

Inflationary universe in $F(R)$ gravity with antisymmetric tensor fields and their suppression during its evolution

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The intriguing question, why the present scale of the universe is free from any perceptible footprints of rank-2 antisymmetric tensor fields (generally known as Kalb-Ramond fields), is addressed. A quite natural explanation of this issue is given from the angle of higher-curvature gravity, both in four- and in five-dimensional spacetime. The results here obtained reveal that the amplitude of the Kalb-Ramond field may be actually large and play a significant role during the early universe, while the presence of higher-order gravity suppresses this field during the cosmological evolution, so that it eventually becomes negligible in the current universe. Besides the suppression of the Kalb-Ramond field, the extra degree of freedom in $F(R)$ gravity, usually known as scalaron, also turns out to be responsible for inflation. Such $F(R)$ gravity with Kalb-Ramond fields may govern the early universe to undergo an inflationary stage at early times (with the subsequent graceful exit) for a wider range of $F(R)$ gravity than without antisymmetric fields. Furthermore, the models—in four- and five-dimensional spacetimes—are linked to observational constraints, with the conclusion that the corresponding values of the spectral index and tensor-to-scalar ratio closely match the values provided by the Planck survey 2018 data.

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I. INTRODUCTION

A surprising feature of the present universe is that it carries no noticeable footprints of higher rank (rank two or higher) antisymmetric tensor fields. Apart from being the massless (1,1) representation of the Lorentz group, such fields also arise naturally as closed string modes [1] and, consequently, are of considerable interest in string theory.

In this context, the second rank antisymmetric tensor fields, generally known as Kalb-Ramond (KR) fields [2], have drawn considerable attention and have been extensively studied. However, dimensional analysis demands that the coupling strength of the KR field to other matter fields should go as $1/M_p$ (M_p being the four-dimensional Planck mass), i.e., share the same dimensional coupling as the graviton. In spite of this, the large scale behavior of the present universe appears to be governed solely by gravity and there is no experimental evidence of any second-rank antisymmetric Kalb-Ramond field being present. Therefore, if the KR field exists at all, it clearly must be severely suppressed at the present scale of our universe.

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This raises a natural question: why are the effects of the KR field less perceptible than the force of gravitation? Some attempts have been made to solve this puzzle both in four-dimensional as well as in higher dimensional braneworld models [3–7]. In the context of higher dimensional models, the warping geometrical nature of extra dimensions causes a huge suppression of the amplitude of the bulk for the KR field on our visible 3-brane.

However, the suppression of the KR field still awaits a proper understanding in the context of cosmology. On the other hand, it has been shown that the energy density associated with the KR field is large and might play a relevant role in the early universe. This fact, along with present day observations, have inspired us to study the cosmological evolution of the KR field from the very early universe, where it is also crucial to investigate whether the universe goes through an inflationary stage or not. In particular, we are interested in analyzing the possible evolution of the Kalb-Ramond field from the very early stages of the universe, and whether a suppression of this field can be achieved, in order to satisfy the observational constraints that we have at present. In addition, we also intend here to study whether inflation can still be realised in the presence of the KR field, including its compulsory graceful exit. The values of the spectral index, n_s , and the tensor-to-scalar ratio, r , are obtained and compared to the most recent Planck data available [8]. The present paper is a serious attempt to provide a natural explanation of the questions above in the framework of $F(R)$ gravity, both in four- and in five-dimensional spacetimes.

It is well known that the Einstein-Hilbert term can be generalized to include higher order curvature terms in the gravitational action, which naturally arise from the diffeomorphism property of the action. Such higher order curvature terms may have their origin in string theory, such that they naturally arise in the gravitational action [9]. $F(R)$ gravity [10–25], Gauss-Bonnet (GB) [26–29] or more generally Lanczos-Lovelock gravity [30,31] are some of the well known higher order curvature gravitational theories. While GB or Lanczos-Lovelock gravity have nontrivial consequences besides in higher dimensions, $F(R)$ gravity survives even in the four-dimensional spacetime model. For some choices of $F(R)$ [for which $F'(R) > 0$], the corresponding model becomes free of ghosts.

On the other hand, over the past two decades, models with extra spatial dimensions [32–38] have been increasingly playing a central role in physics beyond the standard model of particle physics [39] and cosmology [40–44]. In all such models our visible universe is identified with a 3-brane embedded within a higher dimensional spacetime. Among all, the so-called Randall-Sundrum (RS) model [34] has gained special attention as it solves the gauge hierarchy problem without introducing any intermediate scale (between Planck and TeV) in the theory. The RS scenario assumes one extra spatial dimension (in addition to the usual three spatial dimensions) with S^1/Z_2

orbifolding where the orbifolded fixed points are identified with two 3-branes. The intermediate region between the branes is fixed as a bulk which has a curvature of Planck order. In such higher order curvature regime, $F(R)$ gravity is supposed to play a relevant role. However all the higher dimensional braneworld scenarios demand a certain mechanism for stabilization of interbrane separation, also known as modulus or radion [45–48]. Here, we show that higher order curvature degree(s) of freedom (d.o.f.) can generate a potential term for the radion field and fulfill the purpose of modulus stabilization. Keeping this in mind, here we try to address the cosmological evolution of the antisymmetric Kalb-Ramond field in $F(R)$ gravity by analyzing two frameworks: the KR field in four dimensions and in a higher dimensional bulk spacetime.

The paper is organized as follows: in Sec. II we briefly describe the equivalence between the $F(R)$ model and scalar-tensor (ST) theory in D dimensions. Sections III and IV are devoted to the analysis of the cosmological evolution of the KR field in four- and five-dimensional spacetime in $F(R)$ gravity, respectively. The paper ends with some conclusive remarks and discussions in Sec. V.

II. $F(R)$ GRAVITY AND ITS SCALAR-TENSOR COUNTERPART IN D -DIMENSIONS

In this section, we briefly describe $F(R)$ gravity in D -dimensions and its conformal picture in the Einstein frame, which results in the Hilbert-Einstein action with the presence of a scalar field. The $F(R)$ action can be written as follows:

$$S = \int d^D x \sqrt{-G} \left[\frac{F(R)}{2\kappa^2} \right], \quad (2.1)$$

where G is the determinant of D -dimensional metric G_{MN} (M, N runs from 0 to $D - 1$), R is the D -dimensional Ricci scalar and $\frac{1}{2\kappa^2} = M^{D-2}$ with M is the D -dimensional Planck mass. By introducing an auxiliary field $A(x)$, the action (2.1) can be rewritten as

$$S = \int d^D x \sqrt{-G} \frac{1}{2\kappa^2} [F'(A)(R - A) + F(A)]. \quad (2.2)$$

The variation of this action over the auxiliary field $A(x)$ leads to $A = R$, which finally results in the original action (2.1). Moreover, the action (2.2) can be mapped into the Einstein frame by applying the following conformal transformation on the metric $G_{MN}(x)$:

$$G_{MN}(x) \rightarrow \tilde{G}_{MN}(x) = e^{-\left[\sqrt{\frac{4}{(D-1)(D-2)\kappa^2}} \xi \right]} G_{MN}(x), \quad (2.3)$$

where $\xi(x)$ is the conformal factor which is related to the auxiliary field as $F'(A) = e^{-\left[\sqrt{\frac{D-2}{D-1}\kappa^2} \xi \right]}$, while R and \tilde{R} are the Ricci scalars in terms of the metrics G_{MN} and \tilde{G}_{MN} respectively, such that they are related by

$$R = e^{-\left[\sqrt{\frac{4}{(D-1)(D-2)\kappa\xi}}\right]} \left[\tilde{R} - \kappa^2 \tilde{G}^{MN} \partial_M \xi \partial_N \xi + 2\kappa \sqrt{\frac{D-1}{D-2}} \tilde{\square} \xi \right],$$

where $\tilde{\square}$ represents the d'Alembertian operator formed by \tilde{G}_{MN} . Using the above expression along with the aforementioned relation among $\xi(x)$ and $F'(A)$, the following scalar-tensor action is achieved:

$$S = \int d^D x \sqrt{-\tilde{G}} \left[\frac{\tilde{R}}{2\kappa^2} - \frac{1}{2} \tilde{G}^{MN} \partial_\mu \xi \partial_\nu \xi - \left(\frac{AF'(A) - F(A)}{2\kappa^2 F'(A)^{D/(D-2)}} \right) \right]. \quad (2.4)$$

Note that the field $\xi(x)$ acts as a scalar field with the potential $\frac{AF'(A) - F(A)}{2\kappa^2 F'(A)^{D/(D-2)}} [= V(A(\xi))]$. Thus, the higher order curvature d.o.f. manifests itself as a scalar field d.o.f. $\xi(x)$ with the potential $V(\xi)$, which actually depends on the form of $F(R)$.

III. KALB-RAMOND FIELD IN FOUR DIMENSIONS IN $F(R)$ GRAVITY

Let us first consider a four-dimensional spacetime in $F(R)$ gravity. As mentioned earlier, here we are interested in how the higher order terms affect the dynamical evolution of a second rank antisymmetric tensor field, generally known as the Kalb-Ramond field ($B_{\mu\nu}$). Therefore the action of the model is given by

$$S = \int d^4 x \sqrt{-g} \left[\frac{F(R)}{2\kappa^2} - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} \right] = \int d^4 x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{R^2}{m^2} \right) - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} \right]. \quad (3.1)$$

Here we are assuming a particular form for gravitational sector, the so-called Starobinsky model [49], $F(R) = R + \frac{R^2}{m^2}$, where m is a parameter having mass dimension and $\frac{1}{2\kappa^2} = M_{(4)}^2$ ($M_{(4)}$ being the four-dimensional Planck mass). Moreover, $H_{\mu\nu\alpha}$ is the field strength tensor of the Kalb-Ramond (KR) field, defined by $H_{\mu\nu\alpha} = \partial_{[\mu} B_{\nu\alpha]}$. As we may notice, $H_{\mu\nu\alpha}$ is invariant under the KR gauge transformation: $B_{\mu\nu} \rightarrow B_{\mu\nu} + \partial_{[\mu} \omega_{\nu]}$ and thereby the action turns out also invariant under such a transformation.

Using the conformal transformation from Sec. II, the action (3.1) can be expressed as a scalar-tensor theory:

$$S = \int d^4 x \sqrt{-\tilde{g}} \left[\frac{\tilde{R}}{2\kappa^2} - \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \xi \partial_\nu \xi - V(\xi) - \frac{1}{12} e^{-\sqrt{\frac{4}{3}\kappa\xi}} H_{\mu\nu\rho} H_{\alpha\beta\delta} \tilde{g}^{\mu\alpha} \tilde{g}^{\nu\beta} \tilde{g}^{\rho\delta} \right], \quad (3.2)$$

where the scalar potential $V(\xi)$ has the following expression:

$$V(\xi) = \frac{m^2}{8\kappa^2} (1 - e^{\sqrt{\frac{4}{3}\kappa\xi}})^2. \quad (3.3)$$

The potential has a stable minima at $\langle \xi \rangle = 0$ and asymptotically reaches $\frac{m^2}{8\kappa^2}$ as ξ goes to $-\infty$. Figure 9 depicts the form of the potential $V(\xi)$. From Eq. (3.2), it is straightforward to show that the kinetic term of the KR field becomes noncanonical because of the presence of the scalar field $\xi(x)$. In order to make the KR field canonical, we redefine the field as follows:

$$B_{\mu\nu} \rightarrow \tilde{B}_{\mu\nu} = e^{-\frac{1}{2}\sqrt{\frac{4}{3}\kappa\xi}} B_{\mu\nu}. \quad (3.4)$$

Then, the final form of the scalar-tensor action can be expressed as follows:

$$S = \int d^4 x \sqrt{-\tilde{g}} \left[\frac{\tilde{R}}{2\kappa^2} - \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \xi \partial_\nu \xi - V(\xi) - \frac{1}{12} \tilde{H}_{\mu\nu\rho} \tilde{H}^{\mu\nu\rho} \right], \quad (3.5)$$

where we consider $\kappa \tilde{B}_{\mu\nu} < 1$ and $V(\xi)$ is obtained from Eq. (3.3). In the following, we determine the solutions of the cosmological field equations for the scalar-tensor model [see Eq. (3.5)], from which one can extract the corresponding solutions for the original $F(R)$ model (3.1) by taking the inverse conformal transformation.

A. Cosmological field equations and solutions in the scalar-tensor representation

In order to obtain the field equations of the scalar-tensor (ST) action (3.5), first we have to obtain the energy-momentum tensor for $\xi(x)$ and $\tilde{B}_{\mu\nu}(x)$,

$$T_{\mu\nu}[\xi] = \frac{2}{\sqrt{-\tilde{g}}} \frac{\delta}{\delta \tilde{g}^{\mu\nu}} \left[\sqrt{-\tilde{g}} \left(\frac{1}{2} \tilde{g}^{\alpha\beta} \partial_\alpha \xi \partial_\beta \xi + V(\xi) \right) \right] = \partial_\mu \xi \partial_\nu \xi - \tilde{g}_{\mu\nu} \left(\frac{1}{2} \tilde{g}^{\alpha\beta} \partial_\alpha \xi \partial_\beta \xi + V(\xi) \right), \quad (3.6)$$

and

$$T_{\mu\nu}[\tilde{B}] = \frac{2}{\sqrt{-\tilde{g}}} \frac{\delta}{\delta \tilde{g}^{\mu\nu}} \left[\frac{1}{12} \sqrt{-\tilde{g}} \tilde{g}^{\mu\alpha} \tilde{g}^{\nu\beta} \tilde{g}^{\lambda\gamma} \tilde{H}_{\mu\nu\lambda} \tilde{H}_{\alpha\beta\gamma} \right] = \frac{1}{6} \left[3 \tilde{g}_{\nu\rho} \tilde{H}_{\alpha\beta\mu} \tilde{H}^{\alpha\beta\rho} - \frac{1}{2} \tilde{g}_{\mu\nu} \tilde{H}_{\alpha\beta\gamma} \tilde{H}^{\alpha\beta\gamma} \right]. \quad (3.7)$$

Here we are interested in the cosmological evolution of the KR field. For that purpose, we assume the ansatz of a flat Friedmann-Lemaître-Robertson-Walker (FLRW) metric:

$$\begin{aligned} \tilde{d}s^2 &= \tilde{g}_{\mu\nu}(x) dx^\mu dx^\nu \\ &= -dt^2 + a^2(t) [dx^2 + dy^2 + dz^2], \end{aligned} \quad (3.8)$$

where t and $a(t)$ are the cosmic time and the scale factor respectively. However before obtaining the field equations, we would like to emphasize that $\tilde{H}_{\mu\nu\lambda}$ has four independent components in four-dimensional spacetimes due to its antisymmetric nature, such that they can be expressed as

$$\begin{aligned}\tilde{H}_{012} &= h_1, & \tilde{H}^{012} &= h^1, \\ \tilde{H}_{013} &= h_2, & \tilde{H}^{013} &= h^2, \\ \tilde{H}_{023} &= h_3, & \tilde{H}^{023} &= h^3, \\ \tilde{H}_{123} &= h_4, & \tilde{H}^{123} &= h^4.\end{aligned}\quad (3.9)$$

As the KR field tensor $\tilde{H}_{\mu\nu\alpha}$ owns four independent components, it can be equivalently expressed as a vector field (which has also four independent components in four dimensions) [50], $\tilde{H}_{\mu\nu\alpha} = \varepsilon_{\mu\nu\alpha\beta} \Upsilon^\beta$, with Υ^β being the vector field.

Moreover, Eq. (3.9) together with the expressions for the energy-momentum tensor and the FLRW metric lead to the off-diagonal Friedmann equations as follows [23]:

$$h_4 h^3 = h_4 h^2 = h_4 h^1 = h_2 h^3 = h_1 h^3 = h_1 h^2 = 0, \quad (3.10)$$

where the fields are considered homogeneous. The above set of equations has the following solution:

$$h_1 = h_2 = h_3 = 0, \quad h_4 \neq 0. \quad (3.11)$$

Using this solution, one easily obtains the total energy density and pressure for the matter fields (ξ , $\tilde{B}_{\mu\nu}$), which become $\rho_T = [\frac{1}{2}\dot{\xi}^2 + V(\xi) + \frac{1}{2}h_4 h^4]$ and $p_T = [\frac{1}{2}\dot{\xi}^2 - V(\xi) + \frac{1}{2}h_4 h^4]$ respectively (where the dot denotes $\frac{d}{dt}$). As a result, the diagonal Friedmann equations turn out to be

$$H^2 = \frac{\kappa^2}{3} \left[\frac{1}{2}\dot{\xi}^2 + \frac{m^2}{8\kappa^2} \left(1 - e^{\sqrt{\frac{2}{3}}\kappa\xi}\right)^2 + \frac{1}{2}h_4 h^4 \right], \quad (3.12)$$

and

$$2\dot{H} + 3H^2 + \kappa^2 \left[\frac{1}{2}\dot{\xi}^2 - \frac{m^2}{8\kappa^2} \left(1 - e^{\sqrt{\frac{2}{3}}\kappa\xi}\right)^2 + \frac{1}{2}h_4 h^4 \right] = 0, \quad (3.13)$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter. In order to obtain the above equations, we have used the explicit expression of $V(\xi)$ as shown in Eq. (3.3). Furthermore, the field equations for the KR field ($\tilde{B}_{\mu\nu}$) and the scalar field (ξ) are given by

$$\tilde{\nabla}_\mu \tilde{H}^{\mu\nu\lambda} = \frac{1}{a^3(t)} \partial_\mu [a^3(t) \tilde{H}^{\mu\nu\lambda}] = 0, \quad (3.14)$$

and

$$\ddot{\xi} + 3H\dot{\xi} - \sqrt{\frac{2}{3}} \frac{m^2}{4\kappa} e^{\sqrt{\frac{2}{3}}\kappa\xi} (1 - e^{\sqrt{\frac{2}{3}}\kappa\xi}) = 0. \quad (3.15)$$

From Eq. (3.14), we know that the nonzero component of $\tilde{H}_{\mu\nu\alpha}$ (i.e., $\tilde{H}_{123} = h_4$) depends on t only (see Appendix A for the derivation), which is also expected from the gravitational field equations. Differentiating both sides (with respect to t) of Eq. (3.12), one easily obtains

$$6H\dot{H} = \kappa^2 \left[\dot{\xi}\ddot{\xi} - \sqrt{\frac{2}{3}} \frac{m^2}{4\kappa} e^{\sqrt{\frac{2}{3}}\kappa\xi} (1 - e^{\sqrt{\frac{2}{3}}\kappa\xi}) \dot{\xi} + \frac{1}{2} \frac{d}{dt}(h_4 h^4) \right].$$

Furthermore, Eqs. (3.12) and (3.13) lead to the expression $2\dot{H} = -\kappa^2 [\dot{\xi}^2 + h_4 h^4]$. Substituting this expression of \dot{H} in the above equation and using the scalar field equation of motion, we obtain the following cosmic evolution for $h_4 h^4$:

$$\frac{d}{dt}(h_4 h^4) = -6Hh_4 h^4. \quad (3.16)$$

Solving (3.16), we get

$$h_4 h^4 = \frac{h_0}{a^6}, \quad (3.17)$$

with h_0 being an integration constant which must be taken positive in order to ensure a real solution for $h^4(t)$. Recall that the term $\frac{1}{2}h_4 h^4$ represents the energy density contribution from the Kalb-Ramond field. Therefore, Eq. (3.17) clearly indicates that the KR field energy density ($\tilde{\rho}_{\text{KR}}$) is proportional to $1/a^6$ and as a result, $\tilde{\rho}_{\text{KR}}$ gradually decreases with the expansion of the universe. However, Eq. (3.17) also shows that KR energy density is large and may play a significant role in the early universe (when the scale factor is small in comparison to the present one). Therefore, in order to address the dynamical suppression of the KR field, we should study the dynamics of the KR field from the very early universe when it is also important to evaluate whether the early universe undergoes an accelerating stage or not, i.e., an inflationary phase. To check this phenomena, we need to obtain the form of the scale factor at early times.

From the solution $h_4 h^4$ in terms of the scale factor, two independent equations remained,

$$H^2 = \frac{\kappa^2}{3} \left[\frac{1}{2}\dot{\xi}^2 + \frac{m^2}{8\kappa^2} \left(1 - e^{\sqrt{\frac{2}{3}}\kappa\xi}\right)^2 + \frac{1}{2} \frac{h_0}{a^6} \right], \quad (3.18)$$

and

$$\ddot{\xi} + 3H\dot{\xi} - \sqrt{\frac{2}{3}} \frac{m^2}{4\kappa} e^{\sqrt{\frac{2}{3}}\kappa\xi} (1 - e^{\sqrt{\frac{2}{3}}\kappa\xi}) = 0. \quad (3.19)$$

Here, we should mention that Eqs. (3.18) and (3.19) match the field equations when $\tilde{H}_{\mu\nu\alpha}$ is expressed in the vector

representation, i.e., $\tilde{H}_{\mu\nu\alpha} = \varepsilon_{\mu\nu\alpha\beta}\Upsilon^\beta$ (see Appendix B for the derivation of this equivalence). This confirms the equivalence between the two representations at the level of the equations of motion, which is also in agreement with Ref. [50].

However, Eqs. (3.18) and (3.19) are sufficient to determine the evolution of two unknown functions: the scale factor $a(t)$ and the scalar field $\xi(t)$. As mentioned above, we are interested in solving the field equations during the initial phase of the universe where the potential energy of the scalar field is assumed to be greater than that of the kinetic energy, known as the slow-roll approximation, i.e.,

$$V(\xi) \gg \frac{1}{2}\dot{\xi}^2. \quad (3.20)$$

Under such an approximation, Eqs. (3.18) and (3.19) become

$$H^2 = \frac{\kappa^2}{3} \left[\frac{m^2}{8\kappa^2} \left(1 - e^{\sqrt{\frac{2}{3}}\kappa\xi}\right)^2 + \frac{1}{2} \frac{h_0}{a^6} \right], \quad (3.21)$$

and

$$3H\dot{\xi} - \sqrt{\frac{2}{3}} \frac{m^2}{4\kappa} e^{\sqrt{\frac{2}{3}}\kappa\xi} \left(1 - e^{\sqrt{\frac{2}{3}}\kappa\xi}\right) = 0. \quad (3.22)$$

By considering $\frac{\kappa^2 h_0}{m^2} < 1$ (which is also necessary in order to relate the model with the observational constraints, as shown below), Eqs. (3.21) and (3.22) can be written as follows:

$$H = \frac{m}{2\sqrt{6}} \left(1 - e^{\sqrt{\frac{2}{3}}\kappa\xi}\right) \left[1 + \frac{2\kappa^2 h_0}{m^2} \frac{1}{a^6 (1 - e^{\sqrt{\frac{2}{3}}\kappa\xi})^2} \right], \quad (3.23)$$

$$Q(t) = \frac{(5 + 9C(1 + 3C)) \left(1 - \sqrt{\frac{3}{2}} m(t - t_0)\right) + \sqrt{6}(1 + 6C + 81C^2) \left(-m(t - t_0) + \sqrt{\frac{3}{2}} m^2(t - t_0)^2\right)}{\left(1 - \frac{2m(t-t_0)}{3\sqrt{6}C}\right)^{7/2}}. \quad (3.28)$$

Furthermore, C and D are integration constants related to the initial values of $\xi(t)$ and $a(t)$ as follows:

$$\begin{aligned} \xi(t_0) = \xi_0 &= \sqrt{\frac{3}{2\kappa^2}} \left[\ln(1/C) + \frac{\kappa^2 h_0}{9m^2} \frac{C^2}{(1+C)^3} \right], \\ a(t_0) &= D \left[1 + \frac{\kappa^2 h_0}{m^2} (5 + 9C(1 + 3C)) \right]. \end{aligned} \quad (3.29)$$

Note that for $h_0 \rightarrow 0$, both solutions $\xi(t)$ as $a(t)$ go towards the well known Starobinsky solution. Furthermore, Eq. (3.25) leads to the fact that the scalar field increases with time and goes to infinity as $t \rightarrow \frac{9C}{\sqrt{6}m}$. Keeping this in mind, here we consider the initial value of the scalar field (i.e., ξ_0) as negative. The negative initial value of the scalar field is also consistent with the slow-roll condition, as may be noticed from Fig. 1.

and

$$\frac{d\xi}{dt} = \frac{m}{18\kappa} e^{\sqrt{\frac{2}{3}}\kappa\xi} \left[1 - \frac{2\kappa^2 h_0}{m^2} \frac{1}{a^6 (1 - e^{\sqrt{\frac{2}{3}}\kappa\xi})^2} \right], \quad (3.24)$$

where we keep the terms up to the leading order in h_0 . Under the condition $\frac{\kappa^2 h_0}{m^2} < 1$, we can solve the above equations for $\xi(t)$ and $a(t)$ perturbatively where $\frac{\kappa^2 h_0}{m^2}$ is treated as a perturbation parameter. The solutions are (for $m_0 \neq 0$)

$$\xi(t) = \sqrt{\frac{3}{2\kappa^2}} \left[\ln \left(\frac{9}{-\sqrt{6}m(t - t_0) + 9C} \right) + \frac{\kappa^2 h_0}{m^2} P(t) \right], \quad (3.25)$$

and

$$\begin{aligned} a(t) &= D \left(1 - \frac{2m(t - t_0)}{3\sqrt{6}C} \right)^{3/4} \left(1 + \frac{\kappa^2 h_0}{m^2} Q(t) \right) \\ &\times \exp \left[\frac{m(t - t_0)}{2\sqrt{6}} \right]. \end{aligned} \quad (3.26)$$

Here $P(t)$ and $Q(t)$ have the following expressions:

$$P(t) = \frac{(-\sqrt{6}m(t - t_0) + 9C)^2}{(-\sqrt{6}m(t - t_0) + 9C + 9)^3}, \quad (3.27)$$

and

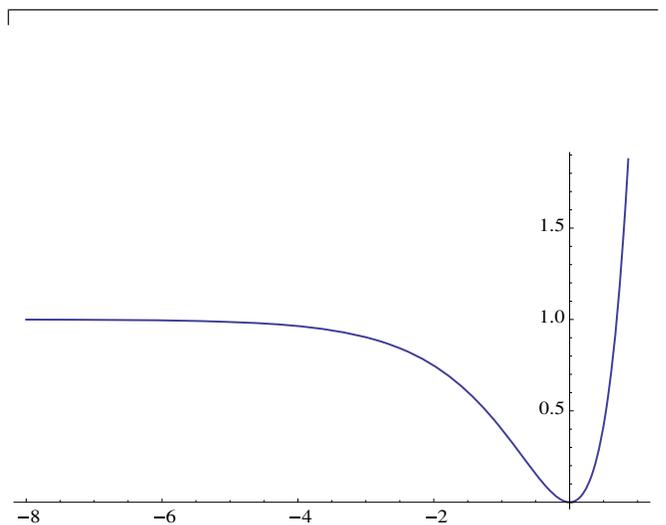


FIG. 1. $V(\xi)$ vs ξ .

1. Beginning of inflation in scalar-tensor model

After obtaining the solution of the scale factor (3.26), we can now analyze whether this form of the scale factor corresponds to an accelerating stage during the early universe (i.e., $t \gtrsim t_0$). For this purpose, we expand $a(t)$ in the form of Taylor series (about $t = t_0$) and keep those terms up to linear order in $t - t_0$:

$$a(t \rightarrow t_0) = D \left(1 - \frac{2m(t-t_0)}{3\sqrt{6}C} \right)^{3/4} \exp \left[\frac{m(t-t_0)}{2\sqrt{6}} \right] \times \left[1 + \frac{\kappa^2 h_0}{m^2} \left((5 + 9C(1 + 3C)) - \frac{m}{\sqrt{6}} (11 + 45C + 513C^2)t \right) \right], \quad (3.30)$$

where we have used the expression of $Q(t)$ at $t \rightarrow t_0$ as $Q(t \rightarrow t_0) = (5 + 9C(1 + 3C)) - \frac{m}{\sqrt{6}} (11 + 45C + 513C^2)t$. Differentiating (twice) both sides of Eq. (3.30) in the limit $t \rightarrow t_0$, one finally gets the following expression for the acceleration:

$$\left. \frac{\ddot{a}}{a} \right|_{t \rightarrow t_0} = \left(\frac{m}{2\sqrt{6}} \right)^2 (1 - e^{-|\sigma_0|}) \left[(1 - e^{-|\sigma_0|}) - 4 \frac{\kappa^2 h_0}{m^2} \left(11 + 45e^{|\sigma_0|} + 513e^{2|\sigma_0|} - \frac{e^{|\sigma_0|}}{18(1 + e^{|\sigma_0|})^3} \right) \right]. \quad (3.33)$$

Note that under the condition

$$\frac{m^2(1 - e^{-|\sigma_0|})}{4(11 + 45e^{|\sigma_0|} + 513e^{2|\sigma_0|} - \frac{e^{|\sigma_0|}}{18(1 + e^{|\sigma_0|})^3})} > \kappa^2 h_0, \quad (3.34)$$

the universe passes through an acceleration phase while it does not when the condition $\frac{m^2(1 - e^{-|\sigma_0|})}{4(11 + 45e^{|\sigma_0|} + 513e^{2|\sigma_0|} - \frac{e^{|\sigma_0|}}{18(1 + e^{|\sigma_0|})^3})} < \kappa^2 h_0$ holds.

Hence, the parameters m and h_0 control the strength of the scalar field and the KR field energy density respectively. Therefore, the interplay among the scalar field and the KR field fixes whether the early universe evolves through an accelerating stage or not. In the next section, we focus again on the cosmological solutions and their possible consequences for the original $F(R)$ model [see Eq. (3.1)] by using the solutions of the corresponding scalar-tensor theory.

B. Cosmological solutions and their possible consequences in the $F(R)$ gravity: Suppression of the Kalb-Ramond field

Recall that the original higher order curvature $F(R)$ model is given by the action (3.1), solutions for the metric can be obtained from the corresponding scalar-tensor

$$\left. \frac{\ddot{a}}{a} \right|_{t \rightarrow t_0} = \left(\frac{m}{2\sqrt{6}} \right)^2 \frac{C-1}{C} \times \left[\frac{C-1}{C} - 4 \frac{\kappa^2 h_0}{m^2} (11 + 45C + 513C^2) \right]. \quad (3.31)$$

By inverting Eq. (3.29), we obtain the explicit expression for the integration constant C in terms of ξ_0 (initial value of the scalar field). For the zeroth order in h_0 , one gets $C^{(0)} = e^{-\sqrt{\frac{2\kappa^2}{3}}\xi_0}$ and up to first order in h_0 , C becomes

$$C = e^{-\sqrt{\frac{2\kappa^2}{3}}\xi_0} + \frac{\kappa^2 h_0}{9m^2} \left(\frac{e^{-3\sqrt{\frac{2\kappa^2}{3}}\xi_0}}{(1 + e^{-\sqrt{\frac{2\kappa^2}{3}}\xi_0})^3} \right) = e^{|\sigma_0|} + \frac{\kappa^2 h_0}{9m^2} \left(\frac{e^{3|\sigma_0|}}{(1 + e^{|\sigma_0|})^3} \right), \quad (3.32)$$

where $\sigma_0 = \sqrt{\frac{2\kappa^2}{3}}\xi_0$ and recall that the initial value of the scalar field is considered to be negative. Then, by using the above expression for C [see Eq. (3.32)], Eq. (3.31) turns out as

theory [see Eqs. (3.25) and (3.26)] with the help of the inverse conformal transformation. Thus, the line element in the $F(R)$ model can be written as

$$ds^2 = e^{\sqrt{\frac{2}{3}}\kappa\xi(t)} [-dt^2 + a^2(t)(dx^2 + dy^2 + dz^2)] = -d\tau^2 + s^2(\tau)(dx^2 + dy^2 + dz^2), \quad (3.35)$$

where $\tau(t)$, $s(\tau)$ are the cosmic time and scale factor respectively in Jordan frame, which are related to the Einstein frame by the conformal transformation:

$$\tau(t) = \int dt e^{\frac{1}{2}\sqrt{\frac{2}{3}}\kappa\xi(t)}, \quad (3.36)$$

and

$$s(\tau(t)) = e^{\frac{1}{2}\sqrt{\frac{2}{3}}\kappa\xi(t)} a(t). \quad (3.37)$$

Equation (3.36) clearly indicates that $\tau(t)$ is a monotonically increasing function of t . However, by integrating Eq. (3.36), one gets the explicit functional form of $\tau(t)$ as follows:

$$\begin{aligned} \tau - \tau_0 = & \frac{1}{4m} \sqrt{\frac{3}{2}} \left[\left(8\sqrt{9C} - 8\sqrt{9C - \sqrt{6m}(t - t_0)} \right), \right. \\ & + \frac{\kappa^2 h_0}{2m^2} \left(\frac{(27 + 45C - 5\sqrt{6m}(t - t_0))\sqrt{9C - \sqrt{6m}(t - t_0)}}{(9 + 9C - \sqrt{6m}(t - t_0))^2} - \frac{(3 + 5C)\sqrt{C}}{3(1 + C)^2} \right), \\ & \left. + \frac{\kappa^2 h_0}{2m^2} \left(\tan^{-1} \left(\frac{3}{\sqrt{9C - \sqrt{6m}(t - t_0)}} \right) - \tan^{-1}(1/\sqrt{C}) \right) \right], \end{aligned} \quad (3.38)$$

where the integration constant [that appeared while integrating Eq. (3.36)] is fixed by the condition $\tau(t_0) = \tau_0$. Moreover, with the solutions of $\xi(t)$ and $a(t)$, Eq. (3.37) immediately leads to the form of $s(\tau)$ [in terms of t , where $\tau(t)$ is given by the above expression] as

$$s(\tau(t)) = D \left(1 - \frac{2m(t - t_0)}{3\sqrt{6C}} \right)^{3/4} \left(1 + \frac{\kappa^2 h_0}{m^2} \left(Q(t) + \frac{1}{2} P(t) \right) \right), \exp \left[\frac{m(t - t_0)}{2\sqrt{6}} + \ln \left(\frac{9}{-\sqrt{6m}(t - t_0) + 9C} \right) \right], \quad (3.39)$$

where $P(t)$ and $Q(t)$ are given by Eqs. (3.27) and (3.28) respectively. However, by using Eq. (3.38), we obtain $\tau(t)$ at $t \rightarrow t_0$,

$$\tau(t \rightarrow t_0) = \tau_0 + \sqrt{\frac{1}{C}} \left[1 + \frac{\kappa^2 h_0}{9m^2} \left(\frac{C^2}{(1 + C)^3} \right) \right] (t - t_0). \quad (3.40)$$

1. Beginning of inflation in $F(R)$ gravity

In this section, we investigate whether the solution of the scale factor [$s(\tau)$, see Eq. (3.39)] corresponds to an inflationary stage of the early universe. In order to analyze this

matter, we expand $s(\tau)$ in the form of a Taylor series (about $\tau = \tau_0$) and keep the terms up to linear order in $\tau - \tau_0$. For this purpose, we need the expression of $\tau(t)$ at $t \rightarrow t_0$, which can be obtained from Eq. (3.38) as

$$\tau(t \rightarrow t_0) = \tau_0 + \sqrt{\frac{1}{C}} \left[1 + \frac{\kappa^2 h_0}{9m^2} \left(\frac{C^2}{(1 + C)^3} \right) \right] (t - t_0). \quad (3.41)$$

Recall that τ goes to τ_0 as $t \rightarrow t_0$, which is also evident from the above expression. Equations (3.39) and (3.41) lead to the expression of the scale factor at $\tau \rightarrow \tau_0$,

$$\begin{aligned} s(\tau \rightarrow \tau_0) = & \frac{D}{\sqrt{C}} \left[1 - \frac{2m\beta(\tau - \tau_0)}{3\sqrt{6C}} \right]^{3/4} \exp \left[\frac{m\beta(\tau - \tau_0)}{2\sqrt{6}} \right] \left[1 + \frac{\kappa^2 h_0}{m^2} \left([5 + 9C(1 + 3C)], \right. \right. \\ & \left. \left. + \frac{C^2}{9(1 + C)^3} - \frac{m\beta}{\sqrt{6}} \left[11 + \frac{1219}{27} C + \frac{13849}{27} C^2 \right] (\tau - \tau_0) \right) \right], \end{aligned} \quad (3.42)$$

with β given by

$$1/\beta = \sqrt{\frac{1}{C}} \left[1 + \frac{\kappa^2 h_0}{9m^2} \left(\frac{C^2}{(1 + C)^3} \right) \right],$$

Differentiating twice both sides with respect to τ , Eq. (3.42) becomes

$$\frac{1}{s} \frac{d^2 s}{d\tau^2} \Big|_{\tau \rightarrow \tau_0} = \beta^2 \left(\frac{m}{2\sqrt{6}} \right)^2 (1 - e^{-|\sigma_0|}) \left[(1 - e^{-|\sigma_0|}) - 4 \frac{\kappa^2 h_0}{m^2} \left(11 + \frac{1219}{27} e^{|\sigma_0|} + \frac{13849}{27} e^{2|\sigma_0|} \right) \right], \quad (3.43)$$

where $C = e^{|\sigma_0|} + \frac{\kappa^2 h_0}{9m^2} \left(\frac{e^{3|\sigma_0|}}{(1 + e^{|\sigma_0|})^3} \right)$ is used. Therefore, it is clear that for

$$\frac{m^2(1 - e^{-|\sigma_0|})}{4 \left(11 + \frac{1219}{27} e^{|\sigma_0|} + \frac{13849}{27} e^{2|\sigma_0|} - \frac{e^{|\sigma_0|}}{18(1 + e^{|\sigma_0|})^3} \right)} > \kappa^2 h_0, \quad (3.44)$$

the early universe undergoes through an inflationary stage (with τ_0 as the onset of inflation), while for $\frac{m^2(1-e^{-|\sigma_0|})}{4(11+\frac{1219}{27}e^{|\sigma_0|}+\frac{13849}{27}e^{2|\sigma_0|}-\frac{e^{|\sigma_0|}}{18(1+e^{|\sigma_0|})^3})} < \kappa^2 h_0$, $\frac{d^2 s}{d\tau^2}|_{\tau \rightarrow \tau_0}$ becomes negative.

Comparison of Eqs. (3.34) and (3.44) makes it clear that the conditions for an early time acceleration in scalar-tensor theory and $F(R)$ gravity are different. However, note that similarly as in scalar-tensor theory, the interplay among the parameters m and h_0 fixes whether the universe evolves through an inflationary stage during the early universe. Furthermore, in order to solve the flatness and horizon problems, the universe must pass through an accelerating stage at early epoch [in the original $F(R)$ model] and from this requirement, here we assume the condition shown in Eq. (3.44).

2. End of inflation in $F(R)$ gravity

In the previous section, we have shown that the very early universe expands with acceleration, a phase generally known as the inflationary epoch. At this stage, it is important to check whether the inflationary era has an end in a finite time. We may define the end of inflation by the condition

$$\frac{d^2 s}{d\tau^2} = 0. \quad (3.45)$$

Recall $s(\tau) = e^{[\sqrt{\frac{3}{8}\kappa\xi(t)}]} a(t)$ is the scale factor in the $F(R)$ model, from which one obtains

$$\frac{d^2 s}{d\tau^2} = e^{[-\frac{1}{2}\sqrt{\frac{3}{8}\kappa\xi(t)}]} a(t) \left[\dot{H} + H^2 + \frac{\kappa}{\sqrt{6}} H \dot{\xi} \right], \quad (3.46)$$

where H is the Hubble parameter in the scalar-tensor picture and the dot represents $\frac{d}{dt}$. Equations (3.45) and (3.46) clearly indicate that the end of the inflationary epoch in the $F(R)$ model can be expressed by the following equation:

$$\dot{H} + H^2 + \frac{\kappa}{\sqrt{6}} H \dot{\xi} = 0. \quad (3.47)$$

Now we analyze whether this condition is consistent with the field equations. Differentiating (with respect to t) both sides of Eq. (3.21), we get

$$\dot{H} = -\frac{m^2}{18} e^{2\sqrt{\frac{3}{8}\kappa\xi(t)}} + \frac{1}{12H} \frac{d}{dt} \left(\frac{\kappa^2 h_0}{a^6(t)} \right). \quad (3.48)$$

Here we have used the scalar field equation. At the end of inflation, the term proportional to $\frac{\kappa^2 h_0}{a^6}$ becomes small enough so that we can apply the method of iteration (with respect to that term) to determine \dot{H} . Up to zeroth order of

iteration, $\dot{H} = -\frac{m^2}{18} e^{[2\sqrt{\frac{3}{8}\kappa\xi(t)}]}$. Consequently, one determines \dot{H} up to first order of iteration as follows:

$$\dot{H} = -\frac{m^2}{18} e^{2\sqrt{\frac{3}{8}\kappa\xi(t)}} + \left(\frac{2m^2}{81} \right) \frac{e^{4\sqrt{\frac{3}{8}\kappa\xi(t)}}}{\left(1 - \sqrt{\frac{2}{3}\kappa\xi(t)} \right)^2}. \quad (3.49)$$

By this expression together with the field equations of motion, Eq. (3.49) leads to the following condition on the scalar field:

$$\frac{1}{4} - \frac{2}{3} e^{[\sqrt{\frac{3}{8}\kappa\xi(t_f)}]} + \frac{1}{9} e^{[2\sqrt{\frac{3}{8}\kappa\xi(t_f)}]} + \frac{8}{81} e^{[4\sqrt{\frac{3}{8}\kappa\xi(t_f)}]} = 0,$$

where $t_f - t_0$ denotes the duration of inflation in the $F(R)$ model (in terms of t). Solving the above algebraic equation [for $\xi(t_f)$], we obtain

$$\xi_f \simeq \sqrt{\frac{3}{2\kappa^2}} \ln\left(\frac{3}{5}\right). \quad (3.50)$$

Equation (3.50) clearly indicates that the inflationary era of the universe continues as long as the value of the scalar field remains greater than $\xi_f (= \sqrt{\frac{3}{2\kappa^2}} \ln(\frac{3}{5}) < 0)$. Correspondingly, the duration of inflation can be calculated from the solution of the scalar field [see Eq. (3.25)] as follows:

$$t_f - t_0 = \frac{9}{\sqrt{6}m} \left[e^{|\sigma_0|} - e^{|\sigma_f|} \left(1 + \frac{\kappa^2 h_0}{9m^2} e^{-|\sigma_0|} \right) \right], \quad (3.51)$$

where $\sigma_0 = \sqrt{\frac{2\kappa^2}{3}} \xi_0$ and $\sigma_f = \sqrt{\frac{2\kappa^2}{3}} \xi_f$. Therefore in terms of the cosmic time τ , the duration of inflation becomes

$$\begin{aligned} \tau_f - \tau_0 = & \frac{1}{4m} \sqrt{\frac{3}{2}} \left[(8\sqrt{9C} - 8\sqrt{d}) \right. \\ & + \frac{\kappa^2 h_0}{2m^2} \left(\frac{(27+5d)\sqrt{d}}{(9+d)^2} - \frac{(3+5C)\sqrt{C}}{3(1+C)^2} \right), \\ & \left. + \frac{\kappa^2 h_0}{2m^2} \left(\tan^{-1}\left(\frac{3}{\sqrt{q}}\right) - \tan^{-1}(1/\sqrt{C}) \right) \right], \quad (3.52) \end{aligned}$$

with $d = e^{\sigma_f} (1 + \frac{\kappa^2 h_0}{9m^2} e^{-|\sigma_0|})$. Moreover, in order to derive the above expression, we have used Eq. (3.38). Note that $\tau_f - \tau_0$ depends on the parameters $\frac{\kappa^2 h_0}{m^2}$ and σ_0 . Therefore, we need the values of such parameters to estimate the duration explicitly, which can be determined from the expression of the spectral index and the tensor to scalar ratio as discussed in the next section.

3. Spectral index, tensor to scalar ratio and number of e -foldings in the $F(R)$ model

In order to test the broad inflationary paradigm as well as particular models vs the observations, we need to calculate the value of spectral index (n_s) and tensor to scalar ratio (r) and for this purpose, here we define a dimensionless parameter (known as slow-roll parameter) as

$$\epsilon_F = -\frac{1}{H_F^2} \frac{dH_F}{d\tau}, \quad (3.53)$$

where H_F is the Hubble parameter in the $F(R)$ model, defined as $H_F = \frac{1}{s(\tau)} \frac{ds}{d\tau}$. By Eqs. (3.36) and (3.37), H_F turns out to be

$$H_F = e^{-\frac{\sqrt{2}\kappa\xi(t)}{2\sqrt{3}}} \left[H + \frac{\sqrt{2}\kappa}{2\sqrt{3}} \dot{\xi} \right]. \quad (3.54)$$

Recall that H is the Hubble parameter in the corresponding scalar-tensor frame. In order to find the explicit expression of ϵ_F , we determine $\frac{dH_F}{d\tau}$ by differentiating with respect to τ , both sides of Eq. (3.54), leading to

$$\frac{dH_F}{d\tau} = e^{-\frac{\sqrt{2}\kappa\xi(t)}{2\sqrt{3}}} \left[\dot{H} - \frac{\sqrt{2}\kappa}{2\sqrt{3}} H \dot{\xi} \right]. \quad (3.55)$$

From the above expressions of H_F and $\frac{dH_F}{d\tau}$ along with the slow-roll field equations, one finally gets the following form of ϵ_F :

$$\begin{aligned} \epsilon_F &= -\frac{1}{H_F^2} \frac{dH_F}{d\tau}, \\ &= \left[\frac{\frac{3\kappa^2 h_0}{2a^6} + \frac{m^2}{12} e^\sigma (1 - e^\sigma)}{\frac{m^2}{8} (1 - e^\sigma)^2 + \frac{\kappa^2 h_0}{2a^6} + \frac{m^2}{6} e^\sigma (1 - e^\sigma)} \right], \end{aligned} \quad (3.56)$$

with $\sigma(\tau) = \sqrt{\frac{2\kappa^2}{3}} \xi(t(\tau))$.

As mentioned above, the second rank antisymmetric KR field can be equivalently expressed as a vector field which can be further recast as the derivative of a massless scalar field (see Appendix B). As a consequence, the spectral index and tensor to scalar ratio in the present context are defined as follows [51–53]:

$$n_s = [1 - 4\epsilon_F - 2\epsilon_2 + 2\epsilon_3 - 2\epsilon_4]_{\tau_0}, \quad (3.57)$$

and

$$r = 8\kappa^2 \frac{\Theta}{F'(R)} \Big|_{\tau_0}. \quad (3.58)$$

Here the slow-roll parameters (ϵ_F , ϵ_2 , ϵ_3 , ϵ_4) are defined by the following expressions:

$$\begin{aligned} \epsilon_F &= -\frac{1}{H_F^2} \frac{dH_F}{d\tau}, & \epsilon_2 &= \frac{1}{2\rho_{\text{KR}} H_F} \frac{d\rho_{\text{KR}}}{d\tau}, \\ \epsilon_3 &= \frac{1}{2F'(R)H_F} \frac{dF'(R)}{d\tau}, & \epsilon_4 &= \frac{1}{2EH_F} \frac{dE}{d\tau}, \end{aligned} \quad (3.59)$$

where Θ and E are given by

$$\Theta = \frac{\rho_{\text{KR}}}{F'(R)H_F^2} \left[F'(R) + \frac{3}{2\kappa^2 \rho_{\text{KR}}} \left(\frac{d}{d\tau} F'(R) \right)^2 \right], \quad (3.60)$$

and

$$E = \frac{\Theta F'(R) H_F^2}{\rho_{\text{KR}}}, \quad (3.61)$$

with $\rho_{\text{KR}} (= H_{123} H^{123})$ being the energy density of the KR field in the $F(R)$ model. However, by virtue of Eq. (3.86), the variation of ρ_{KR} immediately leads to $\rho_{\text{KR}} = e^{-2\sqrt{\frac{2}{3}}\kappa\xi(t)} \frac{h_0}{a^6}$. Keeping this in mind, now we are going to determine the explicit expressions of various terms appearing in the right-hand side of Eqs. (3.57) and (3.58).

(i) ϵ_F : As obtained in Eq. (3.56), ϵ_F is given by

$$\epsilon_F = \left[\frac{\frac{3\kappa^2 h_0}{2a^6} + \frac{m^2}{12} e^\sigma (1 - e^\sigma)}{\frac{m^2}{8} (1 - e^\sigma)^2 + \frac{\kappa^2 h_0}{2a^6} + \frac{m^2}{6} e^\sigma (1 - e^\sigma)} \right]. \quad (3.62)$$

(ii) ϵ_2 : As defined above, ϵ_2 is related to the variation of the KR field energy density and thus to the field equation of the Kalb-Ramond field. However, the KR field energy density in $F(R)$ (ρ_{KR}) and in the corresponding scalar-tensor theory ($\tilde{\rho}_{\text{KR}}$) are connected by $\rho_{\text{KR}} = e^{-2\sigma} \tilde{\rho}_{\text{KR}}$ (with $\sigma = \sqrt{\frac{2}{3}}\kappa\xi$). Differentiating both sides of this expression (with respect to τ), one gets

$$\begin{aligned} \frac{d\rho_{\text{KR}}}{d\tau} &= \frac{d}{dt} [e^{-2\sigma} \tilde{\rho}_{\text{KR}}] \frac{dt}{d\tau} \\ &= \rho_{\text{KR}} e^{-\sigma/2} \left[\frac{1}{\tilde{\rho}_{\text{KR}}} \frac{d\tilde{\rho}_{\text{KR}}}{dt} - 2 \frac{d\sigma}{dt} \right], \end{aligned} \quad (3.63)$$

where we have used the relation among τ and t as shown in Eq. (3.36). Recall that the evolution of $\tilde{\rho}_{\text{KR}}$ [see Eq. (3.16)] is given by

$$\frac{1}{\tilde{\rho}_{\text{KR}}} \frac{d\tilde{\rho}_{\text{KR}}}{dt} + 6H = 0. \quad (3.64)$$

With the help of expression (3.63), the above equation can be written in terms of ρ_{KR} as follows:

$$\frac{\rho'_{\text{KR}}}{2\rho_{\text{KR}}} + e^{-\sigma/2} \dot{\sigma} + 3e^{-\sigma/2} H = 0, \quad (3.65)$$

where prime and dot represent the derivative with respect to τ and t respectively. Equation (3.65) along with the expression of H_F [see Eq. (3.54)] leads to the final form of ϵ_2 as follows:

$$\begin{aligned}\epsilon_2 &= \frac{\rho'_{\text{KR}}}{2\rho_{\text{KR}}H_F} \\ &= -3 + \frac{\dot{\sigma}}{2H_F} e^{-\sigma/2}.\end{aligned}\quad (3.66)$$

(iii) ϵ_3 : Using $F(R) = R + \frac{R^2}{m^2}$ (as we consider in the present context), ϵ_3 can be simplified as

$$\begin{aligned}\epsilon_3 &= \frac{1}{2F'(R)H_F} \frac{dF'(R)}{d\tau} \\ &= \frac{1}{2RH_F} \frac{dR}{d\tau},\end{aligned}\quad (3.67)$$

where we consider $1 + \frac{2R}{m^2} \simeq \frac{2R}{m^2}$ near the beginning of inflation (as n_s and r are calculated at the onset of inflation). Furthermore, for a flat FRW metric, the Ricci scalar takes the form $R \simeq 12H_F^2$, and we get the final form of ϵ_3 as

$$\begin{aligned}\epsilon_3 &= \frac{1}{H_F^2} \frac{dH_F}{d\tau} = -\epsilon_F \\ &= -\left[\frac{\frac{3\kappa^2 h_0}{2a^6} + \frac{m^2}{12} e^\sigma (1 - e^\sigma)}{\frac{m^2}{8} (1 - e^\sigma)^2 + \frac{\kappa^2 h_0}{2a^6} + \frac{m^2}{6} e^\sigma (1 - e^\sigma)} \right].\end{aligned}\quad (3.68)$$

(iv) ϵ_4 : As mentioned above, E is defined as $E = \frac{\Theta F'(R) H_F^2}{\rho_{\text{KR}}}$. Differentiating this expression (with respect to τ), one gets

$$\frac{E'}{EH_F} = \frac{\Theta'}{\Theta H_F} + \frac{1}{F'(R)H_F} \frac{dF'(R)}{d\tau} + 2 \frac{H'_F}{H_F^2} - \frac{\rho'_{\text{KR}}}{\rho_{\text{KR}}H_F}.\quad (3.69)$$

The above expression can be further simplified with the help of Eqs. (3.66) and (3.68),

$$\frac{E'}{EH_F} = \frac{\Theta'}{\Theta H_F} - 4\epsilon_F + 6 - \frac{\dot{\sigma}}{H_F} e^{-\sigma/2}.\quad (3.70)$$

At this stage, we should obtain Θ in order to get the final expression for ϵ_4 as well as for n_s . By its definition, Θ is given by

$$\begin{aligned}\Theta &= \frac{\rho_{\text{KR}}}{H_F^2} + \frac{3}{2\kappa^2 F'(R) H_F^2} \left(\frac{dF'(R)}{d\tau} \right)^2 \\ &= \frac{\rho_{\text{KR}}}{H_F^2} + 6\epsilon_F^2 \left(\frac{F'(R)}{\kappa^2} \right).\end{aligned}\quad (3.71)$$

In the second line of the above expression, we have used Eq. (3.68). Differentiating both sides of Eq. (3.71), the following expression yields

$$\begin{aligned}\frac{\Theta'}{\Theta H_F} &= -2\epsilon_F + 2 \frac{\epsilon'_F}{\epsilon_F H_F} + \frac{\kappa^2 \rho_{\text{KR}}}{6F'(R)\epsilon_F^2 H_F^3} \\ &\times \left[-6H_F + e^{-\sigma/2} \dot{\sigma} - \frac{H'_F}{H_F} \right],\end{aligned}\quad (3.72)$$

where we have used Eqs. (3.66) and (3.68). However, the above expression together with Eq. (3.70) lead to the final form of ϵ_4 as

$$\begin{aligned}\epsilon_4 &= -3\epsilon_F + \frac{\epsilon'_F}{\epsilon_F H_F} + 3 - \frac{\dot{\sigma}}{2H_F} e^{-\sigma/2} \\ &+ \frac{\kappa^2 \rho_{\text{KR}}}{6F'(R)\epsilon_F^2 H_F^3} \left[-6H_F + e^{-\sigma/2} \dot{\sigma} - \frac{H'_F}{H_F} \right].\end{aligned}\quad (3.73)$$

Hence, we can now calculate the spectral index. Introducing the above expressions of ϵ_i [$i = 1, 2, 3, 4$, see Eqs. (3.62), (3.66), (3.68) and (3.73)] into Eq. (3.57), we finally obtain the following form of n_s as

$$n_s = 1 - 2 \frac{\epsilon'_F}{\epsilon_F H_F} + \frac{\kappa^2 \rho_{\text{KR}}}{6F'(R)\epsilon_F^2 H_F^3} \left[-6H_F + e^{-\sigma/2} \dot{\sigma} - \frac{H'_F}{H_F} \right].\quad (3.74)$$

Note that in the absence of the Kalb-Ramond field (i.e., $\rho_{\text{KR}} = 0$), n_s turns out $n_s = 1 - 2 \frac{\epsilon'_F}{\epsilon_F H_F}$, in agreement with the expression of spectral index in a pure $F(R)$ gravity model [51]. However, due to the presence of the KR field, n_s is modified by the terms proportional to ρ_{KR} . Taking these modifications into account, the final form of n_s is given by

$$\begin{aligned}n_s &= 1 - 2 \left[\frac{-\frac{\kappa^2 h_0}{2m^2} + \frac{1}{9} \left(\frac{e^{2\sigma_0} (1 - e^{\sigma_0}) (1 - 2e^{\sigma_0})}{(1 - e^{\sigma_0})^2 + \frac{4\kappa^2 h_0}{m^2}} \right)}{\frac{3\kappa^2 h_0}{2m^2} + \frac{1}{12} e^{\sigma_0} (1 - e^{\sigma_0})} \right] + 2 \left[\frac{-\frac{3\kappa^2 h_0}{m^2} - \frac{1}{9} \left(\frac{e^{2\sigma_0} (1 - e^{2\sigma_0})}{(1 - e^{\sigma_0})^2 + \frac{4\kappa^2 h_0}{m^2}} \right)}{\frac{\kappa^2 h_0}{2m^2} + \frac{1}{8} (1 - e^{\sigma_0})^2 + \frac{1}{6} e^{\sigma_0} (1 - e^{\sigma_0})} \right] \\ &+ \frac{9}{16} \frac{\kappa^2 h_0 / m^2}{\left[\frac{1}{8} (1 - e^{\sigma_0})^2 + \frac{\kappa^2 h_0}{2m^2} + \frac{1}{6} e^{\sigma_0} (1 - e^{\sigma_0}) \right] \left[\frac{3\kappa^2 h_0}{2m^2} + \frac{1}{12} e^{\sigma_0} (1 - e^{\sigma_0}) \right]} - \frac{3}{8} \frac{\kappa^2 h_0 / m^2}{\left[\frac{3\kappa^2 h_0}{2m^2} + \frac{1}{12} e^{\sigma_0} (1 - e^{\sigma_0}) \right]^2}.\end{aligned}\quad (3.75)$$

Let us now calculate the tensor to scalar ratio (r) which is defined as $r = 8\kappa^2 \frac{\rho_{\text{KR}}}{F'(R)}$. By using the explicit expression of Θ , one gets

$$r = 8\kappa^2 \frac{\rho_{\text{KR}}}{H_F^2 F'(R)} + 48 \left(\frac{1}{2H_F F'(R)} \frac{dF'(R)}{d\tau} \right)^2 \Big|_{\tau_0}. \quad (3.76)$$

We need the terms of the right-hand side of Eq. (3.76) which can be obtained as follows:

- (i) First term in the rhs of Eq. (3.76): With $F'(R) = 1 + 2R/m^2 \simeq 24H_F^2/m^2$ along with the expression $H_F = e^{-\sigma/2} [H + \frac{\dot{\sigma}}{2}]$ [see Eq. (3.54)], one gets

$$8\kappa^2 \frac{\rho_{\text{KR}}}{H_F^2 F'(R)} \Big|_{\tau_0} = \frac{\kappa^2 h_0}{\frac{3}{m^2} [H^2 + H\dot{\sigma}]^2}. \quad (3.77)$$

By using the slow-roll field equations, the above expression can be further simplified leading to the following form:

$$8\kappa^2 \frac{\rho_{\text{KR}}}{H_F^2 F'(R)} \Big|_{\tau_0} = \frac{3\kappa^2 h_0/m^2}{[\frac{1}{8}(1 - e^{\sigma_0})^2 + \frac{\kappa^2 h_0}{2m^2} + \frac{1}{6}e^{\sigma_0}(1 - e^{\sigma_0})]^2}. \quad (3.78)$$

- (ii) Second term in the rhs of Eq. (3.76): As obtained in Eq. (3.68), the second term of the rhs of Eq. (3.76) is equal to $48\epsilon_F^2$.

Hence, we obtain the tensor to scalar ratio:

$$r = \frac{3\kappa^2 h_0/m^2}{[\frac{1}{8}(1 - e^{\sigma_0})^2 + \frac{\kappa^2 h_0}{2m^2} + \frac{1}{6}e^{\sigma_0}(1 - e^{\sigma_0})]^2} + 48\epsilon_F^2. \quad (3.79)$$

Note that from Eq. (3.79), whether $\rho_{\text{KR}} = 0$ (or equivalently $h_0 = 0$ i.e., without the KR field), r goes to $48\epsilon_F^2$ —the expression for tensor to scalar ratio in a pure $F(R)$ gravity model [51]. However taking the effect of the Kalb-Ramond field into account and substituting the expression of ϵ_F [obtained in Eq. (3.56)] in Eq. (3.79), we get the final form of r as follows:

$$r = \frac{3\frac{\kappa^2 h_0}{2m^2} + 48[\frac{3\kappa^2 h_0}{2m^2} + \frac{1}{12}e^{\sigma_0}(1 - e^{\sigma_0})]^2}{[\frac{1}{8}(1 - e^{\sigma_0})^2 + \frac{\kappa^2 h_0}{2m^2} + \frac{1}{6}e^{\sigma_0}(1 - e^{\sigma_0})]^2}. \quad (3.80)$$

Thereby the final expressions of n_s and r are shown in Eqs. (3.75) and (3.80) respectively, from which it is evident that both quantities depend on the parameters $\frac{\kappa^2 h_0}{m^2}$ and σ_0 . Equations (3.75) and (3.80) lead to the parametric plot for n_s vs r (with respect to the parameters $\frac{\kappa^2 h_0}{m^2}$ and σ_0), as shown in Fig. 2.

However, observations based on Planck 2018 impose a constraint on n_s and r as $n_s = 0.9650 \pm 0.0066$ and $r < 0.07$ (combining with BICEP2/Keck-Array) respectively. Therefore, Fig. 2 clearly indicates that for

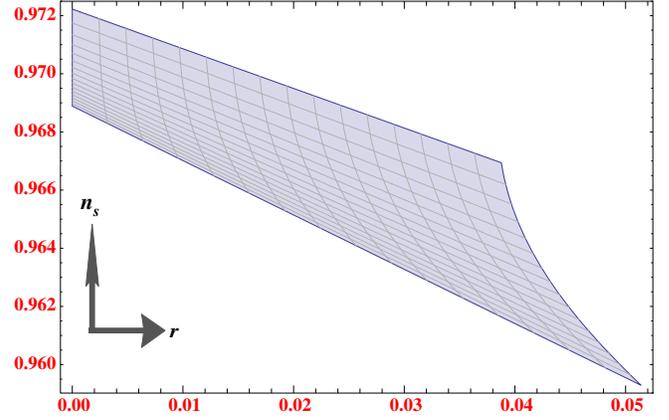


FIG. 2. n_s vs r for $10 \leq |\sigma_0| \leq 14$ and $0.003 \leq \frac{\kappa^2 h_0}{m^2} \leq 0.004$.

$|\sigma_0| > 10$ and $0.003 < \frac{\kappa^2 h_0}{m^2} < 0.004$, the theoretical values of n_s , r (in the present context) match with the observational constraints. In addition, by the estimated values of $\frac{\kappa^2 h_0}{m^2}$ and σ_0 ($\simeq -10$), the duration of inflation [$\tau_f - \tau_0$, see Eq. (3.38)] becomes 10^{-12} (GeV) $^{-1}$ if the mass parameter (m) is separately taken as 10^{-5} (in Planckian units). We also obtain the number of e -foldings, defined by $N = \int_0^{\Delta\tau} H_F d\tau$ ($\Delta\tau = \tau_f - \tau_0$, duration of inflation), numerically, leading to $N \simeq 56$ (with $\sigma_0 = -10$). These results are summarized in Table I.

Table I clearly indicates that the present model may well explain the inflationary scenario of the universe in terms of the observable quantities n_s and r as based on the results of Planck 2018.

Using the solutions of $s(\tau)$ [see Eq. (3.39)] along with the estimated values of the parameters ($\frac{\kappa^2 h_0}{m^2}$, σ_0 , m), we depict the deceleration parameter $q = -\frac{1}{s} \frac{d^2 s}{d\tau^2}$ vs a dimensionless time variable $\tilde{\tau} = \frac{\tau}{\tau_f} N$ in Fig. 3.

Figure 3 shows that the early universe starts from an accelerating stage with a graceful exit at a finite time. However, from Table I, the maximum value for the parameter $\frac{\kappa^2 h_0}{m^2}$ is given by $\frac{\kappa^2 h_0}{m^2} \simeq 0.004$, in order to match the present model with the observations of Planck 2018. Taking $m = 10^{-5}$ (in Planckian unit), we obtain $h_0^{\text{max}} \sim 10^{63}$ (GeV) 4 . Recall that the term $h_0 e^{[-2\sigma_0]}$ [see Eq. (3.4)] denotes the energy density for the KR field (ρ_{KR}) during the early universe in the $F(R)$ model. Therefore, the

TABLE I. Estimated values of various quantities for $\frac{\kappa^2 h_0}{m^2} = 0.0035$, $\sigma_0 = -10$ and $m = 10^{-5}$ (in Planckian unit).

Parameters	Estimated values
n_s	$\simeq 0.9630$
r	$\simeq 0.03$
$\tau_f - \tau_0$	10^{-12} (GeV) $^{-1}$
N	56

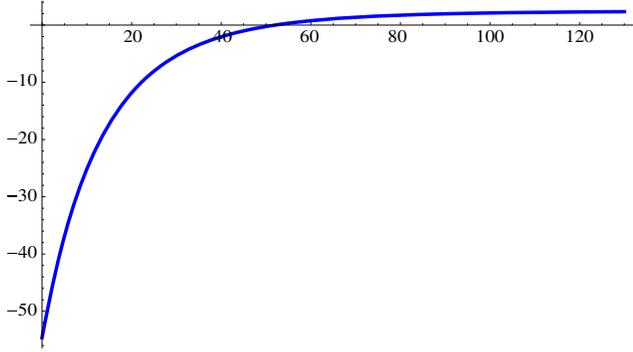


FIG. 3. $q(\tau)$ vs $\bar{\tau}$ for $\frac{\kappa^2 h_0}{m^2} = 0.0035$, $\sigma_0 = -10$ and $m = 10^{-5}$ (in Planckian unit).

present model along with the constraints of Planck 2018 give an upper bound on the KR field energy density during the early universe as $(\rho_{\text{KR}})^{\text{max}} \sim 10^{70} (\text{GeV})^4$ (with $\sigma_0 = -10$). In such a situation, it is important to examine whether the energy density of the KR field [starting with $\sim 10^{70} (\text{GeV})^4$ from the early universe] gets suppressed and leads to a negligible footprint during our present universe. This matter is discussed in the next section.

However, let us discuss briefly the cosmological evolution and the corresponding observable parameters for the cases: (1) quadratic curvature gravity in the absence of the Kalb-Ramond field [i.e., for the pure $F(R) = R + R^2/m^2$ model], (2) in the absence of higher order terms in the gravitational action, i.e., for Einstein gravity with a KR field, and (3) when considering cubic curvature gravity with a KR field.

- (1) *Quadratic curvature gravity in the absence of the KR field.*—In this case, the action of the model becomes

$$S = \int d^4x \sqrt{-g} \frac{1}{2\kappa^2} \left[R + \frac{R^2}{m^2} \right]. \quad (3.81)$$

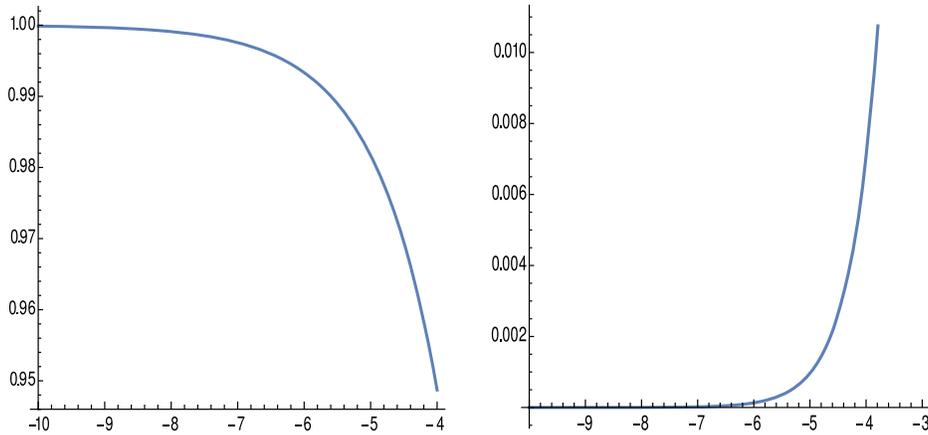


FIG. 4. n_s vs σ_0 (left panel), r vs σ_0 (right panel).

Recall that $\frac{1}{2\kappa^2} = M_p^2$ (where M_p is the four-dimensional Planck mass). For this action, the solution of the FRW scale factor can be obtained by fixing $h_0 = 0$ in the expression obtained in Eq. (3.39), yielding

$$s(\tau(t)) = D \left(1 - \frac{2m(t-t_0)}{3\sqrt{6}C} \right)^{3/4} \times \exp \left[\frac{m(t-t_0)}{2\sqrt{6}} + \ln \left(\frac{9}{-\sqrt{6}m(t-t_0) + 9C} \right) \right], \quad (3.82)$$

where $C = e^{-\sigma_0}$, τ is the cosmic time related to t by Eq. (3.38) with $h_0 = 0$. Equation (3.82) leads to the acceleration of the early universe as

$$\frac{1}{s} \frac{d^2s}{d\tau^2} \Big|_{\tau \rightarrow \tau_0} = C \left(\frac{m}{2\sqrt{6}} \right)^2 (1 - e^{-|\sigma_0|})^2, \quad (3.83)$$

which clearly indicates that the early universe undergoes an inflationary stage, the well known Starobinsky inflation. Correspondingly, the observational parameters as the spectral index and the tensor to scalar ratio depend only on the parameter σ_0 in the absence of the KR field. By introducing $h_0 = 0$ into the Eqs. (3.75) and (3.80), one obtains the variation of n_s and r in terms of the parameter σ_0 , as illustrated in Fig. 4, which clearly shows that in the absence of the Kalb-Ramond field, the spectral index and tensor to scalar ratio lie within the observational constraints for the interval $-5.0 \lesssim \sigma_0 \lesssim -4.5$. However, as mentioned above, even in the presence of the KR field, n_s and r also remain within the constraints but with a bound given by $(\rho_{\text{KR}})^{\text{max}} \sim 10^{70} (\text{GeV})^4$ (with $\sigma_0 = -10$). For clearness, below we illustrate the comparison with/without the antisymmetric KR field in Table II.

TABLE II. Comparison of n_s and r with/without the Kalb-Ramond field.

Parameters	$\frac{\kappa^2 h_0}{m^2} = 0.0035, \sigma_0 = -10$	$h_0 = 0, \sigma_0 = -4.5$
n_s	$\simeq 0.9630$	$\simeq 0.9660$
r	$\simeq 0.03$	$\simeq 0.003$

- (2) *In the absence of higher order curvature gravity.*— Without higher order curvature d.o.f., the action takes the following form:

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa^2} - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} \right]. \quad (3.84)$$

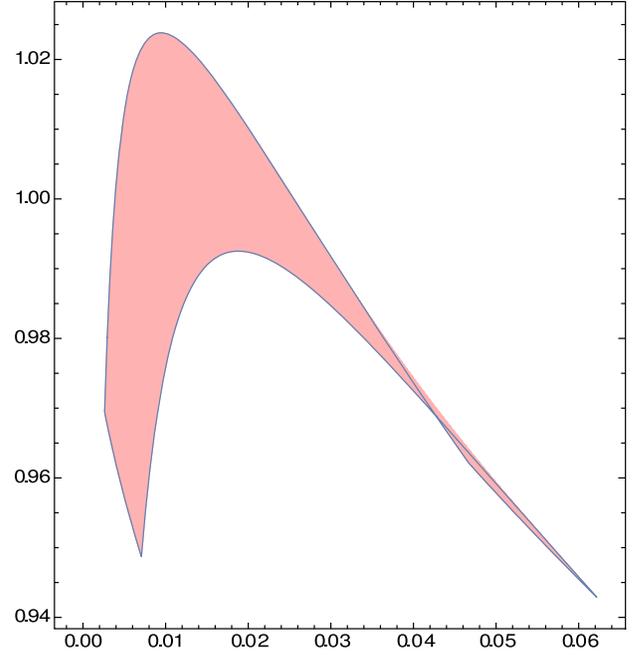
As mentioned above, for a flat FLRW metric, the KR three tensor has only one nonzero component, i.e., H_{123} symbolized by h_4 . With this nonzero component, the pressure and energy density of the KR field turn out to be same and equal to $\frac{1}{2}h_4 h^4$. As a consequence, the FLRW equation becomes $\dot{H} = -3H^2$ which can be solved and the scale factor yields $a(t) = (t - t_0)^{1/3}$. The acceleration of the scale factor turns out negative and inflation does not occur. This result is in agreement with [54], which states that a minimal model with an antisymmetric tensor field (in the Einstein frame) is not consistent with inflation.

However, authors from [54] showed that a stable de-Sitter solution can be achieved in the context of antisymmetric tensor field by introducing a non-minimal coupling between the Ricci scalar and the tensor field. On the other hand, in the present paper, we argue that the minimal prescription (in the presence of an antisymmetric tensor field) can also give rise to an inflationary era, but in the regime of higher order curvature gravity.

- (3) *Cubic curvature gravity with the presence of the KR field.*—In this case, the action is given by

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} [R + \beta R^3] - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} \right]. \quad (3.85)$$

Here β is a free parameter with mass dimension $[-4]$. It is well known that $F(R) = R + \beta R^3$ does not give a *good inflation*, i.e., the theoretical values of n_s and r do not support the observable constraints from Planck 2018. However, in the presence of an antisymmetric Kalb-Ramond field, model (3.85) is consistent with Planck 2018 constraints (i.e., $n_s = 0.9650 \pm 0.0066$ and $r < 0.07$, combining with BICEP2/Keck-Array). Here we present the plot for simultaneous compatibility of n_s, r in Fig. 5:

FIG. 5. n_s vs r for $-5 \lesssim \sigma_0 \lesssim -4$ and $0.03 \lesssim \kappa^2 h_0 \sqrt{\beta} \lesssim 0.3$.

In Fig. 5, $e^{-\sigma_0} = 1 + 3\beta R_0^2$ (with R_0 be the spacetime curvature at the time of horizon crossing). In addition, Fig. 5 clearly reveals that the observable parameters $\{n_s, r\}$ remain within the confident regions provided by Planck 2018.

4. Suppression of the Kalb-Ramond field in $F(R)$ gravity

The energy density of the Kalb-Ramond field ρ_{KR} in our $F(R)$ model in terms of the cosmic time τ is given by

$$\rho_{\text{KR}}(\tau(t)) = e^{-2\sqrt{\frac{2}{3}}\kappa\xi(t)} \frac{h_0}{a^6}, \quad (3.86)$$

where we have used the conformal transformation of the metric along with Eq. (3.17). Therefore, in order to address the effect of the KR field on our present universe, it is important to understand the late time evolution for $\xi(t)$ and $a(t)$. As mentioned above, $\xi(t)$ goes to infinity at late times, starting from a negative value at the early universe. However, this solution $\xi(t)$ is based on the slow-roll approximation which may not hold at late times. Then, let us relax the slow-roll approximation, such that the field equations for $\xi(t)$ and $a(t)$ take the form

$$H^2 = \frac{\kappa^2}{3} \left[\frac{1}{2} \dot{\xi}^2 + \frac{m^2}{8\kappa^2} (1 - e^{\sqrt{\frac{2}{3}}\kappa\xi})^2 + \frac{1}{2} \frac{h_0}{a^6} \right], \quad (3.87)$$

and

$$\ddot{\xi} + 3H\dot{\xi} - \sqrt{\frac{2}{3}} \frac{m^2}{4\kappa} e^{\sqrt{\frac{2}{3}}\kappa\xi} (1 - e^{\sqrt{\frac{2}{3}}\kappa\xi}) = 0. \quad (3.88)$$

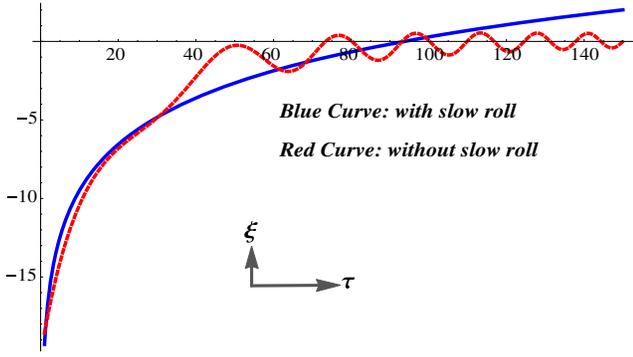


FIG. 6. ξ vs $\tilde{\tau}$ for $\frac{\kappa^2 h_0}{m^2} = 0.0035$, $\sigma_0 = -10$ and $m = 10^{-5}$ (in Planckian unit).

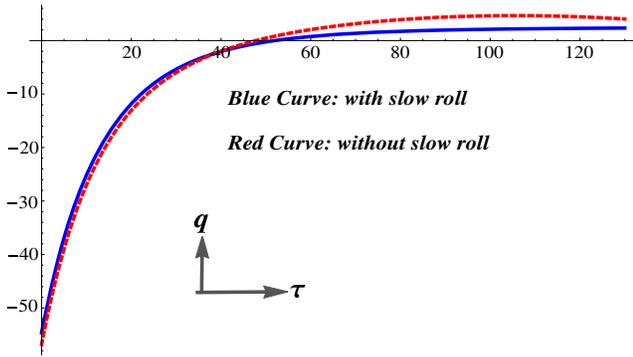


FIG. 7. q vs $\tilde{\tau}$ for $\frac{\kappa^2 h_0}{m^2} = 0.0035$, $\sigma_0 = -10$ and $m = 10^{-5}$ (in Planckian unit).

By solving these equations numerically, the evolution of the KR field and the deceleration parameter are depicted in Figs. 6 and 7 respectively, where we have used the relation of $\tau(t)$ given in Eq. (3.38). Figure 6 shows the evolution of $\xi(\tau)$ for both cases, when assuming the slow-roll approximation and when no approximation is assumed. As shown, the evolution for $\xi(\tau)$ is very similar in both cases. After inflation, the acceleration term for $\xi(\tau)$ starts to contribute and as a result both solutions (with and without slow-roll conditions) differ from each other. A similar conclusion holds for the deceleration parameter. Moreover in the slow-roll approximation, $\xi(\tau)$ does not tend to a finite value asymptotically, but goes to infinity at late times, while in the absence of the slow-roll approximation, $\xi(\tau)$ moves towards $\langle \xi \rangle = 0$ asymptotically, showing an oscillatory behavior at late times.

By using these numerical solutions for $\xi(\tau)$ and $a(\tau)$, the KR field energy density (3.86) is obtained in our $F(R)$ model, ρ_{KR} , as shown in Fig. 8, where the energy density of the KR field gradually decreases with the cosmic time (τ) and the decaying time scale $\tilde{\tau} = 40$ is smaller than the exit time from inflation ($\tilde{\tau} = 56$). This may well explain why the present universe does not show any footprint of the antisymmetric Kalb-Ramond field.

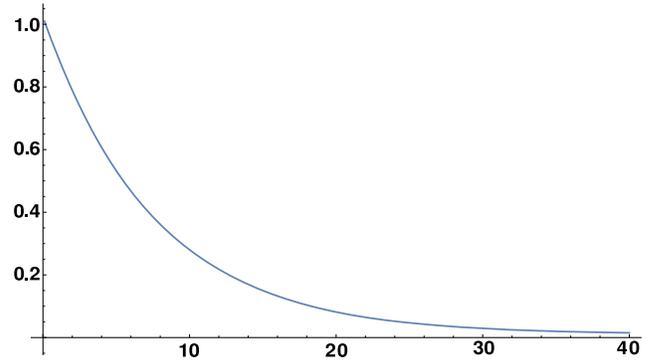


FIG. 8. ρ_{KR} vs $\tilde{\tau}$ for $\frac{\kappa^2 h_0}{m^2} = 0.0035$, $\sigma_0 = -10$ and $m = 10^{-5}$ (in Planckian unit).

However, besides the four-dimensional context, higher dimension spacetimes may provide a natural solution to the hierarchy problem, i.e., apparent mismatch between the fundamental scale and the electroweak symmetry breaking scale [32–34]. In such higher dimensional models, unlike electromagnetic or other matter fields, the Kalb-Ramond field does propagate through extra dimensions and thus have Kaluza-Klein modes. Further attempts to unify gravity and electromagnetism require the inclusion of the Kalb-Ramond field in higher dimensional theories [55,56], such that the KR field may become important in the context of extra dimensional models. In the following sections, we discuss such higher dimensional spacetimes.

IV. KALB-RAMOND FIELD IN FIVE DIMENSIONS IN $F(R)$ GRAVITY

Let us now investigate the cosmological evolution for the Kalb-Ramond field when higher dimensional spacetimes are considered. In particular, here we consider the well known Randall-Sundrum (RS) braneworld model with the presence of a Kalb-Ramond field in the bulk. The RS model consists of one extra spatial dimension. The bulk spacetime is AdS in nature and S^1/Z_2 orbifolded along the extra dimension where the orbifold fixed points are identified with two 3-branes. If φ is taken to be the extra dimensional angular coordinate, then the branes are located at $\varphi = 0$ (hidden brane) and at $\varphi = \pi$ (visible brane) respectively while the latter is identified with our visible universe. However, in such a braneworld scenario, the stabilization of interbrane separation (also known as modulus or radion) is an important issue to address and for this purpose one needs an extra stabilizing agent which is able to generate a stable radion potential. Here, in the present context, we consider the quadratic curvature term in the five-dimensional action together with the Einstein-Hilbert term as the stabilizing agent. Moreover, it is well known that higher order curvature terms become relevant in the limit of large curvature. Thus, in the RS bulk geometry, where the curvature is of the order of the Planck scale, such that

higher order curvature terms have to be included in the action. Hence, the action of the model is given by

$$S = \int d^4x d\varphi \sqrt{-G} \left[\frac{(R + \alpha R^2)}{2\kappa^2} - \Lambda + V_h \delta(\varphi) + V_v \delta(\varphi - \pi) - \frac{1}{12} H_{MNL} H^{MNL} \right],$$

$$= S_g + S_{\text{KR}}, \quad (4.1)$$

where G is the determinant of the five-dimensional metric G_{MN} (M, N , whose indexes runs from 0 to 4 where 0 to 3 are reserved for brane coordinates), α is a constant parameter having mass dimension $[-2]$ and $\frac{1}{2\kappa^2} = M^3$ (with M the five-dimensional Planck mass), while Λ (< 0) symbolizes the bulk cosmological constant and V_h, V_v are the brane tensions on hidden and visible brane respectively. Moreover $H_{MNL} = \partial_{[M} B_{NL]}$ denotes the field strength tensor for the KR field B_{NL} propagating in the five-dimensional spacetime. However, as being allowed to propagate in the extra dimension, the KR field can be decomposed into Kaluza-Klein (KK) modes which are obviously coupled with to the extra dimensional modulus field. The overlap of these KK wave functions with the visible brane actually determines the strength of the KR field in our visible universe. In such a situation, it is important to explore the effects of higher order curvature terms on the dynamics of the modulus field which in turn controls the evolution of the bulk Kalb-Ramond field. These issues are addressed here from the perspective of four-dimensional effective theory. In the following two subsections, we determine the effective four-dimensional action individually for $S_g = \int d^4x d\varphi \sqrt{-G} [\frac{(R + \alpha R^2)}{2\kappa^2} - \Lambda + V_h \delta(\varphi) + V_v \delta(\varphi - \pi)]$ and $S_{\text{KR}} = \int d^4x d\varphi \sqrt{-G} [-\frac{1}{12} H_{MNL} H^{MNL}]$ respectively.

A. Four-dimensional effective action for S_g

In order to find the effective action of S_g , we need the solution for the five-dimensional spacetime metric G_{MN} . For this purpose, first we determine the field(s) solutions in the corresponding scalar-tensor (ST) theory and then transform the solutions back to the Jordan frame by using the inverse conformal transformation. Following Sec. II, the conformal transformation of the spacetime metric can be expressed as $G_{MN} \rightarrow \tilde{G}_{MN} = e^{\frac{\kappa\Phi}{\sqrt{3}}} G_{MN}$, while the action S_g leads to

$$S_g[\Phi, \tilde{G}_{MN}] = \int d^4x d\varphi \sqrt{\tilde{G}} \left[\frac{\tilde{R}}{2\kappa^2} - \frac{1}{2} \tilde{G}^{MN} \partial_M \Phi \partial_N \Phi - V(\Phi) - \Lambda - e^{-\frac{\kappa\Phi}{\sqrt{3}}} V_h \delta(\varphi) - e^{-\frac{\kappa(\Phi + \pi)}{\sqrt{3}}} V_v \delta(\varphi - \pi) \right], \quad (4.2)$$

where Φ is the scalar field in ST theory and $V(\Phi)$ is its potential which takes the following form:

$$V(\Phi) = \frac{1}{8\kappa^2\alpha} \exp\left(-\frac{5}{2\sqrt{3}}\kappa\Phi\right) \left[\exp\left(\frac{3}{2\sqrt{3}}\kappa\Phi\right) - 1 \right]^2 - \Lambda \left[\exp\left(-\frac{5}{2\sqrt{3}}\kappa\Phi\right) + 1 \right]. \quad (4.3)$$

Moreover, the last two terms in Eq. (4.2) are contributions from the brane tensions of hidden and visible branes. However, in order to check the stability of $V(\Phi)$, we take the single derivative with respect to Φ in both sides of Eq. (4.3),

$$V'(\Phi) = \frac{1}{16\sqrt{3}\kappa\alpha} e^{-\frac{5}{2\sqrt{3}}\kappa\Phi} \left[e^{\frac{6}{2\sqrt{3}}\kappa\Phi} + 4e^{\frac{3}{2\sqrt{3}}\kappa\Phi} - (5 - 40\kappa^2\alpha\Lambda) \right], \quad (4.4)$$

which immediately leads to the fact that $V(\Phi)$ is stable only for $\alpha > 0$. Correspondingly the vacuum expectation value $\langle \Phi \rangle$ and the squared mass (m_Φ^2) of Φ are given by

$$\exp\left(\frac{3}{2\sqrt{3}}\kappa\langle \Phi \rangle\right) = \left[\sqrt{9 - 40\kappa^2\alpha\Lambda} - 2 \right], \quad (4.5)$$

and

$$m_\Phi^2 = \frac{1}{8\alpha} \left[\sqrt{9 - 40\kappa^2\alpha\Lambda} \right] \left[\sqrt{9 - 40\kappa^2\alpha\Lambda} - 2 \right]^{-\frac{2}{3}}. \quad (4.6)$$

As we will see below the stability of the modulus field is also ensured by the condition $\alpha > 0$ —same as for the stability of $V(\Phi)$. Thus, it can be argued that the stability of $V(\Phi)$ and of interbrane separation are intimately connected in the higher order curvature RS model. However, note that the minimum value of $V(\Phi)$ is nonzero and is given by

$$V(\langle \Phi \rangle) = \Lambda + \left[\sqrt{9 - 40\kappa^2\alpha\Lambda} - 2 \right]^{-\frac{5}{3}} \times \left[-\Lambda + (1/8\kappa^2\alpha) \left[\sqrt{9 - 40\kappa^2\alpha\Lambda} - 3 \right]^2 \right].$$

This nonzero value of the potential works as a cosmological constant together with Λ and thus the effective cosmological constant in ST theory is given by $\Lambda_{\text{eff}} = \Lambda + V(\langle \Phi \rangle)$ (a simple algebra shows that Λ_{eff} is negative). By considering a small fluctuation of the scalar field around its stable value as $\Phi = \langle \Phi \rangle + \xi$, the action (4.2) can be written as follows:

$$S_g[\Phi, \tilde{G}_{MN}] = \int d^4x d\varphi \sqrt{\tilde{G}} \left[\frac{\tilde{R}}{2\kappa^2} - \frac{1}{2} \tilde{G}^{MN} \partial_M \xi \partial_N \xi - (1/2) m_\Phi^2 \xi^2 - \Lambda_{\text{eff}} - e^{-\frac{\kappa(\langle \Phi \rangle + \xi)}{\sqrt{3}}} V_h \delta(\varphi) - e^{-\frac{\kappa(\langle \Phi \rangle + \xi)}{\sqrt{3}}} V_v \delta(\varphi - \pi) \right], \quad (4.7)$$

where we keep the terms up to quadratic order in ξ . As expected, the scalar-tensor action contains two independent

fields: Φ and \tilde{G}_{MN} . Let us now find the corresponding solutions of the field equations. By assuming a negligible backreaction of the scalar field (Φ) on the background spacetime, the metric \tilde{G}_{MN} is given by the well known Randall-Sundrum solution as

$$d\tilde{s}^2 = e^{-2kr_c|\varphi|} \eta_{\mu\nu} dx^\mu dx^\nu + r_c^2 d\varphi^2, \quad (4.8)$$

where $k = \sqrt{\frac{-\Lambda_{\text{eff}}}{24M^2}}$ and r_c is the compactification radius of the extra dimension in ST theory. Moreover, the brane tensions are given by following expressions:

$$V_h = 24M^3 k * \exp\left[\frac{5}{2\sqrt{3}}\kappa(\langle\Phi\rangle + v_h)\right],$$

$$V_v = -24M^3 k * \exp\left[\frac{5}{2\sqrt{3}}\kappa(\langle\Phi\rangle + v_v)\right].$$

Here v_h and v_v are the boundary values of ξ on the hidden and visible brane respectively. Together with the metric (4.8), the scalar field Φ equation turns out to be

$$-\frac{1}{r_c^2} \partial_\varphi [\exp(-4kr_c|\varphi|) \partial_\varphi \xi] + m_\Phi^2 \exp(-4kr_c|\varphi|) \xi(\varphi) = 0, \quad (4.9)$$

where the scalar field ξ is considered to be the function of an extra dimensional coordinate only. By taking nonzero values of ξ on the branes, the above differential equation has the following solution:

$$\xi(\varphi) = e^{2kr_c|\varphi|} [A e^{\nu kr_c|\varphi|} + B e^{-\nu kr_c|\varphi|}], \quad (4.10)$$

with $\nu = \sqrt{4 + m_\Phi^2/k^2}$. Furthermore, A, B are integration constants that can be obtained from the boundary conditions $\xi(0) = v_h$ and $\xi(\pi) = v_h$, as follows:

$$A = v_v e^{-(2+\nu)kr_c\pi} - v_h e^{-2\nu kr_c\pi},$$

and

$$B = v_h (1 + e^{-2\nu kr_c\pi}) - v_v e^{-(2+\nu)kr_c\pi}.$$

Thus, Eqs. (4.8) and (4.10) specify the field solutions in this spacetime. Recall that the original $F(R)$ model is represented by the action $S_g[G_{MN}] = \int d^4x d\varphi \sqrt{-G} \left[\frac{(R + \alpha R^2)}{2\kappa^2} - \Lambda + V_h \delta(\varphi) + V_v \delta(\varphi - \pi) \right]$. The solution of the spacetime metric (G_{MN}) in the original $F(R)$ model can be obtained from the solutions of the corresponding scalar-tensor theory with the help of the inverse conformal transformation. Thus, the line element turns out to be

$$ds^2 = e^{-\frac{\kappa}{\sqrt{3}}\Phi(\varphi)} [e^{-2kr_c|\varphi|} \eta_{\mu\nu} dx^\mu dx^\nu + r_c^2 d\varphi^2], \quad (4.11)$$

where $\Phi(\varphi) = \langle\Phi\rangle + \xi(\varphi)$ and $\xi(\varphi)$ are obtained in Eq. (4.10). In order to introduce the radion field, r_c is replaced by $T(x)$, known as radion (or modulus) field. For simplicity, here we consider that this new field depends only on the brane coordinates. Thus, the line element becomes

$$ds^2 = e^{-\frac{\kappa}{\sqrt{3}}\Phi(x,\varphi)} [e^{-2kT(x)|\varphi|} g_{\mu\nu}(x) dx^\mu dx^\nu + T(x)^2 d\varphi^2]. \quad (4.12)$$

Here $g_{\mu\nu}(x)$ is the induced on-brane metric and $\Phi(x,\varphi)$ can be obtained from (4.10) by replacing r_c by $T(x)$. Substituting the above solution of G_{MN} into the action $S_g[G_{MN}]$ and integrating over the extra dimensional coordinate φ , the effective four-dimensional on-brane action becomes

$$A_{\text{eff}}^1 = \int d^4x \sqrt{-g} \left[M_{(4)}^2 R_{(4)} - \frac{1}{2} g^{\mu\nu} \partial_\mu \Psi \partial_\nu \Psi - U_{\text{rad}}(\Psi) \right], \quad (4.13)$$

where $M_{(4)}^2 = \frac{M^3}{k} [\sqrt{9 - 40\kappa^2 \alpha \Lambda} - 2]^{1/2}$ is the four-dimensional Planck scale, $R_{(4)}$ is the on-brane Ricci scalar formed by $g_{\mu\nu}(x)$. Moreover, $\Psi(x) = \sqrt{\frac{24M^3}{k}} [1 + \frac{20}{\sqrt{3}} \alpha k^2 \kappa v_h] e^{-k\pi T(x)} = f e^{-k\pi T(x)}$ (with $f = \sqrt{\frac{24M^3}{k}} [1 + \frac{20}{\sqrt{3}} \alpha k^2 \kappa v_h]$), is the canonical radion field and $U_{\text{rad}}(\Psi)$ is the radion potential with the following form [16]:

$$U_{\text{rad}}(\Psi) = \frac{20 \alpha k^5}{\sqrt{3} M^6} \Psi^4 \left[\left(v_h - \frac{\kappa v_h^2}{2\sqrt{3}} + \frac{\kappa v_h v_v}{2\sqrt{3}} \right) (\Psi/f)^\omega - v_v \right]^2, \quad (4.14)$$

where the terms proportional to $\omega (= \frac{m_\Phi^2}{k^2} < 1$, which is also consistent with observational bounds) are neglected. Note that $U_{\text{rad}}(\Psi)$ goes to zero as the higher order curvature parameter α tends to zero. However, as $\alpha \rightarrow 0$, the action contains only the Einstein-Hilbert term which is not able to generate any potential for the modulus field, as shown in [47]. Thereby, the potential term for the radion field is generated entirely due to the presence of the higher order curvature term (αR^2) in the action. Hence, the sign of the higher curvature term comes through the radion potential in the four-dimensional effective action. In this context, the stabilization of the interbrane separation is based on whether the radion potential is stable or not. For $\alpha > 0$, the potential U_{rad} has a minima and a maxima at

$$\Psi_{\text{min}} = \langle\Psi\rangle = \left[\frac{v_v f^\omega}{\left(v_h - \frac{\kappa v_h^2}{2\sqrt{3}} + \frac{\kappa v_h v_v}{2\sqrt{3}} \right)} \right]^{1/\omega}, \quad (4.15)$$

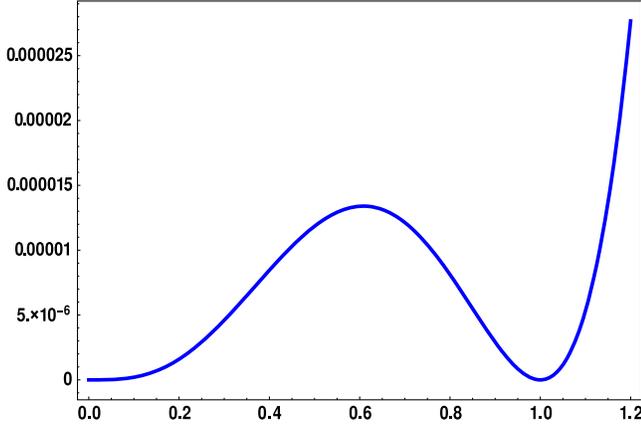


FIG. 9. $U_{\text{rad}}(\Psi)$ vs Ψ for $M = k = 1$ (in Planckian unit), $\omega = 0.04$, $\kappa v_v = 10^{-7}$, $\frac{v_h}{v_v} = 1.2$ and $\alpha = \frac{1}{M^2}$.

and

$$\Psi_{\text{max}} = \left[\left(\frac{2}{2 + \omega} \right) \left(\frac{v_v f^\omega}{v_h - \frac{\kappa v_v^2}{2\sqrt{3}} + \frac{\kappa v_v v_h}{2\sqrt{3}}} \right) \right]^{1/\omega}.$$

The minima of $U_{\text{rad}}(\Psi)$ immediately leads to the stabilization of the interbrane separation,

$$k\pi \langle T(x) \rangle = \frac{4k^2}{m_\Phi^2} \left[\ln \left(\frac{v_h}{v_v} \right) - \frac{\kappa v_v}{2\sqrt{3}} \left(\frac{v_h}{v_v} - 1 \right) \right]. \quad (4.16)$$

The expression of m_Φ^2 as obtained in Eq. (4.6) clearly indicates that $\langle T(x) \rangle$ is proportional to parameter α . Thus, the model considered here would collapse as α tends to zero, as pointed out in the discussion above. Moreover, Eq. (4.14) imposes that $U_{\text{rad}}(\Psi)$ goes to zero at $\Psi = 0$. In Fig. 9, the potential $U_{\text{rad}}(\Psi)$ is depicted.

B. Effective action for S_{KR}

Recall that the 5D KR field action is given by

$$S_H = -\frac{1}{12} \int d^4x d\varphi \sqrt{-G} [H_{MNL} H^{MNL}], \quad (4.17)$$

where the KR field strength tensor H_{MNL} is related to B_{MN} (second rank antisymmetric tensor field) as $H_{MNL} = \partial_{[M} B_{NL]}$, with latin and greek indices running from 0 to 4 and 0 to 3 respectively. It is straightforward to see that the action $S[H]$ is invariant under the gauge transformation $B_{MN} \rightarrow B_{MN} + \partial_{[M} W_{N]}$, with W_N as an arbitrary function of spacetime coordinates. This gauge invariance of the KR field allows us to set $B_{4\mu} = 0$. Then, by using the form of G_{MN} and keeping $B_{4\mu} = 0$, the above action turns out to be

$$S_H = -\frac{1}{12} \int d^4x d\varphi \sqrt{-g} e^{2kT(x)\varphi} T(x) \left[g^{\mu\alpha} g^{\nu\beta} g^{\lambda\gamma} H_{\mu\nu\lambda} H_{\alpha\beta\gamma} - \frac{3}{T(x)^2} e^{-2kT(x)\varphi} g^{\mu\alpha} g^{\nu\beta} B_{\mu\nu} \partial_\phi^2 B_{\alpha\beta} \right]. \quad (4.18)$$

The Kaluza-Klein decomposition for the KR field can be written as

$$B_{\mu\nu}(x, \varphi) = \sum B_{\mu\nu}^{(n)}(x) \chi^{(n)}(x, \varphi), \quad (4.19)$$

where $B_{\mu\nu}^{(n)}(x)$ and $\chi^{(n)}(x, \varphi)$ represent the n th mode of on-brane KR field and extra dimensional KR wave function respectively. The wave function $\chi^{(n)}$ is considered to be a function of the brane coordinates also (apart from the coordinate φ), as we are interested in investigating whether the dynamical evolution of the KR field leads to its invisibility in the present universe.

By substituting the decomposition in the five-dimensional action S_H and integrating over the extra dimension, the four-dimensional effective action turns out to be

$$A_{\text{eff}}^{(2)} = -\frac{1}{12} \int d^4x \sqrt{-g} [g^{\mu\alpha} g^{\nu\beta} g^{\lambda\gamma} H_{\mu\nu\lambda}^{(n)} H_{\alpha\beta\gamma}^{(n)} + 3m_n^2 g^{\mu\alpha} g^{\nu\beta} B_{\mu\nu}^{(n)} B_{\alpha\beta}^{(n)}], \quad (4.20)$$

as far as $\chi^{(n)}(x, \varphi)$ satisfies the following equation of motion:

$$\frac{\partial \chi^{(n)}}{\partial t} \frac{\partial \chi^{(m)}}{\partial t} - \frac{1}{T^2(t)} e^{-2kT(t)\varphi} \chi^{(n)} \frac{\partial^2 \chi^{(m)}}{\partial \varphi^2} = m_n^2 \chi^{(n)} \chi^{(m)}, \quad (4.21)$$

along with the normalization condition,

$$\int_0^\pi d\varphi e^{2kT(t)\varphi} \chi^{(n)} \chi^{(m)} = \frac{1}{T^2(t)} \delta_{mn}, \quad (4.22)$$

where m_n denotes the mass of n th KK mode. As we will see below, obtaining the coupling between the KR field and the Standard Model fields on the visible brane is important. Furthermore, Eq. (4.21) clearly shows that the dynamical evolution of $\chi^{(n)}(x, \varphi)$ is coupled to the modulus (or radion) field $T(x)$. Equations (4.13) and (4.20) immediately lead to the full form of four-dimensional effective action as follows:

$$A_{\text{eff}} = A_{\text{eff}}^{(1)} + A_{\text{eff}}^{(2)} = \int d^4x \sqrt{-g} \left[\frac{M^3}{k} R^{(4)} - \frac{1}{2} g^{\mu\nu} \partial_\mu \Psi \partial_\nu \Psi - U_{\text{rad}}(\Psi) - \frac{1}{12} (g^{\mu\alpha} g^{\nu\beta} g^{\lambda\gamma} H_{\mu\nu\lambda}^{(n)} H_{\alpha\beta\gamma}^{(n)} + 3m_n^2 B_{\mu\nu}^{(n)} B^{\mu\nu(n)}) \right], \quad (4.23)$$

where $U_{\text{rad}}(\Psi)$ is explicitly shown in Eq. (4.14). From now on, we deal with the zeroth Kaluza-Klein mode of Kalb-Ramond field for which $m_{n=0} = 0$. With this lowest KK mode, the four-dimensional effective action turns out to be

$$\begin{aligned} A_{\text{eff}} &= A_{\text{eff}}^{(1)} + A_{\text{eff}}^{(2)} \\ &= \int d^4x \sqrt{-g} \left[\frac{M^3}{k} R^{(4)} - \frac{1}{2} g^{\mu\nu} \partial_\mu \Psi \partial_\nu \Psi - U_{\text{rad}}(\Psi) \right. \\ &\quad \left. - \frac{1}{12} g^{\mu\alpha} g^{\nu\beta} g^{\lambda\gamma} H_{\mu\nu\lambda}^{(0)} H_{\alpha\beta\gamma}^{(0)} \right]. \end{aligned} \quad (4.24)$$

Due to the presence of $U_{\text{rad}}(\Psi)$, the radion field acquires some certain dynamics which affect the dynamical evolution of the KR wave function $\chi^{(0)}(x, \varphi)$, as Ψ and $\chi^{(0)}(x, \varphi)$ are coupled through Eq. (4.21). In such a scenario, our motivation is to investigate whether the evolution of $\chi^{(0)}(x, \varphi)$ leads to a negligible footprint of the KR field in the present visible universe. However, it was shown earlier in [6] that the effect of the KR field may be significant and can play an important role in the early era of the universe. Therefore, in order to address the dynamical suppression of the KR field, it is important to start from the very early universe where we will investigate whether the universe passes through an inflationary era. For these purposes, we try to solve the cosmological Friedmann equations obtained from A_{eff} in the following sections.

C. Effective cosmological equations and solutions

The on-brane metric ansatz that fits our purpose in the present context can be expressed as follows:

$$\begin{aligned} ds_{(4)}^2 &= g_{\mu\nu}(x) dx^\mu dx^\nu \\ &= -dt^2 + b^2(t) [dx^2 + dy^2 + dz^2], \end{aligned} \quad (4.25)$$

where $b(t)$ is the scale factor of our universe. With this ansatz along with the expressions of the energy-momentum tensor for the four-dimensional KR field (as shown in previous section), we obtain the following Einstein's field equations for the action A_{eff} :

$$3H_b^2 = \frac{1}{2} \dot{\Psi}^2 + \frac{20}{\sqrt{3}} \frac{\alpha k^5}{M^6} v_v^2 \Psi^4 [F\Psi^\omega - 1]^2 + \frac{1}{2} h_4 h^4, \quad (4.26)$$

$$\begin{aligned} 2\dot{H}_b + 3H_b^2 + \frac{1}{2} \dot{\Psi}^2 - \frac{20}{\sqrt{3}} \frac{\alpha k^5}{M^6} v_v^2 \Psi^4 [F\Psi^\omega - 1]^2 \\ + \frac{1}{2} h_4 h^4 = 0, \end{aligned} \quad (4.27)$$

where $H_b = \frac{\dot{b}}{b}$ is known as the on-brane Hubble parameter, $F = \frac{1}{v_v f^\omega} (v_h - \frac{\kappa v_h^2}{2\sqrt{3}} + \frac{\kappa v_v v_h}{2\sqrt{3}})$ and $h_4 = H_{123}^{(0)}$ (as the other components of $H_{\mu\nu\lambda}^{(0)}$ vanishes as given by the off-diagonal

Einstein's equations). Moreover, the field equations for $H_{\mu\nu\lambda}^{(0)}$ and for radion field (Ψ) are given by

$$\nabla_\mu H^{\mu\nu\lambda(0)} = \frac{1}{\sqrt{-g}} \partial_\mu [\sqrt{-g} H^{\mu\nu\lambda(0)}] = 0, \quad (4.28)$$

and

$$\ddot{\Psi} + 3H\dot{\Psi} + \frac{80}{\sqrt{3}} \frac{\alpha k^5}{M^6} v_v^2 \Psi^3 [F\Psi^\omega - 1]^2 = 0. \quad (4.29)$$

Following Appendix A, Eq. (4.28) leads to a nonzero component of $H_{\mu\nu\lambda}^{(0)}$, i.e., h_4 depends on the cosmic time t , as also expected from the gravitational field equations. Differentiating both sides of (4.26) with respect to t , the following expression is obtained:

$$6H_b \dot{H}_b = \dot{\Psi} \ddot{\Psi} + \frac{80}{\sqrt{3}} \frac{\alpha k^5}{M^6} v_v^2 \Psi^3 [F\Psi^\omega - 1]^2 \dot{\Psi} + \frac{1}{2} \frac{d}{dt} (h_4 h^4).$$

Furthermore, Eqs. (4.26) and (4.27) immediately give $2\dot{H}_b = \dot{\Psi}^2 - \frac{1}{2} h_4 h^4$. With this expression for \dot{H}_b along with the above equation, we obtain the cosmic evolution for the energy density of the on-brane KR field ($\Omega_{\text{KR}} = \frac{1}{2} h_4 h^4$) as $\frac{d}{dt} \Omega_{\text{KR}} = -6H_b \Omega_{\text{KR}}$. Solving this equation, we get

$$\Omega_{\text{KR}} = \frac{\Omega_0}{b^6}, \quad (4.30)$$

where Ω_0 is an integration constant. Equation (4.30) clearly indicates that the on-brane KR field energy density is proportional to $1/b^6$ (same as previous model) and thus decreases with the expansion of the universe. Moreover, note that Ω_{KR} decreases more rapidly in comparison to normal matter ($\propto 1/b^3$) as well as radiation ($\propto 1/b^4$) energy density. This may well explain why the Kalb-Ramond field has a negligible footprint in the present visible universe. However, at the same time, Eq. (4.30) also reveals that the KR field may have a significant contribution during the early universe [when $b(t)$ is small in comparison to the present one]. On the other hand, recall that the bulk KR field has also an extra dimensional Kaluza-Klein (KK) wave function (besides the on-brane part) which determines the coupling among the on-brane KR field and other matter fields. Thereby, along with the on-brane part, the extra dimensional wave function $\chi^{(0)}(t, \varphi)$ also plays a crucial role to control the signature of the bulk KR field in our visible universe. However, the dynamics of $\chi^{(0)}(t, \varphi)$ are coupled to the evolution of the radion field $\Psi(t)$ and thus we need to obtain $\Psi(t)$ in order to determine the cosmological evolution of the KK wave function.

By Eq. (4.30), there remain two independent equations to fix the evolution of $\Psi(t)$ and $b(t)$,

$$H_b^2 = \frac{1}{3} \left[\frac{1}{2} \dot{\Psi}^2 + \frac{20}{\sqrt{3}} \frac{\alpha k^5}{M^6} v_v^2 \Psi^4 [F\Psi^\omega - 1]^2 \right] + \frac{1}{3} \frac{\Omega_0}{b^6}, \quad (4.31)$$

and

$$\ddot{\Psi} + 3H_b \dot{\Psi} + \frac{80}{\sqrt{3}} \frac{\alpha k^5}{M^6} v_v^2 \Psi^3 [F\Psi^\omega - 1]^2 = 0. \quad (4.32)$$

As mentioned above, we are interested in solving the equations at the early universe where the potential energy of the radion field is considered to be greater than the kinetic term (slow-roll approximation), i.e.,

$$U_{\text{rad}}(\Psi) \gg \frac{1}{2} \dot{\Psi}^2. \quad (4.33)$$

By this approximation, Eqs. (4.31) and (4.32) become

$$H_b^2 = \frac{20}{3\sqrt{3}} \frac{\alpha k^5}{M^6} v_v^2 \Psi^4 [F\Psi^\omega - 1]^2 + \frac{1}{3} \frac{\Omega_0}{b^6}, \quad (4.34)$$

and

$$3H_b \dot{\Psi} + \frac{80}{\sqrt{3}} \frac{\alpha k^5}{M^6} v_v^2 \Psi^3 [F\Psi^\omega - 1]^2 = 0. \quad (4.35)$$

Then, solving the above two equations for $\Psi(t)$ and $b(t)$, we get

$$\Psi(t) = \frac{\Psi_0}{\left[F\Psi_0^\omega - \left(F\Psi_0^\omega - \frac{\sqrt{\Omega_0}}{b_0^3 \xi_0^2} - 1 \right) \exp \left(-8\omega v_v \sqrt{\frac{5}{3\sqrt{3}}} \frac{\alpha k^5}{M^6} (t - t_0) \right) \right]^{1/\omega}}, \quad (4.36)$$

and

$$b(t) = C \left[1 + \sqrt{3\Omega_0} (t - t_0) \right]^{1/3} \exp \left[2v_v \sqrt{\frac{5}{3\sqrt{3}}} \frac{\alpha k^5}{M^6} (g_1(t) - g_2(t)) \right]. \quad (4.37)$$

Recall that $F = \frac{1}{v_v f^\omega} (v_h - \frac{\kappa v_h^2}{2\sqrt{3}} + \frac{\kappa v_v v_h}{2\sqrt{3}})$ and Ψ_0, C are integration constants with $b_0 = C \exp[-\Psi_0^2/8]$. Furthermore, $g_1(t)$ has the following expression:

$$g_1(t) = -\frac{F\Psi_0^\omega}{(F\Psi_0^\omega - 1)} \frac{1}{16\omega v_v \sqrt{\frac{5}{3\sqrt{3}}} \frac{\alpha k^5}{M^6}} \Psi_0^2 {}_2F_1 \left(1, 1, 2 + \frac{2}{\omega}, \frac{F\Psi_0^\omega}{F\Psi_0^\omega - 1} \exp \left(8\omega v_v \sqrt{\frac{5}{3\sqrt{3}}} \frac{\alpha k^5}{M^6} (t - t_0) \right) \right) \\ \times \exp \left(8\omega v_v \sqrt{\frac{5}{3\sqrt{3}}} \frac{\alpha k^5}{M^6} (t - t_0) \right) \left(F\Psi_0^\omega - (F\Psi_0^\omega - 1) \exp \left(-8\omega v_v \sqrt{\frac{5}{3\sqrt{3}}} \frac{\alpha k^5}{M^6} (t - t_0) \right) \right)^{-2/\omega}, \quad (4.38)$$

where ${}_2F_1$ refers to an hypergeometric function. On the other hand, $g_2(t)$ is given by

$$g_2(t) = -\frac{\Psi_0^\omega}{(F\Psi_0^\omega - 1)} \frac{1}{16\omega v_v \sqrt{\frac{5}{3\sqrt{3}}} \frac{\alpha k^5}{M^6}} \Psi_0^2 * \times {}_2F_1 \left(1, 1, 1 + \frac{2}{\omega}, \frac{F\Psi_0^\omega}{F\Psi_0^\omega - 1} \exp \left(8\omega v_v \sqrt{\frac{5}{3\sqrt{3}}} \frac{\alpha k^5}{M^6} (t - t_0) \right) \right) \\ \times \exp \left(8\omega v_v \sqrt{\frac{5}{3\sqrt{3}}} \frac{\alpha k^5}{M^6} (t - t_0) \right) \left(F\Psi_0^\omega - (F\Psi_0^\omega - 1) \exp \left(-8\omega v_v \sqrt{\frac{5}{3\sqrt{3}}} \frac{\alpha k^5}{M^6} (t - t_0) \right) \right)^{1-2/\omega}. \quad (4.39)$$

Note that for $\alpha \rightarrow 0$, the solution of the radion field and Hubble parameter becomes $\Psi(t) = \frac{\Psi_0}{[1 + \frac{\sqrt{\Omega_0}}{b_0^3 \xi_0^2}]^{1/\omega}} = \Psi(t_0)$ and

$H \propto \frac{1}{b^3}$ respectively. However, this result is expected since without any higher order curvature term (i.e., $\alpha = 0$), the radion potential vanishes and the radion field becomes constant $\Psi(t) = \Psi(t_0)$ while the Hubble parameter $\propto 1/b^3$ (solely due to the Kalb-Ramond field has an equation of state parameter $w = 1$). Moreover, for $\Omega_0 = 0$, the solution turn out to be the one for pure $F(R) = R + \alpha R^2$ gravity in the Randall-Sundrum model, as found in [43].

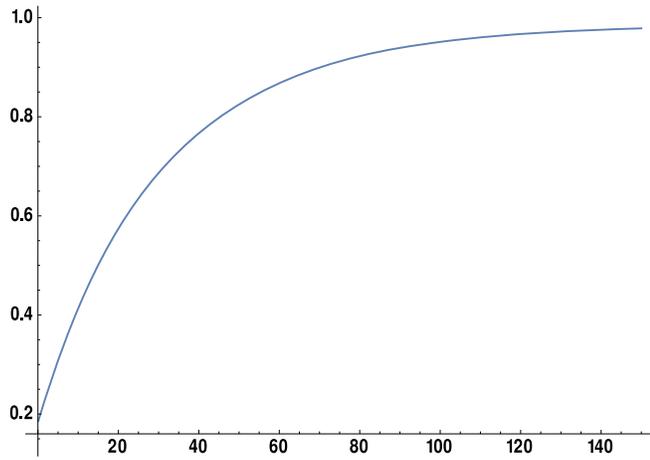


FIG. 10. $\frac{T(t)}{T}$ vs \tilde{t} for $\kappa v_v = \frac{\sqrt{\Omega_0}}{M^2} \simeq 10^{-7}$, $\frac{m_\Phi}{k} = 0.2$ and $\Psi_0 = 36$ (in Planckian unit).

Equation (4.36) shows that the radion field decreases with the cosmic time and finally its bulk leads to [see Eq. (4.15)] asymptotically, i.e.,

$$\Psi(t \gg t_0) = f \left[\frac{v_v}{v_h} \right]^{1/\omega} = \langle \Psi \rangle.$$

Thereby, the dynamics of the interbrane separation ($T(t)$) are as follows: $T(t)$ increases [as $\Psi(t) \propto e^{-k\pi T(t)}$] with the expansion of the universe and gradually goes to a stable value [$k\pi \langle T \rangle = \frac{4k^2}{m_\Phi^2} [\ln(\frac{v_h}{v_v}) - \frac{\kappa v_v}{2\sqrt{3}} (\frac{v_h}{v_v} - 1)]$] asymptotically, as shown in Fig. 10.

Once we obtain the solution for $\Psi(t)$, we can obtain the evolution of the extra dimensional KR wave function $\chi^{(0)}(t, \varphi)$. Nevertheless, let us study whether the solution of the scale factor (4.37) corresponds to an inflationary stage.

D. Beginning of inflation

In order to check whether the solution of the scale factor is consistent with an early inflationary stage, we expand $b(t)$ in the form of Taylor series (about $t = t_0$) and keep the terms up to the linear order in $t - t_0$:

$$b(t \gtrsim t_0) = b_0 [1 + \sqrt{3\Omega_0}(t - t_0)]^{1/3} \times \exp \left[2(F\Psi_0^\omega - 1)\Psi_0^2 v_v \sqrt{\frac{5}{3\sqrt{3}} \frac{\alpha k^5}{M^6}} (t - t_0) \right], \quad (4.40)$$

where b_0 is the value of the scale factor at $t = t_0$ and is related to the integration constant C as

$$b_0 = C \exp[-\Psi_0^2/8].$$

Equation (4.40) leads to an accelerating expansion at $t \rightarrow t_0$ as follows:

$$\frac{\ddot{a}}{a}(t \gtrsim t_0) = \left[2(F\Psi_0^\omega - 1)\Psi_0^2 v_v \sqrt{\frac{5}{3\sqrt{3}} \frac{\alpha k^5}{M^6}} + \sqrt{\Omega_0} \left(1 + \frac{1}{\sqrt{3}} \right) \right] \times \left[2(F\Psi_0^\omega - 1)\Psi_0^2 v_v \sqrt{\frac{5}{3\sqrt{3}} \frac{\alpha k^5}{M^6}} - \sqrt{\Omega_0} \left(1 - \frac{1}{\sqrt{3}} \right) \right]. \quad (4.41)$$

Note that under the condition

$$2(F\Psi_0^\omega - 1)\Psi_0^2 v_v \sqrt{\frac{5}{3\sqrt{3}} \frac{\alpha k^5}{M^6}} > \sqrt{\Omega_0} \left(1 - \frac{1}{\sqrt{3}} \right), \quad (4.42)$$

the early universe expands with an accelerating phase. Otherwise, the acceleration $\frac{\ddot{b}}{b}(t \rightarrow t_0)$ turns out negative. Recall that the on-brane KR field energy density (Ω_{KR}) is proportional to $1/b^6$ as given by Eq. (4.30). Thereby, due to the inflationary expansion of the scale factor, Ω_{KR} rapidly decreases during the very early universe. However, Eq. (4.41) clearly reveals that for $\alpha \rightarrow 0$, $\frac{\ddot{b}}{b}(t \gtrsim t_0)$ becomes less than zero, i.e., the early universe passes through a decelerating phase—solely due to the KR field having equation of state parameter $w = 1$. Therefore, besides stabilizing the interbrane separation, the higher order curvature term also ensures the early inflationary stage subjected to the condition (4.42), which in turn provides a rapid decrease of the Kalb-Ramond field energy density on the visible universe.

E. End of inflation

After obtaining the inflationary solution, it is important to evaluate whether the inflationary phase has a graceful exit in a finite time, as is connected to the resolution of the Horizon problem. The end of inflation can be defined as

$$\frac{\ddot{b}}{b} = \dot{H}_b + H_b^2 = 0. \quad (4.43)$$

Now let us estimate the time interval consistent with this condition. However, at the end of inflation, the term proportional to $1/b^6$ can be safely ignored and thus Eq. (4.31) becomes

$$H_b^2 = \frac{20}{3\sqrt{3}} \frac{\alpha k^5}{M^6} v_v^2 \Psi^4 [F\Psi^\omega - 1]^2.$$

Differentiating both sides of this expression, one gets

$$\dot{H}_b = -\frac{160}{3\sqrt{3}} \frac{\alpha k^5}{M^6} v_v^2 \Psi^2 (F\Psi^\omega - 1)^2. \quad (4.44)$$

Using the above expressions of H_b^2 and \dot{H}_b in Eq. (4.43), we finally get the following condition on the radion field:

$$\Psi = 2\sqrt{2} = \Psi_f = \Psi(t_f), \quad (4.45)$$

where t_f is the time when the radion field takes the value $2\sqrt{2}$ (in Planckian units). Therefore, Eq. (4.45) clearly indicates that the inflationary era continues as long as the

value of the radion field remains greater than $2\sqrt{2}$ (in Planckian units). With this information, one can determine the duration of inflation $t_f - t_0$ from the solution of $\Psi(t)$ [see Eq. (4.36)] as

$$\Psi(t_f) = \frac{\Psi_0}{\left[F\Psi_0^\omega - \left(F\Psi_0^\omega - \frac{\sqrt{\Omega_0}}{b_0^3 \xi_0^2} - 1 \right) \exp\left(-8\omega v_v \sqrt{\frac{5}{3\sqrt{3}} \frac{\alpha k^5}{M^6}} (t_f - t_0)\right) \right]^{1/\omega}}.$$

By simplifying, we get the expression of $t_f - t_0$ as follows:

$$t_f - t_0 = \frac{1}{8\omega v_v \sqrt{\frac{5}{3\sqrt{3}} \frac{\alpha k^5}{M^6}}} \ln \left[\frac{F\Psi_0^\omega - 1 - \frac{\sqrt{\Omega_0}}{b_0^3 \xi_0^2}}{F\Psi_0^\omega - \frac{\Psi_0^\omega}{\Psi_f^\omega}} \right]. \quad (4.46)$$

Recall that $F = \frac{1}{v_v f^\omega} \left(v_h - \frac{\kappa v_h^2}{2\sqrt{3}} + \frac{\kappa v_v v_h}{2\sqrt{3}} \right)$ and $\omega = \frac{m_\Phi^2}{4k^2}$ with m_Φ^2 given in Eq. (4.6). Hence, the duration of inflation depends on the parameters α and Ω_0 , i.e., on the strength of the higher order curvature term and on the energy density of the KR field respectively. Therefore, in order to estimate $t_f - t_0$ explicitly, we need to determine the value of these parameters which, on the other hand, should be consistent with the observational constraints.

F. Spectral index, tensor to scalar ratio and number of e -foldings

As shown in previous sections, the results of Planck 2018 [8] put a certain constraint on the spectral index n_s and the tensor to scalar ratio r as $n_s = 0.9650 \pm 0.00661$ and $r < 0.07$ (combined with BICEP2/Keck-Array) respectively. As shown in Appendix B, the KR tensor $H_{\mu\nu\alpha}^{(0)}$ can

be mapped to a derivative of a massless scalar field and thus n_s , r are defined as follows (in terms of a dimensionless parameter $\epsilon_b = -\frac{\dot{H}_b}{H_b^2}$) [57,58]:

$$\begin{aligned} n_s &= 1 - 6\epsilon_b \Big|_{t=t_0} - 2 \frac{\dot{\epsilon}_b}{H_b \epsilon_b} \Big|_{t=t_0}, \\ r &= 16\epsilon \Big|_{t=t_0}. \end{aligned} \quad (4.47)$$

Thereby, in order to scan the possible values of α and Ω_0 provided by the constraints of Planck 2015, first we need to determine ϵ_b which determines the spectral index and the tensor to scalar ratio. For this purpose, we use the field equation $H_b^2 = \frac{1}{3} U_{\text{rad}}(\Psi) + \frac{\Omega_0}{6b^6}$. Differentiating both sides of this equation with respect to time, we get

$$2\dot{H}_b = -\frac{1}{9H_b^2} \left(\frac{\partial U_{\text{rad}}}{\partial \Psi} \right)^2 - \frac{\Omega_0}{b^6},$$

where we have used the field equation for the radion field. These expressions for \dot{H}_b and H_b^2 lead to the slow-roll parameter ϵ_b as follows:

$$\epsilon_b = \frac{1}{2} \left[\frac{16p^2 v_v^4 \xi_0^6 (F\Psi^\omega - 1)^4 + \frac{3\Omega_0}{b^6} (p v_v^2 \Psi^4 (F\Psi^\omega - 1)^2 + \frac{\Omega_0}{2b^6})}{(p v_v^2 \Psi^4 (F\Psi^\omega - 1)^2 + \frac{\Omega_0}{2b^6})^2} \right], \quad (4.48)$$

where $p = \frac{k^3}{144M^6}$ and $F = \frac{1}{v_v f^\omega} \left(v_h - \frac{\kappa v_h^2}{2\sqrt{3}} + \frac{\kappa v_v v_h}{2\sqrt{3}} \right)$.

By using the expression of ϵ_b along with Eq. (4.47), r and n_s turn out to be

$$r = 8 \left[\frac{16p^2 v_v^4 \xi_0^6 (F\Psi_0^\omega - 1)^4 + \frac{3\Omega_0}{b_0^6} (p v_v^2 \Psi_0^4 (F\Psi_0^\omega - 1)^2 + \frac{\Omega_0}{2b_0^6})}{(p v_v^2 \Psi_0^4 (F\Psi_0^\omega - 1)^2 + \frac{\Omega_0}{2b_0^6})^2} \right], \quad (4.49)$$

and

$$n_s = 1 - \frac{U_1}{U_2}, \quad (4.50)$$

where U_1 and U_2 have the following expressions:

$$U_1 = \left[384p^3 v_v^6 \Psi_0^8 (F\Psi_0^\omega - 1)^6 + \frac{18\Omega_0}{b_0^6} \left(p v_v^2 \Psi_0^4 (F\Psi_0^\omega - 1)^2 + \frac{\Omega_0}{2b_0^6} \right)^2 - \frac{6\Omega_0}{b_0^6} \left(16p^2 v_v^4 \Psi_0^6 (F\Psi_0^\omega - 1)^4 + \frac{3\Omega_0}{b_0^6} \left(p v_v^2 \Psi_0^4 (F\Psi_0^\omega - 1)^2 + \frac{\Omega_0}{2b_0^6} \right) \right) - \frac{144\Omega_0}{b_0^6} p^2 v_v^4 \Psi_0^6 (F\Psi_0^\omega - 1)^4 \right],$$

and

$$U_2 = \left(p v_v^2 \Psi_0^4 (F\Psi_0^\omega - 1)^2 + \frac{\Omega_0}{2b_0^6} \right) \left(16p^2 v_v^4 \Psi_0^6 (F\Psi_0^\omega - 1)^4 + \frac{3\Omega_0}{b_0^6} \left(p v_v^2 \Psi_0^4 (F\Psi_0^\omega - 1)^2 + \frac{\Omega_0}{2b_0^6} \right) \right).$$

As expected, the spectral index and the tensor to scalar ratio depend on the parameters v_v , Ω_0 and Ψ_0 . To fix these parameters, we use the observational results from Planck 2018 [8]. Here we take

$$\kappa v_v = \frac{\sqrt{\Omega_0}}{M^2} \simeq 10^{-7}.$$

These values of v_v and Ω_0 are consistent with the condition that is necessary for neglecting the backreaction of the bulk scalar field and the KR field on the background five-dimensional spacetime. Then, by using Eqs. (4.50) and (4.49) along with the values of v_v and Ω_0 , the parametric plot for n_s vs r is depicted in Fig. 11, which clearly shows that within the interval $34 < \Psi_0 < 38$ (in Planckian unit), both observable quantities n_s and r satisfy the constraints provided by Planck 2018 [8]. Furthermore, with the estimated values of v_v , Ω_0 and Ψ_0 , the duration of inflation $t_f - t_0$ becomes 10^{-10} (Gev) $^{-1}$ as far as the ratio m_Φ/k (bulk scalar field mass to bulk curvature ratio) is taken to be 0.2 [46]. This ratio of m_Φ/k leads to the stabilized interbrane separation as $k\pi\langle T \rangle \simeq 36$ —required for solving the gauge hierarchy problem [46]. We also determine the number of e -foldings, defined by $N = \int_0^{\Delta t} H dt$ ($\Delta t = t_f - t_0$, duration of inflation), numerically and

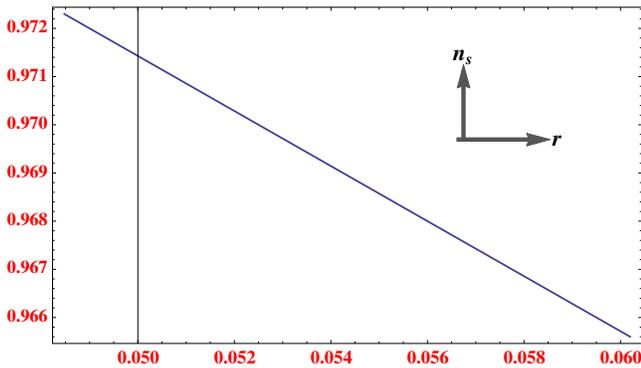


FIG. 11. n_s vs r for $34 < \Psi_0 < 38$ (in Planckian unit).

leading to $N \simeq 58$ (with $\xi_0 = 36$, in Planckian unit). In Table III, the results are summarized.

Table III clearly indicates that the present model may well explain the inflationary scenario of the universe in terms of the observable quantities n_s and r . Moreover, from Tables I and III, n_s lies closer to the observational mean value ($\langle n_s \rangle = 0.9650$) in four dimensions in comparison to the five-dimensional Randall-Sundrum scenario.

By using the solution of $b(t)$ (4.37) along with the estimated values of the parameters (v_v , Ω_0 , Ψ_0), the deceleration parameter is depicted in 12] in terms of the time variable $\tilde{t} = \frac{t}{t_f} N$, which shows that the early universe starts from an accelerating stage with a graceful exit in a finite time.

G. Solution for the Kalb-Ramond extra dimensional wave function

The equation for the zeroth mode of the KR wave function $\chi^{(0)}(t, \varphi)$ follows from (4.21), leading to

$$\left(\frac{\partial \chi^{(0)}}{\partial t} \right)^2 - \frac{1}{T^2(t)} e^{-2kT(t)\varphi} \chi^{(0)} \frac{\partial^2 \chi^{(0)}}{\partial \varphi^2} = 0. \quad (4.51)$$

The dynamics of the interbrane separation controls the evolution of $\chi^{(0)}(t, \varphi)$. The overlap of $\chi^{(0)}(t, \varphi)$ with the brane $\varphi = \pi$ [i.e., $\chi^{(0)}(t, \pi)$] regulates the coupling strengths among the KR field and various Standard Model fields on the visible brane. These interaction terms play the key role to determine the observable signatures of the KR field in our universe, such that we are interested in solving Eq. (4.51) in the vicinity of $\varphi = \pi$ (i.e., near the visible brane). Near the regime of $\varphi \simeq \pi$, Eq. (4.51) can be written as

$$\left(\frac{\partial \chi_v^{(0)}}{\partial t} \right)^2 - \frac{1}{T^2(t)} e^{-2k\pi T(t)} \chi_v^{(0)} \frac{\partial^2 \chi_v^{(0)}}{\partial \varphi^2} = 0, \quad (4.52)$$

TABLE III. Estimated values of various quantities for $\kappa v_v = \frac{\sqrt{\Omega_0}}{M^2} \simeq 10^{-7}$ and $\Psi_0 = 36.5$ (in Planckian unit).

Parameters	Estimated values
n_s	0.9695
r	0.053
$t_f - t_0$	10^{-10} (Gev) $^{-1}$
N	58

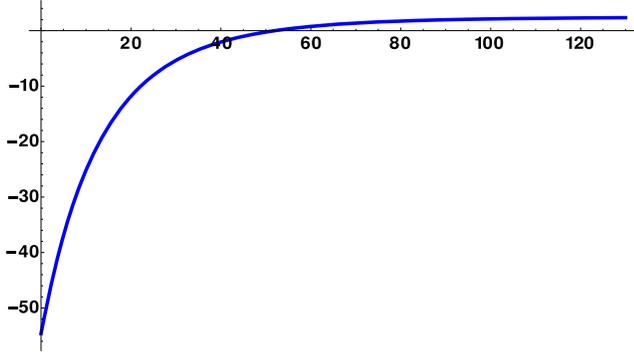


FIG. 12. $q(t)$ vs \tilde{t} for $\kappa v_v = \frac{\sqrt{\Omega_0}}{M^2} \simeq 10^{-7}$ and $\Psi_0 = 36$ (in Planckian unit).

where $\chi_v^{(0)}$ denotes the KR wave function near the visible brane. Equation (4.52) can be solved by using the method of separation of variables as $\chi_v^{(0)}(t, \varphi) = f_1(t)f_2(\varphi)$. By this expression, Eq. (4.52) turns out to be

$$T^2(t)e^{2k\pi T(t)} \frac{1}{f_1^2} \left(\frac{df_1}{dt} \right)^2 = \frac{1}{f_2} \frac{d^2 f_2}{d\varphi^2}. \quad (4.53)$$

As the left- and right-hand sides of Eq. (4.53) are functions of time and φ respectively, both sides can be separately fixed to a constant as follows:

$$T^2(t)e^{2k\pi T(t)} \frac{1}{f_1^2} \left(\frac{df_1}{dt} \right)^2 = \gamma^2, \quad (4.54)$$

and

$$\frac{1}{f_2} \frac{d^2 f_2}{d\varphi^2} = \gamma^2, \quad (4.55)$$

where γ is the constant of separation. The solution for Eq. (4.55) is given by $f_2(\varphi) = e^{-\gamma\varphi}$, while Eq. (4.54) is

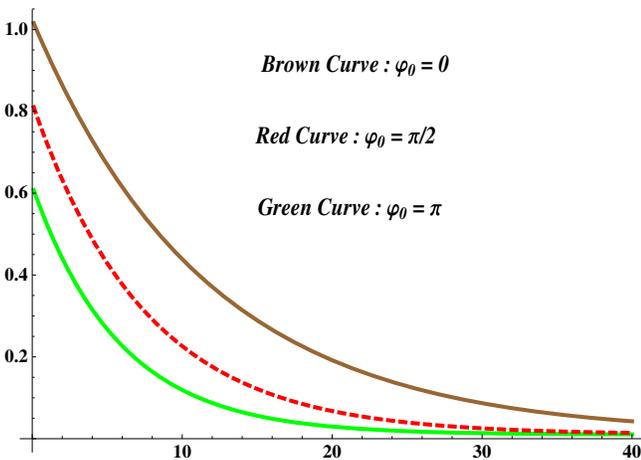


FIG. 13. $\chi_{\varphi_0}^{(0)}(t, \varphi)$ vs \tilde{t} for $\gamma = 0.15$, $\kappa v_v = \frac{\sqrt{\Omega_0}}{M^2} \simeq 10^{-7}$, $\frac{m_\Phi}{k} = 0.2$ and $\Psi_0 = 36$ (in Planckian unit).

solved numerically. Thereby, the solution for $\chi_v^{(0)}(t, \varphi)$ is given by $\chi_v^{(0)}(t, \varphi) = e^{-\gamma\varphi} f_1(t)$. Similarly in the vicinity of a general $\varphi = \text{constant}$ hypersurface within the bulk (i.e., $\varphi \simeq \varphi_0$), the solution of the KR wave function is given by $\chi_{\varphi_0}^{(0)}(t, \varphi) = e^{-\gamma\varphi} f_{\varphi_0}(t)$, where $f_{\varphi_0}(t)$ satisfies the differential equation: $T^2(t)e^{2k\varphi_0 T(t)} \frac{1}{f_{\varphi_0}^2} \left(\frac{df_{\varphi_0}}{dt} \right)^2 = \gamma^2$ [obviously $f_{\varphi_0=\pi}(t) = f_1(t)$]. The solution of $\chi_{\varphi_0}^{(0)}(t, \varphi)$ along with the evolution of brane separation [i.e., $T(t)$, see Eq. (4.36)] leads to the numerical plot for the time evolution of the KR wave function on the $\varphi = \varphi_0$ hypersurface, which is depicted in Fig. 13 for several values of φ_0 . Figure 13 reveals that the zeroth mode of the KR wave function $\chi_{\varphi_0}^{(0)}(t, \varphi)$ decreases with time in the whole five-dimensional bulk, i.e., for $0 \leq \varphi \leq \pi$. However, for a fixed t , $\chi_{\varphi_0}^{(0)}(t, \varphi)$ has different values (in Planckian units) on the hidden ($\varphi_0 = 0$) and visible brane ($\varphi_0 = \pi$) and such hierarchical nature of $\chi_{\varphi_0}^{(0)}(t, \varphi)$ (between the two branes) is controlled by the constant γ .

For $T(t) = \langle T \rangle$, the zeroth mode of the KR wave function acquires a constant value throughout the bulk and given by

$$\chi^{(0)}(t, \varphi) \Big|_{T=\langle T \rangle} = \sqrt{\frac{k}{\langle T \rangle}} e^{-k\pi\langle T \rangle}, \quad (4.56)$$

where we have used the normalization condition as shown in Eq. (4.22). This result is also in agreement with [3]. Using the above expression of $\chi^{(0)}(t, \varphi) \Big|_{T=\langle T \rangle}$, we obtain the coupling strengths of Kalb-Ramond field with $U(1)$ gauge field and fermion field on the visible brane as follows [3]:

$$\lambda_{\text{KR-U}(1)} = \frac{1}{M_p} e^{-k\pi\langle T \rangle}, \quad (4.57)$$

and

$$\lambda_{\text{KR-fer}} = \frac{1}{M_p} e^{-k\pi\langle T \rangle}. \quad (4.58)$$

Here $M_p = \sqrt{M^3/k}$. For $k\langle T \rangle \simeq 12$ (required for solving the gauge hierarchy problem), $e^{-k\pi\langle T \rangle}$ becomes of the order 10^{-16} . Thereby, Eqs. (4.57) and (4.58) clearly indicate that the interaction strengths of the KR field to the matter fields are heavily suppressed over the usual gravity-matter coupling strength $1/M_p$. This may well serve as an explanation about why the behavior of the present universe at large scales is solely governed by gravity and carries practically no observable footprints of antisymmetric Kalb-Ramond field.

V. CONCLUSIONS

We have here addressed the issue of the absence of any perceptible footprints of rank-2 antisymmetric tensor fields, ordinarily known as Kalb-Ramond fields, in the framework

of higher-order curvature gravity, both in four- and in five-dimensional spacetimes. Since all other types of fields, those with scalar, fermion and vector d.o.f., are known to be present in our Universe, the question of the absence of KR fields arises naturally.

We have started from a particular $F(R)$ model, the well known Starobinsky model [49], $F(R) = R + \frac{R^2}{m^2}$, in the presence of a second rank antisymmetric, KR field propagating in a four-dimensional spacetime. In such a scenario, we have obtained the cosmological evolution of the KR field in a flat FLRW universe. Our results reveal that the higher-order curvature term causes a gradual suppression of the energy density ρ_{KR} of the KR field, eventually leading to an imperceptible footprint in the present universe. However, the effect of the KR field might still play a significant role in the early universe. This has led us to study the evolution of the KR field starting at the very early universe, when inflation is supposed to occur. We have shown that inflation is reproduced in our model due to the presence of higher-order terms in the action, so that the early universe expands through an accelerating phase, as far as the condition (3.44) is satisfied. This condition arises owing to an interplay which takes place between the strength of the higher-order curvature terms and the KR field itself, which at the end establishes whether the universe will go through an inflationary stage. In order to test the model with the most recent data (2018 run) from the Planck survey, we have matched the theoretical values for the spectral index of curvature perturbation (n_s) and tensor to scalar ratio (r), which are defined in terms of the slow-roll parameters, with the values coming from the Planck observations. By relying in these definitions, the expressions of n_s and r are explicitly obtained, which provides some suitable values for the remaining free parameters (h_0 , ξ_0), while keeping n_s and r within the confidence regions provided by Planck 2018 (see Table I). In addition, we have also obtained an upper bound for the energy density of the KR field during the early universe, as $\rho_{\text{KR}} \leq 10^{70} \text{ (GeV)}^4$ (see also Ref. [23]).

By contrast, we have proven that in the absence of higher-order curvature terms, the KR field behaves as a stifflike fluid and consequently does not support inflation. However, authors in Ref. [54] showed that a stable de-Sitter solution can be achieved in the context of antisymmetric tensor fields, by introducing a nonminimal coupling between the Ricci scalar and the tensor field. On the other hand, in the present paper, we argue that the minimal prescription (in the presence of an antisymmetric tensor field) can also give rise to an inflationary era, but in the presence of quadratic-curvature gravity. On top of this, we have also considered cubic gravity, where we have shown that, in the presence of the KR field, the spectral index and the tensor to scalar ratio satisfy the observable constraints. However, a successful model for inflation also requires a graceful exit from it, within a finite time with an enough number of e -foldings. Hence, it is important to further

analyze whether R^3 gravity (or a more general R^n gravity with $n \geq 3$) together with the KR field is consistent with an inflationary model, having a graceful exit, which we expected to investigate in a future work.

Moreover, we have also considered the same $F(R)$ model in a five-dimensional Randall-Sundrum warped geometry within a two 3-brane scenario. Such a braneworld scenario requires the stabilization of the interbrane separation (known as modulus or radion), for which one needs a stable potential term for the radion field. Here, the higher-order curvature term αR^2 generates such a stable radion potential, fulfilling the requirement of modulus stabilization, since the radion potential $U_{\text{rad}}(\Psi)$ vanishes as the parameter α goes to zero, which clearly indicates that U_{rad} is generated entirely by the extra gravitational terms in the action. In such a scenario, the cosmological evolution of the KR field is obtained by using a four-dimensional effective theory. However, when the KR field is allowed to propagate along the extra dimension, an additional wave function $\chi^{(0)}$ arises, besides the on-brane part $H_{\mu\nu\lambda}^{(0)}$, which obviously gets coupled to the extra dimensional modulus field. Furthermore, the overlap between $\chi^{(0)}$ and the visible brane determines the coupling strength of the KR field to other matter fields. These interaction terms play a key role in the evaluation of the possible observable effects of the KR field in the current universe.

Due to the presence of $U_{\text{rad}}(\Psi)$, the modulus field $T(t)$ becomes dynamical, since $T(t)$ increases with the cosmic time (t) and finally leads to a stable value asymptotically, as shown in Fig. 10. This dynamics of the radion field triggers such evolution of the extra dimensional KR wave function $\chi^{(0)}(t, \varphi)$ (recall that φ is the extra dimensional coordinate), which decreases with time in the full five-dimensional bulk, i.e., for $0 < \varphi < \pi$. Moreover, for $T(t) = \langle T \rangle$, $\chi^{(0)}(t, \varphi)$ becomes constant throughout the bulk, as obtained in Eq. (4.56).

Consequently, we have obtained the strengths of the couplings of the KR field to several matter fields in the present visible universe, with the result that such interaction strengths come with a heavily suppressed factor over the usual gravity-matter coupling $1/M_p$, thus obtaining a remarkably natural explanation of the absence of any observation of the antisymmetric Kalb-Ramond field at large scales in the current universe.

In addition, the on-brane part, the energy density of the KR field Ω_{KR} has been found to behave as $1/b^6$ (here $b(t)$ is the scale factor of the visible brane), which clearly indicates that Ω_{KR} decreases more rapidly in comparison to radiation and pressureless matter. However, similarly to the four-dimensional case, Eq. (4.30) also entails that Ω_{KR} is large and may play a significant role during the early universe. After exploring the dynamics of the KR field during the early universe, when the scale factor is small compared to the present one, we have found solutions for

the scale factor consistent with an early inflationary stage of the universe. Note that, in the absence of the higher-order curvature term αR^2 , the radion field becomes constant while the Hubble parameter varies as $H_b \propto 1/b^3$. This was to be expected, because for $\alpha \rightarrow 0$, the radion potential tends to zero, and thus the radion field has no dynamics leading to a Hubble parameter that goes as $H_b \propto 1/b^3$ (solely due to the KR field having equation of state parameter = 1). Furthermore, the duration of inflation ($t_f - t_0$) is also obtained by Eq. (4.46), which reveals that the accelerating phase of the universe ends within a finite time. We have also determined the spectral index and tensor to scalar ratio in the present context and found the corresponding constraints on the free parameters when compared to the Planck 2018 values.

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APPENDIX A: TIME DEPENDENCE OF THE KALB-RAMOND FIELD

The field equation for Kalb-Ramond field is given by

$$\partial_\mu[\sqrt{-g}\tilde{H}^{\mu\nu\lambda}] = 0, \quad (\text{A1})$$

where g is the determinant of the on-brane metric. Using the FRW metric ansatz, one obtains $\sqrt{-g} = a^3(t)$, where $a(t)$ is the scale factor of the universe. Thus, Eq. (A1) takes the following form:

$$\begin{aligned} \partial_\mu[a^3(t)\tilde{H}^{\mu\nu\lambda}] &= 0, \\ \Rightarrow \partial_0[a^3(t)\tilde{H}^{0\nu\lambda}] + \partial_1[a^3(t)\tilde{H}^{1\nu\lambda}], \\ \partial_2[a^3(t)\tilde{H}^{2\nu\lambda}] + \partial_3[a^3(t)\tilde{H}^{3\nu\lambda}] &= 0. \end{aligned} \quad (\text{A2})$$

Here the greek indices ν, λ run from 0 to 3.

(i) For $\nu = 2$ and $\lambda = 3$, Eq. (A2) becomes

$$\begin{aligned} \partial_t[a^3(t)\tilde{H}^{023}] + \partial_x[a^3(t)\tilde{H}^{123}], \\ \partial_y[a^3(t)\tilde{H}^{223}] + \partial_z[a^3(t)\tilde{H}^{323}] = 0. \end{aligned} \quad (\text{A3})$$

Due to the antisymmetric nature of the KR field, the last two terms of the above equation identically vanish. Furthermore, from Eq. (3.11), $\tilde{H}^{023} = 0$. As a result, only the second term of Eq. (A3) survives

and leads to the information that the nonzero component of the KR field (\tilde{H}^{123}) is independent of the coordinate x , i.e., $\partial_x[\tilde{H}^{123}] = 0$.

(ii) For $\nu = 1$ and $\lambda = 3$, Eq. (A2) becomes

$$\begin{aligned} \partial_t[a^3(t)\tilde{H}^{013}] + \partial_x[a^3(t)\tilde{H}^{113}], \\ \partial_y[a^3(t)\tilde{H}^{213}] + \partial_z[a^3(t)\tilde{H}^{313}] = 0. \end{aligned} \quad (\text{A4})$$

Here the third term survives, which ensures that \tilde{H}^{123} is independent of y .

(iii) For $\nu = 1$ and $\lambda = 2$, Eq. (A2) becomes

$$\begin{aligned} \partial_t[a^3(t)\tilde{H}^{012}] + \partial_x[a^3(t)\tilde{H}^{112}], \\ \partial_y[a^3(t)\tilde{H}^{212}] + \partial_z[a^3(t)\tilde{H}^{312}] = 0, \end{aligned} \quad (\text{A5})$$

where the fourth term sustains and gives $\partial_z[\tilde{H}^{123}] = 0$.

Therefore it is clear that the nonzero component of the Kalb-Ramond field, i.e., \tilde{H}^{123} , depends only on the time (t) coordinate.

APPENDIX B: EXPRESSING KALB-RAMOND FIELD AS A VECTOR FIELD

Due to the antisymmetric nature, $\tilde{H}_{\mu\nu\alpha}$ has four independent components in four dimensions and thus it can be equivalently expressed as a vector field as

$$\tilde{H}_{\mu\nu\alpha} = \varepsilon_{\mu\nu\alpha\beta}\Upsilon^\beta, \quad (\text{B1})$$

where $\varepsilon_{\mu\nu\alpha\beta}$ is the Levi-Civita symbol and Υ^β is a vector field propagating in four-dimensional spacetime. The four components of Υ^β are connected with the independent components of $\tilde{H}_{\mu\nu\alpha}$ as follows:

$$\begin{aligned} \tilde{H}_{012} = h_1 = \Upsilon^3, & \quad \tilde{H}_{013} = h_2 = -\Upsilon^2, \\ \tilde{H}_{023} = h_3 = \Upsilon^1, & \quad \tilde{H}_{123} = h_4 = -\Upsilon^0. \end{aligned} \quad (\text{B2})$$

Here, we assume the FLRW metric as the ansatz,

$$ds^2 = -dt^2 + a^2(t)[dx^2 + dy^2 + dz^2].$$

By this metric, the off-diagonal Einstein's equations become

$$\Upsilon_3\Upsilon^2 = \Upsilon_3\Upsilon^1 = \Upsilon_2\Upsilon^1 = \Upsilon_0\Upsilon^3 = \Upsilon_0\Upsilon^2 = \Upsilon_0\Upsilon^1 = 0. \quad (\text{B3})$$

The above set of equations clearly indicates that only one component of Υ^β is nonzero which reduces the independent components of $\tilde{H}_{\mu\nu\alpha}$ to 1. Therefore, in a spatially flat FLRW metric in four dimensions, Υ^β can be expressed as a derivative of a massless scalar field $Z(x^\mu)$ (i.e., $\Upsilon^\beta = \partial^\beta Z$), which further relates the KR field tensor with the scalar field as follows:

$$\begin{aligned}\tilde{H}_{\mu\nu\alpha} &= \varepsilon_{\mu\nu\alpha\beta} \Upsilon^\beta \\ &= \varepsilon_{\mu\nu\alpha\beta} \partial^\beta Z.\end{aligned}\quad (\text{B4})$$

Due to the FLRW metric, the scalar field Z is considered to be homogeneous in space and thus its equation of motion turns out to be

$$\ddot{Z} + 3H\dot{Z} = 0, \quad (\text{B5})$$

where H is the Hubble parameter. Then, by solving the above equation, one obtains

$$\frac{\partial Z}{\partial t} \propto \frac{1}{a^3} = \frac{d}{a^3}. \quad (\text{B6})$$

Here d is a proportional constant. By this solution of $\frac{\partial Z}{\partial t}$, the diagonal Friedmann equations take the following form:

$$\begin{aligned}H^2 &= \frac{\kappa^2}{3} \left[\frac{1}{2} \dot{\xi}^2 + \frac{m^2}{8\kappa^2} (1 - e^{\sqrt{\frac{2}{3}\kappa\xi}})^2 + \frac{1}{2} \dot{Z}^2 \right] \\ &= \frac{\kappa^2}{3} \left[\frac{1}{2} \dot{\xi}^2 + \frac{m^2}{8\kappa^2} (1 - e^{\sqrt{\frac{2}{3}\kappa\xi}})^2 + \frac{d}{2a^6} \right],\end{aligned}\quad (\text{B7})$$

and

$$\begin{aligned}2\dot{H} + 3H^2 &= -\kappa^2 \left[\frac{1}{2} \dot{\xi}^2 - \frac{m^2}{8\kappa^2} (1 - e^{\sqrt{\frac{2}{3}\kappa\xi}})^2 + \frac{1}{2} \dot{Z}^2 \right] \\ &= -\kappa^2 \left[\frac{1}{2} \dot{\xi}^2 - \frac{m^2}{8\kappa^2} (1 - e^{\sqrt{\frac{2}{3}\kappa\xi}})^2 + \frac{d}{2a^6} \right].\end{aligned}\quad (\text{B8})$$

Recall that $\xi(t)$ is the scalar field which arises from the higher order curvature d.o.f. Furthermore, the field equation for $\xi(t)$ is given by

$$\ddot{\xi} + 3H\dot{\xi} - \sqrt{\frac{2}{3}} \frac{m^2}{4\kappa} e^{\sqrt{\frac{2}{3}\kappa\xi}} (1 - e^{\sqrt{\frac{2}{3}\kappa\xi}}) = 0. \quad (\text{B9})$$

Note that the above equations match with the field equations obtained in Eqs. (3.18) and (3.19), by identifying the constant d with h_0 . This leads to the argument that the two representations ($\tilde{H}_{\mu\nu\alpha}$ is expressed/is not expressed by a vector field) are equivalent at the level of equation of motion.

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