$B_{(s)} \to D_{s0}^*(2317)P(V)$ decays in perturbative QCD approach

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In this work, we use the pQCD approach to calculate 20 $B_{(s)} \to D_{s0}^*(2317)P(V)$ two body decays by assuming $D_{s0}^*(2317)$ as a $\bar{c}s$ scalar meson, where $P(V)$ denotes a pseudoscalar (vector) meson. These $B_{(s)}$ decays can serve as an ideal platform to probe the valuable information on the inner structure of the charmed-strange meson $D_{s0}^*(2317)$, and to explore the dynamics of strong interactions and signals of new physics. These considered decays can be divided into two types: the Cabibbo-Kobayshi-Maskawa favored decays and the Cabibbo-Kobayshi-Maskawa suppressed decays. The former are induced by a $b \to c$ transition, whose branching ratios are larger than 10^{-5}. The branching fraction of the decay $\bar{B}_s^0 \to D_{s0}^{*+}(2317)\rho^-$ is the largest and reaches about 1.8×10^{-3}, while the branching ratios for the decay $\bar{B}_s^0 \to D_{s0}^{*+}(2317)K^{*-}$ and the other two pure annihilation decays $\bar{B}^0 \to D_{s0}^{*+}(2317)K^-, D_{s0}^{*+}(2317)K^{*-}$ are only at 10^{-5} order. Our predictions are consistent well with the results given by the light cone sum rules approach. These decays are most likely to be measured at the running LHCb and the forthcoming SuperKEKB. The latter are induced by a $b \to u$ transition, among which the channel $\bar{B}^0 \to D^{*-}(2317)\rho^+$ has the largest branching fraction, reaching up to 10^{-5} order. Again the pure annihilation decays $B^- \to D_{s0}^{*-}(2317)\phi, \bar{B}^0 \to D_{s0}^{*-}(2317)K^+(K^{*+})$, and $B^- \to D_{s0}^{*-}(2317)K^0(K^{*0})$ have the smallest branching ratios, which drop to as low as $10^{-10} \sim 10^{-8}$.

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I. INTRODUCTION

The charmed-strange meson $D_{s0}^*(2317)$ was first observed by the *BABAR* Collaboration in the inclusive $D_s^+\pi^0$ invariant mass distribution [1,2], then confirmed by the CLEO [3] and Belle Collaborations [4], respectively. Usually, the $D_{s0}^*(2317)$ meson is suggested as a P-wave $\bar{c}s$ state with spin-parity $J^P = 0^+$. However, there exit two divergences between the data and the theoretical predictions: First, the measured mass for this meson is at least 150 MeV/c^2 lower than the theoretical calculations from a potential model [5,6], lattice QCD, [7] and so on. For example, the authors [8] obtained $M(D_{s0}^*(2317)) = (2480 \pm 30)$ MeV by using the standard Borel-transformed QCD sum rule which was higher than the *BABAR* result by about 160 MeV. Meanwhile, Narsion [9] used the QCD spectral sum rules to get $M(D_{s0}^*(2317)) = (2297 \pm 113)$ MeV and reached the conclusion that $D_s(2317)$ is a $\bar{c}s$ state. Second, the absolute branching ratio of decay $D_{s0}^*(2317)^{\pm} \to D^{\pm}\pi^0$ measured by the BESIII Collaboration [10] showed that $D_{s0}^*(2317)^-$ tends to have a significantly larger branching ratio to $\pi^0 D_s^{*-}$ than to γD_s^{*-}, which differs from the expectation of the conventional $\bar{c}s$ hypothesis. These puzzles inspired various exotic explanations to their inner structure, such as the DK molecule state [11–15], a tetraquark state [16–19], or a mixture of a $\bar{c}s$ state and a tetraquark state [20–22]. In order to further reveal the internal structure of $D_{s0}^*(2317)$, we intend to study the weak production of this charmed-strange meson through the $B_{(s)}$ decays, which can serve as an ideal platform to probe the valuable information on the inner structure of the exotic scalar mesons [23–27]. In the conventional two quark picture, the branching ratios of the decays $B_{(s)} \to D_{s0}^*(2317)P(V)$, where P (V) denotes the light pseudoscalar (vector) meson, are expected to be of the same order of magnitude as those of $B_{(s)} \to D_s P(V)$ decays, since the $D_{s0}^*(2317)$ meson decay constant should be close to that of the pseudoscalar meson D_s as required by the chiral symmetry. On the contrary, in the unconventional picture the corresponding decay amplitudes involve additional hard scattering with the participation of four valence quarks. Then the branching ratios are at least suppressed by the coupling constant and by inverse powers of heavy meson masses, such that they are much smaller than those of $B_{(s)} \to D_s P(V)$ decays by one order. So it is

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meaningful to study the branching ratios of the decays $B_{(s)} \to D_{s0}^*(2317)P(V)$ both in experiment and theory.

$B_{(s)}$ two body nonleptonic decays with the $D_{s0}^*(2317)$ meson involved in the final states have been studied in the light cone sum rules (LCSRs) approach [28], the relativistic quark model (RQM) [29], and the nonrelativistic quark model (NRQM) [30]. Here we would like to use the pQCD approach to study $B_{(s)} \to D_{s0}^*(2317)P(V)$ decays. Studying these decays may shed light on the nature of the $D_{s0}^*(2317)$ meson and explore the dynamics of strong interactions. Furthermore, the study of these weak decays is important for further improvement in the determination of the Cabibbo-Kobayshi-Maskawa (CKM) matrix elements, for testing the prediction of the Standard Model, and searching for possible deviations from theoretical predictions, the so-called "new physics" signals.

The layout of this paper is as follows: we analyze the decay $B_{(s)} \to D_{s0}^*(2317)P(V)$ using the perturbative QCD approach in Sec. II. The numerical results and discussions are given in Sec. III, where the theoretical uncertainties are also considered. The conclusions are presented in the final part.

II. THE PERTURBATIVE CALCULATIONS

In the pQCD approach, the only nonperturbative inputs are the light cone distribution amplitudes and the meson decay constants. For the wave function of the heavy $B_{(s)}$ meson, we take

$$\Phi_{B_{(s)}}(x,b) = \frac{1}{\sqrt{2N_c}}(\not{p}_{B_{(s)}} + m_{B_{(s)}})\gamma_5\phi_{B_{(s)}}(x,b). \quad (1)$$

Here only the contribution of the Lorentz structure $\phi_{B_{(s)}}(x,b)$ is taken into account, since the contribution of the second Lorentz structure $\bar{\phi}_{B_{(s)}}$ is numerically small [31] and has been neglected. For the distribution amplitude $\phi_{B_{(s)}}(x,b)$ in Eq. (1), we adopt the following model:

$$\phi_{B_{(s)}}(x,b) = N_{B_{(s)}}x^2(1-x)^2\exp\left[-\frac{M_{B_{(s)}}^2x^2}{2\omega_b^2} - \frac{1}{2}(\omega_b b)^2\right], \quad (2)$$

where ω_b is a free parameter, we take $\omega_b = 0.4 \pm 0.04(0.5 \pm 0.05)$ GeV for the $B(B_s)$ meson in numerical calculations, and $N_B = 101.445(N_{B_s} = 63.671)$ is the normalization factor for $\omega_b = 0.4(0.5)$. These parameters have been fixed using the rich experimental data on the $B_{(s)}$ decay channels. In this model, the significant feature is the intrinsic transverse momentum dependence, which is essential for the $B_{(s)}$ meson. It can provide additional suppression in the large b region, where the soft dynamics dominates and Sudakov suppression is weaker. Considering a small SU(3) breaking, the s quark momentum fraction is a litter larger than that of the $u(d)$ quark in the lighter B meson, because of the heavier mass for the s quark. From the shape of the distribution amplitude shown in Ref. [32], it is easy to see that the larger ω_b gives a larger momentum fraction to the s quark.

For the wave functions of the scalar meson D_{s0}^*,[1] we use the form defined in Ref. [33]

$$\langle \bar{D}_{s0}^*(2317)^+(p_2)|\bar{c}_\beta(z)s_\gamma(0)|0\rangle = \frac{1}{\sqrt{2N_c}}\int dx e^{ip_2\cdot z}[(\not{p}_2)_{lj} + m_{D_{s0}^*}I_{lj}]\phi_{D_{s0}^*}. \quad (3)$$

It is noticed that the distribution amplitudes which associate with the nonlocal operators $\bar{c}(z)\gamma_\mu s$ and $\bar{c}(z)s$ are different. The difference between them is the order of $\bar{\Lambda}/m_{D_{s0}^*} \sim (m_{D_{s0}^*} - m_c)/m_{D_{s0}^*}$. If we set $m_{D_{s0}^*} \sim m_c$, these two distribution amplitudes are very similar. For the leading power calculation, it is reasonable to parametrize them in the same form as

$$\phi_{D_{s0}^*}(x) = \tilde{f}_{D_{s0}^*}6x(1-x)[1 + a(1-2x)] \quad (4)$$

in the heavy quark limit. Here $\tilde{f}_{D_{s0}^*} = 225 \pm 25$ MeV is determined from the two-point QCD sum rules, and the shape parameter $a = -0.21$ [28] is fixed under the condition that the distribution amplitude $\phi_{D_{s0}^*}(x)$ possesses the maximum at $\bar{x} = (m_{D_{s0}^*} - m_c)/m_{D_{s0}^*}$ with $m_c = 1.275$ GeV. It is worthwhile to point out that the intrinsic b dependence of this charmed meson's wave functions has been neglected in our analysis.

Since the light cone distribution amplitudes of the pseudoscalar mesons $\pi, K, \eta^{(\prime)}$ and the vector mesons ρ, K^*, ω have been well constrained in Refs. [34–40], and been tested systematically in Ref. [32], we will use these light cone distribution amplitudes directly listed in [32], together with the corresponding decay constants.

For these processes considered, the weak effective Hamiltonian $H_{\rm eff}$ can be written as two types:

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}}V_{cb}V_{uq}^*[C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu)], \quad \text{Type I} \quad (5)$$

where the tree operators are given as:

$$O_1 = (\bar{c}_\alpha b_\beta)_{V-A}(\bar{q}_\beta u_\alpha)_{V-A}, \quad O_2 = (\bar{c}_\alpha b_\alpha)_{V-A}(\bar{q}_\beta u_\alpha)_{V-A}, \quad (6)$$

for the CKM favored channels, while the effective Hamiltonian for the CKM suppressed decays is written as

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}}V_{ub}V_{cq}^*[C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu)], \quad \text{Type II} \quad (7)$$

with

[1]From now on, we will use D_{s0}^* to denote $D_{s0}^*(2317)$ for simplicity in some places.

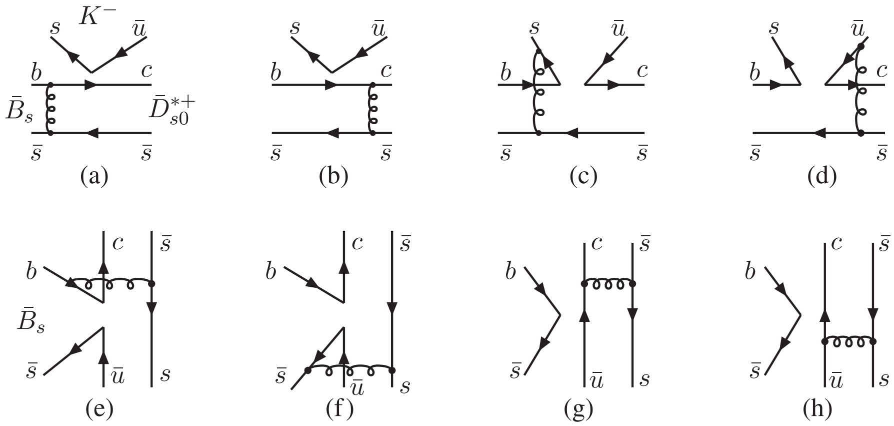

FIG. 1. Diagrams contributing to the $\bar{B}_s^0 \to D_{s0}^{*+}K^-$ decay, where (a) and (b) are the factorizable emission diagrams, (c) and (d) are the nonfactorizable emission diagrams, (e) and (f) are the nonfactorizable annihilation diagrams, (g) and (h) are the factorizable annihilation diagrams.

$$O_1 = (\bar{u}_\alpha b_\beta)_{V-A}(\bar{D}_\beta c_\alpha)_{V-A}, \quad O_2 = (\bar{u}_\alpha b_\alpha)_{V-A}(\bar{D}_\beta c_\alpha)_{V-A}. \tag{8}$$

Here D represents the $d(s)$ quark. The type I channel is induced by the $b \to c$ transition, such as $\bar{B}_s^0 \to D_{s0}^{*+}\pi^-(K^-), D_{s0}^{*+}\rho^-(K^{*-}), \bar{B}^0 \to D_{s0}^{*+}K^-(K^{*-})$. While the type II decay is induced by the $b \to u$ transition, such as $\bar{B}^0 \to D_{s0}^{*-}\pi^+(K^+), D_{s0}^{*-}\rho^+(K^{*+}), B^- \to D_{s0}^{*-}\pi^0(\rho^0, \omega, \phi)$, $D_{s0}^{*-}\eta^{(\prime)}$. For the CKM favored decays, we take the decay $\bar{B}_s^0 \to D_{s0}^{*+}K^-$ as an example, whose Feynman diagrams are given in Fig. 1. The first line Feynman diagrams are for the emission type ones, where Figs. 1(a) and 1(b) are the factorization diagrams, Figs. 1(c) and 1(d) are the nonfactorization ones, and their amplitudes can be written as

$$\begin{aligned}\mathcal{F}^P_{B\to D_{s0}^*} = {} & 8\pi C_F M_B^4 f_P \int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1, b_1)\phi_{D_{s0}^*} \\ & \times \{[1 + x_2 - r_{D_{s0}^*}(2x_2 - 1)]E_e(t_a)S_t(x_2)(t_a)h_e(x_1, x_2(1 - r^2_{D_{s0}^*}), b_1, b_2) \\ & + [(2 - r_{D_{s0}^*})r_{D_{s0}^*} + r_c(1 - 2r_{D_{s0}^*})]E_e(t_b)S_t(x_1)h_e(x_2, x_1(1 - r^2_{D_{s0}^*}), b_2, b_1)]\},\end{aligned} \tag{9}$$

$$\begin{aligned}\mathcal{M}^P_{B\to D_{s0}^*} = {} & -32\pi C_f m_B^4/\sqrt{2N_C} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_B(x_1, b_1) \\ & \times \phi_{D_{s0}^*}(x_2)\phi_P(x_3)[(r_{D_{s0}^*}x_2 - x_3)E_{en}(t_c)h^c_{en}(x_1, x_2, x_3, b_1, b_3) \\ & + (1 + x_2 - x_3 - r_{D_{s0}^*}x_2)E_{en}(t_d)h^d_{en}(x_1, x_2, x_3, b_1, b_3)],\end{aligned} \tag{10}$$

where P denotes a pseudoscalar meson, $r_{D_{s0}^*} = m_{D_{s0}^*}/M_B$, $r_c = m_c/M_B$ and f_P is the decay constant of the pseudoscalar meson. The evolution factors evolving the scale t and the hard functions for the hard part of the amplitudes are listed as

$$E_e(t) = \alpha_s(t)\exp[-S_B(t) - S_{D_{s0}^*}(t)], \tag{11}$$

$$E_{en}(t) = \alpha_s(t)\exp[-S_B(t) - S_P(t) - S_{D_{s0}^*}(t)|_{b_1=b_2}], \tag{12}$$

$$\begin{aligned}h_e(x_1, x_2, b_1, b_2) = {} & K_0(\sqrt{x_1x_2}m_Bb_1)[\theta(b_1 - b_2)K_0(\sqrt{x_2}m_Bb_1)I_0(\sqrt{x_2}m_Bb_2) \\ & +\theta(b_2 - b_1)K_0(\sqrt{x_2}m_Bb_2)I_0(\sqrt{x_2}m_Bb_1)],\end{aligned} \tag{13}$$

$$\begin{aligned}h^j_{en}(x_1, x_2, x_3, b_1, b_3) = {} & [\theta(b_1 - b_3)K_0(\sqrt{A^2}m_Bb_1)I_0(\sqrt{A^2}m_Bb_3) \\ & +(b_1 \leftrightarrow b_3)]\left.\begin{pmatrix} K_0(A_jm_Bb_3) & \text{for } A_j^2 \geq 0 \\ \frac{i\pi}{2}H_0^{(1)}\left(\sqrt{|A_j^2|}m_Bb_3\right) & \text{for } A_j^2 \leq 0 \end{pmatrix}\right|_{j=c,d},\end{aligned} \tag{14}$$

with the variables

$$A^2 = x_2 x_1, \tag{15}$$

$$A_c^2 = x_2(x_1 - x_3(1 - r^2_{D^*_{s0}})), \tag{16}$$

$$A_d^2 = x_2(x_1 - (1 - x_3)(1 - r^2_{D^*_{s0}})). \tag{17}$$

The hard scales t and the expression of the Sudakov factor in each amplitude can be found in the Appendix. As we know that the double logarithms $\alpha_s \ln^2 x$ produced by the radiative corrections are not small expansion parameters when the end point region is important, in order to improve the perturbative expansion, the threshold resummation of these logarithms to all order is needed, which leads to a quark jet function

$$S_t(x) = \frac{2^{1+2c}\Gamma(3/2 + c)}{\sqrt{\pi}\Gamma(1 + c)}[x(1 - x)]^c, \tag{18}$$

with $c = 0.5$. It is effective to smear the end point singularity with a momentum fraction $x \to 0$. This factor will also appear in the factorizable annihilation amplitudes.

As to the amplitudes for the second line, Feynman diagrams can be obtained by the Feynman rules and are given as

$$\begin{aligned}\mathcal{M}_{ann}^{D^*_{s0}} = {} & 32\pi C_f m_B^4/\sqrt{2N_C}\int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_B(x_1, b_1)\phi_{D^*_{s0}}(x_2) \\ & \times \{-[r_{D^*_{s0}} r_P((x_2 - x_3 + 3)\phi_P^p(x_3) - (x_2 + x_3 - 1)\phi_P^T(x_3)) \\ & + x_2\phi_P(x_3)]E_{an}(t_e)h_{an}^e(x_1, x_2, x_3, b_1, b_3) \\ & + [(1 - x_3)\phi_P(x_3) + r_{D^*_{s0}} r_P((x_2 - x_3 + 1)\phi_P^p(x_3) + (x_2 + x_3 - 1)\phi_P^T(x_3))] \\ & \times E_{an}(t_f)h_{an}^f(x_1, x_2, x_3, b_1, b_3)\},\end{aligned} \tag{19}$$

$$\begin{aligned}\mathcal{F}_{ann}^{D^*_{s0}} = {} & -8\pi C_f f_B m_B^4 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \phi_{D^*_{S0}}(x_2)\{[4r_P r_{D^*_{s0}}(1 - x_3)\phi_P^p(x_3) \\ & + r_P(r_c + 2r_{D^*_{S0}} x_3)(\phi_P^p(x_3) + \phi_P^T(x_3)) + (1 + 2r_c r_{D^*_{s0}} - x_3)\phi_P(x_3)]E_{af}(t_g) \\ & \times S_t(x_3)h_{af}(x_2, x_3(1 - r^2_{D^*_{s0}}), b_2, b_3) - [x_2\phi_P(x_3) + 2r_P r_{D^*_{s0}}(1 + x_2)\phi_P^p(x_3)] \\ & \times E_{af}(t_h)S_t(x_2)h_{af}(x_3, x_2(1 - r^2_{D^*_{s0}}), b_3, b_2)\}.\end{aligned} \tag{20}$$

Here $\mathcal{F}_{ann}^{D^*_{s0}}(\mathcal{M}_{ann}^{D^*_{s0}})$ are the (non)factorizable annihilation type amplitudes, where the evolution factors E evolving the scale t and the hard functions of the hard part of factorization amplitudes are listed as

$$E_{an}(t) = \alpha_s(t)\exp[-S_B(t) - S_{D^*_{s0}}(t) - S_P(t)|_{b_2=b_3}], \tag{21}$$

$$E_{af}(t) = \alpha_s(t)\exp[-S_{D^*_{s0}}(t) - S_P(t)], \tag{22}$$

$$\begin{aligned}h_{an}^j(x_{i=1,2,3}, b_1, b_3) = {} & i\frac{\pi}{2}\Big[\theta(b_1 - b_3)H_0^{(1)}\Big(\sqrt{x_2x_3(1 - r^2_{D^*_{s0}})}m_B b_1\Big)J_0\Big(\sqrt{x_2x_3(1 - r^2_{D^*_{s0}})}m_B b_3\Big) \\ & + (b_1 \leftrightarrow b_3)\Big]\left.\begin{pmatrix} K_0(L_j m_B b_1) & \text{for } L_j^2 \geq 0 \\ \frac{i\pi}{2}H_0^{(1)}\Big(\sqrt{|L_j^2|}m_B b_1\Big) & \text{for } L_j^2 \leq 0\end{pmatrix}\right|_{j=e,f},\end{aligned} \tag{23}$$

$$\begin{aligned}h_{af}(x_2, x_3, b_2, b_3) = {} & \left(i\frac{\pi}{2}\right)^2 H_0^{(1)}(\sqrt{x_2x_3}m_B b_2) \\ & \times[\theta(b_2 - b_3)H_0^{(1)}(\sqrt{x_3}m_B b_2)J_0(\sqrt{x_3}m_B b_3) + (b_2 \leftrightarrow b_3)],\end{aligned} \tag{24}$$

where the definition of L_j^2 is written as

$$L_e^2 = r_b^2 - (1 - x_2)[x_3(1 - r^2_{D^*_{s0}}) + r^2_{D^*_{s0}} - x_1], \tag{25}$$

$$L_f^2 = x_2[x_1 - (1-x_3)(1-r_{D_{s0}^*}^2)]. \tag{26}$$

The functions $H_0^{(1)}, J_0, K_0, I_0$ in the upper hard kernels $h_e, h_{en}^j, h_{an}^j, h_{af}$ are the (modified) Bessel functions, which can be obtained from the Fourier transformations of the quark and gluon propagators.

Similarly, we can also give the amplitudes for the CKM suppressed decay channels,

$$\begin{aligned}\mathcal{F}_{B\to P}^{D_{s0}^*} = {}& 8\pi C_F M_B^4 f_{D_{S0}^*} \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_B(x_1, b_1) \\ & \times [(1+x_3)\phi_P(x_3) - r_P(2x_3-1)(\phi_P^p(x_3) + \phi_P^T(x_3))] \\ & \times E_e(t_a) S_t(x_3)(t_a) h_e(x_1, x_3(1-r_D^2), b_1, b_3) \\ & + [2r_P\phi_P^p(x_3)] E_e(t_b) S_t(x_1) h_e(x_3, x_1(1-r_D^2), b_3, b_1)],\end{aligned} \tag{27}$$

$$\begin{aligned}\mathcal{M}_{B\to P}^{D_{s0}^*} = {}& 32\pi C_f m_B^4/\sqrt{2N_C} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \phi_{D_{s0}^*}(x_2) \\ & \times \{[x_2\phi_P(x_3) + r_P x_3(\phi_P^T(x_3) - \phi_P^p(x_3))] E_{en}(t_c) h_{en}^c(x_1, x_3, x_3, b_1, b_3) \\ & - [(1-x_2+x_3)\phi_P(x_3) - r_P x_3(\phi_P^T(x_3) + \phi_P^p(x_3))] \\ & \times E_{en}(t_{D_{s0}^*}) h_{en}^d(x_1, x_2, x_3, b_1, b_3)\},\end{aligned} \tag{28}$$

where these two amplitudes are factorizable and nonfactorizable emission contributions, respectively. The amplitudes $\mathcal{F}_{B_s\to D_{s0}^*}^P, \mathcal{M}_{B_s\to D_{s0}^*}^P$ are the color allowed amplitudes, while $\mathcal{F}_{B\to P}^{D_{s0}^*}, \mathcal{M}_{B\to P}^{D_{s0}^*}$ are the color suppressed ones. The annihilation type amplitudes are listed as

$$\begin{aligned}\mathcal{M}_{\rm ann}^P = {}& 32\pi C_f m_B^4/\sqrt{2N_C} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \phi_{D_{S0}^*}(x_2) \\ & \times \{[r_{D_{S0}^*} r_P((x_2+x_3+2)\phi_P^p(x_3) - (x_2-x_3)\phi_P^T(x_3)) \\ & - x_3\phi_P(x_3)] E_{an}(t_e) h_{an}^e(x_1, x_2, x_3, b_1, b_3) \\ & + [-r_{D_{S0}^*} r_P((x_2+x_3)\phi_P^p(x_3) + (x_2-x_3)\phi_P^T(x_3)) + x_2\phi_P(x_3)] \\ & \times E_{an}(t_f) h_{an}^f(x_1, x_2, x_3, b_1, b_3)\},\end{aligned} \tag{29}$$

$$\begin{aligned}\mathcal{F}_{\rm ann}^P = {}& 8\pi C_f f_B m_B^4 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \phi_{D_{S0}^*}(x_2) \\ & \times \{[2r_{D_{S0}^*} r_P(1+x_2)\phi_P^p(x_3) - x_2\phi_P(x_3)] E_{af}(t_g) S_t(x_3) h_{af}(x_3, x_2(1-r_{D_{S0}^*}^2), b_3, b_2) \\ & + [(x_3 - r_{D_{S0}^*}(2r_c - r_{D_{S0}^*}))\phi_P(x_3) + r_P(r_c - 2r_{D_{S0}^*}(1+x_3))\phi_P^p(x_3) \\ & - r_P(r_c - 2r_{D_{S0}^*}(1-x_3))\phi_P^T(x_3)] E_{af}(t_h) S_t(x_2) h_{af}(x_2, x_3(1-r_{D_{S0}^*}^2), b_2, b_3)\}.\end{aligned} \tag{30}$$

The definitions for the evolution factors, the hard functions and the jet function $S_t(x)$ in Eqs. (27)–(30) can be found in Eqs. (9) and (10) and in Eqs. (19) and (20) with the different parameters in the hard function $h_{en}^{c,d}, h_{an}^{e,f}$, which are listed as

$$A \to A'^2 = x_1 x_3 (1 - r_{D_{s0}^*}^2), \tag{31}$$

$$A_c^2 \to A_c'^2 = (x_1 - x_2) x_3 (1 - r_{D_{s0}^*}^2), \tag{32}$$

$$A_d^2 \to A_d'^2 = r_c^2 - r_{D_{s0}^*}^2 + (x_1 + x_2) r_{D_{s0}^*}^2 - (1 - x_2 - x_1) x_3 (1 - r_{D_{S0}^*}^2), \tag{33}$$

$$L_e^2 \to L_e'^2 = r_b^2 - (1 - x_2)[1 - x_3(1 - r_{D_{S0}^*}^2) - x_2 r_{D_{S0}^*}^2 - x_1], \tag{34}$$

$$L_f^2 \to L_f'^2 = x_2[x_1 - x_3(1 - r_{D_{s0}^*}^2) - x_2 r_{D_{s0}^*}^2]. \quad (35)$$

For the decays $B_{(s)} \to D_{s0}^* V$, their amplitudes can be obtained from the ones of decays $B_{(s)} \to D_{s0}^* P$ with the following substitutions:

$$\phi_P \to \phi_V, \quad \phi_P^p \to \phi_V^s, \quad \phi_P^t \to \phi_V^t,$$
$$r_P \to -r_V, \quad f_P \to f_V. \quad (36)$$

Combining these amplitudes, one can ease to write out the total decay amplitude of each considered channel:

$$\mathcal{A}(\bar{B}_s^0 \to D_{s0}^{*+}\pi^-) = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* (F_{B_s \to D_{s0}^*}^{\pi} a_1 + M_{B_s \to D_{s0}^*}^{\pi} C_1), \quad (37)$$

$$\mathcal{A}(\bar{B}_s^0 \to D_{s0}^{*+}K^-) = \frac{G_F}{\sqrt{2}} V_{cb} V_{us}^* (F_{B_s \to D_{s0}^*}^{K} a_1 + M_{B_s \to D_{s0}^*}^{K} C_1 + M_{\rm ann}^{D_{s0}^*} C_2 + F_{\rm ann}^{D_{s0}^*} a_2), \quad (38)$$

$$\mathcal{A}(\bar{B}^0 \to D_{s0}^{*+}K^-) = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} (M_{ann}^{D_{s0}^*} C_2 + F_{ann}^{D_{s0}^*} a_2), \quad (39)$$

$$A(\bar{B}^0 \to D_{s0}^{*-}\pi^+) = \frac{G_F}{\sqrt{2}} V_{ub} V_{cs}^* (F_{B \to \pi}^{D_{s0}^*} a_1 + M_{B \to \pi}^{D_{s0}^{*-}} C_1), \quad (40)$$

$$\mathcal{A}(B^- \to D_{s0}^{*-}\pi^0) = \frac{G_F}{\sqrt{2}} V_{ub} V_{cs}^* \frac{1}{\sqrt{2}} (F_{B \to \pi}^{D_{s0}^*} a_1 + M_{B \to \pi}^{D_{s0}^{*-}} C_1), \quad (41)$$

$$\mathcal{A}(\bar{B}^0 \to D_{s0}^{*-}K^+) = \frac{G_F}{\sqrt{2}} V_{ub} V_{cd}^* (M_{\rm ann}^{K} C_2 + F_{\rm ann}^{K} a_2), \quad (42)$$

$$\mathcal{A}(B^- \to D_{s0}^{*-}K^0) = \frac{G_F}{\sqrt{2}} V_{ub} V_{cd}^* (M_{\rm ann}^{K} C_1 + F_{\rm ann}^{K} a_1), \quad (43)$$

$$\mathcal{A}(B^- \to D_{s0}^{*-}\eta_{n\bar{n}}) = \frac{G_F}{\sqrt{2}} V_{ub} V_{cs}^* (F_{B \to \eta_{n\bar{n}}}^{D_{s0}^*} a_1 + M_{B \to \eta_{n\bar{n}}}^{D_{s0}^{*-}} C_1), \quad (44)$$

$$\mathcal{A}(B^- \to D_{s0}^{*-}\eta_{s\bar{s}}) = \frac{G_F}{\sqrt{2}} V_{ub} V_{cs}^* (M_{\rm ann}^{\eta_{s\bar{s}}} C_1 + F_{\rm ann}^{\eta_{s\bar{s}}} a_1), \quad (45)$$

where $\eta_{n\bar{n}} = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$ and $\eta_{s\bar{s}}$. The physical states η and η' can be related to these two flavor states $\eta_{n\bar{n}}$ and $\eta_{s\bar{s}}$ through the following mixing mechanism:

$$\begin{pmatrix} |\eta\rangle \\ |\eta\rangle \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} |\eta_{n\bar{n}}\rangle \\ |\eta_{s\bar{s}}\rangle \end{pmatrix}, \quad (46)$$

with the mixing angle $\phi = 39.3° \pm 1.0°$ [41]. The formulas for the $B_{(s)} \to D_{s0}^* V$ can be obtained through the following substitutions in Eqs. (37)–(45),

$$\pi^\pm \to \rho^\pm, \quad \pi^0 \to \rho^0, \omega, \quad K \to K^*, \quad \eta_{s\bar{s}} \to \phi. \quad (47)$$

III. THE NUMERICAL RESULTS AND DISCUSSIONS

We use the following input parameters in the numerical calculations [28,42]:

$$f_B = 190 \text{ MeV}, \quad f_{B_s} = 230 \text{ MeV},$$
$$M_B = 5.28 \text{ GeV}, \quad M_{B_s} = 5.37 \text{ GeV}, \quad (48)$$

$$\tau_B^\pm = 1.638 \times 10^{-12} s, \quad \tau_{B^0} = 1.519 \times 10^{-12} s,$$
$$\tau_{B_s} = 1.512 \times 10^{-12} s, \quad (49)$$

$$M_W = 80.42 \text{ GeV}, \quad M_{D_{s0}^*} = 2.3177 \text{ GeV},$$
$$\tilde{f}_{D_{s0}^*} = (225 \pm 25) \text{ MeV}. \quad (50)$$

For the CKM matrix elements, we adopt the Wolfenstein parametrization with values from the Particle Data Group [42] $A = 0.811 \pm 0.026, \lambda = 0.22506 \pm 0.00050$, $\bar{\rho} = 0.124_{-0.018}^{+0.019}$, and $\bar{\eta} = 0.356 \pm 0.011$.

In the $B_{(s)}$-rest frame, the decay rates of $B_{(s)} \to D_{s0}^*(2317)P(V)$ can be written as

$$\mathcal{BR}(B_{(s)} \to D_{s0}^*(2317)P(V)) = \frac{\tau_{B_{(s)}}}{16\pi M_B}(1 - r_{D_{s0}^*}^2)|\mathcal{A}|^2, \quad (51)$$

where $\mathcal{A}$ is the total decay amplitude of each considered decay, which has been listed in Eqs. (37)–(45). The branching ratios for the CKM favored (type I) decays are given in Table I, where one can find that our predictions are consistent with those calculated in the light cone sum rules approach within errors. However, our predictions are smaller than the results given by the RQM [29] and the NRQM [30], respectively. Especially, for the pure annihilation decay $\bar{B}_s^0 \to D_{s0}^{*+}K^{*-}$, whose branching fraction reaches up to 10^{-3} predicted by NRQM approach, it seems too large to be acceptable.

The Belle Collaboration has measured the product of the branching fractions $\mathrm{Br}(\bar{B}^0 \to D_{s0}^*(2317)^+ K^-) \times \mathrm{Br}(D_{s0}^*(2317)^+ \to D_s^+ \pi^0)$,[2] which is given as $(5.3_{-1.3}^{+1.5} \pm 0.7 \pm 1.4) \times 10^{-5}$ [43]. After rescaling the branching ratio of the decay $D^+ \to \phi\pi^+$, the Particle Data Group reported

[2] Recently, the absolute branching fraction of $D_{s0}^*(2317)^+ \to D_s^+\pi^0$ has been measured by the BESIII Collaboration as $1.00_{-0.14}^{+0.00} \pm 0.14$ [10].

TABLE I. Branching ratios ($\times 10^{-4}$) of the CKM favored (type I) decays obtained in the pQCD approach (this work), where the errors for these entries correspond to the uncertainties in the $w_b = 0.4 \pm 0.04(0.5 \pm 0.05)$ for $B(B_s)$ meson, the hard scale t varying from $0.75t$ to $1.25t$, and the CKM matrix elements. In Ref. [28], the branching ratios are calculated in the factorization assumption (FA) with the form factors obtained in the LCSRs. We also list the results given by the RQM [29] and the NRQM [30], respectively.

Modes	This Work	LCSRs [28]	RQM [29]	NRQM [30]
$\bar{B}^0_s \to D^*_{s0}(2317)^+\pi^-$	$5.49^{+2.64+0.41+0.35}_{-1.68-0.27-0.35}$	$5.2^{+2.5}_{-2.1}$	9	10
$\bar{B}^0_s \to D^*_{s0}(2317)^+K^-$	$0.51^{+0.06+0.01+0.01}_{-0.04-0.01-0.01}$	$0.4^{+0.2}_{-0.2}$	0.7	0.9
$\bar{B}^0_s \to D^*_{s0}(2317)^+\rho^-$	$17.7^{+8.5+1.3+1.2}_{-5.3-0.8-1.1}$	13^{+6}_{-5}	22	27
$\bar{B}^0_s \to D^*_{s0}(2317)^+K^{*-}$	$1.01^{+0.44+0.06+0.05}_{-0.31-0.06-0.07}$	$0.8^{+0.4}_{-0.3}$	1.2	16
$\bar{B}^0 \to D^*_{s0}(2317)^+K^-$	$0.18^{+0.06+0.01+0.01}_{-0.04-0.01-0.01}$	…	…	…
$\bar{B}^0 \to D^*_{s0}(2317)^+K^{*-}$	$0.25^{+0.07+0.02+0.01}_{-0.06-0.02-0.02}$	…	…	…

$\mathrm{Br}(\bar{B}^0 \to D^*_{s0}(2317)^+K^-) \times \mathrm{Br}(D^*_{s0}(2317)^+ \to D_s^+\pi^0) = (4.2^{+1.4}_{-1.3} \pm 0.4) \times 10^{-5}$ [42]. Then the Belle Collaboration improved the measurement for the decay $\bar{B}^0 \to D^*_{s0}(2317)^+K^-$ and renewed the branching ratio as $(3.3 \pm 0.6 \pm 0.7) \times 10^{-5}$ [44], where the authors concluded that the branching ratio for this pure annihilation decay is of the same order of magnitude as $\mathrm{Br}(\bar{B}^0 \to D_s^+K^-)$, which is measured as $(2.7 \pm 0.5) \times 10^{-5}$ [42]. Although the decay $\bar{B}^0 \to D^*_{s0}(2317)^+K^-$ has not been measured accurately by experiment, we believe that our prediction is reasonable.

It is helpful to define the following ratios based on the factorization assumption:

$$R_1 = \frac{\mathrm{Br}(\bar{B}^0_s \to D^{*+}_{s0}\pi^-)}{\mathrm{Br}(\bar{B}^0_s \to D^{*+}_{s0}K^-)} \approx \left|\frac{V_{ud}f_\pi}{V_{us}f_K}\right|^2 \approx 12.4, \quad (52)$$

$$R_2 = \frac{\mathrm{Br}(\bar{B}^0_s \to D^{*+}_{s0}\rho^-)}{\mathrm{Br}(\bar{B}^0_s \to D^{*+}_{s0}K^{*-})} \approx \left|\frac{V_{ud}f_\rho}{V_{us}f_{K^*}}\right|^2 \approx 17.4, \quad (53)$$

which are consistent with the results given by our predictions.

In the following, we list the branching ratios for the CKM suppressed decays $B_{(s)} \to D^*_{s0}(2317)P$ as the following:

$$\mathrm{Br}(\bar{B}^0_s \to D^*_{s0}(2317)^-K^+) = (6.86^{+2.60+0.29+0.45}_{-1.79-0.20-0.43}) \times 10^{-6}, \quad (54)$$

$$\mathrm{Br}(\bar{B}^0 \to D^*_{s0}(2317)^-\pi^+) = (6.91^{+2.93+0.45+0.43}_{-1.95-0.44-0.30}) \times 10^{-6}, \quad (55)$$

$$\mathrm{Br}(B^- \to D^*_{s0}(2317)^-\pi^0) = (3.72^{+1.59+0.23+0.25}_{-1.04-0.16-0.23}) \times 10^{-6}, \quad (56)$$

$$\mathrm{Br}(B^- \to D^*_{s0}(2317)^-\eta) = (6.30^{+2.17+0.41+0.44}_{-1.52-0.40-0.29}) \times 10^{-7}, \quad (57)$$

$$\mathrm{Br}(B^- \to D^*_{s0}(2317)^-\eta') = (4.17^{+1.50+0.33+0.27}_{-1.05-0.18-0.27}) \times 10^{-7}, \quad (58)$$

$$\mathrm{Br}(\bar{B}^0 \to D^*_{s0}(2317)^-K^+) = (5.99^{+0.56+0.33+0.39}_{-0.60-0.31-0.38}) \times 10^{-9}, \quad (59)$$

$$\mathrm{Br}(B^- \to D^*_{s0}(2317)^-K^0) = (0.82^{+0.17+0.15+0.06}_{-0.15-0.09-0.05}) \times 10^{-9}, \quad (60)$$

where the first uncertainty comes from the $w_b = 0.4 \pm 0.04(0.5 \pm 0.05)$ for the $B(B_s)$ meson, the second error is from the hard scale-dependent uncertainty, which we vary from $0.75t$ to $1.25t$, and the third one is from the CKM matrix elements. For these CKM suppressed decays, the factorizable emission diagrams [where the $D^*_{s0}(2317)^-$ meson is emitted from the weak vertex] are the color favored ones with the Wilson coefficients $a_1 = C_2 + C_1/3$, while the nonfactorizable emission diagrams are highly suppressed by the Wilson coefficient $C_1/3$. This means that the dominant amplitudes are nearly proportional to the product of $D^*_{s0}(2317)$ meson decay constant and a B to the light meson form factor. Unfortunately, the decay constant of the scalar meson for the vector current is small, which is defined as $\langle 0|\bar{s}\gamma_\mu c|D^*_{s0}(P)\rangle = f_{D^*_{s0}}P_\mu$. This vector current decay constant $f_{D^*_{s0}}$ can be related with the scale-dependent scalar one $\tilde{f}_{D^*_{s0}}$ by the equation of motion

$$f_{D^*_{s0}} = \frac{(m_c - m_s)}{m_{D^*_{s0}}}\tilde{f}_{D^*_{s0}}, \quad (61)$$

where $m_{c(s)}$ is the current quark $c(s)$ mass and $\tilde{f}_{D^*_{s0}}$ iss defined as $\langle|\bar{s}c|D^*_{s0}(P)\rangle = m_{D^*_{s0}}\tilde{f}_{D^*_{s0}}$. $\tilde{f}_{D^*_{s0}} = (225 \pm 25)$ MeV has been determined from the two-point QCD sum rules. If taking $m_c = 1.275$ GeV, $m_s = 0.096$ GeV, $m_{D^*_{s0}} = 2.3177$ GeV [42], one can find that $f_{D^*_{s0}} = 0.11$ GeV. So we can speculate that the branching ratio of the decay

TABLE II. The amplitudes from the nonfactorizable and factorizable annihilation Feynman diagrams, which denote as NFAA and FAA, respectively. For each amplitude, the value has been given, together with the corresponding Wilson coefficient (WC).

	NFAA		FAA		
Modes	WC	value	WC	value	Total
$B^- \to D_{s0}^{*-} K^0 (\times 10^5)$	C_1	$-3.99 - i2.75$	$C_2 + C_1/3$	$1.72 - i0.57$	$-2.13 - i3.31$
$B^- \to D_{s0}^{*-} K^{*0} (\times 10^5)$		$-1.02 - i4.60$		$1.38 + i0.87$	$-0.50 - i3.72$
$\bar{B}^0 \to D_{s0}^{*-} K^+ (\times 10^5)$	C_2	$8.50 - i8.22$	$C_1 + C_2/3$	$-0.50 - i0.35$	$8.00 - i8.57$
$\bar{B}^0 \to D_{s0}^{*-} K^{*+} (\times 10^5)$		$-8.80 - i13.54$		$0.07 - i0.11$	$-0.26 - i11.5$
$\bar{B}^0 \to D_{s0}^{*+} K^- (\times 10^3)$	C_2	$4.46 - i4.44$	$C_1 + C_2/3$	$-0.32 - i0.16$	$4.14 - i4.60$
$\bar{B}^0 \to D_{s0}^{*+} K^{*-} (\times 10^3)$		$4.83 - i5.47$		$-0.18 - i0.06$	$4.65 - i5.53$

$\bar{B}^0 \to D_s^- \pi^+$ should be much larger than that of $\bar{B}^0 \to D_{s0}^*(2317)^- \pi^+$. This is indeed the case: If we replace the decay constant, the mass, and the wave functions of the scalar meson D_{s0}^{*-} with those of the pseudoscalar meson D_s^- in the calculation program, we find that the branching ratio $\mathrm{Br}(\bar{B}^0 \to D_s^- \pi^+) = (27.6^{+8.23}_{-7.65}) \times 10^{-5}$, which is consistent with the current experimental value $(21.6 \pm 2.6) \times 10^{-5}$ [42] within errors. Since the form factors of $B \to V$ are a little large, one can expect that these tree operator dominant decays $B \to D_{s0}^{*-} V$ have a larger branching ratios than those of $B \to D_{s0}^{*-} P$ decays. While this conclusion is not satisfied to the pure annihilation type decays.

For the decays $\bar{B}^0 \to D_{s0}^{*-} \pi^+, B^- \to D_{s0}^{*-} \pi^0$, their branching ratios are sensitive to the form factor $B \to \pi$. If using the Gegenbauer coefficients $a_2^\pi = 0.44, a_4^\pi = 0.25$, we will get a reasonable form factor $F^{B\to\pi}(0) = 0.22$, which is larger than $F^{B\to\pi}(0) = 0.18$ obtained by using the updated Gegenbauer coefficients $a_2^\pi = 0.115, a_4^\pi = -0.015$. Corresponding to the smaller form factor, the branching ratios of the decays $\bar{B}^0 \to D_{s0}^{*-} \pi^+, B^- \to D_{s0}^{*-} \pi^0$ will have a noticeable decrement and become $\mathrm{Br}(\bar{B}^0 \to D_{s0}^{*-} \pi^+) = 3.78 \times 10^{-6}, \mathrm{Br}(B^- \to D_{s0}^{*-} \pi^0) = 2.05 \times 10^{-6}$. It is similar for the decays $B^- \to D_{s0}^{*-} \eta^{(\prime)}$. While for the CKM favored decay $\bar{B}_s^0 \to D_{s0}^{*+} \pi^-$, the branching ratio is not sensitive to the Gegenbauer coefficients for the π meson wave functions. The difference of the branch ratios by using these two group Gegenbauer coefficients is only about 4%.

Similarly, the branching ratios of the decays $B_{(s)} \to D_{s0}^*(2317)^- V$ are calculated as

$$\mathrm{Br}(\bar{B}_s^0 \to D_{s0}^*(2317)^- K^{*+}) = (7.97^{+2.56+0.49+0.52}_{-2.14-0.64-0.51}) \times 10^{-6}, \quad (62)$$

$$\mathrm{Br}(\bar{B}^0 \to D_{s0}^*(2317)^- \rho^+) = (1.61^{+0.64+0.16+0.11}_{-0.46-0.10-0.10}) \times 10^{-5}, \quad (63)$$

$$\mathrm{Br}(B^- \to D_{s0}^*(2317)^- \rho^0) = (8.70^{+3.42+0.51+1.53}_{-2.34-0.56-0.52}) \times 10^{-6}, \quad (64)$$

$$\mathrm{Br}(B^- \to D_{s0}^*(2317)^- \omega) = (5.44^{+2.16+0.52+0.36}_{-1.48-0.36-0.30}) \times 10^{-6}, \quad (65)$$

$$\mathrm{Br}(B^- \to D_{s0}^*(2317)^- \phi) = (1.74^{+0.67+0.33+0.11}_{-0.50-0.26-0.11}) \times 10^{-8}, \quad (66)$$

$$\mathrm{Br}(\bar{B}^0 \to D_{s0}^*(2317)^- K^{*+}) = (6.38^{+1.25+0.18+0.41}_{-1.02-0.48-0.41}) \times 10^{-9}, \quad (67)$$

$$\mathrm{Br}(B^- \to D_{s0}^*(2317)^- K^{*0}) = (0.73^{+0.19+0.10+0.04}_{-0.21-0.08-0.05}) \times 10^{-9}, \quad (68)$$

where the errors are the same as ones given in Eqs. (54)–(60).

The pure annihilation decays have the smallest branching ratios both for the CKM allowed and the CKM suppressed ones. In Table II, we list the contributions from the nonfactorizable annihilation amplitudes (NFAA) and the factorizable annihilation amplitudes (FAA), where the Wilson coefficients have been included. One can find that the nonfactorizable contributions are more important than the factorizable ones. Even the FAA with the large Wilson coefficient ($a_1 = C_2 + C_1/3$) also has a smaller value because of the destructive interference between the pair of factorizable annihilation Feynman diagrams in each channel, such as Figs. 1(g) and 1(h). For example, in the decay $B^- \to D_{s0}^{*-} K^0$ both of the two factorization annihilation amplitudes have large imaginary parts in magnitude but with opposite signs: One is 3.71×10^{-5}, the other is -4.28×10^{-5}, so the imaginary part of the total FAA becomes -5.7×10^{-6} given in Table II.

IV. CONCLUSION

In summary, we investigate the branching ratios of the decays $B_{(s)} \to D_{s0}^*(2317) P(V)$ within the pQCD approach by assuming $D_{s0}^*(2317)$ as a $\bar{c}s$ scalar meson. For the CKM favored decays, their branching fractions are larger than 10^{-5}, even for the pure annihilation type channels.

Our predictions are consistent with the results given by the light cone sum rules approach. So we consider that these decays can be measured at the running LHCb and the forthcoming SuperKEKB. We may shed light on the nature of the meson $D_{s0}^*(2317)$ by combining with the future data and the theoretical predictions: If they are consistent with each other, one can conclude that this charmed-strange meson is composed (mainly) of $\bar{c}s$. Otherwise, some other component or the DK threshold effect in meson-meson scattering may be important to the dynamic mechanism for the $D_{s0}^*(2317)$ production. As for the CKM suppressed decays, their branching ratios are usually at 10^{-6} order. However, the branching fraction for the decay $\bar{B}^0 \to D_{s0}^*(2317)^-\rho^+$ reaches up to 1.61×10^{-5}. As to the pure annihilation type decays $B^- \to D_{s0}^*(2317)^-\phi, \bar{B}^0 \to D_{s0}^*(2317)^-K^+(K^{*+})$, and $B^- \to D_{s0}^*(2317)^-K^0(K^{*0})$, their branching fractions drop to as low as $10^{-10} \sim 10^{-8}$. Here the decay $B^- \to D_{s0}^*(2317)^-\phi$ has the larger branching ratio because of the large CKM matrix element V_{cs}. The branching ratios of the decays $\bar{B}^0 \to D_{s0}^*(2317)^-K^+(K^{*+})$ are larger than those of $B^- \to D_{s0}^*(2317)^-K^0(K^{*0})$ because of owning the larger nonfactorizable annihilation amplitudes. For these pure annihilation type decays, the magnitudes of the nonfactorizable amplitudes are generally larger than those of factorization amplitudes. It is because there exists the destructive interference between the pair of factorization amplitudes in each decay mode. If this type of pure annihilation decay is observed by the future experiments with larger branching fractions than our predictions, it may indicate that some new physics contributes to these decays.

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APPENDIX

$$t_a = \max(\sqrt{x_2}m_B, 1/b_1, 1/b_2), \tag{A1}$$

$$t_b = \max(\sqrt{x_1}m_B, 1/b_1, 1/b_2), \tag{A2}$$

$$t'_a = \max\left(\sqrt{x_3(1 - r_{D_{s0}^*}^2)}m_B, 1/b_1, 1/b_3\right), \tag{A3}$$

$$t'_b = \max\left(\sqrt{x_1(1 - r_{D_{s0}^*}^2)}m_B, 1/b_1, 1/b_3\right), \tag{A4}$$

$$t_{c,d} = \max\left(\sqrt{x_1 x_2}m_B, \sqrt{|A_{c,d}^2|}m_B, 1/b_1, 1/b_2\right), \tag{A5}$$

$$t'_{c,d} = \max\left(\sqrt{x_1 x_3(1 - r_{D_{s0}^*}^2)}m_B, \sqrt{|A_{c,d}'^2|}m_B, 1/b_1, 1/b_3\right), \tag{A6}$$

$$t_{e,f} = \max\left(\sqrt{x_2(1-x_3)(1 - r_{D_{s0}^*}^2)}m_B, \sqrt{|L_{e,f}^2|}, m_B, 1/b_1, 1/b_3\right), \tag{A7}$$

$$t'_{e,f} = \max\left(\sqrt{x_2 x_3(1 - r_{D_{s0}^*}^2)}m_B, \sqrt{|L_{e,f}'^2|}, m_B, 1/b_1, 1/b_3\right), \tag{A8}$$

$$t_g = \max\left(\sqrt{(1-x_3)(1 - r_{D_{s0}^*}^2)}m_B, 1/b_2, 1/b_3\right), \tag{A9}$$

$$t_h = t'_g = \max\left(\sqrt{x_2(1 - r_{D_{s0}^*}^2)}m_B, 1/b_2, 1/b_3\right), \tag{A10}$$

$$t'_h = \max\left(\sqrt{x_3(1 - r_{D_{s0}^*}^2)}m_B, 1/b_2, 1/b_3\right), \tag{A11}$$

where the definitions of $A_{c,d}^{(\prime)}, L_{e,f}^{(\prime)}$ are listed in Eqs. (16), (17), (25), (26), and (32)–(35). And the $S_j(t)(j = B, D_{D_{s0}^*}, P)$ functions in the Sudakov form factors in Eqs. (11), (12), (21), and (22) are given as

$$S_B(t) = s\left(x_1\frac{m_B}{\sqrt{2}}, b_1\right) + 2\int_{1/b_1}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu})), \tag{A12}$$

$$S_{D_{s0}^*}(t) = s\left(x_3\frac{m_B}{\sqrt{2}}, b_3\right) + 2\int_{1/b_3}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu})), \tag{A13}$$

$$S_P(t) = s\left(x_2\frac{m_B}{\sqrt{2}}, b_2\right) + s\left((1-x_2)\frac{m_B}{\sqrt{2}}, b_2\right) + 2\int_{1/b_2}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu})), \tag{A14}$$

where the quark anomalous dimension $\gamma_q = -\alpha_s/\pi$, and the expression of the $s(Q,b)$ in the one-loop running coupling coupling constant is used

$$s(Q,b) = \frac{A^{(1)}}{2\beta_1}\hat{q}\ln\left(\frac{\hat{q}}{\hat{b}}\right) - \frac{A^{(1)}}{2\beta_1}(\hat{q} - \hat{b}) + \frac{A^{(2)}}{4\beta_1^2}\left(\frac{\hat{q}}{\hat{b}} - 1\right) - \left[\frac{A^{(2)}}{4\beta_1^2} - \frac{A^{(1)}}{4\beta_1}\ln\left(\frac{e^{2\gamma_E - 1}}{2}\right)\right]\ln\left(\frac{\hat{q}}{\hat{b}}\right), \tag{A15}$$

with the variables are defined by $\hat{q} = \ln[Q/(\sqrt{2}\Lambda)], \hat{q} = \ln[1/(b\Lambda)]$ and the coefficients $A^{(1,2)}$ and β_1 are

$$\beta_1 = \frac{33 - 2n_f}{12}, \qquad A^{(1)} = \frac{4}{3}, \tag{A16}$$

$$A^{(2)} = \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27} n_f + \frac{8}{3}\beta_1 \ln\left(\frac{1}{2} e^{\gamma_E}\right), \tag{A17}$$

where n_f is the number of the quark flavors and γ_E the Euler constant.

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