Direct CP violation in multi-body B decays with $a_0^0(980)-f_0(980)$ mixing

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We predict that $a_0^0(980)-f_0(980)$ mixing would lead to large CP violation. We calculate the localized direct CP asymmetry in the decays $B^{\pm}\to f_0(980)[a_0^0(980)]\pi^{\pm}\to\pi^+\pi^-\pi^{\pm}$ via the $a_0^0(980)-f_0(980)$ mixing mechanism based on the hypothetical $q\bar{q}$ structures of $a_0^0(980)$ and $f_0(980)$ in the QCD factorization approach. It is shown that there is a peak for CP violation, which could be as large as 58%, when the invariant mass of $\pi\pi$ is near the masses of $a_0^0(980)$ and $f_0(980)$. Since the CP asymmetry is sensitive to $a_0^0(980)-f_0(980)$ mixing, measuring the CP-violating parameter in the aforementioned decays could provide a new way to verify the existence of $a_0^0(980)-f_0(980)$ mixing and be helpful in clarifying the configuration of the light scalar mesons.

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CP violation is one of the most fundamental and important properties of the weak interactions. Even though it has been known since 1964 [1], we still do not know the source of CP violation completely. In the standard model, CP violation is originated from the weak phase in the Cabibbo-Kobayashi-Maskawa matrix [2,3]. Besides the weak phase, a large strong phase is also needed for direct CP violation in decay processes. Usually, this large phase is provided by the short-distance and long-distance interactions. The short-distance interactions are caused by QCD loop corrections, and the long-distance interaction—which is more sensitive to the structure of the final states—can be obtained through some phenomenological mechanisms. It was suggested a long time ago that large CP violation should be observed in the B-meson systems. In the past few years, more attention has been focused on CP violation in the multibody B-meson decays both theoretically and experimentally. Inspired by the experimental progress, more efforts should be carried out to precisely test the standard model and look for new physics through CP violation in these decay processes.

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The scalars below 1 GeV play an important role in understanding nonperturbative QCD. The $a_0^0(980)$ and $f_0(980)$ mesons aroused considerable theoretical interest, but their structures are still controversial. These two mesons, which have different isospins but the same spin parity quantum numbers and closed masses, lie near the threshold of the $K\bar{K}$ channel and both of them couple to $K\bar{K}$. Due to the fact that the amplitude of the isospin-breaking transition is caused by the mass difference of $K\bar{K}$, a mixing will occur between the $f_0(980)$ and $a_0^0(980)$ intermediate states in the multibody decays. $a_0^0(980)-f_0(980)$ mixing was discovered theoretically in the late 1970s [4] and has recently been studied experimentally in several decays by the BESIII Collaboration [5]. However, the mixing mechanism between these two mesons still lacks firm experimental evidence. In previous works, it was found that ρ-ω mixing, which is also introduced due to the isospin violation, generates large strong phases and thus enhances the CP violation when the invariant mass of the final $\pi\pi$ state is in the ρ-ω interference region. Inspired by ρ-ω mixing, we expect that $a_0^0(980)-f_0(980)$ mixing may lead to large CP violation. Since the CP asymmetry contains more information on the strong phase than the decay width, we propose to test $a_0^0(980)-f_0(980)$ mixing by the much more subtle calculation of CP violation. We will investigate direct CP violation in three-body decays of the B meson and discuss $a_0^0(980)$–$f_0(980)$ mixing.

Both $a_0^0(980)$ and $f_0(980)$ can couple to $K\bar{K}$. Due to the nonzero difference between the K^+K^- and $K^0\bar{K}^0$ masses, the isospin-breaking processes $a_0^0(980)\to(K^+K^-+K^0\bar{K}^0)\to f_0(980)$ and $f_0(980)\to(K^+K^-+K^0\bar{K}^0)\to a_0^0(980)$

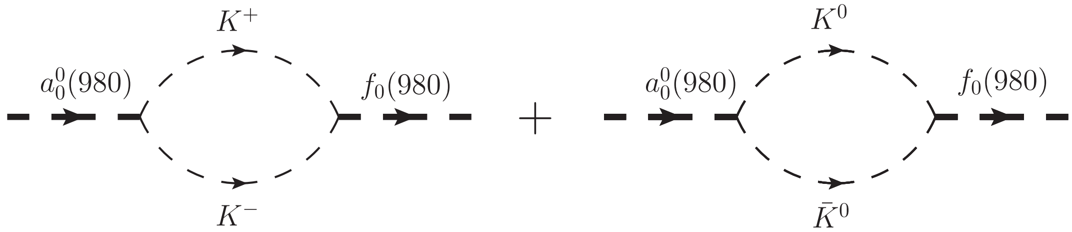

FIG. 1. The Feynman diagram for $a_0^0(980) - f_0(980)$ mixing.

appear in the narrow region of the $K\bar{K}$ thresholds, which are shown in Fig. 1. These $K\bar{K}$ loops would lead to a mixing amplitude. In Ref. [4], this mixing mechanism was investigated phenomenologically. The amplitude of $a_0^0(980) - f_0(980)$ mixing can be written as [4,6]

$$\Pi_{a_0^0 f_0}(m^2) = \frac{g_{a_0^0 K^+K^-} g_{f_0 K^+K^-}}{16\pi}\left\{ \mathrm{i}[\rho_{K^+K^-}(m^2) - \rho_{K^0\bar{K}^0}(m^2)] - \frac{\rho_{K^+K^-}(m^2)}{\pi}\ln\frac{1+\rho_{K^+K^-}(m^2)}{1-\rho_{K^+K^-}(m^2)} + \frac{\rho_{K^0\bar{K}^0}(m^2)}{\pi}\ln\frac{1+\rho_{K^0\bar{K}^0}(m^2)}{1-\rho_{K^0\bar{K}^0}(m^2)}\right\}$$
$$\approx \frac{g_{a_0^0 K^+K^-} g_{f_0 K^+K^-}}{16\pi}\mathrm{i}[\rho_{K^+K^-}(m^2) - \rho_{K^0\bar{K}^0}(m^2)], \tag{1}$$

where $g_{a_0^0(f_0)K^+K^-}$ are the effective coupling constants, $\rho_{K\bar{K}}(m^2) = \sqrt{1-4m_K^2/m^2}$ when m (the invariant masses of scalar resonances) $\geq 2m_K$, and $\rho_{K\bar{K}}(m^2)$ should be replaced by $\mathrm{i}|\rho_{K\bar{K}}(m^2)|$ in the region $0 \leq m \leq 2m_K$.

In recent years, the LHCb Collaboration has focused on multibody final states in the decays of the B mesons and used a novel strategy to probe CP asymmetries in the Dalitz plots [7]. These multibody decays provide much more information on strong phases than the two-body decays. Naturally, $a_0^0(980) - f_0(980)$ mixing, if it exists, would affect the CP violation of the sequential three-body decay $B \to f_0(980)[a_0^0(980)]P \to \pi\pi P$ (where P represents a pseudoscalar meson) when the invariant mass of the final $\pi\pi$ is located around 980 MeV.

For the sequential decays $B \to f_0(980)[a_0^0(980)]P \to \pi\pi P$, the decay width has the form

$$\frac{d\Gamma}{dm} = \frac{m}{16\pi^3 m_B^2}|\mathbf{p_1^*}||\mathbf{p_3}||\mathcal{M}|^2, \tag{2}$$

where m is the invariant mass of $\pi\pi$, m_B is the mass of the B meson, $\mathbf{p_1^*}[= (1-4m_\pi^2/m^2)^{\frac{1}{2}}]$, and $\mathbf{p_3} = [m_B^4 - 2m_B^2(m^2 + m_P^2) + (m_B^2 - m_P^2)^2]/(2m_B)$ with m_P being the mass of P. The amplitudes can be expressed as

$$\mathcal{M} = \langle\pi^+\pi^- P|\mathcal{H}^T|B\rangle + \langle\pi^+\pi^- P|\mathcal{H}^P|B\rangle$$
$$= \langle\pi^+\pi^- P|\mathcal{H}^T|B\rangle[1 + r e^{\mathrm{i}(\delta+\phi)}], \tag{3}$$

where $\mathcal{H}^T$ and $\mathcal{H}^P$ are the tree and penguin operators, respectively, and we also define the strong phase δ, the weak phase ϕ, and the relative magnitude r. Considering $a_0^0(980) - f_0(980)$ mixing, one has

$$\langle\pi^+\pi^- P|\mathcal{H}^T|B\rangle = \frac{g_{f\pi\pi}T_{f_0}}{D_{f_0}} + \frac{g_{f\pi\pi}T_{a_0^0}\Pi_{a_0^0 f_0}}{D_{a_0^0}D_{f_0} - \Pi_{a_0^0 f_0}^2},$$
$$\langle\pi^+\pi^- P|\mathcal{H}^P|B\rangle = \frac{g_{f\pi\pi}P_{f_0}}{D_{f_0}} + \frac{g_{f\pi\pi}P_{a_0^0}\Pi_{a_0^0 f_0}}{D_{a_0^0}D_{f_0} - \Pi_{a_0^0 f_0}^2}, \tag{4}$$

in which $T_{a_0^0(f_0)}$ and $P_{a_0^0(f_0)}$ correspond to the tree and penguin diagram amplitudes for $B \to a_0^0(980)[f_0(980)]P$, respectively, $g_{f\pi\pi}$ is the effective coupling constant of $f_0(980) \to \pi^+\pi^-$, and $D_{a_0^0}[D_{f_0}]$ is the inverse propagator of $a_0^0(980)[f_0(980)]$ constructed with taking into account the finite width. The expression for $D_{a_0^0(f_0)}$ can be written as

$$D_{a_0^0(f_0)}(s) = m_{a_0^0(f_0)}^2 - m^2 + \sum_{ab}[\Re e\Pi_{a_0^0(f_0)}^{ab}(m_{a_0^0(f_0)}^2) - \Pi_{a_0^0(f_0)}^{ab}(m_{a_0^0(f_0)}^2)], \tag{5}$$

where $\Pi_{a_0^0(f_0)}^{ab}$ are the diagonal matrix elements of the polarization operator corresponding to the contribution of the ab intermediate states, with $ab = (\pi\pi, K\bar{K})$ for $f_0(980)$ and $ab = (\eta\pi, K\bar{K})$ for $a_0^0(980)$.

Substituting Eq. (4) into Eq. (3), we have

$$r e^{\mathrm{i}(\delta+\phi)} = \frac{\langle\pi^+\pi^- P|\mathcal{H}^P|B\rangle}{\langle\pi^+\pi^- P|\mathcal{H}^T|B\rangle} \approx \frac{P_{f_0}D_{a_0^0} + P_{a_0^0}\Pi_{a_0^0 f_0}}{T_{f_0}D_{a_0^0} + T_{a_0^0}\Pi_{a_0^0 f_0}}. \tag{6}$$

It can be seen from Eq. (6) that $a_0^0(980) - f_0(980)$ mixing provides additional complex terms which may enlarge the CP-even phase and lead to a peak for CP violation when the invariant mass of $\pi\pi$ is near the $a_0^0(980)$ and $f_0(980)$

mesons. The differential CP-violating parameter can be defined as

$$A_{CP} \equiv \frac{|\mathcal{M}|^2 - |\bar{\mathcal{M}}|^2}{|\mathcal{M}|^2 + |\bar{\mathcal{M}}|^2} = \frac{-2r\sin\delta\sin\phi}{1+2r\cos\delta\cos\phi + r^2}, \quad (7)$$

where m is near 980 MeV in our case, and $\bar{\mathcal{M}}$ is the decay amplitude of the CP-conjugate process.

By defining

$$\frac{P_{a_0^0}}{T_{f_0}} \equiv r' e^{i(\delta_q+\phi)}, \qquad \frac{T_{a_0^0}}{T_{f_0}} \equiv \alpha e^{i\delta_\alpha}, \qquad \frac{P_{a_0^0}}{P_{f_0}} \equiv \beta e^{i\delta_\beta}, \quad (8)$$

to the leading order of $\Pi_{a_0^0 f_0}$ we have

$$r e^{i\delta} = \frac{r' e^{i\delta_q}}{D_{a_0^0}}[\Pi_{a_0^0 f_0} + \beta e^{i\delta_\beta}(D_{a_0^0} - \alpha e^{i\delta_\alpha})]. \quad (9)$$

δ_α, δ_β, and δ_q denote the remaining unknown strong phases at short distance. In the absence of $a_0^0(980) - f_0(980)$ mixing, at least one of these phases would have to be nonzero for CP asymmetry. In the ideal case, we neglect the strong phases at short distance. One has $re^{i\delta} \propto \Pi_{a_0^0 f_0}/D_{a_0^0}$. The isospin breaking for ρ and ω is generated at the quark level (between the u and d quarks), while $a_0^0(980) - f_0(980)$ mixing is caused by the mass difference between $K^0\bar{K}^0$ and K^+K^-. Therefore, the modulus of $\Pi_{a_0^0 f_0}$ ($|\Pi_{a_0^0 f_0}| \approx 0.03\ \mathrm{GeV}^2$ [6]) is larger than that of ρ-ω mixing ($|\Pi_{\rho\omega}| \approx 0.0045\,\mathrm{GeV}^2$ [8]), that is to say, $a_0^0(980) - f_0(980)$ mixing can increase CP violation much more than ρ-ω mixing. We also note that the phase of the mixing amplitude varies from 0 to $\pi/2$ when the invariant mass of $\pi^+\pi^-$ is near 980 MeV [6]. Thus, the CP-violating parameter has a narrow peak located around 980 MeV in the final $\pi^+\pi^-$ spectrum.

Without loss of generality, we will now evaluate CP violation in the decays $B^\pm \to f_0(980)[a_0^0(980)]\pi^\pm \to \pi^+\pi^-\pi^\pm$. Since we are studying local CP violation in the narrow region near the masses of $f_0(980)$ and $a_0^0(980)$, the first step of the sequential decay $B^\pm \to f_0(980)[a_0^0(980)]\pi^\pm$ respects a simple factorization relation. We shall use the quasi-two-body QCD factorization approach to deal with the amplitudes [9]. It is known that the underlying structures of the scalar mesons are not well established. The mesons $f_0(980)$ and $a_0^0(980)$ can be interpreted as conventional $q\bar{q}$ mesons, $q\bar{q}q\bar{q}$ multiquark states, or meson-meson bound states, and even scalar glueballs. If the scalar mesons are four-quark states, one more quark-antiquark pair is required to be produced in the decay process compared with the two-quark picture. It is thus expected that the amplitude would be smaller in the four-quark picture than that in the two-quark picture when a light scalar meson is produced. Therefore, we assume that the $q\bar{q}$ structure dominates in our process and calculate the amplitude based on the $q\bar{q}$ structure.

In the QCD factorization approach, with the $q\bar{q}$ structure assumption, the amplitude of the decays $B^- \to f_0(980)[a_0^0(980)]\pi^-$ can be written as [10,11]

$$\begin{aligned} A(B^- \to f_0\pi^-) = -\frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p\Bigg\{&(a_1\delta_u^p + a_4^p - r_\chi^\pi a_6^p + a_{10}^p - r_\chi^\pi a_8^p)_{f_0^u\pi} \\ &\times f_\pi F_0^{Bf_0^u}(m_\pi^2)(m_B^2 - m_{f_0}^2) + \left(a_6^p - \frac{1}{2}a_8^p\right)_{\pi f_0^u}\bar{r}_\chi^{f_0}\bar{f}_{f_0}^u F_0^{B\pi}(m_{f_0}^2)(m_B^2 - m_\pi^2) \\ &- f_B[(b_2\delta_u^p + b_3 + b_{3,\mathrm{EW}})_{f_0^u\pi} + (b_2\delta_u^p + b_3 + b_{3,\mathrm{EW}})_{\pi f_0^u}]\Bigg\}, \end{aligned} \quad (10)$$

$$\begin{aligned} A(B^- \to a_0^0\pi^-) = -\frac{G_F}{2}\sum_{p=u,c}\lambda_p\Bigg\{&(a_1\delta_u^p + a_4^p - r_\chi^\pi a_6^p + a_{10}^p - r_\chi^\pi a_8^p)_{a_0\pi} \\ &\times f_\pi F_0^{Ba_0}(m_\pi^2)(m_B^2 - m_{a_0}^2) - \left(a_6^p - \frac{1}{2}a_8^p\right)_{\pi a_0}\bar{r}_\chi^{a_0}\bar{f}_{a_0}(m_B^2 - m_\pi^2)F_0^{B\pi}(m_{a_0}^2) \\ &- f_B[(b_2\delta_\mu^p + b_3 + b_{3,\mathrm{EW}})_{a_0\pi} - (b_2\delta_\mu^p + b_3 + b_{3,\mathrm{EW}})_{\pi a_0}]\Bigg\}, \end{aligned} \quad (11)$$

where $\lambda_p = V_{pb}V_{pd}^*$, f_{B,π,f_0,a_0}, and m_{π,f_0,a_0} are the decay constants and the masses of the corresponding mesons, $\bar{f}_S = f_S\frac{2m_\pi^2}{m_b(\mu)(m_u(\mu)+m_s(\mu))}$ (where μ is the scale parameter), $F_0^{Bf_0^u,Ba_0,B\pi}$ are form factors, and

$$\bar{r}_\chi^{a_0} = \frac{2m_{a_0}}{m_b(\mu)}, \qquad \bar{r}_\chi^{f_0} = \frac{2m_{f_0}}{m_b(\mu)}, \qquad r_\chi^\pi = \frac{2m_\pi^2}{m_b(\mu)(m_u(\mu)+m_s(\mu))}. \quad (12)$$

The general form of the coefficients $a_i^p(M_1M_2)$ at the next-to-leading order in α_s are

$$a_i^p(M_1M_2) = c_i + \frac{c_{i\pm1}}{N_c} + \frac{c_{i\pm1}}{N_c}\frac{C_F\alpha_s}{4\pi} \times \left[V_i(M_2) + \frac{4\pi^2}{N_c}H_i(M_1M_2) + P_i^p(M_2)\right], \tag{13}$$

where c_i are the Wilson coefficients, the upper (lower) sign of $\pm$ corresponds to i being odd (even), $C_F = (N_c^2 - 1)/(2N_c)$ with $N_c = 3$, M_2 is the emitted meson, M_1 shares the same spectator quark with the B meson, and $V_i(M_2)$, $P_i(M_2)$, and $H_i(M_1, M_2)$ are vertex corrections, penguin corrections, and hard spectator corrections, respectively, which are listed in the Appendix. As for the weak annihilation contributions, we follow the expressions given in Refs. [10,11],

$$\begin{aligned} b_1 &= \frac{C_F}{N_c^2}c_1A_1^i, \quad b_1 = \frac{C_F}{N_c^2}[c_3A_1^i + c_5(A_3^i + A_3^f) + N_cc_6A_3^f], \\ b_1 &= \frac{C_F}{N_c^2}c_2A_1^i, \quad b_1 = \frac{C_F}{N_c^2}[c_4A_1^i + c_6A_2^f], \\ b_{3,\mathrm{EW}} &= \frac{C_F}{N_c^2}[c_9A_1^i + c_7(A_3^i + A_3^f) + N_cc_8A_3^f], \\ b_{4,\mathrm{EW}} &= \frac{C_F}{N_c^2}[c_{10}A_1^i + c_8A_2^i], \end{aligned} \tag{14}$$

where the subscripts 1, 2, 3 of $A_n^{i,f}$ denote the annihilation amplitudes induced from $(V-A)(V-A)$, $(V-A)(V+A)$, and $(S-P)(S+P)$ operators, respectively, and i and f denote the gluon emission from the initial- and final-state quarks, respectively.

In the naive $q\bar{q}$ model, $f_0(980)$ is a pure $s\bar{s}$ state. However, several experiments indicate that there is a mixture between the light and strange quarks:

$$|f_0(980)\rangle = |s\bar{s}\rangle\cos\theta + |n\bar{n}\rangle\sin\theta,$$
$$|f_0(500)\rangle = -|s\bar{s}\rangle\sin\theta + |n\bar{n}\rangle\cos\theta,$$

where $n\bar{n} \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$ and θ is the mixing angle. We take $\theta = 20°$ as in Refs. [10,11]. In the expressions for the spectator and annihilation corrections, there are end-point divergences $X_{H(A)} \equiv \int_0^1 \mathrm{d}x/(1-x)$. The QCD factorization approach suffers from these end-point divergences, which can be parametrized as [10,11]

$$X_{H(A)} = \ln\frac{m_B}{\Lambda_h}[1 + \rho_{H(A)}\mathrm{e}^{\mathrm{i}\phi_{H(A)}}], \tag{15}$$

where one may take $\rho_{H(A)} \leq 0.5$ and arbitrary strong phases $\phi_{H(A)}$ [10]. In practice, we calculate the CP violation when $\rho_{H(A)} = 0.25, 0.5$ and $\phi_{H(A)} = 0, \pi/2, \pi, 3\pi/2$, respectively.

Then, considering $a_0^0(980) - f_0(980)$ mixing, the total amplitude of the decay $B^- \to f_0(980)[a_0^0(980)]\pi^- \to \pi^+\pi^-\pi^-$ can be written as

$$\begin{aligned} \mathcal{M}(B^- \to \pi^+\pi^-\pi^-) &= \frac{g_{f\pi\pi}}{D_{f_0}(m)}A(B^- \to f_0(980)\pi^-) \\ &\quad + \frac{g_{f\pi\pi}\Pi_{a_0^0f_0}(m)}{D_{a_0^0}(m)D_{f_0}(m) - \Pi^2_{a_0^0f_0}(m)} \\ &\quad \times A(B^- \to a_0^0(980)\pi^-), \end{aligned} \tag{16}$$

and the differential CP-violating parameter is

$$A_{CP} = \frac{|\mathcal{M}(B^- \to \pi^+\pi^-\pi^-)|^2 - |\mathcal{M}(B^+ \to \pi^+\pi^-\pi^+)|^2}{|\mathcal{M}(B^- \to \pi^+\pi^-\pi^-)|^2 + |\mathcal{M}(B^+ \to \pi^+\pi^-\pi^+)|^2}. \tag{17}$$

One can see that $\lambda_p(\lambda_p^*)$ provides a weak CP-violation phase and the strong phases occur in the mixing amplitude, the scalar propagators, and $V_i(M_2)$, $P_i(M_2)$, and $H_i(M_1, M_2)$ in the decays $B^\pm \to f_0(980)[a_0^0(980)]\pi^\pm$. Since we focus on local CP violation in the narrow region near 980 MeV, it is plausible to assume that the intermediate states $f_0(980)$ and $a_0^0(980)$ are on shell. According to the kinematics of the sequential decay, the amplitudes and the CP-violating parameter are dependent on the invariant mass of $\pi^+\pi^-$, m. Substituting Eqs. (10), (11), (13), (14), and (16) into Eq. (17), the differential CP-violating parameter is obtained, which is displayed in Fig. 2 as a function of m. In the effective region of $a_0^0(980) - f_0(980)$ mixing, we can see that the CP-violating parameter can reach as large as 58 percent and an obvious peak exists, which can be used to test the existence of $a_0^0(980) - f_0(980)$ mixing. Also, it is shown that the presence of the peak is not sensitive to the strong phases at short distances mentioned in Eq. (9). If a peak for CP violation is observed in the experiments, it will be a strong evidence for $a_0^0(980) - f_0(980)$ mixing.

At present, $a_0^0(980) - f_0(980)$ mixing is examined by the mixing intensities in some isospin-violating decay processes. For example, the BESIII Collaboration recently investigated the mixing intensities, which are defined as

$$\xi_{fa} = \frac{B(J/\psi \to \phi f_0(980) \to \phi a_0^0(980) \to \phi\eta\pi)}{B(J/\psi \to \phi f_0(980) \to \phi\pi\pi)}, \tag{18}$$

$$\xi_{af} = \frac{B(\chi_{c1} \to \pi a_0^0(980) \to \pi f_0(980) \to \pi\pi\pi)}{B(\chi_{c1} \to \pi a_0^0(980) \to \pi\pi\pi))}. \tag{19}$$

In those studies, the mixing intensities were measured from the decay amplitudes. On the other hand, the CP asymmetry is proportional to the sine of the strong phases, which depends on $a_0^0(980) - f_0(980)$ mixing more sensitively than the amplitudes. We propose to test $a_0^0(980) - f_0(980)$ mixing by measuring the CP-violating parameter. If the branching ratio for $B^\pm \to f_0(980)[a_0^0(980)]\pi^\pm$ is of the

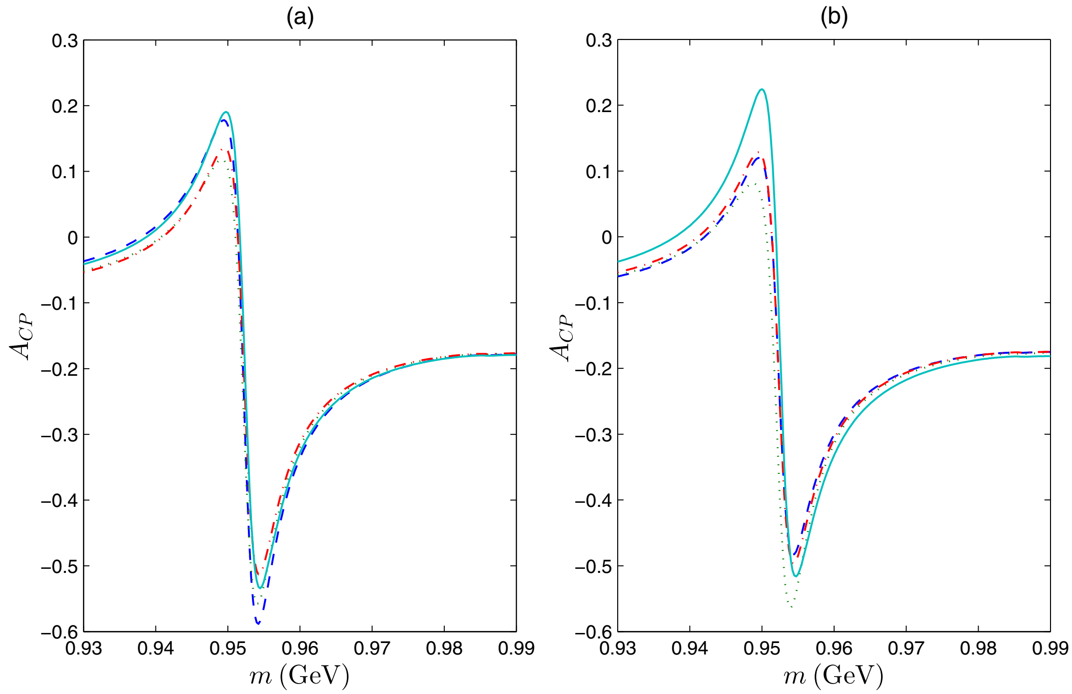

FIG. 2. The differential *CP*-violating parameter as a function of m in the decays $B^{\pm} \to f_0(980)[a_0^0(980)]\pi^{\pm} \to \pi^+\pi^-\pi^{\pm}$. (a) For $\rho_{H(A)} = 0.25$: the dashed, dotted, dot-dashed, and solid lines correspond to the cases when $\phi_{H(A)} = 0, \pi/2, \pi, 3\pi/2$, respectively. (b) The same as (a) but for $\rho_{H(A)} = 0.5$.

order of 10^{-6} [12], then the number of $B\bar{B}$ pairs needed is roughly

$$N = \frac{n^2}{BR * A_{CP}^2}(1 - A_{CP}^2),$$

which is 2×10^7 for 3σ significance and 5×10^7 for 5σ significance. With the increased data provided by the LHC and the forthcoming Belle II experiment, the study of *CP* violation in the *B* meson will reach a high level of precision. The number of $B\bar{B}$ pairs at LHCb could be around 10^{12} per year [13] and the Belle detector has collected 8×10^8 $B\bar{B}$ pairs [14], which is sufficient to investigate the $a_0^0(980) - f_0(980)$ mixing mechanism through *CP* violation.

Like ρ-ω mixing, $a_0^0(980) - f_0(980)$ mixing could occur in a lot of multibody decay modes that contain the *S*-wave $\pi\pi$ final states in the *B*- or *D*-meson decays. We predict that $a_0^0(980) - f_0(980)$ mixing would lead to large *CP* violation. Since the strong phase is more sensitive to $a_0^0(980) - f_0(980)$ mixing than the decay widths, A_{CP} is a more useful parameter to test $a_0^0(980) - f_0(980)$ mixing. We calculated localized direct *CP* asymmetries in the three-body decays $B^{\pm} \to f_0(980)[a_0^0(980)]\pi^{\pm} \to \pi^+\pi^-\pi^{\pm}$ based on the hypothetical $q\bar{q}$ structure of $a_0^0(980)$ and $f_0(980)$ in the QCD factorization approach. It was shown that there is a peak for *CP* violation when the invariant mass of $\pi\pi$ is near 980 MeV. This would be a new way to verify $a_0^0(980) - f_0(980)$ mixing. Some input parameters were not well determined in our calculation, and thus there are some uncertainties. One may also wonder if $a_0^0(980)$ and $f_0(980)$ produced in *B* decays are dominated by the $q\bar{q}$ configuration and whether $a_0^0(980) - f_0(980)$ mixing could also provide a large strong phase and lead to large *CP* violation when $a_0^0(980)$ and $f_0(980)$ have other structures. This will need further investigation.

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APPENDIX: THE EXPRESSIONS OF $V_i(M_2)$, $P_i(M_2)$, $H_i(M_1,M_2)$, AND $A_n^{i,f}$

In order to clearly show how we calculated the amplitudes, in this appendix we present explicit formulas for the vertex corrections $V_i(M_2)$, penguin corrections $P_i(M_2)$, hard spectator corrections $H_i(M_1, M_2)$, and $A_n^{i,f}$ in the weak annihilation contributions.

The twist-2 light-cone distribution amplitude (LCDA) for the scalar meson *S*, Φ_S, has the form [10]

$$\Phi_M(x,\mu) = f_M 6x(1-x)\left[1 + \sum_{n=1}^{\infty} \alpha_n^M(\mu) C_n^{3/2}(2x-1)\right], \tag{A1}$$

where $\alpha_n = \mu_S B_n$ with B_n being the Gegenbauer moments and $C_n^{3/2}$ the Gegenbauer polynomials. For twist-3 LCDAs, we have $\Phi_S^s(x) = \bar{f}_S$ and $\Phi_S^\sigma(x) = \bar{f}_S 6x(1-x)$. The twist-2 and twist-3 distribution amplitudes for the pseudoscalar meson P are

$$\Phi_P(x) = f_P 6x(1-x), \qquad \Phi_P^p(x) = f_P, \qquad \Phi_P^\sigma(x) = f_P 6x(1-x). \tag{A2}$$

The expressions for the vertex corrections $V_i(M_2)$ are (apart from the decay constant f_{M_2}) [10]

$$V_i(M_2) = 12\ln\frac{m_b}{\mu} - 18 - \frac{1}{2} - 3\pi \mathrm{i} + \left(\frac{11}{2} - 3\pi \mathrm{i}\right)\alpha_1^M - \frac{21}{20}\alpha_2^M + \left(\frac{79}{36} - \frac{2\pi \mathrm{i}}{3}\right)\alpha_3^M + \cdots, \tag{A3}$$

for $i = 1$–4, 9, 10,

$$V_i(M_2) = -12\ln\frac{m_b}{\mu} + 6 - \frac{1}{2} - 3\pi \mathrm{i} - \left(\frac{11}{2} - 3\pi \mathrm{i}\right)\alpha_1^M - \frac{21}{20}\alpha_2^M - \left(\frac{79}{36} - \frac{2\pi \mathrm{i}}{3}\right)\alpha_3^M + \cdots, \tag{A4}$$

for $i = 5$, 7. and $V_i(M_2) = -6$ for $i = 6$, 8 in the naive dimensional regularization scheme for γ_5.

The hard spectator corrections read [10]

$$H_i(M_1M_2) = \frac{1}{f_{M_2} F_0^{BM_1}(0) m_B^2}\int_0^1 \frac{d\rho}{\rho}\Phi_B(\rho)\int_0^1 \frac{d\xi}{\bar{\xi}}\Phi_{M_2}(\xi)\int_0^1 \frac{d\eta}{\bar{\eta}}\left[\Phi_{M_1}(\eta) + r_\chi^{M_1}\frac{\bar{\xi}}{\xi}\Phi_{m_1}(\eta)\right], \tag{A5}$$

for $i = 1$–4, 9, 10,

$$H_i(M_1M_2) = -\frac{1}{f_{M_2} F_0^{BM_1}(0) m_B^2}\int_0^1 \frac{d\rho}{\rho}\Phi_B(\rho)\int_0^1 \frac{d\xi}{\xi}\Phi_{M_2}(\xi)\int_0^1 \frac{d\eta}{\bar{\eta}}\left[\Phi_{M_1}(\eta) + r_\chi^{M_1}\frac{\xi}{\bar{\xi}}\Phi_{m_1}(\eta)\right], \tag{A6}$$

for $i = 5, 7$, and $H_i = 0$ for $i = 6, 8$, where $\bar{\xi} \equiv 1 - \xi$ and $\bar{\eta} \equiv 1 - \eta$, Φ_M (Φ_m) is the twist-2 (twist-3) light-cone distribution amplitude of the meson M.

The explicit expressions for $A_n^{i,f}$ in the weak annihilations are given by [10]

$$A_1^i = \int \cdots \begin{cases} \left(\Phi_{M_2}(x)\Phi_{M_1}(y)\left[\frac{1}{y(1-x\bar{y})} + \frac{1}{\bar{x}^2 y}\right] - r_\chi^{M_1} r_\chi^{M_2}\Phi_{m_2}(x)\Phi_{m_1}(y)\frac{2}{\bar{x}y}\right) & \text{for } M_1M_2 = PS, \\ \left(\Phi_{M_2}(x)\Phi_{M_1}(y)\left[\frac{1}{y(1-x\bar{y})} + \frac{1}{\bar{x}^2 y}\right] + r_\chi^{M_1} r_\chi^{M_2}\Phi_{m_2}(x)\Phi_{m_1}(y)\frac{2}{\bar{x}y}\right) & \text{for } M_1M_2 = SP, \end{cases}$$

$$A_2^i = \int \cdots \begin{cases} \left(-\Phi_{M_2}(x)\Phi_{M_1}(y)\left[\frac{1}{\bar{x}(1-x\bar{y})} + \frac{1}{\bar{x}y^2}\right] + r_\chi^{M_1} r_\chi^{M_2}\Phi_{m_2}(x)\Phi_{m_1}(y)\frac{2}{\bar{x}y}\right) & \text{for } M_1M_2 = PS, \\ \left(-\Phi_{M_2}(x)\Phi_{M_1}(y)\left[\frac{1}{\bar{x}(1-x\bar{y})} + \frac{1}{\bar{x}y^2}\right] - r_\chi^{M_1} r_\chi^{M_2}\Phi_{m_2}(x)\Phi_{m_1}(y)\frac{2}{\bar{x}y}\right) & \text{for } M_1M_2 = SP, \end{cases}$$

$$A_3^i = \int \cdots \begin{cases} \left(r_\chi^{M_1}\Phi_{M_2}(x)\Phi_{m_1}(y)\frac{2\bar{y}}{\bar{x}y(1-x\bar{y})} + r_\chi^{M_2}\Phi_{M_1}(y)\Phi_{m_2}(x)\frac{2x}{\bar{x}y(1-x\bar{y})}\right) & \text{for } M_1M_2 = PS, \\ \left(-r_\chi^{M_1}\Phi_{M_2}(x)\Phi_{m_1}(y)\frac{2\bar{y}}{\bar{x}y(1-x\bar{y})} + r_\chi^{M_2}\Phi_{M_1}(y)\Phi_{m_2}(x)\frac{2x}{\bar{x}y(1-x\bar{y})}\right) & \text{for } M_1M_2 = SP, \end{cases}$$

$$A_3^f = \int \cdots \begin{cases} \left(r_\chi^{M_1}\Phi_{M_2}(x)\Phi_{m_1}(y)\frac{2(1+\bar{x})}{\bar{x}^2 y} - r_\chi^{M_2}\Phi_{M_1}(y)\Phi_{m_2}(x)\frac{2(1+y)}{\bar{x}y^2}\right) & \text{for } M_1M_2 = PS, \\ \left(-r_\chi^{M_1}\Phi_{M_2}(x)\Phi_{m_1}(y)\frac{2(1+\bar{x})}{\bar{x}^2 y} - r_\chi^{M_2}\Phi_{M_1}(y)\Phi_{m_2}(x)\frac{2(1+y)}{\bar{x}y^2}\right) & \text{for } M_1M_2 = SP, \end{cases}$$

$$A_1^f = A_2^f = 0, \tag{A7}$$

where $\int \cdots = \pi\alpha_s \int_0^1 dxdy$, $\bar{x} \equiv 1 - x$, and $\bar{y} \equiv 1 - y$.

[1] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Lett. **13**, 138 (1964).
[2] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963).
[3] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
[4] N. N. Achasov, S. A. Devyanin, and G. N. Shestakov, Phys. Lett. **88B**, 367 (1979); **96B**, 168 (1980).
[5] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **121**, 022001 (2018).
[6] N. N. Achasov and G. N. Shestakov, Phys. Rev. D **96**, 016027 (2017); Nucl. Part. Phys. Proc. **287–288**, 89 (2017).
[7] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. Lett. **112**, 011801 (2014).
[8] C. E. Wolfe and K. Maltman, Phys. Rev. D **83**, 077301 (2011).
[9] H. Y. Cheng, Nucl. Part. Phys. Proc. **273–275**, 1290 (2016).
[10] H. Y. Cheng, C. K. Chua, and K. C. Yang, Phys. Rev. D **73**, 014017 (2006).
[11] H. Y. Cheng, C. K. Chua, K. C. Yang, and Z. Q. Zhang, Phys. Rev. D **87**, 114001 (2013).
[12] M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018).
[13] R. Aaij *et al.* (LHCb Collaboration), arXiv:1808.08865.
[14] A. B. Kaliyar *et al.* (Belle Collaboration), Phys. Rev. D **99**, 031102 (2019).