

Gravitational electromotive force in magnetic reconnection around Schwarzschild black holes

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We analytically explore the effects of the gravitational electromotive force on magnetic reconnection around Schwarzschild black holes through a generalized general-relativistic magnetohydrodynamic model that retains two-fluid effects. It is shown that the gravitational electromotive force can couple to collisionless two-fluid effects and drive magnetic reconnection. This is allowed by the departure from quasineutrality in curved spacetime, which is explicitly manifested as the emergence of an effective resistivity in Ohm's law. The departure from quasineutrality is owed to different gravitational pulls experienced by separate parts of the current layer. This produces an enhancement of the reconnection rate due to purely gravitational effects.

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Magnetic fields are ubiquitous in the Universe and they play a major role in a variety of astrophysical systems. At large scales, the behavior of highly conducting magnetized plasmas is well described by the equations of ideal magnetohydrodynamics (MHD), which impose significant constraints on the plasma dynamics. Indeed, an ideal MHD evolution implies the frozen-in condition and therefore the preservation of field line connectivity among fluid elements. This is a remarkably general result, which is valid in nonrelativistic [1], special relativistic [1–3], as well as general relativistic [4] plasmas.

On the other hand, at small spatial scales, physical effects beyond ideal MHD can break the frozen-in condition and allow for a topological rearrangement of the magnetic field configuration that occurs on time scales much faster than the global magnetic diffusion time. This process, known as magnetic reconnection [5], enables a rapid conversion of magnetic energy into plasma particle energy, and is generally believed to be the underlying mechanism that powers some of the most energetic astrophysical phenomena in the Universe, such as solar and stellar flares [6,7], nonthermal signatures of pulsar wind nebulae [8,9], and gamma-ray flares in blazar jets [10,11].

Electrical resistivity due to Coulomb interactions between charged particles is the prototypical effect that can break the frozen-in condition and allow for the reconnection of magnetic field lines. This was indeed employed in many models of magnetic reconnection, from the pioneering Sweet-Parker model [12,13] to the more

recent models of fast magnetic reconnection mediated by the plasmoid instability [14–19]. Anomalous resistivity due to wave-particle interactions and scatterings off the turbulent fluctuations can also enable magnetic reconnection, and they have been considered as a possible agent of fast reconnection [20–22]. Depending on the value of the classical/anomalous resistivity, other nonideal effects can be even more important. For example, electron inertia effects are indeed known to permit nondissipative magnetic reconnection [23–25], and in an analogous fashion, non-gyrotropic electron pressure tensor effects can break the frozen-in constraint and sustain most of the reconnection electric field required for fast reconnection [26–28].

In relativistic plasmas, thermal effects proportional to the relativistic enthalpy density couple to the inertial effects, leading to an increase of the magnetic reconnection rate [29,30]. Furthermore, the Hall terms, that cannot cause magnetic reconnection *per se* in the nonrelativistic case, do allow for a change in the magnetic field line connectivity if there is a significant difference between the enthalpy density of the positively and negatively charged fluids constituting the plasma [31]. The situation is rendered even more complex in the presence of a strong gravitational field, as in the vicinity of compact objects like black holes. Several studies have predicted the formation of reconnection layers in the vicinity of black holes [32–38], and the theoretical investigation of magnetic reconnection in curved spacetime has just started [30,39].

With this manuscript we intend to explore the effects of the gravitational electromotive force on magnetic reconnection in a curved spacetime around a black hole. In previous works [30,39] the role of the radial gravitational

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force due to the black hole was not studied, as it requires a correct definition of the gravitational electromotive forces as well as understanding the influence of the charge density in curved spacetimes (see below). That the gravitational electromotive force contributes to magnetic reconnection was suggested by Koide [40], without working out explicitly its quantitative effects on the reconnection rate. Here we focus on the simplest form of the gravitational field created by a black hole, i.e., a Schwarzschild black hole, and we calculate the reconnection rate due to the gravitational electromotive force.

In order to show that the gravitational field of a Schwarzschild black hole introduces new effects that are relevant for reconnection, we adopt a generalized version of the general relativistic magnetohydrodynamic (GRMHD) equations [30,40] which retain two-fluid effects that are neglected in the simpler single-fluid descriptions. In particular, we employ a set of equations [30] that describes electron-ion plasmas in the thermal-inertia regime [41,42]. This is the regime in which the thermal-inertia terms are larger than the Hall terms. Therefore, by taking into account the proper mass ratio between the positively and negatively charged particles, the same set of equations describes also pair plasmas, where the Hall terms vanish identically.

The considered spacetime $x^\mu = (t, x^1, x^2, x^3)$ is characterized by a metric $g_{\mu\nu}$, where the line element is given by $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$. Note that we choose units in which the speed of light c is unity. The GRMHD equations deal with a single-fluid plasma model with proper enthalpy density $h = n^2(h_+/n_+^2 + h_-/n_-^2)$, where n_\pm indicate the proper particle number density for the positively (+) and negatively (-) charged components fluids. Similarly, the enthalpy density h_\pm of each charged fluid is specified with the corresponding subscript, and $n = n_+ + n_-$. Furthermore, it is assumed that $\Delta h \ll h$, where $\Delta h = mn^2(h_+/m_+n_+^2 - h_-/m_-n_-^2)/2$ is the difference between enthalpy densities of the fluids (with $m = m_+ + m_-$, and m_\pm indicating the mass of the corresponding charged particle). It is also assumed the equation of state $h_\pm = m_\pm n_\pm K_3(m_\pm/k_B T_\pm)/K_2(m_\pm/k_B T_\pm)$ [43,44], where K_2 and K_3 are the modified Bessel functions of the second kind of orders 2 and 3, T_\pm are the temperatures of each fluid, and k_B is the Boltzmann constant.

In this model the momentum equation that retains thermal-inertia effects is [30,40]

$$\nabla_\nu \left[h \left(U^\mu U^\nu + \frac{\xi}{4n^2 e^2} J^\mu J^\nu \right) \right] = -\nabla^\mu p + J_\nu F^{\mu\nu}, \quad (1)$$

where ∇_ν denotes the covariant derivative associated with the spacetime metric $g_{\mu\nu}$, U^μ is the plasma four-velocity, J^μ is the four-current density, and $F^{\mu\nu}$ is the electromagnetic field tensor. Furthermore, $p = p_+ + p_-$ indicates the proper plasma pressure, e is the electron charge, and $\xi = 1 - (\Delta\mu)^2$, with $\Delta\mu = (m_+ - m_-)/(m_+ + m_-)$.

Observe that $\xi \approx 4m_-/m_+$ for an electron-ion plasma, while $\xi = 1$ for a pair plasma.

Furthermore, the generalized Ohm's law in the thermal-inertia regime is [30]

$$U_\nu F^{\mu\nu} = \eta [J^\mu - \rho'_e U^\mu] + \frac{\xi}{4e^2 n} \nabla_\nu \left[\frac{h}{n} \left(U^\mu J^\nu + J^\mu U^\nu - \frac{\Delta\mu}{ne} J^\mu J^\nu \right) \right], \quad (2)$$

where $\rho'_e = -U_\nu J^\nu$ is the charge density observed by the local center-of-mass frame, and η is the electrical resistivity, which is considered as a phenomenological parameter. Notice that, in comparison with the model equations of Ref. [30], we are considering a plasma where the thermal energy exchange rate between the two fluids is negligible [40], i.e., the redistribution coefficient of the thermalized energy to the positively and negatively charged fluids is 0.

The plasma dynamics is completed by the continuity equation

$$\nabla_\nu (n U^\nu) = 0, \quad (3)$$

and Maxwell's equations

$$\nabla_\nu F^{\mu\nu} = J^\mu, \quad \nabla_\nu F^{*\mu\nu} = 0, \quad (4)$$

where $F^{*\mu\nu}$ is the dual of the electromagnetic field tensor.

To explicitly display the gravitational effects in the above plasma model in a familiar fashion, we write the previous equations in the 3 + 1 formalism [45–47]. In such a form, the spacetime curvature effects become apparent in a set of vectorial equations. For a Schwarzschild background, with spherical geometry, the line element becomes

$$ds^2 = -\alpha^2 dt^2 + h_1^2 dr^2 + h_2^2 d\theta^2 + h_3^2 d\phi^2, \quad (5)$$

with $\alpha = \sqrt{1 - 2r_s/r}$, $h_1 = 1/\alpha$, $h_2 = r$, and $h_3 = r \sin \theta$. Here, α is known as the lapse function, r is the radial distance to the black hole, r_s is the half of the Schwarzschild radius (hereafter $G = 1 = c$), $0 \leq \theta \leq \pi$, and $0 \leq \phi \leq 2\pi$. In order to properly describe the plasma dynamics, it is also useful to rewrite the plasma vectorial equations by introducing a locally nonrotating frame called “zero-angular-momentum-observer” (ZAMO) frame [30,39,40,48], which introduces a locally Minkowskian spacetime where the line element (5) can be written as $ds^2 = -\hat{d}t^2 + \sum_{i=1}^3 (d\hat{x}^i)^2$, where $\hat{d}t = \alpha dt$ and $d\hat{x}^i = h_i dx^i$. In the following, quantities observed in the ZAMO frame are denoted with hats.

We first consider the continuity equation (3), which can be rewritten in the ZAMO frame as [30,40]

$$\frac{\partial(\hat{\gamma}n)}{\partial\hat{t}} + \frac{\alpha}{r^2 \sin \theta} \sum_j \frac{\partial}{\partial\hat{x}^j} \left(\frac{r^2 \sin \theta}{h_j} \hat{\gamma} n \hat{v}^j \right) = 0, \quad (6)$$

where \hat{v} is the velocity in the ZAMO frame, and $\gamma = (1 - \hat{v}^2)^{-1/2}$ is the Lorentz factor (we use latin indices for space components). We also consider the spatial components of the generalized momentum equation (1), which lead to the dynamical equation

$$\begin{aligned} \frac{\partial \hat{P}^i}{\partial t} = & -\frac{\alpha}{r^2 \sin \theta} \sum_j \frac{\partial}{\partial x^j} \left(\frac{r^2 \sin \theta}{h_j} \hat{T}^{ij} \right) \\ & - (\epsilon + \gamma \rho) \frac{1}{h_i} \frac{\partial \alpha}{\partial x^i} + \sum_j \alpha [G_{ij} \hat{T}^{ij} - G_{ji} \hat{T}^{jj}], \end{aligned} \quad (7)$$

where

$$\hat{P}^i = h\gamma^2 \hat{v}^i + \frac{h\xi}{4n^2 e^2} \hat{J}^i \hat{J}^0 + \sum_{j,k} \epsilon_{ijk} \hat{E}_j \hat{B}_k, \quad (8)$$

$$\epsilon = h\gamma^2 + \frac{h\xi}{4e^2 n^2} (\hat{J}^0)^2 - p - \rho\gamma + \frac{1}{2} (\hat{B}^2 + \hat{E}^2), \quad (9)$$

and

$$\begin{aligned} \hat{T}^{ij} = & p\delta^{ij} + h\gamma^2 \hat{v}^i \hat{v}^j + \frac{h\xi}{4e^2 n^2} \hat{J}^i \hat{J}^j \\ & + \frac{1}{2} (\hat{B}^2 + \hat{E}^2) \delta^{ij} - \hat{B}_i \hat{B}_j - \hat{E}_i \hat{E}_j. \end{aligned} \quad (10)$$

Here, \hat{J}^0 is the separation of charge density while \hat{J}^i is the current density, both observed in the ZAMO frame. It is the main goal of this work to show (below) that \hat{J}^0 affects the magnetic reconnection process by the gravitational electromotive force. Besides, it is important to notice that \hat{J}^0 is related to the invariant $\rho'_e = -U_\mu J^\mu$. We also specify that \hat{E}_j and \hat{B}_j are the electric and magnetic fields measured in the ZAMO frame, $G_{ij} = -(1/h_i h_j)(\partial h_i / \partial x^j)$, and ϵ_{ijk} is the Levi-Civita symbol.

For the spatial components of the generalized Ohm's law (2), in the ZAMO frame we have

$$\begin{aligned} & \frac{\xi}{en} \frac{\partial}{\partial t} \left[\frac{h\gamma}{4en} (\hat{J}^i + \hat{J}^0 \hat{v}^i) \right] \\ = & -\frac{h\xi\gamma\hat{J}^0}{2e^2 n^2 h_i} \frac{\partial \alpha}{\partial x^i} - \frac{\alpha}{enr^2 \sin \theta} \sum_j \frac{\partial}{\partial x^j} \left(\frac{r^2 \sin \theta}{h_j} \hat{K}^{ij} \right) \\ & + \frac{\alpha}{en} \sum_j (G_{ij} \hat{K}^{ij} - G_{ji} \hat{K}^{jj}) \\ & + \alpha\gamma \hat{F}_{i0} + \alpha\gamma \hat{v}^j \hat{F}_{ij} - \alpha\eta (\hat{J}^i - \rho'_e \gamma \hat{v}^i), \end{aligned} \quad (11)$$

where $\hat{K}^{ij} = (h\xi\gamma/4en)(\hat{v}^i \hat{J}^j + \hat{v}^j \hat{J}^i)$. Similarly, the temporal component of Eq. (2) becomes [40]

$$\begin{aligned} & \frac{\xi}{2en} \frac{\partial}{\partial t} \left(\frac{h\gamma\hat{J}^0}{en} \right) \\ = & -\frac{h\xi\gamma}{4e^2 n^2} \sum_j \frac{1}{h_j} \frac{\partial \alpha}{\partial x^j} (\hat{J}^j + \hat{J}^0 \hat{v}^j) \\ & - \frac{\alpha}{enr^2 \sin \theta} \sum_j \frac{\partial}{\partial x^j} \left(\frac{r^2 \sin \theta}{h_j} \frac{h\xi\gamma}{4en} [\hat{J}^j + \hat{J}^0 \hat{v}^j] \right) \\ & + \alpha\gamma \hat{v}^j \hat{F}_{j0} - \alpha\eta (\hat{J}^0 - \rho'_e \gamma). \end{aligned} \quad (12)$$

Finally, we rewrite Maxwell's equations (4) in the ZAMO frame. These are

$$\sum_j \frac{\partial}{\partial x^j} \left(\frac{r^2 \sin \theta}{ah_j} \hat{B}_j \right) = 0, \quad (13)$$

$$\frac{\alpha}{r^2 \sin \theta} \sum_j \frac{\partial}{\partial x^j} \left(\frac{r^2 \sin \theta}{ah_j} \hat{E}_j \right) = \hat{J}^0, \quad (14)$$

$$\alpha \hat{J}^i + \frac{\partial \hat{E}_i}{\partial t} = \frac{ah_i}{r^2 \sin \theta} \sum_{j,k} \epsilon^{ijk} \frac{\partial}{\partial x^j} (ah_k \hat{B}_k), \quad (15)$$

$$\frac{\partial \hat{B}_i}{\partial t} = \frac{-ah_i}{r^2 \sin \theta} \sum_{j,k} \epsilon^{ijk} \frac{\partial}{\partial x^j} (ah_k \hat{E}_k). \quad (16)$$

The gravitational field of a Schwarzschild black hole introduces effects in the generalized Ohm's law (11) that can be seen as effective electric fields. In particular, terms with the form $G_{ij} \hat{K}^{ij}$ and $G_{ji} \hat{K}^{jj}$ in Eq. (11) can introduce effective resistivities of the order $(h\xi/4en)(\partial_j h_i / h_i h_j)$, where both the gravitational field and the thermal-inertial effects are important. However, as we see below, in the simplest possible geometry for the reconnection layer, both these terms vanish. On the other hand, as noticed by Koide in Ref. [40], the term proportional to $\hat{J}^0 (\partial_i \alpha / h_i)$ in Eq. (11) produces a radial contribution to the generalized Ohm's law that can be interpreted as an effective electric field, as long as \hat{J}^0 does not vanish. Therefore, in this work we analyze this possibility, showing that a reconnection layer around a Schwarzschild black hole allows a solution in which the separation of charge \hat{J}^0 is finite, and that in this case the electromotive force due to gravity can drive magnetic reconnection.

Without loss of generality, let us assume that the reconnection layer is at $\theta = \pi/2$ at some given distance r . We consider a quasi-two-dimensional reconnection layer having characteristic length L and width δ such that $\delta \ll L$. The length L is in the ϕ -direction, while the width δ is in the θ -direction, as depicted in Fig. 1. We also assume that the layer is not close to the black hole, $\delta \ll L \ll r$. This model allows us to study magnetic reconnection using a Sweet-Parker-like approach for a plasma that is supported against the black hole gravity [49–51], as for the model we

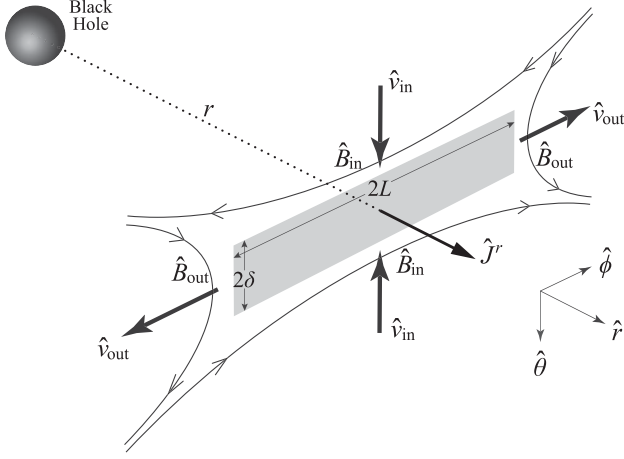


FIG. 1. Sketch of a magnetic reconnection layer showing the studied configuration. The shaded gray area represents the magnetic diffusion region.

investigated in Kerr curved spacetime [30,39]. We also assume that the radial plasma velocity is null or negligible, i.e. $\hat{v}^r = 0$, and that in the diffusion region $\hat{J}^\theta = 0$ and $\hat{J}^\phi = 0$. Then, it is important to observe that $\rho'_e = -U_\mu J^\mu = \gamma \hat{J}^0 \neq 0$, in general [40]. Furthermore, the reconnecting magnetic field has magnitude \hat{B}_{in} in the ϕ -direction (with no radial component in the reconnection layer), while the electric field is in the radial direction (see Fig. 1).

Under the above assumptions, we can readily calculate the outflow velocity of the plasma accelerated through the reconnection channel. This plasma outflow is in the ϕ -direction along the neutral line. By using the momentum equation (7) we find

$$\sum_j \frac{\partial}{\partial x^j} \left(\frac{r^2 \sin \theta}{h_j} \hat{T}^{\phi j} \right) = 0, \quad (17)$$

as other terms identically vanish along the ϕ -direction. The solution for this equation is $T^{\phi\phi} = 0$. Taking the tensor (10) along the neutral line, and using that $p \approx \hat{B}_{in}^2/2 \approx h/4$ in the relativistic regime [30,39], we can readily find that the outflow plasma velocity satisfies $\gamma_{out} \hat{v}_{out} \approx 1/\sqrt{2}$.

Similarly, we can estimate other relevant quantities for this reconnection layer configuration. From the divergenceless equation (13), the outflow magnetic field in the θ -direction is

$$\hat{B}_\theta \Big|_{out} \approx \frac{\delta}{L} \hat{B}_{in}. \quad (18)$$

On the other hand, using the continuity equation (6) for the Schwarzschild geometry, the inflow plasma velocity can be written as

$$\gamma_{in} \hat{v}_{in} \approx \frac{\delta}{L} \gamma_{out} \hat{v}_{out}. \quad (19)$$

Besides, from Eq. (15) we obtain that the radial current density at the X point is simply

$$\hat{J}^r \Big|_X \approx \frac{\hat{B}_{in}}{\delta}. \quad (20)$$

The results (18), (19), and (20) are equivalent to those pertaining to relativistic plasmas in flat spacetimes [29,52]. The explanation for this is the chosen configuration around the Schwarzschild black hole. The simple geometry studied here, with the invoked assumptions, implies that no gravitational effects appear in the momentum equation or Maxwell's equations when they are evaluated in the reconnection layer. As we see now, all the gravitational effects appear in the generalized Ohm's law.

We focus on the spatial part of the generalized Ohm's law (11) along the r -direction. For our geometry, in the current sheet this equation becomes

$$\begin{aligned} \frac{\alpha}{enr^2 \sin \theta} \sum_j \frac{\partial}{\partial x^j} \left(\frac{r^2 \sin \theta}{h_j} \frac{h\xi\gamma}{4en} \hat{v}^j \hat{J}^r \right) + \frac{h\xi\alpha\gamma\hat{J}_0}{2e^2n^2} \frac{\partial\alpha}{\partial r} \\ = +\alpha\gamma\hat{E}_r - \alpha\gamma\hat{v}^\theta\hat{B}_\phi + \alpha\gamma\hat{v}^\phi\hat{B}_\theta - \alpha\eta\hat{J}^r. \end{aligned} \quad (21)$$

We evaluate this equation in the inflow point, where the inflow plasma velocity is in the θ -direction and the term proportional to the resistivity is negligible. Thus, we get

$$\hat{E}_r \Big|_{in} \approx \hat{v}_{in} \hat{B}_{in} + \frac{h\xi\hat{J}_0}{2e^2n^2} \frac{r_s}{\alpha r^2} \Big|_{in}, \quad (22)$$

where we have used that $\partial_r \alpha = r_s/\alpha r^2$. Here, we have neglected the nonlinear terms and considered $\gamma_{in} \approx 1$, in agreement with the results of Refs. [30,39]. We can also evaluate Eq. (21) at the X point (where the plasma velocity vanishes), obtaining

$$\hat{E}_r \Big|_X \approx (\eta + \Lambda) \hat{J}^r + \frac{h\xi\hat{J}_0}{2e^2n^2} \frac{r_s}{\alpha r^2} \Big|_X, \quad (23)$$

where we have introduced the effective relativistic collisionless resistivity [29]

$$\Lambda = \frac{h\xi}{4e^2n^2L}. \quad (24)$$

Both results (22) and (23) reduce to those of Ref. [29] in the flat spacetime limit $r_s \rightarrow 0$.

As there is no quasineutrality, with \hat{J}^0 different from 0, the electric fields $\hat{E}_r \Big|_{in}$ and $\hat{E}_r \Big|_X$ are not equal. This is due to the presence of different gravitational gradients at the inflow and X points. The radial distance of the inflow point $r \Big|_{in}$ is related to the radial distance r of the X point

by $r|_{\text{in}} \approx r + \delta^2/(8r)$, where $r|_X \equiv r$, and thereby the two points experience slightly different gravitational pulls. We can obtain the difference between the electric field at the inflow and X points by using Eq. (14). By integration among these two points in the current layer, and the radial distance of the inflow point, we get

$$\hat{E}_r|_{\text{in}} - \hat{E}_r|_X \approx \frac{\delta^2}{8\alpha r} \hat{J}^0, \quad (25)$$

where the lapse function must be evaluated at the distance r of the X point.

What remains to be done is to obtain a relation between the current density and \hat{J}^0 . This can be achieved through the temporal part of the generalized Ohm's law, namely Eq. (12). We can use that $\rho'_e = \gamma \hat{J}^0$ is an invariant to calculate \hat{J}^0 by evaluation of Eq. (27) at the outflow point. Thereby, assuming that the variations of the current density are negligible in this geometry compared to the gravitational gradient, i.e., neglecting the divergence of the current density with respect to the gradient of the lapse function projected along the current

$$\frac{\alpha}{r^2} \frac{\partial}{\partial r} (\alpha r^2 \hat{J}^r) \ll \alpha \frac{\partial \alpha}{\partial r} \hat{J}^r, \quad (26)$$

from Eq. (12) evaluated in the outflow point we obtain

$$0 \approx -\frac{\gamma \Lambda L r_s}{r^2} \hat{J}^r|_{\text{out}} - \frac{\alpha \gamma \Lambda L}{r} \frac{\partial}{\partial \phi} (\hat{v}^\phi \hat{J}^0)|_{\text{out}} + \alpha \eta \gamma^2 \hat{v}^2 \hat{J}^0|_{\text{out}}, \quad (27)$$

where we have used that $1 - \gamma^2 = -\gamma^2 \hat{v}^2$. As \hat{J}^0 decreases to the X point, the previous equation can be solved for \hat{J}^0 to finally get

$$\hat{J}^0 \approx \frac{2\Lambda L \chi r_s}{\alpha r^2 (\eta + \Lambda)} \hat{J}^r, \quad (28)$$

where $\chi = 1 - L^2/(4r^2) - r_s L^2/(8\alpha^2 r^3)$, and we have used the radial distance of the outflow point $r|_{\text{out}} \approx r + L^2/(8r)$ in terms of the radial distance r of the X point. Notice that the separation of charge \hat{J}^0 is only relevant in curved spacetimes, as it vanishes when $r_s \rightarrow 0$.

Finally, using the above equations, we can obtain the reconnection rate for this configuration. In order to preserve the validity of our result, we restrict ourselves to a plasma sufficiently far from the black hole, $r_s \ll r$. In this case, the reconnection rate becomes simply

$$\hat{v}_{\text{in}} \approx \left(\frac{1}{S} + \frac{\Lambda}{L} \right)^{1/2} \left[1 + \frac{\Lambda L^2 r_s}{8\alpha^2 r^3 (\eta + \Lambda)} \right], \quad (29)$$

where $S = L/\eta \gg 1$ is the relativistic Lundquist number.

The result (29) shows that the gravitational electromotive force increases the reconnection rate due to purely the gravitational attraction of the Schwarzschild black hole, compared to the MHD limit $\hat{v}_{\text{in}} \approx S^{-1/2}$ (when $\Lambda = 0$). In the flat spacetime limit, $r_s \rightarrow 0$, we recover the reconnection rates for special relativistic pair plasmas $\hat{v}_{\text{in}} \approx (1/S + \Lambda/L)^{1/2}$ studied in Ref. [29].

The physical mechanism for the increase of the reconnection rate due to gravity is straightforward to understand. The gravitational force (due to gradients of α) at the inflow point is along the radial direction at an angle $\theta \approx \pi/2 - \delta/(2r)$. This is the force proportional to $\hat{J}^0(r_s/\alpha r^2)|_{\text{in}}$ that appears in Eq. (22). On the other hand, the gravitational force that the plasma experiences at the X point is also along the radial direction but now at an angle $\theta = \pi/2$. Anew, this force is proportional to the term $\hat{J}^0(r_s/\alpha r^2)|_X$ in Eq. (23). These two gradient forces point in the radial direction at different angles, implying the existence of a net force antiparallel to the θ direction, along the plane of the reconnection layer. Therefore, the net force pushes the plasma toward the X point, producing an increase of the reconnection rate.

In the case in which the difference of gravitational forces between the inflow and X points is neglected, the plasma can be considered as quasineutral, with $\hat{J}^0 = 0$. This is the case of the analyses presented in Refs. [30,39], where quasineutral plasma were studied around Kerr black holes, and only the curvature due to spacetime rotation was considered. However, if the most general case for the simplest gravitational effect produced by any compact object is considered in the study of magnetic reconnection in the surrounding plasma, a deviation from quasineutrality is expected.

Finally, the reconnection rate (29) explicitly displays the importance of taking into consideration the collisionless effects. Those effects are the ones coupled to gravity. In particular, the difference of the reconnection rate (29) in the limit $S \rightarrow \infty$ and its flat spacetime counterpart $\hat{v}_{\text{in}} \approx \sqrt{\Lambda/L}$ is proportional to

$$\frac{\hat{v}_{\text{in}}}{\sqrt{\Lambda/L}} - 1 \Big|_{S \rightarrow \infty} \propto \left(\frac{h}{m_- n} \right)^{1/2} \left(\frac{d_e}{16} \right) \left(\frac{L}{r} \right)^2 \left(\frac{2r_s}{r} \right), \quad (30)$$

for pair plasmas, and proportional to

$$\frac{\hat{v}_{\text{in}}}{\sqrt{\Lambda/L}} - 1 \Big|_{S \rightarrow \infty} \propto \left(\frac{h}{m_+ n} \right)^{1/2} \left(\frac{d_e}{8} \right) \left(\frac{L}{r} \right)^2 \left(\frac{2r_s}{r} \right), \quad (31)$$

for ion-electron plasmas (here $d_e = \lambda_e/L$ is the dimensionless electron inertial length, with λ_e indicating the electron skin depth). Both results show that reconnection rates are larger in plasmas around Schwarzschild black holes, depending on the size of the black hole $\propto 2r_s/r$, and on the geometry of the current sheet $\propto L/r$. Nevertheless, the

reconnection rate for pair plasmas is larger according to the fact that positrons contribute as the electrons to the effective relativistic collisionless resistivity Λ .

The presented results complete the theoretical analysis of magnetic reconnection in curved spacetime initiated in Refs. [30,39]. In this way, we have shown that spacetime curvature effects (gravitational pull or rotation) form an intrinsic part of magnetic reconnection processes in

astrophysical plasmas around compact objects. Future high-resolution numerical simulations with general relativistic codes should be able to extend the predictions of the analytic theory to more complex scenarios, as asymmetric reconnection layers, strong field inhomogeneities in all three spatial directions, and nonsteady reconnection processes.

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