

Testing the Etherington distance duality relation at higher redshifts: Combined radio quasar and gravitational wave data

Jing-Zhao Qi,^{1,2} Shuo Cao,^{2,*} Chenfa Zheng,² Yu Pan,³ Zejun Li,² Jin Li,^{4,†} and Tonghua Liu²

¹*Department of Physics, College of Sciences, Northeastern University, Shenyang 110004, China*

²*Department of Astronomy, Beijing Normal University, 100875, Beijing, China*

³*College of Science, Chongqing University of Posts and Telecommunications, Chongqing 400065, China*

⁴*Department of Physics, Chongqing University, Chongqing 400030, China*



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In this paper, we analyze the implications of the latest cosmological data sets to test the Etherington distance duality relation (DDR), which connects the luminosity distance D_L and angular diameter distance D_A at the same redshift. For D_L , we consider the simulated data of gravitational waves from the third-generation gravitational wave detector [the Einstein Telescope (ET)], which can be considered as standard candles (or standard siren), while the angular diameter distances D_A are derived from the newly compiled sample of compact radio quasars observed by very-long-baseline interferometry (VLBI), which represents a new type of cosmological standard ruler. Alleviating the absorption and scattering effects of dust in the Universe, this will create a valuable opportunity to directly test DDR at much higher precision with the combination of gravitational wave (GW) and electromagnetic (EM) signals. Our results show that, with the combination of the current radio quasar observations, the duality-distance relation can be verified at the precision of 10^{-2} . Moreover, the Einstein Telescope would produce more robust constraints on the validity of such distance duality relation (at the precision of 10^{-3}), with a larger sample of compact milliarcsecond radio quasars detected in future VLBI surveys.

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I. INTRODUCTION

Due to the rapid technological advances in observational cosmology, the accumulation of observational data obtained with increasing precision makes it possible to test some fundamental relations in cosmology, one of which is the well-known Etherington distance duality relation (DDR) [1]. In the framework of DDR, two cosmological distances, i.e., the luminosity distance D_L and angular diameter distance D_A at the same redshift z are connected as

$$\frac{D_L(z)}{D_A(z)}(1+z)^{-2} = 1. \quad (1)$$

Theoretically, distances based on standard candles (e.g., supernovae) and standard rulers (e.g., baryon oscillations) agree as long as three conditions are met: the spacetime is characterized by a metric theory, the light propagates along null geodesics, and the number of photons is conserved. Despite its application in all analyses of cosmological observations, it is necessary to test the validity of this relation because of the possible violation of the DD relation. It has been argued in the literature that the violation of the former

two conditions is related to a signal of exotic physics acting as the background gravity theory [2], while cosmic opacity might contribute to the possibility that the number of photons is not conserved in propagation [3,4]. Therefore, a validity check of the DDR not only tests the existing theories of gravity, but also helps us understand some fundamental properties of the Universe.

Up to now, there are many works devoted to validating the DDR with various observational data, in which the conclusions were drawn from recent type-Ia supernovae data as luminous sources of known (or standardizable) intrinsic luminosity in the Universe, while the angular diameter distance was derived from different astrophysical probes, such as baryon acoustic oscillations (BAO) [5–10], the Sunyaev-Zeldovich effect together with x-ray emission of galaxy clusters [11–13], and strong gravitational lensing systems [14–17], etc. The first two tests are always considered as individual standard rulers while the other two probes are treated as statistical standard rulers in cosmology. However, it should be noted that the information of angular diameter distance obtained from the above standard rulers, is strongly model dependent, which will generate systematic uncertainties hard to quantify [18]. For instance, the angular diameter distance determined from x-ray and SZ studies of clusters is sensitive to the underlying geometry of the galaxy clusters (spherical or elliptical) [13] and the assumptions

*caoshuo@bnu.edu.cn

†cqjinli1983@cqu.edu.cn

of the hot gas density profile (simple β or double $-\beta$ profile) [19]. In addition, it was found that other attempts using angular diameter distance data from baryon acoustic oscillations suffer from the limited sample size covering the redshift range $0.35 \leq z \leq 0.74$ [20]. More importantly, one note that with SN Ia and other standard rulers we are only able to probe the relatively lower redshift range $z \leq 1.40$, which still remains challenging with respect to the exploration of the DDR. In order to draw firm and robust conclusions about the validity of DDR, one will need to minimize statistical uncertainties by increasing the depth and quality of observational data sets. In this paper, we will make a cosmological model-independent test for the DD relation with two new methods by using the simulated data of gravitational waves from the third-generation gravitational wave detector (which can be considered as standard siren to provide the information of luminosity distance) and the newly compiled sample of compact radio quasars observed by very-long-baseline interferometry (which represents a type of new cosmological standard ruler).

It is well known that the first direct detection of gravitational waves (GWs) by the LIGO/Virgo Collaboration [21] has opened the era of GW astronomy. The original idea of using the waveform signal to directly measure the luminosity distance D_L to the GW sources (inspiralling and merging double compact binaries) can be traced back to the paper of Schutz [22], which indicates that the GW sources, especially the inspiralling and merging compact binaries consisting of neutron stars (NSs) and black holes (BHs), can also be used to probe the information of the absolute luminosity distances. This constraint from the so-called standard sirens originates from the dependence of the D_L measurements on the so-called chirp mass and the luminosity distance [22]. The gravitational waves (GWs) provide an alternative tool to testing the cosmology and there have been a number of attempts to do so [23–27]. The constraints derived in these works are compatible with cosmological parameter constraints determined by other techniques [15,16,28], if hundreds of GW events are seen. More importantly, as a promising complementary tool to supernovae, the greatest advantage of GWs lies in the fact that the distance calibration of such standard siren is independent of any other distance ladders. In this paper, we present a significant extension of previous works and investigate the possibility of using GW data to test the validity of DDR. Unfortunately, due to the limited size of observed GW events up to date (which makes it impossible to do statistical analysis), we will focus on a large number of simulated GW events based on the third-generation GW ground-based detector, Einstein Telescope (ET) [29].

In the EM window, with the aim of acquiring angular diameter distances, the angular size of the compact structure in radio quasars provides an effective source of standard rulers in the Universe. However, due to their uncertain intrinsic linear sizes, it is controversial whether the compact

radio sources can be calibrated as standard cosmological probes [30–34]. In general, the precise value of the linear size l_m might depend both on redshifts and the intrinsic properties of the source, i.e., the intrinsic luminosity L [35]. On the other hand, the morphology and kinematics of compact structure in radio quasars could be strongly dependent on the nature of the central engine, including the mass of central black hole and the accretion rate [35]. Therefore, the central region may be standard if these parameters are confined within restricted ranges for specific quasars. Considering the possible correlation between black hole mass M_{BH} and radio luminosity L_R [36,37], Cao *et al.* [38] proposed that intermediate-luminosity quasars (10^{27} W/Hz $< L < 10^{28}$ W/Hz) might be used as a new type of cosmological standard ruler with fixed comoving length, based on a 2.29 GHz VLBI all-sky survey of 613 milliarcsecond ultracompact radio sources [39,40]. More recently, the value of l_m was calibrated at ~ 11 pc though a cosmological model-independent method [41], based on which the cosmological application of the intermediate-luminosity quasar sample (in the redshift range $0.46 < z < 2.76$) was also extensively discussed in the framework of different dark energy models [42,43] and modified gravity theories [44,45]. Compared with our previous works focusing on SNe Ia as background sources [11,13], the advantage of this work is that, we achieve a reasonable and compelling test of DDR at much higher redshifts ($z \sim 3.0$), which will help us to verify the fundamental relations in the early Universe.

This paper is organized as follows. We briefly introduce how to handle the gravitational wave data and the quasar data in Sec. II. Then we show the analysis methods and results of our work in Sec. III. Finally, the conclusions and discussions are presented in Sec. IV.

II. OBSERVATIONS AND SIMULATIONS

A. Angular diameter distance from radio quasars

Following the suggestions by Refs. [31,40], the linear size of the compact structure in radio quasars depends on the redshift z and the intrinsic luminosity L of the source

$$l_m = lL^\beta(1+z)^n \quad (2)$$

where β and n are two parameters represent the “angular size-redshift” and “angular size-luminosity” relations, respectively. The parameter l denotes the linear size scaling factor describing the apparent distribution of radio brightness within the core. In one of the more significant studies involving the compact structure of radio quasars, Gurvits [40] showed that the dispersion in linear size is greatly mitigated by retaining only those sources with flat spectrum ($-0.38 < \alpha < 0.18$). These works have formed the basis of many subsequent investigations [30–34]. More recently, based on a sample of 2.29 GHz VLBI survey with 613

milliarcsecond ultracompact radio sources covering the redshift range $0.0035 < z < 3.787$, Cao *et al.* [38] demonstrated that intermediate-luminosity quasars (ILQSO), which show negligible redshift and luminosity dependence ($|n| \simeq 10^{-3}, \beta \simeq 10^{-4}$), may be used to establish some general constraints on cosmological parameters. In the subsequent analysis, the intrinsic linear size of this cosmological standard ruler at 2.29 GHz was determined at $l_m = 11.03 \pm 0.25$ pc, which was obtained through a cosmological model-independent method [41]. This subsample contains 120 intermediate-luminosity quasars covering the redshift range $0.46 < z < 2.80$, whose cosmological application in Λ CDM resulted with stringent constraints on both the matter density Ω_m and the Hubble constant H_0 , in a good agreement with recent Planck 2015 results. It is now understood that these works lead to a second significant advancement with the use of these sources in the study of cosmic acceleration [42–45].

Let us note that the angular sizes θ of these standard rulers were estimated from the ratio of the correlated and total flux densities measured with radio interferometers ($\Gamma = S_c/S_t$), i.e., the visibility modulus Γ defines a characteristic angular size,

$$\theta = \frac{2\sqrt{-\ln \Gamma \ln 2}}{\pi B}, \quad (3)$$

where B is the interferometer baseline measured in wavelengths [46]. It is reasonable to ask if the derived angular sizes are dependent on the intrinsic luminosities of radio quasars. At first sight, this is the case, since they were obtained by combining the measurements of total flux density S_t and the correlated flux density S_c (fringe amplitude). However, one should observe that in Eq. (3) that the flux densities enters into the angular size θ not through an S measure directly, but rather through a ratio of correlated and total flux densities $\Gamma = S_c/S_t$. Therefore, the intrinsic luminosities of radio quasars do not influence the derived angular sizes θ , which implies that the so-called ‘‘circularity problem’’ will not affect our statistical analysis of the distance duality relation. Meanwhile, we also remark here that the ‘‘ θ ’’ value represents a single-parameter Gaussian, which can be assumed to be a rough representation of the complicated source structure (the actual brightness distribution). The previous works convinced us [47,48] that we could reliably define the size of an unresolved source though such technique, while θ is accurate enough to represent source sizes when averaged over a group of sources in a statistical application. More importantly, besides the direct averaging over the set of sources, it also mimics averaging over the different position angles of the interferometer baselines, although the apparent angular size of milliarcsecond structure in radio quasars is less dependent on the orientation relative to the line of sight. Following Ref. [39], another useful method to define

the characteristic angular size of each source is to measure the size between the peak in the map (i.e., the core) and the most distant component exceeding a given relative brightness level (i.e., 2% of the peak brightness of the core), which was extensively used in the investigation of compact structure in radio sources [49,50].

The observable of ILQSO is the angular size of the compact structure θ , which may then be written as

$$\theta(z) = \frac{l_m}{D_A(z)}. \quad (4)$$

where $D_A(z)$ is the model-dependent angular diameter distance at redshift z . Furthermore, we will consider the future observation of radio quasars from VLBI surveys based on better uv-coverage which will significantly reduce the uncertainty of the angular size of compact structure observed. Consequently, one can expect to have a better angular diameter distance information in the future, which will allow us to test DDR more accurately. Here, in the simulation below, we adopt the flat Λ CDM with $H_0 = 67.8 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $\Omega_m = 0.308$ based on the recent Planck results [51]. Taking the linear size of ILQSO as $l_m = 11.03$ pc and following the redshift distribution of QSOs [52], we have simulated 500 $\theta - z$ data in the redshift $0.50 < z < 6.00$, for which the error of the angular size θ was taken at a level of 3%. This reasonable assumption of the ‘‘ θ ’’ measurements will be realized from both current and future VLBI surveys based on better uv-coverage [53]. There are two general reasons for ignoring sources with $z < 0.5$. First, as z falls below 0.5, the epoch of quasar formation comes to an end and the nature of the population changes dramatically, which indicates the possible existence of a correlation between linear size and radio luminosity. Therefore, following the suggestion of Refs. [40,49], only the high-redshift part of radio quasars could be used as a standard rod to fit different cosmological models with experimental data. Second, as z increases a larger Doppler boosting factor \mathcal{D} is required, i.e., the ratio $\mathcal{D}/(1+z)$ is approximately fixed, so that the rest-frame emitted frequency $(1+z)\nu_r/\mathcal{D}$ is also fixed. See Ref. [47] for mathematical and astrophysical details [54]. Moreover, in order to make the simulated data more representative of the experimental expectation, we assume the angular size measurements obey the Gaussian distribution $\theta_{\text{mean}} = \mathcal{N}(\theta_{\text{fid}}, \sigma_\theta)$ as shown in Fig. 1. The more details of the specific procedure of QSO simulation can be found in Ref. [55].

B. Luminosity distance from gravitation wave sources

In this section, we simulate GW events based on the Einstein Telescope, the third generation of the ground-based GW detector. Compared with the current advanced ground-based detectors (i.e., the advanced LIGO and Virgo detectors), the ET is designed to be ten times more sensitive

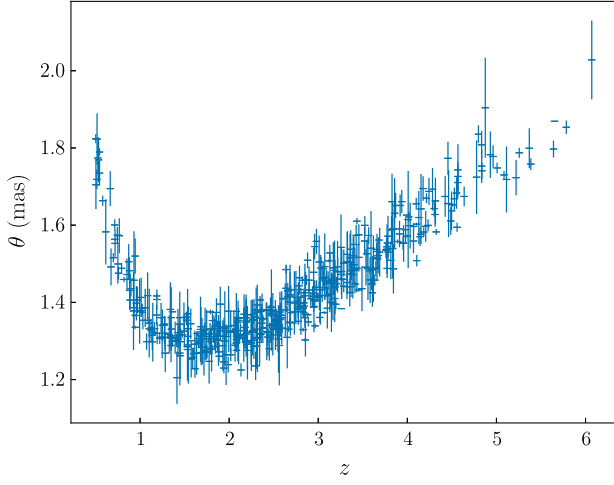


FIG. 1. The angular size measurements from 500 simulated radio quasars.

covering the frequency range of 1–10⁴ Hz. Here, we briefly introduce the GW as standard sirens in the ET observations.

GW detectors based on the ET could measure the strain $h(t)$, which quantifies the change of difference of two optical paths caused by the passing of GWs. It can be expressed as the linear combination of the two polarization states,

$$h(t) = F_+(\theta, \phi, \psi)h_+(t) + F_\times(\theta, \phi, \psi)h_\times(t), \quad (5)$$

where $F_{+, \times}$ are the beam pattern functions, ψ denotes the polarization angle, and (θ, ϕ) are angles describing the location of the source relative to the detector. Following the analysis of Zhao *et al.* [25], the explicit expressions of the beam patterns of the ET are given by

$$\begin{aligned} F_+^{(1)}(\theta, \phi, \psi) &= \frac{\sqrt{3}}{2} \left[\frac{1}{2} (1 + \cos^2(\theta)) \cos(2\phi) \cos(2\psi) \right. \\ &\quad \left. - \cos(\theta) \sin(2\phi) \sin(2\psi) \right], \\ F_\times^{(1)}(\theta, \phi, \psi) &= \frac{\sqrt{3}}{2} \left[\frac{1}{2} (1 + \cos^2(\theta)) \cos(2\phi) \sin(2\psi) \right. \\ &\quad \left. + \cos(\theta) \sin(2\phi) \cos(2\psi) \right]. \end{aligned} \quad (6)$$

Considering the fact that the three interferometers of the ET are arranged in an equilateral triangle, the other two interferometer's antenna pattern functions can also be derived from Eq. (6) [27]

$$F_{+, \times}^{(2)}(\theta, \phi, \psi) = F_{+, \times}^{(1)}(\theta, \phi + 2\pi/3, \psi) \quad (7)$$

$$F_{+, \times}^{(3)}(\theta, \phi, \psi) = F_{+, \times}^{(1)}(\theta, \phi + 4\pi/3, \psi). \quad (8)$$

In this paper, we focus on the GW signals from the merger of binary systems with component masses m_1 and m_2 .

Then the chirp mass can be defined as $\mathcal{M}_c = M\eta^{3/5}$, while the observed counterpart can be written as $\mathcal{M}_{c, \text{obs}} = (1+z)\mathcal{M}_{c, \text{phys}}$, where $M = m_1 + m_2$ is the total mass and $\eta = m_1 m_2 / M^2$ represents the symmetric mass ratio. Following Refs. [25, 56], the Fourier transform $\mathcal{H}(f)$ of the time domain waveform $h(t)$ could be derived by applying the stationary phase approximation,

$$\mathcal{H}(f) = \mathcal{A} f^{-7/6} \exp[i(2\pi f t_0 - \pi/4 + 2\psi(f/2) - \varphi_{(2,0)})], \quad (9)$$

where t_0 is the epoch of the merger, and the definitions of the functions ψ and $\varphi_{(2,0)}$ can be found in [25]. The Fourier amplitude \mathcal{A} is given by

$$\begin{aligned} \mathcal{A} &= \frac{1}{D_L} \sqrt{F_+^2 (1 + \cos^2(\iota))^2 + 4F_\times^2 \cos^2(\iota)} \\ &\quad \times \sqrt{5\pi/96} \pi^{-7/6} \mathcal{M}_c^{5/6}, \end{aligned} \quad (10)$$

where ι represents the angle of inclination of the binary's orbital angular momentum with the line of sight, and D_L is the theoretical luminosity distance in the fiducial cosmological model we choose. It should be noted that the GW sources used in this work are caused by binary merger of a neutron star with either a neutron star or black hole, which can generate an intense burst of γ -rays (SGRB) with measurable source redshift. More specifically, since the SGRB is emitted in a narrow cone, a criterion on the total beaming angle (e.g., $\iota < 20^\circ$) should be applied to detect one specific gravitational wave event [57]. Moreover, as was pointed out in Cai and Yang [27], Li [58], averaging the Fisher matrix over the inclination ι and the polarization ψ with the constraint $\iota < 20^\circ$ is approximately equivalent to taking $\iota = 0$. Therefore, we can take the simplified case of $\iota = 0$ and then the Fourier amplitude \mathcal{A} will be independent of the polarization angle ψ . Given the waveform of GWs, the combined signal-to-noise ratio (SNR) for the network of three independent ET interferometers is

$$\rho = \sqrt{\sum_{i=1}^3 \langle \mathcal{H}^{(i)}, \mathcal{H}^{(i)} \rangle}. \quad (11)$$

Here, the inner product is defined as

$$\langle a, b \rangle = 4 \int_{f_{\text{lower}}}^{f_{\text{upper}}} \frac{\tilde{a}(f)\tilde{b}^*(f) + \tilde{a}^*(f)\tilde{b}(f)}{2} \frac{df}{S_h(f)}, \quad (12)$$

where $\tilde{a}(f)$ and $\tilde{b}(f)$ are the Fourier transforms of the functions $a(t)$ and $b(t)$. $S_h(f)$ is the one-side noise power spectral density (PSD) characterizing the performance of a GW detector [25]. The lower cutoff frequency f_{lower} is fixed at 1 Hz. The upper cutoff frequency, f_{upper} , is decided by the

last stable orbit (LSO), $f_{\text{upper}} = 2f_{\text{LSO}}$, where $f_{\text{LSO}} = 1/(6^{3/2}2\pi M_{\text{obs}})$ is the orbit frequency at the LSO, and $M_{\text{obs}} = (1+z)\mathcal{M}_{\text{phys}}$ is the observed total mass. Here, we simulate many catalogues of NS-NS and NS-BH systems, with the masses of NS and BH sampled by uniform distribution in the intervals of $[1,2] M_{\odot}$ and $[3,10] M_{\odot}$. Meanwhile, the signal is identified as a GW event only if the ET interferometers have a network SNR of $\rho > 8.0$, the SNR threshold currently used by LIGO/Virgo network [27].

Moreover, using the Fisher information matrix, the instrumental uncertainty on the measurement of the luminosity distance can be estimated as

$$\sigma_{D_L}^{\text{inst}} \simeq \sqrt{\left\langle \frac{\partial \mathcal{H}}{\partial D_L}, \frac{\partial \mathcal{H}}{\partial D_L} \right\rangle^{-1}}, \quad (13)$$

if the uncertainty of D_L is independent with the uncertainties of the other GW parameters. Concerning the uncertainty budget, following the strategy described by Cai and Yang [27], the distance precision per GW is taken as $\sigma_{D_L}^2 = (\sigma_{D_L}^{\text{inst}})^2 + (\sigma_{D_L}^{\text{lens}})^2$. In the simplified case of $\iota \simeq 0$, the estimate of the uncertainty of D_L expresses as $\sigma_{D_L}^{\text{inst}} \simeq \frac{2D_L}{\rho}$ [58], while the lensing uncertainty caused by the weak lensing is modeled as $\sigma_{D_L}^{\text{lens}}/D_L = 0.05z$ [27,58,59]. Thus, the total uncertainty of D_L is taken to be

$$\begin{aligned} \sigma_{D_L} &= \sqrt{(\sigma_{D_L}^{\text{inst}})^2 + (\sigma_{D_L}^{\text{lens}})^2} \\ &= \sqrt{\left(\frac{2D_L}{\rho}\right)^2 + (0.05zD_L)^2}. \end{aligned} \quad (14)$$

Finally, we adopt the redshift distribution of the GW sources observed on Earth, which can be written as [59]

$$P(z) \propto \frac{4\pi D_c^2(z)R(z)}{H(z)(1+z)}, \quad (15)$$

where $H(z)$ is the Hubble parameter of the fiducial cosmological model, $D_c = \int_0^z 1/H(z)dz$ is the corresponding comoving distance at redshift z , and $R(z)$ represents the time evolution of the burst rate taken as [60,61]

$$R(z) = \begin{cases} 1 + 2z, & z \leq 1 \\ \frac{3}{4}(5 - z), & 1 < z < 5 \\ 0, & z \geq 5. \end{cases} \quad (16)$$

The final key question required to be answered is: *how many GW events can be detected per year for the ET?* Focusing on the GW sources caused by binary merger of neutron stars with either neutron stars or black holes, recent analysis [27] revealed that the third generation ground-based GW detector can detect up to 1000 GW events in a

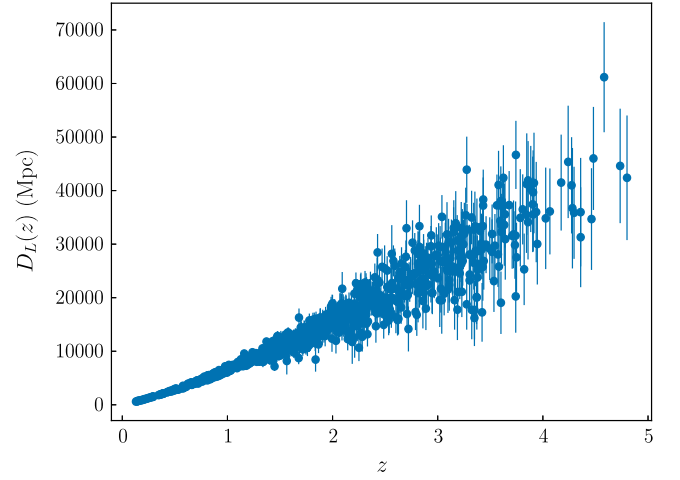


FIG. 2. The luminosity distance measurements from 1000 observed GW events.

10-year observation (with detectable EM counterpart measurable source redshift). Therefore, assuming the luminosity distance measurements obey the Gaussian distribution $D_L^{\text{mean}} = \mathcal{N}(d_L^{\text{fid}}, \sigma_{d_L})$, we simulate 1000 GW events used for statistical analysis in the next section, the redshift distribution of which is shown in Fig. 2.

III. CONSTRAINTS AND RESULTS

From the theoretical point of view, in order to directly test the DDR from GW + EM observations, our analysis will be based on one constant parametrization and two parametric representations for possible redshift dependence of the distance duality expression [13,62], namely,

$$\eta(z) = \frac{D_L}{D_A} (1+z)^{-2}, \quad (17)$$

and

$$\begin{aligned} \eta(z, \eta_0) &= 1 + \eta_0, \\ \eta(z, \eta_1) &= 1 + \eta_1 z, \\ \eta(z, \eta_2) &= 1 + \eta_2 \frac{z}{1+z} \end{aligned} \quad (18)$$

where η_0 , η_1 , and η_2 are constant parameters, the likelihood of which is expected to peak at zero in order to satisfy the DD relation. Such parametrizations are clearly inspired on similar expressions for the equation of state (w) in different dark energy models [63], i.e., the Λ CDM model (where the equation of state parameter for dark energy is a constant) and time-varying dark energy models (where the w parametrizations stem from the first order Taylor expansions in redshift z). Note that the first two expressions are continuous and smooth linear expansion, while the last one may effectively avoid the possible divergence at high redshifts. More importantly, it should be noted that the deviations from DDR may point to a nonmetric spacetime structure

(since the DDR depends only on Lorentzian spacetime geometry apart from geometrical photon conservation), which could possibly generate drastic effects on light propagation [64]. The η parametrizations in Eq. (18) are sufficient to account for such nonmetric behavior at leading orders, which is supported by the recent discussion of the DDR on nonmetric backgrounds [64].

From the observational point of view, for a given D_A^{QSO} data point, in order to check the validity of the DDR, the luminosity distance from an associated GW data point D_L^{GW} should be observed at the same redshift. Following the recent analysis of Cao and Liang [65], the testing results of the DDR may be influenced by the particular choice of the selection criteria for a given pair of data sets; i.e., the choice of δz may play an important role in this model-independent test. The redshifts of GW sample are carefully chosen to coincide with the ones of the associated quasar sample, which may hopefully ease the systematic errors brought by redshift inconsistency between GW and EM observations. More specifically, in our analysis, a selection criterion that bins D_L measurements from GW within the redshift range $|z_{\text{QSO}} - z_{\text{GW}}| < 0.005$ is adopted to get D_L at the redshift of QSO. If D_{L_i} represents the i th appropriate GW luminosity distance with $\sigma_{D_{L_i}}$ denoting its reported observational uncertainty, the weighted mean luminosity distance \bar{D}_L at the QSO redshift and its corresponding uncertainty $\sigma_{\bar{D}_L}$ can be obtained by the standard data reduction framework [66]: $\bar{D}_L = \frac{\sum (D_{L_i}/\sigma_{D_{L_i}}^2)}{\sum 1/\sigma_{D_{L_i}}^2}$, and $\sigma_{\bar{D}_L}^2 = \frac{1}{\sum 1/\sigma_{D_{L_i}}^2}$. Subsequently, the observed $\eta_{\text{obs}}(z)$ can be expressed as

$$\eta_{\text{obs}}(z) = \frac{\bar{D}_L}{D_A} (1+z)^{-2}, \quad (19)$$

and the corresponding statistical error is given by

$$\sigma_{\eta_{\text{obs}}}^2 = \frac{\sigma_{\bar{D}_L}^2}{D_A^2} (1+z)^{-4} + \frac{\bar{D}_L^2}{D_A^4} \sigma_{D_A}^2 (1+z)^{-4}. \quad (20)$$

The likelihood estimator is determined by χ^2 statistics

$$\chi^2(\eta_j) = \sum_i \frac{[\eta(z, \eta_j) - \eta_{i,\text{obs}}(z)]^2}{\eta_{i,\text{obs}}^2} \quad (21)$$

for the three different parametrizations $j = 0, 1, 2$.

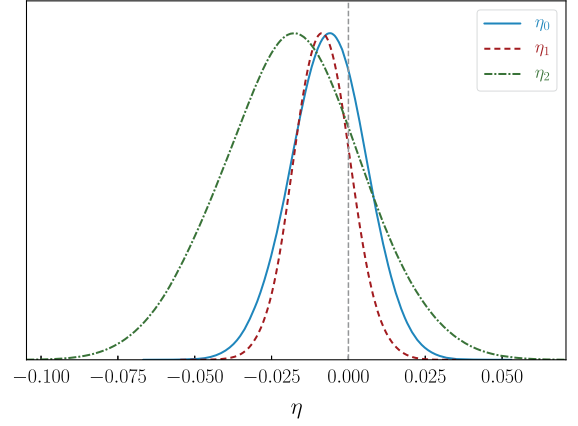


FIG. 3. The likelihood distributions of η_0 , η_1 , and η_2 from the current QSO data and simulated GW data.

To get reasonable D_A , we firstly turn to the recent catalog by Cao *et al.* [41] that contains 120 intermediate-luminosity quasars, with redshifts ranging from 0.46 to 2.80, all observed with Very Large Baseline Interferometry (VLBI). Considering the uncertainties in D_A encountered previously [38], we include the statistical error of observations in $\theta(z)$ and an additional 10% systematical uncertainty accounting for the intrinsic spread in the linear size. The GW data are carefully selected whose redshift is closest to the quasar's redshift, demanding that the difference in redshift is smaller than 0.005. Combining these quasar data together with the GW estimate of the luminosity distances, for model I we obtain the best-fit value $\eta_0 = -0.007 \pm 0.012$ at 68.3% confidence level and plot the likelihood distribution function in Fig. 3. Working on the other two parametrization forms of the DD relation: $\eta(z) = 1 + \eta_1 z$ and $\eta(z) = 1 + \eta_2 z / (1+z)$, the best-fit values are $\eta_1 = -0.0086 \pm 0.0093$ and $\eta_2 = -0.018 \pm 0.023$ at 68.3% confidence level. The results are summarized on Table I and are depicted on Fig. 3, which indicate that $\eta < 1$ tends to be slightly favored by all three parametrizations of $\eta(z)$. Such tendency has been previously noted and extensively discussed in the literature [18,65,67]. We remark here that, compared with the previous works based on observations of D_A on large angular scales (galaxy clusters [62,65,67], BAO [5], galaxy strong lensing systems [18]), using such different technique (compact structure in radio quasars) to estimate D_A opens the interesting possibility to test

TABLE I. Summary of the best-fit values for the DDR parameter obtained from different observations.

Data	η_0 (Model I)	η_1 (Model II)	η_2 (Model III)
QSO (Cur) + GW (Sim) [this work]	-0.007 ± 0.012	-0.0086 ± 0.0093	-0.018 ± 0.023
QSO (Sim) + GW (Sim) [this work]	0.0002 ± 0.0029	-0.0004 ± 0.0018	-0.0007 ± 0.0051
Union2 + galaxy cluster [65]	$-0.03^{+0.05}_{-0.06}$	$-0.01^{+0.15}_{-0.16}$	$-0.01^{+0.21}_{-0.24}$
Union2.1 + BAOs [5]	-0.009 ± 0.033	0.027 ± 0.064	0.039 ± 0.099
Union2.1 + f_g as [67]	□	$-0.08^{+0.11}_{-0.10}$	□
JLA + strong lensing [18]	□	$-0.005^{+0.351}_{-0.215}$	□

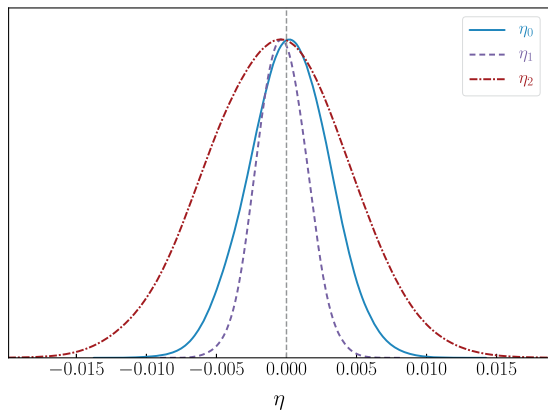


FIG. 4. The likelihood distributions of η_0 , η_1 and η_2 from the simulated QSO and GW data.

the fundamental relations in the early universe ($z \sim 3$). Moreover, it is necessary to compare our results with those of earlier studies using alternative probes at high redshifts (GRBs and SGL systems). More recently, Yang *et al.* [68] tested the DD relation with current strong lensing observations [16] and future luminosity distances from gravitational waves sources, with the final conclusion that the DD relation can be accommodated at 1σ (C.L.). In our analysis, in the framework of model-independent methods testing the DDR, the current compiled quasar sample may achieve constraints with much higher precision of $\Delta\eta = 10^{-2}$.

On the other hand, we also pin our hope on the VLBI observations of more compact radio quasars with higher angular resolution based on better uv-coverage. In order to compare with previous results from the current quasar sample, we also derive the testing results from simulated QSO and GW data in Table I, with the best-fit values of the η parameter in the three DDR models: $\eta_0 = 0.0002 \pm 0.0029$ for model I, $\eta_1 = -0.0004 \pm 0.0018$ for model II, and $\eta_2 = -0.0007 \pm 0.0051$ for model III. The corresponding likelihood distribution function from three one-parameter forms of DDR parametrizations are also shown in Fig. 4. Furthermore, the future VLBI observations of ILQSO combined with the simulated data of GWs using the Einstein Telescope (ET) could extend the test of DDR to much higher redshifts (i.e., $z \sim 5$). More importantly, one can clearly see that the future compiled quasar data improves the constraints on model parameters significantly. With the confrontation between the angular diameter distance (ADD) from quasars and luminosity distance (LD) from GWs, one can expect the validity of the distance duality relation to be confirmed at the precision of $\Delta\eta = 10^{-3}$.

Now it is worthwhile to make some comments on the results obtained above. As was commented earlier, the cosmic opacity caused by the absorption or scattering effects of dust in the Universe might contribute to the possible violation of DDR. In particular, the latest observations of SN Ia, which strongly support the accelerated expansion of the Universe may be affected by the dust in

their host galaxies and Milky Way [3,4,69]. However, it should be emphasized that the luminosity distance derived from waveform and amplitude of the gravitational waves observations is insensitive to nonconservation of the number of photons [68]. Therefore, the method proposed in our analysis opens an interesting possibility to probe exotic physics in the theory of gravity [2], as can be seen from possible deviation from the standard distance duality relation.

IV. CONCLUSIONS

In this paper, we have discussed a new model-independent cosmological test for the distance duality relation. For D_L we consider the simulated data of gravitational waves from the third-generation gravitational wave detector (ET), which can be considered as standard siren, while the angular diameter distances D_A are derived from the newly-compiled sample of compact radio quasars observed by VLBI, which represents a type of new cosmological standard ruler. This creates a valuable opportunity to directly test DDR at much higher precision with the combination of gravitational wave (GW) and electromagnetic (EM) signals. In order to obtain a more reliable result of testing the DDR from GW + EM observations, we use one constant parametrization ($\eta(z) = 1 + \eta_0$) and two parametric representations for possible redshift dependence of the distance duality expression ($\eta(z) = 1 + \eta_1 z$, $\eta(z) = 1 + \eta_2 z/(1+z)$). The redshifts of GW sample are carefully chosen to coincide with the ones of the associated quasar sample, which may hopefully ease the systematic errors brought by redshift inconsistency between GW and EM observations. More specifically, in our analysis, a selection criterion that bins D_L measurements from GW within the redshift range $|z_{\text{QSO}} - z_{\text{GW}}| < 0.005$ is adopted to get D_L at the redshift of QSO.

Firstly of all, we turn to the recent catalog by Cao *et al.* [41] that contains 120 intermediate-luminosity quasars with redshifts ranging from 0.46 to 2.80, all observed with Very Large Baseline Interferometry (VLBI). Combining these quasar data together with the GW estimate of the luminosity distances, we obtain the best-fit value $\eta_0 = -0.007 \pm 0.012$, $\eta_1 = -0.0086 \pm 0.0093$ and $\eta_2 = -0.018 \pm 0.023$ at 68.3% confidence level, which indicate that $\eta < 1$ tends to be slightly favored by all three parametrizations of $\eta(z)$. In the framework of model-independent methods testing the DDR, the current compiled quasar sample may achieve constraints with much higher precision of $\Delta\eta = 10^{-2}$. Moreover, compared with the previous works based on observations of D_A on large angular scales (galaxy clusters, BAO, galaxy strong lensing systems), using such different technique (compact structure in radio quasars) to estimate D_A opens the interesting possibility to test the fundamental relations in the early universe ($z \sim 3$). Therefore, the spacetime characterized by a metric theory and that the light propagates along null geodesics is strongly supported

by the available observations. This is the most unambiguous result of the current data sets.

Working on more simulated compact radio quasars with higher angular resolution based on better uv-coverage, our results show that the future VLBI observations of ILQSO combined with the simulated data of GWs using the Einstein Telescope (ET) could extend the test of DDR to much higher redshifts (i.e., $z \sim 5$). More importantly, one can expect the validity of the DDR to be confirmed at the precision of $\Delta\eta = 10^{-3}$. Since the luminosity distances obtained from GW observations are insensitive to the nonconservation of photon number, any deviation from the standard distance duality relation can be explained as possible existence of exotic physics in the gravity theory. This encourages us to expect the possibility of testing DDR at much higher precision in the future, which reinforces the interest in the observational search for more quasar samples and GW events with smaller statistical and systematic uncertainties.

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