Phenomenological consequences of the refined swampland conjecture

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We discuss phenomenological consequences of the recently introduced refinements of the de Sitter swampland conjecture, which constrains the first and the second derivative of the scalar potential in terms of two $O(1)$ constants c and c'. Contrary to the original de Sitter swampland conjecture, the refinement has no constraints on spontaneous breaking scenarios, such as the Higgs, the chiral symmetry breaking, and the QCD axion. However, the refinement still strongly constrains inflation models. While we can achieve sufficient number of e-foldings, single-field inflationary models have trouble reproducing the observed valued of density perturbations, when c and c' are $O(1)$. We point out that this constraint can be evaded for example by curvaton scenarios, which typically leads to detectable non-Gaussianities. Our work can also be regarded as bottom-up constraints on the values of c and c'.

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I. INTRODUCTION

It has been a fascinating question if theories of quantum gravity give rise to any novel low-energy constraints beyond those discussed in the framework of the low-energy effective quantum field theory. To address this question, several "swampland conjectures" have been proposed (see, e.g., [1] for a recent review). The claim is that these conjectures should be satisfied if the low-energy effective field theory in question has a consistent UV completion with gravity included.

One of the most recent among such swampland conjectures is the striking conjecture (the so-called de Sitter conjecture) by Obied *et al.* [2] (see also [3–7] for related discussion). This conjecture states that the scalar potential V in a low-energy effective theory admitting a consistent UV completion with gravity should satisfy the constraint

$$M_{\rm Pl}|\nabla V| > cV. \tag{1}$$

Here c is an $O(1)$ positive constant independent of the choice of the theory (as long as we are in four dimensions and the scalar field is canonically normalized), and $M_{\rm Pl} = 2.4 \times 10^{18}$ GeV is the reduced Planck mass. Subsequently there have been many papers discussing this conjecture [8–55].

After the initial proposal, several authors examined the bottom-up consequences of the conjecture, in the context of the Higgs field [16,29,32,51] and the QCD axion [29] (see also [23]). While these constraints do not necessarily exclude the de Sitter conjecture (1), some exotic scenarios seem to be inevitable, and one might be tempted to conclude that such scenarios are unlikely.

In view of these results, one natural direction is to weaken/refine the conjecture. Some proposals along these lines have been made in [9,10,12,29]. Very recently, in particular, Ooguri *et al.* [56] proposed a refinement which is closely related with the proposal in [12], which states [57]

$$M_{\rm Pl}|\nabla V| > cV \quad \text{or} \quad M_{\rm Pl}^2 \ \min(\nabla\nabla V) \le -c'V. \tag{2}$$

Here c and c' are $O(1)$ positive constants, and $\min(\nabla\nabla V)$ is the minimal eigenvalue of the Hessian $\nabla_i \nabla_j V$ in an orthonormal frame. We call this the refined de Sitter conjecture. This conjecture is obviously weaker than the original conjecture (1), but is stronger than the conjecture of [29], which corresponds to the $c' = 0$ case of (2):

$$M_{\rm Pl}|\nabla V| > cV \quad \text{when } \nabla\nabla V > 0, \tag{3}$$

namely the conjecture applies only when the Hessian is positive definite (i.e., $\min(\nabla\nabla V) > 0$).

While the authors of [56] point out the connection of the refined de Sitter conjecture to the distance conjecture [59], it is fair to say that the conjecture is still speculative, and it is obviously an important question to find out whether or not there are counterexamples to the conjectures (2) and (3) inside the framework of string/M-theory.

Instead of addressing these questions, in this paper we assume the conjecture [(2) and (3)] as a working hypothesis and set out to discuss some phenomenological consequences of the conjecture. The quantitative constraints of this paper depend on the numerical values of the two positive constants c and c' in the conjecture, which we regard as free parameters [60]. One should keep in mind, however, that small values of c or c' signals some tension with the conjecture. Our discussion is for the most part rather general and does not rely on specific models. We then illustrate our results by several concrete inflationary models.

II. QUINTESSENCE

The stable de Sitter vacua, namely the point where we have

$$V > 0, \qquad \nabla V = 0, \qquad \nabla\nabla V > 0, \tag{4}$$

is clearly excluded by the refined de Sitter conjecture (2). This means that the origin of the dark energy should not be the cosmological constant, but rather be the quintessence [61–63] as in the case of the original conjecture (1) [2,8]. For example, the quintessence potential of the form

$$V_Q(Q) = \Lambda_Q^4 e^{-c_Q \frac{Q}{M_{\rm Pl}}} \tag{5}$$

satisfies the conjecture, as long as $c_Q > c$. For $c_Q < 0.5$–0.9, the quintessence model is compatible with the current cosmological observations [8,45,51].

III. HIGGS, CHIRAL SYMMETRY, QCD AXION AND ALL THAT

The refined version of the de Sitter conjecture removes bottom-up constraints for the Higgs and the QCD axion pointed out in the literature [16,23,29,32,51]. While this is technically an easy consequence, it is worth emphasizing this point, since one of the important motivations for the refinements of the de Sitter conjectures (2), (3) is to evade the bottom-up constraints.

Let us take the Higgs as an example. The potential for the Higgs field $H_{\rm SM}$

$$V_{H_{\rm SM}}(H_{\rm SM}) = \lambda(|H_{\rm SM}|^2 - v^2)^2 \tag{6}$$

has a local maximum at $H_{\rm SM} = 0$, which violates the first condition in (2) for the first derivative. However, the second derivative is nonzero around this point, where we have

$$M_{\rm Pl}^2 \frac{V''}{V} \sim -O\left(\frac{M_{\rm Pl}^2}{(100\ {\rm GeV})^2}\right) \ll -c'. \tag{7}$$

The similar argument applies to the QCD axion, which has a cosine potential, or more generally many scenarios for spontaneous symmetry breaking, such as the chiral symmetry breaking. In fact, as these examples show, the condition (2) is satisfied for a generic potential for energy scale much smaller than the Planck scale, except when there is an (nearly) stable de Sitter vacua.

IV. INFLATION

The new conjecture (9) has more nontrivial implications on inflation, for which the potential is often fine-tuned.

For simplicity let us consider a single-field inflation with the canonical kinetic term. Let us denote the inflaton by ϕ. It is customary to define two slow-roll parameters ϵ_V and η_V by

$$\epsilon_V = \frac{M_{\rm Pl}^2}{2}\left(\frac{V'}{V}\right)^2, \qquad \eta_V = M_{\rm Pl}^2\left(\frac{V''}{V}\right), \tag{8}$$

and then the refined dS conjecture (2) reads

$$[\epsilon]: \epsilon_V \geq \frac{c^2}{2} \quad \text{or} \quad [\eta]: \eta_V \leq -c'. \tag{9}$$

In the slow-roll regime $\epsilon_V \ll 1$, $|\eta_V| \ll 1$, the slow-roll parameters are directly related with the scalar spectral index n_s and the tensor-to-scalar ratio r by the relations,

$$n_s = 1 - 6\epsilon_V + 2\eta_V, \qquad r = 16\epsilon_V. \tag{10}$$

The parameters c and c' are bounded by the conjecture for consistency with a canonical single-field inflation. If we try to satisfy the first condition $[\epsilon]$ in (9), then we have $r \geq 8c^2$. By combining this with the current observational bound $r < 0.064$ [64] we have $c < 0.09$, which is in some tension with the conjecture, as already pointed out in [8,11,12,14,15,24]. The inflation also relates the condition $[\epsilon]$ to another conjecture [11,15], the distance conjecture [59] $\Delta\phi/M_{\rm Pl} < \Delta$ with a constant $\Delta = O(1)$. The restriction is relaxed in the new conjecture (9):

$$\frac{\Delta\phi}{M_{\rm Pl}} = \int \sqrt{2\epsilon_V} {\rm d}N_e > c N_e^{(\rm convex)}, \tag{11}$$

where $N_e^{(\rm convex)}$ is the e-fold elapsed in the convex region of the potential, instead of that measured from the end of inflation.

In the refined version of the conjecture (2) we have another option, namely to satisfy the condition $[\eta]$ during inflation. This in particular implies that the inflaton potential should be concave ($\eta_V < 0$), and this is also favored by recent observations. The condition $\eta_V \leq -c'$ immediately leads to the bound on the size of the tensor-to-scalar ratio,

$$r \leq \frac{8}{3}(1 - 2c' - n_s). \qquad (12)$$

Taking into account the scalar modes as well as the tensor modes, the observational data lead to $\epsilon_V < 0.005$, $\eta_V \simeq -0.01$ without assuming the slow-roll approximation 64]]. We then have $c' < 0.01$.

A small value of c' is also indicated by another argument, namely to get a sufficiently long period of inflation, $N_e = 50$–60. The condition [η] is not a direct obstacle for inflation, but ϵ_V should be tuned to a tiny value because the slope of the potential rapidly increases as the inflaton rolls down the potential. However, in this case, the energy scale of the inflation becomes very low to give the observed value of the scalar power spectrum amplitude $A_s \propto V/\epsilon_V$, which conflicts with the lower bound on the reheating temperature from big bang nucleosynthesis [65] when c' is $O(1)$.

It is noteworthy that, in contrary to the case of the original conjecture, the refined conjecture is compatible with the accelerated expansion of the Universe and constrained solely by observations of the fluctuations. Therefore, the conclusions above depend on the generation mechanism of the curvature fluctuations.

For example, in the curvaton scenario [66–69], the scalar spectral index is given by,

$$n_s = 1 + 2\frac{\dot{H}}{H^2} + 2\eta_{\sigma\sigma}, \qquad (13)$$

for the curvaton σ with $\eta_{\sigma\sigma} \equiv V_{\sigma\sigma}/3H^2$, where the quantities are evaluated at the horizon exit. When there is no coupling with the inflaton, the conjecture (2) reads $|\eta_{\sigma\sigma}| > c'\Omega_\sigma$ for the energy fraction of the curvaton Ω_σ at the horizon exit. Thus, $\eta_{\sigma\sigma}$ can be small when the curvaton is sufficiently subdominat during inflation. The tiny value of ϵ_V for a sufficiently long period of inflation leads to $n_s \simeq 1 + 2\eta_{\sigma\sigma}$. Consistency with the observed value of n_s requires $\eta_{\sigma\sigma} \sim -O(0.01)$, which indicates that the curvaton should have a potential with a hilltop region.

To conclude whether the curvaton scenario makes inflation compatible with the refined conjecture, more investigations will be needed with taking into account non-Gaussianity. In fact, Kawasaki *et al.* [70] showed that large non-Gaussianity can be produced by a curvaton with a hilltop potential even when the curvaton dominates the Universe at its decay. More concretely, a pseudo-Nambu-Goldstone curvaton predicts non-Gaussianity parameter larger than the Planck constraint [71]: $10 \lesssim f_{\rm NL} \lesssim 30$. On the other hand, Mukaida *et al.* [72] pointed out that non-Gaussianity can be suppressed for some class of the curvaton potentials. We leave it to future work to investigate whether the curvaton scenario above can be consistent with observations of non-Gaussianity by suitably designing the curtvaton potential.

V. GLOBAL CONSTRAINTS ON INFLATON POTENTIAL

We have so far discussed the region of the inflaton potential for the accelerated expansion. The condition (2), however, applies to any point in the configuration space (as long as the low-energy effective theory is valid), and hence we need to make sure that there are no extra constraints in other regions of the configuration space for the inflaton.

In inflationary models one often assumes that there is a region of the inflaton potential where the inflaton oscillates around the bottom of the potential and the reheating happens. The potential in this region can for example be taken to be quadratic (where we have chosen the origin $\phi = 0$ to be the minimum of the inflaton potential)

$$V(\phi) \sim \frac{m^2\phi^2}{2}. \qquad (14)$$

Near the origin ($|\phi| < M_{\rm Pl}$) we have $M_{\rm Pl}V'/V = M_{\rm Pl}/\phi$, and the first derivative condition [ϵ] is automatically satisfied as long as $\phi \ll M_{\rm Pl}/c$.

The situation is more nontrivial when the value of the inflaton becomes large, $\phi \sim M_{\rm Pl}/c$. Let us choose the value of the inflaton to be $\phi = \phi_*$ such that we have $V''/V = -c'$. Then we need to satisfy

$$M_{\rm Pl}\frac{V'}{V}\bigg|_{\phi=\phi_*} > c. \qquad (15)$$

This gives extra nontrivial constraints on the parameters of the inflaton potential (See Fig. 1 for the schematic picture of the inflationary potential.). Equivalently, once we fix the inflaton potential we can use this condition as constraints on the parameters c and c'. In the following we will sometimes simplify the analysis by taking $c' = 0$. This corresponds to the weaker conjecture (3), and gives the more conservative estimate of the constraints.

One might notice that in many inflationary models (including pure natural inflation and α-attractors discussed below) have plateau, where one might run into contradictions with the conjecture (2). Since we have argued below (12) that $c' < 0.01$, this would happen at large values of the inflaton, where the effective field theory might break down according to the distance conjecture [59]. Our analysis below is conservative in that we will not take this point fully into account.

VI. NATURAL INFLATION

For our further discussion we need to have the inflation scenario where we know the global form of the inflaton potential. A good example is provided by the natural inflation [73,74]. In this scenario the inflaton potential is generated by the coupling of the inflaton to the non-Abelian Yang-Mills gauge field, and the inflaton potential takes the cosine form, as generated by the one-instanton:

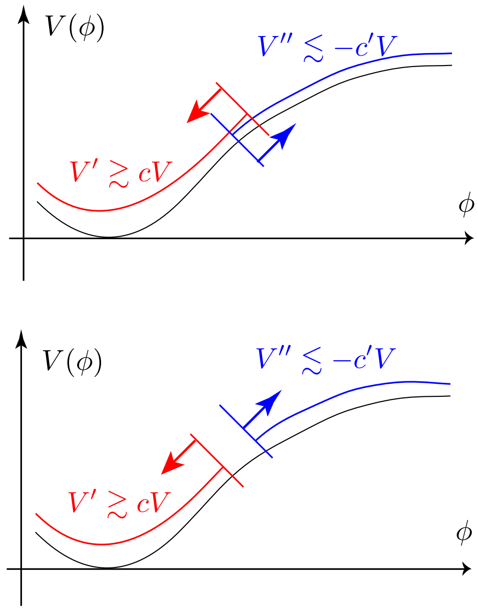

FIG. 1. For a plateau-type inflaton potential, we find that (1) near the bottom of the potential for reheating the condition $[\epsilon]$ is satisfied (region colored red), and (2) the condition $[\eta]$ is satisfied in the regions for accelerated expansion (region colored blue). Depending on the choice of the parameters there could be an intermediate region where neither condition is satisfied, as in the figure below.

$$V(\phi) = V_0\left(1 - \cos\frac{\phi}{f}\right). \tag{16}$$

Here f is the decay constant of the axion and the overall scale V_0 is determined by the dynamical scale of the confined Yang-Mills field.

This potential is $2\pi f$-periodic, has a maximum at $\phi = \pi f$, and is convex when $|\phi| < \pi f/2$. In this range the minimal value of the ratio $M_{\rm Pl}V'/V$ should be larger than the $O(1)$ coefficient c, so that

$$M_{\rm Pl}\frac{V'}{V} = \frac{M_{\rm Pl}}{f}\cot\frac{\phi}{2f} \gtrsim \frac{M_{\rm Pl}}{f} > c, \tag{17}$$

namely we obtain an upper bound on the decay constant

$$f < \frac{M_{\rm Pl}}{c}. \tag{18}$$

For $c \gtrsim 1$ this constraint can be stronger than the constraint from the weak gravity conjecture [75], which imposes $f \lesssim O(M_{\rm Pl})$. Note that we obtain a similar constraint $f < M_{\rm Pl}/\sqrt{c'}$ by applying the second-derivative condition $[\eta]$ near the top of the potential [56]. We can also plot the constraint on the $n_s - r$ plane, see Fig. 2 for exclusion region for $c = 0.3$.

We have chosen $c' = 0$ above for simplicity, but we can analyze the more general case $c' > 0$. We impose consistency with the conjectures for the inflaton potential in the whole region between the initial field value for inflation and the global minimum. In Fig. 3, we show the resulting constraints for c' and c for the natural inflation with e-folding 50.

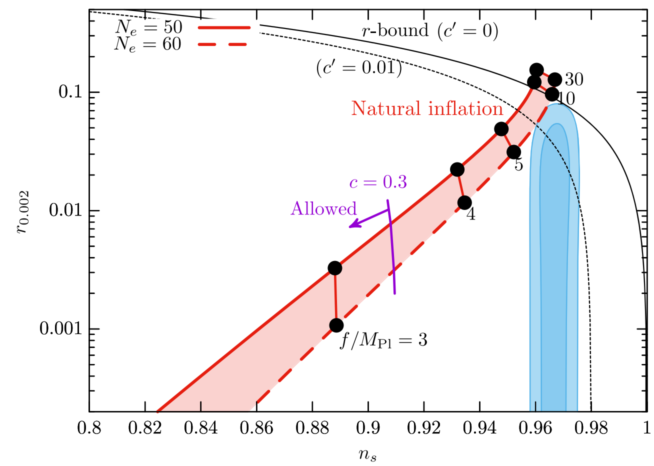

FIG. 2. The $n_s - r$ plane for the natural inflation (16), with e-folding between 50 and 60. The constraint (18) for $c = 0.3$ excludes the region of the right side of the purple line, making the model inconsistent with current observational constraints [64] (blue regions, 68% and 95% CL). The black and solid (dotted) line shows the upper-bound for r (12) with $c' = 0(0.01)$.

VII. STAROBINSKY MODEL

Let us next discuss the R^2 inflation model by Starobinsky [76]. When we choose the canonical kinetic term for the inflaton, the inflaton potential is given by

$$V_S(\phi) = V_0\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}}\right)^2. \tag{19}$$

By repeating the computation as before, we find constraints from the intermediate regions as in Fig. 1 to be

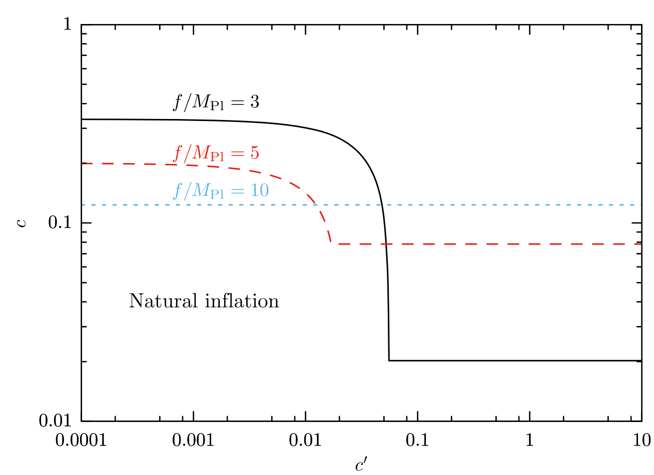

FIG. 3. The constraint of c and c' for the natural inflation model with e-folding 50. The regions below the lines are allowed.

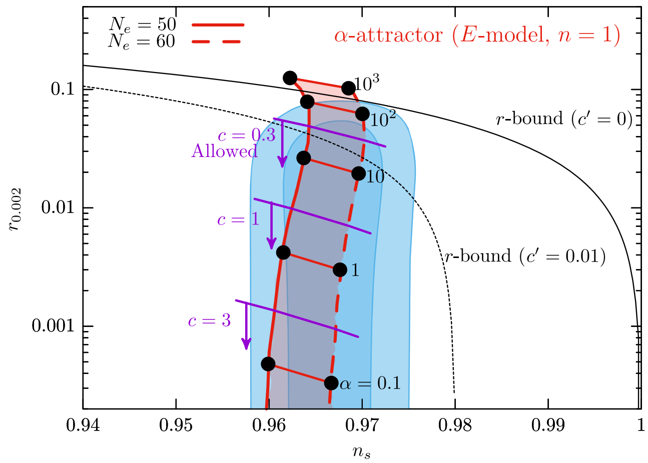

FIG. 4. Same as in Fig. 2 but for the E-model α-attractor (21) with $n = 1$, with e-folding between 50 and 60. This model contains the Starobinsky model (19) as a special case $\alpha = 1$. The constraint (22) for $c = 0.3$, 1, 3 excludes the region above the purple line.

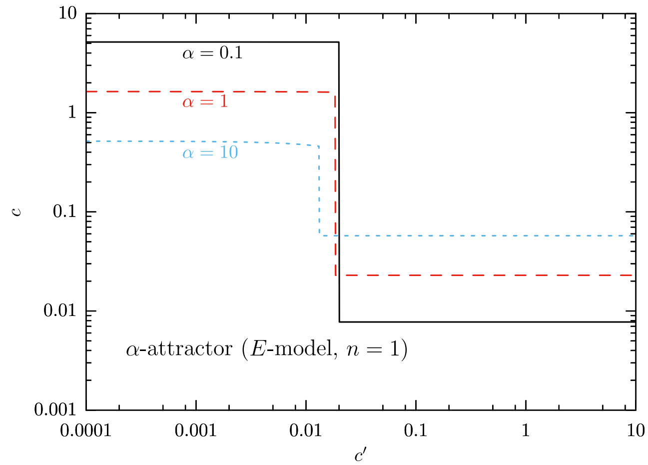

FIG. 5. Same as in Fig. 3 but for the E-model α-attractor (21) with $n = 1$.

$$c < 2\sqrt{\frac{2}{3}} \sim 1.6. \tag{20}$$

VIII. α-ATTRACTOR

The Starobinsky model can be thought of as a special case of the more general models known as the α-attractor [77,78].

The so-called E-model of the α-attractor is

$$V_E(\phi) = V_0\left(1 - e^{-\sqrt{\frac{2}{3\alpha}}\frac{\phi}{M_{\rm Pl}}}\right)^{2n}, \tag{21}$$

which includes the Starobinsky model (19) as a special case $n = 1$, $\alpha = 1$. In this case the constraint on c gives

$$c < 2\sqrt{\frac{2}{3\alpha}}\frac{n}{2n-1}, \tag{22}$$

which is shown in Fig. 4 on the $n_s - r$ plane.

The constraint (22) is strong for larger values of α, such as $\alpha \sim O(10^2)$, which are still marginally consistent with the current observations. Note that the constraints from the field range $[\Delta\phi/M_{\rm Pl} < O(1)]$ is also stronger for larger values of α. We also show the constraint of c and c' in Fig. 5.

In the Supplementary Material [79] we discuss more examples for such constraints. In all these analysis, we obtain interesting bottom-up constraints on values of c and c', and these constraints apply universally to any scalar potential of a low-energy effective theory with UV completion.

IX. SUMMARY AND CONCLUSION

In this paper, we examined phenomenological implications of the refined swampland conjecture. Although the original de Sitter conjecture (1) severely constrains the low-energy spontaneous symmetry breaking scenarios, such as the Higgs potential, chiral symmetry breaking, and QCD axion, the refined conjecture (2) has no such consequences.

The main consequence of the refined conjecture is in inflation. The original conjecture forbids a flat scalar potential, which is an essential ingredient for the exponential expansion during the cosmological inflation. By contrast in the refined conjecture, we can obtain sufficient e-folding for c and c' of $O(1)$, if the initial condition is fine-tuned. However, there is still tension with observed cosmological density fluctuation—we need either c or c' should be much smaller than $O(1)$ for observationally successful single-field slow-roll inflation. If we still require that c and c' are $O(1)$, inflation models beyond a single-field slow-roll model is required, such as noncanonical kinetic term and multifield models (including curvaton). In such cases, we can expect exotic cosmological signatures, such as detectable non-Gaussianity in the curvaton scenario.

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