

Excited states of ϕ meson

Cheng-Qun Pang*

College of Physics and Electronic Information Engineering,
Qinghai Normal University, Xining 810000, China



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In this paper, the excited states of the ϕ meson, especially containing the newly observed $X(2000)$ with $I(J^P) = 0(1^-)$ by the BESIII Collaboration, is studied. In addition, $Y(2175)$ as a ϕ meson excited state is investigated. The mass spectrum and strong decay behaviors of ϕ meson excited states are analyzed, which indicates that $X(2000)$ and $Y(2175)$ are the candidates of $\phi(3S)$ and $\phi(2D)$ states with $I(J^P) = 0(1^-)$, respectively. In addition, $\phi(1D)$ and $\phi(4S)$ are predicted to have the mass of 1.87 and 2.5 GeV and width of 440 and 940 MeV, respectively.

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I. INTRODUCTION

In light meson spectroscopy, there exist two systems: $q\bar{q}$ (q defines the u , d , or s quark) mesons and exotic states (glueballs, hybrids, and multiquark states). Exotic states have exotic quantum numbers [such as $\pi_1(1400)$] or the same quantum numbers as the conventional meson system. In the latter case, i.e., if exotic states have the same quantum numbers as conventional meson system, it is difficult but intriguing to identify the exotica from light meson spectroscopy [1–17]. Thus, some states may be the conventional $q\bar{q}$ mesons or exotic states, which are related to $X(1835)$ [18,19], $X(1860)$ [20], $X(1812)$, $Y(2175)$ [21–23] and so on.

Very recently, the BESIII Collaboration observed a structure [$X(2000)$] appearing in the $\phi\eta'$ invariant mass spectrum of the $J/\psi \rightarrow \phi\eta\eta'$ process [23]. With assumption that the spin-parity quantum number $J^P = 1^-$, its measured resonance parameters are $M = 2002.1 \pm 27.5 \pm 15.0$ MeV and $\Gamma = 129 \pm 17 \pm 7$ MeV with significance of 5.3σ . Assuming the spin-parity quantum number $J^P = 1^+$, the resonance parameters are $M = 2062.8 \pm 13.1 \pm 4.2$ MeV $\Gamma = 177 \pm 36 \pm 20$ MeV with significance of 4.9σ .

Naturally, one can note that the resonance parameters with the first assumption are close to the observed $Y(2175)$ [21–23]. $Y(2175)$ has been studied in various theoretical explanations [1–17]. $X(2000)$ has also been studied by recent work [24–26]. Reference [16] studied $Y(2175)$ as a $\phi(2^3D_1)$ state. Reference [24] treated $X(2000)$ as a $h_1(3^1P_1)$ state with $s\bar{s}$ component under $J^P = 1^+$ assignment,

Cui *et al.* [25] argued that the $X(2000)$ is the partner of the tetraquark state $Y(2175)$ with $J^P = 1^+$, and Ref. [26] assigned $X(2000)$ to be a new $s\bar{s}\bar{s}s$ tetraquark state with the same J^P . As another possibility, i.e., $X(2000)$ has the resonance parameters $M = 2002.1 \pm 27.5 \pm 15.0$ MeV and $\Gamma = 129 \pm 17 \pm 7$ MeV with $J^P = 1^-$, has not been theoretically studied. In addition, a hybrid with the same quantum numbers and a similar mass and width are predicted by the flux-tube model [27–30]. Identifying whether $X(2000)$ is $s\bar{s}$ or $s\bar{s}$ hybrid is a difficult, interesting, and urgent research issue. In $J^{PC} = 1^{--}$ assignment, $X(2000)$ is the candidate of an excited state of ϕ meson in the conventional $s\bar{s}$ meson framework. In fact, Refs. [31,32] predicted a $\phi(3S)$ state with the mass of 2050 and 1900–1960 MeV, Ref. [31] also predicted that the width of $\phi(3S)$ will be 380 MeV. If this X state is considered as the conventional mesons under the $J^P = 1^-$ assignment, what is the relation between $X(2000)$ and $Y(2175)$? Is $X(2000)$ a $\phi(3S)$ state? These questions should be clarified. In addition, the angular excited state of $\phi(1S)$, the mass and the width of $\phi(1D)$ are unclear. A systemic study of excited states of ϕ meson represents an intriguing and important research topic.

This paper is aimed to give a systemic study of excited states of ϕ meson. By using modified Godfrey-Isgur (GI) model and quark pair creation model, the mass spectrum and strong decay behavior of excited states of ϕ meson are analyzed, which indicates that $X(2000)$ is a candidate of the $\phi(3S)$ meson with $I(J^P) = 0(1^-)$ and $Y(2175)$ is a candidate of the $\phi(2D)$ state. At the same time, the mass and the width of $\phi(1D)$, $\phi(3D)$, and $\phi(4S)$ are predicted.

In this work, the spectra of the ϕ meson family are studied using the modified Godfrey-Isgur (MGI) model [33–36], which contains the screening effect. At higher excited states of ϕ meson, the screening effect should be considered for the larger average distance between the

*xuehua45@163.com

quark pair. The former studies [33–35,37–41] show that the GI model works well for describing hadron spectroscopy. Then, for further studying the properties of ϕ mesons, their Okubo-Zweig-Iizuka (OZI)-allowed two-body strong decays are studied, taking input with the spatial wave functions obtained from the mass spectrum by numerical calculation. Their partial and total decay widths are calculated by using a quark pair creation (QPC) model that was proposed by Micu [42] and extensively applied to studies of strong decay of other hadrons [13,16,35,36,39,43–67]. This paper also gives a comparison of $X(2000)$'s two-body decay information between that of $\phi(3S)$ and $s\bar{s}$ hybrid [29]. The effort will be helpful to uncover the structure of $X(2000)$ and $Y(2175)$ and establish ϕ meson families.

This paper is organized as follows. In Sec. II, the models employed in this work are briefly reviewed. The mass spectrum and decay behavior phenomenological analysis of ϕ mesons will be performed in Sec. III. The paper ends with a conclusion in Sec. IV.

II. MODELS EMPLOYED IN THE WORK

In this work, the modified GI quark model and quark pair creation (QPC) model are utilized to calculate the mass spectrum and the two-body strong decays of the meson family, respectively. In the following, these models will be illustrated briefly.

A. The modified GI model

In 1985, Godfrey and Isgur proposed the GI model for describing relativistic meson spectra with great success, specifically for low-lying mesons [37]. Regarding the excited states, the screening potential should be taken into account for its coupled-channel effect [33–36].

The internal interaction of mesons is depicted by the Hamiltonian of the potential model and can be written as

$$\tilde{H} = \sqrt{m_1^2 + \mathbf{p}^2} + \sqrt{m_2^2 + \mathbf{p}^2} + \tilde{V}_{\text{eff}}(\mathbf{p}, \mathbf{r}), \quad (2.1)$$

where m_1 and m_2 denote the mass of quark and antiquark, respectively, the relation between $\tilde{V}_{\text{eff}}(\mathbf{p}, \mathbf{r})$ and $V_{\text{eff}}(\mathbf{p}, \mathbf{r})$ will be illustrated later, and the effective potential has a familiar format in the nonrelativistic limit [37,68]

$$V_{\text{eff}}(r) = H^{\text{conf}} + H^{\text{hyp}} + H^{\text{so}}, \quad (2.2)$$

with

$$\begin{aligned} H^{\text{conf}} &= \left[-\frac{3}{4} \left(\frac{b(1 - e^{-\mu r})}{\mu} + c \right) + \frac{\alpha_s(r)}{r} \right] (\mathbf{F}_1 \cdot \mathbf{F}_2) \\ &= S(r) + G(r). \end{aligned} \quad (2.3)$$

$$\begin{aligned} H^{\text{hyp}} &= -\frac{\alpha_s(r)}{m_1 m_2} \left[\frac{8\pi}{3} \mathbf{S}_1 \cdot \mathbf{S}_2 \delta^3(\mathbf{r}) + \frac{1}{r^3} \left(\frac{3\mathbf{S}_1 \cdot \mathbf{r} \mathbf{S}_2 \cdot \mathbf{r}}{r^2} \right. \right. \\ &\quad \left. \left. - \mathbf{S}_1 \cdot \mathbf{S}_2 \right) \right] (\mathbf{F}_1 \cdot \mathbf{F}_2), \end{aligned} \quad (2.4)$$

$$H^{\text{so}} = H^{\text{so(cm)}} + H^{\text{so(tp)}}, \quad (2.5)$$

where $\mathbf{S}_1/\mathbf{S}_2$ indicates the spin of quark/antiquark and \mathbf{L} is the orbital momentum. \mathbf{F} are related to the Gell-Mann matrices in color space. For a meson, $\langle \mathbf{F}_1 \cdot \mathbf{F}_2 \rangle = -4/3$, the running coupling constant $\alpha_s(r)$ has the following form:

$$\alpha_s(r) = \sum_k \frac{2\alpha_k}{\sqrt{\pi}} \int_0^{\gamma_k r} e^{-x^2} dx, \quad (2.6)$$

where k is from 1 to 3 and the corresponding α_k and γ_k are constant, $\alpha_{1,2,3} = 0.25, 0.15, 0.2$ and $\gamma_{1,2,3} = \frac{1}{2} \cdot \frac{\sqrt{10}}{2} \cdot \frac{\sqrt{1000}}{2}$ [37]. H^{conf} consists of two pieces, the spin-independent linear confinement piece $S(r)$ and Coulomb-like potential $G(r)$. H^{hyp} is the color-hyperfine interaction and also includes two parts, tensor and contact terms; H^{so} denotes the spin-orbit interaction with the color magnetic term due to one-gluon exchange and the Thomas precession term, which can be written as

$$H^{\text{so(cm)}} = \frac{-\alpha_s(r)}{r^3} \left(\frac{1}{m_1} + \frac{1}{m_2} \right) \left(\frac{\mathbf{S}_1}{m_1} + \frac{\mathbf{S}_2}{m_2} \right) \cdot \mathbf{L} (\mathbf{F}_1 \cdot \mathbf{F}_2), \quad (2.7)$$

$$H^{\text{so(tp)}} = -\frac{1}{2r} \frac{\partial H^{\text{conf}}}{\partial r} \left(\frac{\mathbf{S}_1}{m_1^2} + \frac{\mathbf{S}_2}{m_2^2} \right) \cdot \mathbf{L}. \quad (2.8)$$

In the light meson system, relativistic effects in effective potential must be considered; the GI model introduces these relativistic effects in two ways.

First, the GI model introduces a smearing function for a $q\bar{q}$ meson, which includes nonlocal interactions and new \mathbf{r} dependence.

$$\rho(\mathbf{r} - \mathbf{r}') = \frac{\sigma^3}{\pi^{3/2}} e^{-\sigma^2(\mathbf{r} - \mathbf{r}')^2}, \quad (2.9)$$

then, $S(r)$ and $G(r)$ become smeared potentials $\tilde{S}(r)$ and $\tilde{G}(r)$, respectively, by the following procedure:

$$\tilde{f}(r) = \int d^3 r' \rho(\mathbf{r} - \mathbf{r}') f(r'), \quad (2.10)$$

with

TABLE I. Parameters and their values in this work, which are determined by fitting the meson experimental data listed in PDG.

Parameter	Value [36]	Parameter	Value [36]
m_u (GeV)	0.163	σ_0 (GeV)	1.799
m_d (GeV)	0.163	s (GeV)	1.497
m_s (GeV)	0.387	μ (GeV)	0.0635
b (GeV ²)	0.221	c (GeV)	-0.240
ϵ_c	-0.138	ϵ_{sov}	0.157
ϵ_{sos}	0.9726	ϵ_t	0.893

$$\sigma_{12}^2 = \sigma_0^2 \left[\frac{1}{2} + \frac{1}{2} \left(\frac{4m_1 m_2}{(m_1 + m_2)^2} \right)^4 \right] + s^2 \left(\frac{2m_1 m_2}{m_1 + m_2} \right)^2, \quad (2.11)$$

where the values of σ_0 and s are defined in Table I [36].

Second, to make up for the loss of relativistic effects in the nonrelativistic limit, a general potential relying on the center-of-mass of interacting quarks and momentum are applied as

$$\tilde{G}(r) \rightarrow \left(1 + \frac{p^2}{E_1 E_2} \right)^{1/2} \tilde{G}(r) \left(1 + \frac{p^2}{E_1 E_2} \right)^{1/2}, \quad (2.12)$$

and

$$\frac{\tilde{V}_i(r)}{m_1 m_2} \rightarrow \left(\frac{m_1 m_2}{E_1 E_2} \right)^{1/2+\epsilon_i} \frac{\tilde{V}_i(r)}{m_1 m_2} \left(\frac{m_1 m_2}{E_1 E_2} \right)^{1/2+\epsilon_i}, \quad (2.13)$$

where $\tilde{V}_i(r)$ delegates the contact, tensor, vector spin-orbit, and scalar spin-orbit terms, and ϵ_i represents the relevant modification parameters as shown in Table I. After the above revision in two points, $\tilde{V}_{\text{eff}}(\mathbf{p}, \mathbf{r})$ is replaced by $V_{\text{eff}}(\mathbf{p}, \mathbf{r})$.

Diagonalizing and solving the Hamiltonian in Eq. (2.1) by exploiting a simple harmonic oscillator (SHO) basis, the mass spectrum and wave functions will be obtained. In configuration and momentum space, SHO wave functions have explicit forms:

$$\begin{aligned} \Psi_{nLM_L}(\mathbf{r}) &= R_{nL}(r, \beta) Y_{LM_L}(\Omega_r), \\ \Psi_{nLM_L}(\mathbf{p}) &= R_{nL}(p, \beta) Y_{LM_L}(\Omega_p), \end{aligned} \quad (2.14)$$

with

$$\begin{aligned} R_{nL}(r, \beta) &= \beta^{3/2} \sqrt{\frac{2n!}{\Gamma(n+L+3/2)}} (\beta r)^L e^{-\frac{r^2 \beta^2}{2}} \\ &\times L_n^{L+1/2}(\beta^2 r^2). \end{aligned} \quad (2.15)$$

$$\begin{aligned} R_{nL}(p, \beta) &= \frac{(-1)^n (-i)^L}{\beta^{3/2}} e^{-\frac{p^2}{2\beta^2}} \sqrt{\frac{2n!}{\Gamma(n+L+3/2)}} \left(\frac{p}{\beta} \right)^L \\ &\times L_n^{L+1/2} \left(\frac{p^2}{\beta^2} \right), \end{aligned} \quad (2.16)$$

where $Y_{LM_L}(\Omega)$ is spherical harmonic function, $L_{n-1}^{L+1/2}(x)$ is the associated Laguerre polynomial, and $\beta = 0.4$ GeV for the calculation.

After diagonalization of the Hamiltonian matrix, the mass and wave function of the meson that is available to undergo the strong decay process can be obtained.

B. QPC model

The QPC model is used to obtain the Okubo-Zweig-Iizuka (OZI) allowed hadronic strong decays. This model was first proposed by Micu [42] and was further developed by Orsay group [43, 69–72]. The QPC model was widely applied to the OZI-allowed two-body strong decays of hadrons in Refs. [16, 35, 36, 44, 45, 48, 50, 52, 54–59, 61–66, 73–77].

For the process $A \rightarrow B + C$,

$$\langle BC | \mathcal{T} | A \rangle = \delta^3(\mathbf{P}_B + \mathbf{P}_C) \mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}, \quad (2.17)$$

where $\mathbf{P}_{B(C)}$ is a three-momentum of a meson $B(C)$ in the rest frame of a meson A . M_{J_i} ($i = A, B, C$) denotes the magnetic quantum number. The transition operator \mathcal{T} describes a quark-antiquark pair creation from vacuum with $J^{PC} = 0^{++}$, i.e., \mathcal{T} can be written as

$$\begin{aligned} \mathcal{T} &= -3\gamma \sum_m \langle 1m; 1-m | 00 \rangle \int d\mathbf{p}_3 d\mathbf{p}_4 \delta^3(\mathbf{p}_3 + \mathbf{p}_4) \\ &\times \mathcal{Y}_{1m} \left(\frac{\mathbf{p}_3 - \mathbf{p}_4}{2} \right) \chi_{1,-m}^{34} \phi_0^{34} (\omega_0^{34})_{ij} b_{3i}^\dagger(\mathbf{p}_3) d_{4j}^\dagger(\mathbf{p}_4). \end{aligned} \quad (2.18)$$

where the quark and antiquark are denoted by indices 3 and 4, respectively, and γ depicts the strength of the creation of $q\bar{q}$ from vacuum. In this work, $\gamma = 6.57$, which is obtained by fitting the decay width of $\phi(1680)(2S)$ state as shown in Table II and is independent of the decay channels' branch ratios. $\mathcal{Y}_{\ell m}(\mathbf{p}) = |\mathbf{p}|^\ell Y_{\ell m}(\mathbf{p})$ are the solid harmonics. χ , ϕ , and ω denote the spin, flavor, and color wave functions, respectively, which can be separately treated. Subindices i and j denote the color of a $q\bar{q}$ pair. The decay width reads

TABLE II. The decay widths of the $\phi(1680)(2S)$ state.

Decay channel	Expe. (MeV)	This work
Total	150	150
$\phi(1680) \rightarrow KK^*$...	117
$\phi(1680) \rightarrow \eta\phi$...	16.7
$\phi(1680) \rightarrow KK$...	15.5
$\gamma = 6.57$		

$$\Gamma = \frac{\pi |\mathbf{P}|}{4 m_A^2} \sum_{J,L} |\mathcal{M}^{JL}(\mathbf{P})|^2, \quad (2.19)$$

where m_A is the mass of an initial state A and the two decay amplitudes can be related to the Jacob-Wick formula as [78]

$$\begin{aligned} \mathcal{M}^{JL}(\mathbf{P}) = & \frac{\sqrt{4\pi(2L+1)}}{2J_A+1} \sum_{M_{J_B} M_{J_C}} \langle L0; JM_{J_A} | J_A M_{J_A} \rangle \\ & \times \langle J_B M_{J_B}; J_C M_{J_C} | J_A M_{J_A} \rangle \mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}. \end{aligned} \quad (2.20)$$

In the calculation, the spatial wave functions of the discussed mesons can be numerically obtained by the MGI model.

III. NUMERICAL RESULTS AND PHENOMENOLOGICAL ANALYSIS

A. Mass spectrum analysis

Applying the MGI model and the parameters in Table I, the mass spectrum of the ϕ family can be obtained, as shown in Table III. In addition, the mass spectrum of mesons with $J^P = 1^-$ was calculated by the GI model, Ref. [79] also gave a spectrum for the ϕ meson. The mass spectrum of these ϕ states can be obtained by the MGI model which is listed in Table III. The numerical results are compared with the GI model [37], Ref. [79] and experiments in Table III.

1. The spectrum of ϕ meson excitations

The spectrum of ϕ meson excitations is calculated, and the values are listed in Table III. The third radial excited state of $\phi(1S)$ has a mass of 2.5 GeV, which is smaller than the result of GI model and close to that reported in Ref. [79]. For the ground state of a D-wave ϕ meson [$\phi(1D)$], its first and second radial excited states [$\phi(2D)$ and $\phi(3D)$] have the mass of 1.869, 2.276, and 2.6 GeV, respectively, which are also smaller than those reported in Ref. [37].

TABLE III. The mass spectrum of ϕ mesons. “Expe.” represents experimental value. The unit of the mass is GeV.

State	This work	GI [37]	Ebert [79]	Expe.
$\phi(1S)$	1.030	1.016	1.038	1.019
$\phi(2S)$	1.687	1.687	1.698	1.680
$\phi(3S)$	2.149	2.200	2.119	...
$\phi(4S)$	2.498	2.622	2.472	...
$\phi(1D)$	1.869	1.876	1.845	...
$\phi(2D)$	2.276	2.337	2.258	...
$\phi(3D)$	2.593	2.725	2.607	...

2. $Y(2175)$ and $X(2000)$

According to Table III, one can note that $Y(2175)$ tends to be the candidate of $\phi(3S)$ rather than $\phi(2D)$ state; Ref. [79] and the MGI model mass spectrum show that $Y(2175)$ could be the $\phi(3S)$ or $\phi(2D)$ state because the mass of $Y(2175)$ is between their masses. The position of $Y(2175)$ in the ϕ family needs further discussion based on the decay behavior, which will be given in the next section.

As shown in Table III, the mass spectrum of Ref. [79], the GI model and MGI model all indicate that the newly observed state $X(2000)$ [23] may be $\phi(1D)$ or $\phi(3S)$ state. In fact, Ref. [31] estimated the mass of $\phi(3S)$ to be 2050 MeV, which is smaller than the mass obtained with the GI model, Ref. [79] and MGI model. Further discussion based on the decay behaviors on the assignment of $X(2000)$ will be given below.

The above discussions are only from the point of view of the mass spectra. In the next section, their strong decays will be studied.

B. Decay behavior analysis

Applying the QPC model, one can obtain the OZI-allowed two-body strong decay of vector light family, which is shown in Tables IV and V.

1. The radial excited states of S-wave ϕ meson

In this section, the radial excited states of S-wave ϕ meson will be discussed.

$\phi(1680)$ has been established as a $\phi(2S)$ state [31]. As presented in Table II, the branch ratio $\Gamma_{KK}/\Gamma_{KK^*}$ is approximately 0.13, which is closer to the experimental value 0.07 ± 0.01 [80] than the theoretical result of Ref. [31]. The ratio $\Gamma_{\eta\phi}/\Gamma_{KK^*}$ is predicted to be 0.14, which is close to the value (0.18) of Ref. [31].

TABLE IV. The partial decay widths of the $\phi(3S)$, $s\bar{s}$ hybrid and $\phi(4S)$, the unit of widths is MeV.

$\phi(3S)$		$s\bar{s}$ hybrid		$\phi(4S)$	
Mass	2188	2002	2000 [29]	Mass	2498
Channel	Value	Value	Value [29]	Channel	Value
Total	217	133	120	Total	942
$KK^*(1410)$	48.4	45.2	9	$KK_3^*(1780)$	178
KK^*	60	26.9	16	$K^*K_2^*(1430)$	160
KK_1'	4.36	21.4	64	KK_1'	123
KK_1	23.6	19.3	26	K^*K^*	94.9
KK	11.9	10.8	...	$KK_2^*(1430)$	84.6
$KK_2^*(1430)$	39.8	4.48	2	$KK^*(1410)$	80.9
K^*K^*	22.7	4.40	...	KK^*	67.5
$\eta\phi$	6.66	0.363	3	KK_1	54.2
$\eta'\phi$	0.0862	0.0729	0.02	$KK_4^*(2045)$	54.8
				K^*K_1	29.0
				$KK^*(1680)$	14.9

TABLE V. The partial decay widths of the ϕ (D-wave) mesons, the unit of widths is MeV.

	$\phi(1D)$	X(2175) as $\phi(2D)$	$\phi(3D)$
Channel	Value	Value	Value
Total	442	186	229
KK_1	318	70.7	61.4
K^*K^*	11.5	33.4	40.5
KK	40.8	25.4	17.4
KK^*	57.8	18.7	12.6
$\eta\phi$	13.6	0.879	0.3
$\eta'\phi$...	0.0887	0.087
$KK^*(1410)$...	19.6	5.76
$KK_2^*(1430)$...	14.5	12.1
KK_1'	...	2.56	0.59
$K^*K^*(1410)$	45.6
K^*K_1	26.8
$KK_3^*(1780)$	3.27
$f_1(1426)\phi$	2.16
K^*K_1'	0.454

The decay widths of $\phi(3S)$ state with the mass of 2188[Y(2175)] and 2002[X(2000)] are compared in Table IV. Reference [31] also estimated the mass and the width of $\phi(3S)$ to be 2050 and 380 MeV, respectively. If Y(2175) is the second excited state of $\phi(1S)$, its total width is 217 MeV, which does not agree with the experimental value [22]. According to the mass spectrum analysis section and Ref. [31], the mass and the width of Y(2175) will be larger than the theoretical result when it is treated as $\phi(3S)$.

According to Table IV, when X(2000) is treated as the $\phi(3S)$ state, the width (133 MeV) is in very good agreement with the experimental value [23] and smaller than the theoretical result of Ref. [31]. Unfortunately, the width of the corresponding $s\bar{s}$ hybrid is in the range of 100–150 MeV in flux tube model [29,30], which makes it difficult to determine the internal structure of this X state. Table IV gives a comparison of the two-body decay information between $\phi(3S)$ and $s\bar{s}$ hybrid [29]. Under the $\phi(3S)$ assignment, $X(2000) \rightarrow KK^*(1410) \rightarrow KK\pi\pi$ will be the main decay mode with the branch ratio $\frac{\Gamma_{KK^*(1410)}}{\Gamma_{Total}} \approx 0.34$, which is smaller than that of the $s\bar{s}$ hybrid assignment. KK^* , KK_1' and KK_1 are predicted to be its important decay channels, which have the ratios of 0.2, 0.16, and 0.14, respectively. When treated as a $s\bar{s}$ hybrid [29], X(2000) dominantly decays to KK_1' , with the branch ratio $\frac{\Gamma_{KK_1'}}{\Gamma_{Total}} \approx 0.5$. KK^* , KK_1' and KK_1 can decay to $KK\pi\pi$, which indicates that $KK\pi\pi$ will be the dominant final states of X(2000) as the candidate of second excitation of ϕ . We suggest that experimentalists focus on this final channel. KK , $KK_2^*(1430)$, K^*K^* , and $\eta\phi$ are the sizable decay modes as well. To summarize, the branch ratios of $KK^*(1410)$, KK_1' , and KK differ greatly when X(2000) is treated as $\phi(4S)$ and $s\bar{s}$ hybrid. These predictions of the

branch ratios can help reveal the internal structure of this X state.

The total width of $\phi(4S)$ is approximately 940 MeV, which is too wide to be easily observed experimentally. According to Table IV, the main decay modes of $\phi(4S)$ are $KK_3^*(1780)$, $K^*K_2^*(1430)$, and KK_1' , which have the branch ratios of 0.19, 0.17, and 0.13, respectively. K^*K^* , $KK_2^*(1430)$, $KK^*(1410)$, and KK^* are its important decay channels. Considering the final decay channels of $KK_3^*(1780)$, $K^*K_2^*(1430)$, KK_1' , K^*K^* , $KK_2^*(1430)$, $KK^*(1410)$, and KK^* , $KK\pi\pi$, and $KK\pi$ will be the most important final channels in searching for the $\phi(4S)$ state experimentally. KK_1 , $KK_4^*(2044)$, K^*K_1 , and $KK^*(1680)$ also have sizable contributions to the total width of $\phi(4S)$. These predictions can help us search for and establish this $\phi(4S)$ state.

2. D-wave ϕ mesons

The decay information of the D-wave ϕ mesons is listed in Table V.

As shown in the second column of Table V, the strong decay of $\phi(1D)$ is predicted, which is still unobserved. $\phi(1D)$ has the total width of 442 MeV. KK_1 is its dominant decay channel, which is consistent with Ref. [16]. KK^* and KK are the important final states. $\eta\phi$ and K^*K^* have the same ratio of 3%. If X(2000) is treated as the $\phi(1D)$ state, its total width will be larger than 440 MeV, which does not agree with the experimental value [22]. Thus, it can be basically ruled out that X(2000) is the candidate of $\phi(1D)$.

When treated as the $\phi(2D)$ state, Y(2175) has a total width of 186 MeV, which is consistent with that in Ref. [21]. Under this assignment, $Y(2175) \rightarrow KK_1$ will be the dominant decay mode. In the calculation, K^*K^* and KK are the important decay channels. However, K^*K^* and KK modes are not observed in recent experiments [23,81]. If Y(2175) is the $\phi(2D)$ state, this puzzle should be explained in theory or experiment.

The decay information of the $\phi(3D)$ state is also predicted in this work. The total width of $\phi(3D)$ is approximately 230 MeV, with a mass of 2.6 GeV. The channels KK_1 , $K^*K^*(1410)$ and K^*K^* have the branch ratios of 0.27, 0.2, and 0.18, respectively, which are the main decay modes. K^*K_1 and KK are its important decay channels. Their branch ratios are approximately 0.12 and 0.08, respectively. This work suggests that experimentalists should search for this missing state in KK or $KK\pi\pi$ final states. Otherwise, KK^* and $KK_2^*(1430)$ have sizable contributions to the total width of $\phi(3D)$.

IV. CONCLUSION

This paper presents an analysis of mass spectra of the excitations of ϕ meson, in particular the newly observed X(2000) state, using the modified Godfrey-Isgur quark model, and the structure information of these excitations of the ϕ meson is obtained. After comparing our theoretical

results of the two-body strong decays with the experimental data, we can reach the following conclusions under the conventional meson framework.

- (1) Mass and strong decay behavior analysis indicates that the newly observed state $X(2000)$ [23] may be the $\phi(3S)$ state, and $KK^*(1410)$ will be the dominant decay mode.
- (2) Mass analysis supports $Y(2175)$ as a candidate of $\phi(3S)$ or $\phi(2D)$. However, strong decay behavior analysis shows that the $Y(2175)$ is preferably a $\phi(2D)$ state.
- (3) $\phi(4S)$ is predicted to have a mass of 2.5 GeV and a width of 942 MeV. The ground state, $\phi(1D)$ and second radial excited state $\phi(3D)$ have the mass of 1.869 and 2.6 GeV and the widths of 442 and 229 MeV, respectively.

According to the comparison of the two-body strong decays under the $\phi(3S)$ assignment with that of the $s\bar{s}$ hybrid, it is apparent that the study of the branch ratios of

$KK^*(1410)$, KK'_1 and KK in experiment will be very valuable for identifying the nature of $X(2000)$.

This study is crucial not only to establish the ϕ meson family and future search for the missing excitations but also to help us reveal the structure information of the newly observed $X(2000)$ state. Thus, more experimental measurements of the resonance parameters should be conducted by the BESIII and other experiments, which can help us to identify the nature of $X(2000)$ and establish the ϕ meson family in the future.

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