

Hairy Schwarzschild-(A)dS black hole solutions in degenerate higher order scalar-tensor theories beyond shift symmetry

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We derive the general conditions for a large family of shift symmetry breaking degenerate higher order scalar-tensor (DHOST) theories to admit stealth black hole solutions. Such black hole configurations correspond to vacuum solutions of general relativity and admit a scalar hair which does not gravitate, revealing itself only at the perturbative level. We focus our investigation on hairy Schwarzschild-(A)dS or pure Schwarzschild solutions, dressed with a linear time-dependent scalar hair, and assuming a constant kinetic term. We also discuss subclasses of this family which satisfy the observational constraint $c_{\text{grav}} = c_{\text{light}}$, as well as the recent constraint ensuring the absence of graviton decay. We provide at the end concrete examples of DHOST Lagrangians satisfying our conditions. This work provides a first analysis of exact black hole solutions in shift symmetry breaking DHOST theories beyond Horndeski.

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I. INTRODUCTION

Since the discovery of the acceleration of the expansion of the Universe, an important effort has been devoted to constructing large scale modification of general relativity (GR). Scalar-tensor theories are by far the most studied extensions. As any modified gravity theory, these scalar-tensor candidates have to successfully explain the observed acceleration on large scale, while restoring GR on scales in which the theory is experimentally confirmed, typically for the local tests within the Solar System. Additionally to the IR modifications in the cosmological sector, the new scalar-metric coupling allows us to violate some of the assumptions of the famous no hair theorem which strongly restricts the black hole solution in general relativity [1–3].

The search for such hairy black hole solutions beyond general relativity is however quite challenging. Hawking, and later Sotiriou and Faraoni, derived no hair theorems for Brans-Dicke scalar-tensor theories and their generalization [4]. An exact black hole solution with scalar hair was then found by violating some assumptions of these theorems, an example of which is the Bocharova-Bronnikov-Melnikov-Bekenstein hairy black hole [5]. However, such scalar hairy configurations are usually unstable and the scalar field fails to be regular at the horizon. An additional no hair theorem was found later on in the more general shift symmetric Horndeski theory in [6]. Soon after, it was shown how to by

pass the assumptions of this no-go result. Hairy solutions were obtained following two different strategies: by introducing a Gauss-Bonnet-scalar coupling [7–11], or by allowing the scalar field for a linear time-dependent profile [12].

In this work [12], a stealth Schwarzschild-(A)dS black hole dressed with a linear time-dependent scalar field was obtained. Such stealth configuration corresponds to a vacuum metric solution of GR supplemented with a non-trivial scalar hair which does not gravitate. The scalar field is a spectator admitting a vanishing energy momentum tensor and its physical effects only show up at the perturbative level. Such stealth black hole solutions were initially introduced in [13] and represent the simplest example of hairy black hole configuration, i.e., in which the scalar field remains regular on the horizon.

These stealth black hole configurations were then investigated in several scalar-tensor extensions, such as biscalar extension of Horndeski [14], covariant galileons theories [15], shift symmetric beyond Horndeski [16–19], and more recently in shift symmetric breaking Horndeski theory [20,21]. More details on the black holes solutions and stars within the Horndeski and beyond Horndeski classes can be found in [22–25].

For single scalar field extension of GR, the most general theory constructed so far, up to cubic order in the derivative of the scalar field, was presented in [26], and dubbed degenerate higher order scalar tensor theories, i.e., degenerate higher order scalar-tensor (DHOST), owing to the degeneracy property of its Lagrangian which ensures the

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absence of an Ostrogradsky ghost. See [27] for a recent review on DHOST theories and [28–32] for further details, as well as [33] for a recent investigation of their cosmological sector and [34] concerning the Vainshtein mechanism. This DHOST construction encompasses most of the existing scalar-tensor candidates studied so far, among them the Gleyzes–Langlois–Piazza–Vernizzi (GLPV) theory [35,36], and represents therefore an unifying framework to discuss viable scalar-tensor theories and confront them with observational tests. (See [37] for a review with a more general perspective.)

Recently, the joined detection of the events GW170817 and GR170817 led to the new observational constraint that the speed of gravitational wave be equal to the speed of light, i.e., $c_{\text{grav}} = c_{\text{light}}$ (up to deviations of order 10^{-15}), at least on cosmological scales [38,39]. The remaining DHOST theories satisfying this observational constraint were derived in [40], restricting drastically the viable candidates. Additional constraints preventing the potential decay of graviton into dark energy fluctuations were presented in [41].

In this article, we provide a first scan of *shift symmetry breaking* DHOST theories beyond Horndeski and derive the general conditions for a large class of these theories to admit stealth black hole configurations. We focus on hairy Schwarzschild-(A)dS and pure hairy Schwarzschild black hole solutions. The scalar field is assumed to be linearly time dependent while its kinetic term remains constant.

This work is organized as follow. In Sec. I A, we present the general DHOST model. In Sec. I B, we discuss the algorithm to solve the modified field equations in the spherically symmetric case and with our scalar profile. This algorithm is borrowed from [19]. In Sec. I C, we present our general conditions for the hairy Schwarzschild-(A)dS and pure Schwarzschild black hole solutions. Section I D is devoted to the subclasses satisfying the observational constraint $c_{\text{grav}} = c_{\text{light}}$ as well as the subclass free from graviton decay. In Sec. I E, we consider also the reduction to the GLPV and Horndeski subclasses satisfying $c_{\text{grav}} = c_{\text{light}}$. Finally, in Sec. I F, we provide concrete examples of Lagrangian solutions of our conditions.

Our work extends the results obtained in previous works in several subclasses of shift symmetric DHOST theories [16–20] as well as on shift symmetry breaking Horndeski theory satisfying $c_{\text{grav}} = c_{\text{light}}$ [21].

A. The DHOST model

Let us consider the family of DHOST theories given by the action

$$S_{\text{VDHOST}}[g, \phi] = \int d^4x \sqrt{|g|} \sum_I \mathcal{L}^I(g, \phi), \quad (1)$$

where the different Lagrangians read

$$\mathcal{L}_2 = P(\phi, X), \quad (2)$$

$$\mathcal{L}_3 = Q(\phi, X) \square \phi, \quad (3)$$

$$\mathcal{L}_4 = F(\phi, X) \mathcal{R}, \quad (4)$$

$$\begin{aligned} \mathcal{L}_5 = & A_3(\phi, X) \phi^\mu \phi^\nu \phi_{\mu\nu} \square \phi + A_4(\phi, X) \phi^\mu \phi_\lambda \phi_{\mu\nu} \phi^{\nu\lambda} \\ & + A_5(\phi, X) (\phi_{\mu\nu} \phi^\mu \phi^\nu)^2, \end{aligned} \quad (5)$$

where the six potentials (P, Q, F, A_I) with $I \in \{3, 4, 5\}$ are free functions of ϕ and its kinetic term X . We have adopted the notation of [40]. This family of theories corresponds to the quadratic DHOST theories amputated from the Lagrangians $\mathcal{L}_{1,1} = (\square \phi)^2$ and $\mathcal{L}_{1,2} = \phi_{\mu\nu} \phi^{\mu\nu}$, namely $A_1 = A_2 = 0$ in the standard notation [28].

This class of DHOST theories can be made consistent with the recent observational constraint from GW170817 which imposes that the speed of gravitons equal the speed of light (up to deviations of order 10^{-15}), at least on cosmological scales [40]. In order to satisfy this constraint, the last two functions $A_4(\phi, X)$ and $A_5(\phi, X)$ are related to $F(\phi, X)$ and $A_3(\phi, X)$ through

$$A_4 = \frac{1}{8F} (48F_X^2 - 8(F - XF_X)A_3 - X^2A_3^2) \quad (6)$$

$$A_5 = \frac{1}{2F} (4F_X + XA_3)A_3. \quad (7)$$

The potentials $A_3(\phi, X)$ and $F(\phi, X)$ remain free functions, and the viable DHOST theories contain thus only four free potentials (P, Q, F, A_3) .

In the following, we derive general conditions on the potential of the DHOST family (1) to admit stealth black hole solutions without restricting ourselves to the subclass satisfying $c_{\text{grav}} = c_{\text{light}}$. This constraint, together with the constraint derived in [41] concerning the graviton decay, is discussed in the last section. The reduction to the beyond Horndeski (GLPV) and Horndeski theories is also discussed at the end.

B. Solving the fields equations: the algorithm

In order to solve the field equations, we adopt the elegant strategy presented in [19]. Starting from the model (1), we derive the field equations that we write in a compact way,

$$\delta \mathcal{L} = \mathcal{E}_{\alpha\beta}^{(g)} \delta g^{\alpha\beta} + \mathcal{E}^{(\phi)} \delta \phi. \quad (8)$$

The field equations being rather complicated, we do not write them explicitly here. Instead, the equation of motion with respect to the metric $g_{\alpha\beta}$ can be written in the simple form

$$FG_{\alpha\beta} = \mathcal{T}_{\alpha\beta} \quad (9)$$

$$(\mathcal{Q}_\phi - \mathcal{Q}_X - P_X)\square\phi = \zeta, \quad (10)$$

where $G_{\alpha\beta}$ is the standard Einstein tensor and where $\mathcal{T}_{\alpha\beta}$ and ζ account for all the other terms obtained from the variation of the action respectively with respect to $g_{\alpha\beta}$ and ϕ , containing therefore all the higher order terms.

Looking for stealth black hole solutions implies that the scalar field does not gravitate. This can be translated in (9) by $\mathcal{T}_{\alpha\beta} = 0$. Then, one can solve the left-hand side of the equation of motion using a GR black hole solution such that $G_{\alpha\beta} = 0$. A common strategy is to assume e.g., a constant kinetic term for the scalar field profile, such that $X = X_*$. Then, under some specific conditions on the potentials of the Lagrangian, the effective energy momentum tensor $\mathcal{T}_{\alpha\beta}$ can be written as

$$\mathcal{T}_{\alpha\beta} = f(X)T_{\alpha\beta} \quad (11)$$

such that $f(X_*) = 0$, an example of which being $f(X) = \log(X_*/X)$. Below, we should derive the condition on the DHOST Lagrangian (1) to admit such stealth black hole solutions.

We consider therefore a static spherical symmetric metric which reads

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2d\Omega^2, \quad (12)$$

and we choose a linear time-dependent profile for the scalar field

$$\phi(t, r) = \dot{\phi}_c t + \psi(r), \quad (13)$$

where $\dot{\phi}_c$ is assumed to be a constant. Following [19], we introduce the notation $\dot{\phi}_c = Mq$, which implies that the kinetic term reads

$$X = -\frac{g^{\alpha\beta}\partial_\beta\phi\partial_\alpha\phi}{M^2} = q^2e^{-\nu} - e^{-\lambda}\frac{(\psi')^2}{M^2}. \quad (14)$$

In order to further (drastically) simplify the field equations, we also assume that the kinetic term is constant everywhere, such that $X = X_* = q^2$. Notice that the kinetic energy $-M^2X$ is negative since the gradient of the scalar field is a timelike vector. With this assumption, all the unknown potentials, commonly denoted $f(\phi, X)$, can now be written as function

$$f(\phi, X) = f(qt + \psi(r), X_*). \quad (15)$$

In the following, we restrict further to potentials $f(\phi, X)$ satisfying

$$f_\phi(\phi, X_*) = 0 \Rightarrow \partial_\phi^n f(\phi, X_*) = 0 \quad \forall n \in \mathbb{N}. \quad (16)$$

This allows us to simplify our conditions in the beyond shift symmetric case. Notice that while this condition is quite general, it is still possible to find a counterexample in principle, and thus, we are potentially restricting the set of allowed potentials.

The third simplification, inherited from the assumption of a constant kinetic term, lies in that the radial dependent part of the scalar field is given by

$$\psi' = Mq\sqrt{e^\lambda(1 + e^{-\nu})}. \quad (17)$$

Hence, ψ' is directly known in terms of the metric components, as well as its higher order derivatives: ψ'' , ψ''' etc. This can be plugged back in the field equations to further simplify the expression.

In the end, the field equations become lengthy expressions depending on the radial coordinate r . Now these field equations have to be satisfied at any point of space-time, and thus at any couple (t, r) . The elegant strategy followed in [19] is to expand the resulting field equations around a given r_* and check the resulting conditions between the unknown potential $A_I(qt + \psi(r), q^2)$ and their derivatives. Denoting $\epsilon = r - r_*$, the expansion of the equations of motion can be written as

$$\mathcal{E}_{tt}(r, t) = \sum_n^m \mathcal{E}_{tt}^{(n)}(t, r)|_{r_*} \epsilon^n + \mathcal{O}(\epsilon^m) = 0, \quad (18)$$

$$\mathcal{E}_{rr}(r, t) = \sum_n^m \mathcal{E}_{rr}^{(n)}(t, r)|_{r_*} \epsilon^n + \mathcal{O}(\epsilon^m) = 0, \quad (19)$$

$$\mathcal{E}_\phi(r, t) = \sum_n^m \mathcal{E}_\phi^{(n)}(t, r)|_{r_*} \epsilon^n + \mathcal{O}(\epsilon^m) = 0. \quad (20)$$

The conditions we obtain out of this procedure are of the form

$$\mathcal{E}_{tt}^{(n)}(t, r)|_{r_*} = \mathcal{E}_{rr}^{(n)}(t, r)|_{r_*} = \mathcal{E}_\phi^{(n)}(r, t)|_{r_*} = 0, \quad (21)$$

but these conditions are not all independent. Moreover, they are only valid when evaluated at $X = X_* = q^2$. Once conditions on the potentials (P, Q, F, A_I) (and their derivatives) are obtained at a given order, we inject them back in the full field equations and expand once more around the same r_* to obtain new conditions. The algorithm closes when we obtain enough conditions between the potentials such that the full field equations are completely satisfied.

Notice that this perturbative algorithm is rather general, and especially useful when working with such a complicated Lagrangian. Owing to the large freedom in the potentials $A_I(\phi, X)$, the search for black hole solutions

in these theories is somehow reversed, since one can start with any black hole metric and scalar profile, and using this algorithm, look for a specific Lagrangian which admits this ansatz as solution of its field equations.

Obviously, one can in principle proceed to the expansion around any value of r . But in practice, some specific values will allow one to close the algorithm in a quicker way. In the following, we expand the field equations around $r_* = 0$. The set of conditions we obtained being quite involved to reduce, we emphasize that, once the full conditions on the potentials have been found, we have checked the consistency of our solution by injecting it directly in the full (spherically symmetric reduced) field equations and check that they are identically vanishing. Having reviewed the method of resolution of the field equations borrowed from [19], we present now our result.

C. Exact hairy black hole solutions

We consider the Schwarzschild-(A)dS metric given by

$$e^\nu = e^{-\lambda} = 1 - \frac{2m}{r} - \Lambda r^2. \quad (22)$$

The radial dependent part of the scalar field is straightforwardly obtained by integrating (17) and reads

$$\psi'(r) = Mq \sqrt{\frac{(2m + \Lambda r^3)r}{(2m - r + \Lambda r^3)^2}}, \quad (23)$$

and one observes that $\psi'(r) \rightarrow 0$ when $r \rightarrow 0$. We can now inject this in the equations of motion and proceed to the expansion around $r_* = 0$.

1. Stealth Schwarzschild-(A)dS solution

Applying the algorithm reviewed above, we obtain a complete set of conditions on the potentials which fully solve the field's equations. These conditions, valid only when evaluated at the value $X_* = q^2$, read

$$Q_\phi|_{X_*} = F_\phi|_{X_*} = 0, \quad (24)$$

$$Q_X|_{X_*} = -\frac{X}{2}A_{3\phi}|_{X_*}, \quad (25)$$

$$P|_{X_*} = -6\Lambda F|_{X_*}, \quad (26)$$

$$A_3|_{X_*} = \frac{2}{9\Lambda}(P_X + 12\Lambda F_X)|_{X_*}. \quad (27)$$

At this stage, there are no conditions on the potentials A_4 and A_5 , and the class solution of our conditions depends still on six free potentials (P, Q, F, A_I). With the conditions (24) to (27), we have obtained a subset of DHOST theories, larger than the sector satisfying $c_{\text{grav}} = c_{\text{light}}$, which admits a stealth Schwarzschild-(A)dS solution dressed with a

linear time-dependent scalar field (13), assuming a constant kinetic term $X = X_* = q^2$.

2. Stealth Schwarzschild solution: $\Lambda = 0$

It is interesting to investigate the case $\Lambda = 0$ which corresponds to a pure Schwarzschild geometry. In that case, several of the previous conditions are modified because some of them are proportional to Λ and therefore disappear for pure Schwarzschild. The new set of conditions, valid at $X = X_*$, is

$$Q_\phi|_{X_*} = F_\phi|_{X_*} = 0, \quad (28)$$

$$P|_{X_*} = P_X|_{X_*} = 0, \quad (29)$$

$$Q_X|_{X_*} = -\frac{X}{2}A_{3\phi}|_{X_*}. \quad (30)$$

Once again, there are no conditions on the potentials A_4 and A_5 which remain free. In total, this class depends still on six free potentials (P, Q, F, A_I) subject to conditions (28)–(30) at $X = X_*$.

We can now investigate an additional requirement for the subclass of DHOST that we found in order to satisfy the recent observational constraints.

D. Observational constraints

1. Subsector satisfying $c_{\text{grav}} = c_{\text{light}}$

We can now impose the recent observational constraint $c_{\text{grav}} = c_{\text{light}}$ and reduce the above constraints to the viable subsector of DHOST remaining after GW170817 [40].

Schwarzschild-(A)dS: This is done by imposing the relations (6) and (7) to obtain the two last potentials A_4 and A_5 at $X = X_*$. These conditions read for $A_4(\phi, X)$

$$A_4|_{X_*} = \frac{6F_X^2}{F} - \frac{(12\Lambda F_X + P_X)^2}{162\Lambda^2 F_X} + \frac{2(X-1)(12\Lambda F_X + P_X)}{9\Lambda X} \Big|_{X_*}, \quad (31)$$

while for $A_5(\phi, X)$, one has

$$A_5|_{X_*} = \frac{2(12\Lambda F_X + P_X)(30\Lambda F_X + P_X)}{81\Lambda^2 X F} \Big|_{X_*}, \quad (32)$$

which ensure that $c_{\text{grav}} = c_{\text{light}}$ for the Schwarzschild-(A)dS solution sector.

Pure Schwarzschild: In that case, we do not have any constraint on (A_3, A_{3X}, F, F_X) at $X = X_*$; there is no additional constraint on A_4 and A_5 for this subsector. Notice that despite the above constraints on A_4 and A_5 at $X = X_*$, the subset of theories satisfying (31) and (32) has again six free potentials (P, Q, F, A_I).

2. Absence of graviton decay

If one takes into account now the recent constraint derived in [41] for the absence of graviton decay, in addition to the constraint $c_{\text{grav}} = c_{\text{light}}$, then the DHOST action (1) reduces to

$$\mathcal{L} = P(\phi, X) + Q(\phi, X)\Box\phi + F(\phi, X)R + 6F_X^2(\phi, X)/F(\phi, X)\phi^\mu\phi_\lambda\phi_{\mu\nu}\phi^{\nu\lambda}, \quad (33)$$

with only three free potentials (P, Q, F) . It implies that

$$A_3(\phi, X) = 0 \quad A_4(\phi, X) = 6F_X^2(\phi, X)/F(\phi, X). \quad (34)$$

Schwarzschild-(A)dS: The constraint (27) implies that

$$F_X|_{X_*} = -\frac{P_X}{12\Lambda}|_{X_*}, \quad (35)$$

which, as expected, automatically satisfies the constraint on A_4 at $X = X_*$ as seen from (31).

Pure Schwarzschild: In that case, the vanishing of $A_3(\phi, X)$ implies from (30)

$$Q_X|_{X_*} = 0 \quad (36)$$

and no additional restriction other than (28) and (29) occurs.

It implies therefore that the subclass of shift symmetry breaking DHOST theories successfully accounting for the observational constraints $c_{\text{grav}} = c_{\text{light}}$, and satisfying the requirement of absence of graviton decay, still admits nontrivial stealth black hole solutions. In each case, the remaining theory still has three free potentials (P, Q, F) still depending on ϕ and X .

E. Beyond Horndeski and Horndeski subclasses

We consider now the GLPV and Horndeski subclasses satisfying $c_{\text{grav}} = c_{\text{light}}$. These two subclasses are given, in the standard notation, by $P = G_2$, $Q = G_3$ and $F = G_4$ with,

$$A_3 = -A_4 = -4F_X/X \quad \text{GLPV}, \quad (37)$$

$$A_3 = A_4 = F_X = 0 \quad \text{Horndeski}, \quad (38)$$

and $A_5 = 0$ valid for any ϕ and X . The resulting Lagrangian depends on only three free potentials (P, Q, F) .

Schwarzschild-(A)dS: Using (31) and (32), one obtains the following constraints in each case,

$$F_X|_{X_*} = -\frac{P_X}{30\Lambda}|_{X_*} \quad \text{GLPV} \quad (39)$$

$$P_X|_{X_*} = Q_X|_{X_*} = 0 \quad \text{Horndeski}, \quad (40)$$

additionally to the constraint (24) and (26).

Pure Schwarzschild: For this solution, one obtains from (30) the following constraints,

$$\text{No additional constraints} \quad \text{GLPV} \quad (41)$$

$$Q_X|_{X_*} = 0 \quad \text{Horndeski}, \quad (42)$$

additionally to constraints (28) and (29). Notice that in the Horndeski case, since F depends only on ϕ , constraint (28) implies that $F(\phi) = \zeta$ where ζ is a constant [42].

F. Concrete examples

As a last step, we provide concrete examples of potential solutions of our conditions for both the Schwarzschild-(A)dS and pure Schwarzschild cases for the general DHOST case, but also for the GLPV and Horndeski subclasses.

1. Examples for DHOST

Let us first focus on the general DHOST Lagrangian.

Schwarzschild-(A)dS: An example of solution of our conditions (24)–(27) is given by

$$F(\phi, X) = f_1(\phi, X) \log\left(\frac{q^2}{X}\right) + f_2(X), \quad (43)$$

$$P(\phi, X) = f_3(\phi, X) \log\left(\frac{q^2}{X}\right) - 6\Lambda f_2(X), \quad (44)$$

$$A_3(\phi, X) = \frac{2}{9X\Lambda} \left[6\Lambda f_{2X}(X) - \frac{f_3(\phi, X) + 12\Lambda f_1(\phi, X)}{X} \right], \quad (45)$$

$$Q(\phi, X) = -\frac{1}{9\Lambda} [f_{3\phi}(\phi, X) + 12\Lambda f_{1\phi}(\phi, X)] \log\left(\frac{q^2}{X}\right) + \int dX \log\left(\frac{q^2}{X}\right) f_4(X), \quad (46)$$

where (f_1, f_3) are free potentials depending on both ϕ and X while (f_2, f_4) are free potentials depending only on X .

Pure Schwarzschild: For the pure Schwarzschild solution, an example of potentials solving our conditions (28)–(30) is given by

$$F(\phi, X) = f_1(\phi, X) \log\left(\frac{q^2}{X}\right) + f_2(X), \quad (47)$$

$$P(\phi, X) = f_3(\phi, X) \log^2\left(\frac{q^2}{X}\right), \quad (48)$$

$$Q(\phi, X) = \frac{X^2}{2} A_{3\phi}(\phi, X) \log\left(\frac{q^2}{X}\right) + \int dX \log\left(\frac{q^2}{X}\right) f_4(X), \quad (49)$$

where again, (f_1, f_3) are free potentials depending on both ϕ and X while (f_2, f_4) are free potentials depending only on X .

2. Examples for GLPV

We focus now on the GLPV subclass satisfying the observational constraint $c_{\text{grav}} = c_{\text{light}}$.

Schwarzschild-(A)dS: A set of potential solutions of our conditions is given e.g., by

$$F(\phi, X) = f_1(\phi, X) \log\left(\frac{q^2}{X}\right) + f_2(X) \quad (50)$$

$$P(\phi, X) = [-30\Lambda f_1(\phi, X) + 24\Lambda X f_{2X}(X)] \log\left(\frac{q^2}{X}\right) - 6\Lambda f_2(X) \quad (51)$$

$$A_3(\phi, X) = -\frac{4}{X} \left[f_{1X}(\phi, X) \log\left(\frac{q^2}{X}\right) - \frac{f_1(\phi, X)}{X} + f_{2X}(X) \right] \quad (52)$$

$$Q(\phi, X) = 2[f_{1\phi}(\phi, X)] \log\left(\frac{q^2}{X}\right) + \int dX \log\left(\frac{q^2}{X}\right) f_4(X), \quad (53)$$

which depends again on four free potentials.

Pure Schwarzschild solution: For the pure Schwarzschild case, an example of potential solutions of our conditions is given by

$$F(\phi, X) = f_1(\phi, X) \log\left(\frac{q^2}{X}\right) + f_2(X), \quad (54)$$

$$P(\phi, X) = f_3(\phi, X) \log^2\left(\frac{q^2}{X}\right), \quad (55)$$

$$A_3(\phi, X) = -\frac{4}{X} \left(f_{1X}(\phi, X) \log\left(\frac{q^2}{X}\right) - \frac{f_1(\phi, X)}{X} + f_{2X}(X) \right), \quad (56)$$

$$Q(\phi, X) = 2f_{1\phi}(\phi, X) \log\left(\frac{q^2}{X}\right) + \int dX \log\left(\frac{q^2}{X}\right) f_4(X). \quad (57)$$

3. Examples for Horndeski

Finally, let us consider the Horndeski theory with $c_{\text{grav}} = c_{\text{light}}$. An example of a Horndeski Lagrangian admitting a stealth Schwarzschild-(A)dS black hole solution for our specific scalar profile is the standard Einstein-Hilbert

term plus a cosmological constant together with a generalized Galileon term, given by

$$\mathcal{L}_{\text{Horndeski}} = \mathcal{R} - 6\Lambda + \log^2\left(\frac{q^2}{X}\right) q(\phi, X) \square \phi, \quad (58)$$

where $q(\phi, X)$ is a free potential. One can easily check that it satisfies conditions (40). The pure stealth Schwarzschild solution is simply obtained for $\Lambda = 0$. As mentioned in the introduction, the square log term allows one to obtain an effective energy momentum tensor for the scalar field of the form (11), which vanishes for $X = q^2$. Therefore, the scalar field does not gravitate and the metric remains a pure GR solution. This provides a simple example of the mechanism behind stealth black hole configuration in such higher order scalar-tensor theories.

II. CONCLUSION

In this letter, we have provided a first scan of the spherically symmetric sector of a large family of shift symmetry breaking DHOST theories (the quadratic Lagrangian with $A_1 = A_2 = 0$). We have derived general conditions for this subclass to admit the vacuum Schwarzschild-(A)dS solution, Eqs. (24)–(27), or a pure Schwarzschild solution of GR, Eqs. (28)–(30), with a linear time-dependent scalar dressing as well as for a static scalar dressing under the assumption of a constant kinetic term, i.e., $X = q^2$.

Then, we have restricted the class of theories by requiring the remaining shift symmetric DHOST theories, solutions of our conditions, to satisfy the observational constraint $c_{\text{grav}} = c_{\text{light}}$ as discussed in [40], and finally, the recent theoretical requirement of absence of graviton decay discussed in [41]. For all these viable subclasses, we have shown that stealth black hole solutions, with or without a cosmological constant, exist.

Finally, we have considered the restriction to the shift symmetry breaking GLPV and Horndeski subclasses, satisfying the constraint $c_{\text{grav}} = c_{\text{light}}$. We have shown that in each subclass, one can also find stealth black holes solutions upon satisfying the conditions (39)–(42). In the last section, we have provided concrete examples satisfying the conditions we found.

This work extends previous results focusing on stealth black hole solutions in DHOST with a constant scalar profile [20], in shift symmetry breaking Horndeski theory with $c_{\text{grav}} = c_{\text{light}}$ for more general scalar profiles [21], as well as previous results obtained for linear time-dependent scalar dressing in shift symmetric GLPV and DHOST theories with or without $c_{\text{grav}} = c_{\text{light}}$ [16–19].

A crucial step for the future would be to investigate the fate of the stealth black hole solutions considered here at the perturbative level to fully contemplate the scalar field backreaction on the metric and investigate its stability. Indeed, it is well known that the stealth hairy black hole

solutions found in shift symmetric Horndeski theory with $X = \text{Constant}$ are either unstable or strongly coupled as shown in [43]. We leave this stability analysis for future work.

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