Inverse seesaw model with a natural hierarchy at the TeV scale

Takaaki Nomura[*]

School of Physics, KIAS, Seoul 02455, Republic of Korea

Hiroshi Okada[2†]

Asia Pacific Center for Theoretical Physics, Pohang, Gyeongbuk 790-784, Republic of Korea

We propose a new kind of inverse seesaw model without any additional symmetries. Instead of the symmetries, we introduce several fermions and bosons with higher $SU(2)_L$ representations. After formulating the Higgs sector and neutrino sector, we show that the cutoff energy, which is valid for our model, is at around the TeV scale by examining behavior of $SU(2)_L$ gauge coupling. Then we show the validity of our model for testing at collider physics.

I. INTRODUCTION

One of the important topics in particle physics beyond the standard model (SM) is to explore mechanisms generating tiny neutrino masses and their mixing. Sometimes, one relies upon a mechanism in which the tininess originates from the ultra-high-energy scale Λ_H such as in the grand unified theory scale ($\sim 10^{15}$ GeV) or string scale ($\sim 10^{18}$ GeV), by embedding the SM gauge group into a larger one. In this case, the neutrino mass may simply be realized by running a heavy field inside a diagram for the neutrino mass; therefore, the neutrino mass is proportional to Λ_{ew}/Λ_H, where Λ_{ew} is at the electroweak scale ~ 100 GeV. These theories are elegant in a sense that the neutrino mass is directly reflected to the high-energy scale; however, the heavy particle cannot directly be tested by any current experiments such as those at the large hadron collider (LHC). To achieve detectability at the current experiments, a theory should be closed within the TeV scale. To realize such a low-energy scenario, we apply higher multiplet fields under the $SU(2)_L$ group instead of introducing any larger gauge groups [1]. Once the neutrino mass is induced after a higher iso-spin scalar field developing the vacuum expectation value (VEV), the order of the neutrino mass is, at least, suppressed by 2 orders of magnitude compared to the origin of SM Higgs. This result follows from the bound on the ρ parameter that is describe by the mass ratio between the neutral gauge boson and charged gauge boson in the SM. The higher multiplet particles there exist, the more suppression the neutrino mass receives. Thus, we realize the tiny neutrino mass with a natural hierarchy, by applying this feature.

To achieve this mechanism in a more effective manner, an inverse seesaw model [3,4] is a promising candidate to explain the miniscule neutrino masses with mild hierarchy of Majorana mass matrix for neutral Majorana fermions in a theory, and provides a lot of phenomenologies since the neutrino mass structure is more intricate than the other mechanisms such as canonical seesaw [5–8] and linear seesaw [4,9,10]. In order to realize the inverse seesaw model, heavier neutral fermions with both chiralities have to be introduced. In addition to these new fields, most of the cases, one also has to impose an additional symmetry such as the (non-)Abelian local(global) one to control the texture of neutral fermion mass matrix.

In this article, we propose an inverse seesaw model without introducing any additional symmetries to the SM. Instead, we introduce several fermions (quartet and septet) and bosons (quintet and quartet) with *higher* $SU(2)_L$ *representations* [11]. Due to such fields, the behavior of the $SU(2)_L$ gauge coupling g_2 blows up at the TeV scale via the renormalization group equation (RGE). It suggests that our model is tightly relevant at the low-energy scale, and the testability of our model is largely expected at various experiments such as the LHC, the ILC, and future colliders.

This article is organized as follows. In Sec. II, we review our model and formulate the lepton sector. Then we discuss phenomenologies of neutrinos. In Sec. III, we discuss extra charged particles at collider experiments. Finally, we provide the summary of our results and thconclusion.

II. MODEL SETUP AND CONSTRAINTS

In this section, we formulate our model. For the fermion sector, we introduce three families of vectorlike fermions ψ

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[*]nomura@kias.re.kr
[†]okada.hiroshi@apctp.org

with $(\mathbf{4}, -3/2)$ charge under the $SU(2)_L \times U(1)_Y$ gauge symmetry, and right-handed fermions Σ_R with $(\mathbf{7}, 0)$ charge under the same gauge symmetry. For the scalar sector, we add quartet and quintet scalar fields H_4 and H_5 with, respectively, $3/2$ and 2 charges under the $U(1)_Y$ gauge symmetry, where SM-like Higgs field is identified as H_2. Here, we denote each vacuum expectation value(VEV) of the scalar fields to be $\langle H_i \rangle \equiv v_i/\sqrt{2}$ $(i = 2, 4, 5)$ that arises after the electroweak spontaneously symmetry breaking. All the field contents and their assignments are summarized in Table I, where the quark sector is exactly the same as the one of the SM and omitted. The renormalizable Yukawa Lagrangian under these symmetries is given by

$$
\begin{aligned}
-\mathcal{L}_\ell = &\ y_{\ell_{aa}} \bar{L}_L^a H_2 e_R^a + y_{D_{ab}} [\bar{L}_L^a H_5^* (\psi_L^c)^b] \\
&+ f_{L_{ab}} [\bar{\psi}_L^a H_4^* \Sigma_R^b] + f_{R_{ab}} [(\bar{\psi}_R^c)^a H_4 \Sigma_R^b] \\
&+ M_{aa} \bar{\psi}_R^a \psi_L^a + M_{\Sigma_{aa}} (\bar{\Sigma}_R^c)^a \Sigma_R^a + \text{H.c.},
\end{aligned} \tag{1}
$$

where $SU(2)_L$ index is omitted assuming it is contracted to be gauge invariant, and upper indices $(a, b) = 1$–3 are the number of families, and y_ℓ, M, and M_Σ are assumed to be diagonal matrix with real parameters. After spontaneous symmetry breaking we obtain mass matrices $m_\ell = y_\ell v/\sqrt{2}$ and $m_D = y_D v_5/\sqrt{2}$.

Scalar potential and VEVs: The scalar potential in our model is given by

$$
\begin{aligned}
\mathcal{V} = &-\mu_h^2 |H_2|^2 + M_4^2 |H_4|^2 + M_5^2 |H_5|^2 + \lambda_H |H_2|^4 \\
&+ \mu [H_4 H_2 H_5^* + \text{H.c.}] + \lambda_0 [H_4^* H_2 H_2 H_2 + \text{H.c.}] \\
&+ \mathcal{V}_{\text{trivial}},
\end{aligned} \tag{2}
$$

where $\mathcal{V}_{\text{trivial}}$ indicates other trivial 4-point terms and $SU(2)_L$ indices are implicitly contracted in the second line to be gauge invariant. Applying condition $\partial \mathcal{V}/\partial v_i = 0$, we obtain the VEVs as

$$
v_2 \sim \sqrt{\frac{\mu_h^2}{\lambda_H}}, \qquad v_4 \sim \frac{\lambda_0 v^3}{M_4^2}, \qquad v_5 \sim \frac{\mu v_4 v}{M_5^2}, \tag{3}
$$

where we have used $v_4, v_5 \ll v_2$. Thus, v_4 and v_5 can be naturally the $\mathcal{O}(1)$ GeV scale if M_4 and M_5 are the TeV scale.

TABLE I. Charge assignments of the our lepton and scalar fields under $SU(2)_L \times U(1)_Y$, where the upper index a is the number of the family that runs over 1-3, and all of them are singlet under $SU(3)_C$.

	L_L^a	e_R^a	ψ^a	Σ_R^a	H_2	H_4	H_5
$SU(2)_L$	2	1	4	7	2	4	5
$U(1)_Y$	$-\frac{1}{2}$	-1	$-\frac{3}{2}$	0	$\frac{1}{2}$	$\frac{3}{2}$	2

ρ parameter: The VEVs of H_4 and H_5 are restricted by the ρ parameter at tree level that is given by

$$
\rho \approx \frac{v_2^2 + 7v_4^2 + 10v_5^2}{v_2^2 + v_4^2 + 2v_5^2}, \tag{4}
$$

where the experimental value is given by $\rho = 1.0004^{+0.0003}_{-0.0004}$ at 2σ confidence level [20]. On the other hand, we have $v_{\text{SM}} = \sqrt{v_2^2 + 7v_4^2 + 10v_5^2} \simeq v_2 \approx 246$ GeV. Therefore, v_4 and v_5 are restricted via the constraint of the ρ parameter. Here, we take these VEVs to be $v_2 \approx 245.9$, $v_4 \approx 1.67$, and $v_5 \approx 1.72$ GeV, which are the typical scale for the VEVs satisfying the constraint.

Exotic particles: The scalars and fermions with large $SU(2)_L$ multiplet provide exotic charged particles. Here we write components of multiplets as

$$
H_4 = (\phi_4^{+++}, \phi_4^{++}, \phi_4^{+}, \phi_4^{0})^T, \tag{5}
$$

$$
H_5 = (\phi_5^{++++}, \phi_5^{+++}, \phi_5^{++}, \phi_5^{+}, \phi_5^{0})^T, \tag{6}
$$

$$
\psi_{L(R)} = (\psi^0, \psi^-, \psi^{--}, \psi^{---})^T_{L(R)}, \tag{7}
$$

$$
\Sigma_R = (\Sigma^{+++}, \Sigma^{++}, \Sigma^{+}, \Sigma^{0}, \Sigma^{-}, \Sigma^{--}, \Sigma^{---})^T_R. \tag{8}
$$

The masses of components in H_4 and H_5 are respectively given by $\sim M_4$ and $\sim M_5$ since $v_{4,5} \ll M_{4,5}$. The charged components in $\psi_{L(R)}$ have Dirac mass M and neutral component is discussed with neutrino sector below. The septet fermion mass is M_Σ and charged components have Dirac mass term constructed by pairs of positive-negative charged components in the multiplet. Charged particles in the same multiplet have degenerate mass at tree level which will be shifted at loop level [21].

Neutrino sector: After the spontaneously symmetry breaking, neutral fermion mass matrix in basis of $(\nu_L, \psi_R^{0c}, \psi_L^0)^T$ is given by

$$
M_N = \begin{bmatrix} 0 & 0 & m_D^* \\ 0 & \mu_R^* & M \\ m_D^\dagger & M^T & \mu_L^* \end{bmatrix}, \tag{9}
$$

where $\mu_{L/R}$ is given by $v_4^2 f_{L/R} M_\Sigma^{-1} f_{L/R}^T$ on the analogical manner of seesaw mechanism, as shown in Fig. 1. Then the active neutrino mass matrix can approximately be found as

$$
m_\nu \approx m_D^* M^{-1} \mu_R^* (M^T)^{-1} m_D^\dagger, \tag{10}
$$

where $\mu_{L/R} < m_D \ll M$ is naturally expected due to the constraint of ρ parameter and seesawlike mechanism of $\mu_{R/L}$ [22]. We thus obtain correlation among size of neutrino mass and other mass parameters such that
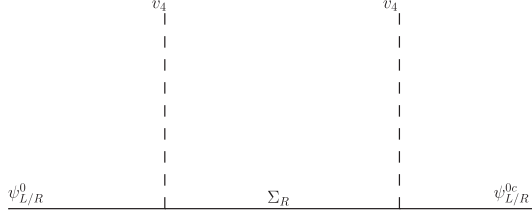
FIG. 1.   Feynman diagram to generate the masses of $\mu_{L/R}$.

$$m_\nu \sim \frac{v_4}{M_\Sigma}\left(\frac{v_5}{M}\right)^2 f_R^2 y_D^2 v_4. \qquad (11)$$

Note that M_Σ and M cannot be much larger than TeV scale, since v_4 and v_5 are GeV scale requiring the perturbative limit for Yukawa coupling constants. The neutrino mass matrix is diagonalized by unitary matrix $U_{\rm MNS}$; $D_\nu = U_{\rm MNS}^T m_\nu U_{\rm MNS}$, where $D_\nu \equiv {\rm diag}(m_1, m_2, m_3)$. Here we apply a convenient method to reproduce neutrino oscillation data as follows [27]:

$$m_D^* \approx U_{\rm MNS}^* \sqrt{D_\nu} O_{\rm mix} \sqrt{I_N} (L_N^T)^{-1}. \qquad (12)$$

Here $O_{\rm mix}$ is an arbitrary 3 by 3 orthogonal matrix with complex values, I_N is a diagonal matrix, and L_N is a lower unit triangular [28], which can uniquely be decomposed to be $M^{-1}\mu_R^*(M^T)^{-1} = L_N^T I_N L_N$, since it is symmetric matrix. Note here that all the components of m_D should not exceed $\mathcal{O}(1)$ GeV, once perturbative limit of y_D is taken to be 1.

Nonunitarity: Constraint of nonunitarity should always be taken into account in the case of the larger neutral mass matrix whose components are greater than three by three, since experimental neutrino oscillation results suggest nearly unitary. In case of the inverse seesaw, when nonunitarity matrix $U'_{\rm MNS}$ is defined, one can typically parametrize it by the following form,

$$U'_{\rm MNS} \equiv \left(1 - \frac{1}{2}FF^\dagger\right) U_{\rm MNS}, \qquad (13)$$

where $F \equiv M^{-1}m_D^*$ is a Hermitian matrix, and $U'_{\rm MNS}$ represents the deviation from the unitarity. Considering several experimental bounds [29], one finds the following constraints [30]:

$$|FF^\dagger| \le \begin{bmatrix} 2.5\times10^{-3} & 2.4\times10^{-5} & 2.7\times10^{-3} \\ 2.4\times10^{-5} & 4.0\times10^{-4} & 1.2\times10^{-3} \\ 2.7\times10^{-3} & 1.2\times10^{-3} & 5.6\times10^{-3} \end{bmatrix}. \qquad (14)$$

Once we conservatively take $F \approx 10^{-5}$, we find $\mu_R \approx$ 1–10 GeV to satisfy the typical neutrino mass scale, which could be easy task.

Beta function of $SU(2)_L$ gauge coupling g_2: Here, it is worth discussing the running of gauge coupling of g_2 in the
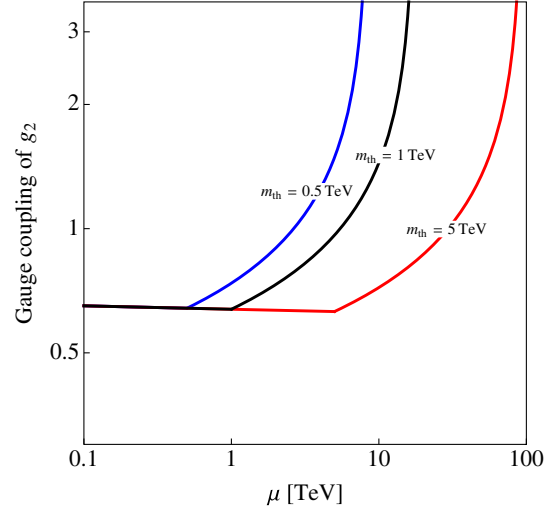
FIG. 2.   The running of g_2 in terms of a reference energy of μ.

presence of several new multiplet fields of $SU(2)_L$ [31]. The new contribution to g_2 for an $SU(2)_L$ quartet fermion (boson) $\psi(H_4)$, septet fermion Σ_R, and quintet boson H_5 are respectively given by

$$\Delta b_{g_2}^\psi = \frac{10}{3}, \quad \Delta b_{g_2}^{\Sigma_R} = \frac{56}{3}, \quad \Delta b_{g_2}^{H_4} = \frac{5}{3}, \quad \Delta b_{g_2}^{H_5} = \frac{10}{3}. \qquad (15)$$

Then one finds the energy evolution of the gauge coupling g_2 as [17,32]

$$\frac{1}{g_{g_2}^2(\mu)} = \frac{1}{g^2(m_{\rm in})} - \frac{b_{g_2}^{\rm SM}}{(4\pi)^2} \ln\left[\frac{\mu^2}{m_{\rm in}^2}\right] - \theta(\mu - m_{th})$$
$$\times \frac{N_f(\Delta b_{g_2}^\psi + \Delta b_{g_2}^\psi) + \Delta b_{g_2}^{H_4} + \Delta b_{g_2}^{H_5}}{(4\pi)^2} \ln\left[\frac{\mu^2}{m_{th}^2}\right], \qquad (16)$$

where $N_f = 3$ is the number of ψ and Σ_R, μ is a reference energy, $b_{g_2}^{\rm SM} = -19/6$, and we assume $m_{\rm in}(= m_Z) < m_{th}$ with m_{th} being threshold masses of exotic fermions and bosons. The resulting flow of $g_2(\mu)$ is then given by the Fig. 2. This figure shows that g_2 is relevant up to the mass scale $\mu = \mathcal{O}(10)$ TeV in the case of $m_{th} = 500$ GeV, while g_2 is relevant up to the mass scale $\mu = \mathcal{O}(100)$ TeV in the case of $m_{th} = 5000$ GeV. Thus, our theory is not spoiled, as long as we work at around the scale of TeV.

III. COLLIDER PHYSICS

Here, let us discuss the collider physics of our model. We have rich phenomenology at collider experiments since there are many exotic charged particles from large $SU(2)$ multiplet scalars and fermions. As the most specific
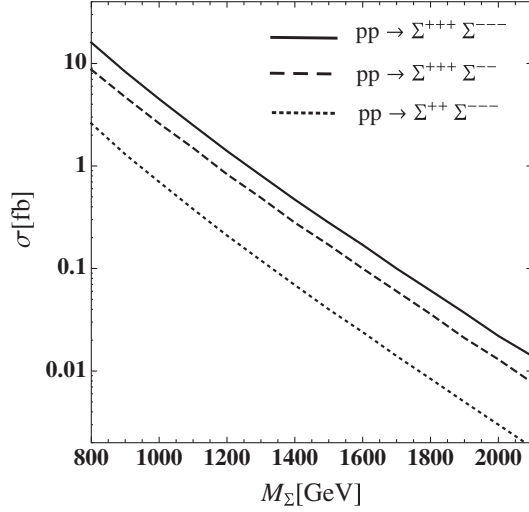
FIG. 3. The cross sections for processes $pp \to Z/\gamma \to \Sigma^{+++}\Sigma^{---}$ and $pp \to W^{\pm} \to \Sigma^{+++(++)}\Sigma^{--(---)}$ as a function of septet fermion mass.

signature, we focus on the production of triply charged lepton $\Sigma^{\pm\pm\pm}$ and its decay at the LHC. The gauge interactions associated with the triply charged lepton are obtained as

$$\bar{\Sigma}_R i\gamma^{\mu} D_{\mu} \Sigma_R \supset \bar{\Sigma}^{+++}\gamma^{\mu}(3g_2 c_W Z_{\mu} + 3eA_{\mu})\Sigma^{+++}$$
$$+ \sqrt{3}g_2\bar{\Sigma}^{+++}\gamma^{\mu}W_{\mu}^{+}\Sigma^{++} + \text{H.c.}, \quad (17)$$

where $c_W = \cos\theta_W$ with Weinberg angle θ_W and e is the electromagnetic coupling: the covariant derivative for the septet can be referred to Ref. [19]. Then, we estimate cross sections for the triply charged lepton production processes using CALCHEP [33] by use of the CTEQ6 parton distribution functions (PDFs) [34], implementing relevant interactions. In Fig. 3, we show production cross section for triply charged lepton as a function of its mass; pair production $pp \to \Sigma^{+++}\Sigma^{---}$ and associate productions $pp \to \Sigma^{+++(++)}\Sigma^{--(---)}$ at the LHC 13 TeV. The cross section for pair production is the largest one and larger than 1 fb for 1 TeV mass due to large charge.

The triply charged lepton can decay via Yukawa coupling in Eq. (1) as $\Sigma^{+++} \to \phi_4^{Q_\phi}\psi^{Q_\psi}$ where $Q_{\phi(\psi)}$ is the electric charge of $\phi(\psi)$ with $Q_\phi + Q_\psi = 3$, and we assume exotic scalars are lighter than exotic fermions in our discussion. In addition, ψ^{Q_ψ} decays as $\psi_{Q_\psi} \to \phi_5^{Q'_\phi}\ell^+(\nu)$ with $Q_\psi = Q'_\phi + 1(0)$. There are several decay modes for exotic charged leptons due to the combination of charges in the final states which have similar size branching ratios (BRs). Here, we discuss the representative decay chain,

$$\Sigma^{+++} \to \phi_4^{++}\psi^+ \to \phi_4^{++}\phi_5^0\ell^+ \to W^+W^+ZZ\ell^+, \quad (18)$$

TABLE II. Number of expected signal events for two final state from $pp \to \Sigma^{+++}\Sigma^{---}$ production where integrated luminosity is taken as 300(3000) fb^{-1}.

Signal	$3\ell^+3\ell^-8j\not{E}_T$	$3\ell^{\pm}12j\not{E}_T$
$\sigma \cdot \text{BR}(M_\Sigma = 0.8 \text{ TeV})$ [fb]	0.73 (7.3)	7.2 (72)
$\sigma \cdot \text{BR}(M_\Sigma = 1.0 \text{ TeV})$ [fb]	0.21 (2.1)	2.0 (20)
$\sigma \cdot \text{BR}(M_\Sigma = 1.2 \text{ TeV})$ [fb]	0.064 (0.64)	0.63 (6.3)

where ϕ_4^{++} and ϕ_5^0 decay into W^+W^+ and ZZ via gauge interaction [17]

$$(D_{\mu}H_4)^{\dagger}(D^{\mu}H_4) \supset \sqrt{\frac{3}{2}}v_4 g_2^2 W_{\mu}^{\pm}W^{\pm\mu}\phi_4^{\mp\mp} \quad (19)$$

$$(D_{\mu}H_5)^{\dagger}(D^{\mu}H_5) \supset \frac{1}{8}\frac{g_2^2}{c_W^2}v_5\phi_5^0 Z_{\mu}Z^{\mu}. \quad (20)$$

Note that the BRs for $\phi_4^{\pm\pm} \to W^{\pm}W^{\pm}$ and $\phi_5^0 \to ZZ$ are dominant when $v_4 \sim v_5 \sim 1$ GeV. When W^+ decays into leptons and Z decays into jets, we obtain the signal of three same-sign charged leptons with jets and missing transverse energy, which provides products of BRs; $\text{BR}(W^+ \to \ell^+\nu)^2\text{BR}(Z \to q\bar{q})^2 \sim 0.02$ with $\ell = \mu, e$. Thus, we can obtain $\sim 6(60)$ signal events containing three same-sign charged leptons for integrated luminosity of 300(3000) fb^{-1} when products of the $\Sigma^{\pm\pm\pm}$ production cross section and $\text{BR}(\phi_4^{\pm\pm}\psi^{\pm})\text{BR}(\psi^{\pm} \to \phi_5^0\ell^{\pm})$ is around 1 fb. This size of the cross section can be obtained for $M_\Sigma \sim 1$ TeV, where we show the expected number of events in Table II for several values of M_Σ considering one or two pairs of same-sign W bosons decaying into leptons. We find that the number of events tends to be too small when both same-sign W boson pairs $W^+W^+(W^-W^-)$ decay into leptons although the signal will be very clear. Thus, the signal of three same-sign charged leptons with jets and $\not{E}_T$ can be a good target in searching for the signature of our model. We expect a sizable discovery significance even if the number of signal events is less than 10 since the SM background is very small for three same-sign charged lepton signals.

IV. SUMMARY AND CONCLUSIONS

We have constructed an inverse seesaw model with large $SU(2)_L$ multiplet fields in which we have formulated the neutrino mass matrix to reproduce current neutrino oscillation data, satisfying the ρ parameter and nonunitarity bound. We have also checked the relevant energy scale of our theory via the RGE of $SU(2)_L$ gauge coupling g_2 that gives the most stringent constraint. Then, we have analyzed the collider physics focusing on triply charged lepton production at the LHC as a representative process of our model and showing a possibility of detection. We have

found the specific signal of the triply charged lepton as three same-sign charged leptons with jets and missing transverse momentum. The number of events of the signal can be at the detectable level with integrated luminosity $300(3000)$ fb^{-1} when the triply charged lepton mass is around 1 TeV. More detailed analysis will be given elsewhere.

ACKNOWLEDGMENTS

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