Rapid Communications

Unruh effect for fermions from the Zubarev density operator

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Using the Zubarev quantum-statistical density operator, we calculated the corrections to the energy-momentum tensor of a massless fermion gas associated with acceleration. It is shown that when fourth-order corrections are taken into account, the energy-momentum tensor in the laboratory frame is equal to zero at a proper temperature measured by a comoving observer equal to the Unruh temperature. Consequently, the Minkowski vacuum is visible to the accelerated observer as a medium filled with a heat bath of particles with the Unruh temperature, which is the essence of the Unruh effect.

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I. INTRODUCTION

According to the standard formulation of the Unruh effect, an accelerated observer sees the Minkowski vacuum state as a thermal bath of particles with a temperature $T_U = \frac{a}{2\pi}$, depending on the proper acceleration a [1–4].

The Unruh effect is most easily derived for scalar particles from consideration of the change in the ratio between positive and negative frequency modes of scalar fields in the proper time of the accelerated observer [1,4]. However, the Unruh effect was also established in the general case of theories with arbitrary spin and with interaction on the basis of the algebraic approach [5,6]. Also, the Unruh effect was established for fermions in the framework of quantum field theory in terms of path integrals [7].

The phenomena associated with the acceleration and the Unruh effect also continue to be the subject of modern research. In particular, the critical accelerations for the spontaneous symmetry breaking of $U(1)$ in the $\lambda\phi^4$ theory and the chiral symmetry breaking in the Nambu-Jona-Lasinio model have been analysed in Refs. [8,9] respectively. The critical acceleration of the Bose-Einstein condensate in accelerated systems has been also analyzed [10]. Spin dynamics under the joint action of the gravitational, inertial, and electromagnetic fields is also considered in Ref. [11] from the point of view of quantum mechanics.

There were also indications that the Unruh effect may be significant when considering the collisions of elementary particles. In Refs. [12,13], it was shown that the hadronization process can be accompanied by accelerated particle motion, which can be a source of thermalization in elementary processes such as e^+e^- annihilation or pp and $p\bar{p}$ collisions, just due to the Unruh effect, and also allows one to explain some features in the multiplicities of hadrons. So, in the elementary processes of electron-positron annihilation or proton-(anti)proton collisions, tremendous accelerations can occur, which may open the way for observing the effects associated with acceleration, while in heavy ion collision also large vorticity may arise, and polarization in heavy ion collisions provides information about both vorticity and acceleration [14–17].

An interesting new look at the Unruh effect was recently obtained from the standpoint of quantum relativistic statistical mechanics [18,19]. In Ref. [18], it was shown by calculating the values of quantum correlators for scalar fields at a finite temperature that the average value of any local operator turns out to be zero after subtracting of the vacuum contribution at the proper temperature, measured by a comoving observer, equal to the Unruh temperature. This fact means that the Minkowski vacuum is perceived by the accelerated observer as a medium filled with a thermal bath of particles with an Unruh temperature $\frac{a}{2\pi}$, which is the essence of the Unruh effect.

The analysis in Ref. [18] is given for scalar particles. Our main result is a generalization of Ref. [18] for the case of massless fermions: we show that for gas of massless fermions with chemical potentials equal to zero, the

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energy-momentum tensor in the laboratory frame is zero at the proper temperature, measured by a comoving observer, equal to the Unruh temperature.

We use a fundamental approach based on the quantum-statistical density operator of Zubarev [20–24] (see also in Ref. [25] the current review of Zubarev's approach and its connection with the Kubo formulas and in Refs. [18,19] discussion of thermodynamic equilibrium with acceleration, as well as the derivation of the entropy current). The effects associated with acceleration in this operator are described by a term with the boost operator and can be investigated in the framework of quantum field perturbation theory. So, the first- and second-order corrections in acceleration (and other velocity derivatives) were calculated in Refs. [22–24], while the third-order corrections in the axial current were analyzed in Ref. [26]. In particular, it was shown that the standard formula for the chiral vortical effect is reproduced [22,24], and corrections to it were obtained [24,26].

The values calculated in this way did not vanish at the Unruh temperature. We show that when fourth-order corrections are taken into account in the energy-momentum tensor, this vanishing occurs. Thus, we generalize the result [18] to the case of fermions (at least on the level of energy-momentum tensor of massless fermions) and at the same time confirm that the Unruh effect can be obtained in the Zubarev approach.

We also proposed a formula for energy density in the form of momentum integrals. The motivation for the introduction of this formula is an exact match with the result of the perturbative calculation based on the density operator in the $T > T_U$ region and the correct limit at $a \to 0$. We also derive the energy density using the covariant Wigner function. The proposed integral representation can be considered as a modification of this formula resulting from the Wigner function.

II. UNRUH EFFECT FROM DENSITY OPERATOR

The most fundamental object describing a medium in a state of local thermodynamic equilibrium is the density operator, introduced by Zubarev [20–25]. In the case of zero vorticity $\omega^\mu = 0$, zero chemical potentials $\mu = \mu_5 = 0$, and global thermodynamic equilibrium with acceleration [18,24,27], this operator is reduced to the next form [18,22,24]

$$\hat{\rho} = \frac{1}{Z}\exp\{-\beta_\mu \hat{P}^\mu - \alpha_\mu \hat{K}^\mu_x\}, \tag{2.1}$$

where $\hat{P}^\mu$ is a four-momentum operator, $\hat{K}^\mu_x$ is a boost operator translated to the vector x^μ, and $\alpha^\mu = \frac{a^\mu}{T}$ is the vector of thermal acceleration, which is proportional to the usual kinematic acceleration vector in the case of global equilibrium. The considered case of global equilibrium corresponds to motion with constant proper acceleration.

In the fourth order of perturbation theory, the operator (2.1) leads to the energy-momentum tensor of the following form,

$$\langle \hat{T}^{\mu\nu}\rangle = (\rho_0 + A_1 a^2 T^2 + A_2 a^4) u^\mu u^\nu - (p_0 + B_1 a^2 T^2 + B_2 a^4)\Delta^{\mu\nu} + (A_3 T^2 + A_4 a^2) a^\mu a^\nu + \mathcal{O}(a^6) \qquad \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu, \tag{2.2}$$

where we assume $a = \sqrt{-a_\mu a^\mu}$ and ρ_0, p_0, A_1, A_2, A_3, A_4, B_1, B_2 are coefficients to be defined. The formula (2.2) is the most general expression allowed by parity requirements in the fourth order of perturbation theory (see Ref. [19]). In Ref. [22], coefficients up to the second order of the perturbation theory were calculated. In the chiral limit $m \to 0$ and at $\mu = \mu_5 = 0$,

$$\rho_0 = \frac{7\pi^2 T^4}{60}, \qquad A_1 = \frac{1}{24}, \qquad p_0 = \frac{\rho_0}{3} = \frac{7\pi^2 T^4}{180},$$
$$B_1 = \frac{A_1}{3} = \frac{1}{72}, \qquad A_3 = 0. \tag{2.3}$$

Obviously, in the second order of the perturbation theory, the energy-momentum tensor does not vanish. For example, the corresponding expression for energy density (index *Den* means that the value is calculated using the density operator)

$$\rho_{\mathrm{Den}} = \frac{7\pi^2 T^4}{60} + \frac{T^2 a^2}{24} + \mathcal{O}(a^4), \tag{2.4}$$

and it is obvious that it is always positive and does not vanish at $T = T_U$.

We show that taking into account the fourth-order corrections leads to $\langle \hat{T}^{\mu\nu}\rangle(T = T_U) = 0$ (third-order corrections are forbidden by parity). Next, we follow the algorithm for calculating corrections in thermal vorticity in quantum statistical averages using the density operator (2.1), described in Refs. [22,24] and also in Ref. [26]. So, for A_2, we have

$$A_2 = \frac{1}{4!}\int_0^{|\beta|} d\tau_x d\tau_y d\tau_z d\tau_f \langle T_\tau \hat{K}^3_{-i\tau_x u}\hat{K}^3_{-i\tau_y u}\hat{K}^3_{-i\tau_z u}\hat{K}^3_{-i\tau_f u}\hat{T}^{00}(0)\rangle_{\beta(x),c}, \tag{2.5}$$

where the appearance of four boost operators $\hat{K}$ corresponds to an expansion of up to the fourth-order perturbation theory, and the operator $\hat{T}^{00}$ appears, since we calculate the mean value of the energy density. The index $\beta(x), c$ means that averaging is performed by means of the operator (2.1) with $\alpha^\mu = 0$ and only connected correlators are taken, T_τ means ordering by inverse temperature, and $|\beta| = \frac{1}{T}$. Expressing $\hat{K}$ through the energy-momentum tensor, we obtain that the coefficients will be expressed in terms of the quantities (compare with Refs. [22,24,26])

$$C^{\alpha_1\alpha_2|\alpha_3\alpha_4|\alpha_5\alpha_6|\alpha_7\alpha_8|\alpha_9\alpha_{10}|ijkl} = \int_0^{|\beta|} d\tau_x d\tau_y d\tau_z d\tau_f d^3x d^3y d^3z d^3f \times x^i y^j z^k f^l \langle T_\tau \hat{T}^{\alpha_1\alpha_2}(\tau_x, \mathbf{x}) \hat{T}^{\alpha_3\alpha_4}(\tau_y, \mathbf{y}) \hat{T}^{\alpha_5\alpha_6}(\tau_z, \mathbf{z}) \hat{T}^{\alpha_7\alpha_8}(\tau_f, \mathbf{f}) \hat{T}^{\alpha_9\alpha_{10}}(0) \rangle_{\beta(x),c}, \tag{2.6}$$

where the operator of the energy-momentum tensor is the Belinfante energy-momentum tensor for fermions in the split form

$$\hat{T}^{\alpha\beta}(X) = \lim_{X_1, X_2 \to X} \mathcal{D}^{\alpha\beta}_{ab}(\partial_{X_1}, \partial_{X_2}) \bar{\Psi}_a(X_1) \Psi_b(X_2),$$
$$\mathcal{D}^{\alpha\beta}_{ab}(\partial_{X_1}, \partial_{X_2}) = \frac{i^{\delta_{0\alpha}+\delta_{0\beta}}}{4} [\tilde{\gamma}^\alpha_{ab}(\partial_{X_2} - \partial_{X_1})^\beta + \tilde{\gamma}^\beta_{ab}(\partial_{X_2} - \partial_{X_1})^\alpha], \tag{2.7}$$

where $\tilde{\gamma}$ are Euclidean gamma matrices and $X = (\tau_x, \mathbf{x})$. Then, we obtain for the coefficients arising in the fourth order of perturbation theory

$$A_2 = \frac{1}{4!} C^{00|00|00|00|00|3333}, \qquad B_2 = \frac{1}{4!} C^{00|00|00|00|33|2222},$$
$$A_4 = -B_2 + \frac{1}{4!} C^{00|00|00|00|33|3333}. \tag{2.8}$$

The calculation of the quantities C differs from the similar calculation in Refs. [22,24,26] only by the larger number of operators under the averaging. We give the final answer right away:

$$A_2 = -\frac{17}{960\pi^2}, \qquad B_2 = \frac{A_2}{3} = -\frac{17}{2880\pi^2}, \qquad A_4 = 0. \tag{2.9}$$

Now, it is easy to see that the energy-momentum tensor (2.2) with coefficients (2.3) and (2.9) vanishes at T_U,

$$\langle \hat{T}^{\mu\nu} \rangle = 0 \qquad (T = T_U). \tag{2.10}$$

In particular, the energy density takes the form

$$\rho_{\mathrm{Den}} = \frac{7\pi^2 T^4}{60} + \frac{T^2 a^2}{24} - \frac{17 a^4}{960\pi^2} + \mathcal{O}(a^6)$$
$$= \frac{1}{240}\left(T^2 - \left(\frac{a}{2\pi}\right)^2\right)(17a^2 + 28\pi^2 T^2) + \mathcal{O}(a^6) \tag{2.11}$$

and goes to zero at $T = T_U$. The pressure, as it should, satisfies the equation of state for massless particles $p = \frac{\rho}{3}$ and is also equal to 0 at $T = T_U$.

Other observables are zero due to parity requirements. In particular, in the case of zero vorticity, which we consider, the axial current turns out to be zero—this follows from the direct calculation [26], as well as from parity considerations. When calculating the coefficients (2.3) and (2.9), the Belinfante energy-momentum tensor (2.7) was used, which corresponds to the spin-tensor $S^{\lambda,\mu\nu} = 0$ equal to zero. However, the canonical spin tensor is also zero—this is evident from the fact that it is expressed through an axial current. The vector current is also equal to zero, since we consider the simplest case of $\mu = 0$, and the vector current is an odd quantity in chemical potential.

The equality to zero of the observables corresponds to the Minkowski vacuum. Thus, from (2.10), it follows that the Minkowski vacuum corresponds to the proper temperature $T_U = \frac{a}{2\pi}$, which is the essence of the Unruh effect.

From Eq. (2.11), one can express temperature as a function of acceleration and energy density. Normalizing the temperature and acceleration to the temperature value in the absence of acceleration, we obtain

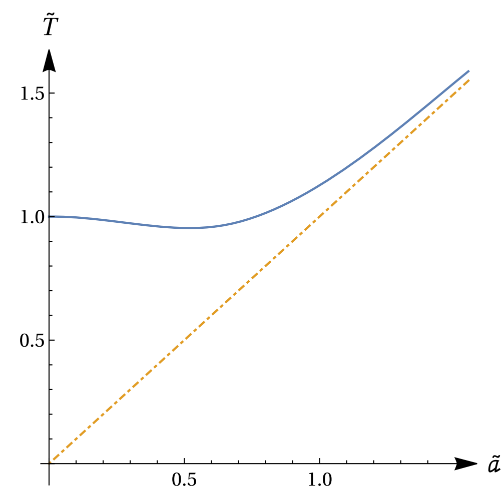

FIG. 1. Temperature (2.12) as a function of acceleration corresponds to a blue solid line. The orange dashed line corresponds to $\tilde{T} = \tilde{a}$.

$$\tilde{T}(\tilde{a}) = \sqrt{\sqrt{1+\frac{144}{49}\tilde{a}^4} - \frac{5}{7}\tilde{a}^2}, \qquad \tilde{T} = \frac{T}{T_0},$$
$$\tilde{a} = \frac{a}{2\pi T_0}, \qquad T_0 = T(a=0) = \left(\frac{60\rho}{7\pi^2}\right)^{1/4}. \quad (2.12)$$

Function (2.12) is shown in Fig. 1. It has a minimum at the point $\tilde{a} = \frac{1}{2}(175/153)^{1/4} \simeq 0.52$. From Fig. 1, one can see that the temperature becomes equivalent to acceleration at $\rho \to 0$, that is, when there is a vacuum in the laboratory system.

III. INTEGRAL REPRESENTATION AND WIGNER FUNCTION

The formula (2.11) has the form of a polynomial with rather unusual numerical coefficients obtained as a result of calculating quantum correlators of the form (2.6). However, it can be shown that these coefficients can be obtained naturally from integrals with Fermi and Bose distributions. One can check that in the region $T > T_U$ equality

$$\begin{aligned}\rho &= \frac{7\pi^2T^4}{60} + \frac{T^2a^2}{24} - \frac{17a^4}{960\pi^2} \\ &= 2\int\frac{d^3p}{(2\pi)^3}\left(\frac{|\mathbf{p}|+ia}{1+e^{\frac{|\mathbf{p}|}{T}+\frac{ia}{2T}}} + \frac{|\mathbf{p}|-ia}{1+e^{\frac{|\mathbf{p}|}{T}-\frac{ia}{2T}}}\right) \\ &\quad + 4\int\frac{d^3p}{(2\pi)^3}\frac{|\mathbf{p}|}{e^{\frac{2\pi|\mathbf{p}|}{a}}-1} \qquad (T > T_U) \end{aligned} \quad (3.1)$$

is exactly satisfied.

The uniqueness of the integral representation (3.1) shall be the subject of special investigation. However, it looks natural and simple. Also, this representation leads to the correct result in the limit $a \to 0$. In this case, only the first term in (3.1) remains, which for $a = 0$ has the form of the integral of the product of particle energy and the Fermi distribution, as it should, according to statistical physics.

On the other hand, Eq. (3.1) can be motivated from the point of view of another approach, based on the covariant Wigner function for particles with spin 1/2 [28] (see also Refs. [26,29–33]). In particular, according to (3.1), there is a substitution $\mu \to \mu \pm \frac{ia}{2}$, which means that the acceleration plays the role of an imaginary chemical potential, which was previously also observed for axial current in the approach with the Wigner function [33].

Using the Wigner function from Ref. [28], one can calculate the mean values of different observables. In particular, in Refs. [32,33], a method for obtaining exact nonperturbative formulas is described using the example of axial current. In our case, it is necessary to calculate the energy density. According to Ref. [32], it is necessary to decompose the energy density twice into a Taylor series,

$$\begin{aligned}\rho_{Wig} &= \frac{1}{2}\int\frac{d^3p}{(2\pi)^3}\varepsilon\Bigg(\sum_{n=0}^{\infty}(-1)^n\exp[t(n+l)(\beta\cdot p-\xi)] \\ &\quad \times\sum_{m=0}^{\infty}\frac{1}{m!}\left(-\frac{1}{2}t(n+l)\right)^m \mathrm{tr}[(\varpi:\Sigma)^m] \\ &\quad + (\xi\to-\xi, \varpi\to-\varpi)\Bigg), \end{aligned} \quad (3.2)$$

where index Wig means that the value is calculated using the Wigner function [28]; $\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu)$ is thermal vorticity tensor; $\xi = \frac{\mu}{T}$ is chemical potential divided by temperature; $t=1,\ l=0$ or $t=-1,\ l=1$; and in brackets there is the contribution of antiparticles, distinguished by signs of chemical potential and vorticity. Next, one needs to calculate a trace in each term of the series and sum up them back. As a result, we obtain the following expression for the energy density,

$$\rho_{\mathrm{Wig}} = 2\int\frac{d^3p}{(2\pi)^3}\varepsilon\left(\frac{1}{1+e^{\frac{\varepsilon}{T}+\frac{ia}{2T}}} + \frac{1}{1+e^{\frac{\varepsilon}{T}-\frac{ia}{2T}}}\right), \quad (3.3)$$

where the result is given for the case of $\xi = 0$ and global thermodynamic equilibrium. As far as we know, the result (3.3) is new and has never appeared before.

The form of the expression (3.3) is the motivation for the integral representation (3.1). Note that in (3.3), the condition $\rho(T=T_U)=0$ cannot be achieved, unlike (3.1). This fact was previously shown in Ref. [29], where the Boltzmann limit was investigated.[1] Apparently, this indicates the limited possibility of using the Wigner function [28] in describing the effects associated with acceleration (while it works well for the effects of vorticity, as was shown in Ref. [26]). In (3.1), the condition $\rho(T=T_U)=0$ is ensured by adding the second integral with the Bose distribution with a temperature equal to the Unruh temperature, and the number of bosonic degrees of freedom is equal to the number of fermion ones. In general, the integral representation (3.1) can be considered as a modification of the formula (3.3) obtained from the Wigner function.

Let us discuss the properties of energy density, following from (3.1). Integrals in (3.1) lead to an additional contribution at $T < T_U$, containing the function $\lfloor\cdot\rfloor$, which takes the integer part

$$\begin{aligned}\rho &= \frac{7\pi^2T^4}{60} + \frac{T^2a^2}{24} - \frac{17a^4}{960\pi^2} + \left(\frac{\pi T^3 a}{3} + \frac{Ta^3}{4\pi}\right)\left\lfloor\frac{1}{2}+\frac{a}{4\pi T}\right\rfloor \\ &\quad - \left(\frac{T^2a^2}{2} + 2\pi^2T^4\right)\left\lfloor\frac{1}{2}+\frac{a}{4\pi T}\right\rfloor^2 - \frac{4\pi T^3a}{3}\left\lfloor\frac{1}{2}+\frac{a}{4\pi T}\right\rfloor^3 \\ &\quad + 4\pi^2T^4\left\lfloor\frac{1}{2}+\frac{a}{4\pi T}\right\rfloor^4. \end{aligned} \quad (3.4)$$

[1] We are grateful to W. Florkowski and E. Speranza for the discussion.

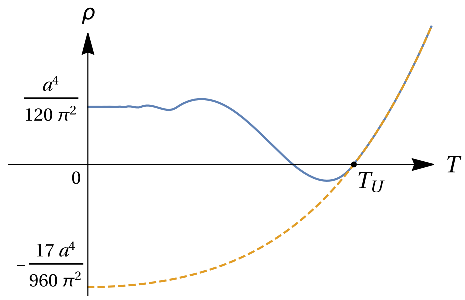

FIG. 2. Energy density as a function of temperature. The proposed formulas (3.1) and (3.4) correspond to the solid blue line; the perturbative result (2.11), obtained from the density operator, is shown by the dashed orange line.

The appearance of an additional contribution at $T < \frac{a}{2\pi}$ does not contradict (2.11), since the formula (2.11) was obtained in the framework of the perturbative approach.

The formula (3.1) admits an interesting interpretation. In the first term, describing fermions, the effects of motion of the medium lead to the appearance of an imaginary contribution to the energy $\varepsilon \to \varepsilon \pm ia$ and the chemical potential $\mu \to \mu \pm i\frac{a}{2}$. The latter leads to the fact that at $T = T_U/(2k+1), k = 0, 1\ldots$, in (3.4) instabilities arise; namely, discontinuities appear in second-, third-, and fourth-order derivatives with respect to temperature $\frac{\partial^2 \rho}{\partial T^2}$, $\frac{\partial^3 \rho}{\partial T^3}$, $\frac{\partial^4 \rho}{\partial T^4}$, which is typical for the theories with imaginary chemical potential [34,35]. The instability at $T = T_U$ can be considered as a manifestation of the Unruh effect. The same situation was in the case of axial current in Ref. [33], in which the acceleration also played the role of an imaginary chemical potential, which led to the appearance of instabilities at $T < \frac{a}{2\pi}$ due to terms with an integer part.

The appearance of an additional contribution with the integer part in (3.4) leads to the fact that below T_U the behavior of (2.11) and (3.4) is different, which is shown in Fig. 2. Formula (3.4) in contrast to (2.11) leads to a nonmonotonic oscillating behavior, which is associated with additional terms with an integer part. However, according to Ref. [18], the Unruh temperature has to be considered as minimal, and the region $T < T_U$ is forbidden.

Note that according to (3.1) and (3.4), when $T > T_U$, the maximum power of acceleration in the energy density is 4. This is also supported by the fact that, starting with the sixth order in acceleration, negative powers of temperature would have arisen, which is necessary to preserve the correct dimension. Thus, we would expect that the perturbative result (2.11) is exact at $T > T_U$ and all the corrections above T_U are zero, although this fact requires more rigorous justification.

IV. CONCLUSIONS

We have shown on the basis of the Zubarev density operator that in the fourth order of perturbation theory, the energy-momentum tensor of the inertial observer vanishes at the proper temperature, measured by a comoving observer, equal to the Unruh temperature. Also in the case under consideration of zero chemical potentials and zero vorticity, the spin tensor and the vector and axial currents are also equal to zero. Thus, the Minkowski vacuum is visible to the accelerated observer as a medium filled with a thermal bath with a temperature $T_U = \frac{a}{2\pi}$, which is the essence of the Unruh effect and generalizes the results of Ref. [18] to the case of fermions.

We present the obtained perturbative result in the form of momentum integrals. The proposed formula exactly coincides with the perturbative result at $T > T_U$ and can be motivated by a formula derived from the Wigner function.

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