Study of CP violation in $B^- \to K^-\pi^+\pi^-$ and $B^- \to K^-\sigma(600)$ decays in the QCD factorization approach

Jing-Juan Qi* and Xin-Heng Guo†
College of Nuclear Science and Technology, Beijing Normal University, Beijing 100875, China

Zhen-Yang Wang‡
Physics Department, Ningbo University, Zhejiang 315211, China

Zhen-Hua Zhang§
School of Nuclear and Technology, University of South China, Hengyang, Hunan 421001, China

Chao Wang‖
Center for Ecological and Environmental Sciences, Key Laboratory for Space Bioscience and Biotechnology, Northwestern Polytechnical University, Xi'an 710072, China

(Received 7 November 2018; revised manuscript received 18 March 2019; published 16 April 2019)

In this work, we study the localized CP violation in $B^- \to K^-\pi^+\pi^-$ and $B^- \to K^-\sigma(600)$ decays by employing the quasi-two-body QCD factorization approach. Both the resonance and the nonresonance contributions are studied for the $B^- \to K^-\pi^+\pi^-$ decay. The resonance contributions include those not only from $[\pi\pi]$ channels including $\sigma(600)$, $\rho^0(770)$ and $\omega(782)$ but also from $[K\pi]$ channels including $K_0^*(700)(\kappa)$, $K^*(892)$, $K_0^*(1430)$, $K^*(1410)$, $K^*(1680)$ and $K_2^*(1430)$. By fitting the four experimental data $\mathcal{A}_{CP}(K^-\pi^+\pi^-) = 0.678 \pm 0.078 \pm 0.0323 \pm 0.007$ for $m^2_{K^-\pi^+} < 15\,\mathrm{GeV}^2$ and $0.08 < m^2_{\pi^+\pi^-} < 0.66\,\mathrm{GeV}^2$, $\mathcal{A}_{CP}(B^- \to K_0^*(1430)\pi^-) = 0.061 \pm 0.032$, $\mathcal{B}(B^- \to K_0^*(1430)\pi^-) = (39^{+6}_{-5}) \times 10^{-6}$ and $\mathcal{B}(B^- \to \sigma(600)\pi^- \to \pi^-\pi^+\pi^-) < 4.1 \times 10^{-6}$, we get the end-point divergence parameters in our model, $\phi_S \in [1.77, 2.25]$ and $\rho_S \in [2.39, 4.02]$. Using these results for ρ_S and ϕ_S, we predict that the CP asymmetry parameter $\mathcal{A}_{CP} \in [-0.34, -0.11]$ and the branching fraction $\mathcal{B} \in [6.53, 17.52] \times 10^{-6}$ for the $B^- \to K^-\sigma(600)$ decay. In addition, we also analyze contributions to the localized CP asymmetry $\mathcal{A}_{CP}(B^- \to K^-\pi^+\pi^-)$ from $[\pi\pi]$, $[K\pi]$ channel resonances and nonresonance individually, which are found to be $\mathcal{A}_{CP}(B^- \to K^-[\pi^+\pi^-] \to K^-\pi^+\pi^-) = 0.509 \pm 0.042$, $\mathcal{A}_{CP}(B^- \to [K^-\pi^+]\pi \to K^-\pi^+\pi^-) = 0.174 \pm 0.025$ and $\mathcal{A}_{CP}{}^{\mathrm{NR}}(B^- \to K^-\pi^+\pi^-) = 0.061 \pm 0.0042$, respectively. Comparing these results, we can see that the localized CP asymmetry in the $B^- \to K^-\pi^+\pi^-$ decay is mainly induced by the $[\pi\pi]$ channel resonances while contributions from the $[K\pi]$ channel resonances and nonresonance are both very small.

DOI: 10.1103/PhysRevD.99.076010

I. INTRODUCTION

Nonleptonic decays of hadrons containing a heavy quark play an important role in testing the Standard Model (SM) picture of the charge-parity (CP) violation mechanism in flavor physics, improving our understanding of nonperturbative and perturbative QCD and exploring new physics beyond the SM. CP violation is related to the weak complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix, which describes the mixing of different generations of quarks [1,2]. Besides the weak phase, a large strong phase is also needed for a large CP asymmetry. Generally, this strong phase is provided by QCD loop corrections and some phenomenological models.

Three-body decays of heavy mesons are more complicated than the two-body case as they receive resonant and nonresonant contributions and involve three-body matrix elements. The direct nonresonant three-body decay of mesons generally receives two separate contributions: one from the pointlike weak transition and the other from

*jjqi@mail.bnu.edu.cn
†Corresponding author. xhguo@bnu.edu.cn
‡wangzhenyang@mail.nbu.edu.cn
§Corresponding author. zhangzh@usc.edu.cn
‖chaowang@nwpu.edu.cn

the pole diagrams that involve three-point or four-point strong vertices. The nonresonant background in charmless three-body B decays due to the transition $B \to M_1 M_2 M_3$ has been studied extensively based on heavy meson chiral perturbation theory (HMChPT) [3–5]. However, the predicted decay rates are, in general, unexpectedly large and not recovered in the soft meson region. Therefore, it is important to reexamine and clarify the existing calculations. In this work we will follow Ref. [6] to assume the momentum dependence of nonresonance amplitudes in the exponential form $e^{-\alpha_{\rm NR} p_B \cdot (p_i + p_j)}$ ($\alpha_{\rm NR}$ is unknown parameter, p_B, p_i and p_j are the four momenta of the B, i and j mesons, respectively) so that the HMChPT results are recovered in the soft meson limit p_i, $p_j \to 0$. At any rate, it is important to understand and identify the underlying mechanism for nonresonant decays.

Besides the nonresonance background, the three-body meson decays are generally dominated by intermediate resonances, namely, they proceed via quasi-two-body decays containing resonance states. LHCb also observed the large CP asymmetry in the localized region of the phase space [7,8], i.e., $\mathcal{A}_{CP}(K^-\pi^+\pi^-) = 0.678 \pm 0.078 \pm 0.0323 \pm 0.007$ for $m^2_{K^-\pi^+} < 15\,\mathrm{GeV}^2$ and $0.08 < m^2_{\pi^+\pi^-} < 0.66\,\mathrm{GeV}^2$, which spans the $[\pi\pi]$ channel and $[K\pi]$ channel resonances, such as $\sigma(600)$, $\rho^0(770)$, $\omega(782)$, $K_0^*(700)(\kappa)$, $K^*(892)$, $K^*(1410)$, $K_0^*(1430)$, $K^*(1680)$ and $K_2^*(1430)$ mesons. Some other considerations also motivate a precise analysis of $B^- \to K^-\pi^+\pi^-$ decays. The CP asymmetries in the decays $B \to K^*(892)\pi$, $B \to K^*(1430)\pi$ and $B \to K_2^*(1430)\pi$ are predicted to be negligible [9,10] compared to the current precision, since these are mediated by $b \to s$ loop (penguin) transitions only, with no $b \to u$ tree component. It is worthwhile to study the contributions from $K\pi$ channel resonances in the $B^- \to K^-\pi^+\pi^-$ decays. The underlying structures of light scalar mesons are still under controversy. Scalar mesons could be identified as ordinary $\bar{q}q$ states, four-quark states, meson-meson bound states, or even those supplemented with a scalar glueball. In our work we will use the simple $\bar{q}q$ quark model for scalar mesons [11].

Theoretically, to calculate the hadronic matrix elements of hadronic B weak decays, some approaches, including QCD factorization (QCDF) [10,12], perturbative QCD (pQCD) [13] and soft-collinear effective theory (SCET) [14], have been fully developed and extensively employed in recent years. Even though the annihilation contributions are formally power suppressed in the heavy quark limit, they may be numerically important for realistic hadronic B decays, particularly for pure annihilation processes and direct CP asymmetries. Unfortunately, in the collinear factorization approximation, the calculation of annihilation corrections always suffers from end-point divergence. In the pQCD approach, such divergence is regulated by introducing the parton transverse momentum k_T and the Sudakov factor at the expense of modeling the additional k_T dependence of meson wave functions, and large complex annihilation corrections are presented [15]. In the SCET approach, such divergence is removed by separating the physics at different momentum scales and using zero-bin subtraction to avoid double counting the soft degrees of freedom [16,17]. In the QCDF approach, such divergence is usually parametrized in a model-independent manner [10,12] and will be explicitly expressed in Sec. III.

There are many experimental studies which have been successfully carried out at B factories (*BABAR* and Belle), Tevatron (CDF and D0) and LHCb, and are being continued at LHCb and Belle experiments. These experiments provide highly fertile ground for theoretical studies and have yielded many exciting and important results, such as measurements of pure annihilation $B_s \to \pi\pi$ and $B_d \to KK$ decays reported recently by CDF, LHCb and Belle [18–20], which may suggest the existence of unexpected large annihilation contributions and have attracted much attention [21–23]. So it is also important to consider the annihilation contributions to B decays.

The remainder of this paper is organized as follows. In Sec. II, we present the form factors, decay constants and distribution amplitudes of different mesons. In Sec. III, we present the formalism for B decays in the QCDF approach. In Sec. IV, we present detailed calculations of CP violation for $B^- \to K^-\pi^+\pi^-$ and $B^- \to K^-\sigma(600)$ decays. The numerical results are given in Sec. V and we summarize our work in Sec. VI.

II. FORM FACTORS, DECAY CONSTANTS AND LIGHT-CONE DISTRIBUTION AMPLITUDES

Since the form factors for $B \to P$, $B \to V$, $B \to S$ and $B \to T$ (P, V, S and T represent pseudoscalar, vector, scalar and tensor mesons, respectively) weak transitions and light-cone distribution amplitudes and decay constants of P, V, S and T will be used in treating B decays, we first discuss them in this section.

The form factors of B to a meson weak transition can be decomposed as [24,25]

$$
\begin{aligned}
\langle P(p')|\hat{V}_\mu|B(p)\rangle &= \left(p_\mu - \frac{m_B^2 - m_P^2}{q^2} q_\mu\right) F_1^{BP}(q^2) + \frac{m_B^2 - m_P^2}{q^2} q_\mu F_0^{BP}(q^2),\\
\langle V(p')|\hat{V}_\mu|B(p)\rangle &= \frac{2}{m_B + m_V} \varepsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu} p^\rho p'^\sigma V^{BV}(q^2),\\
\langle V(p')|\hat{A}_\mu|B(p)\rangle &= i\left\{(m_B + m_V)\epsilon_\mu^* A_1^{BV}(q^2) - \frac{\epsilon^*\cdot q}{m_B + m_V} P_\mu A_2^{BV}(q^2) - 2m_V \frac{\epsilon^*\cdot P}{q^2} q_\mu [A_3^{BV}(q^2) - A_0^{BV}(q^2)]\right\},\\
\langle S(p')|\hat{A}_\mu|B(p)\rangle &= -i\left[\left(P_\mu - \frac{m_B^2 - m_S^2}{q^2} q_\mu\right) F_1^{BS}(q^2) + \frac{m_B^2 - m_S^2}{q^2} q_\mu F_0^{BS}(q^2)\right],\\
\langle T(p')|\hat{V}_\mu|B(p)\rangle &= \frac{2}{m_B + m_T} \varepsilon_{\mu\nu\rho\sigma} e^{*\nu} p^\rho p'^\sigma V^{BT}(q^2),\\
\langle T(p')|\hat{A}_\mu|B(p)\rangle &= i\left\{(m_B + m_T) e_\mu^* A_1^{BT}(q^2) - \frac{e^*\cdot q}{m_B + m_T} P_\mu A_2^{BT}(q^2) - 2m_T \frac{e^*\cdot P}{q^2} q_\mu [A_3^{BT}(q^2) - A_0^{BT}(q^2)]\right\},
\end{aligned} \tag{1}
$$

where $P_\mu = (p + p')_\mu$, $q_\mu = (p - p')_\mu$, $\hat{V}_\mu$, $\hat{A}_\mu$ and $\hat{S}_\mu$ are the weak vector, axial-vector and scalar currents, respectively, i.e., $\hat{V}_\mu = \bar{q}_2\gamma_\mu q_1$, $\hat{A}_\mu = \bar{q}_2\gamma_\mu\gamma_5 q_1$, $\hat{S} = \bar{q}_2 q_1$, ϵ_μ is the polarization vector of V, $e^{*\mu} \equiv \epsilon^{*\mu\nu} p_\nu/m_B$ ($\epsilon_{\mu\nu}$ is the polarization tensor of T), $F_i^{BP}(q^2)$ ($i = 0,\ 1$) and $A_i^{BV(T)}(q^2)$ ($i = 0,\ 1,\ 2,\ 3$) are the weak form factors. The form factors included in our calculations satisfy $F_1^{BP}(0) = F_0^{BP}(0)$, $A_3^{BV(T)}(0) = A_0^{BV(T)}(0)$, $A_3^{BV(T)}(q^2) = [(m_B + m_{V(T)})/(2m_{V(T)})]A_1^{BV(T)}(q^2) - [(m_B + m_{V(T)})/(2m_{V(T)})]A_2^{BV(T)}(q^2)$ and $F_1^{BS}(q^2) = F_0^{BS}(q^2)$.

The decay constants are defined as [25,26]

$$
\begin{aligned}
\langle P(p')|\hat{A}_\mu|0\rangle &= -if_P p'_\mu,\\
\langle V(p')|\hat{V}_\mu|0\rangle &= f_V m_V \epsilon_\mu^*,\\
\langle V(p')|\bar{q}\sigma_{\mu\nu}q'|0\rangle &= f_V^\perp (p'_\mu\epsilon_\nu^* - p'_\nu\epsilon_\mu^*) m_V,\\
\langle S(p')|\hat{V}_\mu|0\rangle &= f_S p'_\mu, \qquad \langle S(p')|\hat{S}|0\rangle = m_S \bar{f}_S,\\
\langle T(p')|J_{\mu\nu}(0)|0\rangle &= f_T m_T^2 \epsilon_{\mu\nu}^*,\\
\langle T(p')|J_{\mu\nu\alpha}^\perp(0)|0\rangle &= -if_T^\perp (p'_\nu\epsilon_{\mu\alpha}^* - p'_\mu\epsilon_{\mu\alpha}^*) m_T,
\end{aligned} \tag{2}
$$

where $J_{\mu\nu}(0)$ and $J_{\mu\nu\alpha}^\perp(0)$ are local currents involving covariant derivatives which take the following forms:

$$
\begin{aligned}
J_{\mu\nu}(0) &= \frac{1}{2}(\bar{q}_1(0)\gamma_\mu i\overleftrightarrow{D}_\nu q_2(0) + \bar{q}_1(0)\gamma_\nu i\overleftrightarrow{D}_\mu q_2(0)),\\
J_{\mu\nu\alpha}^\perp(0) &= \bar{q}_1(0)\sigma_{\mu\nu} i\overleftrightarrow{D}_\alpha q_2(0),
\end{aligned} \tag{3}
$$

and $\overleftrightarrow{D} = \vec{D}_\mu - \overleftarrow{D}_\mu$ with $\vec{D}_\mu = \vec{\partial}_\mu + ig_s A_\mu^a\lambda^a/2$ and $\overleftarrow{D}_\mu = \overleftarrow{\partial}_\mu - ig_s A_\mu^a\lambda^a/2$ (g_s is the QCD coupling constant, A_μ^a is the vector field and λ^a are the Gellman matrices).

The twist-2 light-cone distribution amplitudes for the pseudoscalar, vector and tensor mesons are respectively [10,25]

$$
\Phi_M(x,\mu) = 6x(1-x)\left[\sum_{m=0}^{\infty} \alpha_m^M(\mu) C_m^{3/2}(2x-1)\right], \qquad M = P, V, T \tag{4}
$$

and the twist-3 ones are respectively

$$
\Phi_m(x) = \begin{cases} 1 & m = p,\\ 3\left[2x - 1 + \sum_{m=1}^{\infty} \alpha_{m,\perp}^V(\mu) P_{m+1}(2x-1)\right] & m = v,\\ 5(1 - 6x + 6x^2), & m = t, \end{cases} \tag{5}
$$

where $C_m^{3/2}$ and P_m are the Gegenbauer and Legendre polynomials in Eqs. (4) and (5), respectively, $\alpha_m(\mu)$ are Gegenbauer moments which depend on the scale μ.

The twist-2 light-cone distribution amplitude for a scalar meson reads [11,27]

$$
\Phi_S(x,\mu)^{(n,s)} = \bar{f}_S^{n,s} 6x(x-1) \sum_{m=1,3,5}^{\infty} B_m(\mu) C_m^{3/2}(2x-1), \tag{6}
$$

where B_m are Gegenbauer moments, $\bar{f}_S$ is the decay constant of the scalar mesons, n denotes the u, d quark component of the scalar meson, $n = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$, and s denotes the components $s\bar{s}$. As for the twist-3 ones, we shall take the asymptotic forms [11,27]

$$\Phi_s(x)^{(n,s)} = \bar{f}_S^{n,s}. \tag{7}$$

III. B DECAYS IN QCD FACTORIZATION

In the SM, the effective weak Hamiltonian for nonleptonic B-meson decays is given by [28]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}}\left[\sum_{p=u,c}\sum_{D=d,s}\lambda_p^{(D)}\left(c_1 O_1^p + c_2 Q_2^p + \sum_{i=3}^{10} c_i O_i + c_{7\gamma}O_{7\gamma} + c_{8g}O_{8g}\right)\right] + \text{H.c.}, \tag{8}$$

where $\lambda_p^{(D)} = V_{pb}V_{pD}^*$, V_{pb} and V_{pD} are the CKM matrix elements, G_F represents the Fermi constant, c_i ($i = 1$–10, 7γ, $8g$) are Wilson coefficients, $O_{1,2}^p$ are the tree level operators, O_{3-6} are the QCD penguin operators, O_{7-10} arise from electroweak penguin diagrams, and $O_{7\gamma}$ and O_{8g} are the electromagnetic and chromomagnetic dipole operators, respectively.

Within the framework of QCD factorization [10,12], the effective Hamiltonian matrix elements are written in the form

$$\langle M_1M_2|\mathcal{H}_{\text{eff}}|B\rangle = \sum_{p=u,c}\lambda_p^{(D)}\langle M_1M_2|\mathcal{T}_A^p + \mathcal{T}_B^p|B\rangle, \tag{9}$$

where $\mathcal{T}_A^p$ describes the contribution from naive factorization, vertex correction, penguin amplitude and spectator scattering expressed in terms of the parameters a_i^p, while $\mathcal{T}_B^p$ contains annihilation topology amplitudes characterized by the annihilation parameters b_i^p.

The flavor parameters a_i^p are basically the Wilson coefficients in conjunction with short-distance nonfactorizable corrections such as vertex corrections and hard spectator interactions. In general, they have the expressions [10]

$$a_i^p(M_1M_2) = \left(c_i' + \frac{c_{i\pm1}'}{N_c}\right)N_i(M_2) + \frac{c_{i\pm1}'}{N_c}\frac{C_F\alpha_s}{4\pi}\left[V_i(M_2) + \frac{4\pi^2}{N_c}H_i(M_1M_2)\right] + P_i^p(M_2), \tag{10}$$

where c_i' are effective Wilson coefficients which are defined as $c_i(m_b)\langle O_i(m_b)\rangle = c_i'\langle O_i\rangle^{\text{tree}}$, with $\langle O_i\rangle^{\text{tree}}$ being the matrix element at the tree level, the upper (lower) signs apply when i is odd (even), $N_i(M_2)$ is leading-order coefficient, $C_F = (N_c^2 - 1)/2N_c$ with $N_c = 3$, the quantities $V_i(M_2)$ account for one-loop vertex corrections, $H_i(M_1M_2)$ describe hard spectator interactions with a hard gluon exchange between the emitted meson and the spectator quark of the B meson, and $P_i^p(M_1M_2)$ are from penguin contractions [10].

The expressions of the quantities $N_i(M_2)$ read [10,25]

$$N_i(V) = \begin{cases} 0 & i = 6, 8, \\ 1 & \text{else}, \end{cases} \qquad N_i(P) = 1, \qquad N_i(T) = 0, \tag{11}$$

and $N_i(S) = 1$ for charged scalar mesons, while for neutral scalar mesons [29]

$$N_i(S) = \begin{cases} 1 & i = 6, 8, \\ 0 & \text{else}. \end{cases} \tag{12}$$

When $M_1M_2 = VP, PV$, the correction from the hard gluon exchange between M_2 and the spectator quark is given by [10,12]

$$H_i(M_1M_2) = \frac{f_B f_{M_1}}{2m_V\epsilon_V^*\cdot p_B F_0^{B\to M_1}(0)}\int_0^1\frac{d\xi}{\xi}\Phi_B(\xi)\int_0^1 dx\int_0^1 dy\left[\frac{\Phi_{M_2}(x)\Phi_{M_1}(y)}{\bar{x}\bar{y}} + r_\chi^{M_1}\frac{\Phi_{M_2}(x)\Phi_{m_1}(y)}{x\bar{y}}\right], \tag{13}$$

for $i = 1$–4, 9, 10,

$$H_i(M_1M_2) = -\frac{f_B f_{M_1}}{2m_V\epsilon_V^*\cdot p_B F_0^{B\to M_1}(0)}\int_0^1\frac{d\xi}{\xi}\Phi_B(\xi)\int_0^1 dx\int_0^1 dy\left[\frac{\Phi_{M_2}(x)\Phi_{M_1}(y)}{x\bar{y}} + r_\chi^{M_1}\frac{\Phi_{M_2}(x)\Phi_{m_1}(y)}{\bar{x}\bar{y}}\right], \tag{14}$$

for $i = 5$, 7 and $H_i(M_1M_2) = 0$ for $i = 6$, 8.

When $M_1M_2 = SP, PS$ [10,11,27],

$$H_i(M_1M_2) = \frac{f_B f_{M_1}}{f_{M_2}F_0^{B\to M_1}m_B^2}\int_0^1\frac{d\xi}{\xi}\Phi_B(\xi)\int_0^1 dx\int_0^1 dy\left[\frac{\Phi_{M_2}(x)\Phi_{M_1}(y)}{\bar{x}\bar{y}} + r_\chi^{M_1}\frac{\Phi_{M_2}(x)\Phi_{m_1}(y)}{x\bar{y}}\right], \tag{15}$$

for $i = 1$–4, 9, 10,

$$H_i(M_1M_2) = -\frac{f_B f_{M_1}}{f_{M_2}F_0^{B\to M_1}m_B^2}\int_0^1 \frac{d\xi}{\xi}\Phi_B(\xi)\int_0^1 dx\int_0^1 dy\left[\frac{\Phi_{M_2}(x)\Phi_{M_1}(y)}{x\bar{y}} + r_\chi^{M_1}\frac{\Phi_{M_2}(x)\Phi_{m_1}(y)}{\bar{x}\bar{y}}\right], \tag{16}$$

for $i = 5, 7$ and $H_i(M_1M_2) = 0$ for $i = 6, 8$.

When $M_1M_2 = TP, PT$ [25,30]

$$H_i(M_1M_2) = \frac{f_B f_{M_1}}{2m_B p_c}\int_0^1 \frac{d\xi}{\xi}\Phi_B(\xi)\int_0^1 dx\int_0^1 dy \times \begin{cases} \frac{m_{M_1}}{2\sqrt{\frac{2}{3}}p_c A_0^{B\to M_1}(m_{M_2}^2)}\left[\sqrt{\frac{2}{3}}\frac{\Phi_{M_2}(x)\Phi_{M_1}(y)}{\bar{x}\bar{y}} + r_\chi^{M_1}\frac{\Phi_{M_2}(x)\Phi_{m_1}(y)}{\sqrt{\frac{2}{3}}x\bar{y}}\right], & (M_1M_2 = TP) \\ \frac{1}{F_1^{B\to M_1}(m_{M_2}^2)}\left[\frac{\Phi_{M_2}(x)\Phi_{M_1}(y)}{\bar{x}\bar{y}} + r_\chi^{M_1}\frac{\Phi_{M_2}(x)\Phi_{m_1}(y)}{x\bar{y}}\right], & (M_1M_2 = PT) \end{cases} \tag{17}$$

for $i = 1$–4, 9, 10,

$$H_i(M_1M_2) = -\frac{f_B f_{M_1}}{2m_B p_c}\int_0^1 \frac{d\xi}{\xi}\Phi_B(\xi)\int_0^1 dx\int_0^1 dy \times \begin{cases} \frac{m_{M_1}}{2\sqrt{\frac{2}{3}}p_c A_0^{B\to M_1}(m_{M_2}^2)}\left[\sqrt{\frac{2}{3}}\frac{\Phi_{M_2}(x)\Phi_{M_1}(y)}{x\bar{y}} + r_\chi^{M_1}\frac{\Phi_{M_2}(x)\Phi_{m_1}(y)}{\sqrt{\frac{2}{3}}\bar{x}\bar{y}}\right], & (M_1M_2 = TP) \\ \frac{1}{F_1^{B\to M_1}(m_{M_2}^2)}\left[\frac{\Phi_{M_2}(x)\Phi_{M_1}(y)}{x\bar{y}} + r_\chi^{M_1}\frac{\Phi_{M_2}(x)\Phi_{m_1}(y)}{\bar{x}\bar{y}}\right], & (M_1M_2 = PT) \end{cases} \tag{18}$$

for $i = 5, 7$ and $H_i(M_1M_2) = 0$ for $i = 6, 8$.

In Eqs. (13)–(18) $\bar{x} = 1 - x$, $\bar{y} = 1 - y$, and $r_\chi^{M_i}$ $(i = 1, 2)$ are "chirally enhanced" terms which are defined as

$$r_\chi^P(\mu) = \frac{2m_P^2}{m_b(\mu)(m_{q_1} + m_{q_2})(\mu)}, \qquad r_\chi^{V,T} = \frac{2m_{V,T}}{m_b(\mu)}\frac{f_{V,T}^\perp(\mu)}{f_{V,T}},$$
$$r_\chi^S = \frac{2m_S}{m_b(\mu)}\frac{\bar{f}_S(\mu)}{f_S} = \frac{2m_S^2}{m_b(\mu)(m_2(\mu) - m_1(\mu))}, \qquad \bar{r}_\chi^S = \frac{2m_S}{m_b(\mu)}. \tag{19}$$

The weak annihilation contributions to $B \to M_1M_2$ can be described in terms of b_i and $b_{i,\mathrm{EW}}$, which have the following expressions:

$$b_1 = \frac{C_F}{N_c^2}c_1'A_1^i, \qquad b_2 = \frac{C_F}{N_c^2}c_2'A_1^i,$$
$$b_3^p = \frac{C_F}{N_c^2}[c_3'A_1^i + c_5'(A_3^i + A_3^f) + N_c c_6'A_3^f], \qquad b_4^p = \frac{C_F}{N_c^2}[c_4'A_1^i + c_6'A_2^i],$$
$$b_{3,\mathrm{EW}}^p = \frac{C_F}{N_c^2}[c_9'A_1^i + C_7'(A_3^i + A_3^f) + N_c c_8'A_3^f], \qquad b_{4,\mathrm{EW}}^p = \frac{C_F}{N_c^2}[c_{10}'A_1^i + c_8'A_2^i], \tag{20}$$

where the subscripts 1, 2, 3 of $A_n^{i,f}$ $(n = 1, 2, 3)$ stand for the annihilation amplitudes induced from $(V - A)(V - A)$, $(V - A)(V + A)$, and $(S - P)(S + P)$ operators, respectively, the superscripts i and f refer to gluon emission from the initial- and final-state quarks, respectively. Their explicit expressions are given by [10,11,25,27,30]

$$A_1^i = \pi\alpha_s \int_0^1 dxdy \begin{cases} \left(\Phi_{M_2}(x)\Phi_{M_1}(y)\left[\frac{1}{y(1-x\bar{y})}+\frac{1}{\bar{x}^2 y}\right] - r_\chi^{M_1} r_\chi^{M_2}\Phi_{m_2}(x)\Phi_{m_1}(y)\frac{2}{\bar{x}y}\right), & \text{for } M_1M_2 = VP, PS, \\ \left(\Phi_{M_2}(x)\Phi_{M_1}(y)\left[\frac{1}{y(1-x\bar{y})}+\frac{1}{\bar{x}^2 y}\right] + r_\chi^{M_1} r_\chi^{M_2}\Phi_{m_2}(x)\Phi_{m_1}(y)\frac{2}{\bar{x}y}\right), & \text{for } M_1M_2 = PV, SP, \\ \sqrt{\frac{2}{3}}\left(\Phi_{M_2}(x)\Phi_{M_1}(y)\left[\frac{1}{y(1-x\bar{y})}+\frac{1}{\bar{x}^2 y}\right] - \frac{3}{2} r_\chi^{M_1} r_\chi^{M_2}\Phi_{m_2}(x)\Phi_{m_1}(y)\frac{2}{\bar{x}y}\right), & \text{for } M_1M_2 = TP, \\ \sqrt{\frac{2}{3}}\left(\Phi_{M_2}(x)\Phi_{M_1}(y)\left[\frac{1}{y(1-x\bar{y})}-\frac{1}{\bar{x}^2 y}\right] + \frac{3}{2} r_\chi^{M_1} r_\chi^{M_2}\Phi_{m_2}(x)\Phi_{m_1}(y)\frac{2}{\bar{x}y}\right), & \text{for } M_1M_2 = PT, \end{cases}$$

$$A_2^i = \pi\alpha_s \int_0^1 dxdy \begin{cases} \left(-\Phi_{M_2}(x)\Phi_{M_1}(y)\left[\frac{1}{\bar{x}(1-x\bar{y})}+\frac{1}{\bar{x}y^2}\right] + r_\chi^{M_1} r_\chi^{M_2}\Phi_{m_2}(x)\Phi_{m_1}(y)\frac{2}{\bar{x}y}\right), & \text{for } M_1M_2 = VP, PS, \\ \left(-\Phi_{M_2}(x)\Phi_{M_1}(y)\left[\frac{1}{\bar{x}(1-x\bar{y})}+\frac{1}{\bar{x}y^2}\right] - r_\chi^{M_1} r_\chi^{M_2}\Phi_{m_2}(x)\Phi_{m_1}(y)\frac{2}{\bar{x}y}\right), & \text{for } M_1M_2 = PV, SP, \\ \sqrt{\frac{2}{3}}\left(\Phi_{M_2}(x)\Phi_{M_1}(y)\left[\frac{1}{\bar{x}(1-x\bar{y})}+\frac{1}{\bar{x}y^2}\right] + \frac{3}{2} r_\chi^{M_1} r_\chi^{M_2}\Phi_{m_2}(x)\Phi_{m_1}(y)\frac{2}{\bar{x}y}\right), & \text{for } M_1M_2 = TP, \\ \sqrt{\frac{2}{3}}\left(\Phi_{M_2}(x)\Phi_{M_1}(y)\left[\frac{1}{\bar{x}(1-x\bar{y})}+\frac{1}{\bar{x}y^2}\right] - \frac{3}{2} r_\chi^{M_1} r_\chi^{M_2}\Phi_{m_2}(x)\Phi_{m_1}(y)\frac{2}{\bar{x}y}\right), & \text{for } M_1M_2 = PT, \end{cases}$$

$$A_3^i = \pi\alpha_s \int_0^1 dxdy \begin{cases} \left(r_\chi^{M_1}\Phi_{M_2}(x)\Phi_{m_1}(y)\frac{2\bar{y}}{\bar{x}y(1-x\bar{y})} + r_\chi^{M_2}\Phi_{M_1}(y)\Phi_{m_2}(x)\frac{2x}{\bar{x}y(1-x\bar{y})}\right), & \text{for } M_1M_2 = VP, PS, \\ \left(-r_\chi^{M_1}\Phi_{M_2}(x)\Phi_{m_1}(y)\frac{2\bar{y}}{\bar{x}y(1-x\bar{y})} + r_\chi^{M_2}\Phi_{M_1}(y)\Phi_{m_2}(x)\frac{2x}{\bar{x}y(1-x\bar{y})}\right), & \text{for } M_1M_2 = PV, SP, \\ \sqrt{\frac{2}{3}}\left(\frac{3}{2} r_\chi^{M_1}\Phi_{M_2}(x)\Phi_{m_1}(y)\frac{2\bar{y}}{\bar{x}y(1-x\bar{y})} + r_\chi^{M_2}\Phi_{M_1}(y)\Phi_{m_2}(x)\frac{2x}{\bar{x}y(1-x\bar{y})}\right), & \text{for } M_1M_2 = TP, PT, \end{cases}$$

$$A_3^f = \pi\alpha_s \int_0^1 dxdy \begin{cases} \left(r_\chi^{M_1}\Phi_{M_2}(x)\Phi_{m_1}(y)\frac{2(1+\bar{x})}{\bar{x}^2 y} - r_\chi^{M_2}\Phi_{M_1}(y)\Phi_{m_2}(x)\frac{2(1+y)}{\bar{x}y^2}\right), & \text{for } M_1M_2 = VP, PS, \\ \left(-r_\chi^{M_1}\Phi_{M_2}(x)\Phi_{m_1}(y)\frac{2(1+\bar{x})}{\bar{x}^2 y} - r_\chi^{M_2}\Phi_{M_1}(y)\Phi_{m_2}(x)\frac{2(1+y)}{\bar{x}y^2}\right), & \text{for } M_1M_2 = PV, SP, \\ \sqrt{\frac{2}{3}}\left(\frac{3}{2} r_\chi^{M_1}\Phi_{M_2}(x)\Phi_{m_1}(y)\frac{2(1+\bar{x})}{\bar{x}^2 y} - r_\chi^{M_2}\Phi_{M_1}(y)\Phi_{m_2}(x)\frac{2(1+y)}{\bar{x}y^2}\right), & \text{for } M_1M_2 = TP, PT, \end{cases}$$

$$A_1^i = A_2(M_1M_2)^f = 0. \tag{21}$$

When dealing with the weak annihilation contributions and the hard spectator contributions, one has to deal with the infrared end-point singularity $X = \int_0^1 dx/(1-x)$. The treatment of this end-point divergence is model dependent, and we follow Ref. [10] to parametrize this end-point divergence in the annihilation and hard spectator diagrams as

$$X_{A,H}^{M_1M_2} = (1 + \rho_{A,H}^{M_1M_2} e^{i\phi_{A,H}^{M_1M_2}}) \ln\frac{m_B}{\Lambda_h}, \tag{22}$$

where Λ_h is a typical scale of order 0.5 GeV, $\rho_{A(H)}^{M_1M_2}$ is an unknown real parameter and $\phi_{A(H)}^{M_1M_2}$ is a free strong phase in the range $[0, 2\pi]$ for the annihilation (hard spectator) process.

In our work, we will follow the assumption $X_H^{M_1M_2} = X_A^{M_1M_2} = X^{M_1M_2}$ for the $B \to PV(PT)$ decays [25,31,32], but for the $B \to SP$ decays, we will further assume that $X^{M_1M_2} = X^{M_2M_1}$ compared with the $B \to PV(PT)$ decays.

IV. CALCULATION OF *CP* VIOLATION

A. FRAMEWORK

1. Nonresonance background

In the absence of resonances, the factorizable nonresonance amplitude for the $B^- \to K^-\pi^+\pi^-$ decay has the expression [6,33]

$$A_{\mathrm{NR}} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^s [\langle \pi^+\pi^- | (\bar{u}b)_{V-A} | B^- \rangle \langle K^- | (\bar{s}u)_{V-A} | 0 \rangle [a_1 \delta_{pu} + a_4^p + a_{10}^p - (a_6^p + a_8^p) r_\chi^K] + \langle \pi^- | \bar{d}b | B^- \rangle \langle K^-\pi^+ | \bar{s}d | 0 \rangle (-2a_6^p + 2a_8^p)]. \tag{23}$$

For the parameters a_i which contain effective Wilson coefficients, we take the following values [6,33]:

$$\begin{aligned} &a_1 = 0.99 \pm 0.037i, \quad a_2 = 0.19 - 0.11i, \quad a_3 = -0.002 + 0.004i, \quad a_5 = 0.0054 - 0.005i, \\ &a_4^u = -0.03 - 0.02i, \quad a_4^c = -0.04 - 0.008i, \quad a_6^u = -0.006 - 0.02i, \quad a_6^c = -0.006 - 0.006i, \\ &a_7 = 0.54 \times 10^{-4} i, \quad a_8^u = (4.5 - 0.5i) \times 10^{-4}, \quad a_8^c = (4.4 - 0.3i) \times 10^{-4}, \quad a_9 = -0.010 - 0.0002i, \\ &a_{10}^u = (-58.3 + 86.1i) \times 10^{-5}, \quad a_{10}^c = (-60.3 + 88.8i) \times 10^{-5}. \end{aligned} \tag{24}$$

For the current-induced process, the amplitude $\langle \pi^+\pi^- | (\bar{u}b)_{V-A} | B^- \rangle \langle K^- | (\bar{s}u)_{V-A} | 0 \rangle$ can be expressed in terms of three unknown form factors [6,33,34]

$$\begin{aligned} A_{\text{current-ind}}^{\mathrm{HMChPT}} &\equiv \langle \pi^+(p_1)\pi^-(p_2) | (\bar{u}b)_{V-A} | B^- \rangle \langle K^-(p_3) | (\bar{s}u)_{V-A} | 0 \rangle \\ &= -\frac{f_\pi}{2} [2m_3^2 r + (m_B^2 - s_{12} - m_3^2)\omega_+ + (s_{23} - s_{13} - m_2^2 + m_1^2)\omega_-], \end{aligned} \tag{25}$$

where r, $\omega_\pm$, and h are form factors which can be evaluated in the framework of HMChPT and the results read [34,35]

$$\begin{aligned} \omega_+ &= -\frac{g}{f_\pi^2} \frac{f_{B^*} m_{B^*} \sqrt{m_B m_{B^*}}}{s_{23} - m_{B^*}^2} \left[1 - \frac{(p_B - p_1)\cdot p_1}{m_{B^*}^2}\right] + \frac{f_B}{2f_\pi^2}, \\ \omega_- &= \frac{g}{f_\pi^2} \frac{f_{B^*} m_{B^*} \sqrt{m_B m_{B^*}}}{s_{23} - m_{B^*}^2} \left[1 + \frac{(p_B - p_1)\cdot p_1}{m_{B^*}^2}\right], \\ r &= \frac{f_B}{2f_\pi^2} - \frac{f_B}{f_\pi^2} \frac{p_B \cdot (p_2 - p_1)}{(p_B - p_1 - p_2)^2 - m_{B^2}} + \frac{2g f_{B^*}}{f_\pi^2} \sqrt{\frac{m_B}{m_{B^*}}} \frac{(p_B - p_1)\cdot p_1}{s_{23} - m_{B^*}^2} - \frac{4g^2 f_B}{f_\pi^2} \frac{m_B m_{B^*}}{(p_B - p_1 - p_2)^2 - m_B^2} \\ &\quad \times \frac{p_1 \cdot_2 - p_1 \cdot (p_B - p_1) p_2 \cdot (p_B - p_1)/m_{B^*}^2}{s_{23} - m_{B^*}^2}, \end{aligned} \tag{26}$$

where $s_{ij} \equiv (p_i + p_j)^2$, g is a heavy-flavor-independent strong coupling which can be extracted from the CLEO measurement of the D^{*+} decay width, $|g| = 0.59 \pm 0.01 \pm 0.07$ [36], which sign is fixed to be negative in Ref. [3].

However, the predicted nonresonance results based on HMChPT are not recovered in the soft meson region and lead to decay rates that are too large which are in disagreement with experiment [37]. For example, the branching fraction is found to be of order 7.5×10^{-5}, which is 1 order of magnitude larger than the *BABAR* result, 5.3×10^{-6} [38]. The issue is related to the applicability HMChPT, which requires the two mesons in the final state in the $B \to M_1 M_2$ transition have to be soft and hence an exponential form of the amplitudes is necessary [33,39],

$$A_{\text{current-ind}} = A_{\text{current-ind}}^{\mathrm{HMChPT}} e^{-\alpha_{\mathrm{NR}} p_B \cdot (p_1 + p_2)} e^{i\phi_{12}}, \tag{27}$$

where α_{NR} is constrained from the tree dominated decay $B^- \to \pi^+\pi^-\pi^-$ to be $\alpha_{\mathrm{NR}} = 0.081^{+0.015}_{-0.009}\ \mathrm{GeV}^{-2}$, and the phase ϕ_{12} of the nonresonant amplitude will be set to zero for simplicity [33,39].

The matrix element of $\langle K^-\pi^+ | \bar{s}d | 0 \rangle^{\mathrm{NR}}$ is related to $\langle K^+K^- | \bar{s}s | 0 \rangle^{\mathrm{NR}}$ via SU(3) symmetry, i.e., $\langle K^-\pi^+ | \bar{s}d | 0 \rangle^{\mathrm{NR}} = \langle K^+K^- | \bar{s}s | 0 \rangle^{\mathrm{NR}}$, we shall adopt Ref. [6] to assume that final state interactions amount to giving a large strong phase δ to the nonresonance component of the matrix element of $\langle K^-\pi^+ | \bar{s}d | 0 \rangle^{\mathrm{NR}}$ and a fit to the data of direct CP asymmetries in $B^- \to K^-\pi^+\pi^-$ yields

$$\begin{aligned} &\langle K^-(p_1)\pi^+(p_2) | \bar{s}d | 0 \rangle^{\mathrm{NR}} \\ &\quad = \frac{\nu}{3}(3F_{\mathrm{NR}} + 2F'_{\mathrm{NR}}) + \sigma_{\mathrm{NR}} e^{-\alpha s_{12}} e^{i\delta} \\ &\quad \approx \frac{\nu}{3}(3F_{\mathrm{NR}} + 2F'_{\mathrm{NR}}) + \sigma_{\mathrm{NR}} e^{-\alpha s_{12}} e^{i\pi} \left(1 + 4\frac{m_K^2 - m_\pi^2}{s_{12}}\right), \end{aligned} \tag{28}$$

where the parameter $\sigma_{\mathrm{NR}} = (3.39^{+0.18}_{-0.21}) e^{i\pi/4}$ GeV, and $\nu = \frac{m_{K^+}^2}{m_u + m_s} = \frac{m_K^2 - m_\pi^2}{m_s - m_d}$ characterizes the quark-operator parameter

$\langle\bar{q}q\rangle$ which spontaneously breaks the chiral symmetry and the experimental measurement leads to $\alpha = (0.14 \pm 0.02)\ \mathrm{GeV}^{-2}$ [40]. Motivated by the asymptotic constraints from pQCD, namely, $F(t)^{(\prime)} \to (1/t)[\ln(t/\tilde{\Lambda}^2)]^{-1}$ in the large-t limit [41], the nonresonance form factors in Eq. (28) can be parametrized as [6]

$$F_{\mathrm{NR}}(s_{23}) = \left(\frac{x_1}{s_{23}} + \frac{x_2}{s_{23}^2}\right)\left[\ln\left(\frac{s_{23}}{\tilde{\Lambda}^2}\right)\right]^{-1},$$
$$F'_{\mathrm{NR}}(s_{23}) = \left(\frac{x'_1}{s_{23}} + \frac{x'_2}{s_{23}^2}\right)\left[\ln\left(\frac{s_{23}}{\tilde{\Lambda}^2}\right)\right]^{-1}, \tag{29}$$

where $\tilde{\Lambda} \approx 0.3$ GeV is the QCD scale parameter, the unknown parameters x_i and x'_i are fitted from the kaon electromagnetic data, giving the following best-fit values [42]:

$$x_1 = -3.26\ \mathrm{GeV}^2, \qquad x_2 = 5.02\ \mathrm{GeV}^4,$$
$$x'_1 = 0.47\ \mathrm{GeV}^2, \qquad x'_2 = 0. \tag{30}$$

2. Resonance contributions

LHCb has observed large CP asymmetries in localized regions of phase space $m^2_{K^-\pi^+} < 15\ \mathrm{GeV}^2$ and $0.08 < m^2_{\pi^+\pi^-} < 0.66\ \mathrm{GeV}^2$ [7,8], which contains the $[\pi\pi]$ and $[K\pi]$ channel resonances including $\sigma(600)$, $\rho^0(770)$, $\omega(782)$, $K_0^*(700)$, $K_0^*(1430)$, $K_2^*(1430)$ and $(K^*)^i$ [$K^*(892)$, $K^*(1410)$, $K^*(1680)$ for $i = 1, 2, 3$] which will be denoted as σ, ρ, ω, κ, K_0^*, K_2^* and $(K^*)^i$ for simplicity, respectively. The total resonance amplitude including the $\rho-\omega$ mixing effect can be written as [6,43]

$$\sum_R A_R = A_\sigma + A_{\rho,\omega} + A_\kappa + \sum_i A_{(K^*)^i} + A_{K_0^*} + A_{K_2^*}$$
$$= A_{[\pi\pi]} + A_{[K\pi]}, \tag{31}$$

where the sum over R refers to that over the aforementioned resonances including the $\rho-\omega$ mixing effect.

$\rho-\omega$ mixing has the dual advantages that the strong phase difference is large and well known [44,45]. In order to deal with the large localized CP violation, we need to appeal this mechanism to the $B^- \to K^-\pi^+\pi^-$ decay. In this scenario one has [46–48]

$$A_{\rho,\omega} = \langle K^-\pi^+\pi^-|\mathcal{H}^T|B^-\rangle + \langle K^-\pi^+\pi^-|\mathcal{H}^P|B^-\rangle$$
$$= \epsilon^\lambda\cdot(p_{\pi^-} - p_{\pi^+})\left[\left(\frac{g_\rho}{s_\rho s_\omega}\tilde{\Pi}_{\rho\omega}t_\omega + \frac{g_\rho}{s_\rho}t_\rho\right)\right.$$
$$\left. + \left(\frac{g_\rho}{s_\rho s_\omega}\tilde{\Pi}_{\rho\omega}p_\omega + \frac{g_\rho}{s_\rho}p_\rho\right)\right], \tag{32}$$

where $\mathcal{H}^T$ and $\mathcal{H}^P$ are the Hamiltonians for the tree and penguin operators, respectively, t_V ($V = \rho$ or ω) is the tree amplitude and p_V is the penguin amplitude for producing an intermediate vector meson V, g_ρ is the coupling for $\rho\to\pi^+\pi^-$, $\tilde{\Pi}_{\rho\omega}$ is the effective $\rho-\omega$ mixing amplitude, and s_V is from the inverse propagator of the vector meson V, $s_V = s - m_V^2 + i m_V\Gamma_V$ and $\sqrt{s}$ is the invariant mass of the $\pi^+\pi^-$ pair. The direct coupling $\omega\to\pi^+\pi^-$ is effectively absorbed into $\tilde{\Pi}_{\rho\omega}$ [49], leading to the explicit s dependence of $\tilde{\Pi}_{\rho\omega}$. Making the expansion $\tilde{\Pi}_{\rho\omega}(s) = \tilde{\Pi}_{\rho\omega}(m_\omega^2) + (s - m_\omega^2)\tilde{\Pi}'_{\rho\omega}(m_\omega^2)$, the $\rho-\omega$ mixing parameters were determined in the fit of Gardner and O'Connell [50]: $\mathfrak{Re}\tilde{\Pi}_{\rho\omega}(m_\omega^2) = -3500 \pm 300\ \mathrm{MeV}^2$, $\mathfrak{Im}\tilde{\Pi}_{\rho\omega}(m_\omega^2) = -300 \pm 300\ \mathrm{MeV}^2$, $\tilde{\Pi}'_{\rho\omega}(m_\omega^2) = 0.03 \pm 0.04$. In practice, the effect of the derivative term is negligible.

Because of its large width, σ cannot be modeled by a naive Breit-Wigner distribution. In this paper, we will adopt the Bugg model to parametrize the distribution of σ which is given by [51–53]

$$R_\sigma(s) = M\Gamma_1(s)\Big/\left[M^2 - s - g_1^2(s)\frac{s - s_A}{M^2 - s_A}z(s) - iM\Gamma_{\mathrm{tot}}(s)\right], \tag{33}$$

where $z(s) = j_1(s) - j_1(M^2)$ with $j_1(s) = \frac{1}{\pi}[2 + \rho_1\ln(\frac{1-\rho_1}{1+\rho_1})]$, $\Gamma_{\mathrm{tot}}(s) = \sum_{i=1}^4\Gamma_i(s)$ and

$$M\Gamma_1(s) = g_1^2(s)\frac{s - s_A}{M^2 - s_A}\rho_1(s),$$
$$M\Gamma_2(s) = 0.6 g_1^2(s)(s/M^2)\exp(-\alpha|s - 4m_K^2|)\rho_2(s),$$
$$M\Gamma_3(s) = 0.2 g_1^2(s)(s/M^2)\exp(-\alpha|s - 4m_\eta^2|)\rho_3(s),$$
$$M\Gamma_4(s) = M g_{4\pi}\rho_{4\pi}(s)/\rho_{4\pi}(M^2),$$
$$g_1^2(s) = M(c_1 + c_2 s)\exp[-(s - M^2)/A],$$
$$\rho_{4\pi}(s) = 1.0/[1 + \exp(7.082 - 2.845s)]. \tag{34}$$

The parameters in Eqs. (33) and (34) are fixed to be $M = 0.953$ GeV, $s_A = 0.14 m_\pi^2$, $c_1 = 1.302\ \mathrm{GeV}^2$, $c_2 = 0.340$, $A = 2.426\ \mathrm{GeV}^2$ and $g_{4\pi} = 0.011$ GeV, which are given in the fourth column of Table I in Ref. [51]. The parameters $\rho_{1,2,3}$ are the phase-space factors of the decay channels $\pi\pi$, KK and $\eta\eta$, respectively, which are defined as [51]

$$\rho_i(s) = \sqrt{1 - 4\frac{m_i^2}{s}}, \tag{35}$$

with $m_1 = m_\pi$, $m_2 = m_K$ and $m_3 = m_\eta$. Other resonants in Eq. (31) will be modeled by the naive Breit-Wigner distribution.

Within the QCDF, we derive the tree and penguin amplitudes of ρ and ω in Eq. (32) and obtain

$$t_\rho = -iG_F m_\rho \epsilon_\rho^*\cdot p_B\lambda_u^{(s)}[\alpha_1(\rho K)A_0^{B\to\rho}(0)f_K$$
$$+ \alpha_2(K\rho)F_0^{B\to K}(0)f_\rho + b_2(\rho K)f_B f_\rho f_K], \tag{36}$$

$$t_{\omega} = -iG_F m_{\omega} \epsilon_{\omega}^{*} \cdot p_B \lambda_u^{(s)} [\alpha_1(\omega K) A_0^{B\to\omega}(0) f_K + \alpha_2(K\omega) F_0^{B\to K}(0) f_{\omega} + b_2(\omega K) f_B f_{\omega} f_K], \tag{37}$$

$$p_{\rho} = -iG_F m_{\rho} \epsilon_{\rho}^{*} \cdot p_B \sum_{p=u,c} \lambda_p^{(s)} \bigg\{ [\alpha_4^p(\rho K) + \alpha_{4,\mathrm{EW}}^p(\rho K)] A_0^{B\to\rho}(0) f_K + \frac{3}{2} \alpha_{3,\mathrm{EW}}^p(K\rho) F_0^{B\to K}(0) f_{\rho} + [b_3^p(\rho K) - b_{3,\mathrm{EW}}^p(\rho K)] f_B f_{\rho} f_K \bigg\}, \tag{38}$$

$$p_{\omega} = -iG_F m_{\omega} \epsilon_{\omega}^{*} \cdot p_B \sum_{p=u,c} \lambda_p^{(s)} \bigg\{ \bigg[2\alpha_3(K\omega) + \frac{1}{2}\alpha_3^p(K\omega)\bigg] F_0^{B\to K}(0) f_{\omega} + \bigg[\alpha_4^p(\omega K) + \frac{3}{2}\alpha_{4,\mathrm{EW}}^p(\omega K)\bigg] \times A_0^{B\to\omega}(0) f_K + [b_3^p(\omega K) + b_{3,\mathrm{EW}}^p(\omega K)] f_B f_{\omega} f_K \bigg\}. \tag{39}$$

The polarization vectors of a vector meson V with mass m_V and momentum p satisfies

$$\sum_{\lambda=0,\pm1} \epsilon_{\mu}^{\lambda}(p)(\epsilon_{\nu}^{\lambda}(p))^{*} = -\bigg(g_{\mu\nu} - \frac{p_{\mu}p_{\nu}}{m_V^2}\bigg), \tag{40}$$

from which one obtains [54]

$$\sum_{\lambda=0,\pm1} \epsilon^{\lambda} \cdot (p_2 - p_3)(\epsilon^{\lambda})^{*} \cdot p_B = \hat{s}_{13} - s_{13}, \tag{41}$$

$\hat{s}_{13}$ is the midpoint of the allowed range of s_{13}, i.e., $\hat{s}_{13} = (s_{13,\mathrm{max}} + s_{13,\mathrm{min}})/2$, with $s_{13,\mathrm{max}}$ and $s_{13,\mathrm{min}}$ being the maximum and minimum values of s_{13} for fixed s_{12}.

As for the polarization vectors of a tensor meson we have [43]

$$\sum_{-2}^{2} \epsilon_{\alpha\beta}(\lambda) p_2^{\alpha} p_3^{\beta} \epsilon_{\mu\nu}^{*}(\lambda) p_B^{\nu} p_1^{\mu} = \frac{1}{3}(|\vec{p}_1||\vec{p}_2|)^2 - (\vec{p}_1 \cdot \vec{p}_2)^2, \tag{42}$$

where $\vec{p}_1$ and $\bar{p}_2$ are three momenta of $\pi^{-}(p_1)$ and $\pi^{+}(p_2)$, respectively, in the rest frame of $\pi^{+}(p_2)$ and $K^{-}(p_3)$. One obtains, with $m_{23} = \sqrt{s_{23}}$ [43],

$$\begin{aligned} |\vec{p}_1| &= \frac{1}{2m_{23}} \sqrt{[m_B^2 - (m_{23} + m_1)^2][m_B^2 - (m_{23} - m_1)^2]}, \\ |\vec{p}_2| &= \frac{1}{2m_{23}} \sqrt{[s_{23} - (m_3 + m_2)^2][s_{23} - (m_3 - m_2)^2]}, \\ \vec{p}_1 \cdot \vec{p}_2 &= s_{12} - s_{23} + \frac{(m_B^2 - m_1^2)(m_3^2 - m_2^2)}{s_{23}}. \end{aligned} \tag{43}$$

Inserting Eqs. (36)–(39) into Eq. (32), one can get the amplitude from $\rho - \omega$ mixing contribution,

$$\begin{aligned} A_{\rho,\omega} = {} & -iG_F(\hat{s}_{K\pi} - s_{K\pi}) \bigg\{ \frac{g_{\rho}}{s_{\rho}s_{\omega}} \tilde{\Pi}_{\rho\omega} [m_{\omega}\lambda_u^{(s)}(\alpha_1(\omega K) A_0^{B\to\omega}(0) f_K + \alpha_2(K\omega) F_0^{B\to K}(0) f_{\omega} + b_2(\omega K) f_B f_{\omega} f_K m_{\omega}/(m_B p_c))] \\ & + \frac{g_{\rho}}{s_{\rho}} [m_{\rho}\lambda_u^{(s)}(\alpha_1(\rho K) A_0^{B\to\rho}(0) f_K + \alpha_2(K\rho) F_0^{B\to K}(0) f_{\rho} + b_2(\rho K) f_B f_{\rho} f_K m_{\omega}/(m_B p_c))] \bigg\} \\ & + \bigg\{ \frac{g_{\rho}}{s_{\rho}s_{\omega}} \tilde{\Pi}_{\rho\omega} \times \bigg[m_{\omega} \sum_{p=u,c} \lambda_p^{(s)} \bigg\{ \bigg(2\alpha_3(K\omega) + \frac{1}{2}\alpha_3^p(K\omega)\bigg) F_0^{B\to K}(0) f_{\omega} + \bigg(\alpha_4^p(\omega K) + \frac{3}{2}\alpha_{4,\mathrm{EW}}^p(\omega K)\bigg) A_0^{B\to\omega}(0) f_K \\ & + (b_3^p(\omega K) + b_{3,\mathrm{EW}}^p(\omega K)) f_B f_{\omega} f_K m_{\omega}/(m_B p_c) \bigg\} \bigg] \\ & + \frac{g_{\rho}}{s_{\rho}} \bigg[m_{\rho} \sum_{p=u,c} \lambda_p^{(s)} \bigg\{ (\alpha_4^p(\rho K) + \alpha_{4,\mathrm{EW}}^p(\rho K)) A_0^{B\to\rho}(0) f_K + \frac{3}{2}\alpha_{3,\mathrm{EW}}^p(K\rho) F_0^{B\to K}(0) f_{\rho} \\ & + (b_3^p(\rho K) - b_{3,\mathrm{EW}}^p(\rho K)) f_B f_{\rho} f_K m_{\omega}/(m_B p_c) \bigg\} \bigg] \bigg\}, \end{aligned} \tag{44}$$

where p_c is the magnitude of the three momentum of either final state meson in the rest frame of the B meson, $\alpha_i^p(M_1M_2)$ can be expressed in terms of the coefficients a_i^p defined in Eq. (10) and have the following expressions:

$$
\begin{aligned}
\alpha_1(M_1M_2) &= a_1(M_1M_2),\\
\alpha_2(M_1M_2) &= a_2(M_1M_2),\\
\alpha_3^p(M_1M_2) &= \begin{cases} a_3^p(M_1M_2) - a_5^p(M_1M_2), & \text{if } M_1M_2 = VP, SP, TP,\\ a_3^p(M_1M_2) + a_5^p(M_1M_2), & \text{if } M_1M_2 = PV, PS, PT,\end{cases}\\
\alpha_4^p(M_1M_2) &= \begin{cases} a_4^p(M_1M_2) + r_\chi^{M_2} a_6^p(M_1M_2), & \text{if } M_1M_2 = PV, PT,\\ a_4^p(M_1M_2) - r_\chi^{M_2} a_6^p(M_1M_2), & \text{if } M_1M_2 = VP, PS, SP, TP,\end{cases}\\
\alpha_{3,\mathrm{EW}}^p(M_1M_2) &= \begin{cases} a_9^p(M_1M_2) - a_7^p(M_1M_2), & \text{if } M_1M_2 = VP, SP, TP,\\ a_9^p(M_1M_2) + a_7^p(M_1M_2), & \text{if } M_1M_2 = PV, PS, PT,\end{cases}\\
\alpha_{4,\mathrm{EW}}^p(M_1M_2) &= \begin{cases} a_{10}^p(M_1M_2) + r_\chi^{M_2} a_8^p(M_1M_2), & \text{if } M_1M_2 = PV, PT,\\ a_{10}^p(M_1M_2) - r_\chi^{M_2} a_8^p(M_1M_2), & \text{if } M_1M_2 = VP, PS, SP, TP.\end{cases}
\end{aligned}
\tag{45}
$$

Meanwhile, it is straightforward get the amplitudes contributed by others resonances, including σ, κ, $(K^*)^i$, K_0^* and K_2^*, respectively,

$$
\begin{aligned}
A_\sigma = -iG_F g_{\sigma\pi\pi} R_\sigma \sum_{p=u,c} \lambda_p^{(s)} \Big\{ & (m_\sigma^2 - m_B^2) F_0^{B\to\sigma}(m_K^2) f_K [\delta_{pu}\alpha_1(\sigma K) + \alpha_4^p(\sigma K) + \alpha_{4,\mathrm{EW}}^p(\sigma K)]\\
& - f_B f_K \bar{f}_\sigma^u [\delta_{pu} b_2(\sigma K) + b_3^p(\sigma K) + b_{3,\mathrm{EW}}^p(\sigma K)] + \left[\delta_{pu}\alpha_2(K\sigma) + 2\alpha_3^p(K\sigma) + \frac{1}{2}\alpha_{3,\mathrm{EW}}^p(K\sigma)\right]\\
& \times (m_B^2 - m_K^2) F_0^{B\to K}(0) \bar{f}_\sigma^u + \left[\sqrt{2}\alpha_3^p(K\sigma) + \sqrt{2}\alpha_4^p(K\sigma) - \frac{1}{\sqrt{2}}\alpha_{3,\mathrm{EW}}^p(K\sigma) - \frac{1}{\sqrt{2}}\alpha_{4,\mathrm{EW}}^p(K\sigma)\right]\\
& \times (m_B^2 - m_K^2) F_0^{B\to K}(m_\sigma^2) \bar{f}_\sigma^s - f_B f_K \bar{f}_\sigma^s [\sqrt{2}\delta_{pu} b_2(K\sigma) + \sqrt{2} b_3^p(K\sigma) + \sqrt{2} b_{3,\mathrm{EW}}^p(K\sigma)] \Big\},
\end{aligned}
\tag{46}
$$

$$
\begin{aligned}
A_\kappa = -iG_F \frac{g_{\kappa K\pi}}{s_\kappa} \sum_{p=u,c} \lambda_p^{(s)} \Big\{ & b_2(\pi\kappa) f_B f_\pi \bar{f}_\kappa - \left(\alpha_4^p(\pi\kappa) - \frac{1}{2}\alpha_{4,\mathrm{EW}}^p(\pi\kappa)\right)((m_B^2 - m_\pi^2) F_0^{B\to\pi}(m_\kappa^2) \bar{f}_\kappa)\\
& - (b_3^p(\pi\kappa) + b_{3,\mathrm{EW}}^p(\pi\kappa)) f_B f_\pi \bar{f}_\kappa \Big\}.
\end{aligned}
\tag{47}
$$

$$
\begin{aligned}
A_{(K^*)^i} = -iG_F (\hat{s}_{\pi\pi} - s_{\pi\pi}) \frac{g_{(K^*)^i K\pi}}{s_V} \sum_{p=u,c} \lambda_p^{(s)} \Big\{ & b_2(\pi(K^*)^i) f_B f_\pi f_{(K^*)^i} m_{(K^*)^i}/(m_B p_c) - \left(\alpha_4^p(\pi(K^*)^i) - \frac{1}{2}\alpha_{4,\mathrm{EW}}^p(\pi(K^*)^i)\right)\\
& \times (-2 m_V F_1^{B\to\pi} f_{(K^*)^i}) - (b_3^p(\pi(K^*)^i) + b_{3,\mathrm{EW}}^p(\pi(K^*)^i)) \times f_B f_\pi f_{(K^*)^i} m_{(K^*)^i}/(m_B p_c) \Big\},
\end{aligned}
\tag{48}
$$

where $(K^*)^i = K^*(892)$, $K^*(1410)$, $K^*(1680)$ corresponding to $i = 1, 2, 3$, respectively, and

$$
\begin{aligned}
A_{K_0^*} = -iG_F \frac{g_{K_0^* K\pi}}{s_{K_0^*}} \sum_{p=u,c} \lambda_p^{(s)} \Big\{ & b_2(\pi K_0^*) f_B f_\pi \bar{f}_{K_0^*} - \left(\alpha_4^p(\pi K_0^*) - \frac{1}{2}\alpha_{4,\mathrm{EW}}^p(\pi K_0^*)\right)\\
& \times ((m_B^2 - m_\pi^2) F_0^{B\to\pi}(m_{K_0^*}^2) \bar{f}_{K_0^*}) - (b_3^p(\pi K_0^*) + b_{3,\mathrm{EW}}^p(\pi K_0^*)) f_B f_\pi \bar{f}_{K_0^*} \Big\}.
\end{aligned}
\tag{49}
$$

$$
\begin{aligned}
A_{K_2^*} = -iG_F \left[\frac{1}{3}(|\vec{p}_{\pi^-}||\vec{p}_{\pi^+}|)^2 - (\vec{p}_{\pi^-}\cdot\vec{p}_{\pi^+})^2\right] \frac{g_{K_2^* K\pi}}{s_{K_2^*}} \sum_{p=u,c} \lambda_p^{(s)} \Big\{ & b_2(\pi K_2^*) f_B f_\pi f_{K_2^*} m_{K_2^*}/(m_B p_c)\\
& - \left(\alpha_4^p(\pi K_2^*) - \frac{1}{2}\alpha_{4,\mathrm{EW}}^p(\pi K_2^*)\right)(-2 m_T F_1^{B\to\pi} f_{K_2^*}) - (b_3^p(\pi K_2^*) + b_{3,\mathrm{EW}}^p(\pi K_2^*)) f_B f_\pi f_{K_2^*} m_{K_2^*}/(m_B p_c) \Big\}.
\end{aligned}
\tag{50}
$$

Combining Eq. (44) with Eq. (46), one obtains the amplitude of $B^- \to K^-[\pi^+\pi^-] \to K^-\pi^+\pi^-$:

$$
\begin{aligned}
A_{[\pi\pi]} = &-iG_F(\hat{s}_{K\pi} - s_{K\pi})\bigg\{\frac{g_\rho}{s_\rho s_\omega}\tilde{\Pi}_{\rho\omega}[m_\omega \lambda_u^{(s)}(\alpha_1(\omega K)A_0^{B\to\omega}(0)f_K + \alpha_2(K\omega)F_0^{B\to K}(0)f_\omega + b_2(\omega K)f_B f_\omega f_K m_\omega/(m_B p_c))] \\
&+\frac{g_\rho}{s_\rho}[m_\rho \lambda_u^{(s)}(\alpha_1(\rho K)A_0^{B\to\rho}(0)f_K + \alpha_2(K\rho)F_0^{B\to K}(0)f_\rho + b_2(\rho K)f_B f_\rho f_K m_\omega/(m_B p_c))]\bigg\} \\
&+\bigg\{\frac{g_\rho}{s_\rho s_\omega}\tilde{\Pi}_{\rho\omega}\times\bigg[m_\omega\sum_{p=u,c}\lambda_p^{(s)}\bigg\{\bigg(2\alpha_3(K\omega)+\frac{1}{2}\alpha_3^p(K\omega)\bigg)F_0^{B\to K}(0)f_\omega + \bigg(\alpha_4^p(\omega K)+\frac{3}{2}\alpha_{4,\mathrm{EW}}^p(\omega K)\bigg)A_0^{B\to\omega}(0)f_K \\
&+(b_3^p(\omega K)+b_{3,\mathrm{EW}}^p(\omega K))f_B f_\omega f_K m_\omega/(m_B p_c)\bigg\}\bigg] \\
&+\frac{g_\rho}{s_\rho}\bigg[m_\rho\sum_{p=u,c}\lambda_p^{(s)}\bigg\{(\alpha_4^p(\rho K)+\alpha_{4,\mathrm{EW}}^p(\rho K))A_0^{B\to\rho}(0)f_K + \frac{3}{2}\alpha_{3,\mathrm{EW}}^p(K\rho)F_0^{B\to K}(0)f_\rho + (b_3^p(\rho K) \\
&-b_{3,\mathrm{EW}}^p(\rho K))f_B f_\rho f_K m_\omega/(m_B p_c)\bigg\}\bigg]\bigg\} \\
&+iG_F g_{\sigma\pi\pi}R_\sigma\sum_{p=u,c}\lambda_p^{(s)}\bigg\{(m_\sigma^2 - m_B^2)F_0^{B\to f}(m_K^2)f_K[\delta_{pu}\alpha_1(\sigma K)+\alpha_4^p(\sigma K)+\alpha_{4,\mathrm{EW}}^p(\sigma K)] \\
&-f_B f_K \bar{f}_\sigma^u[\delta_{pu}b_2(\sigma K)+b_3^p(\sigma K)+b_{3,\mathrm{EW}}^p(\sigma K)] + \bigg[\delta_{pu}\alpha_2(K\sigma)+2\alpha_3^p(K\sigma)+\frac{1}{2}\alpha_{3,\mathrm{EW}}^p(K\sigma)\bigg] \\
&\times(m_B^2 - m_K^2)F_0^{B\to K}(0)\bar{f}_\sigma^u + \bigg[\sqrt{2}\alpha_3^p(K\sigma)+\sqrt{2}\alpha_4^p(K\sigma) - \frac{1}{\sqrt{2}}\alpha_{3,\mathrm{EW}}^p(K\sigma) - \frac{1}{\sqrt{2}}\alpha_{4,\mathrm{EW}}^p(K\sigma)\bigg](m_B^2 - m_K^2)F_0^{B\to K}(m_\sigma^2)\bar{f}_\sigma^s \\
&-f_B f_K \bar{f}_\sigma^s[\sqrt{2}\delta_{pu}b_2(K\sigma)+\sqrt{2}b_3^p(K\sigma)+\sqrt{2}b_{3,\mathrm{EW}}^p(K\sigma)]\bigg\},
\end{aligned}
\tag{51}
$$

Meanwhile, using Eqs. (48)–(50), we get the amplitude of $B^- \to [K^-\pi^+]\pi^- \to K^-\pi^+\pi^-$:

$$
\begin{aligned}
A_{[K\pi]} = &-iG_F(\hat{s}_{\pi\pi} - s_{\pi\pi})\frac{g_{(K^*)^i K\pi}}{s_V}\sum_{p=u,c}\lambda_p^{(s)}\bigg\{b_2(\pi(K^*)^i)f_B f_\pi f_{(K^*)^i}m_{(K^*)^i}/(m_B p_c) \\
&-\bigg(\alpha_4^p(\pi(K^*)^i) - \frac{1}{2}\alpha_{4,\mathrm{EW}}^p(\pi(K^*)^i)\bigg)(-2m_V F_1^{B\to\pi}f_{(K^*)^i}) - (b_3^p(\pi(K^*)^i)+b_{3,\mathrm{EW}}^p(\pi(K^*)^i))f_B f_\pi f_{(K^*)^i}m_{(K^*)^i}/(m_B p_c)\bigg\} \\
&-iG_F\frac{g_{\kappa K\pi}}{s_\kappa}\sum_{p=u,c}\lambda_p^{(s)}\bigg\{b_2(\pi\kappa)f_B f_\pi \bar{f}_\kappa - \bigg(\alpha_4^p(\pi\kappa)-\frac{1}{2}\alpha_{4,\mathrm{EW}}^p(\pi\kappa)\bigg) \\
&\times((m_B^2 - m_\pi^2)F_0^{B\to\pi}(m_\kappa^2)\bar{f}_\kappa) - (b_3^p(\pi\kappa)+b_{3,\mathrm{EW}}^p(\pi\kappa))f_B f_\pi \bar{f}_\kappa\bigg\} - iG_F\frac{g_{K_0^* K\pi}}{s_{K_0^*}}\sum_{p=u,c}\lambda_p^{(s)} \\
&\times\bigg\{b_2(\pi K_0^*)f_B f_\pi \bar{f}_{K_0^*} - \bigg(\alpha_4^p(\pi K_0^*)-\frac{1}{2}\alpha_{4,\mathrm{EW}}^p(\pi K_0^*)\bigg)((m_B^2 - m_\pi^2)F_0^{B\to\pi}(m_{K_0^*}^2)\bar{f}_{K_0^*}) - (b_3^p(\pi K_0^*)+b_{3,\mathrm{EW}}^p(\pi K_0^*))f_B f_\pi \bar{f}_{K_0^*}\bigg\} \\
&-iG_F\bigg[\frac{1}{3}(|\vec{p}_{\pi^-}||\vec{p}_{\pi^+}|)^2 - (\vec{p}_{\pi^-}\cdot\vec{p}_{\pi^+})^2\bigg]\frac{g_{K_2^* K\pi}}{s_{K_2^*}}\sum_{p=u,c}\lambda_p^{(s)} \\
&\times\bigg\{b_2(\pi K_2^*)f_B f_\pi f_{K_2^*}m_{K_2^*}/(m_B p_c) - \bigg(\alpha_4^p(\pi K_2^*)-\frac{1}{2}\alpha_{4,\mathrm{EW}}^p(\pi K_2^*)\bigg) \\
&\times(-2m_T F_1^{B\to\pi}f_{K_2^*}) - (b_3^p(\pi K_2^*)+b_{3,\mathrm{EW}}^p(\pi K_2^*))f_B f_\pi f_{K_2^*}m_{K_2^*}/(m_B p_c)\bigg\}.
\end{aligned}
\tag{52}
$$

In addition, we can obtain the amplitude of the $B^- \to \sigma\pi^-$, which has the following form:

$$
\begin{aligned}
A(B^- \to [\pi^+\pi^-]_\sigma \pi^-) = iG_F \sum_{p=u,c} \lambda_p^d \Bigg\{ &(m_\sigma^2 - m_B^2) F_0^{B\to\sigma}(m_\pi^2) f_\pi [\delta_{pu}\alpha_1(\sigma\pi) + \alpha_4^p(\sigma\pi) + \alpha_{4,\mathrm{EW}}^p(\sigma\pi)] \\
&- f_B f_\pi \bar{f}_\sigma^u [\delta_{pu} b_2(\sigma\pi) + b_3^p(\sigma\pi) + b_{3,\mathrm{EW}}^p(\sigma\pi)] + \Bigg[\delta_{pu}\alpha_2(\pi\sigma) + 2\alpha_3^p(\pi\sigma) + \alpha_4^p(\pi\sigma) \\
&+ \frac{1}{2}\alpha_{3,\mathrm{EW}}^p(\pi\sigma) - \frac{1}{2}\alpha_{4,\mathrm{EW}}^p(\pi\sigma)\Bigg](m_B^2 - m_\pi^2) F_0^{B\to\pi}(0)\bar{f}_\sigma^u + [\sqrt{2}\alpha_3^p(\pi\sigma) - \sqrt{2}\alpha_{3,\mathrm{EW}}^p(\pi\sigma)] \\
&\times (m_B^2 - m_\pi^2) F_0^{B\to\pi}(0)\bar{f}_\sigma^s + f_B f_\pi \bar{f}_\sigma^u \Bigg[\delta_{pu} b_2(\pi\sigma) + b_3^p(\pi\sigma) - \frac{1}{2} b_{3,\mathrm{EW}}^p(\pi\sigma)\Bigg]\Bigg\}. \qquad (53)
\end{aligned}
$$

3. Total result for the amplitude of $B^- \to K^-\pi^+\pi^-$

In the QCDF, both the resonance and nonresonance contributions have been considered, inserting Eqs. (51) and (52) to Eq. (31) then combining Eqs. (23)–(30), the decay amplitude via $B^- \to R + NR \to K^-\pi^+\pi^-$ can be finally obtained as

$$
\begin{aligned}
A = iG_F g_{\sigma\pi\pi} R_\sigma \sum_{p=u,c} \lambda_p^{(s)} \Bigg\{ &(m_\sigma^2 - m_B^2) F_0^{B\to\sigma}(m_K^2) f_K [\delta_{pu}\alpha_1(\sigma K) + \alpha_4^p(\sigma K) + \alpha_{4,\mathrm{EW}}^p(\sigma K)] \\
&- f_B f_K \bar{f}_\sigma^u [\delta_{pu} b_2(\sigma K) + b_3^p(\sigma K) + b_{3,\mathrm{EW}}^p(\sigma K)] + \Bigg[\delta_{pu}\alpha_2(K\sigma) + 2\alpha_3^p(K\sigma) + \frac{1}{2}\alpha_{3,\mathrm{EW}}^p(K\sigma)\Bigg] \\
&\times (m_B^2 - m_K^2) F_0^{B\to K}(0)\bar{f}_\sigma^u + \Bigg[\sqrt{2}\alpha_3^p(K\sigma) + \sqrt{2}\alpha_4^p(K\sigma) - \frac{1}{\sqrt{2}}\alpha_{3,\mathrm{EW}}^p(K\sigma) - \frac{1}{\sqrt{2}}\alpha_{4,\mathrm{EW}}^p(K\sigma)\Bigg] \\
&\times (m_B^2 - m_K^2) F_0^{B\to K}(m_\sigma^2)\bar{f}_\sigma^s - f_B f_K \bar{f}_\sigma^s [\sqrt{2}\delta_{pu} b_2(K\sigma) + \sqrt{2} b_3^p(K\sigma) + \sqrt{2} b_{3,\mathrm{EW}}^p(K\sigma)]\Bigg\} \\
&- iG_F(\hat{s}_{K\pi} - s_{K\pi})\Bigg\{\frac{g_\rho}{s_\rho s_\omega}\tilde{\Pi}_{\rho\omega}[m_\omega \lambda_u^{(s)}(\alpha_1(\omega K) A_0^{B\to\omega}(0) f_K + \alpha_2(K\omega) F_0^{B\to K}(0) f_\omega \\
&+ b_2(\omega K) f_B f_\omega f_K m_\omega/(m_B p_c))] + \frac{g_\rho}{s_\rho}[m_\rho \lambda_u^{(s)}(\alpha_1(\rho K) A_0^{B\to\rho}(0) f_K + \alpha_2(K\rho) F_0^{B\to K}(0) f_\rho \\
&+ b_2(\rho K) f_B f_\rho f_K m_\omega/(m_B p_c))]\Bigg\} + \Bigg\{\frac{g_\rho}{s_\rho s_\omega}\tilde{\Pi}_{\rho\omega} \times \Bigg[m_\omega \sum_{p=u,c}\lambda_p^{(s)}\Bigg\{\Bigg(2\alpha_3(K\omega) + \frac{1}{2}\alpha_3^p(K\omega)\Bigg) F_0^{B\to K}(0) f_\omega \\
&+ \Bigg(\alpha_4^p(\omega K) + \frac{3}{2}\alpha_{4,\mathrm{EW}}^p(\omega K)\Bigg) A_0^{B\to\omega}(0) f_K + (b_3^p(\omega K) + b_{3,\mathrm{EW}}^p(\omega K)) f_B f_\omega f_K m_\omega/(m_B p_c)\Bigg\}\Bigg] \\
&+ \frac{g_\rho}{s_\rho}\Bigg[m_\rho \sum_{p=u,c}\lambda_p^{(s)}\Bigg\{(\alpha_4^p(\rho K) + \alpha_{4,\mathrm{EW}}^p(\rho K)) A_0^{B\to\rho}(0) f_K + \frac{3}{2}\alpha_{3,\mathrm{EW}}^p(K\rho) F_0^{B\to K}(0) f_\rho \\
&+ (b_3^p(\rho K) - b_{3,\mathrm{EW}}^p(\rho K)) f_B f_\rho f_K m_\omega/(m_B p_c)\Bigg\}\Bigg]\Bigg\} - iG_F \frac{g_{(K^*)^i K\pi}}{s_V}\sum_{p=u,c}\lambda_p^{(s)}\Bigg\{b_2(\pi(K^*)^i) f_B f_\pi f_{(K^*)^i} \\
&- \Bigg(\alpha_4^p(\pi(K^*)^i) - \frac{1}{2}\alpha_{4,\mathrm{EW}}^p(\pi(K^*)^i)\Bigg)(-2m_B p_c F_1^{B\to\pi} f_{(K^*)^i}) - (b_3^p(\pi(K^*)^i) + b_{3,\mathrm{EW}}^p(\pi(K^*)^i)) f_B f_\pi f_{(K^*)^i}\Bigg\} \\
&- iG_F \frac{g_{\kappa K\pi}}{s_\kappa}\sum_{p=u,c}\lambda_p^{(s)}\Bigg\{b_2(\pi\kappa) f_B f_\pi \bar{f}_\kappa - \Bigg(\alpha_4^p(\pi\kappa) - \frac{1}{2}\alpha_{4,\mathrm{EW}}^p(\pi\kappa)\Bigg)((m_B^2 - m_\pi^2) F_0^{B\to\pi}(m_\kappa^2)\bar{f}_\kappa) \\
&- (b_3^p(\pi\kappa) + b_{3,\mathrm{EW}}^p(\pi\kappa)) f_B f_\pi \bar{f}_\kappa\Bigg\} - iG_F(\hat{s}_{\pi\pi} - s_{\pi\pi})\frac{g_{K_0^* K\pi}}{s_{K_0^*}}\sum_{p=u,c}\lambda_p^{(s)}\Bigg\{b_2(\pi K_0^*) f_B f_\pi \bar{f}_{K_0^*} \\
&- \Bigg(\alpha_4^p(\pi K_0^*) - \frac{1}{2}\alpha_{4,\mathrm{EW}}^p(\pi K_0^*)\Bigg)((m_B^2 - m_\pi^2) F_0^{B\to\pi}(m_{K_0^*}^2)\bar{f}_{K_0^*}) - (b_3^p(\pi K_0^*) + b_{3,\mathrm{EW}}^p(\pi K_0^*)) f_B f_\pi \bar{f}_{K_0^*}\Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
&- iG_F\left[\frac{1}{3}(|\vec{p}_{\pi^-}||\vec{p}_{\pi^+}|)^2 - (\vec{p}_{\pi^-}\cdot\vec{p}_{\pi^+})^2\right]\frac{g_{K_2^*K\pi}}{s_{K_2^*}}\sum_{p=u,c}\lambda_p^{(s)}\Big\{b_2(\pi K_2^*)f_B f_\pi f_{K_2^*}m_{K_2^*}/(m_B p_c)\\
&-\left(\alpha_4^p(\pi K_2^*) - \frac{1}{2}\alpha_{4,\mathrm{EW}}^p(\pi K_2^*)\right)(-2m_T F_1^{B\to\pi}f_{K_2^*}) - (b_3^p(\pi K_2^*) + b_{3,\mathrm{EW}}^p(\pi K_2^*))f_B f_\pi f_{K_2^*}m_{K_2^*}/(m_B p_c)\Big\}\\
&-\frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^s\frac{f_\pi}{2}[2m_K^2 r + (m_B^2 - s_{\pi\pi} - m_K^2)\omega_+ + (s_{\pi K} - s_{\pi K})\omega_-][a_1\delta_{pu} + a_4^p + a_{10}^p - (a_6^p + a_8^p)r_\chi^K]\\
&\times e^{-\alpha_{\mathrm{NR}}(s_{\pi\pi}+s_{K\pi}-m_\pi^2-m_K^2)} + \left(\frac{m_B^2 - m_\pi^2}{m_d - m_b}F_0^{B\to\pi}(0)\right)(-2a_6^p + 2a_8^p)\left[\frac{\nu}{3}(3F_{\mathrm{NR}} + 2F'_{\mathrm{NR}}) + \sigma_{\mathrm{NR}}e^{-\alpha s_{\pi\pi}}e^{i\pi}\left(1 + 4\frac{m_K^2 - m_\pi^2}{s_{\pi\pi}}\right)\right].
\end{aligned}
\tag{54}
$$

4. Localizd CP violation

Totally, the decay amplitude for $B \to K^-\pi^+\pi^-$ is the sum of resonant (R) contributions and the nonresonant (NR) background [6]

$$A = \sum_R A_R + A_{\mathrm{NR}}. \tag{55}$$

The differential *CP* asymmetry parameter can be defined as

$$\mathcal{A}_{CP} = \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2}. \tag{56}$$

In this work, we will consider eight resonances in a certain phase region Ω which includes $m_{K^-\pi^+}^2 < 15\ \mathrm{GeV}^2$ and $0.08 < m_{\pi^+\pi^-}^2 < 0.66\ \mathrm{GeV}^2$ for the $B^- \to K^-\pi^+\pi^-$ decay. By integrating the denominator and numerator of $\mathcal{A}_{CP}$ in this region, we get the localized integrated *CP* asymmetry, which can be measured by experiments and takes the following form:

$$\mathcal{A}_{CP}^{\Omega} = \frac{\int_\Omega ds_{12}ds_{13}(|A|^2 - |\bar{A}|^2)}{\int_\Omega ds_{12}ds_{13}(|A|^2 + |\bar{A}|^2)}. \tag{57}$$

B. Calculation of differential *CP* violation and branching fraction of $B^- \to K^-\sigma$ decay

Using Eq. (56), the differential *CP* asymmetry parameter of $B \to M_1M_2$ can be expressed as

$$\mathcal{A}_{CP}(B\to M_1M_2) = \frac{|A(B\to M_1M_2)|^2 - |\bar{A}(B\to M_1M_2)|^2}{|A(B\to M_1M_2)|^2 + |\bar{A}(B\to M_1M_2)|^2}. \tag{58}$$

The branching fraction of the $B \to M_1M_2$ decay has the following form:

$$\mathcal{B}(B\to M_1M_2) = \tau_B\frac{p_c}{8\pi m_B^2}|A(B\to M_1M_2)|^2, \tag{59}$$

where τ_B and m_B are the lifetime and the mass of the B meson, respectively, p_c is the magnitude of the three momentum of either final state meson in the rest frame of the B meson which can be expressed as

$$p_c = \frac{1}{2m_B}\sqrt{[m_B^2 - (m_{M_1} + m_{M_2})^2][m_B^2 - (m_{M_1} - m_{M_2})^2]}, \tag{60}$$

with m_{M_1} and m_{M_2} being the two final state mesons' masses, respectively.

The amplitude of $B^- \to K^-\sigma$ has the following form:

$$
\begin{aligned}
A(B^-\to\sigma K^-) &= \langle\sigma K^-|\mathcal{H}_{\mathrm{eff}}|B^-\rangle\\
&= \sum_{p=u,c}\lambda_p^{(s)}\frac{G_F}{2}\Big\{[\alpha_1(\sigma K)\delta_{pu} + \alpha_4^p(\sigma K) + \alpha_{4,\mathrm{EW}}^p(\sigma K)]\times(m_\sigma^2 - m_B^2)F_0^{B\to\sigma}(m_K^2)f_K\\
&\quad + \left[\alpha_2(K\sigma)\delta_{pu} + 2\alpha_3(K\sigma) + \frac{1}{2}\alpha_{3,\mathrm{EW}}^p(K\sigma)\right]\times(m_B^2 - m_K^2)F_0^{B\to K}(m_\sigma^2)\bar{f}_\sigma^u\\
&\quad + \left[\sqrt{2}\alpha_3^p(K\sigma) + \sqrt{2}\alpha_4^p(K\sigma) - \frac{1}{\sqrt{2}}\alpha_{3,\mathrm{EW}}^p(K\sigma) - \frac{1}{\sqrt{2}}\alpha_{4,\mathrm{EW}}^p(K\sigma)\right]\\
&\quad \times(m_B^2 - m_K^2)F_0^{B\to K}(m_\sigma^2)\bar{f}_\sigma^s - [b_2(\sigma K)\delta_{pu} + b_3^p(\sigma K) + b_{3,\mathrm{EW}}^p(\sigma K)]\\
&\quad \times f_B f_K\bar{f}_\sigma^u - \sqrt{2}[b_2(K\sigma)\delta_{pu} + b_3^p(K\sigma) + b_{3,\mathrm{EW}}^p(K\sigma)]\times f_B f_K\bar{f}_\sigma^s\Big\}.
\end{aligned}
\tag{61}
$$

Substituting Eq. (61) into Eq. (58) we can get the expression of $\mathcal{A}_{\mathcal{CP}}(B^- \to K^-\sigma)$. Substituting Eqs. (61) and (60) into Eq. (59), one can obtain the branching fraction of $B^- \to K^-\sigma$.

V. NUMERICAL RESULTS

The theoretical results obtained in the QCDF approach depend on many input parameters. The values of the Wolfenstein parameters are given as $\bar{\rho} = 0.117 \pm 0.021$, $\bar{\eta} = 0.353 \pm 0.013$ [55].

The effective Wilson coefficients used in our calculations are taken from Ref. [56]:

$$\begin{aligned}
&C_1' = -0.3125, \qquad C_2' = -1.1502,\\
&C_3' = 2.120\times 10^{-2} + 5.174\times 10^{-3} i, \qquad C_4' = -4.869\times 10^{-2} - 1.552\times 10^{-2} i,\\
&C_5' = 1.420\times 10^{-2} + 5.174\times 10^{-3} i, \qquad C_6' = -5.792\times 10^{-2} - 1.552\times 10^{-2} i,\\
&C_7' = -8.340\times 10^{-5} - 9.938\times 10^{-5} i, \qquad C_8' = 3.839\times 10^{-4},\\
&C_9' = -1.017\times 10^{-2} - 9.938\times 10^{-5} i, \qquad C_{10}' = 1.959\times 10^{-3}.
\end{aligned} \tag{62}$$

For the masses appearing in B decays, we shall use the following values (in units of GeV) [55]:

$$\begin{aligned}
&m_u = m_d = 0.0035, \qquad m_s = 0.119, \qquad m_b = 4.2, \qquad m_q = \frac{m_u + m_d}{2}, \qquad m_{\pi^\pm} = 0.14,\\
&m_{B^-} = 5.279, \qquad m_\omega = 0.782, \qquad m_{\rho^0(770)} = 0.775, \qquad m_{K^-} = 0.494, \qquad m_\kappa = 0.824, \qquad m_{K^*}(892) = 0.895,\\
&m_{K^*}(1410) = 1.414, \qquad m_{K_0^*}(1430) = 1.425, \qquad m_{K^*}(1680) = 1.717, \qquad m_{K_2^*}(1430) = 1.426,
\end{aligned} \tag{63}$$

while for the widths we shall use (in units of GeV) [55]

$$\begin{aligned}
&\Gamma_\rho = 0.149, \qquad \Gamma_\omega = 0.00849, \qquad \Gamma_{\sigma(600)} = 0.5, \qquad \Gamma_\kappa = 0.478, \qquad \Gamma_{K^*(892)} = 0.047,\\
&\Gamma_{K^*(1410)} = 0.232, \qquad \Gamma_{K^*(1680)} = 0.322, \qquad \Gamma_{K_0^*(1430)} = 0.270, \qquad \Gamma_{K_2^*(1430)} = 0.109,\\
&\Gamma_{\rho\to\pi\pi} = 0.149, \qquad \Gamma_{\omega\to\pi\pi} = 0.00013, \qquad \Gamma_{\sigma(600)\to\pi\pi} = 0.3, \qquad \Gamma_{K^*(892)\to K\pi} = 0.0487,\\
&\Gamma_{K^*(1410)\to K\pi} = 0.015, \qquad \Gamma_{K^*(1680)\to K\pi} = 0.10, \qquad \Gamma_{K_0^*(1430)\to K\pi} = 0.251, \qquad \Gamma_{K_2^*(1430)\to K\pi} = 0.054.
\end{aligned} \tag{64}$$

The strong coupling constants are determined from the measured widths through the relations [6,43,57]

$$\begin{aligned}
g_{S\to M_1'M_2'} &= \sqrt{\frac{8\pi m_S^2}{p_c(S)}\Gamma_{S\to M_1'M_2'}},\\
g_{V\to M_1'M_2'} &= \sqrt{\frac{6\pi m_V^2}{p_c(V)^3}\Gamma_{V\to M_1'M_2'}},\\
g_{T\to M_1'M_2'} &= \sqrt{\frac{60\pi m_T^2}{p_c(T)^5}\Gamma_{T\to M_1'M_2'}},
\end{aligned} \tag{65}$$

where $p_c(S, V, T)$ are the magnitudes of the three momenta of the final state meson in the rest frame of S, V, and T mesons, respectively.

The following numerical values for the decay constants will be used (in units of GeV) [6,11,25]:

$$\begin{aligned}
&f_{\pi^\pm} = 0.131, \qquad f_{B^-} = 0.21 \pm 0.02, \qquad f_{K^-} = 0.156 \pm 0.007,\\
&f_{\rho^0(770)} = 0.216 \pm 0.003, \qquad f^\perp_{\rho^0(770)} = 0.165 \pm 0.009, \qquad f_\omega = 0.187 \pm 0.005, \qquad f^\perp_\omega = 0.151 \pm 0.009,\\
&\bar{f}_\kappa = 0.34 \pm 0.02, \qquad f_{K^*(892)} = 0.22 \pm 0.005, \qquad f^\perp_{K^*(892)} = 0.185 \pm 0.010,\\
&\bar{f}_{K_0^*(1430)} = -0.300 \pm 0.030, \qquad f_{K_2^*(1430)} = 0.118 \pm 0.005, \qquad f^\perp_{K_2^*(1430)} = 0.077 \pm 0.014.
\end{aligned} \tag{66}$$

As for the form factors, we use [6,11,25]

$$\begin{aligned}
&F_0^{B\to K}(0) = 0.35 \pm 0.04, \qquad A_0^{B\to\rho}(0) = 0.303 \pm 0.029, \qquad F_0^{B\to\pi}(0) = 0.25 \pm 0.03,\\
&A_0^{B\to K^*(892)}(0) = 0.374 \pm 0.034, \qquad A_0^{B\to K_2^*(1430)}(0) = 0.25 \pm 0.04,\\
&A_1^{B\to K_2^*(1430)}(0) = 0.14 \pm 0.02, \qquad F_0^{B\to K_0^*(1430)}(0) = 0.21.
\end{aligned} \tag{67}$$

The values of Gegenbauer moments at $\mu = 1$ GeV are taken from [6,11,25,58]:

$$\begin{aligned}
&\alpha_1^\rho = 0, \qquad \alpha_2^\rho = 0.15 \pm 0.07, \qquad \alpha_{1,\perp}^\rho = 0, \qquad \alpha_{2,\perp}^\rho = 0.14 \pm 0.06,\\
&\alpha_1^\omega = 0, \qquad \alpha_2^\omega = 0.15 \pm 0.07, \qquad \alpha_{1,\perp}^\omega = 0, \qquad \alpha_{2,\perp}^\omega = 0.14 \pm 0.06,\\
&\alpha_1^{K_2^*(1430)} = \frac{5}{3}, \qquad \alpha_{1,\perp}^{K_2^*(1430)} = \frac{5}{3},\\
&\alpha_1^{K^*(892)} = 0.03 \pm 0.02, \qquad \alpha_{1,\perp}^{K^*(892)} = 0.04 \pm 0.03, \qquad \alpha_2^{K^*(892)} = 0.11 \pm 0.09, \qquad \alpha_{2,\perp}^{K^*(892)} = 0.10 \pm 0.08,\\
&B_{1,\sigma(600)}^u = -0.42 \pm 0.074, \qquad B_{3,\sigma(600)}^u = -0.58 \pm 0.23,\\
&B_{1,\sigma(600)}^s = -0.35 \pm 0.061, \qquad B_{3,\sigma(600)}^s = -0.43 \pm 0.18.\\
&B_{1,\kappa} = -0.92 \pm 0.11, \qquad B_{3,\kappa} = 0.15 \pm 0.09, \qquad B_{1,K_0^*(1430)} = 0.58 \pm 0.07, \qquad B_{3,K_0^*(1430)} = -1.20 \pm 0.08.
\end{aligned} \tag{68}$$

Using the large energy effective theory (LEET) technique, Refs. [59,60] formulate the $B \to K_J^*$ ($J \geq 1$) form factors in the large recoil region. All the form factors can be expressed in terms of two independent LEET functions, $\xi_\perp$ and $\xi_\parallel$. Explicitly, we have

$$A_0^{B\to K_J^*}(q^2)\left(\frac{|\vec{p}_{K_J^*}|}{m_{K_J^*}}\right)^{J-1} \simeq \left(1 - \frac{m_{K_J^*}^2}{m_B E}\right)\xi_\parallel^{K_J^*}(q^2) + \frac{m_{K_J^*}}{m_B}\xi_\perp^{K_J^*}(q^2), \tag{69}$$

where have used $|\vec{p}_{K_J^*}|/E \simeq 1$, $|\vec{p}_{K_J^*}|$ is the magnitude of the three momentum of the K_J^* meson in the rest frame of the B meson. With $\xi_\parallel^{K^*(1410)}(0) = 0.22 \pm 0.03$, $\xi_\perp^{K^*(1410)}(0) = 0.28 \pm 0.04$, $\xi_\parallel^{K^*(1680)}(0) = 0.18 \pm 0.03$ and $\xi_\perp^{K^*(1680)}(0) = 0.24 \pm 0.05$ derived from the Bauer-Stech-Wirbel model [24], we can obtain $A_0^{B\to K^*(1410)}(0) = 0.26 \pm 0.0275$ and $A_0^{B\to K^*(1680)}(0) = 0.2154 \pm 0.0281$, respectively. For $F_0^{B\to\sigma}(m_K^2)$, we take $F_0^{B\to\sigma}(m_K^2) = 0.45 \pm 0.15$ [61]. In our work, all the form factors are evaluated at $q^2 = 0$ due to the smallness of m_π^2 and m_K^2 compared with m_B^2 [10]. As for the decay constants and Gegenbauer moments of the $K^*(1410)$ and the $K^*(1680)$ mesons, we assume they have the same central values as $K^*(892)$ and assign their uncertainties to be ± 0.1. In fact, the magnitudes of these errors have negligible influences.

A general fit of the parameters ρ and ϕ to the $B \to VP$ and $B \to PV$ data indicates $X^{PV} \neq X^{VP}$, i.e., $\rho^{PV} = 0.87$, $\rho^{VP} = 1.07$, $\phi^{VP} = -30^0$ and $\phi^{PV} = -70^0$ [31]. For the $B \to PT$ and $B \to TP$ cases, we will use the values in Ref. [25]: $\rho^{TP} = 0.83$, $\rho^{PT} = 0.75$, $\phi^{TP} = -70^0$ and $\phi^{PT} = -30^0$. We shall assign an error of ± 0.1 to $\rho^{M_1M_2}$ and $\pm 20^0$ to $\phi^{M_1M_2}$ for estimation of theoretical uncertainties. We calculate the branching ratios and CP asymmetries for B to a vector meson or a tensor meson plus a pseudoscalar meson involved in our work, which are shown in Tables I and II, respectively. As can be seen from these two tables, our results are consistent with the available experimental data. However, for the $B \to PS$ and $B \to SP$ decays, there is few experimental data so the values of ρ_S and ϕ_S are not determined well, to make an estimation about $\mathcal{A}_{\mathcal{CP}}(B^- \to K^-\sigma)$ and $\mathcal{B}(B^- \to K^-\sigma)$, we will adopt $X^{PS} = X^{SP} = (1 + \rho_S e^{i\phi_S}) \ln \frac{m_B}{\Lambda_h}$ as described in Sec. III. Now we are left with only two free parameters with all the above considerations, which are the divergence parameters ρ_S and ϕ_S for $\mathcal{A}_{\mathcal{CP}}(B^- \to R + NR \to K^-\pi^+\pi^-)$. By fitting the theoretical result to the experimental data $\mathcal{A}_{\mathcal{CP}}(B^- \to K^-\pi^+\pi^-) = 0.678 \pm 0.078 \pm 0.0323 \pm 0.007$ in the region $m_{K^-\pi^+}^2 < 15\ \mathrm{GeV}^2$ and $0.08 < m_{\pi^+\pi^-}^2 < 0.66\ \mathrm{GeV}^2$, $\mathcal{A}_{\mathcal{CP}}(B^- \to K_0^*(1430)\pi^-) = 0.061 \pm 0.032$, $\mathcal{B}(B^- \to K_0^*(1430)\pi^-) = (39_{-5}^{+6}) \times 10^{-6}$ and $\mathcal{B}(B^- \to \sigma(600)\pi^- \to \pi^-\pi^+\pi^-) < 4.1 \times 10^{-6}$ [62], and varying ϕ_S and ρ_S by 0.01 each time in the range $\phi_S \in [0, 2\pi]$ and $\rho_S \in [0, 8]$ [63,64], i.e., $\Delta\rho_S = 0.01$ and $\Delta\phi_S = 0.01$, it is found that there exist ranges of the parameters ρ_S and ϕ_S which satisfy the above experimental data. The allowed ranges are $\phi_S \in [1.77, 2.25]$ and $\rho_S \in [2.39, 4.02]$. Therefore, the interference of resonances $[\pi\pi]$ resonances including $\sigma(600)$, $\rho^0(770)$, $\omega(782)$ mesons, $[K\pi]$ resonances including κ, $K^*(892)$, $K^*(1410)$, $K_0^*(1430)$, $K^*(1680)$ and $K_2^*(1430)$ mesons together with the nonresonance

TABLE I. Branching fractions (in units of 10^{-6}) of resonant and nonresonant (NR) contributions to $B^- \to K^-\pi^+\pi^-$. The theoretical errors corresponding to the uncertainties due to the form factors, decay constants Gegenbauer moments and divergence parameters.

Decay mode	*BABAR* [68]	Belle [69]	This work
σK^-	$\cdots$	$\cdots$	[6.53,17.52]
$\rho^0 K^-$	$3.56 \pm 0.45 \pm 0.43^{+0.38}_{-0.15}$	$3.89 \pm 0.47 \pm 0.29^{+0.32}_{-0.29}$	2.84 ± 0.26
ωK^-	$0.09 \pm 0.13 \pm 0.02^{+0.03}_{-0.04}$	$\cdots$	0.072 ± 0.012
$\kappa\pi^-$	$\cdots$	$\cdots$	[4.11,13.46]
$K^{*0}(892)\pi^-$	$7.2 \pm 0.4 \pm 0.7^{+0.3}_{-0.5}$	$6.45 \pm 0.43 \pm 0.48^{+0.25}_{-0.35}$	5.77 ± 0.35
$K^{*0}(1410)\pi^-$	$\cdots$	$\cdots$	1.58 ± 1.01
$K^{*0}(1680)\pi^-$	$\cdots$	$\cdots$	1.09 ± 0.73
$K_2^{*0}(1430)\pi^-$	$1.85 \pm 0.41 \pm 0.28^{+0.54}_{-0.08}$	$\cdots$	1.02 ± 0.11
NR	$9.3 \pm 1.0 \pm 1.2^{+6.7}_{-0.4} \pm 1.2$	$16.9 \pm 1.3 \pm 1.3^{+1.1}_{-0.9}$	13.35 ± 2.3

contribution can indeed induce the data for the localized CP asymmetry in the $B^- \to K^-\pi^+\pi^-$ decay. It is noted that the range of $\rho_S \in [2.39, 4.02]$ is larger than the previously conservative choice of $\rho \leq 1$ [10,12]. Since the QCDF itself cannot give information about the parameters ρ and ϕ, there is no reason to restrict ρ to the range $\rho \leq 1$ [23,31,65,66]. In the pQCD approach, the possible un-negligible large weak annihilation contributions were noticed first in Refs. [15,67]. In fact, there are many experimental studies which have been successfully carried out at B factories (*BABAR* and Belle), Tevatron (CDF and D0) and LHCb in the past and will be continued at LHCb and Belle experiments. These experiments provide highly fertile ground for theoretical studies and have yielded many exciting and important results, such as measurements of pure annihilation $B_s \to \pi\pi$ and $B_d \to KK$ decays reported recently by CDF, LHCb and Belle [18–20], which suggest the existence of unexpected large annihilation contributions and have attracted much attention [21–23]. Thus larger values of ρ_S are acceptable when dealing with the divergence problems for $B \to SP(PS)$ decays. With the large values of ρ_S, it is certain that both the weak annihilation and the hard spectator scattering processes can make large contributions to $B^- \to K^-\sigma$ decays. Many more experimental and theoretical efforts are expected to understand the underlying QCD dynamics of annihilation and spectator scattering contributions. In the obtained allowed ranges for ρ_S and ϕ_S, i.e., $\rho_S \in [2.39, 4.02]$ and $\phi_S \in [1.77, 2.25]$, we calculate the CP asymmetry parameter and the branching fraction for the $B^- \to K^-\sigma$ decay modes using Eqs. (58)–(60). Similarly, we can also get the corresponding results of the $B^- \to \kappa\pi^-$ decay. We obtain that $\mathcal{A}_{CP}(B^- \to K^-\sigma) \in [-0.34, -0.11]$, $\mathcal{B}(B^- \to K^-\sigma) \in [6.53, 17.52] \times 10^{-6}$, $\mathcal{A}_{CP}(B^- \to \kappa\pi^-) \in [-0.18, 0.10]$ and $\mathcal{B}(B^- \to \kappa\pi^-) \in [4.11, 13.46] \times 10^{-6}$ when ρ_S and ϕ_S vary in their allowed ranges, which are shown in Tables I and II, respectively. Moreover, with the obtained values of ρ_S and ϕ_S, we can also get the localized CP asymmetry $\mathcal{A}_{CP}(B^- \to K^-\pi^+\pi^-)$ induced by only $[\pi\pi]$ and only $[K\pi]$ resonances, respectively, in the same region $m^2_{K^-\pi^+} < 15\ \mathrm{GeV}^2$ and $0.08 < m^2_{\pi^+\pi^-} < 0.66\ \mathrm{GeV}^2$. Inserting Eqs. (51) and (52) into Eq. (57) respectively, the results are $\mathcal{A}_{CP}(B^- \to [K^-\pi^+]\pi^- \to K^-\pi^+\pi^-) = 0.174 \pm 0.025$ and $\mathcal{A}_{CP}(B^- \to K^-[\pi^+\pi^-] \to K^-\pi^+\pi^-) = 0.509 \pm 0.042$. Comparing these two results, we can see the contribution from the $[K\pi]$ resonances are much smaller than that from the $[\pi\pi]$

TABLE II. Direct CP asymmetries (in units of 10^{-2}) of resonant and nonresonant (NR) contributions to $B^- \to K^-\pi^+\pi^-$. The theoretical errors corresponding to the uncertainties due to the form factors, decay constants, Gegenbauer moments and divergence parameters.

Decay mode	*BABAR* [68]	PDG [62]	This work
σK^-	$\cdots$	$\cdots$	[−34, −11]
$\rho^0 K^-$	$44 \pm 10 \pm 4^{+5}_{-13}$	37 ± 10	32 ± 1.2
ωK^-	$\cdots$	-2 ± 4	-1 ± 0.1
$\kappa\pi^-$	$\cdots$	$\cdots$	[−18,10]
$K^{*0}(892)\pi^-$	$3.2 \pm 5.2 \pm 1.1^{+1.2}_{-0.7}$	$\cdots$	2.6 ± 1.7
$K^{*0}(1410)\pi^-$	$\cdots$	$\cdots$	2.4 ± 2.1
$K^{*0}(1680)\pi^-$	$\cdots$	$\cdots$	3.0 ± 2.5
$K_2^{*0}(1430)\pi^-$	$5 \pm 23 \pm 4^{+18}_{-7}$	5^{+29}_{-24}	3.5 ± 1.9
NR	$\cdots$	$16.9 \pm 1.3 \pm 1.3^{+1.1}_{-0.9}$	10.4 ± 1.2

resonances. This is because $B^- \to [K^-\pi^+]\pi$ decays are mediated by the $b \to s$ loop (penguin) transition without the $b \to u$ tree component as shown in Eqs. (44) and (46)–(50) and also because the resonance regions of $[K\pi]$ channel mesons have smaller widths and are further away from $[\pi\pi]$ channel mesons (ρ, ω and σ). Therefore, the contributions from the $[K\pi]$ channel resonances are much smaller than that from $[\pi\pi]$ channel resonances. Furthermore, using Eqs. (23)–(30) and Eq. (57), we also get that the nonresonance contribution as $\mathcal{A}_{\mathcal{CP}}{}^{\mathrm{NR}}(B^- \to K^-\pi^+\pi^-) = 0.061 \pm 0.0042$ which is also much smaller than that from the $[\pi\pi]$ resonances in our studied region $m^2_{K^-\pi^+} < 15\ \mathrm{GeV}^2$ and $0.08 < m^2_{\pi^+\pi^-} < 0.66\ \mathrm{GeV}^2$. Since both $\mathcal{A}_{\mathcal{CP}}(B^- \to [K^-\pi^+]\pi^- \to K^-\pi^+\pi^-)$ and $\mathcal{A}_{\mathcal{CP}}{}^{\mathrm{NR}}(B^- \to K^-\pi^+\pi^-)$ are much smaller than $\mathcal{A}_{\mathcal{CP}}(B^- \to K^-[\pi^+\pi^-] \to K^-\pi^+\pi^-)$, we can conclude that the large localized *CP* asymmetry $\mathcal{A}_{\mathcal{CP}}(B^- \to K^-\pi^+\pi^-) = 0.678 \pm 0.078 \pm 0.0323 \pm 0.007$ is mainly induced by the contributions from the $[\pi\pi]$ channel resonances.

VI. SUMMARY

In this work, within a quasi-two-body QCD factorization approach, we study the localized integrated *CP* violation in the $B^- \to K^-\pi^+\pi^-$ decay in the region $m^2_{K^-\pi^+} < 15\ \mathrm{GeV}^2$ and $0.08 < m^2_{\pi^+\pi^-} < 0.66\ \mathrm{GeV}^2$ by including the contributions from both resonances including $\sigma(600)$, $\rho^0(770)$ and $\omega(782)$ mesons from the $[\pi\pi]$ channel and $K^*(892)$, $K^*(1410)$, $K_0^*(1430)$, $K^*(1680)$ and $K_2^*(1430)$ mesons from the $[K\pi]$ channel. By fitting the experimental data $\mathcal{A}_{\mathcal{CP}}(B^- \to K^-\pi^+\pi^-) = 0.678 \pm 0.078 \pm 0.0323 \pm 0.007$ in the above experimental region, $\mathcal{A}_{\mathcal{CP}}(B^- \to K_0^*(1430)\pi^-) = 0.061 \pm 0.032$, $\mathcal{B}(B^- \to K_0^*(1430)\pi^-) = (39^{+6}_{-5}) \times 10^{-6}$ and $\mathcal{B}(B^- \to \sigma(600)\pi^- \to \pi^-\pi^+\pi^-) < 4.1 \times 10^{-6}$, it is found that there exist ranges of parameters ρ_S and ϕ_S which satisfy the above experimental data. Thus, the resonance and nonresonance contributions can indeed induce the data for the localized *CP* asymmetry in the $B^- \to K^-\pi^+\pi^-$ decay. The allowed ranges for ϕ_S and ρ_S are $\phi_S \in [1.77, 2.25]$ and $\rho_S \in [2.39, 4.02]$ which is larger than the previously conservative choice of $\rho \le 1$. In fact, there is no reason to restrict ρ to the range $\rho \le 1$ because the QCDF itself cannot give information and constraint on the parameter ρ and it can only be obtained through the experimental data. Large values of ρ_S reveal that the contributions from the weak annihilation and the hard spectator scattering processes are both large for the $B^- \to K^-\pi^+\pi^-$ decay. Especially, the contribution from the weak annihilation part should not be neglected. In fact, the large weak annihilation contributions have been observed and predicted in experimental and theoretical studies. So the larger values of ρ_S are acceptable when dealing with the divergence problems for the $B \to SP(PS)$ decays. With the obtained allowed ranges for ρ_S and ϕ_S, we predict the *CP* asymmetry parameter and the branching fraction for $B^- \to K^-\sigma$. The results are $\mathcal{A}_{\mathcal{CP}}(B^- \to K^-\sigma) \in [-0.34, -0.11]$ and $\mathcal{B}(B^- \to K^-\sigma) \in [6.53, 17.52] \times 10^{-6}$ when ρ_S and ϕ_S vary in their allowed ranges, respectively. In addition, we also calculate the localized *CP* asymmetry $\mathcal{A}_{\mathcal{CP}}(B^- \to K^-\pi^+\pi^-)$ only considering the $[\pi\pi]$, $[K\pi]$ resonances and nonresonance, respectively, in the same region $m^2_{K^-\pi^+} < 15\ \mathrm{GeV}^2$ and $0.08 < m^2_{\pi^+\pi^-} < 0.66\ \mathrm{GeV}^2$. The results are $\mathcal{A}_{\mathcal{CP}}(B^- \to [K^-\pi^+]\pi \to K^-\pi^+\pi^-) = 0.174 \pm 0.025$, $\mathcal{A}_{\mathcal{CP}}(B^- \to K^-[\pi^+\pi^-] \to K^-\pi^+\pi^-) = 0.509 \pm 0.042$ and $\mathcal{A}_{\mathcal{CP}}{}^{\mathrm{NR}}(B^- \to K^-\pi^+\pi^-) = 0.061 \pm 0.0042$, respectively. Therefore, the large localized *CP* asymmetry in the $B^- \to K^-\pi^+\pi^-$ is mainly induced by the contributions from the $[\pi\pi]$ channel resonances and the contributions from $[K\pi]$ channel resonances and nonresonance are very small.

ACKNOWLEDGMENTS

One of the authors (J.-J. Q.) is very grateful to Professor Hai-Yang Cheng, Professor Hsiang-nan Li and Professor Zhi-Gang Wang for valuable discussions. This work was supported by National Natural Science Foundation of China (Projects No. 11575023, No. 11775024, No. 11705081 and No. 11881240256).

[1] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963).
[2] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
[3] T. M. Yan, H. Y. Cheng, C. Y. Cheung, G. L. Lin, Y. C. Lin, and H. L. Yu, Phys. Rev. D **46**, 1148 (1992).
[4] M. B. Wise, Phys. Rev. D **45**, R2188 (1992).
[5] G. Burdman and J. F. Donoghue, Phys. Lett. B **280**, 287 (1992).
[6] H. Y. Cheng and C. K. Chua, Phys. Rev. D **88**, 114014 (2013).
[7] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. Lett. **111**, 101801 (2013).
[8] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. Lett. **112**, 011801 (2014).
[9] C. W. Chiang, M. Gronau, Z. Luo, J. L. Rosner, and D. A. Suprun, Phys. Rev. D **69**, 034001 (2004).
[10] M. Beneke and M. Neubert, Nucl. Phys. **B675**, 333 (2003).
[11] H. Y. Cheng, C. K. Chua, and K. C. Yang, Phys. Rev. D **73**, 014017 (2006).

[12] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, Nucl. Phys. **B606**, 245 (2001).
[13] Y. Y. Keum, H. n. Li, and A. I. Sanda, Phys. Lett. B **504**, 6 (2001).
[14] C. W. Bauer, S. Fleming, and M. E. Luke, Phys. Rev. D **63**, 014006 (2000).
[15] C. D. Lu, K. Ukai, and M. Z. Yang, Phys. Rev. D **63**, 074009 (2001).
[16] A. V. Manohar and I. W. Stewart, Phys. Rev. D **76**, 074002 (2007).
[17] C. M. Arnesen, Z. Ligeti, I. Z. Rothstein, and I. W. Stewart, Phys. Rev. D **77**, 054006 (2008).
[18] T. Aaltonen *et al.* (CDF Collaboration), Phys. Rev. Lett. **108**, 211803 (2012).
[19] R. Aaij *et al.* (LHCb Collaboration), J. High Energy Phys. 10 (2012) 037.
[20] Y.-T. Duh *et al.* (Belle Collaboration), Phys. Rev. D **87**, 031103 (2013).
[21] Z. J. Xiao, W. F. Wang, and Y. y. Fan, Phys. Rev. D **85**, 094003 (2012).
[22] M. Gronau, D. London, and J. L. Rosner, Phys. Rev. D **87**, 036008 (2013).
[23] Q. Chang, J. Sun, Y. Yang, and X. Li, Phys. Rev. D **90**, 054019 (2014).
[24] M. Wirbel, B. Stech, and M. Bauer, Z. Phys. C **29**, 637 (1985).
[25] H. Y. Cheng and K. C. Yang, Phys. Rev. D **83**, 034001 (2011).
[26] H. Y. Cheng and K. C. Yang, Phys. Rev. D **71**, 054020 (2005).
[27] H. Y. Cheng, C. K. Chua, and K. C. Yang, Phys. Rev. D **77**, 014034 (2008).
[28] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).
[29] H. Y. Cheng, C. K. Chua, K. C. Yang, and Z. Q. Zhang, Phys. Rev. D **87**, 114001 (2013).
[30] H. Y. Cheng, Y. Koike, and K. C. Yang, Phys. Rev. D **82**, 054019 (2010).
[31] H. Y. Cheng and C. K. Chua, Phys. Rev. D **80**, 114008 (2009).
[32] C. Wang, Z. Y. Wang, Z. H. Zhang, and X. H. Guo, Phys. Rev. D **93**, 116008 (2016).
[33] H. Y. Cheng, C. K. Chua, and A. Soni, Phys. Rev. D **76**, 094006 (2007).
[34] C. L. Y. Lee, M. Lu, and M. B. Wise, Phys. Rev. D **46**, 5040 (1992).
[35] S. Fajfer, R. J. Oakes, and T. N. Pham, Phys. Rev. D **60**, 054029 (1999).
[36] S. Ahmed *et al.* (CLEO Collaboration), Phys. Rev. Lett. **87**, 251801 (2001).
[37] H. Y. Cheng and K. C. Yang, Phys. Rev. D **66**, 054015 (2002).
[38] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. D **79**, 072006 (2009).
[39] H. Y. Cheng, Nucl. Part. Phys. Proc. **273**, 1290 (2016).
[40] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **99**, 161802 (2007).
[41] S. J. Brodsky and G. R. Farrar, Phys. Rev. D **11**, 1309 (1975).
[42] C. K. Chua, W. S. Hou, S. Y. Shiau, and S. Y. Tsai, Phys. Rev. D **67**, 034012 (2003).
[43] J.-P. Dedonder, A. Furman, R. Kaminski, L. Lesniak, and B. Loiseau, Acta Phys. Pol. B **42**, 2013 (2011).
[44] S. Gardner, H. B. O'Connell, and A. W. Thomas, Phys. Rev. Lett. **80**, 1834 (1998).
[45] X. H. Guo and A. W. Thomas, Phys. Rev. D **58**, 096013 (1998).
[46] C. Wang, Z. H. Zhang, Z. Y. Wang, and X. H. Guo, Eur. Phys. J. C **75**, 536 (2015).
[47] X. H. Guo, O. M. A. Leitner, and A. W. Thomas, Phys. Rev. D **63**, 056012 (2001).
[48] I. Bediaga, G. Guerrer, and J. M. de Miranda, Phys. Rev. D **76**, 073011 (2007).
[49] H. B. O'Connell, A. W. Thomas, and A. G. Williams, Nucl. Phys. **A623**, 559 (1997); K. Maltman, H. B. O'Connell, and A. G. Williams, Phys. Lett. B **376**, 19 (1996).
[50] S. Gardner and H. B. O'Connell, Phys. Rev. D **57**, 2716 (1998).
[51] D. V. Bugg, J. Phys. G **34**, 151 (2007).
[52] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. D **92**, 032002 (2015).
[53] Y. Li, A. J. Ma, W. F. Wang, and Z. J. Xiao, Eur. Phys. J. C **76**, 675 (2016).
[54] Z. H. Zhang, X. H. Guo, and Y. D. Yang, Phys. Rev. D **87**, 076007 (2013).
[55] K. A. Olive *et al.* (Particle Data Group), Chin. Phys. C **38**, 090001 (2014).
[56] C. Wang, X. H. Guo, Y. Liu, and R. C. Li, Eur. Phys. J. C **74**, 3140 (2014).
[57] J.-P. Dedonder, R. Kaminski, L. Lesniak, and B. Loiseau, Phys. Rev. D **89**, 094018 (2014).
[58] J. J. Qi, Z. Y. Wang, X. H. Guo, Z. H. Zhang, and J. Xu, Eur. Phys. J. C **78**, 845 (2018).
[59] H. Hatanaka and K. C. Yang, Phys. Rev. D **79**, 114008 (2009).
[60] H. Hatanaka and K. C. Yang, Eur. Phys. J. C **67**, 149 (2010).
[61] A. Deandrea and A. D. Polosa, Phys. Rev. Lett. **86**, 216 (2001).
[62] M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018).
[63] C. Bobeth, M. Gorbahn, and S. Vickers, Eur. Phys. J. C **75**, 340 (2015).
[64] M. Ciuchini, E. Franco, A. Masiero, and L. Silvestrini, Phys. Rev. D **67**, 075016 (2003).
[65] J. Sun, Q. Chang, X. Hu, and Y. Yang, Phys. Lett. B **743**, 444 (2015).
[66] G. Zhu, Phys. Lett. B **702**, 408 (2011).
[67] Y. Y. Keum, H. N. Li, and A. I. Sanda, Phys. Rev. D **63**, 054008 (2001).
[68] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. D **78**, 012004 (2008).
[69] A. Garmash *et al.* (Belle Collaboration), Phys. Rev. Lett. **96**, 251803 (2006).