What can PSR J1640-4631 tell us about the internal physics of this neutron star?

Quan Cheng,[1,2,*] Shuang-Nan Zhang,[2,4,†] Xiao-Ping Zheng,[3,5,‡] and Xi-Long Fan[1,5,§]

[1]*School of Physics and Technology, Wuhan University, Wuhan 430072, China*
[2]*Key Laboratory of Particle Astrophysics, Institute of High Energy Physics,
Chinese Academy of Sciences, Beijing 100049, China*
[3]*Institute of Astrophysics, Central China Normal University, Wuhan 430079, China*
[4]*University of Chinese Academy of Sciences, Beijing 100049, China*
[5]*School of Physics and Mechanical & Electrical Engineering,
Hubei University of Education, Wuhan 430205, China*

Gravitational wave emissions (GWEs) of pulsars could not only make them promising targets for continuous gravitational wave searches but also leave imprints in their timing data. We interpret the measured braking index of PSR J1640-4631 with a model involving both the GWE and dipole magnetic field decay. Combining the timing data of PSR J1640-4631 and the theory of magnetic field decay, we propose a new approach of constraining the number of precession cycles, ξ, which is highly uncertain currently but can be tightly related to the interior physics of a neutron star and its GWE. We suggest that future observation of the tilt angle χ of PSR J1640-4631 would not merely help to constrain ξ but also possibly provide information about the internal magnetic field configuration of this pulsar. We find that ξ would be larger than previous estimates unless a tiny angle $\chi \lesssim 5°$ is observed. Furthermore, a measured angle $\chi \gtrsim 12°$ would indicate $\xi \gtrsim 10^6$, which is at least 10 times larger than that suggested previously.

I. INTRODUCTION

The braking indices of pulsars are indicative of the spin-down mechanisms of neutron stars (NSs), which can be related to various aspects of NS physics. Traditional scenarios of a rotating magnetic dipole in vacuo show that pulsars should have braking indices $n = 3$ (e.g., [1]). However, this simple model is inconsistent with the observations of braking indices for all nine young pulsars, of which eight pulsars have $n < 3$ (see [2] and references therein) and only one has $n > 3$ [3]. To explain the $n < 3$ braking indices, several models have been invoked, including accretion of the fallback disk around a NS [4], braking torques due to relativistic particle winds and magnetic dipole radiation (MDR) [5], spin-down caused by quantum vacuum friction and MDR [6], a decrease in the effective moment of inertia of a NS as its interior normal matter becomes superfluid [7], and an increase in the surface dipole magnetic field due to either reemergence of the magnetic field buried after birth [8] or evolution of the crustal magnetic field [9].

The only young pulsar PSR J1640-4631 with $n > 3$ [3] observed hitherto[1] has attracted great attention and various models have been proposed to elucidate the large braking index, for instance, magnetic dipole spin-down of a pulsar with a plasma-filled magnetosphere [11], a combination of dipole and wind braking [12], spin-down of a conventional NS (or even an exotic low-mass NS [13]) due to MDR and gravitational wave emission (GWE) [14,15], classical MDR braking but with dipole field decay involved [16]. Theoretically, both GWE and dipole field decay may be inevitable for a NS with a strong magnetic field and a finitely conductive crust.

The strong magnetic fields of NSs could deform them into a quadruple ellipsoid (see [17] for a recent review), making them promising sources for continuous GW searches using ground-based GW detectors, such as advanced LIGO [18], Virgo [19], and the planned Einstein Telescope [20]. Although no GW signals from known pulsars have been detected during the first observing run of advanced LIGO [21], the magnetically induced GWE could indeed affect the spin evolution of NSs and leave some imprints in their

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[*]qcheng@ihep.ac.cn
[†]zhangsn@ihep.ac.cn
[‡]zhxp@mail.ccnu.edu.cn
[§]fanxilong@outlook.com

[1]Recently, it has been claimed that another young x-ray pulsar PSR J0537-6910 may have $n = 7$ as inferred from its complete timing data [10]. However, the result is inconclusive because of frequent glitches of this pulsar.

timing data. Moreover, for a deformed NS that is not in the minimum spin energy state, to minimize its spin energy, free-body precession of the star's magnetic axis around the spin axis will unavoidably occur, which could lead to the change of the tilt angle between the two axes.

Generally, the tilt angle evolution of a NS with a plasma-filled magnetosphere [22] is determined by the MDR [23], the GWE reaction [24], and damping of the free-body procession due to internal dissipation [25,26]. Among them, the angle evolution result from damping of the free-body precession can be related to a critical parameter called the number of precession cycles, ξ [25,27]. Since the damping mechanisms are not clearly understood, only quite rough estimates for ξ have been proposed hitherto. For instance, as a possible damping mechanism, Alpar and Sauls [25] studied the core-crust coupling due to scattering of electrons off the neutron vortices and obtained $\xi \approx 10^{2-4}$. On the other hand, damping of the stellar free-body precession caused by elastic dissipation in the crust gives a relatively large $\xi \lesssim 10^5$ [26]. However, this parameter is extremely important in discussing the GWE of a NS (e.g., [28,29]), because ξ could significantly affect the timescale over which the optimal (unfavorable) configuration for GWE can be achieved, provided that the star has a prolate (oblate) shape.

It has long been suggested that the dipole field that possibly associated with the crustal field of a NS could decay due to Hall drift and Ohmic dissipation (e.g., [30,31]). The specific timescale for the field decay is still uncertain, though typical timescales of $\sim 10^4$ yr (depending on the dipole field strength and the density at the base of the crust) [32,33] and $\sim 10^6$ yr (depending on the electrical conductivity of the crust) [31,34,35] were proposed for Hall drift and Ohmic dissipation, respectively. Furthermore, population synthesis studies of isolated radio pulsars suggested an extremely long decay timescale of $\gtrsim 10^8$ yr if field decay could indeed occur [36].

In this paper, we explain the braking index of PSR J1640-4631 based on a model involving both GWE and dipole field decay, which are natural consequences with the presence of strong magnetic fields of a NS. We propose a new approach of estimating ξ by using the timing data of PSR J1640-4631 and the magnetic field decay theory. We suggest that once the tilt angle of this pulsar is measured, we could not only put constraints on the highly uncertain parameter ξ but also possibly know about its internal magnetic field configuration. Interestingly, the value of ξ would be larger than previous results unless a tiny tilt angle ($\lesssim 5°$) is observed. The paper is organized as follows. The evolutionary model for PSR J1640-4631 is presented in Sec. II. We introduce the theory of magnetic field decay in Sec. III. Our results are given in Sec. IV. Finally, a conclusion and some brief discussions about possible physical explanations of a large ξ and its influence on the GWEs from newborn magnetars are provided in Sec. V.

II. EVOLUTION OF PSR J1640-4631

Using the *NuSTAR* X-ray observatory, Gotthelf *et al.* [37] discovered the pulsar PSR J1640-4631, whose period and first period derivative are $P = 206$ ms and $\dot{P} = 9.758 \times 10^{-13}$ s/s, respectively. Recently, by performing a phase-coherent timing analysis of the x-ray timing data of PSR J1640-4631 observed with *NuSTAR*, Archibald *et al.* [3] obtained its second period derivative and braking index $n = 3.15(3)$.

For a pulsar with a corotating plasma magnetosphere [22] that spins down mainly due to MDR and magnetic deformation-induced GWE, its angular frequency evolution has the following form [24,38]:

$$\dot{\omega} = -\frac{2G\epsilon_B^2 I \omega^5}{5c^5}\sin^2\chi(1+15\sin^2\chi) - \frac{kB_d^2 R^6 \omega^3}{Ic^3}(1+\sin^2\chi),$$

(1)

where ϵ_B is the ellipticity of magnetic deformation, I the moment of inertia, χ the tilt angle, k the coefficient related to MDR, B_d the surface dipole magnetic field at the magnetic pole, and R the stellar radius. Hereafter, we adopt $k = 1/6$, and take canonical values for the parameters of the presumed 1.4 $M_\odot$ NS as $I = 10^{45}$ g cm^2 and $R = 10$ km.[2] We define a ratio $\eta = \dot{\omega}_{MDR}/\dot{\omega}_{GWE} = 5kc^2 B_d^2 R^6(1 + \sin^2\chi)/[2G\epsilon_B^2 I^2\omega^2(1 + 15\sin^2\chi)\sin^2\chi]$, where $\dot{\omega}_{MDR}$ and $\dot{\omega}_{GWE}$ are the MDR-induced and GWE-induced spin-down rate, respectively. Though the GWE braking becomes maximal when $\chi = \pi/2$ is taken, one still has $\eta \gg 1$ for $|\epsilon_B| \ll 8.69 \times 10^{-3}(B_d/10^{13}$ G$)$, as ω is known for PSR J1640-4631. We will show that no matter whether the internal fields of this NS are poloidal dominated (PD) or toroidal dominated (TD), the theoretically estimated ϵ_B is far beneath this limit.

Previous studies have shown that the NS equation of state, the magnetic energy, the internal magnetic configuration, and the presence of proton superconductivity in the core (which may change the interior magnetic field distribution) could all affect the magnetic deformation of a NS (e.g., Refs. [41,42]). Lots of theoretical calculations have been made to obtain the ellipticity (see, e.g., Refs. [26,29,41–51]). For a young NS like PSR J1640-4631, its interior temperature is probably lower than the critical temperature for proton superconductivity [52], even if only modified Urca cooling occurs [53]. Hence, to estimate ϵ_B of PSR J1640-4631, the effect of proton superconductivity should be involved, as that done in Ref. [47].

After considering type-II proton superconductivity in the interior of a NS, Lander [47] self-consistently obtained an

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[2]We note that the value of k is still in debate (see Refs. [23, 38–40]). However, adopting different values for k (= 1/4) and R (= 12 km) could affect the value of ξ by at most a factor of 2.

equilibrium configuration that consists of a mixed poloidal-toroidal field and derived the corresponding magnetic ellipticity,

$$\epsilon_{\rm B} = 3.4 \times 10^{-7} \left(\frac{B_{\rm d}}{10^{13}\,{\rm G}}\right)\left(\frac{H_{\rm c1}(0)}{10^{16}\,{\rm G}}\right), \qquad (2)$$

where the central critical field strength is taken to be $H_{\rm c1}(0) = 10^{16}$ G [47]. In this field configuration, since the dominant part is the poloidal component, the NS has an oblate shape ($\epsilon_{\rm B} > 0$). This configuration is partially akin to the twisted-torus configuration found in numerical simulations [54]. The main difference is that in the latter configuration, the toroidal field may be dominant [55], the NS possibly has a prolate shape ($\epsilon_{\rm B} < 0$). With type-II proton superconductivity involved, and based on the twisted-torus configuration, a calculation of $\epsilon_{\rm B}$ is presented in Ref. [48]. However, the results are very rough and only upper limits are given for $\epsilon_{\rm B}$ because the superconducting stellar interior is assumed to have a homogeneous magnetic permeability, which is in fact physically implausible. Since there is no self-consistent calculations for the ellipticity of a superconducting NS that has a TD twisted-torus field configuration inside currently, we simply adopt $\epsilon_{\rm B}$ derived for the pure toroidal configuration as a substitution, which takes the form [56]

$$\epsilon_{\rm B} \approx -10^{-8} \left(\frac{H}{10^{15}\,{\rm G}}\right)\left(\frac{\bar{B}_{\rm in}}{10^{13}\,{\rm G}}\right), \qquad (3)$$

where $H \approx 10^{15}$ G is the critical field strength and $\bar{B}_{\rm in}$ the volume-averaged strength of the internal toroidal field. It is generally hard to determine $\bar{B}_{\rm in}$ of a NS. Fortunately, the observed positive correlation between the surface temperatures and dipole magnetic fields of isolated NSs (with $B_{\rm d} \gtrsim 10^{13}$ G) indicate that strong toroidal fields with volume-averaged strengths of $\sim 10 B_{\rm d}$ possibly exist in NS crusts [57]. We thus assume that the strengths of the crustal toroidal fields are representative of $\bar{B}_{\rm in}$ of the whole stars, that is, $\bar{B}_{\rm in} \simeq 10 B_{\rm d}$. Internal fields that are 1 order of magnitude (or more) higher than dipole fields may indeed be present in young pulsars (see Ref. [58]).

It should be noted that the internal fields which determine the ellipticity may also decrease as the star evolves. Here we assume that the relation between the internal fields and $B_{\rm d}$ remains unchanged and the expression for $\epsilon_{\rm B}$ given by Eq. (2) or (3) still holds with the decay of $B_{\rm d}$, though a global long-term numerical simulation is needed to reveal how internal fields and $\epsilon_{\rm B}$ vary with time. Interestingly, a time-dependent $\epsilon_{\rm B}$, as also considered in Ref. [15], can hardly change our results in comparison with the case of a time-independent $\epsilon_{\rm B}$. The reason is that adopting a time-dependent $\epsilon_{\rm B}$ results in a factor $(1 + 1/\eta) \simeq 1$ just before the term $\dot{B}_{\rm d}/B_{\rm d}$ in Eq. (5), which is 1 for the case of a time-independent $\epsilon_{\rm B}$. From Eqs. (2) and (3), we can see that

these estimated $\epsilon_{\rm B}$ are consistent with the requirement of $\eta \gg 1$. The GWE braking can therefore be neglected due to its little effect on the spin-down of PSR J1640-4631. However, the GWE could still affect the pulsar's tilt angle evolution.

The tilt angle evolution of a magnetically deformed NS with a plasma magnetosphere is given by [23,24,42,59]

$$\dot{\chi} = \begin{cases} -\frac{2G}{5c^5}I\epsilon_{\rm B}^2\omega^4\sin\chi\cos\chi(15\sin^2\chi + 1) - \frac{\epsilon_{\rm B}}{\xi P}\tan\chi \\ \quad -\frac{kB_{\rm d}^2R^6\omega^2}{Ic^3}\sin\chi\cos\chi, \quad \text{for } \epsilon_{\rm B} > 0 \\ -\frac{2G}{5c^5}I\epsilon_{\rm B}^2\omega^4\sin\chi\cos\chi(15\sin^2\chi + 1) - \frac{\epsilon_{\rm B}}{\xi P}\cot\chi \\ \quad -\frac{kB_{\rm d}^2R^6\omega^2}{Ic^3}\sin\chi\cos\chi, \quad \text{for } \epsilon_{\rm B} < 0. \end{cases} \qquad (4)$$

The first and third terms of the above formula represent the alignment effects caused by the GWE and MDR, respectively. The second term represents the angular evolution from damping of the stellar free-body procession due to internal dissipation. Depending on the shape of a NS (or the sign of $\epsilon_{\rm B}$), this effect could either decrease or increase χ. Actually, Eq. (4) stands for the main difference as compared to previous models [13–15], in which these mechanisms for tilt angle evolution were not considered.

By taking both the field decay and tilt angle evolution into account, the braking index reads

$$n = 3 - \frac{2P}{\dot{P}}\left\{\frac{\dot{B}_{\rm d}}{B_{\rm d}} + \dot{\chi}\sin\chi\cos\chi\left[\frac{1}{1 + \sin^2\chi}\right.\right.$$
$$\left.\left. + \frac{1 + 30\sin^2\chi}{\eta\sin^2\chi(1 + 15\sin^2\chi)}\right]\right\}, \qquad (5)$$

where $\dot{B}_{\rm d}$ is the decay rate of $B_{\rm d}$. We will see below Eq. (5) is a critical link that relates ξ in Eq. (4) to the timing data of PSR J1640-4631 and the field decay timescale $\tau_{\rm D} = -B_{\rm d}/\dot{B}_{\rm d}$ determined by the field decay theory.

III. THE THEORY OF MAGNETIC FIELD DECAY

The decay rate of $B_{\rm d}$ is determined by the specific field decay mechanisms, which are generally considered to be Hall drift and Ohmic dissipation if the dipole field has a crustal origin. However, the mathematical form of field decay is still not clearly known. For simplicity, we consider two typical decay forms that introduce the least parameters. The first one is the exponential form [33,57]

$$\frac{dB_{\rm d}}{dt} = -\frac{B_{\rm d}}{\tau_{\rm D}}, \qquad (6)$$

where $\tau_{\rm D}$ is the dipole field decay timescale. The second one is the nonlinear form [16,33,60]

$$\frac{dB_d}{dt} = -\frac{B_d}{\tau_D + t},\qquad(7)$$

where t is the actual age of the pulsar. Generally, τ_D may be determined by both Hall drift and Ohmic dissipation in the crust as $1/\tau_D = 1/\tau_{Hall} + 1/\tau_{Ohmic}$ (see, e.g., [16]), where τ_{Hall} and τ_{Ohmic} are Hall drift and Ohmic dissipation timescales, respectively. It should also be noted that Hall drift itself is a nondissipative process, however, could substantially accelerate the field decay by changing the large scale magnetic field into small scale components, which would decay rapidly due to Ohmic dissipation [31,61]. In this case, the field decay timescale may be set by the Hall timescale in the crust as $\tau_D = \tau_{Hall} \simeq 1.2 \times 10^4 (10^{15}\,G/B_d)$ yr [32,33].

Furthermore, if Ohmic dissipation dominates the crustal field decay process, as indicated by the positive correlation between the surface temperatures and dipole fields of isolated NSs [57], the dipole fields which are assumed to be proportional to the crustal fields may decay on the same timescale $\tau_D = \tau_{Ohmic} \simeq 5 \times 10^5$ or 10^6 yr as the latter [57]. Lastly, numerical modeling of the coupled magnetic field evolution in the crust and the core of a NS shows that B_d could decay over a timescale $\tau_D \simeq 150$ Myr due to the combined effects of flux tube drift in the core and Ohmic dissipation in the crust [62,63]. This may represent the longest field decay timescale predicted theoretically, and it is also consistent with the results of pulsar population synthesis [36].

In Fig. 1 we show τ_D as a function of χ. The latter is related to B_d via Eq. (1) by neglecting the term of GWE. From the timing data of PSR J1640-4631, we obtain $B_d \sim 2 \times 10^{13}$ G. Thus $\tau_{Hall}(\chi)$ (black solid line) is approximately equal to $\tau_{Ohmic} \simeq 5 \times 10^5$ yr (black dashed line). If $\tau_D(\chi)$ follows the form $\tau_D(\chi) = 1/[1/\tau_{Hall}(\chi) + 1/\tau_{Ohmic}]$, its minimum value at χ can be obtained by taking $\tau_{Ohmic} = 5 \times 10^5$ yr, as shown by the black dash-dot-dotted line (also the lower boundary of the blank region) in Fig. 1. A larger τ_{Ohmic} can shift this boundary upwards, but should not surpass $\tau_{Hall}(\chi)$. The maximum value of $\tau_D(\chi)$ at χ could be determined by τ_{Ohmic}, which may be 5×10^5, 10^6 (black dotted line), or 1.5×10^8 yr (black dash-dotted line) if Ohmic dissipation dominates the field decay.[3] The upper boundary of the blank region in Fig. 1 corresponds to $\tau_D(\chi) = 1.5 \times 10^8$ yr, above which should be excluded following the field decay theory.

From Eqs. (6) and (7), we have $\tau_D = -B_d/\dot{B}_d$ and $\tau_D = -B_d/\dot{B}_d - t$, respectively. The actual age t of PSR J1640-4631 remains unconstrained from observations currently, though an estimate of $t \sim 3000$ yr (close to its

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[3]Here we attribute $\tau_D(\chi) \simeq 150$ Myr to the effect of crustal Ohmic dissipation but keep in mind that flux tube drift in the core region also plays an important role.

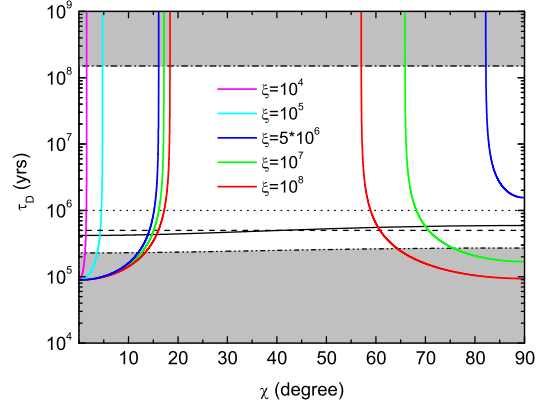FIG. 1. Dipole field decay timescale τ_D versus tilt angle χ. This figure shows a comparison between $\tau_D(\chi)$ derived using timing data (colored lines) and that obtained based on the magnetic field decay theory (black lines). The NS is assumed to have PD internal fields. See the text for details.

characteristic age $\tau_c = 3350$ yr [37]) was proposed on the basis of the dipole field decay [16]. Assuming $t \simeq \tau_c$, from Fig. 1 we can see that t is far below the lower boundary of $\tau_D(\chi)$. Therefore, hereafter we can safely neglect the term t and determine the decay timescale via $\tau_D = -B_d/\dot{B}_d$.

IV. RESULTS

By substituting the observed P, $\dot{P}$, $n = 3.15$, and Eq. (5) into Eq. (5), and taking ξ as a free parameter, one can solve for $\tau_D = -B_d/\dot{B}_d$ versus χ. The evolution curves $\tau_D(\chi)$ for different ξ are shown by the colored curves in Fig. 1. Since the evolution of χ depends on the shape of the NS, in Fig. 1, we first show the results for the PD case with ϵ_B given by Eq. (2).

The constraint on ξ is set by the fact that at a certain χ, $\tau_D(\chi)$ derived from timing data of PSR J1640-4631 should be equal to $\tau_D(\chi)$ obtained based on the field decay theory. That is, it requires that the colored curve should at least intersect with one of the black curves, as presented in Fig. 1. If the internal fields of this pulsar are PD, for the number of precession cycles in a wide range of $10^4 \lesssim \xi \lesssim 10^8$, each of the colored curves has at least one intersection with the black lines. The interactions are distributed within $2° \lesssim \chi \lesssim 18°$ and $57° \lesssim \chi \lesssim 90°$. Specifically, for $\xi \lesssim 10^5$, all the intersections are within $\chi \lesssim 5°$. For $5 \times 10^6 \lesssim \xi \lesssim 10^8$, $\tau_D(\chi)$ derived via Eq. (5) splits into two branches, of which the left one has interactions at $12° \lesssim \chi \lesssim 18°$, and the right one has interaction(s) at $57° \lesssim \chi \lesssim 82°$. Even if $\xi \gtrsim 10^9$ (which might be unphysical) is taken, no interactions could be found for intermediate angles $18° \lesssim \chi \lesssim 57°$.

We also investigate another possibility that this NS has TD internal fields with ϵ_B given by Eq. (3). The results are presented in Fig. 2, which shows that in order to have at least one intersection between the curve $\tau_D(\chi)$ obtained based on the timing data and the black dash-dot-dotted
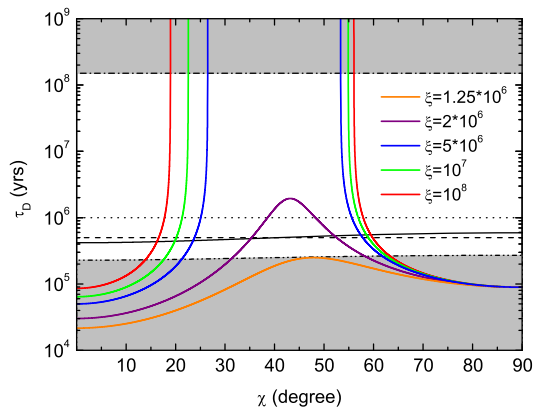
FIG. 2. The same as Fig. 1, however, the NS is assumed to have TD internal fields. See the text for details.

line, the lower limit for the number of precession cycles can be set as $\xi \gtrsim 1.25 \times 10^6$ (the orange curve). All the intersections are distributed within $14° \lesssim \chi \lesssim 63°$ for $1.25 \times 10^6 \lesssim \xi \lesssim 10^8$. For the tilt angle in the ranges $\chi \lesssim 14°$ and $\chi \gtrsim 63°$, there are no intersections even though an (unphysically) large $\xi \gtrsim 10^9$ is adopted. The same as in the PD case, $\tau_D(\chi)$ derived from the timing data also shows a bifurcation for $5 \times 10^6 \lesssim \xi \lesssim 10^8$.

Therefore, we suggest that future observations of the tilt angle of PSR J1640-4631 would probably help to probe its internal magnetic field configuration and put constraints on the number of precession cycles. For instance, a small measured angle $\chi \lesssim 14°$ possibly supports a PD internal field configuration because no intersections are found for χ in this range in the TD case. Moreover, a small value for the number of precession cycles $\xi \lesssim 10^5$ as suggested in previous work [25,26,29,59] could be confirmed only if a tiny angle $\chi \lesssim 5°$ is observed. Beyond this angle, ξ would be larger than previous estimates no matter whether the internal fields are PD or TD. With some more calculations we find that as long as an angle $\chi \gtrsim 12°$ is observed,[4] one would have $\xi \gtrsim 10^6$, irrespective of the internal field configuration. A large angle $\chi \gtrsim 63°$ may also indicate the PD scenario, however, the required ξ is in the range $10^6 \lesssim \xi \lesssim 10^8$, at least $\sim 10-10^3$ larger than previous results. In contrast, an intermediate angle $18° \lesssim \chi \lesssim 57°$ seems to favor a TD internal field configuration, and a large ξ whose lower limit is 1.25×10^6. Only for the measured angle in two small ranges $14° \lesssim \chi \lesssim 18°$ and $57° \lesssim \chi \lesssim 63°$, we could not deduce whether the poloidal or the toroidal field is dominant in the NS interior.

V. CONCLUSION AND DISCUSSIONS

Based on the timing data of PSR J1640-4631 and the magnetic field decay theory, we propose a new method of

estimating a vital but presently highly unknown parameter called the number of precession cycles, ξ. In the modeling, we considered different internal magnetic field configurations, field decay formulas, and field decay timescales. We conclude that if the tilt angle χ of PSR J1640-4631 could be measured through polarization observation using future x-ray telescopes (e.g., eXTP [64]), we may get quite valuable information about ξ and the internal magnetic fields of this pulsar. Most importantly, irrespective of the internal field configuration, as long as the angle is observed to be $\chi \gtrsim 5°$, ξ should be constrained to be larger than previous results [25,26,29,59]. As a conservative estimate, a measured angle $\chi \gtrsim 12°$ would indicate $\xi \gtrsim 10^6$, which is at least 10 times larger than that suggested previously.

Physically, a large ξ indicates that some rather weak damping mechanisms are responsible for the dissipation of the precessional energy. In the crust, if phonon excitations govern the interactions between vortices and lattices, the mutual friction parameter, whose reciprocal is approximately equal to ξ, could be as large as $\mathcal{B} \approx 10^{-8}$ (e.g., [65,66]). Therefore, an inferred large $\xi \approx 10^8$ may suggest that most of the precessional energy is dissipated in the crust due to vortex-lattice interaction controlled by phonon excitations. On the other hand, in the core some (*unknown*) weak damping mechanisms rather than electron-vortex interaction may be dominant, as recently found in [66] that in the core $\mathcal{B} \sim 10^{-7}-10^{-6}$ is required to interpret the rising processes of three large Crab glitches. If ξ is constrained to be large in the future, it would greatly expedite our understanding of complex interactions in NSs.

Furthermore, a large ξ means a long timescale for a prolate NS (e.g., newborn magnetars) to achieve the orthogonal configuration [28] provided that χ could not rapidly increase during the very early period [42]. Thus, if newborn magnetars have a large ξ, their GWEs may be weak and not easy to be detected.

Finally, though we only performed a case study for PSR J1640-4631, we should stress that our new method of estimating ξ also applies to other eight pulsars with a measured braking index. The derived constraints on ξ for these pulsars may be different from that for PSR J1640-4631. This is reasonable because for different pulsars the dominant interior interactions and the internal magnetic field configurations are possibly various. A detailed analysis for other pulsars will be presented in a subsequent paper.

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[4]This is the largest lower limit required to satisfy $\xi \gtrsim 10^6$, which is derived for the PD case and by taking $\tau_D(\chi) = 150$ Myr.

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