

Gravitational waves from inspiralling compact binaries in conformal gravity

Patric Hölscher^{*} and Dominik J. Schwarz[†]

Fakultät für Physik, Bielefeld University, Postfach 100131, 33501 Bielefeld, Germany



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We investigate the production of gravitational waves during the inspiral of compact binaries close to their merger in the context of a conformal gravity model. The model incorporates five massive polarisation degrees of freedom, besides the two massless gravitational wave polarizations of general relativity. For small graviton mass, we find that the amplitude of the gravitational waves is strongly suppressed as compared to general relativity and decreases as coalescence is approached, which contradicts the observational fact. We conclude that this model with small graviton mass, including a regime that can explain galaxy rotation curves without dark matter, cannot describe the observed gravitational wave events. For a large graviton mass, the modifications to the waveform, compared to the one from general relativity, are negligible on the relevant distance scales and hence a conformal gravity model with a large graviton mass is in agreement with LIGO/VIRGO observations and leads to chirp mass and distance estimates that agree with those from general relativity.

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I. INTRODUCTION

With the detection of gravitational waves (GW) by the aLIGO and VIRGO observatories the era of GW astronomy started. Several binary black hole mergers were detected [1–6]. The analysis of these events shows that the predictions of general relativity (GR) are in very accurate agreement with these observations, which at the same time open a fantastic window of opportunity to test the physics of gravity beyond GR. Recently also the first binary neutron star merger (GW170817) has been detected with electromagnetic follow-up measurements in nearly the whole electromagnetic spectrum coming from GRB 170817A [7–9]. From the measured difference in the arrival time of the gravitational and electromagnetic signal strong constraints on the speed of the GWs are given. This rules out several theories of modified gravity [10–17], especially when the graviton has a small mass.

In this work we investigate the wave form of GWs in conformal gravity (CG) to investigate if the GW detections of aLIGO and VIRGO can be explained within these theories. This theory is special because it is based on a Weyl invariant action, which is unique up to the choice of the matter content, the coupling constants and their signs. Explicit mass scales are forbidden by this symmetry, and become manifest only after fixing Weyl symmetry. The Einstein-Hilbert term and the cosmological constant appear then together with all other masses and the CG models exhibit two gravitational modes, a massless and a massive

one. Thus these models cannot be constrained based on the signal travel time alone, as the massless mode behaves as in GR and a more detailed analysis of the wave forms is required.

A special case is Mannheim’s CG model for which an exact solution for the gravitational potential for a static, spherically symmetric system was found [18]. In addition to the $1/r$ -term in the Schwarzschild solution it contains a term linear in r , which can be used to fit a large number of galaxy rotation curves without the need for dark matter [19–21]. However, cosmic structure formation has not been analyzed yet.

The model of CG studied in this work is a fourth-order derivative theory. This makes them power-counting renormalizable [22,23], which means that they are candidates for a theory of quantum gravity. But CG models with higher derivatives suffer from the Weyl ghost, although it is claimed that the ghost issue can be solved [24–31].

In [32] the authors have analyzed the gravitational radiation of stellar binary systems in a CG model (CGM) that includes Mannheim’s CG as a special case. A neutron star-white dwarf system in slow stationary inspiral on circular orbits in the Newtonian limit was studied and two parameter regimes, corresponding to a small and a large graviton mass, were identified. It was shown that for the case of a small graviton mass the radiated power cannot explain the decrease of the orbital period. However, it turned out there is a parameter regime with a large graviton mass which is in agreement with the observations and which contains a GR limit.

For a small graviton mass the energy lost by the binary system could be stored in the near field by some

^{*}patric.hoelscher@physik.uni-bielefeld.de
[†]dschwarz@physik.uni-bielefeld.de

mechanism, which means that this parameter regime is not necessarily ruled out. In the present work we use the measurements of GWs in the late phase of the evolution of a binary system. aLIGO and VIRGO measured GW signals consistent with the predictions of GR. Hence, we investigate if it is possible to also fit the observed GW signal with wave forms predicted in the CGM. As we show below, this turns out to be impossible and we can thus rule out small graviton masses and therefore also Mannheim's model of CG.

In Sec. II we give an introduction to the CGM. In Sec. III we recap the results from [32], where the linearized equations of motion and their GW solutions for a binary system on circular orbits and in the Newtonian limit were worked out. We discuss the modifications to Kepler's third law in Sec. IV and calculate the wave form in the late inspiral phase and compare it to GR. Lastly, we discuss our findings and conclude.

In the first two sections and in the beginning of Sec. III we use natural units, hence $c = \hbar = 1$. Greek indices take values 0 to 3, whereas latin indices run from 1 to 3. Other conventions are specified in the Appendix.

II. CONFORMAL GRAVITY MODELS

The action of models of CG is given by

$$I = -\alpha_g \int d^4x \sqrt{-g} C_{\lambda\mu\nu\kappa} C^{\lambda\mu\nu\kappa} + I_M \quad (1)$$

where α_g is a dimensionless coupling constant, x denotes the spacetime coordinates, g is the determinant of the metric tensor $g_{\mu\nu}$, $C_{\lambda\mu\nu\kappa}$ is the Weyl tensor and I_M denotes the matter action. Using the Gauss-Bonnet term, which is a total derivative in four space-time dimensions and hence does not contribute to the field equations for the metric, we can rewrite (1) as

$$I = -2\alpha_g \int d^4x \sqrt{-g} \left(R_{\mu\kappa} R^{\mu\kappa} - \frac{1}{3} R^2 \right) + I_M, \quad (2)$$

where $R_{\mu\kappa}$ and R are the Ricci tensor and scalar. The gravitational action is invariant under Weyl transformations of the metric,

$$g_{\mu\nu}(x) \rightarrow \Omega^2(x) g_{\mu\nu}(x), \quad (3)$$

where the conformal factor Ω is real, positive and smooth.

Variation of the action (2) with respect to $g_{\mu\nu}$ leads to the equations of motion for the gravitational field [33]

$$4\alpha_g W^{\mu\nu} = 4\alpha_g [2C^{\mu\lambda\nu\kappa}{}_{;\lambda\kappa} - C^{\mu\lambda\nu\kappa} R_{\lambda\kappa}] = T_M^{\mu\nu}, \quad (4)$$

where $W_{\mu\nu}$ is the Bach tensor and

$$T_M^{\mu\nu} \equiv \frac{2}{(-g)^{1/2}} \frac{\delta I_M}{\delta g_{\mu\nu}} \quad (5)$$

is the matter energy-momentum tensor.

The matter energy-momentum tensor should also be Weyl invariant. Hence, the most general local matter action for a generic scalar and a spinor field coupled conformally to gravity is [34]

$$I_M = - \int d^4x \sqrt{-g} \left[\epsilon \left(-\frac{S^\mu S_{,\mu}}{2} + \frac{S^2 R}{12} \right) + \lambda S^4 + i\bar{\psi} \gamma^\mu(x) [\partial_\mu + \Gamma_\mu(x)] \psi - \xi S \bar{\psi} \psi \right]. \quad (6)$$

$S(x)$ represents a self-interacting real scalar field and $\psi(x)$ is a generic spin-1/2 fermion field. ξ and λ are dimensionless coupling constants, $\gamma^\mu(x)$ are the vierbein-dependent Dirac-gamma matrices, $\bar{\psi} = \psi^\dagger \gamma^0$ and $\Gamma_\mu(x)$ is the fermion spin connection [35]. To be invariant under local Weyl transformations the several fields have to transform as $S(x) \rightarrow \Omega^{-1}(x) S(x)$, $\psi(x) \rightarrow \Omega^{-3/2}(x) \psi(x)$ and $g_{\mu\nu}(x) \rightarrow \Omega^2(x) g_{\mu\nu}(x)$. The exponent of the conformal factor is called conformal weight.

In the action (6) only the combination of the two terms in round brackets is Weyl invariant. Hence, we introduce the parameter ϵ , which can take the values -1 or $+1$. In the first case, the theory corresponds to the model advocated by Mannheim and Kazanas [18]. The case $\epsilon = +1$ was called massive conformal gravity in our previous work [32], but it was pointed out in [36] that this naming is in conflict with previous terminology. In order to avoid any further confusion, we refer to both cases as the same conformal gravity model (referred to as CGM below) which has two distinct regimes specified by $\epsilon = \pm 1$.

Since the action I is Weyl invariant, it is always possible to choose a frame in which the scalar field is constant

$$S(x) \rightarrow S'(x) = \Omega^{-1}(x) S(x) = S_0 = \text{const}, \quad (7)$$

with $\Omega(x) = S(x)/S_0$. This is called the Higgs or unitary gauge [37,38]. In order to make connection to GR, one chooses

$$8\pi G \equiv \frac{6}{S_0^2}, \quad (8a)$$

$$\Lambda \equiv 6\lambda S_0^2, \quad (8b)$$

where G denotes Newton's constant and Λ is interpreted as the cosmological constant. This is a reasonable choice because for $\epsilon = +1$ and in the limit $\alpha_g \rightarrow 0$, the action I becomes the Einstein-Hilbert action with minimally coupled matter.

Variation of the action (6) with respect to $g_{\mu\nu}$ leads to the matter energy-momentum tensor

$$T_{\mu\nu}^M = T_{\mu\nu} + \frac{\epsilon}{8\pi G} \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) - g_{\mu\nu} \frac{\Lambda}{8\pi G}, \quad (9)$$

where

$$T_{\mu\nu} = \frac{1}{2} [i\bar{\psi}\gamma_\mu(x)[\partial_\nu + \Gamma_\nu(x)]\psi + (\mu \leftrightarrow \nu)] \quad (10)$$

is the fermionic energy-momentum tensor.

The equation for the gravitational field now becomes [39,40],

$$4\alpha_g W_{\mu\nu} = T_{\mu\nu} + \frac{1}{8\pi G} [\epsilon G_{\mu\nu} - g_{\mu\nu} \Lambda], \quad (11)$$

where $G_{\mu\nu}$ denotes the Einstein tensor. From the trace of (11) we get

$$\epsilon R + 4\Lambda = 8\pi GT, \quad (12)$$

where T is the trace of the fermionic matter energy-momentum tensor. Note that the fermion energy-momentum tensor is covariantly conserved,

$$T_{;\nu}^{\mu\nu} = 0, \quad (13)$$

due to the Bianchi identities for the Bach and Einstein tensors.

We observe that it is convenient to introduce a new parameter m_g with the dimensions of a mass,

$$m_g^2 \equiv \frac{1}{32\pi G\alpha_g}. \quad (14)$$

We can then write

$$-\epsilon G_{\mu\nu} + g_{\mu\nu} \Lambda + \frac{1}{m_g^2} W_{\mu\nu} = 8\pi GT_{\mu\nu}, \quad (15)$$

and observe that in the limit $m_g \rightarrow \infty$, the Einstein equations are recovered for $\epsilon = +1$. Mannheim's case ($\epsilon = -1$) does not contain general relativity as a limit.

III. GRAVITATIONAL WAVES

In [32] we linearized the equations of motion for the metric and derived the gravitational wave solutions for conformal gravity with both signs of ϵ and for different parameter regions. In a flat Minkowski background we found a massless and a massive mode of the metric perturbation given by

$$h_{\mu\nu} = \epsilon(H_{\mu\nu} + \Psi_{\mu\nu}), \quad (16)$$

where

$$\Psi_{\mu\nu} = \frac{1}{m_g^2} \left(\square h_{\mu\nu} - \frac{1}{3} \eta_{\mu\nu} R \right) \quad (17)$$

represents the massive mode and $H_{\mu\nu}$ is the massless mode. Using (16) and (17) in the linearized version of (15) leads to

$$\square \bar{H}_{\mu\nu} = -16\pi GT_{\mu\nu}, \quad \partial^\mu \bar{H}_{\mu\nu} = 0, \quad (18a)$$

$$(\square - \epsilon m_g^2) \hat{\Psi}_{\mu\nu} = 16\pi GT_{\mu\nu}, \quad \partial_\rho \partial_\sigma \hat{\Psi}^{\rho\sigma} = 0, \quad (18b)$$

where $\bar{H}_{\mu\nu} \equiv H_{\mu\nu} - 1/2 \eta_{\mu\nu} H$ and $\hat{\Psi}_{\mu\nu} \equiv \Psi_{\mu\nu} - \eta_{\mu\nu} \Psi$. This result holds in the Teyssandier gauge

$$Z_\mu = -m_g^{-2} [(\square - \epsilon m_g^2) \partial_\rho \bar{h}_\mu^\rho + (1/3) \partial_\mu R] = 0, \quad (19)$$

see [41,42]. It can be shown that there are two massless polarization degrees of freedom (d.o.f.) encoded in $H_{\mu\nu}$ and five massive ones encoded in $\Psi_{\mu\nu}$. The solutions for the massless modes are identical to the case of general relativity. For the massive modes we found that binary systems can excite only two of the five modes and, as in general relativity, the gravitational waves are sourced by time varying quadrupole moments of the mass distribution.

The solutions for the metric perturbation for a binary system of two compact objects of mass m_1 and m_2 in circular motion in the Newtonian limit and in the center of mass frame were derived in [32]. The dynamics of the system can be reduced to a one-body problem with reduced mass $\mu = m_1 m_2 / (m_1 + m_2)$. The relative coordinates for a circular path in the xy-plane are given by

$$x(t) = -R \sin(\omega_s t), \quad (20a)$$

$$y(t) = R \cos(\omega_s t), \quad (20b)$$

where R is the distance between the two compact objects and ω_s is the orbital frequency. The corresponding circular frequency of the emitted gravitational waves is $\omega_{\text{gw}} = 2\omega_s$. To find an approximate analytic solutions to Eq. (18b) we distinguish two cases. The relation between the graviton mass m_g and the frequency ω_s determines the physical behavior of the massive mode.

For a small graviton mass, $m_g c^2 < \hbar \omega_{\text{gw}}$, in [32] we found

$$\begin{aligned} h_{11}(t, r) &= -h_{22}(t, r) \\ &= \frac{\epsilon G \mu R^2 \omega_{\text{gw}}^2}{r c^4} [\cos(\omega_{\text{gw}} t_0) - \cos(\omega_{\text{gw}} t_m)], \end{aligned} \quad (21a)$$

$$\begin{aligned} h_{12}(t, r) &= h_{21}(t, r) \\ &= \frac{\epsilon G \mu R^2 \omega_{\text{gw}}^2}{r c^4} [\sin(\omega_{\text{gw}} t_0) - \sin(\omega_{\text{gw}} t_m)], \end{aligned} \quad (21b)$$

where now t denotes the time of an observer at distance r to the source, $t_0 = t - r/c$ is the retarded time for the massless mode and $t_m = t - v_{g,\epsilon} r/c^2$ is the retarded time for the massive mode. $v_{g,\epsilon} = c[1 - \epsilon m_g^2 c^4 / \hbar^2 \omega_{\text{gw}}^2]^{1/2}$ is the group velocity of the massive gravitational wave.

For $\epsilon = +1$ and $m_g c^2 > \hbar \omega_{\text{gw}}$, in [32] we found

$$\begin{aligned} h_{11}(t, r) &= -h_{22}(t, r) \\ &= \frac{1}{r} \frac{G\mu R^2 \omega_{\text{gw}}^2}{c^4} [\cos(\omega_{\text{gw}} t_0) - e^{-k_\omega r} \cos(\omega_{\text{gw}} t)], \end{aligned} \quad (22a)$$

$$\begin{aligned} h_{12}(t, r) &= h_{21}(t, r) \\ &= \frac{1}{r} \frac{G\mu R^2 \omega_{\text{gw}}^2}{c^4} [\sin(\omega_{\text{gw}} t_0) - e^{-k_\omega r} \sin(\omega_{\text{gw}} t)], \end{aligned} \quad (22b)$$

where $k_\omega = \sqrt{(m_g c / \hbar)^2 - (\omega_{\text{gw}} / c)^2}$. This is the GR solution modified by an exponentially damped term. We do not consider the case of a large graviton mass for $\epsilon = -1$, because in the Newtonian limit this model leads to repulsive gravity.

IV. KEPLER'S THIRD LAW AND THE WAVE FORM OF INSPIRALLING BINARIES

To investigate how gravitational radiation influences the orbits of the binary system, we have to find Kepler's third law in the CGM. For the analysis we consider an idealized system of two compact objects of mass m_1 and m_2 in the Newtonian limit and in the center of mass frame with reduced mass μ and total mass $M = m_1 + m_2$. Assuming that the Newtonian approximation is applicable in the stationary and quasircular phase of the merger, we can use the Newtonian potential energy. To find Kepler's third law it is necessary to discuss on which scales the modifications to the $1/R$ -potential become important.

For the following estimates we consider the typical gravitational waves as observed with the LIGO/VIRGO detectors. These gravitational waves are in the audio band, thus have a typical frequency of 1 kHz and a typical wavelength of 300 km.

Let us first consider $\epsilon = +1$, for which the Newtonian potential is modified by a Yukawa potential. From the analysis of the GWs we found that we have to distinguish two regimes. The case of a large graviton mass $m_g \gg \hbar \omega_{\text{gw}} / c^2$ applies for the reduced Compton wavelength $\lambda_g \equiv \hbar / m_g c \ll c / \omega_{\text{gw}} \approx 50$ km. The Newtonian potential of conformal gravity is then given as

$$E_{\text{pot}, \epsilon=+1} = -\frac{G\mu M}{R} \left(1 - \frac{4}{3} e^{-R/\lambda_g}\right). \quad (23)$$

Tests of the inverse square law from terrestrial to sub-mm distances lead to the constraint $\lambda_g < 10^{-5}$ m [43] (corresponding to $m_g > 10^{-38}$ kg and frequencies $f_g > 4.8 \times 10^{12}$ Hz).

For small graviton mass $m_g \ll \hbar \omega_{\text{gw}} / c^2$, the reduced Compton wavelengths are much longer than 50 km and the modified Newtonian potential must read

$$E_{\text{pot}, \epsilon=+1} = -\frac{4G\mu M}{R} \left(e^{-R/\lambda_g} - \frac{3}{4}\right). \quad (24)$$

Modifications on large length scales are constrained by terrestrial and Solar System tests of the inverse square law, which lead to $\lambda_g > 10^{16}$ m ≈ 0.3 pc [43] (corresponding to $m_g < 10^{-58}$ kg and $f_g < 4.8 \times 10^{-9}$ Hz).

For $\epsilon = -1$,

$$E_{\text{pot}, \epsilon=-1} = -\frac{G\mu M}{R} + \frac{\gamma_\odot \mu M}{2} c^2 R, \quad (25)$$

where γ_\odot is an integration constant with dimension of inverse mass times inverse distance.

The term linear in R in Eq. (25) was used to fit galaxy rotation curves without dark matter leading to $\gamma_\odot = 2.4 \times 10^{-69}$ kg $^{-1}$ m $^{-1}$ [21]. This corresponds to typical length scale $\lambda = (4G/\gamma_\odot c^2)^{1/2} \approx 36$ kpc and therefore contributes noticeably to the potential energy on galactic distance scales ($R > 1$ kpc), but is negligible on Solar System scales.

Assuming that the semimajor axis of the binary systems is well below galactic distance scales (much smaller than the parsec scale) and well above the sub-mm scale, the modifications in (23)–(25) to the $1/R$ -term are negligible. Hence, Kepler's third law for circular orbits is approximately given by

$$\omega_s^2 \approx \frac{GM}{R^3}, \quad (26)$$

and for the orbital energy we find

$$E_{\text{orbit}} = E_{\text{kin}} + E_{\text{pot}} \approx -\frac{G\mu M}{2R} \quad (27)$$

as in the Newtonian approximation of GR. Inserting (26) into (27) we get

$$E_{\text{orbit}} \approx -\left(\frac{G^2 M_c^5 \omega_{\text{gw}}^2}{32}\right)^{1/3}, \quad (28)$$

where $\omega_{\text{gw}} = 2\omega_s$ and $M_c = \mu^{3/5} M^{2/5}$ is the chirp mass. The energy loss of the system is given by

$$\dot{E}_{\text{orbit}} \approx -\frac{2}{3} \left(\frac{G^2 M_c^5}{32 \omega_{\text{gw}}}\right)^{1/3} \dot{\omega}_{\text{gw}}. \quad (29)$$

A. Large graviton mass

In [44,45] it is pointed out that the CGM with a large graviton mass cannot fit galaxy rotation curves without dark matter, as can also be seen most easily from Eq. (23). Nevertheless, the large graviton mass case is very interesting because of its GR limit.

The massive part of the gravitational wave is damped and effectively only the massless graviton, which is the same as in GR, contributes. Nevertheless, there is a profound difference to GR, since this theory is power-counting renormalizable [22,23] and hence could provide a viable theory of quantum gravity.

In this case the Newtonian potential is given by Eq. (23). This means that the Yukawa term in (23) becomes important only on sub-mm distance scales. For binary systems in the inspiral phase the distance between the objects is always macroscopic (at least larger than two Schwarzschild radii) and hence we can neglect this term for the analysis of gravitational radiation from macroscopic binary systems.

The radiated power is given by [32]

$$P = P_{GR} = \frac{32c^5}{5G} \left(\frac{GM_c \omega_{gw}}{2c^3} \right)^{10/3}. \quad (30)$$

In the following we go beyond the quasistationary case that we have considered before.

Equating $\dot{E}_{orbit} = -P$ and solving for $\dot{\omega}_{gw}$ yields

$$\dot{\omega}_{gw} = \frac{12}{5} 2^{1/3} \left(\frac{GM_c}{c^3} \right)^{5/3} \omega_{gw}^{11/3}. \quad (31)$$

Integrating (31) we get

$$\omega_{gw}(\tau) = \frac{1}{4} \left(\frac{5}{\tau} \right)^{3/8} \left(\frac{GM_c}{c^3} \right)^{-5/8}, \quad (32)$$

where $\tau = (t_{coal} - r/c) - (t - r/c) = t_{coal} - t$ is the time to coalescence as measured by a signal that propagates with the speed of light.

Figure 1 shows how the frequency of a gravitational wave, $f_{gw} = \omega_{gw}/2\pi$, evolves over the last 100 seconds before coalescence (in the approximation used in this work). Our approximation breaks down at frequencies above the LIGO/VIRGO frequency band (10 Hz to 10 kHz). To give two examples for the CGM with a large graviton mass we show the evolution of gravitational wave frequency for chirp masses of $1.2 M_\odot$, corresponding to a system of two neutron stars with masses of $1.4 M_\odot$ each, and $30 M_\odot$, a value typical for a pair of black holes.

We look at the wave form of the GWs produced by an object in quasircular motion which is on an orbit in the xy -plane

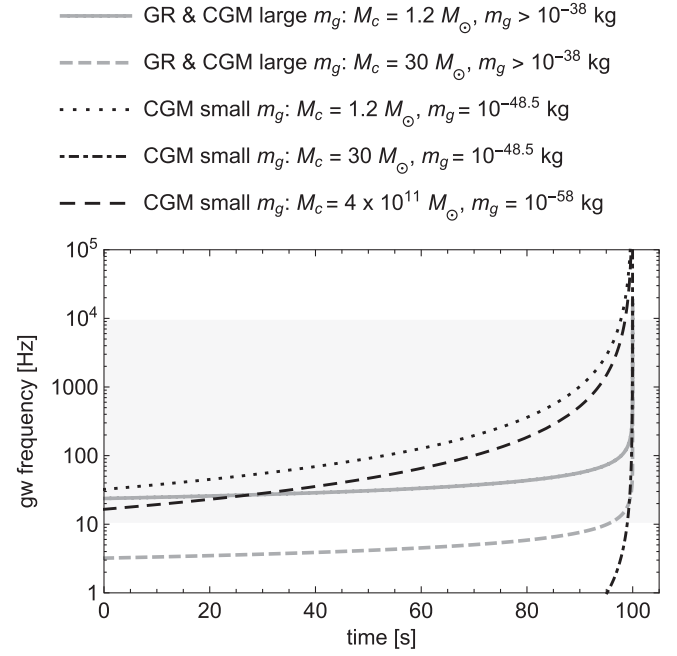


FIG. 1. Time evolution of the gravitational wave frequency f_{gw} of massless modes during the last 100 seconds before coalescence. The shaded region indicates the LIGO/VIRGO frequency band. The grey lines show the GR prediction, which is identical to the CGM prediction with a large graviton mass. The black lines show three examples for the CGM with a small graviton mass. Solar system tests of gravity imply that $m_g < 10^{-58}$ kg.

$$x(\tau) = R(\tau) \sin\left(\frac{\Phi(\tau)}{2}\right), \quad (33a)$$

$$y(\tau) = R(\tau) \cos\left(\frac{\Phi(\tau)}{2}\right). \quad (33b)$$

Since the orbit is not stationary, we have to do the following replacements: $R \rightarrow R(\tau)$, which leads to terms proportional to \dot{R} in the GW solutions when calculating the time derivatives of the quadrupole moment, but as long as $\dot{\omega}_s \ll \omega_s^2$ we can neglect radial velocities. Using (31) this condition translates to $\omega_{gw} \ll 0.51(c^3/GM_c)$. This means we can neglect \dot{R} as long as $f_{gw} \ll 16.6 \text{ kHz}(M_\odot/M_c)$.

Additionally, we have to replace ω_{gw} with $\omega_{gw}(\tau)$ in the amplitude of (22a)–(22b) and $\omega_{gw}t_0$ becomes $\Phi(\tau)$ in the argument of the trigonometric functions. The phase is defined as the circular frequency of the GW integrated over time

$$\Phi(\tau) = \int_{\tau_i}^{\tau} d\tau' \omega_{gw}(\tau') + \Phi_i, \quad (34)$$

where Φ_i is the phase at some initial τ_i . This leads to

$$\Phi(\tau) = \Phi_i - 2 \left(\frac{GM_c}{c^3} \right)^{-5/8} \left[\left(\frac{\tau_i}{5} \right)^{5/8} - \left(\frac{\tau}{5} \right)^{5/8} \right]. \quad (35)$$

Finally, we can find the wave form in terms of the time to coalescence τ ,

$$h_{11}(t, r) = -h_{22}(t, r) = \frac{c}{r} \left(\frac{GM_c}{c^3} \right)^{5/4} \left(\frac{5}{\tau} \right)^{1/4} \cos[\Phi(\tau)], \quad (36a)$$

$$h_{12}(t, r) = h_{21}(t, r) = \frac{c}{r} \left(\frac{GM_c}{c^3} \right)^{5/4} \left(\frac{5}{\tau} \right)^{1/4} \sin[\Phi(\tau)]. \quad (36b)$$

This is the same result as in GR and the predictions are consistent with the observed GW events. Hence, it leads to the same predictions on the chirp mass and the distance to the source.

B. Small graviton mass

For the case of a small graviton mass ($m_g < \hbar\omega_{\text{gw}}/c^2$), the power radiated into GWs is given by [32]

$$P = \frac{1}{2} \left(\frac{m_g c^2}{\hbar\omega_{\text{gw}}} \right)^2 \frac{32c^5}{5G} \left(\frac{GM_c \omega_{\text{gw}}}{2c^3} \right)^{10/3}. \quad (37)$$

We now go beyond the quasistationary limit by equating $\dot{E}_{\text{orbit}} = -P$ and solving for $\dot{\omega}_{\text{gw}}$ yields

$$\dot{\omega}_{\text{gw}} = \frac{6}{5} 2^{1/3} \left(\frac{m_g c^2}{\hbar} \right)^2 \left(\frac{GM_c}{c^3} \right)^{5/3} \omega_{\text{gw}}^{5/3}. \quad (38)$$

We follow the same procedure as above and find with $\tau = t_{\text{coal}} - t$ being the time interval to coalescence as measured by a fiducial observer at the position of the source,

$$\omega_{\text{gw}}(\tau) = \left(\frac{m_g c^2}{\hbar} \right)^{-3} \frac{1}{32} \left(\frac{5}{\tau} \right)^{3/2} \left(\frac{GM_c}{c^3} \right)^{-5/2}. \quad (39)$$

Note that ω_{gw} diverges at $t = t_{\text{coal}}$. This is no problem, since the two objects merge before the divergence takes place and further the Newtonian approximation breaks down, when the two objects come to close together. Inserting numerical values in (39) we find

$$f_{\text{gw}} = 1.67 \times 10^{33} \text{ Hz} \left(\frac{10^{-58} \text{ kg}}{m_g} \right)^3 \left(\frac{M_{\odot}}{M_c} \right)^{5/2} \left(\frac{1 \text{ s}}{\tau} \right)^{3/2}. \quad (40)$$

The evolution of the gravitational wave frequency towards coalescence is shown in Fig. 1. The dotted, dash-dotted and long dashed lines show the case of the CGM with small graviton mass. For the dotted and dash-dotted line we have tuned the gravitational mass in order to

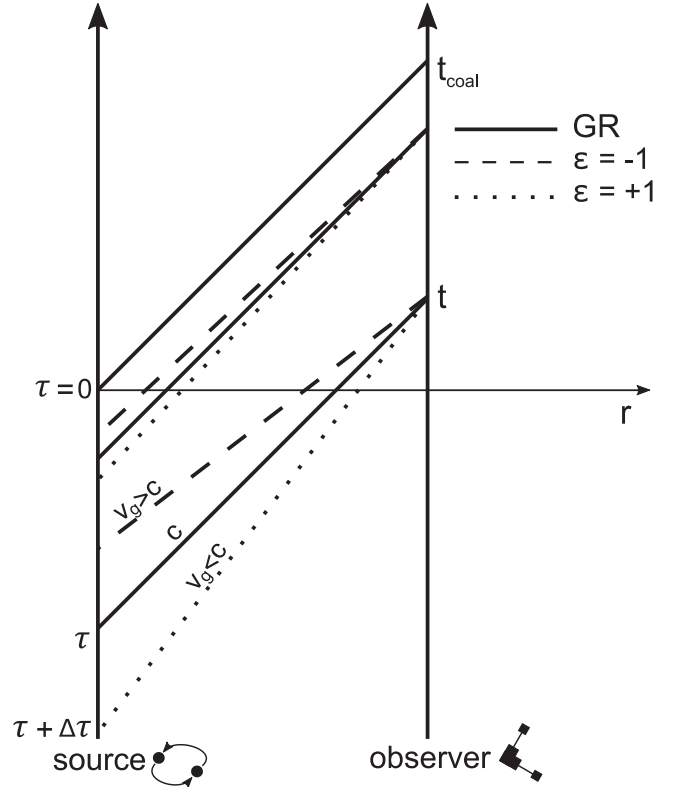


FIG. 2. World lines of massless and massive gravitational wave modes emitted by a binary source and observed by a detector. The observer sees a superposition of massless modes emitted at τ and massive modes emitted at $\tau + \Delta\tau$. Note that $\Delta\tau > 0$ for $\epsilon = +1$ and $\Delta\tau < 0$ for $\epsilon = -1$. It is also shown that $\Delta\tau \rightarrow 0$ for $\tau \rightarrow 0$ meaning that the speed of the massive modes approaches the speed of light at coalescence.

obtain a signal in the LIGO/VIRGO frequency band and stuck to the chirp masses of two typical neutron stars and a black hole pair respectively. The value of the graviton mass used in that cases exceeds the experimental upper limits on m_g by many orders of magnitude. For the long dashed line we fix m_g to its maximum value allowed by Solar System tests of gravity and adapt the chirp mass, which now has to be on the order of the mass of a galaxy. Thus we see already that the observed GW events are not easily explained in the context of the CGM with small graviton mass. But note that for a proper comparison with data we also have to consider the modified propagation of the gravitational wave signal in the CGM and thus the results shown in Fig. 1 would actually only be observed by a fiducial observer very close to the source, but not by us. Thus we work out the details of the wave form below.

For the massless mode of the gravitational wave $\tau = t_{\text{coal}} - t = t_{\text{coal},0} - t_0$, as for the large mass limit of conformal gravity. However, in the case of small m_g we find for the massive mode of the gravitational radiation that the modified speed of propagation must be taken into account when we evaluate what a distant observer sees.

Figure 2 illustrates the situation. Well before coalescence at time t , the observer sees a superposition of the massless mode emitted at retarded time $t_0 = t - r/c$ and the massive mode emitted at time $t_m = t - v_{g,\epsilon}r/c^2$. The time lag between the emission of the two modes is $\Delta\tau \equiv (1 - v_{g,\epsilon}/c)(r/c) \approx \epsilon(r/2c)(m_g c^2/\hbar\omega_{\text{gw}})^2$, where we expanded the expression for the group velocity, as the condition $m_g c^2/\hbar\omega_{\text{gw}} \ll 1$ holds in the small mass regime.

In the following we will assume that $\Delta\tau/\tau \ll 1$. This is an excellent assumption for the interesting parameter space as

$$\frac{\Delta\tau}{\tau} = \epsilon \frac{512}{5} \frac{r}{c} \left(\frac{m_g c^2}{\hbar}\right)^8 \left(\frac{GM_c}{c^3}\right)^5 \left(\frac{\tau}{5}\right)^2 \quad (41)$$

$$= \epsilon 4.24 \times 10^{-66} \left[\frac{r}{1 \text{ Gpc}}\right] \left[\frac{m_g}{10^{-58} \text{ kg}}\right]^8 \left[\frac{M_c}{M_\odot}\right]^5 \left[\frac{\tau}{1 \text{ s}}\right]^2. \quad (42)$$

Since the value assumed for m_g is an upper limit, and the value assumed for the distance of the source is of the order of the size of the observable universe, the condition is met for all chirp masses below $1.4 \times 10^{13} M_\odot$. It is even met for larger chirp masses if the source is located at distance < 1 Gpc. Note that the condition is not met for $m_g > 2 \times 10^{-50}$ kg, but those values are excluded based on Solar System tests.

The next step is to calculate the frequency at the time of the emission of the massive mode,

$$\omega_{\text{gw}}(\tau + \Delta\tau) \approx \omega_{\text{gw}}(\tau) + \dot{\omega}_{\text{gw}}\Delta\tau. \quad (43)$$

Using (38) we find

$$\omega_{\text{gw}}(\tau + \Delta\tau) \approx \omega_{\text{gw}}(\tau) \left[1 - \epsilon \frac{384}{125} \left[\frac{GM_c}{c^3}\right]^5 \left[\frac{m_g c^2}{\hbar}\right]^8 \frac{r}{c} \tau^2\right]. \quad (44)$$

For the massless mode we obtain the phase by integrating (39) from τ_i to τ ,

$$\begin{aligned} \Phi(\tau) - \Phi_i &= \int_{\tau_i}^{\tau} d\tau' \omega_{\text{gw}}(\tau') \\ &= \frac{5}{16} \left(\frac{m_g c^2}{\hbar}\right)^{-3} \left(\frac{GM_c}{c^3}\right)^{-5/2} \left[\left(\frac{5}{\tau_i}\right)^{1/2} - \left(\frac{5}{\tau}\right)^{1/2}\right], \end{aligned} \quad (45)$$

where $\Phi_i = \Phi(\tau_i)$.

For the massive mode we have to integrate (39) from $\tau_i + \Delta\tau_i$ to $\tau + \Delta\tau$. Keeping only contributions that are linear in $\Delta\tau$, we find

$$\begin{aligned} &\Phi(\tau + \Delta\tau) - \Phi(\tau_i + \Delta\tau_i) \\ &= \int_{(\tau+\Delta\tau)_i}^{\tau+\Delta\tau} d\tau' \omega_{\text{gw}}(\tau') \\ &\approx \int_{\tau_i}^{\tau} d\tau' \omega_{\text{gw}}(\tau') + \omega_{\text{gw}}(\tau)\Delta\tau - \omega_{\text{gw}}(\tau_i)\Delta\tau_i \\ &\approx \Phi(\tau) - \Phi_i \\ &\quad + \epsilon 16 \frac{r}{c} \left(\frac{m_g c^2}{\hbar}\right)^5 \left(\frac{GM_c}{c^3}\right)^{5/2} \left[\left(\frac{\tau}{5}\right)^{3/2} - \left(\frac{\tau_i}{5}\right)^{3/2}\right]. \end{aligned} \quad (46)$$

To find the waveform we start from Eqs. (21a) and (21b) in which we have to replace $\omega_{\text{gw}}t_0$ with $\Phi(\tau)$ and $\omega_{\text{gw}}t_m$ with $\Phi(\tau + \Delta\tau)$. After making use of Kepler's 3rd law (26), by which we eliminate the orbital radius, we must take care of the fact that the gravitational wave frequency has to be evaluated at different retarded times for the massless and massive modes, which yields

$$\begin{aligned} h_{11}(t, r) &= -h_{22}(t, r) \\ &= \frac{\epsilon c}{4r} \left(\frac{\hbar}{m_g c^2}\right)^2 \left[\frac{5}{\tau} \cos[\Phi(\tau)] - \frac{5}{\tau + \Delta\tau} \cos[\Phi(\tau + \Delta\tau)]\right], \end{aligned} \quad (47a)$$

$$\begin{aligned} h_{12}(t, r) &= h_{21}(t, r) \\ &= \frac{\epsilon c}{4r} \left(\frac{\hbar}{m_g c^2}\right)^2 \left[\frac{5}{\tau} \sin[\Phi(\tau)] - \frac{5}{\tau + \Delta\tau} \sin[\Phi(\tau + \Delta\tau)]\right]. \end{aligned} \quad (47b)$$

Now we assume that the delay between the massive and massless modes is small and keep all terms linear in $\Delta\tau$. Let us demonstrate this for the h_{11} component,

$$\begin{aligned} &\left[\frac{1}{\tau} \cos[\Phi(\tau)] - \frac{1}{\tau + \Delta\tau} \cos[\Phi(\tau + \Delta\tau)]\right] \\ &\approx \frac{\Delta\tau}{\tau} \left[\sin[\Phi(\tau)]\Phi(\tau)' - \frac{1}{\tau} \cos[\Phi(\tau)]\right]. \end{aligned} \quad (48)$$

With $\Phi(\tau)' = \omega_{\text{gw}}(\tau)$, we find

$$\begin{aligned} h_{11}(t, r) &= -h_{22}(t, r) \\ &\approx 4 \left(\frac{m_g c^2}{\hbar}\right)^3 \left(\frac{GM_c}{c^3}\right)^{5/2} \left(\frac{\tau}{5}\right)^{1/2} \\ &\quad \times \left[\sin[\Phi(\tau)] - \frac{32}{5} \left(\frac{m_g c^2}{\hbar}\right)^3 \left(\frac{GM_c}{c^3}\right)^{5/2} \right. \\ &\quad \left. \times \left(\frac{\tau}{5}\right)^{1/2} \cos[\Phi(\tau)]\right], \end{aligned} \quad (49a)$$

$$\begin{aligned}
 h_{12}(t, r) &= h_{21}(t, r) \\
 &\approx 4 \left(\frac{m_g c^2}{\hbar} \right)^3 \left(\frac{GM_c}{c^3} \right)^{5/2} \left(\frac{\tau}{5} \right)^{1/2} \left[-\cos[\Phi(\tau)] \right. \\
 &\quad \left. - \frac{32}{5} \left(\frac{m_g c^2}{\hbar} \right)^3 \left(\frac{GM_c}{c^3} \right)^{5/2} \left(\frac{\tau}{5} \right)^{1/2} \sin[\Phi(\tau)] \right].
 \end{aligned} \tag{49b}$$

At leading order, the massless and massive mode cancel each other and the emission of gravitational waves is strongly suppressed when compared to GR or the large graviton mass situation. Another important observation is that the result does not depend on the distance of the observer from the source, which is a reflection of the Weyl symmetry of the model. For small graviton mass, the emitted gravitational waves are in the high energy regime of the theory where typically symmetries are seen more explicitly than at low energy scales. Moreover the result does not depend on the sign of ϵ . And most importantly, the amplitude of the signal decreases towards coalescence and vanishes at $\tau = 0$, in stark contrast to the GR and large mass CGM predictions.

To estimate the order of magnitude of the gravitational wave amplitude we can write the factor in front of the square bracket as

$$\begin{aligned}
 &4 \left(\frac{m_g c^2}{\hbar} \right)^3 \left(\frac{GM_c}{c^3} \right)^{5/2} \left(\frac{\tau}{5} \right)^{1/2} \\
 &= 6.00 \times 10^{-35} \left(\frac{m_g}{10^{-58} \text{ kg}} \right)^3 \left(\frac{M_c}{M_\odot} \right)^{5/2} \left(\frac{\tau}{1 \text{ s}} \right)^{1/2}.
 \end{aligned} \tag{50}$$

For the typically assumed chirp masses this is many orders of magnitudes smaller than the amplitude observed by the LIGO/VIRGO detectors. For chirp masses below the mass of a typical galaxy, the second terms in the square brackets are negligible and it is sufficient to compare LIGO/VIRGO observations to the leading terms for both polarizations.

Thus the predicted wave form for gravitational radiation from a coalescing binary for the CGM in the small graviton mass regime reads,

$$\begin{aligned}
 h_{11}(t, r) &= -h_{22}(t, r) \\
 &\approx 4 \left(\frac{m_g c^2}{\hbar} \right)^3 \left(\frac{GM_c}{c^3} \right)^{5/2} \left(\frac{\tau}{5} \right)^{1/2} \sin[\Phi(\tau)],
 \end{aligned} \tag{51a}$$

$$\begin{aligned}
 h_{12}(t, r) &= h_{21}(t, r) \\
 &\approx -4 \left(\frac{m_g c^2}{\hbar} \right)^3 \left(\frac{GM_c}{c^3} \right)^{5/2} \left(\frac{\tau}{5} \right)^{1/2} \cos[\Phi(\tau)].
 \end{aligned} \tag{51b}$$

In the limit of vanishing m_g , there is no gravitational radiation emitted to the far field of the source.

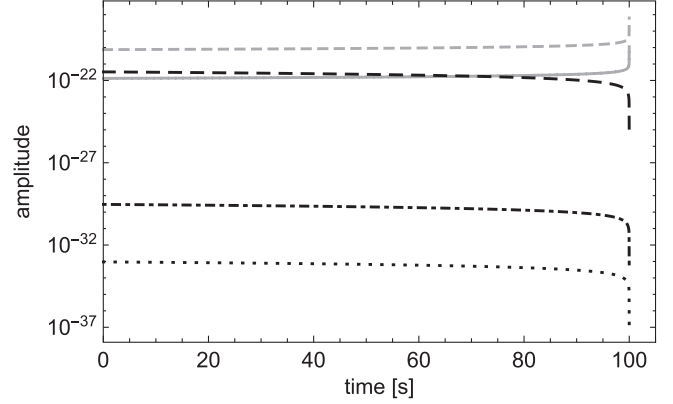
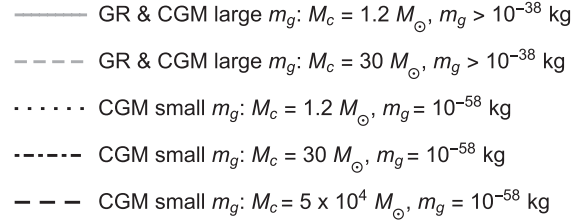


FIG. 3. Time evolution of the gravitational wave amplitudes during the last 100 seconds before coalescence. The grey solid and dashed lines show the GR prediction for two different chirp masses at an assumed distance of 10 Mpc. The GR prediction is identical to the CGM prediction with a large graviton mass. The black lines show three examples for the CGM with a small graviton mass with the maximally allowed value of the graviton mass.

In Fig. 3 we show the time evolution of the GW amplitudes for GR or the CGM with large m_g (grey lines) and for the CGM for $m_g = 10^{-58}$ kg (black lines). The two GR examples are located at a distance of 10 Mpc and show chirp masses of $1.2 M_\odot$ and $30 M_\odot$. The CGM results are valid for arbitrary distance. For comparison we show the chirp masses typical for the observed GW events in the context of GR as dotted and dashed-dotted lines. They give rise to amplitudes that are many orders of magnitude below those discovered by the LIGO/VIRGO collaboration. In order to match the typical amplitudes found, one would require a chirp mass of $5 \times 10^4 M_\odot$, which in turn would predict a gravitational wave frequency $\approx 3 \times 10^{21}$ Hz $(1 \text{ s}/\tau)^{3/2}$, which is not at all in the frequency band. It is not possible to find values of m_g and M_c that fit both the frequency and amplitude range of the LIGO/VIRGO detectors and evolve on a typical timescale of a fraction of a second to about a few minutes.

V. CONCLUSION

We have calculated the effect of gravitational radiation on the orbit of a binary system of compact objects in the late inspiral phase in a conformal gravity model for large and small graviton masses. For the binary system we used the center of mass frame on a quasicircular orbit and in the

Newtonian limit, which is justified because in the early phase of observed merger events the two bodies are still quite far apart. Furthermore, binary systems, although they may possess large eccentricity in the early phase of the inspiral, tend to circularize in the late inspiral phase very rapidly.

This work builds on the results obtained in [32], where the decrease of the orbital period of stellar binary systems in the CGM in the early stationary inspiral phase was investigated.

The conformal models of gravity studied in this work allow for seven radiative d.o.f. Two of them are massless and behave very similar as in general relativity. The other five d.o.f. are massive. For a conserved energy-momentum tensor of the source only two of those five d.o.f. can be excited by a binary system [32]. These two modes propagate to a distant detector if the frequency of the gravitational wave is above the characteristic frequency that corresponds to the mass m_g . We referred to this as the small graviton mass case.

For large graviton mass we found that the observed wave form is (at least at leading order in a post-Newtonian expansion) indistinguishable from the GR result. Corrections from higher-derivative contributions of the CGM become important only on microscopic scales, which are irrelevant on the distance scales of binary systems. All modifications to the GR results are negligible, since they are much smaller than the error of measurement.

In the case of a small graviton mass the GW solutions look very different than in GR. In the parameter regime with $\Delta\tau/\tau \ll 1$ the amplitude is independent of the distance to the binary system and is decreasing as coalescence is approached. We conclude that for small graviton mass both regimes of the studied CGM ($\epsilon = \pm 1$) cannot explain the observed gravitational wave events.

Suppressed GWs have also been found in a version of torsionfree gravitational Yang-Mills gauge theories based on the $SO(4,2)$ conformal group of the Minkowski spacetime [46,47], which coincides with the considered CGM for $\epsilon = +1$. The retarded Green's function in a de Sitter vacuum derived in [48] resembles our results in flat spacetime, which we found in [32].

Our new findings, together with the results from [32], restrict the CGM to large graviton masses and the regime $\epsilon = +1$, which cannot explain galaxy rotation curves without dark matter. The large graviton masses give rise

to a modification of GR at high energies and small distances and therefore the CGM remains an interesting target for further studies.

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APPENDIX: CONVENTIONS

The signature of the metric is $g = \text{diag}(-, +, +, +)$. The Riemann tensor in terms of Christoffel symbols is given by

$$R^\lambda_{\mu\nu\kappa} = -(\partial_\nu\Gamma^\lambda_{\mu\kappa} - \partial_\kappa\Gamma^\lambda_{\mu\nu} + \Gamma^\lambda_{\nu\alpha}\Gamma^\alpha_{\mu\kappa} - \Gamma^\lambda_{\kappa\alpha}\Gamma^\alpha_{\mu\nu}). \quad (\text{A1})$$

From this we find the Ricci tensor $R_{\mu\kappa} = g^{\lambda\nu}R_{\lambda\mu\nu\kappa}$ and the Ricci scalar $R = g^{\mu\kappa}R_{\mu\kappa}$. The Weyl tensor is given by the expression

$$C_{\lambda\mu\nu\kappa} = R_{\lambda\mu\nu\kappa} + \frac{1}{6}R[g_{\lambda\nu}g_{\mu\kappa} - g_{\lambda\kappa}g_{\mu\nu}] - \frac{1}{2}[g_{\lambda\nu}R_{\mu\kappa} - g_{\lambda\kappa}R_{\mu\nu} - g_{\mu\nu}R_{\lambda\kappa} + g_{\mu\kappa}R_{\lambda\nu}]. \quad (\text{A2})$$

The Einstein equations in the convention used in [49] reads

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi GT_{\mu\nu} + \Lambda g_{\mu\nu}. \quad (\text{A3})$$

Finally, the Bach tensor can be written as

$$W^{\mu\nu} = -\frac{1}{6}g^{\mu\nu}R^{;\beta}_{;\beta} + R^{\mu\nu;\beta}_{;\beta} - R^{\mu\beta;\nu}_{;\beta} - R^{\nu\beta;\mu}_{;\beta} - 2R^{\mu\beta}R^\nu_{\beta} + \frac{1}{2}g^{\mu\nu}R_{\alpha\beta}R^{\alpha\beta} + \frac{2}{3}R^{;\mu;\nu} + \frac{2}{3}RR^{\mu\nu} - \frac{1}{6}g^{\mu\nu}R^2. \quad (\text{A4})$$

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