Kerr–de Sitter and Kerr–anti–de Sitter black holes as accelerators for spinning particles

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It is shown that the Kerr black hole could act as an accelerator for spinning particles. In principle, it could obtain arbitrarily high energy for an extremal Kerr black hole. In this paper, we extend the previous research to the Kerr–(anti–)de Sitter background and find that the cosmological constant plays an important role on the result. For the case of Kerr–anti–de Sitter black holes, like the Kerr background, only the extremal Kerr–anti–de Sitter black holes can have a divergent center-of-mass energy of collision. While, for the case Kerr–de Sitter black holes, the collision of two spinning particles can take place on the outer horizon as well as cosmological horizon of the black holes and the center-of-mass energy of collision can blow up if one of the collision particle takes the critical angular momentum. Hence, nonextremal Kerr–de Sitter black holes could also act as accelerators with arbitrarily high center-of-mass energy for spinning particles.

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I. INTRODUCTION

In 2009, Bañados, Silk, and West(BSW) first showed that rotating black hole can act as particle accelerators [1]. They show that for critical Kerr black hole, the collision center-of-mass energy can be arbitrarily high if two test particles at rest at infinity collide near the event horizon [1]. Along this line, a lot of progress has been made in the decade to further study the issue of BSW mechanism [2–21]. So far, most authors focus on point particles where trajectory is an geodesic. However a real particle is an extended body with self-interaction. It has been shown [22–25] that the trajectory of a spinning test particle is no longer a geodesic. And it orbits of spinning particles around black holes background have governed by the Mathisson-Papapetrou-Dixon (MPD) equations [10–12]. By employing MPD equations, in [8], the authors show that for an extremal Kerr black hole, the collision energy could be divergent with some additional critical condition be satisfied. However, as shown in [13], the spin of astrophysical black holes should be less than $0.998M$ (M is the mass of the black hole) which means there is no extremal Kerr black hole existed in the nature.

In the past years, people have found many evidences coming from the cosmological oberservation which shows our present universe is in a state of accelerated expansion. Although many plausible models are constructed to explain the existence of such accelerated expansion, most observations favor the cold dark matter model with a cosmological constant (Λ-CDM model)[14]. At least phenomenologically the Einstein equations should be modified with a cosmological constant Λ at cosmological scale. With a cosmological constant, the Kerr black hole should be generalized to the Kerr–de–Sitter(Kerr–dS) background.

On the other hand, the AdS/CFT correspondence becomes a fruitful field, and had many interesting results has been made in the past decades [26–28]. In [29], the authors investigate the issues of conformal field theory (CFT) dual to collision particles on a given black hole background. Therefore, it is also interesting to extend Kerr black hole to Kerr–anti–de Sitter background and study the process of collisions of spinning particles. Moreover, in [15], the authors showed that unlike the Kerr case, for the Kerr–dS black hole, even the nonextremal Kerr–dS background can serve as an accelerator for point particles without spin, and its corresponding collision energy can be divergent when some critical conditions are satisfied.

With all these strong motivations in hand, in this paper, we study the possibility of Kerr–de Sitter and Kerr–anti–de Sitter black holes as accelerator for spinning particles. By using the MPD equations, we investigate the BSW process of the Kerr–(anti–)de Sitter black hole. Because the Kerr–(anti–)de Sitter black holes have a more complicated horizon structure than Kerr black holes, it is very hopeful that some novel features will be emergent.

The paper is organized as follows. In Sec. II, we introduce the equations of motion for spinning particles. In Sec. III, the four momentum of a spinning particle are solved in Kerr–(anti)–dS background. In Sec. IV, we obtained the collision center-of-mass energy of two spinning particles. And then we discuss the Kerr–dS and Kerr–AdS background separately in Sec. V and Sec. VI. The summary and conclusion are given in Sec. VII.

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Throughout the paper, we adopt the convention that gravitational constant G and the speed of light c are equal to unity.

II. EQUATIONS OF MOTION FOR SPINNING PARTICLES

The trajectory of a spin particle in the curved spacetime is described by the Mathission-Papapetrou-Dixon(MPD) equations [8,16]

$$\frac{DP^a}{D\tau} = -\frac{1}{2}R^a_{bcd}v^b S^{cd},$$
$$\frac{DS^{ab}}{D\tau} = P^a v^b - P^b v^a, \tag{2.1}$$

where

$$v^a = \left(\frac{\partial}{\partial\tau}\right)^a \tag{2.2}$$

is the tangent vector of the center-of-mass world line, $\frac{D}{D\tau}$ is the covariant derivative along v^a, and P^a is the canonical 4-momentum of the spinning particles satisfying the condition

$$m^2 = -P^a P_a. \tag{2.3}$$

Moreover, S^{ab} is the antisymmetric spin tensor in which its square related to the spin and mass of the particle are as follows [11]

$$\frac{1}{2}S^{ab}S_{ab} = S^2 = m^2 s^2, \tag{2.4}$$

here m and s represent the mass and spin of the particles, respectively. In order to simplify the calculation in the main text and to easily gain the physical insight, people are usually working in a specific frame which only 3-components of the spin tensor is nonzero [7]. This adds to the spin supplementary condition [7,8]

$$S^{ab}P_b = 0 \tag{2.5}$$

or equivalently set $S^{0i} = 0$. Again for latter convenience, we normalize the parameter τ in Eq. (2.2) as,

$$u^a v_a = -1 \tag{2.6}$$

which means τ is not the proper time of the spin particle. The detailed calculation shows the relation between u^a and v^a can be written as [8,11]

$$v^a - u^a = \frac{S^{ab}R_{bcde}u^c S^{de}}{2(m^2 + \frac{1}{4}R_{bcde}S^{bc}S^{de})}. \tag{2.7}$$

Furthermore, for the spacetime with a Killing vector field ξ^a, we can define the following conserved quantity for spinning particles,

$$Q_\xi = P^a\xi_a - \frac{1}{2}S^{ab}\nabla_b\xi^a, \tag{2.8}$$

which is very helpful to find the trajectory of the spinning particle.

III. SPINNING PARTICLES IN KERR–ADS AND KERR–DS BACKGROUND

In this section, we focus ourself on the Kerr–dS and Kerr–AdS case. The corresponding spacetime metric in the Boyer-Lindquist coordinates is [6,21]

$$ds^2 = -\frac{\Delta_r}{\Sigma}\left(dt - \frac{a\sin^2\theta}{\Xi}d\varphi\right)^2 + \frac{\Sigma}{\Delta_r}dr^2 + \frac{\Sigma}{\Delta_\theta}d\theta^2 + \frac{\Delta_\theta\sin^2\theta}{\Sigma}\left(adt - \frac{r^2+a^2}{\Xi}d\varphi\right)^2 \tag{3.1}$$

where

$$\Delta_r = (r^2+a^2)\left(1 - \frac{\Lambda}{3}r^2\right) - 2Mr, \tag{3.2}$$

$$\Sigma = r^2 + a^2\cos^2\theta, \tag{3.3}$$

$$\Delta_\theta = 1 + \frac{\Lambda a^2}{3}\cos^2\theta, \tag{3.4}$$

$$\Xi = 1 + \frac{\Lambda a^2}{3},. \tag{3.5}$$

Here parameters M and a correspond to the mass and angular momentum per unit rest mass of the black hole, Λ is the positive(negative) cosmological constant. The roots of $\Delta_r = 0$ give the horizon of Kerr–dS and Kerr–AdS black hole. From Fig. 1, we can see that the Kerr–dS case behaves differently with the Kerr and Kerr–AdS case.

Figure 2 depicts the roots of Δ_r in the Kerr–dS case. For Kerr–dS black hole, there are three roots which correspond to the inner, outer, and the cosmological horizon, respectively. The particle collision near the cosmological horizon with Reissner-Nordstrom de Sitter black hole is investigated in [30].

The tetrad reads

$$e^{(0)}_a = \sqrt{\frac{\Delta_r}{\Sigma}}\left(dt_a - \frac{a\sin^2\theta}{\Xi}d\varphi_a\right), \tag{3.6}$$

$$e^{(1)}_a = \sqrt{\frac{\Sigma}{\Delta_r}}dr_a, \tag{3.7}$$

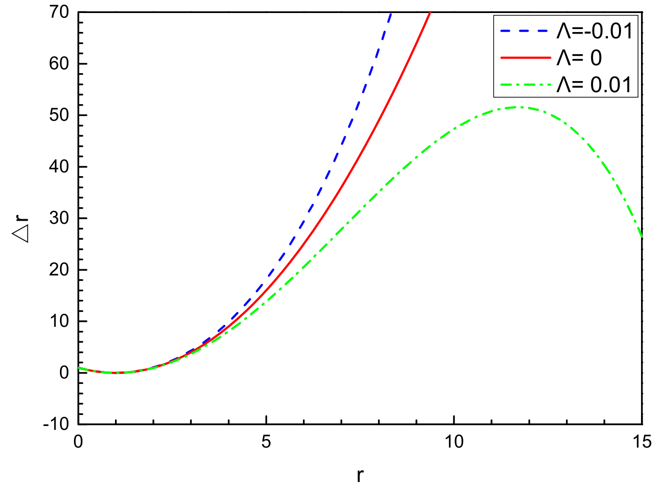

FIG. 1. The function of Δ_r for the Kerr, Kerr–dS, and Kerr–AdS black hole ($a = 0.99$).

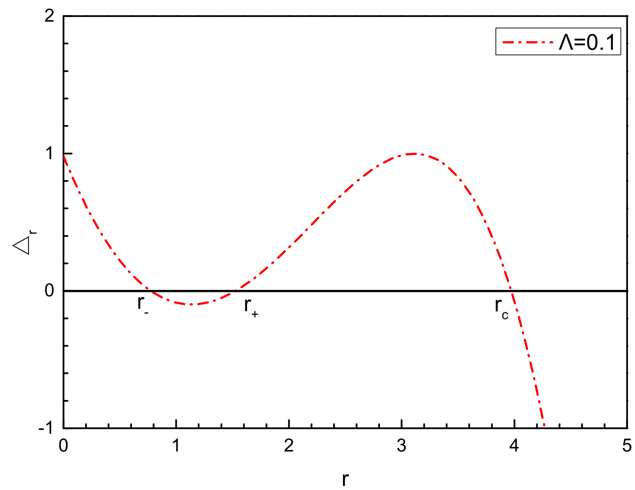

FIG. 2. The roots of Δ_r for the Kerr–dS black hole with a cosmological constant($a = 0.99$).

$$e_a^{(2)} = \sqrt{\frac{\Sigma}{\Delta_\theta}} d\theta_a, \tag{3.8}$$

$$e_a^{(3)} = \sqrt{\frac{\Delta_\theta}{\Sigma}} \sin\theta \left(-adt_a + \frac{r^2 + a^2}{\Xi} d\varphi_a\right). \tag{3.9}$$

The stationary and rotational symmetry properties of the Kerr–dS(AdS) metric are characterized by two Killing vector fields: the timelike Killing vector $\xi^a = (\frac{\partial}{\partial t})^a$ and the axial Killing vector $\phi^a = (\frac{\partial}{\partial \phi})^a$. For the convenience of the following calculation, we expanded these two Killing vectors in the tetrad formalism as

$$\xi_a = -\left(\sqrt{\frac{\Delta_r}{\Sigma}} e_a^{(0)} + \sqrt{\frac{\Delta_\theta}{\Sigma}} a \sin\theta e_a^{(3)}\right),$$

$$\phi_a = \sqrt{\frac{\Delta_r}{\Sigma}} e_a^{(0)} + \sqrt{\frac{\Delta_\theta}{\Sigma}} (r^2 + a^2) \sin\theta e_a^{(3)}. \tag{3.10}$$

They are two conserved quantities corresponding to these two Killing vectors, namely the energy of per unit mass of the particle $e = \frac{E}{m}$, and the z component of total angular momentum per unit mass of the particle $j = \frac{J}{m}$. By applying Eq. (2.7) and the tetrad formalism we have

$$e = -u^a \xi_a + \frac{1}{2m} S^{ab} \nabla_b \xi_a,$$
$$j = u^a \phi_a - \frac{1}{2m} S^{ab} \nabla_b \phi_a. \tag{3.11}$$

For simplicity, we only consider the situation where a spinning particle moves on the orbits in the equatorial plane ($\theta = \frac{\pi}{2}$). First, we introduce a special spin vector $s^{(a)}$ as [8,11]

$$s^{(a)} = -\frac{1}{2m} \epsilon^{(a)}{}_{(b)(c)(d)} u^{(b)} S^{(c)(d)}, \tag{3.12}$$

or equivalently

$$S^{(a)(b)} = m\epsilon^{(a)(b)}{}_{(c)(d)} u^{(c)} s^{(d)}, \tag{3.13}$$

where $\epsilon_{(a)(b)(c)(d)}$ is the completely antisymmetric tensor with the component $\epsilon_{(0)(1)(2)(3)} = 1$. And the only nonvanishing component of $s^{(a)}$ to be [8,11]

$$s^{(2)} = -s, \tag{3.14}$$

where s indicates not only the magnitude of spin but also includes the spin direction. The particle spin is parallel to the black hole spin for $s > 0$, while it is antiparallel for $s < 0$. Therefore the remaining nonvanishing tetrad components of the spin angular momentum are

$$S^{(0)(1)} = -msu^{(3)},$$
$$S^{(0)(3)} = -msu^{(1)},$$
$$S^{(1)(3)} = -msu^{(0)}. \tag{3.15}$$

By calculating the tetrad components of Eq. (3.15) and substituting it to Eq. (3.11), we obtain the expression of the energy and the angular momentum per unit mass e and j as

$$e = \frac{\sqrt{\Delta_r}}{r} u^{(0)} + \frac{a}{r} u^{(3)} + \frac{M - \frac{\Lambda}{3} r^3}{r^2} su^{(3)}, \tag{3.16}$$

$$j = \frac{a\sqrt{\Delta_r}}{r} u^{(0)} + \frac{r^2 + a^2}{r} u^{(3)} + \frac{a(M + r - \frac{\Lambda}{3} r^3)}{r^2 \Xi} su^{(3)} + \frac{\sqrt{\Delta_r}}{r\Xi} su^{(0)}. \tag{3.17}$$

Solving the Eq. (3.16) and Eq. (3.17) gives

$$u^{(0)} = \frac{\tilde{K}}{\sqrt{\Delta_r} K}, \tag{3.18}$$

$$u^{(3)} = \frac{\bar{K}}{K}, \tag{3.19}$$

where

$$K = -3Ms(3s + a^3\Lambda) + r^3(9 + 3a^2\Lambda + 3s^2\Lambda + a^3 s\Lambda^2), \tag{3.20}$$

$$\tilde{K} = r(-j(3 + a^2\Lambda)(3ar + 3Ms - r^3 s\Lambda) + 3e(3r^3 + a^4 r\Lambda + a^2 r(3 + r^2\Lambda) + as(3M + 3r - r^3\Lambda))), \tag{3.21}$$

$$\bar{K} = 3r^2(j(3 + a^2\Lambda) - e(3a + 3s + a^3\Lambda)). \tag{3.22}$$

Note that we are working on the equatorial plane ($\theta = \frac{\pi}{2}$), so $u^{(2)} = 0$. The normalization condition of $u^{(a)}u_{(a)} = -1$ gives us

$$-(u^{(0)})^2 + (u^{(1)})^2 + (u^{(3)})^2 = -1. \tag{3.23}$$

By substituting Eqs. (3.18) and (3.19) to Eq. (3.23), we obtain

$$(u^{(1)})^2 = \frac{\tilde{K}^2 - \Delta_r(\bar{K}^2 + K^2)}{\Delta_r K^2}. \tag{3.24}$$

IV. CENTER-OF-MASS ENERGY

Now we turn to the center-of-mass energy of the collision particles. For simplicity, we consider the two equal-mass particles with masses $m_1 = m_2 = m$. The collision center-of-mass energy of two spinning particles falling from infinity with angular momentum l_1, l_2 reads [8]

$$E_{cm} = \sqrt{2}m\sqrt{1 - g_{ab}u^a_{(1)}u^b_{(2)}}, \tag{4.1}$$

Substituting Eqs. (3.18), (3.19), and (3.22) to Eq. (4.1), one can easily obtain

$$g_{ab}u^a_{(1)}u^b_{(2)} = u^0_{(1)}u^0_{(2)} - u^1_{(1)}u^1_{(2)} - u^3_{(1)}u^3_{(2)} = \frac{\tilde{K}_{(1)}\tilde{K}_{(2)} - \sqrt{\tilde{K}^2_{(1)} - \Delta_r(\bar{K}^2_{(1)} + K^2_{(1)})}\sqrt{\tilde{K}^2_{(2)} - \Delta_r(\bar{K}^2_{(2)} + K^2_{(2)})}}{\Delta_r K_{(1)}K_{(2)}} - \frac{\bar{K}_{(1)}\bar{K}_{(2)}}{K_{(1)}K_{(2)}}, \tag{4.2}$$

where

$$K_{(i)} = K|_{s=s_i, j=j_i},$$
$$\tilde{K}_{(i)} = \tilde{K}|_{s=s_i, j=j_i},$$
$$\bar{K}_{(i)} = \bar{K}|_{s=s_i, j=j_i}, \qquad i = 1, 2 \tag{4.3}$$

are the quantities corresponding to the particle 1 or 2. One can easily see that for the case $\Lambda = 0$ our result of the center-of-mass energy is the same as Kerr spacetime with spinning particles [8]. For $\Lambda = 0$ and $a = 0$, our result reduces to the situation of the Schwarzschild black hole with spinning particles [7]. And of course the case of the Kerr black hole with spinless particles is recovered when we set $\Lambda = 0$ and $s = 0$ [1].

At first sight, one may naively think that E_{cm} could diverge when the particles approach the horizon since the value of Δ_r is zero at the horizon. However, the denominator of E_{cm} could also be divergent. Therefore, we need to carefully analyze the asymptotic behavior of E_{cm} when it approaches horizon. To this aim, we define

$$E_0 = \tilde{K}_{(1)}\tilde{K}_{(2)} - \sqrt{\tilde{K}^2_{(1)} - \Delta_r(\bar{K}^2_{(1)} + K^2_{(1)})} \times \sqrt{\tilde{K}^2_{(2)} - \Delta_r(\bar{K}^2_{(2)} + K^2_{(2)})}. \tag{4.4}$$

The first fraction of Eq. (4.2) may be divergence when particles collide at the horizon $r = r_+$. In order to ensure if Eq. (4.2) could be infinite at the horizon, we expand E_0 near the horizon $r = r_+$

$$E_0 = a + b(r - r_+) + \cdots \tag{4.5}$$

where the first coefficients of the Taylor expansion a reads

$$a = E_0|_{r=r_+} = \left[\tilde{K}_{(1)}\tilde{K}_{(2)} - \sqrt{\tilde{K}^2_{(1)}}\sqrt{\tilde{K}^2_{(2)}}\right]_{r=r_+} = 0. \tag{4.6}$$

On the other hand, from Eq. (4.4), we can write that

$$b = \left.\frac{dE_0}{dr}\right|_{r=r_+} = \frac{K^2_{(2)}(\bar{K}^2_{(1)} + K^2_{(1)}) + K^2_{(1)}(\bar{K}^2_{(2)} + K^2_{(2)})}{2\tilde{K}_{(1)}\tilde{K}_{(2)}}\Delta'_r \tag{4.7}$$

where $\Delta'_r = \frac{d\Delta_r}{dr}$. By using the expression of Δ_r, and note that near the horizon $\Delta_r \sim r - r_+$(nonextremal) or $\Delta_r \sim (r - r_+)^2$(extremal). The Eq. (4.2) at horizon ($r = r_+$) can be written as

$$g_{ab}u^a_{(1)}u^b_{(2)} = \frac{\Delta'_r[K^2_{(2)}(\bar{K}^2_{(1)} + K^2_{(1)}) + K^2_{(1)}(\bar{K}^2_{(2)} + K^2_{(2)})]}{2\tilde{K}_{(1)}\tilde{K}_{(2)}K_{(1)}K_{(2)}} - \frac{\bar{K}_{(1)}\bar{K}_{(2)}}{K_{(1)}K_{(2)}}. \tag{4.8}$$

Therefore, if we want the E_{cm} to blow up for the nonextremal black holes, the only possibility is that $\tilde{K}_{(i)} = 0$ or $K_{(i)} = 0$, where $i = 1, 2$ at horizon. On the other hand we note that for the non-extremal Kerr–dS black hole, for a given cosmological constant Λ, there always exists some a such that $\Delta_r' < 0$ [15]; while, for the nonextremal Kerr–AdS black hole, the $\Delta_r' > 0$. We will discuss these two cases separately in the following sections.

V. KERR–DE SITTER BLACK HOLE

In this section, we consider the spin particles are accelerated in Kerr–dS background($\Lambda > 0$). Let $j = l + s$ as the total angular momentum, with l being the orbital angular momentum, where ($l = j - s$) [7]. Since v^a is a timelike vector, we know that $\frac{dt}{d\tau} > 0$ near the horizon $r = r_+$. Using Eq. (2.7) and the normalization condition Eq. (3.23), the relation between v^a and u^a can be solved

$$v^{(0)} = X^{-1}\left(1 - \frac{Ms^2}{r^3}\right)u^{(0)},$$
$$v^{(1)} = X^{-1}\left(1 - \frac{Ms^2}{r^3}\right)u^{(1)},$$
$$v^{(3)} = X^{-1}\left(1 + \frac{2Ms^2}{r^3}\right)u^{(3)}, \tag{5.1}$$

where

$$X = 1 - \frac{Ms^2}{r^3}[1 + 3(u^{(3)})^2]. \tag{5.2}$$

The general form of 4-velocity v^a of a spinning particle can be expressed as [8]

$$v^a = \frac{dt}{d\tau}\left(\frac{\partial}{\partial t}\right)^a + \frac{dr}{d\tau}\left(\frac{\partial}{\partial r}\right)^a + \frac{d\varphi}{d\tau}\left(\frac{\partial}{\partial \varphi}\right)^a. \tag{5.3}$$

Plugging Eqs. (3.6)–(3.9) into Eq. (5.3), we have

$$v^{(0)} = \sqrt{\frac{\Delta_r}{\Sigma}}\left(\frac{dt}{d\tau} - \frac{a\sin^2\theta}{\Xi}\frac{d\varphi}{d\tau}\right),$$
$$v^{(1)} = \sqrt{\frac{\Sigma}{\Delta_r}}\frac{dr}{d\tau},$$
$$v^{(3)} = \sqrt{\frac{\Delta_\theta}{\Sigma}}\sin\theta\left(-a\frac{dt}{d\tau} + \frac{r^2 + a^2}{\Xi}\frac{d\varphi}{d\tau}\right). \tag{5.4}$$

Combining Eqs. (5.1) and (5.4), we can get

$$\frac{dt}{d\tau} = \frac{(a^2 + r^2)(1 - \frac{Ms^2}{r^3})\sqrt{\Delta_r}u^{(0)} + a(1 + \frac{2Ms^2}{r^3})\Delta_r u^{(3)}}{Xr\Delta_r}, \tag{5.5}$$

$$\frac{dr}{d\tau} = \frac{\sqrt{\Delta_r}(1 - \frac{Ms^2}{r^3})u^{(1)}}{Xr}, \tag{5.6}$$

$$\frac{d\varphi}{d\tau} = \frac{\Xi(a\sqrt{\Delta_r}(1 - \frac{Ms^2}{r^3})u^{(0)} + \Delta_r(1 + \frac{2Ms^2}{r^3})u^{(3)})}{Xr\Delta_r}. \tag{5.7}$$

Substituting Eqs. (3.18), (3.19), and (5.2) to Eq. (5.5), we can write it as

$$\frac{dt}{d\tau} = \frac{(a^2 + r^2)(r^3 - Ms^2)\tilde{K} + a(r^3 + 2Ms^2)\Delta_r\bar{K}}{[r^3 - Ms^2(1 + (\frac{\bar{K}}{K})^2)]Kr\Delta_r} \tag{5.8}$$

Since $\Delta_r|_{r=r_+} = 0$, the numerator can be simplified as $(a^2 + r^2)(r^3 - Ms^2)\tilde{K}$ at the horizon, and note that $s \ll M$ [8,16]. The condition $dt/d\tau > 0$ is equivalent to

$$\tilde{K} > 0. \tag{5.9}$$

Solving this equation gives us an upper limit of the total angular momentum j

$$j < \frac{3(3Mas + \Lambda a^4 r_+ + \Lambda a^2 r_+^3 + 3a^2 r_+ - \Lambda a r_+^3 s + 3ar_+ s + 3r_+^3)}{(\Lambda a^2 + 3)(3Ms + 3ar_+ - \Lambda s r_+^3)} = j_c. \tag{5.10}$$

When $\Lambda = 0$, this upper limit of total angular momentum j_c is coincide with the critical angular momentum in Kerr black hole background [8]. Although at first glance, the limit case $j = j_c$(which corresponds $\tilde{K} = 0$) cannot be obtained. However, similar to [8], in the case of $j = j_c$, we still have $v^a v_a < 0$, namely the collision particles with critical angular momentum $j = j_c$ is still timelike.

Figure 3 shows the effect of the different a and cosmological constants Λ on the critical angular momentum j_c. We can see that the j_c becomes bigger as cosmological constants Λ(spin of the black hole a) increase.

Therefore, for a nonextremal Kerr–dS background, the collision particles with critical angular momentum $j = j_c$ has $\tilde{K} = 0$. By using Eq. (4.8), we have a blow up $g_{ab}u^a_{(1)}u^b_{(2)}$, which in turn implies an arbitrarily high center-of-mass energy E_{cm}.

Now, we verify that for the critical situation whether the spinning particle can really reach the horizon. It is found

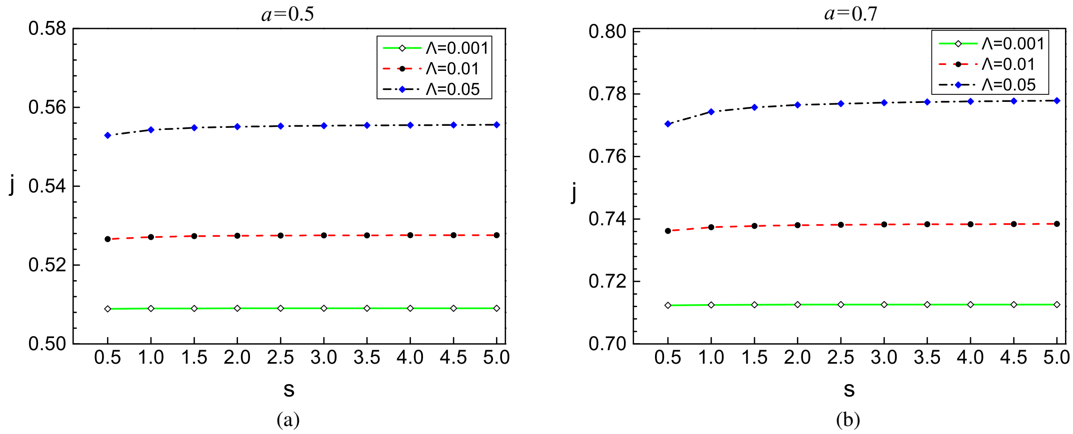

FIG. 3. (a) Critical angular momentum j as a function of spin s for the Kerr–dS background with different value of Λ, ($a = 0.5$). (b) Critical angular momentum j as a function of spin s for the nonextremal Kerr–dS background with different value of Λ, ($a = 0.7$).

that the radial turning points occurred at the point of $u^r = 0$ [7]. Since $u^r = \frac{P^r}{m}$, at the turning points, $(P^r)^2 = 0$. Using Eqs. (3.18), (3.19), and (3.24), the nonvanishing components of the momentum are

$$\frac{P^t}{m} = \frac{(r^2 + a^2)\tilde{K} + a\bar{K}\Delta_r}{\Delta_r K r}, \tag{5.11}$$

$$\frac{P^\phi}{m} = \frac{(a\tilde{K} + \bar{K}\Delta_r)\Xi}{\Delta_r K r}, \tag{5.12}$$

$$\left(\frac{P^r}{m}\right)^2 = \frac{\tilde{K}^2 - \Delta_r(\bar{K}^2 + K^2)}{K^2 r^2}. \tag{5.13}$$

Note that for the nonextremal Kerr–dS black hole, $\Delta_r \sim (r - r_+)$ and $\tilde{K} = 0$, we get $(\frac{P^r}{m})^2|_{r=r_+} = 0$. Obviously, the spinning particles can reach the horizon and the collision energy E_{cm} is divergent near the horizon. Therefore, the collision center-mass-energy E_{cm} blow up at $r = r_+$ when the critical collision angular momentum $j = j_c$ is satisfied even for the non-extremal Kerr–dS black hole.

On the other hand, from Eq. (4.8), it seems that the center-mass-energy E_{cm} could also be divergent when $K = 0$. However, in this case, the particle cannot approach the horizon because of $(\frac{P^r}{m})^2|_{r=r_+} \neq 0$ when $K = 0$.

Next, we focus on the point in which the collision of spinning particles takes place on the cosmological horizon $r = r_c$. Similarly, if we ensure the Eq. (4.2) could diverge, the E_0 of Eq. (4.4) near the cosmological radius r_c is expanded as

$$E_0 = a + b(r - r_c) + \cdots \tag{5.14}$$

We find that the first coefficients of the Taylor expansion a are still equal to zero. And, from Eq. (4.4), we can rewrite that

$$b = \frac{dE_0}{dr}\bigg|_{r=r_c} = \frac{K_{(2)}^2(\bar{K}_{(1)}^2 + K_{(1)}^2) + K_{(1)}^2(\bar{K}_{(2)}^2 + K_{(2)}^2)}{2\tilde{K}_{(1)}\tilde{K}_{(2)}}\Delta_r'. \tag{5.15}$$

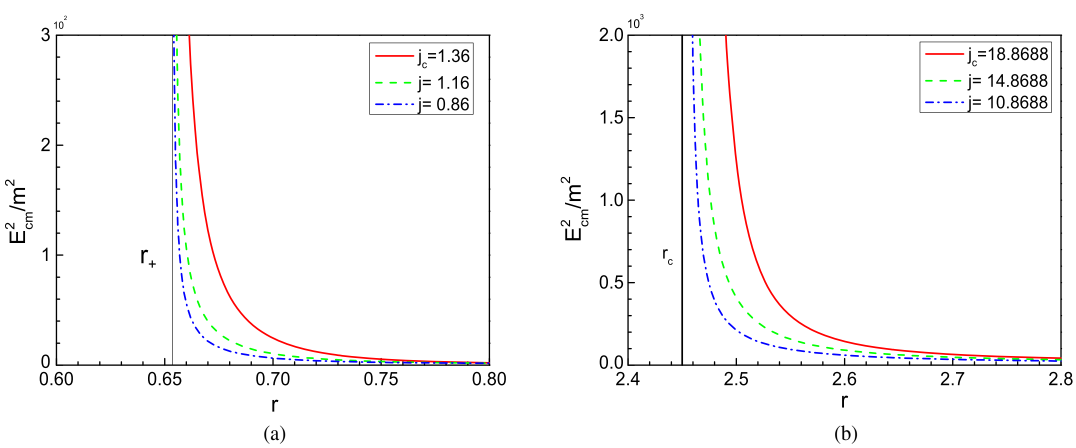

FIG. 4. (a) The collision center-of-mass energy E_{cm} as a function of different angular momentum near the horizon for Kerr–dS background ($a = 0.99$, $\Lambda = 0.5$). (b) The collision center-of-mass energy E_{cm} as a function of different angular momentum near the cosmological horizon for Kerr–dS background ($a = 0.99$, $\Lambda = 0.5$).

Therefore, we can obtain the same equation as Eq. (4.8). In addition, we must consider the sign of Δ_r' near the cosmological horizon r_c. From the Eq. (3.2), which reads

$$\Delta_r' = -\frac{4}{3}\Lambda r^3 + \left(2 - \frac{2}{3}\Lambda a^2\right) r - 2M. \tag{5.16}$$

Since we know the cosmological horizon $r_c \sim \frac{1}{\sqrt{\Lambda}}$ is big enough when cosmological constant Λ takes vanishingly small value and the cosmological constant is positive in Kerr–dS background. Therefore, the first item of Eq. (5.16) is negative and the absolute value of the first item is far greater than the other items, this makes $\Delta_r' < 0$. Therefore we have the same conclusion as the case of $r = r_+$. Namely, when $r = r_c$, the collision particles with critical angular momentum $j = j_c|_{r=r_c}$, the collision center-of-mass energy E_{cm} also diverges at $r = r_c$.

From Fig. 4 we can clearly see that one of the two collision particles with critical angular momentum j_c can be accelerated to arbitrarily high energies near the inner horizon as well as cosmological horizon for Kerr–dS background.

VI. KERR–ANTI–DE SITTER BLACK HOLE

Now we turn to Kerr–anti–de Sitter background. In this case unlike Kerr–dS background, the $\Delta_r' \le 0$. The E_{cm} blow up means that $\tilde{K}_{(i)} = 0$ or $K_{(i)} = 0$, where $i = 1, 2$. This condition can be realized when $r = r_+$. However, in order to ensure the particle can escape to infinity, we need to require that the derivative of $(P^r)^2$ respect to r must be positive at the horizon r_+ [15].

$$\left.\frac{d(\frac{P^r}{m})^2}{dr}\right|_{r=r_+} > 0. \tag{6.1}$$

Substituting Eq. (5.13) to the above formula, we can get

$$\left.\frac{d(\frac{P^r}{m})^2}{dr}\right|_{r=r_+} = \frac{2\tilde{K}\tilde{K}' - \Delta_r'(\bar{K}^2 + K^2)}{2Kr(K'r + K)}. \tag{6.2}$$

Since for the nonextremal Kerr–anti–de Sitter black hole, the $\Delta_r' > 0$, then the $\frac{d(\frac{P^r}{m})^2}{dr}|_{r=r_+}$ is always negative at the horizon, which means the center of the energy E_{cm} cannot reach arbitrarily high. For the extremal Kerr–AdS black hole $\Delta_r' = 0$, which means $\frac{d(\frac{P^r}{m})^2}{dr}|_{r=r_+} = 0$, however, we found that $\frac{d^2(\frac{P^r}{m})^2}{dr^2} > 0$, the collision particles can still escape to infinity. This phenomenon also found for the extremal Kerr–AdS black hole with spinless collision particles [15]. From these analysis, we get a conclusion that E_{cm} can only blow up for extremal Kerr–AdS black hole.

VII. CONCLUSIONS

In this paper, we have analyzed the possibility that Kerr–dS and Kerr–AdS black holes could act as accelerators for spinning particle. We find that the result is very different from the case of Kerr black holes due to the existence of the nonvanishing cosmological constant. On the one hand, it turns out that two particles to collide in the outer horizon with the critical spinning angular momentum $j = j_c$ can reach arbitrary high center-of-mass energy. Moreover, the center-of-mass energy is also divergent when particles collide on the cosmological horizon with the critical spinning angular momentum $j = j_c|_{r=r_c}$. Besides, for the case of the Kerr black hole, it has to be extremal. However, for the case of the Kerr–dS black hole, it does not need to be extremal. Hence, nonextremal Kerr–dS black holes could also serve as particle accelerators with arbitrarily high center-of-mass energy E_{cm}, which is very different from the cases of the Kerr and Kerr-AdS black holes. By detailed analysis, the sign of Δ_r' is different for Kerr–AdS and Kerr–dS case, and this is exactly why the Kerr–dS case is so different.

Why is the Kerr–dS case so different with the Kerr and Kerr–AdS black holes? In particular, when the cosmological constant is very small, people will usually expect the final center-of-mass energy E_{cm} has only small deviation from Kerr background. However, since we know that the cosmological constant Λ set a scale for the universe $r_h = \frac{3}{\sqrt{\Lambda}}$. For short distance($r < r_h$), the physics are almost the same as in zero cosmological constant case. However, for the large distance case ($r > r_h$), the cosmological constant becomes very relevant and the physics will be strongly influenced by the value of the cosmological constant [31,32]. Note that the positive cosmological constant represents a repulsive force and we are considering particles falling off from infinity. Therefore the acceleration effect of the cosmological constant of the Kerr–dS black hole becomes dominant and finally makes the center-of-mass energy E_{cm} blow up.

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