a physical gauge fixing acting only on the gauge



modes, makes the contributions of different modes very transparent. We find that the dominant contributions in the gravitational sector come from the fluctuations of the graviton or traceless transverse tensor fluctuations.

The quantity needed for a reliable computation of the dominant graviton contribution is the exact propagator of the graviton. This is not directly available, and at this point, an approximation is made. In the Einstein-Hilbert truncation employed in the present paper, the inverse graviton propagator is given by

$$G^{-1}(q^2) = \frac{M_{\text{p}}^2 q^2}{4} - \frac{U}{2}, \quad (89)$$

and the question arises if this is a reasonable approximation. The inverse graviton propagator at zero momentum is given by the effective potential

$$G^{-1}(q^2 = 0) = -\frac{U}{2}. \quad (90)$$

For a diffeomorphism invariant formulation, this is an exact relation enforced by diffeomorphism symmetry [78]. The right-hand side of the flow equation involves a momentum integral that is dominated by momenta with  $q^2 \approx k^2$ . We may therefore *define* the parameter  $M_{\text{p}}^2(k)$  by the graviton propagator at  $q^2 = k^2$ , more precisely by

$$M_{\text{p}}^2(k) = \frac{4}{k^2} (G^{-1}(q^2 = k^2) - G^{-1}(q^2 = 0)). \quad (91)$$

This definition goes beyond the Einstein-Hilbert truncation since the contribution of higher-derivative invariants as the squared Weyl tensor can be included in  $G^{-1}(q^2 = k^2)$ . A similar definition was already used in Refs. [41,43–45, 48,50,52,53], and the contributions of higher-derivative invariants were in particular investigated in Refs. [48,53]. Since we treat the fixed-point value of  $M_{\text{p}}^2(k)/k^2$  as a free parameter, we may reinterpret our results as reflecting the definition (91). We note that pure graviton vertices do not appear in the computation of the flow of the effective potential. They would differentiate between different terms in the gravitational sector. All this suggests that our estimate of the dominant graviton contribution, which is not affected by any gauge-fixing issues, is quantitatively rather robust.

Another robust result is the contribution from the measure sector (gauge fluctuations and ghosts) if physical gauge fixing is employed. The measure contribution results in a simple field-independent term for the flow of  $U$ . This term is necessary in order to account for the correct counting of physical d.o.f. The contributions from the matter sector are well understood as well. These are the well-tested standard contributions to the flow in flat space. One can extend their contribution to include Yukawa and gauge interactions with the Higgs field. As long as gauge

couplings and Yukawa couplings remain small in the UV fixed point region, these interaction effects on the flow of  $U$  correspond to the standard perturbative beta function. The effect of the matter fluctuations on the field dependence of  $U$  is much smaller than the gravitational contribution. As far as the critical exponents for  $\tilde{\lambda}_H$  and  $\tilde{m}_H^2$  are concerned, the interactions in the matter sector give only tiny corrections.

In the limit of a constant  $\tilde{M}_{\text{p}^*}^2$  and constant scalar wave function renormalization, the only term that is perhaps subject to somewhat larger truncation errors is the contribution from the physical scalar fluctuations of the metric  $\tilde{\pi}_{0,g}$ . Unless strong cancellations occur, this contribution is only a rather small fraction of the graviton, matter, and measure contributions. The uncertainty in the computation of this term will only result in a rather modest quantitative uncertainty for our overall results.

Replacing for the scaling form of the coefficient of the curvature scalar the constant  $\tilde{M}_{\text{p}^*}^2$  by a function  $F(\tilde{\rho})$  of the scalar field induces additional terms in the flow equation for the mass term and quartic coupling, without affecting much the anomalous dimension  $A$ . A small shift in the fixed-point value of  $\tilde{\lambda}_H$  and in the location of the critical surface has no sizeable impact on the critical exponents. Finally, the omitted flow of the scalar wave function renormalization shifts  $\theta_m$  from  $2 - A$  to  $2 - A - \eta_\phi$  and  $\theta_\lambda$  from  $-A$  to  $-A - 2\eta_\phi$ . This would be a sizeable effect only if  $\eta_\phi$  is of a similar magnitude as  $A$ .

We conclude that at least our Euclidean computation of the flow of the effective potential for the Higgs scalar seems rather reliable. A computation directly in Minkowski space along the lines discussed in Ref. [76] would be welcome, but we do not expect important modifications as compared to the Euclidean results. The scalar potential is particularly robust with respect to analytic continuation since no momenta are involved. The remaining big issue is the determination of the fixed-point value of the dimensionless Planck mass  $\tilde{M}_{\text{p}}^2$ . This depends on  $v_0$ , rendering the flow of  $\tilde{M}_{\text{p}}^2$  and  $v_0$  a coupled system. Only once the fixed point of the combined system is found, it can be decided where a model is situated in the plane of Fig. 1 or on the curves of Figs. 2–5. Various computations give values of  $\tilde{M}_{\text{p}}^2$ . In the context of the present paper, the use of a physical gauge fixing is appropriate, and results from a gauge invariant formulation of the flow equation would be most welcome.

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## APPENDIX: EFFECTIVE ACTION AND FORMULATION

In this Appendix, we derive the flow equation for the effective scalar potential (2) as an approximation to the exact flow equation for the effective average action. The exact flow [2,89–92] takes a simple one-loop form:

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \text{Tr}[(\Gamma_k^{(2)}[\Phi] + \mathcal{R}_k)^{-1} \partial_t \mathcal{R}_k]. \quad (\text{A1})$$

Here,  $\mathcal{R}_k$  is an infrared regulator function, and  $\partial_t = k \partial_k$ . The trace in (A1) sums over momenta and internal space indices of a multifield  $\Phi$ , and the matrix of second functional derivatives  $\Gamma_k^{(2)}$  is the full inverse propagator of  $\Phi$ . For reviews, see Refs. [93–101]. We apply this equation to a model of a singlet real scalar field and gravity and derive the flow equation for the effective scalar field. Generalizations for additional fields are found in the main text.

### 1. Setup

We investigate an effective action of the type

$$\Gamma_k = \Gamma_k^{\text{gravity}} + \Gamma_k^{\text{Higgs}}. \quad (\text{A2})$$

In the gravity sector, we employ the Einstein-Hilbert truncation

$$\Gamma_k^{\text{gravity}} = -\frac{M_p^2}{2} \int d^4x \sqrt{g} R + S_{\text{gf}} + S_{\text{gh}}, \quad (\text{A3})$$

where  $M_p$  is the reduced Planck mass related to Newton's constant by  $M_p^2 = 1/8\pi G_N$ . The cosmological constant is included in the scalar effective potential. The metric is linearly expanded around a fixed background,

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \quad (\text{A4})$$

where  $\bar{g}_{\mu\nu}$  is a constant background metric and  $h_{\mu\nu}$  is a fluctuation field. We will later use a flat Euclidean background,  $\bar{g}_{\mu\nu} = \delta_{\mu\nu}$ . The gauge fixing and the ghost action for diffeomorphism symmetry are given by

$$S_{\text{gf}} = \frac{1}{2\alpha} \int d^4x \sqrt{\bar{g}} \bar{g}^{\mu\nu} \Sigma_\mu \Sigma_\nu, \quad S_{\text{gh}} = - \int d^4x \sqrt{\bar{g}} \bar{C}_\mu \left[ \bar{g}^{\mu\rho} \bar{\nabla}^2 + \frac{1-\beta}{2} \bar{\nabla}^\mu \bar{\nabla}^\rho + \bar{R}^{\mu\rho} \right] C_\rho, \quad (\text{A5})$$

where  $C$  and  $\bar{C}$  are ghost and antighost fields. A class of general gauge fixings is given by

$$\Sigma_\mu = \bar{\nabla}^\nu h_{\nu\mu} - \frac{\beta+1}{4} \bar{\nabla}_\mu h, \quad (\text{A6})$$

where  $h = \bar{g}^{\mu\nu} h_{\mu\nu}$  is the trace mode. Bars denote covariant derivatives, etc., formed with the background metric. Note that there are two gauge-fixing parameters for diffeomorphism symmetry,  $\alpha$  and  $\beta$ . The parameter  $\beta$  is dimensionless, whereas  $\alpha$  has mass dimension minus 2. For the physical gauge fixing, they are given by  $\beta = -1$  and  $\alpha \rightarrow 0$ . We first keep general  $\alpha$  and  $\beta$  in order to see the particular role of the physical gauge fixing explicitly.

Next, we turn to the effective action for the Higgs sector. In the SM, the Higgs field is a component of the doublet field, coupled to the  $SU(2)_L$  and  $U(1)_Y$  gauge fields as well as to quarks and leptons. Near the UV fixed point, the contributions from these couplings to the beta function are smaller than the ones of the graviton. All essential points can be understood by restricting the discussion to a single real scalar field with  $\mathbb{Z}_2$  symmetry as representing the physical mode of the Higgs boson. The effective action takes the standard form

$$\Gamma_k^{\text{Higgs}} = \int d^4x \sqrt{g} \left[ U(\rho) + \frac{Z_\phi}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right]. \quad (\text{A7})$$

We subsequently extend our findings to the SM or possible extensions.

The effective potential  $U(\rho)$  depends only on the invariant  $\rho = \phi^2/2$ . The value of  $U$  at the minimum can be identified with the cosmological constant. We are interested in momenta much larger than the Fermi scale. The expectation value of  $\phi$  can be neglected in this range, and we expand

$$U = V + m_H^2 \rho + \frac{1}{2} \lambda_H \rho^2 + \dots, \quad (\text{A8})$$

where  $m_H^2$  is the mass term of the Higgs boson and  $\lambda_H$  is the quartic coupling. The field-renormalization factor of  $\phi$  is denoted by  $Z_\phi$ .

### 2. Physical metric fluctuations

A crucial quantity for the flow equation is the inverse propagator, i.e., the matrix of second functional derivatives of  $\Gamma_k$ . The physical understanding as well as calculational simplicity are greatly enhanced if we split the metric fluctuations into physical and gauge fluctuations [68]. In flat space,  $\bar{g}_{\mu\nu} = \delta_{\mu\nu}$ , one can use a momentum space representation.

Let us start with splitting the metric fluctuations into

$$h_{\mu\nu} = f_{\mu\nu} + a_{\mu\nu}, \quad (\text{A9})$$

where  $f_{\mu\nu}$  are the physical metric fluctuations, which satisfy the transverse constraint  $q^\mu f_{\mu\nu} = 0$ . The physical metric fluctuations can be decomposed into two independent fields as

$$f_{\mu\nu} = t_{\mu\nu} + s_{\mu\nu}, \quad (\text{A10})$$

where the graviton  $t_{\mu\nu}$  is the transverse and traceless (TT) tensor, i.e.,  $q^\mu t_{\mu\nu} = \delta^{\mu\nu} t_{\mu\nu} = 0$ . The tensor  $s_{\mu\nu}$  is given as a linear function of a scalar field  $\sigma$  such that

$$s_{\mu\nu} = \frac{1}{3} P_{\mu\nu} \sigma, \quad (\text{A11})$$

where we define the projection operator

$$P_{\mu\nu} = \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}. \quad (\text{A12})$$

Similarly, the gauge modes or unphysical metric fluctuations  $a_{\mu\nu}$  are decomposed into a transverse vector mode  $\kappa_\mu$  satisfying  $q^\mu \kappa_\mu = 0$  and a scalar mode  $u$ . In summary, the metric fluctuations (A9) are parametrized by

$$\begin{aligned} f_{\mu\nu} &= t_{\mu\nu} + \frac{1}{3} P_{\mu\nu} \sigma, \\ a_{\mu\nu} &= i(q_\mu \kappa_\nu + q_\nu \kappa_\mu) + \frac{q_\mu q_\nu}{q^2} u. \end{aligned} \quad (\text{A13})$$

Using the linear combinations

$$\sigma = \frac{3}{4}(h + q^2 s), \quad u = \frac{1}{4}(h - 3q^2 s), \quad (\text{A14})$$

we obtain the York decomposition [102] of the fluctuation field

$$\begin{aligned} h_{\mu\nu} &= t_{\mu\nu} + i(q_\mu \kappa_\nu + q_\nu \kappa_\mu) \\ &\quad - \left( q_\mu q_\nu - \frac{1}{4} \eta_{\mu\nu} q^2 \right) s + \frac{1}{4} \eta_{\mu\nu} h, \end{aligned} \quad (\text{A15})$$

where  $h = \delta^{\mu\nu} h_{\mu\nu}$ . We see that the scalar modes  $s$  and  $h$  in the York decomposition are given as a mixture of the physical scalar mode  $\sigma$  and the gauge mode  $u$ . The connection of the physical metric fluctuations  $f_{\mu\nu}$  to the gauge invariant Bardeen potentials generally used in cosmology can be found in Ref. [68].

The decomposition yields Jacobians that read

$$J_{\text{grav}} = [\det'_{(1)}(q^2)]^{1/2}, \quad J_{\text{gh}} = [\det''_{(0)}(q^2)]^{-1}, \quad (\text{A16})$$

where a prime denotes a subtraction of the zero eigenmode. These contributions are taken into account by introducing auxiliary fields

$$\begin{aligned} J_{\text{grav}} &= \int \mathcal{D}\chi \mathcal{D}\zeta \mathcal{D}\bar{\zeta} \\ &\quad \times \exp \left\{ - \int_q \left[ \frac{1}{2} \chi_\mu (q^2)' \chi^\mu - \bar{\zeta}_\mu (q^2)' \zeta^\mu \right] \right\}, \end{aligned} \quad (\text{A17})$$

$$J_{\text{gh}} = \int \mathcal{D}\bar{\varphi} \mathcal{D}\varphi \exp \left[ - \int_q \bar{\varphi}(q^2)' \varphi \right], \quad (\text{A18})$$

where  $\chi_\mu$  is a real bosonic vector field,  $(\bar{\varphi}, \varphi)$  are complex bosonic scalar fields, and  $(\bar{\zeta}_\mu, \zeta^\mu)$  are vector anticommuting ghosts.

In flat space, the matrix of second functional derivatives  $\Gamma_k^{(2)}$  becomes block diagonal in the different representations of the Lorentz group, e.g.,  $t_{\mu\nu}$ ,  $\kappa_\mu$ , and the scalar fields  $(\phi, \sigma, u)$ . We will see that for the physical gauge,  $\beta = -1$  and  $\alpha \rightarrow 0$ , it also becomes block diagonal in the physical fluctuations and gauge fluctuations. Thus, the scalar sectors decouple into separate sectors of  $(\phi, \sigma)$  and the gauge mode  $u$ . Finally, for a vanishing expectation value of  $\phi$ , the physical scalar sector becomes also block diagonal since  $\sigma$  and  $\phi$  belong to different representations of the  $\mathbb{Z}_2$  symmetry. This reflects the different representations of the Higgs doublet and the singlet contained in the metric.

For the  $t_{\mu\nu}$ -mode, we get

$$(\Gamma_{(tt)}^{(2)})^{\mu\nu\rho\sigma} = \frac{M_{\text{p}}^2}{4} \left[ q^2 - \frac{2U}{M_{\text{p}}^2} \right] P^{(t)\mu\nu\rho\sigma}, \quad (\text{A19})$$

where the TT-projection operator reads

$$P^{(t)\mu\nu\rho\sigma} = \frac{1}{2} (P^{\mu\rho} P^{\nu\sigma} + P^{\mu\sigma} P^{\nu\rho}) - \frac{1}{3} P^{\mu\nu} P^{\rho\sigma}. \quad (\text{A20})$$

The Hessian for  $\kappa_\mu$  is given by

$$(\Gamma_{(\kappa\kappa)}^{(2)})^{\mu\nu} = \frac{1}{\alpha} q^2 [q^2 - \alpha U] P^{(v)\mu\nu}, \quad (\text{A21})$$

with  $P^{(v)}$  the projection operator on the vector mode,  $P^{(v)\mu}{}_\mu = 3$ .

In the  $(\sigma, u, \phi)$ -basis, the Hessian for the scalar modes becomes

$$\Gamma_{(00)}^{(2)} = \begin{pmatrix} & \frac{U'\phi}{2} & \\ (\Gamma_{(00)}^{(2)})_{\text{grav}} & & \frac{U'\phi}{2} \\ \frac{U'\phi}{2} & \frac{U'\phi}{2} & Z_\phi q^2 + U' + 2\rho U'' \end{pmatrix}, \quad (\text{A22})$$

where the spin-0 gravitational part is given by the  $2 \times 2$  matrix

$$(\Gamma_{(00)}^{(2)})_{\text{grav}} = \begin{pmatrix} -\frac{M_p^2}{6} \left( q^2 - \frac{U}{2M_p^2} \right) + \frac{(\beta+1)^2}{16\alpha} q^2 & \frac{U}{4} + \frac{(\beta+1)(\beta-3)}{16\alpha} q^2 \\ \frac{U}{4} + \frac{(\beta+1)(\beta-3)}{16\alpha} q^2 & -\frac{U}{4} + \frac{(\beta-3)^2}{16\alpha} q^2 \end{pmatrix}. \quad (\text{A23})$$

The choice of the gauge parameter  $\beta = -1$  eliminates the off-diagonal terms in the matrix (A23). Furthermore, the  $\sigma$ -mode becomes independent of  $\alpha$ . Thus, the whole sector of physical metric fluctuations becomes independent of  $\alpha$ . For the choice  $\beta = -1$ , the gauge fixing function (A6) becomes  $\Sigma_\mu = \bar{\nabla}^\nu h_{\nu\mu}$ . Therefore, the choice  $\beta = -1$  is a gauge fixing for which the gauge  $\bar{\nabla}^\nu h_{\nu\mu} = 0$  satisfies the transverse condition within the Faddeev-Popov method.

Taking furthermore the limit  $\alpha \rightarrow 0$ , and therefore realizing the physical gauge, the  $u$ -mode, i.e., the lower right element of the matrix (A23), is dominated by  $q^2/\alpha$ . In this limit, the finite part of this element (which involves  $U$ ) no longer contributes after the inversion of  $(\Gamma_k^{(2)} + \mathcal{R}_k)$ . The same thing holds for the mixing given as  $U'\phi/2$  with the physical modes in the  $3 \times 3$  matrix (A22). For the physical gauge, one therefore deals with a decoupled gauge mode with inverse propagator  $q^2/\alpha$  and two physical scalars with inverse propagator matrix

$$(\Gamma_{(00)}^{(2)})_{\text{ph}} = \begin{pmatrix} -\frac{M_p^2}{6} \left( q^2 - \frac{U}{2M_p^2} \right) & U'\phi/2 \\ U'\phi/2 & Z_\phi q^2 + U' + 2\rho U'' \end{pmatrix}. \quad (\text{A24})$$

Finally, for  $\alpha \rightarrow 0$ , the inverse propagator for the gauge-vector mode also becomes independent of  $U$ ,

$$(\Gamma_k^{(2)})_{\kappa\kappa}^{\mu\nu} = \frac{q^4}{\alpha} P^{(v)\mu\nu}. \quad (\text{A25})$$

We conclude that the total contribution of the gauge modes (vector + scalar) gives a contribution that is independent of  $\phi$  and independent of all parameters in the effective action.

A similar property holds for the ghost contribution and the contribution from the Jacobians, (A17) and (A18). For an explicit computation, we decompose the ghost field as

$$C_\mu = C_\mu^\perp + iq_\mu C, \quad \bar{C}_\mu = \bar{C}_\mu^\perp + iq_\mu \bar{C}, \quad (\text{A26})$$

where  $C_\mu^\perp$  ( $\bar{C}_\mu^\perp$ ) is the transverse (anti)ghost field and  $C$  ( $\bar{C}$ ) is the scalar (anti)ghost field. The Hessians for the ghost fields is

$$\begin{aligned} (\Gamma_{(\bar{C}^\perp C^\perp)}^{(2)})^{\mu\nu} &= -q^2 P^{(v)\mu\nu}, \\ \Gamma_{(\bar{C}C)}^{(2)} &= \left( 2 - \frac{1+\beta}{2} \right) q^4. \end{aligned} \quad (\text{A27})$$

We have demonstrated the decoupling of the gauge fluctuations for the physical gauge choice for the particular case of a flat background geometry. This property holds actually for a general background geometry, as advocated in Ref. [59].

### 3. Flow generator from physical fluctuations

Let us next investigate the structure of the flow equation in case of the decomposition presented in the previous subsection. Since the sector of physical fluctuations decouples from the ones for the gauge modes, we can treat their contributions separately, provided we choose a cutoff function that respects this decomposition. This is achieved by a block-diagonal cutoff scheme with a physical cutoff  $\mathcal{R}_k^{(\text{ph})}$  that only acts on the fluctuations  $f_{\mu\nu}$  and  $\phi$  and a gauge cutoff  $\sim \alpha^{-1} \mathcal{R}_k^{(g)}$  that only involves the gauge modes of the metric. The contributions of the gauge fluctuations can be combined with the contributions of ghosts and Jacobians to a total measure contribution. We write the general structure as

$$\partial_t \Gamma_k = \zeta_k = \pi_k + \eta_k, \quad (\text{A28})$$

with  $\pi_k$  the physical mode contribution and  $\eta_k$  the measure contribution.

For flat spacetime, we employ cutoff functions that replace for each mode  $q^2$  by  $P_k(q^2) = q^2 + R_k(q^2)$ . This can be generalized by replacing  $q^2$  with an appropriate covariant differential operator. From the TT-mode  $t_{\mu\nu}$ , one finds a contribution

$$\pi_2 = \frac{1}{2} \text{Tr}_{(2)} \frac{\partial_t \mathcal{R}_k}{\Gamma_k^{(2)} + \mathcal{R}_k} \Big|_{tt} = \frac{5}{2} \int_q \frac{\partial_t (M_p^2 R_k)}{M_p^2 (P_k - k^2 v)}. \quad (\text{A29})$$

Here,  $\int_q = (2\pi)^{-4} \int d^4 q$ , and the factor 5 comes from the trace of  $P^{(t)}$ , corresponding to the five independent d.o.f. in  $t_{\mu\nu}$ . The dimensionless quantity  $v(\rho)$  is defined as

$$v(\rho) = \frac{2U(\rho)}{M_p^2 k^2}. \quad (\text{A30})$$

The contribution of the two physical scalar fluctuations takes the form



$$\pi_0 = \frac{1}{2} \int_q \tilde{\partial}_t \ln \left\{ 3\rho U^2 + \left( M_p^2 P_k - \frac{U}{2} \right) (Z_\phi P_k + U' + 2\rho U'') \right\}, \quad (\text{A31})$$

where  $\tilde{\partial}_t$  is the logarithmic derivative  $k\partial_k$  acting only on  $M_p^2 P_k$  and  $Z_\phi P_k$ , not on  $U$  or  $\rho$ . The mixing effects are small for the range of  $\rho$  where

$$3\rho U^2 \ll \left( M_p^2 P_k - \frac{U}{2} \right) (Z_\phi P_k + U' + 2\rho U''). \quad (\text{A32})$$

In this range,  $\pi_0 = \pi_{0,g} + \pi_{0,\phi}$  decouples into two separate parts. The gravitational scalar contributes

$$\pi_{0,g} = \frac{1}{2} \int_q \frac{\partial_t (M_p^2 R_k)}{M_p^2 (P_k - k^2 v/4)}. \quad (\text{A33})$$

This contribution is similar to the tensor contribution  $\pi_2$ . It is suppressed by a factor 1/5, reflecting the single d.o.f., and a smaller enhancement of the denominator for positive  $v$ . The contribution of  $\phi$ ,

$$\pi_{0,\phi} = \frac{1}{2} \int_q \frac{\partial_t (Z_\phi R_k)}{Z_\phi P_k + U' + 2\rho U''}, \quad (\text{A34})$$

is the standard expression for a real scalar theory. For the full SM, it will be supplemented by contributions from the Goldstone directions, the gauge bosons, and fermions.

#### 4. Measure contribution

We next turn to the measure contribution  $\eta_k$ . We have already seen that for a physical gauge fixing it is independent of  $\rho$  and  $U$ . This contribution depends therefore only on the background metric. We will establish that the total measure contribution takes for physical gauge fixing the simple overall form

$$\eta_k = -\frac{1}{2} \text{Tr}_{(1)} \frac{\partial_t P_k(\mathcal{D}_1)}{P_k(\mathcal{D}_1)} - \frac{1}{2} \text{Tr}_{(0)} \frac{\partial_t P_k(\mathcal{D}_0)}{P_k(\mathcal{D}_0)}, \quad (\text{A35})$$

with  $\mathcal{D}_1$  and  $\mathcal{D}_0$  appropriate differential operators formed with the background metric. For a general metric, they take the following forms:

$$\mathcal{D}_1 = -\bar{\nabla}^2 - \frac{\bar{R}}{4}, \quad \mathcal{D}_0 = -\bar{\nabla}^2 - \frac{\bar{R}}{4}. \quad (\text{A36})$$

Such a simple form has been proposed in Ref. [59], based on a direct regularization of the Faddeev-Popov determinant.

Let us explicitly see that the contribution of the gauge modes and ghosts are given by (A35) in the present setup. We first look at the contributions from the spin-1 gauge and ghost modes and the auxiliary field:

$$\eta_1 = \frac{1}{2} \text{Tr}_{(1)} \frac{\partial_t \mathcal{R}_k}{\Gamma_k^{(2)} + \mathcal{R}_k} \Big|_{\kappa\kappa} - \text{Tr}_{(1)} \frac{\partial_t \mathcal{R}_k}{\Gamma_k^{(2)} + \mathcal{R}_k} \Big|_{\bar{c}^\perp c^\perp} + \frac{1}{2} \text{Tr}_{(1)} \frac{\partial_t \mathcal{R}_k}{\Gamma_k^{(2)} + \mathcal{R}_k} \Big|_{\chi\chi} - \text{Tr}_{(1)} \frac{\partial_t \mathcal{R}_k}{\Gamma_k^{(2)} + \mathcal{R}_k} \Big|_{\bar{\zeta}\zeta}. \quad (\text{A37})$$

The last two terms on the right-hand side are the contributions from the Jacobian (A17) associated with the gauge mode  $\kappa_\mu$ . For  $\alpha \rightarrow 0$ , the transverse vector metric fluctuation and the contributions from the Jacobian become

$$\delta_k^{(1)} = \lim_{\alpha \rightarrow 0} \frac{1}{2} \text{Tr}_{(1)} \frac{\partial_t \mathcal{R}_k}{\Gamma_k^{(2)} + \mathcal{R}_k} \Big|_{\kappa\kappa} + \frac{1}{2} \text{Tr}_{(1)} \frac{\partial_t \mathcal{R}_k}{\Gamma_k^{(2)} + \mathcal{R}_k} \Big|_{\chi\chi} - \text{Tr}_{(1)} \frac{\partial_t \mathcal{R}_k}{\Gamma_k^{(2)} + \mathcal{R}_k} \Big|_{\bar{\zeta}\zeta} = \frac{1}{2} \text{Tr}_{(1)} \frac{\partial_t P_k}{P_k}. \quad (\text{A38})$$

The contribution from the vector ghost mode takes the form

$$-\epsilon_k^{(1)} = -\text{Tr}_{(1)} \frac{\partial_t \mathcal{R}_k}{\Gamma_k^{(2)} + \mathcal{R}_k} \Big|_{\bar{c}^\perp c^\perp} = -\text{Tr}_{(1)} \frac{\partial_t P_k}{P_k}. \quad (\text{A39})$$

One finds a simple relation between the contributions from the gauge mode and the ghost field [59],

$$\epsilon_k^{(1)} = 2\delta_k^{(1)}. \quad (\text{A40})$$

The total contribution from spin-1 gauge modes is given by

$$\eta_1 = \delta_k^{(1)} - \epsilon_k^{(1)} = -\frac{1}{2} \text{Tr}_{(1)} \frac{\partial_t P_k}{P_k}. \quad (\text{A41})$$

A different normalization of the vector field does not change this result. If we redefine the transverse vector metric fluctuation as  $\tilde{\kappa}_\mu = \sqrt{q^2} \kappa_\mu$ , the contributions from the Jacobian in (A37) are eliminated. Instead, the contributions from the transverse vector metric fluctuation (A38) should be multiplied by a factor 1/2. Consequently, the contributions from the spin-1 modes yield the result (A41) independently of the field normalization.

Next, we discuss the contributions from the spin-0 modes involved in  $\eta_k$ . We have

$$\eta_0 = \frac{1}{2} \text{Tr}_{(0)} \frac{\partial_t \mathcal{R}_k}{\Gamma_k^{(2)} + \mathcal{R}_k} \Big|_{\text{gauge}} - \text{Tr}_{(0)} \frac{\partial_t \mathcal{R}_k}{\Gamma_k^{(2)} + \mathcal{R}_k} \Big|_{\bar{c}c} + \text{Tr}_{(0)} \frac{\partial_t \mathcal{R}_k}{\Gamma_k^{(2)} + \mathcal{R}_k} \Big|_{\bar{\varphi}\varphi}. \quad (\text{A42})$$

The last term on the right-hand side corresponds to the contribution from the auxiliary fields for the Jacobian (A18) associated with the spin-0 ghost mode. Here, we

denote the first term on the right-hand side corresponding to the gauge scalar mode of the metric fluctuation as

$$\begin{aligned}\delta_k^{(0)} &= \lim_{\alpha \rightarrow 0} \frac{1}{2} \text{Tr}_{(0)} \frac{\partial_t \mathcal{R}_k}{\Gamma_k^{(2)} + \mathcal{R}_k} \Big|_{\text{gauge}} \\ &= \frac{1}{2} \text{Tr}_{(0)} \frac{\partial_t P_k}{P_k}.\end{aligned}\quad (\text{A43})$$

The spin-0 ghost and the auxiliary fields give

$$\begin{aligned}-\epsilon_k^{(0)} &= -\text{Tr}_{(0)} \frac{\partial_t \mathcal{R}_k}{\Gamma_k^{(2)} + \mathcal{R}_k} \Big|_{\bar{c}c} + \text{Tr}_{(0)} \frac{\partial_t \mathcal{R}_k}{\Gamma_k^{(2)} + \mathcal{R}_k} \Big|_{\bar{\varphi}\varphi} \\ &= -\text{Tr}_{(0)} \frac{\partial_t P_k}{P_k}.\end{aligned}\quad (\text{A44})$$

Within the spin-0 modes, a relation similar to (A40) holds, i.e.,

$$\epsilon_k^{(0)} = 2\delta_k^{(0)}.\quad (\text{A45})$$

The total spin-0 contribution from the gauge and the ghost modes is

$$\eta_0 = \delta_k^{(0)} - \epsilon_k^{(0)} = -\frac{1}{2} \text{Tr}_{(0)} \frac{\partial_t P_k}{P_k}.\quad (\text{A46})$$

Again, we may redefine the gauge scalar mode as  $\tilde{u} = u/\sqrt{q^2}$ ; the contribution from the gauge scalar mode (A43) has to be multiplied by the factor 2. On the other hand, the contributions from the Jacobian corresponding to the gauge scalar mode are modified correspondingly. Therefore, the total contributions from the gauge scalar mode and the contributions from the Jacobian do not change from (A43), and accordingly the relation (A40) holds.

In a flat background, the contributions from the gauge modes are given by

$$\eta_k = \eta_1 + \eta_0,\quad (\text{A47})$$

with

$$\eta_1 = -\frac{3}{2} \int_q \frac{\partial_t P_k(q^2)}{P_k(q^2)}, \quad \eta_0 = -\frac{1}{2} \int_q \frac{\partial_t P_k(q^2)}{P_k(q^2)},\quad (\text{A48})$$

since the differential operators  $\mathcal{D}_1$  and  $\mathcal{D}_0$  are simply given by  $q^2$  multiplied with appropriate projectors.

## 5. Flow of the scalar potential

To summarize, the flow generator (A28) consists of the four components

$$\zeta_k = \pi_2 + \pi_0 + \eta_1 + \eta_0.\quad (\text{A49})$$

With the approximation (A32), we have  $\pi_0 = \pi_{0,g} + \pi_{0,\phi}$ . In the high-momentum range and for constant  $M_p^2$  and  $Z_\phi$  and small  $v$ , all fluctuations behave as for massless particles, with contributions  $\sim \int_q (\partial_t P_k)/P_k$ . One can easily count the d.o.f.: The physical modes ( $\pi_2 + \pi_{0,g} + \pi_{0,\phi}$ ) have  $7 = 5 + 1 + 1$  d.o.f. From these degrees are subtracted the  $4 = 3 + 1$  d.o.f. from  $(\eta_1 + \eta_0)$ . The remaining  $7 - 4 = 3$  d.o.f. correspond to the three propagating modes of the system, namely, the two helicities from the graviton and 1 d.o.f. from the real scalar.

We finally evaluate the explicit form of the flow generator (A28) and (A49) for a flat background and consider a constant scalar field. Dividing out a total volume factor,  $\zeta_k$  generates directly the flow of the effective potential  $U$ ,

$$\partial_t U = \tilde{\pi}_2 + \tilde{\pi}_0 + \tilde{\eta}_1 + \tilde{\eta}_0.\quad (\text{A50})$$

We employ the Litim-type cutoff function [77] for the regulator

$$R_k = (k^2 - p^2)\theta(k^2 - p^2).\quad (\text{A51})$$

With this regulator, one can perform the momentum integrations analytically and obtain the explicit form of the beta functions. Verifying that the  $\delta$  function in  $\partial_t R_k$  does not contribute, we can use  $\partial_t P_k = \partial_t R_k = 2k^2\theta(k^2 - p^2)$ . The momentum integrations of Eqs. (A29), (A31), and (A48) yields the flow equation (3) for the effective potential, with  $\tilde{\eta}_k = \tilde{\eta}_0 + \tilde{\eta}_1$ . The measure contributions are simply given by

$$\tilde{\eta}_1 = -\frac{3k^4}{32\pi^2}, \quad \tilde{\eta}_0 = -\frac{k^4}{32\pi^2}.\quad (\text{A52})$$

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