Rare nonleptonic \bar{B}^0_s decays as probes of new physics behind $b \rightarrow s\mu\bar{\mu}$ anomalies

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The anomalous results of recent measurements on various $b \to s\mu^+\mu^-$ processes could be initial evidence of physics beyond the standard model (SM). Assuming this to be the case, we entertain the possibility that the underlying new physics also affects the rare nonleptonic decays of the \bar{B}_s^0 meson. We consider in particular new physics arising from the interactions of a heavy Z' boson and investigate their influence on the decay modes $\bar{B}_s^0 \to (\eta, \eta', \phi)\omega$, which receive sizable QCD- and electroweak-penguin contributions. These decays are not yet observed, and their rates are estimated to be relatively small in the SM. Taking into account the pertinent constraints, we find that the Z' effects can greatly increase the rates of $\bar{B}_s^0 \to (\eta, \phi)\omega$, by as much as two orders of magnitude, with respect to the SM expectations. We have previously shown that $\bar{B}_s^0 \to (\eta, \phi)\pi^0$, with similarly suppressed SM rates, could also undergo substantial Z'-induced enhancement. These rare modes can therefore serve as complementary probes of the potential new physics which may be responsible for the $b \to s\mu^+\mu^-$ anomalies.

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I. INTRODUCTION

The current data on a number of $b \rightarrow s\mu^+\mu^-$ transitions have manifested several tantalizing deviations from the expectations of the standard model (SM). Specifically, the LHCb Collaboration [1] found moderate tensions with the SM in an angular analysis of the decay $B^0 \rightarrow K^{*0}\mu^+\mu^-$, which were later corroborated in the Belle experiment [2]. Moreover, LHCb reported [3] that the ratio R_K of the branching fractions of $B^+ \rightarrow K^+\mu^+\mu^-$ and $B^+ \rightarrow K^+e^+e^$ decays and the corresponding ratio R_{K^*} for $B^0 \rightarrow K^{*0}\mu^+\mu^$ and $B^0 \rightarrow K^{*0}e^+e^-$ decays are a couple of sigmas below their SM predictions [4]. Also, the existing measurements [5,6] on the branching fractions of $B \rightarrow K^{(*)}\mu^+\mu^-$ and $B_s \rightarrow \phi\mu^+\mu^-$ favor values below their SM estimates.

These anomalies may be harbingers of physics beyond the SM, although their statistical significance is still insufficient for drawing a definite conclusion. In fact, model-independent theoretical studies have pointed out that new physics (NP) could explain them [7,8]. This would

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suggest that they might be experimentally confirmed in the near future to have originated from beyond the SM. Thus, it seems timely to explore what might happen if the same underlying NP could have an appreciable influence on some other $b \rightarrow s$ processes.

Previously we have entertained such a possibility in a scenario where a new, electrically neutral and uncolored, spin-one particle, the Z' boson, is behind the $b \rightarrow s\mu^+\mu^$ anomalies [9]. In particular, we investigated the potential implications for the nonleptonic decays of the \bar{B}^0_s meson which are purely isospin-violating, namely $\bar{B}_s^0 \rightarrow$ $(\eta, \eta', \phi)(\pi^0, \rho^0)$, most of which are not yet observed [6]. In the SM limit, they are not affected by QCD-penguin operators, which conserve isospin, while the effects of tree operators are suppressed by a factor $|V_{us}V_{ub}|/|V_{ts}V_{tb}| \sim$ 0.02 involving Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, and thus the amplitudes for these decays tend to be dominated by electroweak-penguin contributions [10]. Accordingly, their rates in the SM are comparatively small [10–19], which motivated earlier works suggesting that one or more of these decay modes could be sensitive to NP signals [19–24]. Incorporating the relevant constraints, we demonstrated in Ref. [9] that the Z' influence could cause the rates of two of the modes, $\bar{B}_s^0 \to (\eta, \phi) \pi^0$, to rise by up to an order of magnitude above their SM expectations. It follows that these modes could offer valuable complementary information about the NP which may be responsible for the $b \rightarrow s\mu^+\mu^-$ anomalies.

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Extending our preceding analysis, the present paper covers $\bar{B}_s \rightarrow (\eta, \eta', \phi)\omega$, which are also not yet observed [6]. Here, as in the $\bar{B}_s^0 \rightarrow (\eta, \eta', \phi)(\pi^0, \rho^0)$ case, the tree operators suffer from the CKM suppression, again allowing the penguin operators to become important. However, unlike the latter modes, $\bar{B}_s \rightarrow (\eta, \eta', \phi)\omega$ preserve isospin and therefore receive both electroweak- and QCD-penguin contributions. In the SM, the rates of these decays turn out to be relatively small as well [12–17], and so they could be expected to serve as additional probes of the potential NP behind the anomalies. We will show that this can indeed be realized in the aforementioned Z' model, especially for the two modes $\bar{B}_s \rightarrow (\eta, \phi)\omega$.

The remainder of the paper is organized as follows. In Sec. II we describe the Z' interactions which impact the various processes of concern. In Sec. III we address how the Z'-induced effects on the considered rare nonleptonic decays could raise some of their rates significantly. We will impose appropriate restraints on the Z' couplings, including from other $b \rightarrow s$ transitions, such as $\bar{B}_s \rightarrow \phi \rho^0$ and $B \rightarrow \pi K$ decays.¹ Our numerical work will also involve $\bar{B}_s \rightarrow (\eta, \phi) \pi^0$, which we investigated before, to see if there might be any correlation between their rate enlargement and that of $\bar{B}_s \rightarrow (\eta, \phi) \omega$. In Sec. IV we give our conclusions. An Appendix contains extra formulas.

II. Z' INTERACTIONS

In our Z' scenario of interest, the mass eigenstates of the u, d, s, and b quarks have nonstandard interactions described by [9]

$$\mathcal{L}_{Z'} \supset -[\bar{s}\gamma^{\kappa}(\Delta_L^{sb}P_L + \Delta_R^{sb}P_R)bZ'_{\kappa} + \text{H.c.}] - \Delta_V^{\mu\mu}\bar{\mu}\gamma^{\kappa}\mu Z'_{\kappa} - [\bar{u}\gamma^{\kappa}(\Delta_L^{uu}P_L + \Delta_R^{uu}P_R)u + \bar{d}\gamma^{\kappa}(\Delta_L^{dd}P_L + \Delta_R^{dd}P_R)d]Z'_{\kappa},$$
(1)

where the constants $\Delta_{L,R}^{sb}$ are generally complex, while $\Delta_V^{\mu\mu}$ and $\Delta_{L,R}^{uu,dd}$ are real due to the Hermiticity of $\mathcal{L}_{Z'}$, and $P_{L,R} = (1 \mp \gamma_5)/2$. As in Ref. [9], we suppose that any other possible couplings of the Z' to SM fermions are negligible and that it does not mix with SM gauge bosons but is not necessarily a gauge boson.² Moreover, for simplicity we concentrate on the special case in which

$$\Delta_L^{sb} = \lambda_t \rho_L, \quad \Delta_R^{sb} = \lambda_t \rho_R, \quad \lambda_q = V_{qs}^* V_{qb}, \tag{2}$$

where $\rho_{L,R}$ are real numbers.

For the Z' being heavy, the couplings to $b\bar{s}$ and $\mu\bar{\mu}$ in Eq. (1) contribute to the effective Lagrangian

$$\mathcal{L}_{\text{eff}} \supset \frac{\alpha_{\text{e}}\lambda_{t}G_{\text{F}}}{\sqrt{2\pi}} (C_{9\mu}\bar{s}\gamma^{\kappa}P_{L}b + C_{9'\mu}\bar{s}\gamma^{\kappa}P_{R}b)\bar{\mu}\gamma_{\kappa}\mu + \text{H.c.}, \quad (3)$$

where $\alpha_{\rm e}$ and $G_{\rm F}$ are the usual fine-structure and Fermi constants, and $C_{9\mu} = C_{9\ell}^{\rm SM} + C_{9\mu}^{\rm NP}$ and $C_{9'\mu} = C_{9'\mu}^{\rm NP}$ are the Wilson coefficients, with $C_{9\ell}^{\rm SM}$ being the flavor-universal SM part ($\ell = e, \mu, \tau$) and [9]

$$C_{9\mu}^{\rm NP} = \frac{-\sqrt{2}\pi\rho_L \Delta_V^{\mu\mu}}{\alpha_{\rm e}G_{\rm F}m_{Z'}^2}, \qquad C_{9'\mu}^{\rm NP} = \frac{-\sqrt{2}\pi\rho_R \Delta_V^{\mu\mu}}{\alpha_{\rm e}G_{\rm F}m_{Z'}^2}.$$
 (4)

According to model-independent analyses [7], one of the best fits to the anomalous $b \to s\mu^+\mu^-$ data corresponds to $C_{9\mu}^{\rm NP} \sim -1.1$ and $C_{9'\mu}^{\rm NP} \sim 0.4$, with no NP in $b \to se^+e^-$.

The bsZ' couplings in $\mathcal{L}_{Z'}$ above also affect $B_s - \bar{B}_s$ mixing at tree level and hence need to satisfy the restrictions inferred from its data. As elaborated in Ref. [9], the requirements from $b \rightarrow s\mu^+\mu^-$ processes and $B_s - \bar{B}_s$ mixing together imply that the left-handed bsZ' coupling must be roughly ten times stronger than the right-handed one, and so $\rho_L \sim 10\rho_R$. This will be taken into account later on.

Additionally, $\mathcal{L}_{Z'}$ can yield modifications to nonleptonic transitions, such as $\bar{B}_s \rightarrow (\eta, \eta', \phi)\omega$. In the SM, their amplitudes proceed from $b \rightarrow s$ four-quark operators $O_{1,2}^u$, $O_{3,4,5,6}$, and $O_{7,8,9,10}$ derived from charmless tree, QCD-penguin, and electroweak-penguin diagrams, respectively.³ In models beyond the SM, new ingredients may alter the Wilson coefficients C_j of O_j and/or generate extra operators \tilde{O}_j which are the chirality-flipped counterparts of O_j . In our Z' case, at the W-mass (m_W) scale only $C_{3,5,7,9}$ and $\tilde{C}_{3,5,7,9}$ get Z' contributions given by [20–22,27]

$$\mathcal{L}_{\text{eff}} \supset \sqrt{8}\lambda_{t}G_{\text{F}}\sum_{q=u,d} \left\{ \bar{s}\gamma^{\kappa}P_{L}b\left[\left(C_{3} + \frac{3}{2}C_{9}e_{q} \right)\bar{q}\gamma_{\kappa}P_{L}q + \left(C_{5} + \frac{3}{2}C_{7}e_{q} \right)\bar{q}\gamma_{\kappa}P_{R}q \right] \right. \\ \left. + \bar{s}\gamma^{\kappa}P_{R}b\left[\left(\tilde{C}_{3} + \frac{3}{2}\tilde{C}_{9}e_{q} \right)\bar{q}\gamma_{\kappa}P_{R}q + \left(\tilde{C}_{5} + \frac{3}{2}\tilde{C}_{7}e_{q} \right)\bar{q}\gamma_{\kappa}P_{L}q \right] \right\},$$

$$(5)$$

¹The possibility of NP in $b \to s\mu^+\mu^-$ producing detectable implications for the $B \to \pi K$ decays has previously been brought up in [25]. ²In the literature pertaining to the anomalies, different Z' models have been explored, some of which can be found in [26]. ³The expressions for O_j , j = 1, 2, ..., 10, are available from, e.g., [18].

where $C_i = C_i^{\text{SM}} + C_i^{Z'}$ and $\tilde{C}_i = \tilde{C}_i^{Z'}$ for i = 3, 5, 7, 9 are the Wilson coefficients with [9]

$$C_{3,5}^{Z'} = \frac{\rho_L(-\delta_{L,R} - 3\Delta_{L,R}^{dd})}{6\sqrt{2}G_F m_{Z'}^2}, \qquad C_{7,9}^{Z'} = \frac{-\rho_L \delta_{R,L}}{3\sqrt{2}G_F m_{Z'}^2},$$
$$\tilde{C}_{3,5}^{Z'} = \frac{\rho_R(-\delta_{R,L} - 3\Delta_{R,L}^{dd})}{6\sqrt{2}G_F m_{Z'}^2}, \qquad \tilde{C}_{7,9}^{Z'} = \frac{-\rho_R \delta_{L,R}}{3\sqrt{2}G_F m_{Z'}^2}, \quad (6)$$

$$\delta_L = \Delta_L^{uu} - \Delta_L^{dd}, \qquad \delta_R = \Delta_R^{uu} - \Delta_R^{dd}, \qquad (7)$$

we have assumed that renormalization group evolution (RGE) between the $m_{Z'}$ and m_W scales can be neglected,

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and $e_u = -2e_d = 2/3$. At the *b*-quark mass (m_b) scale, all the penguin coefficients acquire Z' terms via RGE, which we treat in the next section.

III. Z' EFFECTS ON RARE NONLEPTONIC \bar{B}_s DECAYS

To estimate the Z' impact on $\bar{B}_s \rightarrow (\eta, \eta', \phi)\omega$, following Ref. [9] we employ the soft-collinear effective theory (SCET) [16–18,28]. For any one of them, we can write the SCET amplitude at leading order in the strong coupling $\alpha_s(m_b)$ as [17]

$$\mathcal{A}_{\bar{B}_s \to M_1 M_2} = \frac{f_{M_1} G_{\rm F} m_{B_s}^2}{\sqrt{2}} \left[\int_0^1 d\nu (\zeta_J^{BM_2} T_{1J}(\nu) + \zeta_{Jg}^{BM_2} T_{1Jg}(\nu)) \phi_{M_1}(\nu) + \zeta^{BM_2} T_1 + \zeta_g^{BM_2} T_{1g} \right] + (1 \leftrightarrow 2), \tag{8}$$

where f_M denotes the decay constant of meson M, the ζ s are nonperturbative hadronic parameters extractable from experiment, the Ts represent hard kernels containing the Wilson coefficients C_j and \tilde{C}_j at the m_b scale, and $\phi_M(\nu)$ is the light-cone distribution amplitude of M normalized as $\int_0^1 d\nu \phi_M(\nu) = 1$. We collect the hard kernels, from Refs. [16–18], in Table I, where the flavor states $\eta_q \sim (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\eta_s \sim s\bar{s}$ are linked to the physical meson states η and η' by $\eta = \eta_q \cos \theta - \eta_s \sin \theta$ and $\eta' = \eta_q \sin \theta + \eta_s \cos \theta$ with mixing angle $\theta = 39.3^{\circ}$ [17,18,29]. We note that the so-called charming-penguin contribution is absent from $\mathcal{A}_{\bar{B}_s \to M_1 M_2}$, which is one of the reasons why these decays have low rates [16,17].

In the presence of NP which also gives rise to O_j , the quantities c_k and b_k in Table I depend not only on C_j and \tilde{C}_j but also on the final mesons $M_{1,2}$ besides the CKM factors $\lambda_{u,t}$. The dependence on $M_{1,2}$ is due to the fact that, in view of the nonzero kernels in this table, for each 4-quark operator the contraction of the $\bar{B}_s \rightarrow M_1$ and vacuum $\rightarrow M_2$ matrix elements in the amplitude can lead to an overall negative or positive sign for the contribution of the operator, the sign being fixed by the chirality combination of the operator and by whether $M_{1,2}$ are pseudoscalars (PP'), vectors (VV'), or PV. Thus, for $\bar{B}_s \rightarrow PP'$ and $\bar{B}_s \rightarrow \phi \omega$ we have

$$c_{2} = \lambda_{u} \left(C_{2}^{-} + \frac{C_{1}^{-}}{N_{c}} \right) - \frac{3\lambda_{t}}{2} \left(C_{9}^{-} + \frac{C_{10}^{-}}{N_{c}} \right), \qquad c_{3} = -\frac{3\lambda_{t}}{2} \left(C_{7}^{-} + \frac{C_{8}^{-}}{N_{c}} \right), \qquad c_{5,6} = -\lambda_{t} \left(C_{3,5}^{-} + \frac{C_{4,6}^{-}}{N_{c}} - \frac{C_{9,7}^{-}}{2} - \frac{C_{10,8}^{-}}{2N_{c}} \right), \\ b_{2} = \lambda_{u} \left[C_{2}^{-} + \left(1 - \frac{m_{b}}{\omega_{3}} \right) \frac{C_{1}^{-}}{N_{c}} \right] - \frac{3\lambda_{t}}{2} \left[C_{9}^{-} + \left(1 - \frac{m_{b}}{\omega_{3}} \right) \frac{C_{10}^{-}}{N_{c}} \right], \qquad b_{3} = -\frac{3\lambda_{t}}{2} \left[C_{7}^{-} + \left(1 - \frac{m_{b}}{\omega_{2}} \right) \frac{C_{8}^{-}}{N_{c}} \right], \\ b_{5,6} = -\lambda_{t} \left[C_{3,5}^{-} + \left(1 - \frac{m_{b}}{\omega_{3}} \right) \frac{C_{4,6}^{-}}{N_{c}} - \frac{C_{9,7}^{-}}{2} - \left(1 - \frac{m_{b}}{\omega_{3}} \right) \frac{C_{10,8}^{-}}{2N_{c}} \right], \qquad (9)$$

where $N_c = 3$ is the color number, $C_j = C_j - \tilde{C}_j$, and $b_{2,3,5,6}$, which enter $T_{2J,2J_g}(\nu)$, are also functions of ν because $\omega_2 = \nu m_{B_s}$ and $\omega_3 = (\nu - 1)m_{B_s}$ [17]. However, for $\bar{B}_s \to (\eta_q, \eta_s)\omega$ we need to make the sign change $C_j \to C_j^+ = C_j + \tilde{C}_j$ in $c_{2,3,5,6}$ and $b_{2,3,5,6}$.

TABLE I. Hard kernels $T_{1,2,1g,2g}$ for $\bar{B}_s \to (\eta, \eta', \phi)\omega$ decays. The hard kernels $T_{rJ,rJg}(\nu)$ for r = 1, 2 are obtainable from $T_{r,rg}$, respectively, with the replacement $c_k \to b_k$, where b_k depends on ν .

Decay mode	T_1	T_2	T_{1g}	T_{2g}
$\overline{\bar{B}_s \to \eta_s \omega}$	0	$\frac{1}{\sqrt{2}}(c_2+c_3+2c_5+2c_6)$	0	$\frac{1}{\sqrt{2}}(c_2+c_3+2c_5+2c_6)$
$\bar{B}_s \to \eta_q \omega$	0	0	0	$c_2 + c_3 + 2c_5 + 2c_6$
$\bar{B}_s \to \phi \omega$	0	$\frac{1}{\sqrt{2}}(c_2 + c_3 + 2c_5 + 2c_6)$	0	0

The formulas in Eq. (9) generalize the SM ones from Refs. [16–18], which also provide the C_j^{SM} values at the m_b scale, $C_{1,2}^{\text{SM}} = (1.11, -0.253)$ and $C_{7,8,9,10}^{\text{SM}} = (0.09, 0.24, -10.3, 2.2) \times 10^{-3}$, calculated at leading-logarithm order in the naive dimensional regularization scheme [30] with the prescription of Ref. [31]. We will incorporate these numbers into c_k and b_k . The Z'-generated coefficients in Eq. (6) contribute to Eq. (9) via $C_i = C_i^{\text{SM}} + \delta C_i$ and $\tilde{C}_i = \delta \tilde{C}_i$ for i = 3, 4, ...10, where δC_i are linear combinations of $C_{3,5,7,9}^{Z'}$ due to RGE from the m_W scale to the m_b scale and $\delta \tilde{C}_i$ are analogously related to $\tilde{C}_{3,5,7,9}^{Z'}$.

To evaluate $\mathcal{A}_{\bar{B}_s \to M_1 M_2}$, in light of Table I, we employ the decay constant $f_{\omega} = 192$ MeV [16] and treat the integral in Eq. (8) with the aid of $\int_0^1 d\nu \phi_M(\nu)/\nu = \int_0^1 d\nu \phi_M(\nu)/(1-\nu) \equiv \langle \chi^{-1} \rangle_M$ for $M = \eta_{q,s}, \phi$, in which cases $\langle \chi^{-1} \rangle_{\eta_{q,s}} = 3.3$ and $\langle \chi^{-1} \rangle_{\phi} = 3.54$ [17,18]. Furthermore, for the ζ 's in $\mathcal{A}_{\bar{B}_s \to (\eta_q, \eta_s)\omega}$, we adopt the two solutions from the fit to data performed in Ref. [17]:

$$\begin{aligned} &(\zeta^P, \zeta^P_J, \zeta_g, \zeta_{Jg})_1 = (0.137, 0.069, -0.049, -0.027), \\ &(\zeta^P, \zeta^P_J, \zeta_g, \zeta_{Jg})_2 = (0.141, 0.056, -0.100, 0.051). \end{aligned} \tag{10}$$

It is worth remarking here that, since they were the outcome of fitting to $B \rightarrow PP'$, PV data with SM Wilson coefficients [17], using these solutions in an investigation of NP is justifiable provided that the impact of the NP on the channels which dominate the fit is small compared the SM contribution. This requisite can be met by our Z' scenario, as we set the mass of the Z' to be $\mathcal{O}(1 \text{ TeV})$ and ensure that its couplings comply with the various constraints described in this study. From Eq. (10), we then have $\zeta_{(J)g}^{B\eta_{q,s}} = \zeta_{(J)g}$, assuming flavor-SU(3) symmetry [17]. Other input parameters are CKM matrix elements from Ref. [32] as well as the meson masses $m_{\eta} = 547.862$, $m_{\eta'} = 957.78$, $m_{\omega} = 782.65$, $m_{\phi} = 1019.461$, and $m_{B_s} = 5366.89$, all in

TABLE II. Branching fractions, in units of 10^{-6} , of $\bar{B}_s \rightarrow (\eta, \eta', \phi)\omega$ decays in the SM. For the first two modes, the last two columns correspond to the two solutions of SCET parameters in Eq. (10). The second and third columns exhibit numbers computed with QCDF [12] and PQCD [14,15].

			SCET		
Decay mode	QCDF	PQCD	Solution 1	Solution 2	
$\overline{\bar{B}_s \to \eta \omega}$	$0.03\substack{+0.12+0.06\\-0.02-0.01}$	$0.11\substack{+0.04 \\ -0.03}$	$0.04\substack{+0.04 \\ -0.02}$	$0.007\substack{+0.011\\-0.002}$	
$\bar{B}_s \to \eta' \omega$	$0.15\substack{+0.27+0.15\\-0.08-0.06}$	$0.35\substack{+0.06 \\ -0.04}$	$0.001\substack{+0.095\\-0.000}$	$0.20\substack{+0.34 \\ -0.17}$	
$\bar{B}_s \to \phi \omega$	$0.18\substack{+0.44+0.47\\-0.12-0.04}$	$0.22\substack{+0.15 \\ -0.10}$	0.04 =	± 0.01	

units of MeV, and the B_s mean lifetime $\tau_{B_s} = 1.509 \times 10^{-12}$ s, which are their central values from Ref. [6]. For the third $(\phi \omega)$ mode, we choose the CKM and SCET parameters supplied recently in Ref. [16].

Before dealing with the Z' influence on $\bar{B}_s \rightarrow (\eta, \eta', \phi)\omega$ numerically, with the above SCET prescription we arrive at their SM branching fractions, listed in Table II. For $\bar{B}_s \rightarrow (\eta, \eta')\omega$, the entries in the last two columns correspond to the two solutions of SCET parameters in Eq. (10). The central values of the SCET predictions agree with those in Refs. [16,17], from which we have added the errors shown. For comparison, in the second and third columns we quote results from the QCD factorization (QCDF) [12] and perturbative QCD (PQCD) [14,15] approaches. Evidently, the SCET numbers can be compatible with the QCDF and PQCD ones within the sizable errors. One concludes that for NP to be unambiguously noticeable in the rates it would have to amplify them relative to their SM ranges by significantly more than a factor of two.

In the presence of the Z' contributions, we find the changes of the Wilson coefficients C_j at the m_b scale to be

$$\delta C_{1,2} = 0, \qquad \delta C_3 \simeq 1.13 C_3^{Z'}, \qquad \delta C_4 \simeq -0.29 C_3^{Z'}, \qquad \delta C_5 \simeq 0.93 C_5^{Z'}, \qquad \delta C_6 \simeq 0.29 C_5^{Z'}, \\ \delta C_7 \simeq 0.93 C_7^{Z'}, \qquad \delta C_8 \simeq 0.31 C_7^{Z'}, \qquad \delta C_9 \simeq 1.11 C_9^{Z'}, \qquad \delta C_{10} \simeq -0.25 C_9^{Z'}, \qquad (11)$$

where in each coefficient we have kept only the Z' term with the biggest numerical factor, upon applying the RGE at leading-logarithm order [30] with $\alpha_e = 1/128$, $\alpha_s(m_Z) = 0.119$, $m_b = 4.8$ GeV, and $m_t = 174.3$ GeV [18]. Furthermore, at the m_b scale $\tilde{C}_j = \delta \tilde{C}_j$ are related to $\tilde{C}_{3,5,7,9}^{Z'}$ in an analogous manner. Combining the SM and Z' portions, for $m_{Z'} = 1$ TeV and the central values of the input parameters we derive the amplitudes for $\bar{B}_s \rightarrow (\eta, \eta', \phi)\omega$ to be, in units of 10^{-9} GeV,

$$\begin{aligned} \mathcal{A}_{\bar{B}_{s} \to \eta \omega}^{(1)} &\simeq -0.63 + 2.05i + [(6.06 - 0.12i)\delta_{+} + (12.10 - 0.23i)\Delta_{+}]\rho_{+} + (0.02\delta_{-} + 0.01\Delta_{-})\rho_{-}, \\ \mathcal{A}_{\bar{B}_{s} \to \eta' \omega}^{(1)} &\simeq 0.05 - 0.33i - [(1.22 - 0.02i)\delta_{+} + (2.44 - 0.05i)\Delta_{+}]\rho_{+}, \\ \mathcal{A}_{\bar{B}_{s} \to \eta \omega}^{(2)} &\simeq 0.52 + 0.74i + [(5.83 - 0.11i)\delta_{+} + (11.70 - 0.22i)\Delta_{+}]\rho_{+} + 0.01\Delta_{-}\rho_{-}, \\ \mathcal{A}_{\bar{B}_{s} \to \eta' \omega}^{(2)} &\simeq 3.23 - 3.83i - [(1.37 - 0.03i)\delta_{+} + (2.72 - 0.05i)\Delta_{+}]\rho_{+} - (0.05\delta_{-} + 0.01\Delta_{-})\rho_{-}, \\ \mathcal{A}_{\bar{B}_{s} \to \phi \omega}^{(2)} &\simeq -1.69 - 1.41i - [(14.00 - 0.26i)\delta_{+} + (28.10 - 0.53i)\Delta_{+}]\rho_{-} - (0.01\delta_{-} + 0.03\Delta_{-})\rho_{+}, \end{aligned}$$
(12)

where the superscripts (1) and (2) refer to SCET Solutions 1 and 2, respectively, Z' terms with numerical factors below 0.005 in size are not displayed, and

$$\delta_{\pm} = \delta_L \pm \delta_R, \qquad \Delta_{\pm} = \Delta_L^{dd} \pm \Delta_R^{dd}, \qquad \rho_{\pm} = \rho_L \pm \rho_R.$$
(13)

Given that δ_{\pm} and ρ_{\pm} participate in the amplitudes for $\bar{B}_s^0 \rightarrow (\eta, \eta', \phi)(\pi^0, \rho^0)$ as well [9], as Eqs. (A1)–(A3) in the Appendix show, it is germane to include them in this analysis. What is more, as discussed in Ref. [9], the LHCb finding $\mathcal{B}(\bar{B}_s \rightarrow \phi \rho^0)_{exp} = (0.27 \pm 0.08) \times 10^{-6}$ [6,33], which is in line with some of its SM estimates albeit within large errors [12–14,16,19,21], translates into an important constraint on the Z' couplings. Additionally, treating all these rare decays at the same time would allow us to see if there might be correlations among their rate increases/decreases compared to the SM expectations. Such correlations would constitute Z' predictions potentially testable in upcoming experiments.

The couplings in Eq. (13) also affect other nonleptonic $b \rightarrow s$ processes which have been observed and hence need to respect the restrictions implied by their data. Here we focus on the well-measured decays $B^- \rightarrow \pi^0 K^-, \pi^- \bar{K}^0$ and $\bar{B}^0 \rightarrow \pi^0 \bar{K}^0, \pi^+ K^-$ plus their antiparticle counterparts. Their rates in the SM, with ~40% errors [18], agree with their measurements [6]. Incorporating the Z' terms, we have calculated the $B \rightarrow \pi K$ amplitudes and collected their expressions in Eq. (A6) and Eq. (A7).

To illustrate how the Z' interactions contribute to the decays of interest, we put together 5,000 randomly generated benchmarks fulfilling the following conditions. We impose

$$0.11 \le 10^6 \mathcal{B}(\bar{B}_s \to \phi \rho^0) \le 0.43,$$
 (14)

which is the 2σ range of $\mathcal{B}(\bar{B}_s \to \phi \rho^0)_{exp}$. For the $B \to \pi K$ requirement, since the SM predictions have uncertainties of around 40% and are compatible with their data, we demand that the Z' effects alter the $B \to \pi K$ rates relative to their SM values by no more than 20%.⁴ For the Z' parameters, we select $\rho_R = 0.1\rho_L$ as in Ref. [9] and let the products of ρ_L and the other nonleptonic couplings vary within the intervals

$$\delta_{\pm}\rho_L \in [-1, 1], \qquad \Delta_{\pm}\rho_L \in [-1, 1],$$
(15)

having already set $m_{Z'} = 1$ TeV in Eq. (12) and in the other amplitudes written down in the Appendix. We present

the results in the two figures below which depict twodimensional projections of the benchmarks for a number of quantities.

In Fig. 1 we display the distributions of the enhancement factor

$$\mathcal{R}(MM') = \frac{\Gamma_{\bar{B}_s \to MM'}}{\Gamma^{\rm SM}_{\bar{B}_s \to MM'}} \tag{16}$$

of the $\bar{B}_s \to MM'$ rate with respect to its SM prediction for a few pairs of final states MM'. As the top-left plot reveals, $\mathcal{R}(\eta\omega)$ and $\mathcal{R}(\phi\omega)$ go up or down simultaneously and can reach roughly 50 and 150 (270 and 170), respectively, for Solution 1 (2). It follows that, in light of the SCET central values in Table II, the Z' influence can boost the branching fractions of $\bar{B}_s \to \eta\omega$ and $\bar{B}_s \to \phi\omega$ to $\sim 2 \times 10^{-6}$ and $\sim 6 \times 10^{-6}$, respectively, for both solutions. Accordingly, these decay channels are potentially sensitive to NP signals, and moreover the correlation between $\mathcal{R}(\eta\omega)$ and $\mathcal{R}(\phi\omega)$ may be experimentally checked.

As regards $\bar{B}_s \to \eta'\omega$, for which we do not provide any graphs, with Solution 1 (2) we get at most $\mathcal{R}(\eta'\omega) \sim 80$ (only 2.5), which translates into $\mathcal{B}(\bar{B}_s \to \eta'\omega) \lesssim$ $0.08(0.5) \times 10^{-6}$ based on its SCET central values in Table II. However, the corresponding upper error in this table for Solution 1 suggests that $\mathcal{B}(\bar{B}_s \to \eta'\omega)$ might still undergo a Z'-mediated boost to the 10^{-6} level, thereby offering an additional window to the Z' interactions.

The top-right plot in Fig. 1 indicates that $\mathcal{R}(\eta\pi^0)$ and $\mathcal{R}(\phi\pi^0)$, like $\mathcal{R}(\eta\omega)$ and $\mathcal{R}(\phi\omega)$, increase/decrease at the same time, although the former two can rise to only about 8.0 and 4.5 (10 and 7.3), respectively, for Solution 1 (2). Nevertheless, as elaborated in Ref. [9], such enhancement factors are sufficiently sizable to make $\bar{B}_s \to (\eta, \phi)\pi^0$ promising as extra tools in the quest for the potential NP behind the $b \to s\mu^+\mu^-$ anomalies. The correlation between $\mathcal{R}(\eta\pi^0)$ and $\mathcal{R}(\phi\pi^0)$ is obviously a testable prediction as well. We notice that the preceding Solution-2 numbers are roughly 20% less than their counterparts (12 and 9.1) in Ref. [9], mostly due to the aforementioned $B \to \pi K$ requisite.

From the bottom plots in Fig. 1, unlike the top ones, it is not evident if there is a connection between $\mathcal{R}(\eta\omega)$ or $\mathcal{R}(\phi\omega)$ and $\mathcal{R}(\eta\pi^0)$. The former two also do not seem to have clear correlations with $\mathcal{R}(\phi\pi^0)$, although this is not illustrated here. We will ignore possibly related consequences for $\bar{B}^0_s \to \eta' \pi^0$, $(\eta, \eta') \rho^0$ because the Z' impact on their rates is only modest [9].

Information about relationships between $\mathcal{R}(MM')$ and the Z' couplings is highly valuable for examining the latter if one or more of these decays are observed. For our decay channels of greatest interest, it turns out that there are a few relationships that are more or less plain, which we exhibit in Fig. 2. As might be expected, the curves in the fourth plot resemble the corresponding ones in Ref. [9].

⁴The *CP* asymmetry difference $\Delta A_{CP}^{\exp} = A_{CP}^{\exp}(B^- \to \pi^0 K^-) - A_{CP}^{\exp}(\bar{B}^0 \to \pi^+ K^-) = 0.122 \pm 0.022$ [6] might be another restraint. It excludes the SM central values $\Delta A_{CP}^{SM} \simeq -0.01$ for the two SCET solutions, but the theoretical error is large, ~0.15 [18], implying that the predictions are still consistent with ΔA_{CP}^{\exp} . Similarly, although our Z' benchmarks yield $-0.025 \lesssim \Delta A_{CP}^{SM+Z'} \lesssim 0.002$, they are not in conflict with ΔA_{CP}^{\exp} , considering the substantial theoretical uncertainty.



FIG. 1. Distributions of the ratio $\mathcal{R}(MM') = \Gamma_{\bar{B}_s \to MM'} / \Gamma_{\bar{B}_s \to MM'}^{SM}$ among different pairs of final states MM' for the benchmarks corresponding to SCET Solutions 1 (blue, S1) and 2 (red, S2).

It is worth noting that the restriction we imposed above from the $B \rightarrow \pi K$ sector is of significance to some extent, although how stringent the condition should be is unclear in view of the 40% uncertainties of the SM rate predictions [18]. For illustration, making it stricter so that the Z'-induced changes to the $B \rightarrow \pi K$ rates in the SM not exceed 10%, we arrive at the graphs in Fig. 3. In this instance, $\mathcal{R}(\eta \pi^0)$ and $\mathcal{R}(\phi \pi^0)$, especially the latter, become somewhat less remarkable than before, but $\mathcal{R}(\eta\omega)$ and $\mathcal{R}(\phi\omega)$ are still considerable, and so all these decays remain useful for probing the Z' effects. We further find, however, that if the $B \to \pi K$ rates are allowed to deviate from the SM expectations by up to 25% or higher, the impact on the maxima of these \mathcal{R} s will start to diminish and they can have the wider spreads depicted in Fig. 4.



FIG. 2. Distributions of $\mathcal{R}(\eta\omega)$ and $\mathcal{R}(\phi\omega)$ versus $\Delta_+\rho_L$ and of $\mathcal{R}(\eta\pi^0)$ and $\mathcal{R}(\phi\pi^0)$ versus $\delta_-\rho_L$ for SCET Solutions 1 (S1) and 2 (S2).



FIG. 3. The same as the top two plots in Fig. 1, but with the stronger requirement that the Z'-induced modifications to the $B \rightarrow \pi K$ rates in the SM not exceed 10%.



FIG. 4. The same as Fig. 3, but with the weaker requirement that the Z'-induced modifications to the $B \rightarrow \pi K$ rates in the SM not exceed 25%.

Lastly, we comment that the Z' couplings in our benchmarks are consistent with collider constraints, as discussed in Ref. [9], including those from LHC searches for new highmass phenomena in the dilepton final states [34]. This is partly because the products of the muon-Z' coupling $\Delta_V^{\mu\mu}$ and the quark-Z' flavor-diagonal couplings ($\delta_{\pm}, \Delta_{\pm}$) can be rendered small enough to evade the bounds by sufficiently increasing the size of ρ_L while maintaining $\Delta_V^{\mu\mu}\rho_L$ in Eq. (4) and ($\delta_{\pm}, \Delta_{\pm}$) ρ_L to stay within their desired respective ranges.

IV. CONCLUSIONS

We have explored the possibility that the anomalies detected in the current $b \rightarrow s\mu^+\mu^-$ data arise from physics beyond the SM and that the same new physics also affects the rare nonleptonic decays of the \bar{B}_s meson, most of which are not yet observed. Since the rates of these modes in the SM are comparatively low, one or more of them may be sensitive to NP signals. Adopting a scenario in which the NP is due to the interactions of a heavy Z' boson, we investigate the implications for the rare decays $\bar{B}_s^0 \rightarrow (\eta, \eta', \phi)(\pi^0, \rho^0, \omega)$. Taking into account the pertinent restraints, we demonstrate that the Z' effects can hugely amplify the rates of $\bar{B}_s^0 \rightarrow (\eta, \phi)\omega$ above their SM predictions, by as much as two orders of magnitude. The corresponding enhancement factors for $\bar{B}_s^0 \rightarrow (\eta, \phi)\pi^0$ could be substantial as well, up to an order of magnitude, in line with our previous work. Thus, these four rare modes are potentially very consequential should future experiments establish that the $b \rightarrow s\mu^+\mu^-$ anomalies are really manifestations of NP.

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APPENDIX: ADDITIONAL AMPLITUDES

We have derived the main formulas for the Z' contributions to $\bar{B}_s \rightarrow (\eta, \eta', \phi)(\pi^0, \rho^0)$ in Ref. [9] under the SCET framework. Therein we did not include Solution 1 in the evaluation of the Z' effects and neglected renormalization group evolution for simplicity. In the present paper, we include the latter and give results for

both Solutions 1 and 2. Thus, summing the SM and Z' terms, with the central values of the input parameters and $m_{Z'} = 1$ TeV, we calculate the amplitudes to be, in units of 10^{-9} GeV,

$$\begin{aligned} \mathcal{A}_{\bar{B}_{s} \to \eta \pi^{0}}^{(1)} &\simeq 1.43 + 1.31i + 0.03\delta_{+}\rho_{-} + (4.15 - 0.08i)\delta_{-}\rho_{+}, \\ \mathcal{A}_{\bar{B}_{s} \to \eta' \pi^{0}}^{(1)} &\simeq -0.31 - 0.21i - (0.84 - 0.02i)\delta_{-}\rho_{+}, \\ \mathcal{A}_{\bar{B}_{s} \to \eta' \pi^{0}}^{(1)} &\simeq -1.63 - 2.53i - (5.57 - 0.11i)\delta_{-}\rho_{-} - 0.08\delta_{+}\rho_{+}, \\ \mathcal{A}_{\bar{B}_{s} \to \eta \rho^{0}}^{(1)} &\simeq 2.18 + 2.17i + 0.06\delta_{-}\rho_{-} + (6.60 - 0.13i)\delta_{+}\rho_{+}, \\ \mathcal{A}_{\bar{B}_{s} \to \eta' \rho^{0}}^{(1)} &\simeq -0.47 - 0.34i - 0.01\delta_{-}\rho_{-} - (1.33 - 0.03i)\delta_{+}\rho_{+}, \\ \mathcal{A}_{\bar{B}_{s} \to \eta' \rho^{0}}^{(2)} &\simeq 1.67 + 0.47i + (3.98 - 0.08i)\delta_{-}\rho_{+}, \\ \mathcal{A}_{\bar{B}_{s} \to \eta' \pi^{0}}^{(2)} &\simeq 0.48 - 2.48i - 0.09\delta_{+}\rho_{-} - (1.00 - 0.02i)\delta_{-}\rho_{+}, \\ \mathcal{A}_{\bar{B}_{s} \to \eta' \pi^{0}}^{(2)} &\simeq -2.88 - 1.69i - (7.58 - 0.15i)\delta_{-}\rho_{-} - 0.03\delta_{+}\rho_{+}, \\ \mathcal{A}_{\bar{B}_{s} \to \eta' \rho^{0}}^{(2)} &\simeq 2.56 + 0.77i + (6.35 - 0.12i)\delta_{+}\rho_{+}, \\ \mathcal{A}_{\bar{B}_{s} \to \eta' \rho^{0}}^{(2)} &\simeq 0.78 - 4.12i - 0.16\delta_{-}\rho_{-} - (1.52 - 0.03i)\delta_{+}\rho_{+}, \\ \mathcal{A}_{\bar{B}_{s} \to \eta' \rho^{0}}^{(2)} &\simeq -6.53 - 1.47i - (15.50 - 0.29i)\delta_{+}\rho_{-} + 0.01\delta_{-}\rho_{+}, \end{aligned}$$

where the superscripts (1) and (2) refer to Solutions 1 and 2, respectively, Z' terms with numerical factors below 0.005 are not displayed, and δ_{\pm} and ρ_{\pm} were already defined in Eq. (13). We note that in SCET at leading order the rates of $\bar{B}_s^0 \rightarrow (\eta, \eta', \phi)(\pi^0, \rho^0, \omega)$ are equal to their antiparticle counterparts [16–18]. We have checked that under the same requirements as in Ref. [9] the resulting maximal enhancement factor of the $\bar{B}_s^0 \rightarrow \eta \pi^0(\phi \pi^0)$ rate is around 12 (8.6), which is almost identical to (5% below) what we determined earlier [9] ignoring RGE. Imposing also the $B \rightarrow \pi K$ condition as discussed in Sec. III may lead to smaller enhancement factors, depending on how stringent it is.

In the SCET approach the SM amplitudes for the $B \rightarrow \pi K$ channels are dominated by the so-called charming-penguin terms. The relevant hard kernels are available from Ref. [18], with the quantities $c_{1,2,3,4}$ and $b_{1,2,3,4}$ now involving $C_j^- = C_j - \tilde{C}_j$, analogously to the $\bar{B}_s \rightarrow PP'$ case. Including the Z' parts similarly to the previous paragraph, with Solution 1 we obtain, in units of 10^{-9} GeV,

$$\begin{aligned} \mathcal{A}_{B^- \to \pi^0 K^-}^{(1)} &\simeq -35.10 + 1.45i - [(3.32 - 0.06i)\delta_+ + (3.19 - 0.06i)\Delta_+]\rho_- - [(5.24 - 0.10i)\delta_- + 0.21\Delta_-]\rho_+, \\ \mathcal{A}_{B^- \to \pi^- \bar{K}^0}^{(1)} &\simeq -47.10 + 9.63i + [0.06\delta_+ - (4.51 - 0.09i)\Delta_+]\rho_- + [0.02\delta_- - (0.30 - 0.01i)\Delta_-]\rho_+, \\ \mathcal{A}_{\bar{B}^0 \to \pi^0 \bar{K}^0}^{(1)} &\simeq 31.60 - 8.40i - [0.08\delta_+ - (3.19 - 0.06i)\Delta_+]\rho_- - [(5.01 - 0.10i)\delta_- - 0.21\Delta_-]\rho_+, \\ \mathcal{A}_{\bar{B}^0 \to \pi^+ K^-}^{(1)} &\simeq -47.30 + 4.30i - [(4.63 - 0.09i)\delta_+ + (4.51 - 0.09i)\Delta_+]\rho_- - [(0.33 - 0.01i)\delta_- + (0.30 - 0.01i)\Delta_-]\rho_+, \end{aligned}$$
(A4)

$$\begin{aligned} \mathcal{A}_{B^{+} \to \pi^{0} K^{+}}^{(1)} &\simeq -35.10 + 11.90i - [(2.94 + 0.06i)\delta_{+} + (2.81 + 0.05i)\Delta_{+}]\rho_{-} - [(4.86 + 0.09i)\delta_{-} - 0.17\Delta_{-}]\rho_{+}, \\ \mathcal{A}_{B^{+} \to \pi^{+} K^{0}}^{(1)} &\simeq -47.60 + 9.15i + [0.06\delta_{+} - (3.97 + 0.08i)\Delta_{+}]\rho_{-} + (0.02\delta_{-} + 0.24\Delta_{-})\rho_{+}, \\ \mathcal{A}_{B^{0} \to \pi^{0} K^{0}}^{(1)} &\simeq 32.00 - 4.89i - [0.09\delta_{+} - (2.81 + 0.05i)\Delta_{+}]\rho_{-} - [(5.02 - 0.10i)\delta_{-} + 0.17\Delta_{-}]\rho_{+}, \\ \mathcal{A}_{B^{0} \to \pi^{-} K^{+}}^{(1)} &\simeq -47.20 + 14.60i - [(4.10 + 0.08i)\delta_{+} + (3.97 + 0.08i)\Delta_{+}]\rho_{-} + (0.20\delta_{-} + 0.24\Delta_{-})\rho_{+}, \end{aligned}$$
(A5)

and with Solution 2

$$\begin{aligned} \mathcal{A}_{B^{-} \to \pi^{0} K^{-}}^{(2)} &\simeq -35.10 + 2.04i - [(2.96 - 0.06i)\delta_{+} + (2.85 - 0.05i)\Delta_{+}]\rho_{-} - (4.88 - 0.09i)\delta_{-}\rho_{+}, \\ \mathcal{A}_{B^{-} \to \pi^{-} \bar{K}^{0}}^{(2)} &\simeq -47.40 + 9.95i + [0.06\delta_{+} - (4.02 - 0.08i)\Delta_{+}]\rho_{-} + 0.02\delta_{-}\rho_{+}, \\ \mathcal{A}_{\bar{B}^{0} \to \pi^{0} \bar{K}^{0}}^{(2)} &\simeq 31.70 - 8.38i - [0.07\delta_{+} - (2.85 - 0.05i)\Delta_{+}]\rho_{-} - (4.87 - 0.09i)\delta_{-}\rho_{+}, \\ \mathcal{A}_{\bar{B}^{0} \to \pi^{+} K^{-}}^{(2)} &\simeq -47.20 + 4.77i - [(4.14 - 0.08i)\delta_{+} + (4.02 - 0.08i)\Delta_{+}]\rho_{-} - 0.03\delta_{-}\rho_{+}, \end{aligned}$$
(A6)

$$\begin{split} \mathcal{A}_{B^{+} \to \pi^{0} K^{+}}^{(2)} &\simeq -35.10 + 11.80i - [(2.65 + 0.05i)\delta_{+} + (2.54 + 0.05i)\Delta_{+}]\rho_{-} - [(4.57 + 0.09i)\delta_{-} - (0.31 + 0.01i)\Delta_{-}]\rho_{+}, \\ \mathcal{A}_{B^{+} \to \pi^{+} K^{0}}^{(2)} &\simeq -47.80 + 9.49i + [0.06\delta_{+} - (3.59 + 0.07i)\Delta_{+}]\rho_{-} + [0.02\delta_{-} + (0.44 + 0.01i)\Delta_{-}]\rho_{+}, \\ \mathcal{A}_{B^{0} \to \pi^{0} K^{0}}^{(2)} &\simeq 32.00 - 5.37i - [0.07\delta_{+} - (2.54 + 0.05i)\Delta_{+}]\rho_{-} - [(4.87 + 0.09i)\delta_{-} + (0.31 + 0.01i)\Delta_{-}]\rho_{+}, \\ \mathcal{A}_{B^{0} \to \pi^{-} K^{+}}^{(2)} &\simeq -47.20 + 14.80i - [(3.71 + 0.07i)\delta_{+} + (3.59 + 0.07i)\Delta_{+}]\rho_{-} + [(0.40 + 0.01i)\delta_{-} + (0.44 + 0.01i)\Delta_{-}]\rho_{+}. \end{split}$$
(A7)

In the SM limit, for Solution 1(2) these amplitudes yield the *CP*-averaged branching fractions $\mathcal{B}(B^+ \to \pi^0 K^+)_{SM} = 12.1(12.1)$, $\mathcal{B}(B^+ \to \pi^+ K^0)_{SM} = 21.7(21.9)$, $\mathcal{B}(B^0 \to \pi^0 K^0)_{SM} = 9.1(9.2)$, and $\mathcal{B}(B^0 \to \pi^- K^+)_{SM} = 20.3(20.3)$, all in units of 10⁻⁶, with uncertainties of about 40% [18]. In view of the errors, these predictions are compatible with their experimental counterparts [6] $\mathcal{B}(B^+ \to \pi^0 K^+) = 12.9 \pm 0.5$, $\mathcal{B}(B^+ \to \pi^+ K^0) = 23.7 \pm 0.8$, $\mathcal{B}(B^0 \to \pi^0 K^0) = 9.9 \pm 0.5$, and $\mathcal{B}(B^0 \to \pi^- K^+) = 19.6 \pm 0.5$, all in units of 10⁻⁶.

- R. Aaij *et al.* (LHCb Collaboration), J. High Energy Phys. 02 (2016) 104.
- [2] S. Wehle *et al.* (Belle Collaboration), Phys. Rev. Lett. **118**, 111801 (2017).
- [3] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. Lett. 113, 151601 (2014); J. High Energy Phys. 08 (2017) 055.
- [4] G. Hiller and F. Kruger, Phys. Rev. D 69, 074020 (2004); C. Bouchard, G. P. Lepage, C. Monahan, H. Na, and J. Shigemitsu (HPQCD Collaboration), Phys. Rev. Lett. 111, 162002 (2013); 112, 149902(E) (2014); M. Bordone, G. Isidori, and A. Pattori, Eur. Phys. J. C 76, 440 (2016).
- [5] R. Aaij *et al.* (LHCb Collaboration), J. High Energy Phys. 06 (2014) 133; 09 (2015) 179.
- [6] M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D 98, 030001 (2018).
- [7] B. Capdevila, A. Crivellin, S. Descotes-Genon, J. Matias, and J. Virto, J. High Energy Phys. 01 (2018) 093; W. Altmannshofer, P. Stangl, and D. M. Straub, Phys. Rev. D 96, 055008 (2017).
- [8] G. D'Amico, M. Nardecchia, P. Panci, F. Sannino, A. Strumia, R. Torre, and A. Urbano, J. High Energy Phys. 09 (2017) 010; G. Hiller and I. Nisandzic, Phys. Rev. D 96, 035003 (2017); L. S. Geng, B. Grinstein, S. Jäger, J. Martin Camalich, X. L. Ren, and R. X. Shi, Phys. Rev. D 96, 093006 (2017); M. Ciuchini, A. M. Coutinho, M. Fedele, E. Franco, A. Paul, L. Silvestrini, and M. Valli, Eur. Phys. J. C.

77, 688 (2017); A. Celis, J. Fuentes-Martin, A. Vicente, and J. Virto, Phys. Rev. D **96**, 035026 (2017); T. Hurth, F. Mahmoudi, D. Martinez Santos, and S. Neshatpour, Phys. Rev. D **96**, 095034 (2017).

- [9] G. Faisel and J. Tandean, J. High Energy Phys. 02 (2018) 074.
- [10] R. Fleischer, Phys. Lett. B 332, 419 (1994); N.G. Deshpande, X.G. He, and J. Trampetic, Phys. Lett. B 345, 547 (1995).
- [11] B. Tseng, Phys. Lett. B 446, 125 (1999); Y.H. Chen, H. Y. Cheng, and B. Tseng, Phys. Rev. D 59, 074003 (1999).
- [12] H. Y. Cheng and C. K. Chua, Phys. Rev. D 80, 114026 (2009).
- [13] A. Ali, G. Kramer, Y. Li, C. D. Lu, Y. L. Shen, W. Wang, and Y. M. Wang, Phys. Rev. D 76, 074018 (2007).
- [14] D. C. Yan, X. Liu, and Z. J. Xiao, Nucl. Phys. B935, 17 (2018).
- [15] D. C. Yan, P. Yang, X. Liu, and Z. J. Xiao, Nucl. Phys. B931, 79 (2018).
- [16] C. Wang, S. H. Zhou, Y. Li, and C. D. Lu, Phys. Rev. D 96, 073004 (2017).
- [17] W. Wang, Y. M. Wang, D. S. Yang, and C. D. Lu, Phys. Rev. D 78, 034011 (2008).
- [18] A. R. Williamson and J. Zupan, Phys. Rev. D 74, 014003 (2006); 74, 039901(E) (2006).

- **731**, 279 (2014).
- [20] J. Hua, C. S. Kim, and Y. Li, Phys. Lett. B 690, 508 (2010).
- [21] L. Hofer, D. Scherer, and L. Vernazza, J. High Energy Phys. 02 (2011) 080.
- [22] Q. Chang, X. Q. Li, and Y. D. Yang, J. Phys. G 41, 105002 (2014).
- [23] G. Faisel, Eur. Phys. J. C 77, 380 (2017).
- [24] C. Bobeth, M. Gorbahn, and S. Vickers, Eur. Phys. J. C 75, 340 (2015).
- [25] N. B. Beaudry, A. Datta, D. London, A. Rashed, and J. S. Roux, J. High Energy Phys. 01 (2018) 074; R. Fleischer, R. Jaarsma, and K. K. Vos, Phys. Lett. B 785, 525 (2018).
- [26] A. Crivellin, G. D'Ambrosio, and J. Heeck, Phys. Rev. Lett. 114, 151801 (2015); Phys. Rev. D 91, 075006 (2015); G. Belanger, C. Delaunay, and S. Westhoff, Phys. Rev. D 92, 055021 (2015); K. Fuyuto, W. S. Hou, and M. Kohda, Phys. Rev. D 93, 054021 (2016); C. W. Chiang, X. G. He, and G. Valencia, Phys. Rev. D 93, 074003 (2016); D. Becirevic, O. Sumensari, and R. Zukanovich Funchal, Eur. Phys. J. C 76, 134 (2016); C. S. Kim, X. B. Yuan, and Y. J. Zheng, Phys. Rev. D 93, 095009 (2016); B. Bhattacharya, A. Datta, J.-P. Guévin, D. London, and R. Watanabe, J. High Energy Phys. 01 (2017) 015; P. Ko, T. Nomura, and H. Okada, Phys. Lett. B 772, 547 (2017); Phys. Rev. D 95, 111701 (2017); P. Ko, Y. Omura, Y. Shigekami, and C. Yu, Phys. Rev. D 95, 115040 (2017); W. Wang and S. Zhao, Chin. Phys. C 42, 013105 (2018); A. K. Alok, B. Bhattacharya, A. Datta, D. Kumar, J. Kumar, and D. London, Phys. Rev. D 96, 095009 (2017); C. Bonilla, T. Modak, R. Srivastava, and J. W. F. Valle, Phys. Rev. D 98, 095002 (2018); A. Datta, J. Kumar, J. Liao, and D. Marfatia, Phys. Rev. D 97, 115038 (2018); C. W. Chiang, X. G. He, J. Tandean, and X. B. Yuan, Phys. Rev. D 96, 115022 (2017); S. F. King, J. High Energy Phys. 08 (2017) 019; 09 (2018) 069; R.S. Chivukula, J. Isaacson, K.A. Mohan, D. Sengupta, and E. H. Simmons, Phys. Rev. D 96, 075012 (2017); J. M. Cline and J. Martin Camalich, Phys. Rev. D 96, 055036 (2017); C. H. Chen and T. Nomura, Phys. Lett. B 777, 420 (2018); S. Baek, Phys. Lett. B 781,

376 (2018); L. Bian, S. M. Choi, Y. J. Kang, and H. M. Lee, Phys. Rev. D 96, 075038 (2017); M. Abdullah, M. Dalchenko, B. Dutta, R. Eusebi, P. Huang, T. Kamon, D. Rathjens, and A. Thompson, Phys. Rev. D 97, 075035 (2018); S. Antusch, C. Hohl, S. F. King, and V. Susic, Nucl. Phys. B934, 578 (2018); K. Fuyuto, H. L. Li, and J. H. Yu, Phys. Rev. D 97, 115003 (2018); M. Chala and M. Spannowsky, Phys. Rev. D 98, 035010 (2018); A. Falkowski, S. F. King, E. Perdomo, and M. Pierre, J. High Energy Phys. 08 (2018) 061; L. Mu, H. Okada, and C. Q. Geng, Chin. Phys. C 42, 123106 (2018); M. Kohda, T. Modak, and A. Soffer, Phys. Rev. D 97, 115019 (2018); Fayyazuddin, M. J. Aslam, and C. D. Lu, Int. J. Mod. Phys. A 33, 1850087 (2018); A. Nasrullah, F. M. Bhutta, and M. J. Aslam, J. Phys. G 45, 095007 (2018); S. Baek and C. Yu, J. High Energy Phys. 11 (2018) 054; B. Allanach and J. Davighi, J. High Energy Phys. 12 (2018) 075; F. Z. Xu, W. Zhang, J. Li, and T. Li, Phys. Rev. D 98, 115033 (2018); B.C. Allanach, T. Corbett, M.J. Dolan, and T. You, arXiv:1810.02166.

- [27] V. Barger, L. L. Everett, J. Jiang, P. Langacker, T. Liu, and C. E. M. Wagner, J. High Energy Phys. 12 (2009) 048.
- [28] C. W. Bauer, S. Fleming, D. Pirjol, and I. W. Stewart, Phys. Rev. D 63, 114020 (2001); J. Chay and C. Kim, Phys. Rev. D 68, 071502 (2003); Nucl. Phys. B680, 302 (2004); C. W. Bauer, D. Pirjol, I. Z. Rothstein, and I. W. Stewart, Phys. Rev. D 70, 054015 (2004); C. W. Bauer, I. Z. Rothstein, and I. W. Stewart, Phys. Rev. D 74, 034010 (2006).
- [29] T. Feldmann, P. Kroll, and B. Stech, Phys. Rev. D 58, 114006 (1998); Phys. Lett. B 449, 339 (1999).
- [30] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, Rev. Mod. Phys. 68, 1125 (1996).
- [31] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, Nucl. Phys. B606, 245 (2001).
- [32] M. Bona *et al.* (UTfit Collaboration), J. High Energy Phys. 03 (2008) 049, Online updates available at http://www.utfit .org.
- [33] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. D **95**, 012006 (2017).
- [34] M. Aaboud *et al.* (ATLAS Collaboration), J. High Energy Phys. 10 (2017) 182.