Testing the cosmic distance-duality relation from future gravitational wave standard sirens

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A validation of the cosmic distance-duality relation (CDDR) is crucial because any observational departure from it could be a signal of new physics. In this work, we explore the potentialities of luminosity distance data from the gravitational wave (GW) standard sirens of the future Einstein Telescope (ET) to test the CDDR. The angular diameter distance data are used from the galaxy cluster samples and the baryon acoustic oscillation (BAO) measurements. The basic advantage of GW measurement substituting for the observation from the type Ia supernovae (SNIa) is that the luminosity distance from GW is insensitive to the nonconservation of the number of photons. By simulating 550 and 1000 data points of future GW measurements in the low redshift range $0 < z < 1$, we show that the measurements of future GW events will be a powerful tool to test the CDDR.

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I. INTRODUCTION

The cosmic distance-duality relation (CDDR), which relates luminosity distance (LD) D_L and angular diameter distance (ADD) D_A to a given source at redshift z through the identity

$$\frac{D_L}{D_A}(1+z)^{-2} = 1, \tag{1}$$

was firstly proved by Etherington in 1933 [1] based on two fundamental hypotheses, namely, that light travels always along null geodesics in a Riemannian geometry and that the number of photons is conserved [2,3]. This equation has been used, without any doubt, in astronomical observations and modern cosmology, since it is independent of Einstein field equations and the nature of matter. However, the possibilities of the CDDR violation may result from nonconservation of the number of photons or a nonmetric theory of gravity, in which the light does not travel along null geodesics [4,5]. The nonconservation of the total number of photons may result from the presence of scattering and absorption of some opacity sources [6,7] or nonstandard mechanisms such as scalar fields with a nonminimal coupling to the electromagnetic Lagrangian [8–11] or oscillation of photons propagating in extragalactic magnetic fields into light axions [6,12–14]. Any violation of CDDR from astronomical observations may even be considered as a signal of exotic physics or the existence of some unaccounted errors in the observations [15]. Thus, testing the validity of this relation with different observational data sets and methods is worthy and necessary.

To check the validity of the CDDR with astronomical observations, one should in principle obtain the LD and ADD of some objects at the same redshift. The LD can be generally obtained through the observation of the type Ia supernovae (SNIa) standard candles, and the ADD can be estimated from the observation of galaxy clusters, the cosmic microwave background (CMB), and baryon acoustic oscillation (BAO). Since the redshifts of LD and ADD data points in the present observations usually do not match, some tests on the CDDR are performed through comparing the observed values with the corresponding theoretical ones from an assumed cosmological model. With the LDs from the ΛCDM model, Uzan *et al.* [4] and DeBernardis *et al.* [16] tested successively the CDDR with galaxy cluster samples [17,18] and found no violation from the CDDR. Then, combining the SNIa data with the standard rulers from the CMB and BAO measurements, Lazkoz *et al.* verified the validity of the CDDR at the 2σ confidence level (CL) [19]. Using the galaxy cluster data from the elliptical and spherical β models [18,20], Holanda *et al.* obtained that the CDDR is compatible with the elliptical and spherical β models at 1σ and 3σ CL, respectively [21].

Recently, in order to match the redshifts of the galaxy cluster samples [18,20] with those of SNIa data by the cosmological-model-independent method, Holanda *et al.* [22] adopted a criterion ($\Delta z = |z_{\mathrm{ADD}} - z_{\mathrm{SNIa}}| < 0.005$) and chose the closest one. From the Constitution SNIa compilation [23], they found that the CDDR is marginally

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consistent with the elliptical β model at 2σ, but it indicates a strong violation from the spherical β model even at 3σ CL. Using the Union 2 SNIa compilation, Li *et al.* also performed tests on the CDDR and found that the CDDR is consistent with the elliptical β model at 1σ CL and the spherical β model at 3σ [24]. In order to avoid larger statistical errors brought by using merely one SNIa data point from all those available which meet the selection criterion, Meng *et al.* [25], instead of using the nearest point of SNIa compilation, binned these data available to obtain the LD to match the corresponding ADD sample. They studied different morphological models of galaxy clusters and found that the marked triaxial ellipsoidal model is a better geometrical hypothesis to describe the structure of a galaxy cluster than the spherical model if the CDDR is valid. Then, Wu *et al.* tested the CDDR by combining the Union 2.1 compilation and five ADD data points from the BAO measurements and found that the BAO measurement is a very powerful tool to test the CDDR due to the precision of the BAO measurements [26]. Still some other tests, involving the ADD of galaxy clusters [18,20], current cosmic microwave background (CMB) observations [27], Hubble parameter data $H(z)$ from cosmic chronometers, gas mass fraction measurements in galaxy clusters [28], strong gravitational lensing (SGL) [29,30], and time delay lenses [31], are performed to investigate the validity of the CDDR by assuming a deformed CDDR, such as $D_L(1+z)^{-2}/D_A = \eta(z)$, in different redshift ranges, and the results show that the CDDR is consistent with the observations at certain CLs [32–44].

It is worth noting that the LD from SNIa measurements is dependent on the conservation of the number of photons. Any kind of violation from the conservation of the number of photons, such as absorption and scattering of photon or axion-photon mixing, sensibly imprints its impact on the test of the CDDR [45]. So a common limitation of the aforementioned tests involving LD from SNIa compilations is that, if the evidence of the CDDR violation $\eta \neq 1$ is obtained, the fundamental reason for the departure may not be known because the results from these tests are sensitive to both fundamental hypotheses for the CDDR.

More recently, the joint detections of the gravitational-wave (GW) event GW170817 with an electromagnetic (EM) counterpart (GRB 170817A) from the merger of binary neutron stars (NSs) [46–49] have opened a new era of multimessenger cosmology, and it means for the first time that a cosmic event can be investigated in both EM waves and GWs. The application of GW information in cosmology was first proposed by Schutz [50], who suggested that the Hubble constant can be determined from GW observations using the fact that the waveform signals of GWs from inspiraling and merging compact binaries encode distance information. So, GW sources can be considered as standard sirens in astronomy, analogous to supernova standard candles. Unlike the distance estimation from SNIa observations, one can, from the GW observations, obtain the luminosity distances directly without the need of cosmic distance ladder since standard sirens are self-calibrating. This advantage of GW measurements can help us dodge the influence of the nonconservation of the number of photons on the test of CDDR. If compact binaries are NS-NS or black-hole-(BH-)NS binaries, the source redshift may be observed from EM counterparts that occur coincidentally with the GW events [51–53]. Thus, the LD-redshift relation can be constructed in a cosmological-model-independent way through combining the measurements of the sources' redshifts from the EM counterpart, and it provides us with an opportunity to make constraints on the cosmological parameters and the possible departures from the CDDR. It is worth mentioning that the propagation of GWs in modified gravity theories is in general different from that in general relativity and the LDs from GWs are different from those for the electromagnetic signals [54,55]. Therefore, if one tests the CDDR with LDs from GW measurements and distances from electromagnetic observations, the violation of CDDR might indicate deviations of gravity theory from general relativity besides the existence of new physics. In this work, the main motivation is to employ the GWs as an alternative for the SNIa to test CDDR in the frame of general relativity.

Up to now, the simulated GW data have been used to measure the cosmological parameters [51,56–59], determine the total mass of neutrinos [60], investigate the anisotropy of the Universe [61,62], and set constraints on the evolving Newton's constant G [63]. More recently, Yang *et al.* explored the potentialities of future GW detections to constrain a possible departure from the CDDR through combining the LD of simulated gravitational wave data from the Einstein Telescope (ET) and the ADD from SGL systems in a relatively high redshift range $z \sim 3.6$ [64]. They obtained that future results from GW data will be at least competitive with current constraints on CDDR from SNIa + GRB + SGL analyses. However, it is shown that the mass profile of lensed galaxies and dynamical structure may bring forth significant changes in lensing studies [29,30], and its impact on the test of CDDR needs further investigation. Thus, one needs more relevant ADD data to explore the potentialities of GW measurements on the test of the validity of CDDR. It is well known that BAO measurement is a very precise astronomical observation [26,65] and can be used as a very powerful tool to test the CDDR [26]. In addition, the measurement of galaxy clusters also plays an important role in testing this relation. So, it is worth confirming the ability of future GW measurements jointly with the ADD data from BAO and galaxy clusters to constrain a possible departure from this reciprocal relation.

In this work, we detect the potentialities of future GW measurements to test CDDR. The analyses are carried out with the LD (D_L) from simulated GW data jointly with

ADD from BAO and galaxy cluster samples. We simulate 550 and 1000 data points of GWs from the ET in the redshift range $0 < z < 1$ as a reference and impose limits on $\eta(z)$ to estimate the possible departures from the CDDR. In order to compare our results with that from Refs. [25,26], we also employ the binning method to obtain the corresponding LDs from simulated GW data for each BAO or galaxy cluster system. The results indicate that measurement of future GW events will be a very powerful tool to realize the validation of this reciprocal relation.

II. SAMPLES AND SIMULATED GW DATA

The structure of galaxy clusters is essential for the cosmological probe [66,67]. Generally speaking, to get reasonable results of ADD from galaxy cluster observation, one has to assume certain cluster morphologies and employ the joint analysis of the Sunyaev-Zel'dovich effect (SZE) and x-ray brightness measurements [68]. Two galaxy cluster samples are utilized to obtain the ADD through different morphological assumptions. The first one involves 25 x-ray-selected galaxy clusters [20] described as an isothermal elliptical β model. The second sample includes 38 galaxy clusters [18], whose plasma and matter distributions were analyzed by assuming a hydrostatic equilibrium model and spherical symmetry. Therefore, the CDDR tests are sensitive to the models for the cluster gas distribution, since the ADD data are closely related to the assumption of cluster models. For the galaxy cluster samples, the statistical and systematic errors account for about 20% and 24% [18,69] and they are combined in quadrature for the ADD [18,22].

The observational ADD data can be also obtained from the BAO measurements [65]. The early Universe consisted of a hot, dense plasma of electrons and baryons. Photons were coupled with the baryons via Thomson scattering. A system of standing sound waves within the plasma can be created on account of the existence of a competition between radiation pressure and gravity, so-called BAOs. As the Universe expanded, the plasma cooled to below 3000 K—a low enough energy such that the electrons and protons in the plasma could combine to form neutral hydrogen atoms, i.e., recombination. The free electrons were quickly captured and the coupling between photons and baryons ended abruptly, which led to an overdensity of baryons at the scale of about 150 Mpc today. This scale can be observed in the clustering distribution of galaxies today and can be used as a standard ruler. One can obtain the ADD by combining of the measurements of the baryon acoustic peak and the Alcock-Paczynski distortion from galaxy clustering (see [65] for a review). The five ADD data points from BAO measurements were released by the WiggleZ Dark Energy Survey [70], the Sloan Digital Sky Survey (SDSS) [71] and Data Release 11 [72] (also listed in Table I of Ref. [26]).

The ET is the third generation of the ground-based GW detector, and it, as proposed by the program, consists of three collocated underground arms with a length of 10 km and a 60° opening angle. The ET would be able to detect GW signals to be 10 times more sensitive in amplitude than the advanced ground-based detectors, covering a wide frequency range of $1 \sim 10^4$ Hz with the redshift range $z \sim 2$ for the NS-NS and $z > 2$ for the BH-NS merger systems. If compact mergers are NS-NS or BH-NS binaries, the source redshift may be observed from EM counterparts that occur coincidentally with the GW events [51–53]. Thus, the LD-redshift relation can be constructed in a cosmological-model-independent way, and it can be employed to make constraints on the basic parameters of cosmology. The ratio between NS-NS and BH-NS binaries, in this work, is taken to be 0.03, as illustrated by the Advanced LIGO-Virgo network [73]. Here, for brevity, we only summarize the process of Refs. [51,53,61,64] in which observations of GWs from the future ET are simulated, and then we will forecast the constraints on CDDR.

For the waveform of GWs, the stationary phase approximation is applied to compute the Fourier transform $\mathcal{H}(f)$ of the time domain waveform $h(t)$,

$$\mathcal{H}(f) = \mathcal{A} f^{-7/6} \exp[i(2\pi f t_0 - \pi/4 + 2\psi(f/2) - \varphi_{(2.0)})], \tag{2}$$

where the Fourier amplitude $\mathcal{A}$ is given by

$$\mathcal{A} = \frac{1}{D_L}\sqrt{F_+^2(1+\cos^2(\iota))^2 + 4F_\times^2\cos^2(\iota)} \times \sqrt{5\pi/96}\pi^{-7/6}\mathcal{M}_c^{5/6}, \tag{3}$$

where $\mathcal{M}_c$ denotes the observational chirp mass and D_L is the LD which plays the most important role in this test. The definition of the beam pattern functions $F_{+,\times}$, the polarization angle ψ, the epoch of the merger t_0, the phase parameter such as the angle of inclination ι, and $\varphi_{(2.0)}$ are introduced in Refs. [51,53,61,64]. The cosmological parameters of the fiducial concordant model are adopted with the most recent Planck results [74]:

$$h_0 = 0.678, \qquad \Omega_m = 0.308, \qquad \Omega_k = 0, \qquad w = -1, \tag{4}$$

where $H_0 = 100h_0$ kms^{-1} Mpc^{-1}, Ω_m, Ω_k, and w denote the Hubble constant, dark matter density parameter, the cosmic curvature parameter today, and the dark energy equation of state, respectively. It is known that the redshift range of ADD data from the galaxy cluster and BAO is in the region $0 < z < 1$, and the corresponding number of data points from the SNIa Union 2 and Union 2.1 compilation is 537 and 551, respectively. In order to compare our results relevantly with the number density of data points from the Union compilation in this redshift region, we first simulate 550 data points (set A) from future

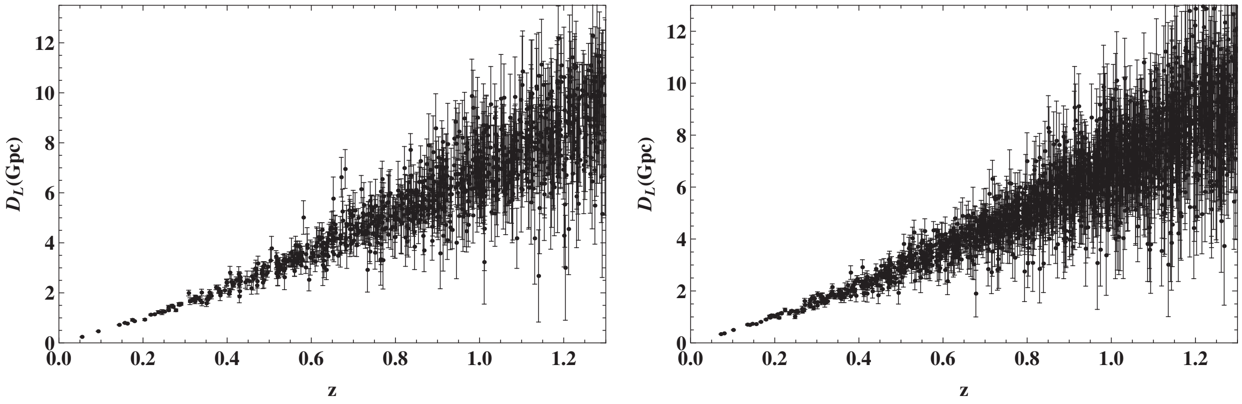

FIG. 1. The example catalogs with 930 (left) and 1700 (right) observed GW events of redshifts, LD, and the error of LD from the fiducial model in the redshift region $0 < z < 1.3$, in which 550 and 1000 data points are located in the redshift region $0 < z < 1$, respectively.

GW events. We also simulate 1000 data points (set B) to study the impact of the quantity of GW data on the test. The mock results are shown in Fig. 1.

III. METHODS

The most straightforward method to test CDDR is to compare the LD with the ADD at the same redshifts through the identity of Eq. (1). Generally, in the checking process, some departures from CDDR are allowed through defining the following function:

$$\frac{D_L(1+z)^{-2}}{D_A} = \eta(z). \tag{5}$$

The CDDR holds while $\eta(z) = 1$. All deviations from CDDR, which occur possibly at some redshifts, will be encoded in the function $\eta(z)$. In this work, four potential parametrizations for the $\eta(z)$ are adopted, namely, a linear one $\eta(z) = 1 + \eta_0 z$ (P_1), and three nonlinear ones, $\eta(z) = 1 + \eta_0 z/(1+z)$ (P_2), $\eta(z) = 1 + \eta_0 z/(1+z)^2$ (P_3), and $\eta(z) = 1 + \eta_0 \ln(1+z)$ (P_4).

In principle, given an ADD sample from each galaxy cluster or BAO system, one should select a LD [$D_L(z)$] data point from GW data that shares the same redshift z with the given ADD data to test the CDDR. However, this condition usually cannot be met in recent astronomical observations. To achieve this goal, a number of methods have been proposed [22,24,25,40,64]. In order to compare our results with that from Refs. [25,26], we employ a cosmological-model-independent binning method to obtain the LD [$D_L(z)$] from certain GW data points.

A. Method: Binning the GW data

In order to test the validity of CDDR in a cosmological-model-independent way, Holanda *et al.* [21,22], Li *et al.* [24], and Liao *et al.* [40] adopted a selection criterion $\Delta z = |z_{\mathrm{ADD}} - z_{\mathrm{SNIa}}| < 0.005$, where z_{ADD} and z_{SNIa} denote the redshift of an ADD sample and SNIa data, respectively, and chose the nearest SNIa data to match an ADD sample. However, using merely one SNIa data point from all those available which meet the selection criterion will lead to larger statistical errors. In order to avoid them, instead of using the nearest point of Union 2.1 SNIa, Wu *et al.* [26] and Meng *et al.* [25] bin the available data to obtain a LD to match the corresponding ADD sample. Following the process, we bin the simulated GW data which meet the criterion. In order to avoid correlations among the individual CDDR tests, we choose the LD samples with a procedure that the data points will not be used again if they have been matched to some cluster or BAO samples. In this method, we employ an inverse variance weighted average of all the selected data. If D_{Li} denotes the ith appropriate luminosity distance data points with $\sigma_{D_{Li}}$ representing the corresponding observational uncertainty, we can straightforwardly obtain the following with the conventional data reduction techniques given in Chapter 4 of Ref. [75]:

$$\bar{D}_L = \frac{\sum (D_{Li}/\sigma^2_{D_{Li}})}{\sum 1/\sigma^2_{D_{Li}}}, \tag{6}$$

$$\sigma^2_{\bar{D}_L} = \frac{1}{\sum 1/\sigma^2_{D_{Li}}}, \tag{7}$$

where $\bar{D}_L$ represents the weighted mean luminosity distance and $\sigma_{\bar{D}_L}$ corresponds to its uncertainty.

The selection criterion can be generally satisfied with all the samples of the spherical β model and BAO. However, for the elliptical β model, only 20 samples are satisfied with this selection criteria for data set A from the simulated GW data, and 21 samples are satisfied for data set B, since there are few simulated GW data points in the low redshift region $0 < z < 0.1$. The distributions of ADD samples and LD data obtained using this method are shown in Fig. 2.

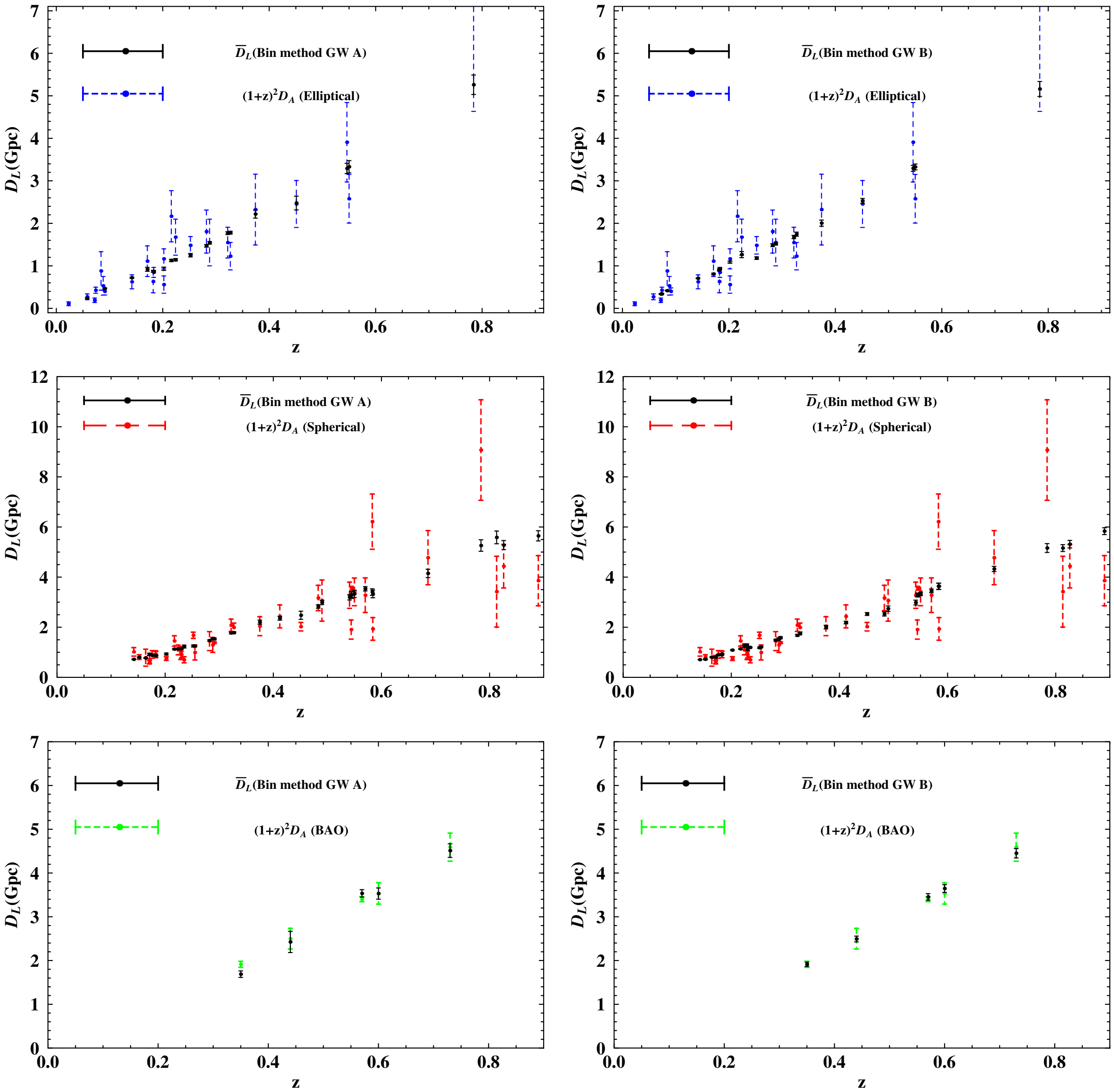

FIG. 2. The distribution of ADD samples from the galaxy cluster of the elliptical β model (row 1), spherical β model (row 2), and BAO (row 3) corresponding to the LD obtained with this method from set A (left panel) and set B (right panel) of simulated GW data. The samples of elliptical β model in the redshift range $z < 0.1$, which do not satisfy the selection criteria, should be discarded in the analysis.

IV. ANALYSIS AND RESULTS

To place constraints on η_0, one must firstly obtain $\eta_{\rm obs}(z)$ with

$$\eta_{\rm obs}(z) = \bar{D}_L(z)(1+z)^{-2}/D_A(z) \tag{8}$$

from the galaxy cluster or BAO samples and the luminosity distance from the binning method. The corresponding error of $\eta_{\rm obs}$ can be obtained through

$$\sigma^2_{\eta_{\rm obs}} = \eta^2_{\rm obs}\left[\left(\frac{\sigma_{D_A(z)}}{D_A(z)}\right)^2 + \left(\frac{\sigma_{D_L(z)}}{D_L(z)}\right)^2\right]. \tag{9}$$

Thus, using the equation

$$\chi^2 = \sum_i \frac{[\eta(z) - \eta_{\rm obs}(z)]^2}{\sigma^2_{\eta_{\rm obs}}}, \tag{10}$$

one can obtain the constraints on η_0. The results are shown in Fig. 3 and Table I.

For the galaxy cluster, seen from Fig. 3 and Table I, it can be obtained that the CDDR is consistent with the elliptical β model and the simulated GW data set A or B at 1σ CL or 2σ CL, respectively, which is consistent with those from Refs. [22,24,25]. However, for the spherical β model, the

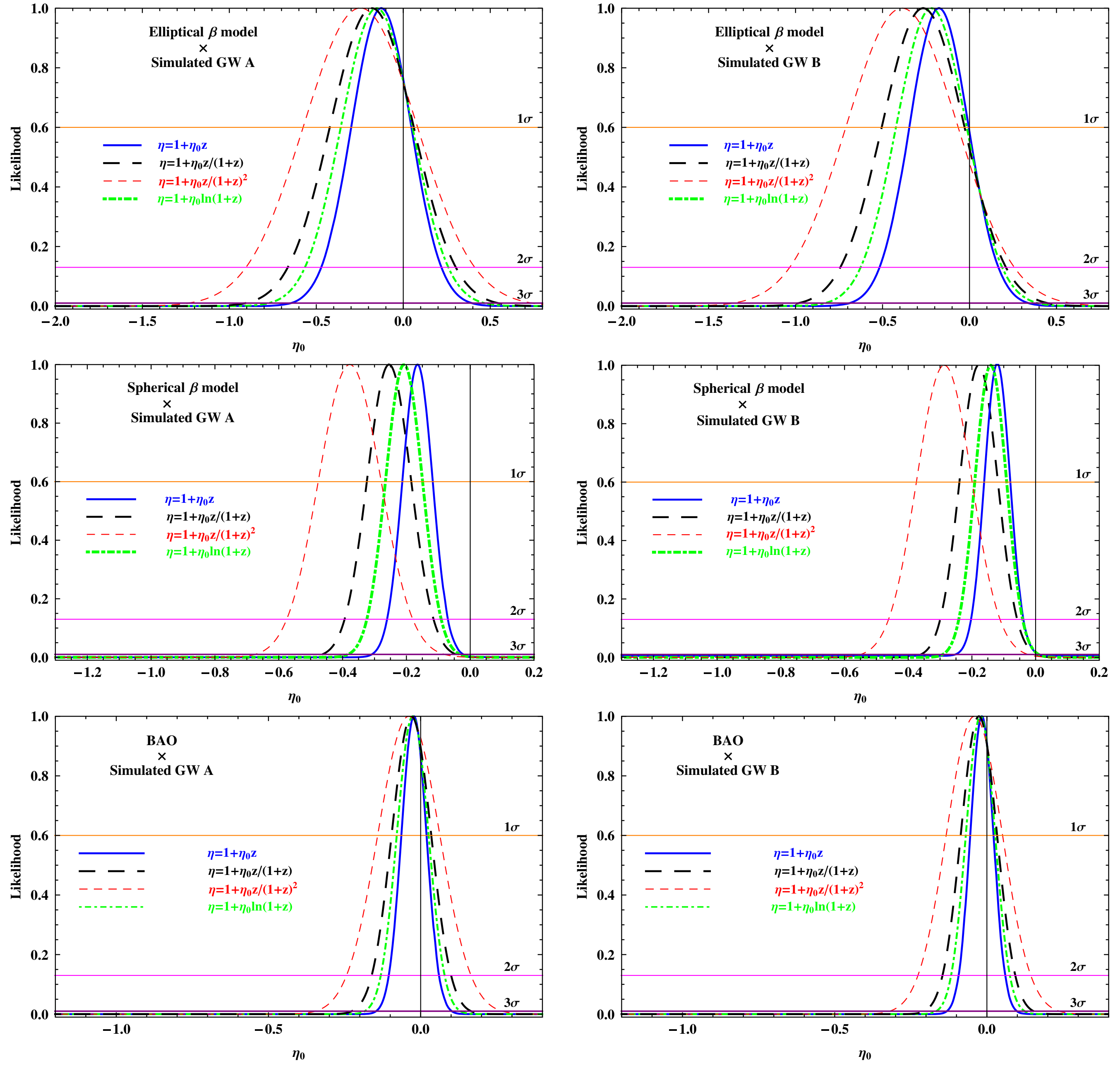

FIG. 3. The likelihood distribution functions obtained from set *A* (left panel) and set *B* (right panel) of the simulated GW data points.

CDDR is not compatible with the observational data even at 3σ CL. This result suggests a stronger violation than those from Refs. [22,24,25]. It should be noted that, unlike SNIa observations, the violation from the CDDR is obtained from the LD of the GW measurements, which is insensitive to the conservation of photon number. If a photon does follow along the null geodesics in a Riemannian geometry, the CDDR is valid in the test of GW measurements. One may conclude that the violation of the CDDR may result from the existence of a large deviation while the spherical β model is used to describe the structure of a galaxy cluster. For BAO samples, the CDDR is compatible with the observational data at 1σ CL, which is consistent with the results from Ref. [26]. As for the four parametrizations, one can conclude that the linear form can provide the strictest constraints and the best fits on the test for the CDDR.

By comparing the constraints on the η_0 at 1σ CL, for the spherical β model, we obtain error bars about 50% smaller than those obtained from Ref. [25] where the Union 2 SNIa compilation is used, regardless of the $\eta(z)$ functions adopted. For the BAO samples, the results are about

TABLE I. The summary of maximum likelihood estimation results of η_0 for four parametrizations. The η_0 is represented by the best fit value at 1σ CL for each data set. The superscripts A and B represent the case obtained from set A and set B of the simulated GW data, respectively.

Parametrization	η_0^A (elliptical)	η_0^B (elliptical)	η_0^A (spherical)	η_0^B (spherical)	η_0^A (BAO)	η_0^B (BAO)
$1+\eta_0 z$	-0.126 ± 0.173	-0.175 ± 0.171	-0.165 ± 0.048	-0.113 ± 0.041	-0.022 ± 0.041	-0.016 ± 0.037
$1+\eta_0 \frac{z}{1+z}$	-0.180 ± 0.242	-0.267 ± 0.238	-0.254 ± 0.070	-0.178 ± 0.060	-0.030 ± 0.065	-0.026 ± 0.060
$1+\eta_0 \frac{z}{(1+z)^2}$	-0.247 ± 0.328	-0.388 ± 0.323	-0.378 ± 0.100	-0.287 ± 0.087	-0.043 ± 0.101	-0.041 ± 0.092
$1+\eta_0 \ln(1+z)$	-0.153 ± 0.206	-0.219 ± 0.203	-0.207 ± 0.058	-0.141 ± 0.050	-0.026 ± 0.052	-0.021 ± 0.048

35% smaller than those from Ref. [26], where $\eta_0 = -0.027 \pm 0.064$ and -0.039 ± 0.099 with P_1 and P_2 of the $\eta(z)$ functions, respectively. For the elliptical β model, the error bars are similar to those from Ref. [25]. However, it should be noted that the number of available samples is only 20, which is less than that used in Ref. [25]. As one may see, much tighter constraints can be obtained by using future measures of GW events while the same number of ADD samples is considered. By comparing the results from simulated GW data set A with the results from set B at 1σ CL, one may find that the tests are almost independent of the quantity of simulated GW data, which may show that the errors of galaxy clusters and BAO samples dominate the constraints on the CDDR.

V. CONCLUSION AND DISCUSSION

The cosmic distance-duality relation plays a fundamental role in astronomical observations and modern cosmology. Its validation with various observational data is an important issue in modern cosmology, as any violation of it could be a signal of new physics in the modern theory of gravity or in particle physics. However, most of the previous tests involving the luminosity distance from SNIa on the CDDR are sensitive to the conservation of the number of photons.

The direct detections of gravitational wave (GW) events have thrown the observational cosmology into a new era of multimessenger. More precisely, for this astronomical observation, the luminosity distances can be measured from the waveform and amplitude of gravitational waves, and they are insensitive to the possible nonconservation of the number of photons.

In this work, we have simulated 550 and 1000 data points of future GW measurements from the Einstein Telescope in the low redshift region $0 < z < 1$. The angular diameter distances are from the galaxy cluster samples [18,20] obtained by using SZE and x-ray surface brightness observations and the BAO data [26]. In order to compare our results to those from Refs. [25,26] where Union 2 or Union 2.1 SNIa are adopted, 550 data points are adopted to ensure that the average number density of the GWs is nearly equal to the number density of the SNIa Union compilation in this redshift range, and the binning method is employed to obtain the corresponding LDs from simulated GW data for each BAO or galaxy cluster system. Then we detect the potentialities of future GW measurements to test the CDDR. The results show that future GW measurements may provide much tighter constraints on the CDDR, while we compare the results at 1σ CL with previous ones from SNIa Union 2.1 or Union 2 [25,26] if the same number of ADD samples is adopted. With the increase of the measuring quality and quantity of future observations, we can forecast that future GW measurement can be considered as a powerful tool to validate this reciprocal relation.

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