

Hamiltonian BRST formalism for gauge fields on black hole spacetimes

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(Received 15 January 2019; published 9 April 2019)

We investigate the Becchi-Rouet-Stora-Tyutin (BRST) formalism for gauge theories on spherically symmetric black hole spacetimes, with or without a cosmological constant ($\Lambda \geq 0$). This is illustrated through the example of scalar electrodynamics. We first demonstrate that the horizons contribute additional surface terms to the Gauss law constraint of the theory when gauge transformations are not required to vanish on the horizons. We then consider the BRST invariant path integral including these surface terms following the Hamiltonian BRST formalism. We fix a radiationlike gauge which involves null components of the electromagnetic field at the horizons. We find that the presence of the surface terms in the constraint forces the ghost and gauge fixing actions to include additional terms at the horizons. The null combination of the gauge fields at the horizons is shown to modify the ghost number charge of the theory through additional terms at the horizons. We also demonstrate how one can construct a gauge-fixing fermion which generates its own nilpotent symmetry transformations, called co-BRST transformations, that leave the theory invariant. The BRST and co-BRST transformations are further used to identify dressed (gauge invariant) fields of theory, whose dressings are affected by the presence of Killing horizons. We conclude with a discussion of potential applications of our results in soft limits and thermal field theories on black hole backgrounds.

DOI: [10.1103/PhysRevD.99.085004](https://doi.org/10.1103/PhysRevD.99.085004)

I. INTRODUCTION

Gauge field theories play a central role in our understanding of interactions between elementary particles. The reasons behind their usefulness as quantum field theories valid to very short distances are renormalizability and unitarity. The Becchi-Rouet-Stora-Tyutin (BRST) symmetry [1,2] provides a very convenient tool for verifying these properties of a gauge theory. While any theory with a Hermitian Hamiltonian operator is necessarily unitary, a gauge field theory has redundant degrees of freedom (d.o.f.) which have to be eliminated by gauge fixing. Apart from a few exceptions, gauge-fixing terms introduce states of negative norm into the theory, which in turn must be eliminated by the introduction of ghost fields. A test of unitarity of a gauge theory is to verify if the action, including the gauge-fixing and ghost terms, is invariant under BRST symmetry. The corresponding conserved charge Q_{BRST} is nilpotent and defines a cohomology on the Fock space of the theory, leading to a consistent separation of physical and unphysical states, and the theory is unitary on the physical subspace. Since the redundancy

of gauge theories is manifested in the constraints, one way of constructing the BRST charge is to start with the constrained Hamiltonian of the theory and introduce ghost fields and their conjugate momenta as Lagrange multipliers for the constraints in the BRST charge [3,4]. In this paper we follow this route for gauge theories on black hole spacetimes, paying close attention to the effect of horizons.

It is known that the presence of spatial boundaries on the manifold can significantly modify the dynamics and quantization of gauge theories. However, the consequences on gauge fields arising from spatial and null boundaries can significantly differ due to the properties of gauge fields and the underlying symmetries of these surfaces. For instance, it has been recently understood that the infinite-dimensional symmetry groups of gauge and gravitational fields at null infinity \mathcal{I} on asymptotically flat spacetimes imply the existence of an infinite number of soft charges on the sphere at null infinity [5–7]. Of central importance to these results are the allowed nonvanishing gauge parameters which depend only on the angular variables. The presence of soft charges on the sphere at null infinity, along with the requirement of charge conservation, has been used to argue that soft electromagnetic and gravitational hairs should also exist on the horizons of black holes on asymptotically flat backgrounds [8,9]. These results have

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further motivated the investigation of conserved charges and currents on null surfaces in general [10–12]. Thus gauge and gravitational fields, or more generally constrained field theories, might provide further insight into our understanding of black holes.

Constrained field theories can be understood within the well established Dirac-Bergmann and BRST formalisms. Ordinarily in the case of curved backgrounds with spatial boundaries, surface terms are introduced in the Hamiltonian to provide constraints without any surface contributions [13–15]. This is consistent with the requirement that gauge parameters vanish at spatial infinity. More generally, the regularity of fields, including gauge fields, can be invoked to fix the parameters of gauge transformations at spatial boundaries [16–18]. As a consequence, the constraints of the theory involve no corrections from the boundary and any surface terms which arise from the Dirac-Bergmann formalism are identified with additional boundary conditions which must be imposed on the fields [19–21].

In the case of BRST invariant actions on manifolds with spatial boundaries, boundary conditions on the gauge fields imply a general class of boundary conditions on the ghosts in order to ensure the BRST invariance of the theory [22–25]. There have also been recent considerations of spatial bounding surfaces within the manifold, as in the case of entangling surfaces used to investigate the entanglement entropy. While the consideration of edge modes and boundary conditions on the gauge fields have important implications on entanglement entropy calculations [26–32], in all known cases with spatial boundaries the constraints of gauge theories are not modified by the presence of spatial boundaries.

The situation is different for Killing horizons, such as black hole event horizons or cosmological horizons. These are not physical boundaries even though the (timelike) Killing vector field becomes null on these 3-surfaces. Specifically, this implies that fields need not be set to zero on horizons, and therefore the parameters of gauge transformations need not vanish on these “null boundaries” as they do on spatial boundaries. In the case of null and spatial infinities, which represent the boundaries of a compactified manifold, we can however identify fall-off conditions on the fields defined on the background which can help restrict the behavior of gauge fields there. This is not the case for Killing horizons, which can represent globally defined null surfaces on black hole backgrounds. This includes the case of black hole de Sitter backgrounds, where the black hole event horizon and cosmological horizon are finitely located within the global manifold. As such, we cannot *a priori* impose any conditions on gauge fields at Killing horizons. We can only insist that gauge invariant scalars, such as those appearing in the stress tensor, are finite at the horizons.

The finiteness of gauge-invariant scalars and the arbitrariness of gauge parameters at the horizons served as the

basis of our recent work on the constrained dynamics of field theories on curved backgrounds with Killing horizons [33,34]. Using the Dirac-Bergmann formalism, it was shown there that the Gauss law constraint of gauge theories gets additional surface contributions from the horizons of the background. This modifies the charges and also allows for interesting gauge fixing choices involving surface terms at the horizons. In this paper, we continue our investigation of constrained field theories on curved backgrounds with horizons, now using the Hamiltonian BRST formalism. In particular, using the example of scalar electrodynamics, we will investigate the effect of horizons on interacting gauge theories. We will find that like the Gauss law constraint, the BRST charge will also pick up a horizon contribution. While we do not consider all possible implications of our results in this paper, we will construct a co-BRST operator and use the invariance under BRST and co-BRST to identify dressed fields as in [35]. In our work, the spacetime is treated as a fixed background; we have not considered gravitational constraints.

We first demonstrate, using the Dirac-Bergmann formalism, that the Gauss law constraint of the theory receives additional surface contributions from the black hole horizon and also from the cosmological horizon, if it is present. We then extend the phase space and consider the Hamiltonian BRST formalism for the theory on spherically symmetric backgrounds with horizons. The BRST charge inherits the horizon terms which are present in the Gauss law constraint of the theory. The horizon terms in the Gauss law constraint ensure that BRST transformations have their usual expressions on curved backgrounds without boundaries. However, the BRST transformations which we derive also act on fields at the horizons. Thus in general, we require a gauge with surface terms to fix the theory at the horizons of the spacetime. For fixing the gauge, we use a modified radiation gauge which include null components of the electromagnetic field at the horizons. This choice leads to surface integrals at the horizons in the BRST invariant action and a ghost number charge which involves additional terms from the horizons of the background.

The gauge fixing fermion in the Hamiltonian BRST formalism can be chosen such that it is a generator of a new set of nilpotent symmetry transformations, different from the BRST transformations, which leave the theory invariant. These are known as co-BRST transformations, which have been considered for scalar electrodynamics in flat spacetime [35,36]. Invariance under both BRST and co-BRST transformations help identify dressed scalar fields as physical fields in flat spacetime. In the context of the present work, we find an additional contribution to the dressing arising from the horizons. We conclude our paper with a discussion on the potential implications of our results on soft limits at the horizons of black holes and thermal field theories on black hole spacetimes.

The organization of our paper is as follows. In Sec. II we set up our notations and conventions for gauge theories on spherically symmetric backgrounds with horizons. In Sec. III we consider scalar electrodynamics on spacetimes with horizons and show that the Gauss law constraint has to include an additional surface term. We also discuss consistent gauge fixing choices which involve surface terms at the horizons, to be used in the Hamiltonian BRST formalism. In Sec. IV, we describe the derivation of the BRST path integral from the Hamiltonian and derive the BRST invariant action in a gauge which involve null components of the gauge field at the horizons. We show that the ghost number charge of the theory in this gauge involves additional terms at the horizons of the background.

In Sec. V, we define a gauge fixing fermion which generates nilpotent co-BRST transformations that leave the theory invariant. The BRST and co-BRST transformations are used to identify the dressing of scalar fields of the theory on backgrounds which are not asymptotically flat. In Sec. VI, we describe how our results could be used to further investigate infrared limits and thermal gauge theories on black hole backgrounds.

II. GEOMETRIC FRAMEWORK

We begin by considering some essential preliminaries needed for the remaining sections. The BRST construction will be carried out on a static, spherically symmetric and torsion-free manifold \mathcal{M} endowed with at least one horizon. In other words, we only assume that the spacetime possesses a timelike Killing vector field ξ^a normalized as $\xi^a \xi_a = -\lambda^2$, which satisfies

$$\xi_{[a} \nabla_b \xi_{c]} = 0. \quad (2.1)$$

It follows that there exists a spacelike hypersurface Σ which is everywhere orthogonal to ξ^a . The horizon is defined by ξ^a becoming null, $\lambda = 0$. For an asymptotically flat or anti-de Sitter space, Σ is the region “outside the horizon,” while for backgrounds with a positive cosmological constant, such as static de Sitter black hole spacetimes, Σ is the region “between the horizons.”

The induced metric h_{ab} and projection operator h_b^a on Σ are given by

$$h_{ab} = g_{ab} + \lambda^{-2} \xi_a \xi_b, \quad h_b^a = \delta_b^a + \lambda^{-2} \xi^a \xi_b, \quad (2.2)$$

leading to the following expression for the determinant of spacetime metric

$$\sqrt{-g} = \lambda \sqrt{h}. \quad (2.3)$$

We will denote the Killing horizons of the spacetime as \mathcal{H} . The intersection of \mathcal{H} with Σ is topologically a 2-sphere

(or the union of two 2-spheres, if there is a cosmological horizon) with an induced metric σ_{ab} which can be written as

$$\sigma_{ab} = h_{ab} - n_a n_b, \quad (2.4)$$

where n^a is the outward (inward) pointing unit spatial normal to the inner (outer) horizon, which points into Σ and satisfies $n_a n^a = 1$. We will also refer to these 2-spheres as the “horizon” and write them as $\partial\Sigma$. Note that $\partial\Sigma$ is not a physical boundary space in any sense, fields or their functions do not need to vanish or diverge there in general. In the context of gauge theories, we only require gauge invariant scalars constructed out of the fields to be finite on $\partial\Sigma$.

If we need to refer to \mathcal{H} , which is of course null, we will call it the spacetime horizon. In this paper we will also make use of the null normals of \mathcal{H} , defined in spacetime. On the spherically symmetric backgrounds we are considering, we can define the two null normals l_a and k_a in terms of the normalized timelike Killing vector field $\lambda^{-1} \xi_a$ and the unit spatial normal n_a to the spatial sections of the null hypersurface,

$$l_a = \frac{1}{\sqrt{2}} (\lambda^{-1} \xi_a + n_a), \quad k_a = \frac{1}{\sqrt{2}} (\lambda^{-1} \xi_a - n_a). \quad (2.5)$$

For this choice of the null normals, we find that l_a and k_a satisfy

$$l_a l^a = 0 = k_a k^a, \quad l_a k^a = -1, \quad (2.6)$$

and we can write the metric on the null hypersurface as

$$\tilde{\sigma}_{ab} = g_{ab} + l_a k_b + l_b k_a. \quad (2.7)$$

These expressions hold for all null hypersurfaces of the background, including \mathcal{H} .

The BRST formalism requires fields which belong to the Grassmann algebra, of both even and odd Grassmann parity. Denoting Grassmann parity by ϵ , we say that the field Φ_A is “even” when $\epsilon_{\Phi_A} = 0 \pmod{2}$ and that it is “odd” when $\epsilon_{\Phi_A} = 1 \pmod{2}$. Lagrangians and Hamiltonians will always be an even functional of the fields. Because Grassmann parity is additive for composite fields, given any two functionals of the fields $F(\Phi_A)$ and $G(\Phi_A)$, we have

$$FG = (-1)^{\epsilon_F \epsilon_G} GF. \quad (2.8)$$

Due to the presence of odd Grassmannian fields, the derivatives of functionals have to be handled carefully. The derivative of a functional $F(\Phi_A)$ of a field Φ_A can be written in two possible ways

$$\text{either } \frac{\delta_L F}{\delta \Phi_A}, \quad \text{or } \frac{\delta_R F}{\delta \Phi_A}, \quad (2.9)$$

where $\frac{\delta_L}{\delta\Phi_A}$ and $\frac{\delta_R}{\delta\Phi_A}$ denote the left and right functional derivatives with respect to Φ_A , respectively. For the left functional derivative $\frac{\delta_L}{\delta\Phi_A}$, we vary F with respect to Φ_A , with $\delta\Phi_A$ moved to the extreme left using Eq. (2.8) and then deleted. Likewise the right functional derivative $\frac{\delta_R}{\delta\Phi_A}$ means that F is varied with respect to Φ_A , with $\delta\Phi_A$ moved to the extreme right and then deleted. These derivatives are identical when the field Φ_A is even. In the following, functional derivatives will always be taken to mean “left” unless specified otherwise.

The action functional for N fields Φ_A , $A = 1, \dots, N$, is given by the time integral of the Lagrangian L

$$S[\Phi_A] = \int dt L = \int dt \int_{\Sigma} dV_x \mathcal{L}(\Phi_A(x), \nabla_a \Phi_A(x)), \quad (2.10)$$

where dV_x is the volume element on Σ , and $\mathcal{L}(\Phi_A(x), \nabla_a \Phi_A(x))$ is the Lagrangian density. The Lagrangian density can be written in terms of the “spatial” and “temporal” derivatives of the fields,

$$\mathcal{L} \equiv \mathcal{L}(\Phi_A(x), \mathcal{D}_a \Phi_A(x), \dot{\Phi}_A(x)), \quad (2.11)$$

where $\mathcal{D}_a \Phi_A = h_a^b \nabla_b \Phi_A$ are the Σ -projected derivatives of the fields Φ_A , and $\dot{\Phi}_A$ are their time derivatives, defined as their Lie derivatives with respect to ξ ,

$$\dot{\Phi}_A := \mathcal{L}_{\xi} \Phi_A. \quad (2.12)$$

The momenta Π^A canonically conjugate to the fields Φ_A are defined as

$$\Pi^A = \frac{\delta L}{\delta \dot{\Phi}_A}, \quad (2.13)$$

where the functional derivative in this definition is taken on the hypersurface Σ , i.e., it is an “equal-time” functional derivative, defined as

$$\frac{\delta \Phi_A(\vec{x}, t)}{\delta \Phi_B(\vec{y}, t)} = \delta_A^B \delta(x, y) = \frac{\delta \dot{\Phi}_A(\vec{x}, t)}{\delta \dot{\Phi}_B(\vec{y}, t)}. \quad (2.14)$$

The $\delta(x, y)$ in Eq. (2.14) is the three-dimensional covariant delta function defined on Σ ,

$$\int_{\Sigma} dV_y \delta(x, y) f(\vec{y}, t) = f(\vec{x}, t). \quad (2.15)$$

Given a Lagrangian L we can construct the canonical Hamiltonian through the Legendre transform

$$H_C = \int_{\Sigma} dV_x (\Pi^A \dot{\Phi}_A) - L. \quad (2.16)$$

The generalized Poisson bracket for two functionals $F(\Phi_A, \Pi^A)$ and $G(\Phi_A, \Pi^A)$ is defined as

$$[F, G]_P = \int dV_z \left(\frac{\delta_R F}{\delta \Phi_A(z)} \frac{\delta_L G}{\delta \Pi^A(z)} - \frac{\delta_R F}{\delta \Pi^A(z)} \frac{\delta_L G}{\delta \Phi_A(z)} \right). \quad (2.17)$$

In accounting for Grassmann fields, the generalized Poisson bracket reduces to a commutator when any one of the fields is even and an anticommutator when both fields are odd. We will henceforth refer to this bracket simply as the Poisson bracket. With the choice of $F(x) = \Pi^B(\vec{x}, t)$ and $G(y) = \Phi_A(\vec{y}, t)$, we recover the canonical relation between the fields and their momenta

$$[\Pi^B(\vec{x}, t), \Phi_A(\vec{y}, t)]_P = -\delta_A^B \delta(x, y). \quad (2.18)$$

The time evolution of any functional of the fields can also be determined from its Poisson bracket with the Hamiltonian.

$$\dot{F}(x) = [F(x), H_C]_P. \quad (2.19)$$

Using the above definitions, we can now consider Hamiltonian formalisms for constrained field theories on spherically symmetric backgrounds with horizons. There exist several excellent textbooks and reviews which cover these topics (see, for example, [4,37–40] for the Dirac-Bergmann formalism and [3,4] for the Hamiltonian BRST approach), and we will not review them here.

III. SCALAR ELECTRODYNAMICS

Before proceeding to the Hamiltonian BRST treatment in the next section, we will first need to identify all the constraints of the theory using the Dirac-Bergmann formalism in order to define the BRST charge operator. The procedure will follow the standard treatment in [4,37–40]. In the case of scalar electrodynamics, the results are simply curved spacetime generalizations of those known in flat spacetime, with the exception of the form of the Gauss law constraint. As promised earlier, we will find that the Gauss law constraint receives a contribution from the horizons of the background. For a similar result in the case of the free Maxwell field, we refer the reader to [33] for spherically symmetric backgrounds and [34] for a certain class of axisymmetric backgrounds.

The action for scalar quantum electrodynamics on the spherically symmetric black hole spacetime is

$$S_{\text{SQED}} = - \int dV_4 \left(D_a \Phi (D_b \Phi)^* g^{ab} + m^2 \Phi \Phi^* + \frac{1}{4} F_{ab} F_{cd} g^{ac} g^{bd} \right), \quad (3.1)$$

where $dV_4 = \lambda dV_x$ is the four dimensional volume form on the manifold $\Sigma \times \mathbb{R}$ (with metric g_{ab}), Φ is a complex scalar field, $D_a = \partial_a + igA_a$ is the gauge covariant derivative and $F_{ab} = 2\partial_{[a}A_{b]}$ is the electromagnetic field strength tensor. We can now project this action on to the hypersurface described in the previous section. Time derivatives are given by the Lie derivative with respect to ξ^a , as in Eq. (2.12). In particular

$$\mathcal{L}_\xi a_b = \dot{a}_b = -\lambda e_b + \mathcal{D}_b \phi, \quad (3.2)$$

where \mathcal{D}_a is the spatial derivative defined in Eq. (2.11) and we have defined $e_d = -\lambda^{-1} \xi^c F_{cd}$. By further defining $a_a = h_a^b A_b$, $\phi = A_a \xi^a$, $\bar{D}_a = \partial_a + iga_a$, $D_0 = \mathcal{L}_\xi + ig\phi$ and $f_{ab} = F_{cd} h_a^c h_b^d$, we can rewrite Eq. (3.1) as

$$S_{\text{SQED}} = \int dt \int_\Sigma dV_x \lambda \left(\lambda^{-2} D_0 \Phi (D_0 \Phi)^* - h^{ab} \bar{D}_a \Phi (\bar{D}_b \Phi)^* - m^2 \Phi \Phi^* - \frac{1}{4} f_{ab} f^{ab} + \frac{1}{2} e_a e^a \right). \quad (3.3)$$

Denoting the conjugate momenta of a_b , ϕ , Φ , and Φ^* by π^b , π^ϕ , Π , and Π^* , respectively, we have

$$\begin{aligned} \pi^b &= \frac{\partial L_{\text{SQED}}}{\partial \dot{a}_b} = -e^b, & \pi^\phi &= \frac{\partial L_{\text{SQED}}}{\partial \dot{\phi}} = 0, \\ \Pi &= \frac{\partial L_{\text{SQED}}}{\partial \dot{\Phi}} = \lambda^{-1} (D_0 \Phi)^*, & \Pi^* &= \frac{\partial L_{\text{SQED}}}{\partial \dot{\Phi}^*} = \lambda^{-1} D_0 \Phi. \end{aligned} \quad (3.4)$$

Thus the only primary constraint of the theory is

$$\Omega_1 = \pi^\phi. \quad (3.5)$$

The canonical Hamiltonian can be constructed from the Legendre transform

$$\begin{aligned} \int_\Sigma dV_y \epsilon(y) \dot{\pi}^\phi(y) &= \int_\Sigma dV_y \epsilon(y) [\pi^\phi(y), \tilde{H}]_P \\ &= \int_\Sigma dV_y \epsilon(y) \left[\pi^\phi(y), \int_\Sigma dV_x \pi^b(x) \mathcal{D}_b^x \phi(x) + ig\phi(x) (\Phi^*(x) \Pi^*(x) - \Phi(x) \Pi(x)) \right]_P \\ &= - \oint_{\partial \Sigma} da_y \epsilon(y) n_b^y \pi^b(y) + \int_\Sigma dV_y \epsilon(y) (\mathcal{D}_b^y \pi^b(y) - ig(\Phi^*(y) \Pi^*(y) - \Phi(y) \Pi(y))). \end{aligned} \quad (3.11)$$

Here we have used the canonical Poisson brackets given in Eq. (3.10) and an integration by parts. The smearing function ϵ is assumed to be well behaved, but ϵ or its first derivative are not required to vanish on the horizon (or

$$\begin{aligned} H_C &= \int_\Sigma dV_x (\pi^b \dot{a}_b + \Pi \dot{\Phi} + \Pi^* \dot{\Phi}^*) - L \\ &= H_0 + \int_\Sigma dV_x (\pi^b \mathcal{D}_b \phi + ig\phi (\Phi^* \Pi^* - \Phi \Pi)), \end{aligned} \quad (3.6)$$

where H_0 is defined as

$$\begin{aligned} H_0 &= \int_\Sigma dV_x \lambda \left(\frac{1}{2} \pi^b \pi_b + \frac{1}{4} f_{ab} f^{ab} + \Pi \Pi^* \right. \\ &\quad \left. + m^2 \Phi \Phi^* + \bar{D}_a \Phi (\bar{D}^a \Phi)^* \right). \end{aligned} \quad (3.7)$$

Apart from the involvement of a curved manifold and its covariant derivatives, the definition of H_0 is the usual one known in flat spacetime which is used in the BRST treatment of this theory.

Using a multiplier v_ϕ , we will now include the primary constraint $\pi^\phi \approx 0$ to the canonical Hamiltonian to define a new Hamiltonian

$$\tilde{H} = H_C + \int_\Sigma dV_x v_\phi \pi^\phi. \quad (3.8)$$

The canonical Poisson brackets of the theory are

$$\begin{aligned} [\phi(x), \pi^\phi(y)]_P &= \delta(x, y), & [a_b(x), \pi^a(y)]_P &= \delta_b^a \delta(x, y), \\ [\Phi(x), \Pi(y)]_P &= \delta(x, y), & [\Phi^*(x), \Pi^*(y)]_P &= \delta(x, y). \end{aligned} \quad (3.9)$$

We now arrive at a key result used in this paper, namely that the Gauss law constraint of this theory is modified through the presence of horizons. This is determined by requiring that the primary constraint π^ϕ is satisfied at all times. The consistency check of the primary constraint $\dot{\pi}^\phi \approx 0$, is evaluated through the Poisson bracket of π^ϕ and \tilde{H} with the help of a smearing function ϵ as follows,

horizons, if Σ is the region between the horizons in a de Sitter black hole spacetime). By not requiring such conditions on the smearing functions at the horizons, we can equivalently state that we are not *a priori* adopting either

Dirichlet or Neumann boundary conditions. Then the Schwarz inequality demonstrates that the surface integral is finite

$$|n_b \pi^b| \leq \sqrt{|n_b n^b| |\pi_b \pi^b|}. \quad (3.12)$$

In this expression, $n_b n^b = 1$ by definition, since n_b is the ‘‘unit normal’’ to spatial sections of the horizon(s) and points in the direction of increasing time. Likewise, $\pi_b \pi^b = e_b e^b$ appears in the energy momentum tensor (more precisely in invariant scalars such as $T^{ab} T_{ab}$), and therefore may not diverge at the horizon. It follows that the surface integral in Eq. (3.11) is finite at the horizon. We can thus read off the Gauss law constraint from the last equality of Eq. (3.11),

$$\Omega_2 = n_b \pi^b|_{\mathcal{H}} - \mathcal{D}_b \pi^b + ig(\Phi^* \Pi^* - \Phi \Pi) \approx 0. \quad (3.13)$$

The vertical bar $|_{\mathcal{H}}$ on the first term in Eq. (3.13) denotes that it is a contribution restricted to the horizon(s) of the spacetime. In other words, while the bulk contribution holds for all points of Σ , the additional surface contribution of Eq. (3.13) must be considered for all points at the horizons $\partial\Sigma$. Like the constraint $\pi^\phi \approx 0$, this constraint also needs to be smeared with a well behaved function, regular at the horizons, for the purpose of Poisson bracket calculations. We thus understand the constraint as

$$\begin{aligned} \int_{\Sigma} dV_x \epsilon(x) \Omega_2(x) &= \oint_{\partial\Sigma} \epsilon(x) n_b^x \pi^b(x) - \int_{\Sigma} dV_x \epsilon(x) (\mathcal{D}_b^x \pi^b(x) \\ &\quad - ig(\Phi(x)^* \Pi(x)^* - \Phi(x) \Pi(x))) \\ &\approx 0. \end{aligned} \quad (3.14)$$

Including the constraint Eq. (3.13) in the Hamiltonian with its own multiplier v_2 , we can write the total Hamiltonian as

$$H_T = H_0 + \int_{\Sigma} dV_x ((v_2 + \phi) \Omega_2 + v_\phi \pi^\phi). \quad (3.15)$$

It is straightforward to verify that $\dot{\Omega}_2 \approx 0$, which reveals that there are no further constraints of the theory.

Since $[\pi^\phi, \Omega_2]_P = 0$, the constraints are first class and generate gauge transformations of the fields. These transformations follow from the Poisson brackets of the fields with the general linear combination of the first class constraints, $\epsilon_1 \pi^\phi + \epsilon_2 \Omega_2$

$$\begin{aligned} \delta\phi &= \epsilon_1, & \delta a_b &= \mathcal{D}_b \epsilon_2, \\ \delta\Phi &= -ig\epsilon_2 \Phi, & \delta\Pi &= ig\epsilon_2 \Pi, \\ \delta\Phi^* &= ig\epsilon_2 \Phi^*, & \delta\Pi^* &= -ig\epsilon_2 \Pi^*. \end{aligned} \quad (3.16)$$

We see that the gauge transformation of a_b , i.e., δa_b in Eq. (3.16), takes its usual form as on curved backgrounds without boundaries. This is a consequence of the surface term in the Gauss law constraint. We also note that the gauge transformations in Eq. (3.16) hold for the fields throughout Σ , including the horizons. This in particular suggests the need to consider gauge fixing choices with surface terms at the horizons.

We can also determine the multipliers on the space of solutions of Hamilton’s equations from the equations of motion. By considering $[\phi, H_T]_P$ we see that $v_\phi = \dot{\phi}$. Likewise, we note that $[a_b, H_T]_P$ gives the expression of Eq. (3.2) provided $\partial_b v_2 = 0$. This allows us to set $v_2 = 0$ without any loss of generality. With this choice for v_ϕ and v_2 , we have

$$H_T = H_0 + \int_{\Sigma} dV_x (\phi \Omega_2 + \dot{\phi} \pi^\phi). \quad (3.17)$$

A. Gauge fixing choices

Before applying the Hamiltonian BRST formalism to this theory, it will be instructive to first describe how consistent gauge fixing choices can be determined within the Dirac-Bergmann formalism. Gauge fixing can be carried out through the introduction of additional constraints which have nonvanishing Poisson brackets with the first-class constraints of the theory. Given the two first-class constraints of the theory in Eqs. (3.5) and (3.13)

$$\begin{aligned} \Omega_1 &= \pi^\phi, \\ \Omega_2 &= n_b \pi^b|_{\mathcal{H}} - \mathcal{D}_b \pi^b + ig(\Phi^* \Pi^* - \Phi \Pi), \end{aligned} \quad (3.18)$$

we need to introduce two additional constraints which should have nonvanishing Poisson brackets with those in Eq. (3.18) and be consistent with them. The Gauss law constraint and its surface terms motivate the following gauge-fixing constraint

$$\Omega_3 = \mathcal{D}_b (\lambda^{-1} a^b) - n_b \lambda^{-1} a^b|_{\mathcal{H}}. \quad (3.19)$$

This constraint has the desired property of providing a nonvanishing Poisson bracket with Ω_2 . The consistency of this constraint requires that its time derivative with the Hamiltonian weakly vanishes. We find the following Poisson bracket of Ω_3 with H_T given in Eq. (3.17)

$$\begin{aligned} [\Omega_3, H_T]_P &= n_b \pi^b|_{\mathcal{H}} - \mathcal{D}_b \pi^b + (n_b \lambda^{-1} \mathcal{D}^b \phi)|_{\mathcal{H}} \\ &\quad - \mathcal{D}_b (\lambda^{-1} \mathcal{D}^b \phi). \end{aligned} \quad (3.20)$$

This weakly vanishes on account of Ω_2 , provided we impose

$$\begin{aligned}\Omega_4 &= \mathcal{D}_b(\lambda^{-1}\mathcal{D}^b\phi) - (n_b\lambda^{-1}\mathcal{D}^b\phi)|_{\mathcal{H}} \\ &+ ig(\Phi^*\Pi^* - \Phi\Pi).\end{aligned}\quad (3.21)$$

Thus Ω_3 and Ω_4 are consistent with the constraints of Eq. (3.18) and have nonvanishing brackets with them. The construction of Dirac brackets on spherically symmetric backgrounds with horizons, following gauge fixing choices which involve surface terms at the horizons, has been described in further detail in [33]. Equations (3.19) and (3.21), apart from surface terms at the horizons, is the familiar choice of the radiation gauge for scalar electrodynamics [38].

An alternative choice of gauge fixing, as far as surface terms at the horizons are concerned, is to consider null components of the fields at the horizons. This can be done by considering in place of Eq. (3.19) the following constraint

$$\Omega_3 = \mathcal{D}_b(\lambda^{-1}a^b) - (\lambda^{-1}n_b a^b - \lambda^{-2}\phi)|_{\mathcal{H}}.\quad (3.22)$$

Using Eq. (2.5), we recognize the surface term as a null component of A_a , specifically $\sqrt{2}\lambda^{-1}k_a A^a$. We now find that Ω_3 weakly vanishes provided

$$\begin{aligned}\Omega_4 &= \mathcal{D}_b(\lambda^{-1}\mathcal{D}^b\phi) - (\lambda^{-1}(n_b\mathcal{D}^b\phi - \lambda^{-1}v^\phi))|_{\mathcal{H}} \\ &+ ig(\Phi^*\Pi^* - \Phi\Pi).\end{aligned}\quad (3.23)$$

Since $v^\phi = \dot{\phi}$ on the space of solutions of Hamilton's equations, we can think of the surface term in Eq. (3.23) as equivalent to $\sqrt{2}\lambda^{-1}k^a\nabla_a\phi$. A few comments about the surface term in Eq. (3.23) may be in order. On the one hand, since the surface term contains the gauge dependent fields a_b and ϕ and their derivatives, we cannot *a priori* impose any conditions on their finiteness be it at the horizon or elsewhere. On the other hand, we also note that regardless of the behavior of a_b and ϕ at \mathcal{H} , the null vectors k^a and l^a in Eq. (2.5) have a reparametrization invariance which can always be used to produce a finite result. For example, we can consider the change in normalization $k^a \rightarrow \lambda^{-1}k^a$ and $l^a \rightarrow \lambda l^a$ in Eq. (2.5), under which the relations in Eq. (2.6) continue to hold. Thus the surface terms in Eqs. (3.22) and (3.23) can always be adjusted so that they do not lead to divergences in any gauge-invariant quantity.

We thus see that fixing null components at the horizons is a consistent choice within the Dirac-Bergmann formalism and can admit an interesting set of Dirac brackets. In the following sections, we will derive results for the BRST invariant action where the effect of such gauge fixing choices will be shown to have more pronounced effects on the surface action at the horizon. We will also demonstrate that the use of Eq. (3.22) in the Hamiltonian BRST formalism can provide horizon corrections to the ghost number charge and dressed gauge invariant fields of scalar electrodynamics.

IV. HAMILTONIAN BRST FORMALISM

We will now apply the Hamiltonian BRST formalism to derive the BRST invariant effective action and path integral for this theory. We will follow the standard treatment given in [3,4], but on the black hole spacetimes with horizon contributions to the Gauss law and gauge fixing constraints, as described above. We first extend the phase space of the previous section to include additional Grassmann odd fields, namely the ghosts and their momenta. Thus in addition to the fields considered in the previous section, we now introduce the ghost \mathcal{C} and antighost $\bar{\mathcal{C}}$ and their conjugate momenta \mathcal{P} , $\bar{\mathcal{P}}$, which satisfy

$$[\bar{\mathcal{P}}(x), \bar{\mathcal{C}}(y)]_{\mathcal{P}} = -\delta(x, y) = [\mathcal{P}(x), \mathcal{C}(y)]_{\mathcal{P}}.\quad (4.1)$$

All brackets involving the ghosts other than those given in Eq. (4.1) vanish. The ghost number can be determined from the ghost number charge

$$Q_C = \int_{\Sigma} dV_x (\mathcal{C}\mathcal{P} + \bar{\mathcal{P}}\bar{\mathcal{C}}).\quad (4.2)$$

Given a functional of the fields F in the extended phase space, we have

$$[F, Q_C]_{\mathcal{P}} = \text{gh}(F)F,\quad (4.3)$$

where $\text{gh}(F)$ denotes the ghost number of F . Then

$$\begin{aligned}\text{gh}(\mathcal{C}) &= 1 = \text{gh}(\bar{\mathcal{P}}), \\ \text{gh}(\mathcal{P}) &= -1 = \text{gh}(\bar{\mathcal{C}}).\end{aligned}\quad (4.4)$$

Apart from the fields \mathcal{C} , $\bar{\mathcal{C}}$, \mathcal{P} , $\bar{\mathcal{P}}$, all other canonical fields in the extended phase space have vanishing ghost number.

The generator of BRST transformations Q_{BRST} in the extended phase space can be directly constructed from the first-class constraints of a theory resulting from the Dirac-Bergmann formalism. Following the procedure in [3] we have

$$Q_{\text{BRST}} = \int_{\Sigma} dV_x (\mathcal{C}(x)\Omega_2(x) - i\lambda\bar{\mathcal{P}}(x)\pi^\phi(x)).\quad (4.5)$$

In Eq. (4.5) we have included a factor of λ in the second term. This is merely a convenient choice for what follows and does not result from more fundamental grounds. Nor does it contradict any known result, as $\lambda = 1$ in flat spacetime.

The BRST charge is Grassmann odd and has ghost number $\text{gh}(Q_{\text{BRST}}) = 1$. BRST transformations of the fields are generated by its Poisson bracket with Q_{BRST} . Given a functional of the fields F , we will denote its BRST transformation by sF

$$sF = [F, Q_{\text{BRST}}]_P. \quad (4.6)$$

If F has ghost number $\text{gh}(F)$ and mass dimension d_F , then sF has ghost number $\text{gh}(F) + 1$ and mass dimension $d_F + 1$. By evaluating the Poisson brackets of the fields with Q_{BRST} we find

$$\begin{aligned} sa_b &= \mathcal{D}_b \mathcal{C}, & s\phi &= -i\lambda\bar{\mathcal{P}}, \\ s\bar{\mathcal{C}} &= i\lambda\pi^\phi, & s\mathcal{P} &= -\Omega_2, \\ s\Phi &= -ig\mathcal{C}\Phi, & s\Pi &= ig\mathcal{C}\Pi, \\ s\Phi^* &= ig\mathcal{C}\Phi^*, & s\Pi^* &= -ig\mathcal{C}\Pi^*, \\ s\bar{\mathcal{P}} &= 0 = s\mathcal{C}, & s\pi^\phi &= 0 = s\pi^a. \end{aligned} \quad (4.7)$$

Just as in the case of the gauge transformations generated by the first class constraints, the BRST transformations of the fields given in Eq. (4.7) are the same as those on backgrounds without boundaries. The BRST charge is nilpotent,

$$[Q_{\text{BRST}}, Q_{\text{BRST}}]_P \equiv Q_{\text{BRST}}^2 = 0, \quad (4.8)$$

i.e., $s^2 F = 0$ for all F .

It is straightforward to verify that H_0 defined in Eq. (3.7) is invariant under the BRST transformations given in Eq. (4.7). Further, since the BRST transformation is nilpotent, any BRST invariant quantity is known up to the addition of a term $sF = [F, Q_{\text{BRST}}]_P$ for any F . In particular, we can define the following BRST invariant Hamiltonian

$$H_{\text{BRST}} = H_0 - s\Psi, \quad (4.9)$$

where Ψ must have odd Grassmann parity and $\text{gh}(\Psi) = -1$, but can be arbitrary otherwise. As we will see shortly, Ψ is used to implement gauge fixing choices within the Hamiltonian BRST formalism and hence is aptly known as the gauge fixing fermion.

From the Legendre transform with the Hamiltonian in Eq. (4.9), we can also define the following BRST invariant action

$$\begin{aligned} S_{\text{BRST}} &= \int dt \int_{\Sigma} dV_x [\dot{a}_b \pi^b + \dot{\phi} \pi^\phi + \dot{\Phi} \Pi + \dot{\Phi}^* \Pi^* \\ &\quad + \dot{\bar{\mathcal{C}}} \bar{\mathcal{P}} + \dot{\mathcal{C}} \mathcal{P} - H_{\text{BRST}}], \end{aligned} \quad (4.10)$$

Since Ψ can be specified arbitrarily, physical processes are independent of the choice Ψ , and hence independent of any gauge choice. The invariance of the partition function is expressed by the Fradkin-Vilkovisky theorem [41,42], which says that the path integral over all the canonical variables of the extended phase space $\mu^A \equiv (a_b, \pi^b, \phi, \pi^\phi, \Phi, \Pi, \Phi^*, \Pi^*, \mathcal{C}, \mathcal{P}, \bar{\mathcal{C}}, \bar{\mathcal{P}})$

$$Z = \int [\mathcal{D}\mu^A] \exp(iS_{\text{BRST}}), \quad (4.11)$$

is independent of the choice of Ψ . The following Ψ is customarily chosen

$$\Psi = \int_{\Sigma} dV_x (i\bar{\mathcal{C}}(x)\chi(x) + \mathcal{P}(x)\phi(x)), \quad (4.12)$$

where χ is independent of the ghosts and their momenta, but can be specified arbitrarily otherwise. In the following we will also assume that χ is independent of all momenta other than π^ϕ . Subject to this assumption, we now adopt Eq. (3.22) in the BRST formalism through the following choice of χ

$$\chi = \mathcal{D}_a(\lambda^{-1}a^a) - (\lambda^{-1}n_a a^a - \lambda^{-2}\phi)|_{\mathcal{H}} - \frac{1}{2}\pi^\phi, \quad (4.13)$$

where as before, the symbol $|_{\mathcal{H}}$ indicates that the term is evaluated at the horizons. Then the BRST transformation of Ψ has the following expression

$$s\Psi = \int_{\Sigma} dV_x (\lambda\pi^\phi\chi + i\lambda\mathcal{P}\bar{\mathcal{P}} + \phi\Omega_2 + i\bar{\mathcal{C}}s\chi), \quad (4.14)$$

where $\int_{\Sigma} dV_x i\bar{\mathcal{C}}s\chi$ explicitly has the form

$$\begin{aligned} \int_{\Sigma} dV_x i\bar{\mathcal{C}}s\chi &= i \int_{\Sigma} dV_x \bar{\mathcal{C}} \mathcal{D}_a(\lambda^{-1}\mathcal{D}^a \mathcal{C}) \\ &\quad - i \oint_{\partial\Sigma} da_x \lambda^{-1} \bar{\mathcal{C}} (n^a \mathcal{D}_a \mathcal{C} + i\bar{\mathcal{P}}). \end{aligned} \quad (4.15)$$

We can now use Eq. (4.14) to define H_{BRST} and thus S_{BRST} , using which we have the following path integral from Eq. (4.11)

$$\begin{aligned} Z &= \int [\mathcal{D}\mu^A] \exp(iS_{\text{BRST}}) \\ &= \int [\mathcal{D}\mu^A] \exp\left(i \int dt \int_{\Sigma} dV_x \left[\dot{a}_a \pi^a + \dot{\phi} \pi^\phi + \dot{\Phi} \Pi \right. \right. \\ &\quad \left. \left. + \dot{\Phi}^* \Pi^* + \dot{\bar{\mathcal{C}}} \bar{\mathcal{P}} + \dot{\mathcal{C}} \mathcal{P} - \phi\Omega_2 - i\bar{\mathcal{C}}s\chi \right. \right. \\ &\quad \left. \left. - \lambda \left(\frac{1}{2} \pi^b \pi_b + \frac{1}{4} f_{ab} f^{ab} + \Pi \Pi^* + m^2 \Phi \Phi^* \right. \right. \right. \\ &\quad \left. \left. \left. + \bar{D}_a \Phi (\bar{D}^a \Phi)^* + \pi^\phi \chi + i\mathcal{P}\bar{\mathcal{P}} \right) \right] \right). \end{aligned} \quad (4.16)$$

We can now integrate out the momenta $\mathcal{P}, \bar{\mathcal{P}}, \Pi, \Pi^*$, and π^a to find

$$Z = \int [D a_a D \phi D \Phi D \Phi^* D \bar{C} D C D \pi^\phi] \exp(i S_{\text{BRST}}), \quad (4.17)$$

where S_{BRST} is now written as $S_{\text{BRST}} = S_{\text{SQED}} + S_{gh} + S_{gf}$, with

$$\begin{aligned} S_{\text{SQED}} &= \int dt \int_{\Sigma} dV_x \lambda \left(\frac{1}{2} e^a e_a - \frac{1}{4} f_{ab} f^{ab} + \lambda^{-2} D_0 \Phi (D_0 \Phi)^* \right. \\ &\quad \left. - \bar{D}_a \Phi (\bar{D}^a \Phi)^* - m^2 \Phi \Phi^* \right), \\ S_{gh} &= -i \int dt \int_{\Sigma} dV_x (\lambda^{-1} \dot{\bar{C}} \dot{C} + \mathcal{D}_a (\lambda^{-1} \mathcal{D}^a C)) \\ &\quad - i \int dt \oint_{\partial \Sigma} da_x \lambda^{-1} \bar{C} (\lambda^{-1} \dot{C} - n^a \mathcal{D}_a C), \\ S_{gf} &= \int dt \int_{\Sigma} dV_x \lambda \pi^\phi \left(\lambda^{-1} \dot{\phi} - \mathcal{D}_a (\lambda^{-1} a^a) + \frac{1}{2} \pi^\phi \right) \\ &\quad - \int dt \oint_{\partial \Sigma} da_x \pi^\phi (\lambda^{-1} \phi - n^a a_a). \end{aligned} \quad (4.18)$$

In deriving Eq. (4.17), we performed a Gaussian integration over π^a and made use of Eq. (3.2). The integration over \mathcal{P} , $\bar{\mathcal{P}}$, Π , and Π^* simply involve delta functions which enforce the relations $\bar{\mathcal{P}} = i\lambda^{-1} \dot{\bar{C}}$ and $\mathcal{P} = -i\lambda^{-1} (\dot{C} + (\lambda^{-1} \bar{C})|_{\mathcal{H}})$, while Π and Π^* have their expressions given in Eq. (3.4). Due to the surface term present in S_{gh} in Eq. (4.18), we cannot integrate out π^ϕ in the path integral as in the absence of a horizon, for example in flat spacetime. To identify the BRST transformations which leave $S_{\text{SQED}} + S_{gh} + S_{gf}$ in Eq. (4.18) invariant, we can in Eq. (4.17) simply substitute for all momenta other than π^ϕ their value at the extremum. This gives

$$\begin{aligned} s a_b &= \mathcal{D}_b C, & s \phi &= \dot{C}, \\ s \Phi &= -i g C \Phi, & s \Phi^* &= i g C \Phi^*, \\ s \bar{C} &= i \lambda \pi^\phi. \end{aligned} \quad (4.19)$$

Likewise, we also find that the ghost number charge in Eq. (4.2) (or equivalently, the Noether charge from S_{gh} in Eq. (4.18) resulting from the scaling transformation $\bar{C} \rightarrow e^{-s} \bar{C}$ and $C \rightarrow e^s C$) now has the following expression

$$Q_C = i \int_{\Sigma} dV_x \lambda^{-1} (\dot{C} \bar{C} - \dot{\bar{C}} C) - i \oint_{\partial \Sigma} da_x \lambda^{-2} C \bar{C}. \quad (4.20)$$

We note that just as in the case of gauge dependent fields, we cannot assume any particular behavior for the ghost fields on the background. The surface integral in Eq. (4.20) is absent in the ghost number charge on backgrounds

without horizons. The implications of the surface integral for fields at the horizon can only be checked through the calculation of physical observables. Our procedure seems to find explicitly the ghost d.o.f. on the horizons of black holes. The presence of the horizon contributions to the ghost number charge in Eq. (4.20) could be particularly relevant in the context of thermal gauge theories, as we will discuss later.

V. THE CO-BRST OPERATOR AND DRESSED CHARGES

We will now explore a construction in which the gauge fixing fermion Ψ is the generator of nilpotent symmetry transformations and a conserved charge of the theory. We will follow the construction made previously in the context of quantum electrodynamics in flat spacetime [35]. There it was shown that a nilpotent operator Q_{BRST}^\perp , different from Q_{BRST} , exists which preserves the gauge fixing action and generates nonlocal and noncovariant transformations, that reduce the ghost number of the fields it acts on by one. It was also argued that physical states of the theory $|\Phi\rangle$ need to satisfy $Q_{\text{BRST}}|\Phi\rangle = 0$ and $Q_{\text{BRST}}^\perp|\Phi\rangle = 0$.

Within the Hamiltonian BRST formalism, it was shown that this conserved charge can be identified with a gauge fixing fermion which generates the nilpotent transformations of Q_{BRST}^\perp and which in addition can be used to identify singlet states belonging to the BRST invariant inner product space [36]. The gauge fixing fermion in this case is known as the co-BRST charge. In this subsection, we will demonstrate how we can choose a gauge fixing fermion with these properties, which will generalize the results of [35,36] to curved backgrounds with horizons.

We begin by noting that we are free to modify H_0 and Ψ of Sec. IV by the BRST differential of an arbitrary functional A as

$$\tilde{H}_0 = H_0 + sA \quad \tilde{\Psi} = \Psi + A, \quad (5.1)$$

Under this modification, H_{BRST} in Eq. (4.9) remains invariant and the results in Sec. IV are not affected. Let us thus consider the following expressions for \tilde{H}_0 and $\tilde{\Psi}$

$$\begin{aligned} \tilde{H}_0 &= H_0 - \int_{\Sigma} dV_x \int_{\Sigma} dV_y \frac{1}{2} \Omega_2(x) G(x, y) \Omega_2(y) \\ \tilde{\Psi} &= \Psi + \int_{\Sigma} dV_x \int_{\Sigma} dV_y \frac{1}{2} \mathcal{P}(x) G(x, y) \Omega_2(y), \end{aligned} \quad (5.2)$$

where Ω_2 is as in Eq. (3.13), Ψ is as in Eq. (4.12) [with χ in Ψ as in Eq. (4.13)] and $G(x, y)$ is a Green function which satisfies

$$F(\mathcal{D})G(x, y) = -\delta(x, y), \quad (5.3)$$

with respect to a differential operator $F(\mathcal{D})$ which will be determined shortly. Since $s\mathcal{P} = -\Omega_2$ and $s\Omega_2 = 0$, we see that $\tilde{H}_0 - s\tilde{\Psi} = H_0 - s\Psi$. Unlike Ψ , we can now show that $\tilde{\Psi}$ can generate its own nilpotent symmetry transformations. We denote $[\mu^\alpha, \tilde{\Psi}]_P = \bar{s}\mu^\alpha$ as the transformations generated by $\tilde{\Psi}$, where $\mu^\alpha \equiv (a_a, \pi^a, \phi, \pi^\phi, \Phi, \Pi, \Phi^*, \Pi^*, \bar{\mathcal{C}}, \bar{\mathcal{P}}, \mathcal{C}, \mathcal{P})$ represents the set of all fields in the extended phase space. Evaluating the Poisson brackets of the fields with $\tilde{\Psi}$ we find the following set of transformations

$$\begin{aligned}\bar{s}a_b(x) &= \int_{\Sigma} dV_y \frac{1}{2} \mathcal{P}(y) \mathcal{D}_b^x(G(x, y)), & \bar{s}\phi(x) &= -\frac{1}{2} i \bar{\mathcal{C}}(x), \\ \bar{s}\mathcal{C}(x) &= -\phi(x) - \int_{\Sigma} dV_y \frac{1}{2} \Omega_2(y) G(x, y), \\ \bar{s}\bar{\mathcal{P}}(x) &= -i\chi(x), \\ \bar{s}\pi_a(x) &= i\lambda^{-1}(x) \mathcal{D}_a^x \bar{\mathcal{C}}(x), \\ \bar{s}\pi_\phi(x) &= -\mathcal{P}(x) - (i\lambda^{-2}(x) \bar{\mathcal{C}}(x))|_{\mathcal{H}}, \\ \bar{s}\Phi(x) &= - \int_{\Sigma} dV_y \frac{1}{2} ig\mathcal{P}(y) \Phi(x) G(x, y), \\ \bar{s}\Pi(x) &= \int_{\Sigma} dV_y \frac{1}{2} ig\mathcal{P}(y) \Pi(x) G(x, y), \\ \bar{s}\Phi^*(x) &= \int_{\Sigma} dV_y \frac{1}{2} ig\mathcal{P}(y) \Phi^*(x) G(x, y), \\ \bar{s}\Pi^*(x) &= - \int_{\Sigma} dV_y \frac{1}{2} ig\mathcal{P}(y) \Pi^*(x) G(x, y), \\ \bar{s}\bar{\mathcal{C}}(x) &= 0 = \bar{s}\mathcal{P}(x).\end{aligned}\quad (5.4)$$

The Poisson bracket with $\tilde{\Psi}$ reduces by 1 the ghost number of the field it acts on. The nilpotence of these transformations on all fields other than \mathcal{C} and $\bar{\mathcal{P}}$ follow trivially. The transformations of \mathcal{C} and $\bar{\mathcal{P}}$ are nilpotent provided the Green function $G(x, y)$ satisfies

$$\begin{aligned}\int_{\Sigma} dV_x f(y) \mathcal{D}_a^x(\lambda^{-1}(x) \mathcal{D}_x^a G(x, y)) \\ - \oint_{\partial\Sigma} da_x f(y) n_x^a \lambda^{-1}(x) \mathcal{D}_a^x G(x, y) = -f(x),\end{aligned}\quad (5.5)$$

where $f(x)$ is any well-behaved function on the hypersurface Σ . We can equivalently write Eq. (5.5) in the following way

$$\mathcal{D}_a^x(\lambda^{-1}(x) \mathcal{D}_x^a G(x, y)) - (\lambda^{-1} n_x^a \mathcal{D}_a^x G(x, y))|_{\mathcal{H}} = -\delta(x, y).\quad (5.6)$$

Thus we can write $F(\mathcal{D}) = \mathcal{D}_a(\lambda^{-1} \mathcal{D}^a) - (\lambda^{-1} n^a \mathcal{D}_a)|_{\mathcal{H}}$. We note that the solution of $\mathcal{D}_a^x(\lambda^{-1}(x) \mathcal{D}_x^a G(x, y)) = -\delta(x, y)$ provides the electrostatic potential on spherically symmetric backgrounds [43–47], whose algebraic expressions are known about the Schwarzschild [46] and Reissner-Nordström spacetimes [47].

Since the transformations in Eq. (5.2) do not affect the expression of H_{BRST} , the path integral in Eq. (4.17) and the actions in Eq. (4.18) are not modified. One can now verify that $S_{\text{BRST}} = S_{\text{SQED}} + S_{gh} + S_{gf}$ is not only invariant under the BRST transformations given in Eq. (4.19), but also under the following co-BRST transformations

$$\begin{aligned}\bar{s}a_b(x) &= -i \int_{\Sigma} dV_y \frac{1}{2} \lambda^{-1}(y) \dot{\bar{\mathcal{C}}}(y) \mathcal{D}_b^x(G(x, y)) \\ &\quad - i \oint_{\partial\Sigma} da_y \frac{1}{2} \lambda^{-2}(y) \bar{\mathcal{C}}(y) \mathcal{D}_b^x(G(x, y)), \\ \bar{s}\Phi(x) &= - \int_{\Sigma} dV_y \frac{1}{2} g \lambda^{-1}(y) \dot{\bar{\mathcal{C}}}(y) \Phi(x) G(x, y) \\ &\quad - \oint_{\partial\Sigma} da_y \frac{1}{2} g \lambda^{-2}(y) \bar{\mathcal{C}}(y) \Phi(x) G(x, y), \\ \bar{s}\Phi^*(x) &= \int_{\Sigma} dV_y \frac{1}{2} g \lambda^{-1}(y) \dot{\bar{\mathcal{C}}}(y) \Phi^*(x) G(x, y) \\ &\quad + \oint_{\partial\Sigma} da_y \frac{1}{2} g \lambda^{-2}(y) \bar{\mathcal{C}}(y) \Phi^*(x) G(x, y), \\ \bar{s}\mathcal{C}(x) &= \int_{\Sigma} dV_y \frac{1}{2} (\mathcal{D}_a^y(\lambda^{-1}(y) \dot{a}^a(y)) \\ &\quad - ig \lambda^{-1}(y) (\Phi^*(y) \mathcal{D}_0^y \Phi(y) - \Phi(y) \mathcal{D}_0^y \Phi^*(y))) G(x, y) \\ &\quad - \frac{1}{2} \phi(x) - \oint_{\partial\Sigma} da_y \frac{1}{2} \lambda^{-1}(y) n_a^y \dot{a}^a(y) G(x, y), \\ \bar{s}\phi(x) &= -\frac{1}{2} i \bar{\mathcal{C}}(x), & \bar{s}\pi_\phi(x) &= i\lambda^{-1}(x) \dot{\bar{\mathcal{C}}}(x), \\ \bar{s}\bar{\mathcal{C}}(x) &= 0.\end{aligned}\quad (5.7)$$

The transformations in Eq. (5.7) were determined from Eq. (5.4) after substituting the values of the momenta at their extremum, specifically $\pi_a = \lambda^{-1} \dot{a}_a - \lambda^{-1} \mathcal{D}_a \phi$ and $\mathcal{P} = -i\lambda^{-1} \dot{\bar{\mathcal{C}}} - i(\lambda^{-2} \bar{\mathcal{C}})|_{\mathcal{H}}$. These are the same expressions for the momenta as those in the previous section. The co-BRST transformations in Eq. (5.7) are the curved spacetime generalizations of those presented in [35], where they were used to demonstrate the invariance of dressed scalar fields. Specifically in flat spacetime, the dressed scalar fields defined as

$$\Phi_{\text{phys}} = \Phi \exp\left(-ig \frac{\partial_i A^i}{\nabla^2}\right), \quad \Phi_{\text{phys}}^* = \Phi^* \exp\left(ig \frac{\partial_i A^i}{\nabla^2}\right), \quad (5.8)$$

satisfy the conditions $s\Phi_{\text{phys}} = 0 = \bar{s}\Phi_{\text{phys}}$ and $s\Phi_{\text{phys}}^* = 0 = \bar{s}\Phi_{\text{phys}}^*$, while Φ and Φ^* do not. In Eq. (5.8), the index i denotes spatial coordinates, while ∇^{-2} is the inverse Laplacian of flat spacetime which satisfies $\nabla_x^2 \nabla^{-2}(\vec{x}, \vec{y}) = -\delta(\vec{x} - \vec{y})$, where $\delta(\vec{x} - \vec{y})$ is the Dirac delta function on flat spacetime. Given the transformations in Eq. (5.7) and the equation satisfied by the Green function $G(x, y)$ in Eq. (5.6), the following dressed fields

$$\begin{aligned} \Phi_{\text{phys}}(x) &= \Phi(x) \exp\left(-ig \int_{\Sigma} dV_y \mathcal{D}_a^y(\lambda^{-1}(y) a^a(y)) G(x, y) \right. \\ &\quad \left. + ig \oint_{\partial\Sigma} da_y \lambda^{-1}(y) n_a^y a^a(y) G(x, y)\right), \\ \Phi_{\text{phys}}^*(x) &= \Phi^*(x) \exp\left(ig \int_{\Sigma} dV_y \mathcal{D}_a^y(\lambda^{-1}(y) a^a(y)) G(x, y) \right. \\ &\quad \left. - ig \oint_{\partial\Sigma} da_y \lambda^{-1}(y) n_a^y a^a(y) G(x, y)\right), \end{aligned} \quad (5.9)$$

can be seen to satisfy $s\Phi_{\text{phys}} = 0 = \bar{s}\Phi_{\text{phys}}$ and $s\Phi_{\text{phys}}^* = 0 = \bar{s}\Phi_{\text{phys}}^*$. In the flat limit, Eq. (5.9) reduces to Eq. (5.8). The additional surface integrals now account for contributions from the horizons of the spacetime. In particular, the above expressions for dressed matter also hold for backgrounds with cosmological horizons. Thus the co-BRST charge can be used to identify dressed matter fields on black hole backgrounds which are not asymptotically flat.

In the next section, we will discuss how the dressed fields in Eq. (5.9) could be relevant in consideration of soft photon limits on backgrounds with horizons. Before proceeding to this discussion, we would like to provide a few comments on the above construction, specifically with regards to the choice of gauge. As we noted in Sec. IV, the nilpotence of the BRST charge Q_{BRST} allows us to choose Ψ in any way we please. In the case of the co-BRST construction, an arbitrary choice can also be considered, but this would in general require modifying Ω_2 in the expressions given in Eq. (5.2) and the need for additional conditions on the Green function $G(x, y)$ apart from Eq. (5.3). In hindsight, we can state that the gauge as chosen in this section provides the simplest generalization of the known construction of the co-BRST charge in flat spacetime. Furthermore, as mentioned previously, in the absence of the surface terms in Eq. (5.6), the solution for the Green function is simply that of the electrostatic potential on spherically symmetric backgrounds, for which there exist known closed form expressions. Thus from the

standpoint of determining physical observables from the path integral, the gauge considered in this paper will prove useful. As is well known, the results for physical observables will in any case be independent of the choice of gauge.

VI. SUMMARY AND DISCUSSION

In this work, we considered the Hamiltonian BRST formalism for constrained theories on spherically symmetric backgrounds with horizons. We first provided the geometric framework needed to perform the Hamiltonian analysis on spacelike hypersurfaces orthogonal to the timelike Killing vector field of the spacetime. By considering the action for scalar quantum electrodynamics as an example, we then derived the constraints using the Dirac-Bergmann formalism. Keeping with our consideration of backgrounds with horizons, we were careful to evaluate Poisson brackets with smearing functions that are regular at the horizons. The Gauss law constraint, derived from the evaluation of Poisson brackets, was shown to involve terms from the horizon(s) of the spacetime. We then considered the Hamiltonian BRST formalism of the theory in the extended phase space involving the ghosts and their momenta. By fixing null components of the gauge fields at the horizons, we demonstrated that the ghost number charge of the theory involve additional corrections from the horizons of the background. We further considered gauge fixing fermions that generate their own nilpotent symmetry transformations which leave the action invariant. The gauge fixing fermion in this case is identified with the co-BRST operator. The requirement that physical fields are invariant under BRST and co-BRST transformations led us to identify dressed gauge invariant scalar fields of scalar electrodynamics, whose dressing function depends on the electromagnetic fields at the horizons of the background.

One of the avenues for further investigation following the results in this paper involves the quantization of gauge theories on black hole backgrounds. In identifying that the constraints are modified at the horizons, it is clear that as operator relations the constraints must be satisfied by states in the bulk and at the horizons. The mode expansion for gauge fields at the horizon and in particular their polarizations can be expected to be along the null directions at the horizons. This could be used to further explore the nature of ‘‘edge modes’’ at the horizon, along the lines of that which has been considered on the spatial boundaries of manifolds [17, 18, 27, 30, 31]. A consideration of the Hilbert spaces and the independent modes at the horizons and in the bulk of the spacetime lie outside the scope of the present work. We do note that in this regard the dressed gauge invariant fields, co-BRST operator and the corrections of the ghost number charge at the horizons, as considered in this paper, will be particularly useful. We should mention that a standard application of the BRST symmetry is its use in proving the renormalizability of a theory, using the

Zinn-Justin equation for example. In our example above, the BRST transformations on spacetimes with horizons turn out to be the same as on those without horizons, so the Zinn-Justin equation is unaffected.

The use of dressed gauge invariant fields in quantum electrodynamics was originally considered by Dirac [48]. The infrared properties of dressed fields were initiated in [49,50] and their relevance in providing a finite S-matrix for quantum electrodynamics was provided by Faddeev and Kulish [51]. More recently, dressed fields have been shown to provide a realization of the soft charges at null infinity on asymptotically flat spacetimes which are consistent with Weinberg’s soft photon theorem [52]. While soft hairs on the horizons of black holes of asymptotically flat spacetime have been argued for in [8], a similar realization of such soft hairs in terms of dressed fields and their implications on black hole information remain open problems. The dressed fields in Eq. (5.9) could be useful in this respect.

Ordinarily, there is a considerable amount of freedom in choosing the gauge dressing of fields in a given theory. For instance, the following dressed field in flat spacetime is perfectly legitimate

$$\Phi_{\text{phys}}(x) = \Phi(x) \exp \left(\int_{\Gamma} dz^i A_i(x_0, z) \right), \quad (6.1)$$

where the integral in the exponent is over some path Γ . This represents the “Wilson dressing” for a given scalar field Φ . While such a dressing can be appropriate in the context of QCD and within holography, we note that in QED this dressing defines an infinitely excited state, where the electric flux is confined along Γ . On the other hand, the field given in Eq. (5.8) does provides the correct expression for the electric field of a static charge. It is particularly important to identify physically viable dressings in order to further investigate infrared properties and soft limits, which in the case of the dressed fields of Eq. (5.8) were studied in

[53,54]. By involving horizon corrections to the dressing function of static scalar fields, Eq. (5.9) in particular allows for the consideration of scattering processes near the horizon following the expansion of the exponential.

The modification of the ghost number charge could also have interesting implications. This is particularly true for thermal gauge theories, whose partition function in the thermofield double formalism is known to depend on the ghost number charge [55,56]. Specifically we note that while $\text{Tr}e^{-\beta H}$ provides the correct partition function for theories without gauge invariance, this is not the case in gauge theories whose state space involves unphysical d.o.f. such as the longitudinal modes of the gauge fields and the ghosts. One can proceed to determine the physical state space either through the co-BRST construction or by adopting a special gauge, such as the Coulomb gauge and axial gauge, in which no physical particles appear. However a much simpler alternative was provided in [55,56], where it was shown that $\text{Tr}e^{-\beta H - \pi Q_c}$ describes the thermal partition function for gauge theories, consistent with the correct (periodic) boundary conditions of the ghosts. Thus by simply substituting $\text{Tr}e^{-\beta H}$ with $\text{Tr}e^{-\beta H - \pi Q_c}$, we can proceed with gauge theories just as one does in theories without gauge invariance. In the context of our paper, we demonstrated that both the Hamiltonian and ghost number charge involve surface corrections at the horizons of the background. This implies that known correlation functions and thermal propagators in flat spacetime could also be involve corrections from the horizons of the background. We look forward to performing these and related investigations in future work.

ACKNOWLEDGMENTS

The research of K. F. is supported in part by Max Planck Partner Group “Quantum Black Holes” between CMI Chennai and AEI Potsdam.

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- [1] C. Becchi, A. Rouet, and R. Stora, Renormalization of gauge theories, *Ann. Phys. (N.Y.)* **98**, 287 (1976).
 - [2] I. V. Tyutin, Gauge invariance in field theory and statistical physics in operator formalism, [arXiv:0812.0580](https://arxiv.org/abs/0812.0580).
 - [3] M. Henneaux, Hamiltonian form of the path integral for theories with a gauge freedom, *Phys. Rep.* **126**, 1 (1985).
 - [4] M. Henneaux and C. Teitelboim, *Quantization of Gauge Systems* (Princeton University, Princeton, NJ, 1992), p. 520.
 - [5] T. He, P. Mitra, A. P. Porfyriadis, and A. Strominger, New symmetries of massless QED, *J. High Energy Phys.* **10** (2014) 112.
 - [6] T. He, V. Lysov, P. Mitra, and A. Strominger, BMS supertranslations and Weinbergs soft graviton theorem, *J. High Energy Phys.* **05** (2015) 151.
 - [7] D. Kapec, M. Pate, and A. Strominger, New symmetries of QED, *Adv. Theor. Math. Phys.* **21**, 1769 (2017).
 - [8] S. W. Hawking, M. J. Perry, and A. Strominger, Soft Hair on Black Holes, *Phys. Rev. Lett.* **116**, 231301 (2016).
 - [9] S. W. Hawking, M. J. Perry, and A. Strominger, Superrotation charge and supertranslation hair on black holes, *J. High Energy Phys.* **05** (2017) 161.
 - [10] F. Hopfmüller and L. Freidel, Null conservation laws for gravity, *Phys. Rev. D* **97**, 124029 (2018).

- [11] V. Chandrasekaran, E. E. Flanagan, and K. Prabhu, Symmetries and charges of general relativity at null boundaries, *J. High Energy Phys.* **11** (2018) 125.
- [12] M. Blau and M. O’Loughlin, Horizon shells and BMS-like soldering transformations, *J. High Energy Phys.* **03** (2016) 029.
- [13] T. Regge and C. Teitelboim, Improved Hamiltonian for general relativity, *Phys. Lett.* **53B**, 101 (1974).
- [14] T. Regge and C. Teitelboim, Role of surface integrals in the Hamiltonian formulation of general relativity, *Ann. Phys. (N.Y.)* **88**, 286 (1974).
- [15] R. Benguria, P. Cordero, and C. Teitelboim, Aspects of the Hamiltonian dynamics of interacting gravitational gauge and Higgs fields with applications to spherical symmetry, *Nucl. Phys.* **B122**, 61 (1977).
- [16] A. P. Balachandran, L. Chandar, E. Ercolessi, T. R. Govindarajan, and R. Shankar, Maxwell-Chern-Simons electrodynamics on a disk, *Int. J. Mod. Phys. A* **09**, 3417 (1994).
- [17] A. P. Balachandran and P. Teotonio-Sobrinho, The edge states of the BF system and the London equations, *Int. J. Mod. Phys. A* **08**, 723 (1993).
- [18] A. P. Balachandran, L. Chandar, and B. Sathiapalan, Chern-Simons duality and the quantum Hall effect, *Int. J. Mod. Phys. A* **11**, 3587 (1996).
- [19] J. Isenberg and J. Nester, Canonical gravity, edited by A. Held, in *General Relativity and Gravitation* (Plenum Press, New York, 1980), Vol. 1, pp. 23–97.
- [20] M. M. Sheikh-Jabbari and A. Shirzad, Boundary conditions as Dirac constraints, *Eur. Phys. J. C* **19**, 383 (2001).
- [21] M. Zabzine, Hamiltonian systems with boundaries, *J. High Energy Phys.* **10** (2000) 042.
- [22] I. Moss and S. J. Poletti, Boundary conditions for quantum cosmology, *Nucl. Phys.* **B341**, 155 (1990).
- [23] I. Moss and S. J. Poletti, Quantum cosmology with antisymmetric tensor fields, *Phys. Lett. B* **245**, 355 (1990).
- [24] I. G. Moss and P. J. Silva, BRST invariant boundary conditions for gauge theories, *Phys. Rev. D* **55**, 1072 (1997).
- [25] I. G. Moss, BRST-invariant boundary conditions and strong ellipticity, *Phys. Rev. D* **88**, 104039 (2013).
- [26] W. Donnelly and A. C. Wall, Unitarity of Maxwell theory on curved spacetimes in the covariant formalism, *Phys. Rev. D* **87**, 125033 (2013).
- [27] W. Donnelly and A. C. Wall, Entanglement Entropy of Electromagnetic Edge Modes, *Phys. Rev. Lett.* **114**, 111603 (2015).
- [28] W. Donnelly and A. C. Wall, Geometric entropy and edge modes of the electromagnetic field, *Phys. Rev. D* **94**, 104053 (2016).
- [29] T. Nishioka, Entanglement entropy: Holography and renormalization group, *Rev. Mod. Phys.* **90**, 035007 (2018).
- [30] A. Blommaert, T. G. Mertens, H. Verschelde, and V. I. Zakharov, Edge state quantization: Vector fields in rindler, *J. High Energy Phys.* **08** (2018) 196.
- [31] A. Blommaert, T. G. Mertens, and H. Verschelde, Edge dynamics from the path integral: Maxwell and Yang-Mills, *J. High Energy Phys.* **11** (2018) 080.
- [32] G. Barnich, Black hole entropy from non-proper gauge degrees of freedom: II. The charged vacuum capacitor, *Phys. Rev. D* **99**, 026007 (2019).
- [33] K. Fernandes, S. Ghosh, and A. Lahiri, Constrained field theories on spherically symmetric spacetimes with horizons, *Phys. Rev. D* **95**, 045012 (2017).
- [34] K. Fernandes and A. Lahiri, Constrained field theories on Kerr backgrounds, *Eur. Phys. J. C* **79**, 160 (2019).
- [35] M. Lavelle and D. McMullan, A New Symmetry for QED, *Phys. Rev. Lett.* **71**, 3758 (1993).
- [36] R. Marnelius, Time evolution in general gauge theories, in *New Non-Perturbative Methods and Quantization on the Light Cone, Les Houches School, 1997*, edited by P. Grangé *et al.* (Springer-Verlag Berlin Heidelberg, 1997), pp. 63–69.
- [37] P. A. M. Dirac, *Lectures on Quantum Mechanics* (Yeshiva University, New York, 1964).
- [38] K. Sundermeyer, Constrained dynamics with applications to Yang-mills theory, general relativity, classical spin, dual string model, *Lect. Notes Phys.* **169**, 1 (1982).
- [39] A. J. Hanson, T. Regge, and C. Teitelboim, *Constrained Hamiltonian Systems* (Accademia Nazionale dei Lincei, Rome, 1976).
- [40] N. Mukunda and G. Sudarshan, *Classical Dynamics: A Modern Perspective* (Wiley, New York, 1974).
- [41] E. S. Fradkin and G. A. Vilkovisky, Quantization of relativistic systems with constraints, *Phys. Lett.* **55B**, 224 (1975).
- [42] I. A. Batalin and G. A. Vilkovisky, Relativistic S matrix of dynamical systems with boson and fermion constraints, *Phys. Lett.* **69B**, 309 (1977).
- [43] E. T. Copson, On electrostatics in a gravitational field, *Proc. R. Soc. A* **118**, 184 (1928).
- [44] J. M. Cohen and R. Wald, Point charge in the vicinity of a Schwarzschild black hole, *J. Math. Phys. (N.Y.)* **12**, 1845 (1971).
- [45] R. S. Hanni and R. Ruffini, Lines of force of a point charge near a Schwarzschild black hole, *Phys. Rev. D* **8**, 3259 (1973).
- [46] B. Linet, Electrostatics and magnetostatics in the Schwarzschild metric, *J. Phys. A* **9**, 1081 (1976).
- [47] B. Leaute and B. Linet, Electrostatics in a Reissner-Nordstrom space-time, *Phys. Lett.* **58A**, 5 (1976).
- [48] P. A. M. Dirac, Gauge invariant formulation of quantum electrodynamics, *Can. J. Phys.* **33**, 650 (1955).
- [49] V. Chung, Infrared divergence in quantum electrodynamics, *Phys. Rev.* **140**, B1110 (1965).
- [50] T. W. B. Kibble, Coherent soft-photon states and infrared divergences I: Classical currents, *J. Math. Phys. (N.Y.)* **9**, 315 (1968); Coherent soft-photon states and infrared divergences ii. Mass-shell singularities of green’s functions, *Phys. Rev.* **173**, 1527 (1968); Coherent soft-photon states and infrared divergences. iii. Asymptotic states and reduction formulas, *Phys. Rev.* **174**, 1882 (1968); Coherent soft-photon states and infrared divergences. iv. The scattering operator, *Phys. Rev.* **175**, 1624 (1968).
- [51] P. P. Kulish and L. D. Faddeev, Asymptotic conditions and infrared divergences in quantum electrodynamics, *Teor. Mat. Fiz.* **4**, 153 (1970) [*Theor. Math. Phys.* **4**, 745 (1971)].
- [52] D. Kapec, M. Perry, A. M. Raclariu, and A. Strominger, Infrared divergences in QED, revisited, *Phys. Rev. D* **96**, 085002 (2017).

- [53] E. Bagan, M. Lavelle, and D. McMullan, Charges from dressed matter: Construction, *Ann. Phys. (N.Y.)* **282**, 471 (2000).
- [54] E. Bagan, M. Lavelle, and D. McMullan, Charges from dressed matter: Physics and renormalization, *Ann. Phys. (N.Y.)* **282**, 503 (2000).
- [55] H. Hata and T. Kugo, An operator formalism of statistical mechanics of gauge theory in covariant gauges, *Phys. Rev. D* **21**, 3333 (1980).
- [56] I. Ojima, Gauge fields at finite temperatures: Thermo field dynamics, KMS condition and their extension to gauge theories, *Ann. Phys. (N.Y.)* **137**, 1 (1981).