Quasiparton distribution functions: Two-dimensional scalar and spinor QCD

Xiangdong Ji,[1,2,*] Yizhuang Liu,[2,3,†] and Ismail Zahed[3,‡]

[1]*Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742, USA*
[2]*Tsung-Dao Lee Institute, Shanghai Jiao University, Shanghai, 200240, China*
[3]*Department of Physics and Astronomy, Stony Brook University, Stony Brook, New York 11794-3800, USA*

(Received 6 November 2018; published 15 March 2019)

We construct the quasiparton distributions of mesons for two-dimensional QCD with either scalar or spinor quarks using the $1/N_c$ expansion. We show that in the infinite momentum limit, the parton distribution function is recovered in both leading and subleading order in $1/N_c$.

DOI: 10.1103/PhysRevD.99.054008

I. INTRODUCTION

Light-cone distribution amplitudes are central to the description of hard exclusive processes with large momentum transfer. They account for the nonperturbative quark and gluon content of a hadron in the infinite momentum frame. Using factorization, hard cross sections can be split into soft partonic distributions convoluted with perturbativly calculable processes. The partonic distributions are inherently nonperturbative. They are currently estimated using experiments, lattice simulations, or models.

Recently, one of us [1] has suggested that the light-cone hadronic wave functions can be recovered from Euclidean correlators in hadronic states using instead quasiparton distribution functions through pertinent renormalization in the infinite momentum limit. Preliminary lattice simulations have proven very promising [2,3]. The purpose of this letter is to explore this construct in two-dimensional scalar and spinor QCD in the nonperturbative $1/N_c$ expansion.

Two-dimensional scalar QCD has a smooth large N_c limit with a confining spectrum [4–6]. In this model, the current correlators exhibits many features of four-dimensional QCD in contrast to two-dimensional spinor QCD [7]. In the deep inelastic regime, the results exhibit expected scaling laws, and are overall in support of the Feynman partonic picture and the light-cone expansion. In this paper, these two models will be used interchangeably to test the concept of the quasidistributions in a nonperturbative context, as they differ by a minor change in the algebra of the pertinent bosonic operators. Specifically, we construct the quasiparton distributions for both scalar and spinor QCD in leading and subleading order in $1/N_c$ and show that they merge with the expected light-cone distributions in the infinite momentum limit without additional renormalization. Our leading conclusion for two-dimensional spinor QCD is in agreement with a recent study [8].

This paper consists of several new results: (i) a bosonization of two-dimensional scalar and spinor QCD in the axial gauge, based on a closed form algebra valid to all orders in $1/N_c$; (ii) a derivation of the parton quasidistribution function for two-dimensional scalar QCD in leading order in $1/N_c$ with both leading and subleading order in $1/P^2$; (iii) a smooth reduction of the parton quasidistribution function to the distribution function in the infinite momentum frame except at $x = 0, 1$; (iv) a derivation of the parton distribution and quasidistribution functions in spinor QCD at subleading order in $1/N_c$.

The organization of the paper is as follows: in Sec. II, we discuss a canonical quantization of two-dimensional scalar QCD in the axial gauge. We make explicit the Hamiltonian of the model in leading order in $1/N_c$ using bosonized fields. Some renormalization issues are also discussed. In Sec. III, we make explicit the wave function for scalar QCD in the light-cone limit. In Sec. IV, we construct the quasiparton distribution function in leading order in $1/N_c$, and show that it reduces to the light-cone wave function in the infinite momentum limit. We also discuss the leading correction in $1/P$. In Sec. V, we show how to generalize the bosonization scheme algebraically for both scalar and spinor QCD, and use it for a systematic organization of the operators in $1/N_c$. This scheme is used in Secs. VI and VII to correct the light-cone parton distribution and quasidistribution in spinor two-dimensional QCD through standard perturbation theory.

*xji@physics.umd.edu
†yizhuang.liu@sju.edu.cn
‡ismail.zahed@stonybrook.edu

We show that the subleading corrections to the quasiparton distribution function merges with the parton distribution function in the infinite momentum limit without renormalization. Our conclusions are in Sec. VIII. In the Appendix, we summarized some elements of two-dimensional spinor QCD pertinent for our canonical analysis both in light-cone and axial gauge.

II. QUANTIZATION OF SCALAR QCD IN AXIAL GAUGE

We first discuss the general structure of the Hamiltonian in two dimensions for scalar SU(N_c) QCD in the axial gauge $A_1 = 0$. The same discussion for two-dimensional spinor QCD in both the light-cone and axial gauge is summarized briefly in the Appendix. The starting Lagrangian is

$$\mathcal{L} = \frac{1}{2}\mathrm{Tr}F_{01}^2 + (D^\mu\phi)^\dagger D_\mu\phi - m^2\phi^\dagger\phi, \tag{1}$$

where $\phi_\alpha(x)$ with $\alpha = 1, \ldots, N_c$ is a charged scalar field in the fundamental representation of SU(N_c). In terms of the canonical momenta $\pi^\dagger = \Pi_\phi = (D_0\phi)^\dagger$ and $\pi = \Pi_{\phi^\dagger} = D_0\phi$, the corresponding Hamiltonian reads

$$H = \int dx\bigg(\pi^\dagger\pi + |\partial_1\phi|^2 + m^2|\phi|^2 + ig\mathrm{Tr}A_0(\pi\phi^\dagger - \phi\pi^\dagger) - \frac{1}{2}\mathrm{Tr}(\partial_1 A_0)^2\bigg). \tag{2}$$

The equation of motion for A_0 is a constraint equation that can be solved in terms of ϕ, π to yield the canonical Hamiltonian

$$\begin{aligned} H &= H_0 + H_{\mathrm{int}} \\ &= \int dx(\pi^\dagger\pi + |\partial_1\phi|^2 + m^2|\phi|^2) \\ &\quad + \frac{g^2}{2}\int dx\left(J^a\frac{-1}{\partial_1^2}J^a\right), \end{aligned} \tag{3}$$

with the current $J^a = i(\phi^\dagger T^a\pi - \pi^\dagger T^a\phi)$. To proceed further, we will use a freelike representation for the charged field and its conjugate

$$\begin{aligned} \phi_\alpha &= \int\frac{dk}{\sqrt{4\pi E_k}}e^{-ikx}(a_k + b_{-k}^\dagger)_\alpha \\ (\pi^\dagger)_\alpha &= i\int\frac{dk}{\sqrt{4\pi E_k}}e^{ikx}E_k(a_k^\dagger - b_{-k})_\alpha, \end{aligned} \tag{4}$$

with $\alpha = 1, \ldots, N_c$ the color index. Instead of the free dispersion law $E_k = \sqrt{k^2 + m^2}$, we will use an arbitary $E(k)$ that will be fixed self-consistently below in the large N_c limit (planar approximation), with $E_k \to |k|$ asymptotically.

A. Hamiltonian to order $1/\sqrt{N_c}$

The Hamiltonian (3) is up to quartic in a_k, $a_k^\dagger$, b_k, $b_k^\dagger$. To analyze it we use the bosonization method as developed in the context of nonrelativistic many-body systems [9], and adapted to relativistic QCD in $1+1$ dimensions [10] to which we refer for more details. More specifically, we define the bilocal and color-singlet operators

$$\begin{aligned} M(k_1, k_2) &= \frac{1}{\sqrt{N_c}}\sum_\alpha a_\alpha(k_1)b_\alpha(k_2) \\ N(k_1, k_2) &= \sum_\alpha a_\alpha^\dagger(k_1)a_\alpha(k_2) \\ \bar{N}(k_1, k_2) &= \sum_\alpha b_\alpha^\dagger(k_1)b_\alpha(k_2), \end{aligned} \tag{5}$$

which are readily shown to form a closed algebra,

$$\begin{aligned} &[M_{12}, M_{34}^\dagger] = \delta_{13}\delta_{24} + \frac{s}{N_c}(\delta_{13}\bar{N}_{42} + \delta_{42}N_{31}) \\ &[M_{12}, N_{34}] = \delta_{13}M_{42} \qquad [M_{12}, \bar{N}_{34}] = \delta_{23}M_{14} \\ &[M_{12}, M_{34}] = [N_{12}, \bar{N}_{34}] = 0 \\ &[N_{12}, N_{34}] = \delta_{23}N_{14} - \delta_{14}N_{32}, \end{aligned} \tag{6}$$

with $N_{12}^\dagger = N_{21}$, and the short hand notation $M_{12} \equiv M(k_1, k_2)$ and so on. The sign assignment for the bosonization of scalar QCD is $s = +1$ as all underlying operators are bosonic. It is $s = -1$ for the bosonization of spinor QCD. $M^\dagger$ creates a mesonlike state composed of a pair of a charged scalar quark and its conjugate in the color-singlet representation, while M annihilates the corresponding pair. Using (5) and the identity

$$\sum_a (T^a)_{ij}(T^a)_{kl} = \delta_{il}\delta_{kj} - \frac{1}{N_c}\delta_{ij}\delta_{kl}, \tag{7}$$

the Hamiltonian (3) now reads to order $1/\sqrt{N_c}$ as

$$\begin{aligned} H &\approx H_2 + H_4 \\ H_2 &= \int dk\,\Pi^+(k)(N(k) + \bar{N}(k)) \\ &\quad + \sqrt{N_c}\int dk\,\Pi^-(k)(M(k) + M^\dagger(k)) \\ H_4 &= \frac{\lambda}{16\pi}\int dk_1dk_2dk_3dk_4\frac{\delta(k_1 + k_2 + k_3 + k_4)}{(k_1 + k_2)^2} \\ &\quad \times(-2f_+(k_1, k_2)f_+(k_3, k_4)M^\dagger(k_1, k_4)M(-k_2, -k_3) \\ &\quad + f_-(k_1, k_2)f_-(k_3, k_4)M^\dagger(k_1, k_4)M^\dagger(k_3, k_2) \\ &\quad + f_-(k_1, k_2)f_-(k_3, k_4)M(k_1, k_4)M(k_3, k_2)). \end{aligned} \tag{8}$$

Here, $\lambda = g^2 N_c$ is the standard t′ Hooft coupling. We have made use of the notation $M(k) \equiv M(k, -k)$, $N(k) \equiv N(k, k)$, and defined

$$\Pi^{\pm}(k) = \frac{1}{2}\left(\frac{k^2+m^2}{E_k} \pm E_k\right) + \lambda \int \frac{dk_1}{8\pi} \frac{\frac{E_{k_1}}{E_k} \pm \frac{E_k}{E_{k_1}}}{(k+k_1)^2}$$
$$f_{\pm}(k_1, k_2) = \sqrt{\frac{E_2}{E_1}} \pm \sqrt{\frac{E_1}{E_2}}. \tag{9}$$

For a consistent expansion in $1/N_c$, we can eliminate the $\sqrt{N_c}$ term in (8) by setting $\Pi^-(k) = 0$. The result is an integral equation for $E(k)$

$$\frac{k^2+m^2}{E_k} - E_k + \frac{\lambda}{4\pi} \int dk_1 \left(\frac{E_{k_1}}{E_k} - \frac{E_k}{E_{k_1}}\right) \frac{1}{|k+k_1|^2} = 0. \tag{10}$$

We now follow [10] and note that the bilocal and color singlet operators N, $N^{\dagger}$ can be recast in terms of the bilocal mesonic and color singlet operators M, $M^{\dagger}$

$$N(k,p) = \int dq\, M^{\dagger}(k,q) M(p,q)$$
$$\bar{N}(k,p) = \int dq\, M^{\dagger}(q,k) M(q,p), \tag{11}$$

without affecting the commutation rules (6) in leading order in $1/N_c$. This replacement is justified in the confined sector. Confinement implies that the creation of a charged scalar through $a^{\dagger}_{\alpha}(k_1)$ has to be always stringed to a meson creation, say $M^{\dagger}(k_1, q)$, and its annihilation through $a_{\alpha}(k_2)$ stringed to a meson annihilation, say $M(q, k_2)$. In non-relativistic many-body physics, the representation (11) is known as the Holstein-Primakoff representation [9]. Inserting (11) into (8) and using the gap equation (9) yields the leading Hamiltonian to order $1/\sqrt{N_c}$

$$\begin{aligned} H \approx & \int dp dq (\Pi^+(p) + \Pi^+(q)) M^{\dagger}(p,q) M(p,q) \\ & + \frac{\lambda}{16\pi} \int dk_1 dk_2 dk_3 dk_4 \frac{\delta(k_1+k_2+k_3+k_4)}{(k_1+k_2)^2} \\ & \times (-2 f_+(k_1,k_2) f_+(k_3,k_4) M^{\dagger}(k_1,k_4) M(-k_2,-k_3) \\ & + f_-(k_1,k_2) f_-(k_3,k_4) M^{\dagger}(k_1,k_4) M^{\dagger}(k_3,k_2) \\ & + f_-(k_1,k_2) f_-(k_3,k_4) M(k_1,k_4) M(k_3,k_2)). \end{aligned} \tag{12}$$

B. Renormalization

The integral in (10) and subsequently the Hamiltonian contain a divergence and require regularization. For that, we regularize $\frac{1}{(k+k_1)^2}$ using the standard principal value (PV) prescription

$$\int dx \frac{f(x)}{(x-y)^2} \to \mathrm{PV} \int dx \frac{f(x) - f(y)}{(x-y)^2} + \frac{2f(y)}{\epsilon}. \tag{13}$$

It is readily seen that Π^- is finite but Π^+ diverges as

$$\Pi^+ = \Pi^+_r + \frac{\lambda}{2\pi\epsilon}, \tag{14}$$

with Π_r finite. We have checked that, for physical states (on mass shell), the ϵ contributions cancel out (see below).

The solution to (10) that asymptotes $E_k \to |k|$ still suffers from a logarithmic divergence even after the PV prescription, namely,

$$\frac{\lambda}{8\pi E_k} \int dk_1 \frac{E_{k_1}}{k_1^2}. \tag{15}$$

This is actually related to the mass divergence for the scalar one-loop self energy and renormalizes the scalar mass

$$m_r^2 = m^2 + \frac{\lambda}{4\pi} \int dk_1 \frac{E_{k_1}}{k_1^2}. \tag{16}$$

From here on, we will refer to Π^+ as the renormalized momentum operator, and m as the renormalized mass, and omit the r-label for convenience. With this in mind, the renormalized integral equation (10) now reads

$$\begin{aligned} & \frac{k^2+m^2}{E_k} - E_k \\ & + \frac{\lambda}{4\pi} \int dk_1 \left(\left(\frac{E_{k_1}}{E_k} - \frac{E_k}{E_{k_1}}\right) \frac{\mathrm{PV}}{|k+k_1|^2} - \frac{E_{k_1}}{E_k} \frac{1}{k_1^2} \right) = 0. \end{aligned} \tag{17}$$

III. WAVE-FUNCTION AND LIGHT-CONE LIMIT

To construct the light-cone wave function of the scalar quarks, it is useful to recast the leading-order Hamiltonian in (12) in the form

$$\begin{aligned} H \approx & \int dp dq (\Pi^+(p) + \Pi^+(q)) M^{\dagger}(p,q) M(p,q) \\ & - \frac{\lambda}{16\pi} \int dP \int dk dp \frac{A+B}{(p-k)^2}, \end{aligned} \tag{18}$$

using the compact notation

$$\begin{aligned} A &= 2 S_+(p,k,P) M^{\dagger}(p-P,p) M(k-P,k) \\ B &= S_-(p,k,P)(M^{\dagger}(p,p-P) M^{\dagger}(k-P,k) + \mathrm{c.c}) \\ S_{\pm}(p,k,P) &= f_{\pm}(p-P,k-P) f_{\pm}(p,k). \end{aligned} \tag{19}$$

The bilocal mesonic operator $M(p,q)$ can be decomposed in suitably normalized modes

$$M(p-P,p)=\frac{1}{\sqrt{|P|}}\sum_n(m_n(P)\phi_n^+(p,P)\\ -m_n^\dagger(-P)\phi_n^-(p-P,-P)), \quad (20)$$

The first contribution refers to the light-cone wave function describing a pair of scalar quarks moving forward in the light front, while the second contribution refers to a pair moving backward in the light front. The pair is characterized by a relative momentum p and a center of mass momentum P.

Here m_n, $m_n^\dagger$ are canonical bosonic annihilation and creation operators that satisfy the standard commutation rules, e.g.,

$$[m_n(P),m_l^\dagger(P')]=\delta_{nl}\delta(P-P'), \quad (21)$$

and when applied to the ground state creates a meson state

$$|P\rangle\equiv|E_n(P),P\rangle=\sqrt{2E_n(P)}m_n^\dagger(P)|0\rangle, \quad (22)$$

on mass shell $E_n(P)=(P^2+M_n^2)^{\frac{1}{2}}$. Equation (21) is seen to follow from the leading $1/N_c$ commutation rules (6) provided that the wave functions $\phi_n^\pm$ satisfy pertinent ortonormality conditions, e.g.,

$$\int dp(\phi_n^+(p,P)\phi_l^+(p,P)-\phi_n^-(p,P)\phi_l^-(p,P))=\delta_{nl}|P|. \quad (23)$$

The Heisenberg equation of motion follows by commutation

$$i\partial_t M^\dagger(p-P,p)|0\rangle=[M^\dagger(p-P,p),H]|0\rangle, \quad (24)$$

which mode-by-mode translates to

$$(\Pi^+(p)+\Pi^+(P-p)\mp P_n^0)\phi_n^\pm(p,P)\\ =\frac{\lambda}{8\pi}\int\frac{dk}{(p-k)^2}\\ \times(S_+(p,k,P)\phi_n^\pm(k,P)-S_-(p,k,P)\phi_n^\mp(k,P)). \quad (25)$$

We checked that the ϵ-dependent divergences noted in the momentum operator cancel out. Indeed, using (14) the LHS in (25) produces $\frac{\lambda}{\pi\epsilon}\phi^\pm$, while the RHS in (25) produces $\frac{\lambda}{4\pi\epsilon}S^+(k,k)\phi^\pm=\frac{\lambda}{\pi\epsilon}\phi^\pm$, both of which cancel out. This checks the consistency of the renormalization procedure for scalar QCD. No such renormalization is needed for spinor QCD.

In the large momentum limit P, the equation simplifies. For that we set $p=xP$, $k=yP$, and take $P\to\infty$ on both sides of (25). In this limit, the backward wave function vanishes $\phi_-\to 0$. Since

$$\Pi^+(Px)+\Pi^+((1-x)P)-\sqrt{P^2+M_n^2}\\ =\frac{1}{2P}\left(\frac{m^2}{x}+\frac{m^2}{1-x}-M_n^2\right)+\mathcal{O}\left(\frac{1}{P^2}\right), \quad (26)$$

and

$$S_+(xP,yP,P)=\frac{(2-x-y)(x+y)}{\sqrt{x(1-x)y(1-y)}}, \quad (27)$$

the equation of motion (25) involves only the forward wave function in the form

$$\left(\frac{m^2}{x}+\frac{m^2}{1-x}-M_n^2\right)\phi_n(x)\\ =\frac{\lambda}{4\pi}\mathrm{PV}\int\frac{dy}{(x-y)^2}\frac{(2-x-y)(x+y)}{\sqrt{x(1-x)y(1-y)}}\phi_n(y), \quad (28)$$

where we have defined $\phi_n^+(xP,P)=\phi_n(x)$, and PV refers to the principal value of the integral. (28) was obtained initially in the light-cone gauge in [5] using different arguments.

IV. QUASI-PARTON DISTRIBUTION FUNCTION

The light-cone distribution for scalar quarks is just $|\phi_n^+(x)|^2$ in leading order in $1/N_c$. We now show that to the same order, the light-cone distribution function and the quasidistribution function as defined in [1] are in agreement without further normalization. For that, we define the quasidistribution function

$$\tilde{q}(x,P)=+i\int\frac{dz}{4\pi}e^{iPxz}\langle P|(\partial_1\phi(z))^\dagger W[z,0]\phi(0)|P\rangle\\ -i\int\frac{dz}{4\pi}e^{iPxz}\langle P|(\phi(z))^\dagger W[z,0]\partial_1\phi(0)|P\rangle, \quad (29)$$

where $|P\rangle$ refers to the meson state. In the axial gauge, the Wilson line $W[z,0]=1$. Using the mode decomposition (4) and the relations (11) we obtain for the quasidistribution

$$\tilde{q}(x,P)=\frac{E_n(P)}{P}\frac{xP}{E(xP)}\\ \times(|\phi_n^+(xP,P)|^2+|\phi_n^+(-xP,P)|^2\\ +|\phi_n^-((xP,P)|^2+|\phi_n^-(-xP,P)|^2). \quad (30)$$

For $P\to\infty$, we have $E_n(P)\to P$ and $E(xP)\to xP$ and all ϕ_- vanish. The quasiparton distribution function reduces identically to the parton distribution function $|\phi_n^+(x)|^2$.

For finite P, (30) shows that the backward moving pair in ϕ^- contributes. To assess this quantitatively, we now expand in $\frac{1}{P}$ the contributions $\phi^\pm$ in (30). For that, we go back to (25) and expand in $\frac{1}{P}$, namely

$$\Pi^+ = |P| + \frac{m^2}{2|P|} + \frac{\beta_1}{2|P|^3} + \mathcal{O}\left(\frac{1}{|P|^4}\right)$$
$$E(P) = |P| + \frac{\beta_2}{|P|} + \mathcal{O}\left(\frac{1}{|P|^2}\right). \quad (31)$$

The coefficients β_1 is fixed through a straightforward Taylor expansion of Π^+, while β_2 is fixed by the integral equation (10). Their explicit form is not needed for the general arguments to follow. With this in mind, the leading correction to ϕ^- is

$$\phi_1^- = P^2\phi_n^-(x) = \frac{\lambda}{24\pi\sqrt{x(1-x)}}\int_0^1 dy \frac{\phi_n(y)}{\sqrt{y(1-y)}}, \quad (32)$$

and the subleading correction for $\phi^+ = \phi(x) + \frac{1}{P^2}\phi_1^+(x)$ formally solves

$$\tilde{\phi} \equiv (K_0 - H_0)\phi_1^+ = -K_1\phi + H_1\phi - H_0^-\phi_1^-. \quad (33)$$

Here we have defined

$$K_0(x) = \frac{m^2}{x} + \frac{m^2}{\bar{x}} - M_n^2$$
$$H_0(x,y) = \frac{\lambda}{4\pi}\frac{(\bar{x}+\bar{y})(x+y)}{\sqrt{xy\bar{x}\,\bar{y}}}\frac{1}{(x-y)^2}$$
$$K_1(x) = \frac{\beta_1}{x^3} + \frac{\beta_1}{\bar{x}^3}$$
$$H_0^-(x,y) = -\frac{\lambda}{4\pi\sqrt{xy\bar{x}\,\bar{y}}}$$
$$H_1(x,y) = \frac{1}{(x-y)^2\sqrt{x\bar{x}y\bar{y}}} + \beta_2(x^2-y^2)\left(\frac{1}{\bar{y}^2} - \frac{1}{\bar{x}^2}\right) + \beta_2(\bar{x}^2-\bar{y}^2)\left(\frac{1}{y^2} - \frac{1}{x^2}\right), \quad (34)$$

with $\bar{x} = 1-x$ and $\bar{y} = 1-y$. In general, this equation is solved in the same Hilbert space that defines $K_0 - H_0$, if we note that $K_0 - H_0$ is hermitian in the space defined with the measure $\int \phi^\dagger\phi$ where the set of ϕ_n forms a complete basis set. The formal solution to (34) is

$$\phi_1^+(x) = \sum_{m\neq n}\frac{\phi_m(x)\int_0^1 dy\phi_m^\dagger(y)\tilde{\phi}(y)}{M_m^2 - M_n^2}. \quad (35)$$

The $\frac{1}{P}$ expansion now clearly shows that the rate at which the quasidistribution (30) approaches the asymptotic light-cone distribution $|\phi_n(x)|^2$ is smooth for all $x \neq 0, 1$. It is singular for $x = 0, 1$ through the contribution of the backward moving pair ϕ^- in (32). So the large P limit should be taken before the $x \to 0, 1$ limits at the edges.

V. ALGEBRAIC STRUCTURE

The algebraic framework (6) allows us to go beyond the leading order in $1/N_c$, and therefore check the proposal in [1] beyond the leading order we have so far established. A solution to the algebraically closed set (6) can be found by organizing the bilocal operator in $1/N_c$,

$$M = M^0 + \frac{1}{N_c}M^1 + \mathcal{O}\left(\frac{1}{N_c^2}\right)$$
$$N = N^0 + \frac{1}{N_c}N^1 + \mathcal{O}\left(\frac{1}{N_c^2}\right), \quad (36)$$

where M^0 satisfies the commutation relation

$$[M^0(k_1,k_2), M^{0\dagger}(k_3,k_4)] = \delta(k_1-k_3)\delta(k_2-k_4), \quad (37)$$

in the large N_c limit. In terms of (36)–(37), the solution to (6) can be found by inspection in leading and next to leading order

$$N_{12}^0 = \int d3\, M_{13}^{0\dagger}M_{23}^0$$
$$\bar{N}_{12}^0 = \int d3\, M_{31}^{0\dagger}M_{32}^0$$
$$M_{12}^1 = \mp\frac{1}{2}\int d3d4\, M_{34}^{0\dagger}M_{14}^0M_{32}^0$$
$$N^1 = 0. \quad (38)$$

Here, we are using the short-hand notations $d3,4 \equiv dk_{3,4}$, $M_{13} \equiv M(k_1,k_3)$ and so on. It is important to note that the expansion of the N's starts at second order. From now on to avoid cluttering, we omit the 0 for the large N_c asymptotic operator.

When the operators in (38) are inserted back into the Hamiltonian, we obtain a complete expression for the first three terms of the $1/N_c$ expanded Hamiltonian in terms of the large N_c asymptotic operators that define the Hilbert space. Specifically and to order $\frac{1}{N_c^2}$, the various contributions to the Hamiltonian can be schematically written as

$$H \approx K_{MM}^{00}M^\dagger M + \frac{1}{N_c}K_{MM}^{01}(M^{\dagger 1}M + M^\dagger M^1) + \frac{1}{N_c^2}K_{MM}^{11}M^{1\dagger}M^1 + \frac{K_{NM}^{00}}{\sqrt{N_c}}NM + \frac{K_{NM}^{01}}{N_c\sqrt{N_c}}NM^1 + \frac{K_{NN}^{00}}{N_c}NN, \quad (39)$$

with the K's referring to pertinent coefficients (integrals of the wave functions). Thus, up to order $1/N_c^2$ we encounter six M interactions, but up to oder $1/N_c\sqrt{N_c}$ we are still dealing with more tractable quartic and qubic terms. Our algebraic treatment differs notably from the one presented in [11] in that in ours the algebra is corrected which is required for a consistent expansion. The resulting effective hadronic Hamiltonian is different.

VI. CORRECTION TO THE PDF IN SPINOR QCD

Thus far, our discussion has concentrated on two-dimensional scalar QCD, where we have established that the quasiparton distribution function reduces to the parton distribution function in leading order in $1/N_c$. We have checked that this is also the case for two-dimensional spinor QCD, in agreement with a recent study [8]. In the Appendix, we have briefly summarized the key changes from scalar to spinor in the light-cone and axial gauge.

Since in the spinor version, the underlying fields are fermionic and not bosonic, the algebraic structure (6) differs from scalar to spinor QCD only in the sign switch $s = +1 \to -1$, with exactly the same bosonized Hamiltonian (39). Also, to avoid unecessary long formula we will only discuss the $1/N_c$ corrections to the parton distribution function in two-dimensional spinor instead of scalar QCD. The arguments for both models are similar, but the formula for scalar QCD are laboriously long as we have checked, with exactly the same conclusion.

Using the definitions for spinor QCD in the Appendix, the pertinent bilocal mesonic operator M in the light-cone gauge takes now the form

$$M(xP,(1-x)P) = \frac{1}{\sqrt{|P|}}\sum_n m_n(P)\phi_n(x) \tag{40}$$

which satisfies (6) with $s = -1$. To order $1/N_c$, the Hamiltonian for two-dimensional spinor QCD is the same as in (39), which after inserting (40) yields the first two leading contributions to the interaction of the form

$$\begin{aligned}&\frac{\lambda}{4\pi\sqrt{N_c}}\int\frac{dPdP_1}{P^{\frac{3}{2}}}\\&\times\left(m_i^\dagger(P_1)m_j^\dagger(P-P_1)m_k(P)f_{ijk}\left(\frac{P_1}{P}\right)+\mathrm{c.c}\right)\\&+\frac{1}{N_c}m^\dagger m^\dagger mm.\end{aligned} \tag{41}$$

The quartic contribution in (41) is only shown schematically. It is of order $1/N_c$, and apparently relevant for the $1/N_c$ correction to the parton distribution function. However, by simple inspection it gives zero contribution when acting on a free and leading meson contribution to the state, i.e.,

$$\left(\frac{1}{N_c}m^\dagger m^\dagger mm\right)m^\dagger|0\rangle = 0. \tag{42}$$

It will be dropped. Therefore, the leading correction to the parton distribution function is given by

$$\begin{aligned}&\sum_{kl}\int\frac{dkdq}{2\pi}\phi_k\left(\frac{xP}{xP+q}\right)\phi_l\left(\frac{k}{k+q}\right)\\&\times{}^1\langle P_i|\left(\frac{m_k^\dagger(xP+q)m_l(k+q)}{\sqrt{(xP+q)(k+q)}}\right)|P_i\rangle^1.\end{aligned} \tag{43}$$

Here, $|P\rangle^1$ is the first-order perturbation of the meson state $m_i^\dagger(P)|0\rangle$, which by standard perturbation theory reads

$$\begin{aligned}|P\rangle^1 &= \frac{\lambda}{2\sqrt{2\pi N_c}}\int dP_1\\&\times\sum_{kl}\left(\frac{f_{kli}(\frac{P_1}{P})m_k^\dagger(P_1)m_l^\dagger(P-P_1)}{\frac{m_k^2}{x}+\frac{m_l^2}{1-x}-m_i^2}\right)|0\rangle.\end{aligned} \tag{44}$$

Inserting (44) into (43) and carrying out the contractions yields

$$\delta q_i(x) = \frac{\lambda^2}{\pi^2 N_c}\int_0^{1-x}dy\sum_{kk'l}\frac{F_{kli}(x,y)F_{k'li}(x,y)}{x+y} \tag{45}$$

as a correction to the leading parton distribution function $q_i(x) = |\phi_i^+(x)|^2$, with

$$F_{kli}(x,y) = \frac{f_{kli}(x+y)\phi_k(\frac{x}{x+y})}{\frac{m_k^2}{x+y}+\frac{m_l^2}{1-x-y}-m_i^2}, \tag{46}$$

and

$$\begin{aligned}\frac{f_{ijk}(x)}{\sqrt{x(1-x)}} &= \int dx_1dx_2\frac{\phi_i(x_1)\phi_j(x_2)\phi_k(x+x_2-xx_2)}{(xx_1+xx_2-x-x_2)^2}\\&\quad-\int dx_1dx_2\frac{\phi_i(x_1)\phi_j(x_2)\phi_k(x_2-xx_2)}{(xx_1+xx_2-x-x_2)^2}.\end{aligned} \tag{47}$$

VII. CORRECTION TO THE QUASI-PDF IN SPINOR QCD

In this section, we derive the $1/N_c$ correction to the quasiparton distribution function for two-dimensional spinor QCD and show that it is in agreement with the $1/N_c$ correction to the parton distribution we just established in the large momentum limit. For that, we switch to the description of two-dimensional spinor QCD in the axial gauge using the changes in the Appendix.

In the axial gauge, the Hamiltonian is written in terms of $m_n(P)$ and $\phi_\pm$. The structure of the Hamiltonian is still of the form (39). We now note that the contributions to the first order shift of the state $|P\rangle^1$ of the form $m^\dagger m^\dagger m^\dagger$ always carries ϕ^-. In the large momentum limit, these terms drop out as we have shown earlier, so they will be ignored.

The only surviving terms in the Hamiltonian at large momentum are also of the form $m^\dagger m^\dagger m + \text{c.c.}$

With the above in mind and to be more specific, the parts of the Hamiltonian (39) that will contribute to the quasi-parton distribution function in leading order in perturbation theory are of the form

$$H_1 = \frac{1}{\sqrt{N_c}} \sum_{123} f_{123} m_1^\dagger m_2^\dagger m_3$$
$$H_2 = \frac{1}{N_c} \sum_{1234} f_{1234} m_1^\dagger m_2^\dagger m_3^\dagger m_4. \tag{48}$$

The ensuing shifts caused by (48) on the mesonic state to first order in $\frac{1}{N_c}$ are, respectively, of the form

$$|i\rangle^1 = \frac{1}{\sqrt{N_c}} \sum_{12} |12\rangle \alpha_{12i} \qquad |i\rangle^2 = \frac{1}{N_c} \sum_{123} |123\rangle \alpha_{123i}, \tag{49}$$

with

$$\alpha_{12i} = \frac{f_{12i}}{E_1 + E_2 - E_i}$$
$$\alpha_{123i} = \frac{f_{123i}}{E_1 + E_2 + E_3 - E_i} + \sum_4 \frac{f_{123} f_{34i}}{(E_1 + E_2 + E_3 - E_i)(E_3 + E_4 - E_i)}, \tag{50}$$

and the coefficients f_{ijk} and f_{ijkl} are

$$f_{ijk}(P_1, P_2, P_3) = \frac{\lambda}{4\pi} \int dk_1 dk_2 dk_3 dk_4 dq \delta(k_1 + k_2 + k_3 - k_4) \delta(k_1 + k_2 - P_1) \delta(k_3 + q - P_2) \delta(k_4 + q - P_3)$$
$$\times \left(\frac{\phi_i^+(k_1, P_1) \phi_j^+(k_3, P_2) \phi_k^+(k_4, P_3) S(k_1, k_2, k_3, k_4)}{(k_1 - k_4)^2} - \frac{\phi_i^+(k_1, P_1) \phi_j^+(q, P_2) \phi_k^+(q, P_3) S(k_2, k_1, k_3, k_4)}{(k_1 + k_3)^2} \right)$$
$$+ f_{ijk}^-, \tag{51}$$

where we have set

$$S(k_1, k_2, k_3, k_4) = \cos\left(\frac{\theta(k_1) - \theta(k_4)}{2}\right) \sin\left(\frac{\theta(k_2) + \theta(k_3)}{2}\right). \tag{52}$$

The last contribution f_{ijk}^- involves at least one ϕ^- and therefore drops out in the large momentum limit, so it will not be quoted.

All contributions of the form f_{ijkl} involve at least one ϕ^- and also drop out in the large momentum limit. More specifically, in the large momentum limit, we set $P_i = P \to +\infty$, and we change our variables to $P_1 = xP$, $P_2 = yP$, and $P_3 = zP$, then any term which contains $\phi^-(x_1 P, x_2 P)$ vanishes in this limit, an example is the f_{1234} term.

The parton fractions are constrained kinematically. For instance, the energy denominator

$$\frac{1}{E_{xP} + E_{yP} + E_z - E_p}$$

implies $0 < x, y, z < 1$ in leading order in $1/P$, otherwise the contribution is subleading. In this case, the only term in H^1 which contains only ϕ^+ [first contribution in (39)] will reduce to the light-cone gauge term if one identifies the creation operators in both cases using

$$\phi_n^+(xP, P) \to \phi_n(x)$$
$$\frac{1}{E_{xP} + E_{(1-x)P} - E_P} \to \frac{2P}{\frac{m_1^2}{x} + \frac{m_2^2}{1-x} - m_i^2}. \tag{53}$$

More specifically, the first order correction to the quasi-parton distribution function is proportional to

$$\langle P| \int dp dq \, \sin\left[\frac{\theta(xP) + \theta(p)}{2}\right] M^\dagger(xP, q) M(p, q) |P\rangle$$
$$+ \langle P| \int dp dq \, \sin\left[\frac{\theta(xP) + \theta(p)}{2}\right]$$
$$\times M^\dagger(q, -p) M(q, -xP) |P\rangle, \tag{54}$$

with $|P\rangle$ corrected to first order. There are two type of contributions in (54) as we now discuss.

First, the $m^\dagger m$ term. For this only the $|i\rangle^1$ in the shift of the state contributes, and the specific contribution with only ϕ^+ is

$$2 \sin \frac{\theta(xP)}{2} \sum_{kk'l} \frac{\alpha_{kli} \alpha_{k'li}}{|k|} \phi_k^+(xP, p_k) \phi_{k'}^+(xP, p_k)$$
$$+ 2 \sin \frac{\theta(xP)}{2} \sum_{kk'l} \frac{\alpha_{kli} \alpha_{k'li}}{|k|}$$
$$\times \phi_k^+(xP + p_k, p_k) \phi_{k'}^+(xP + p_k, p_k). \tag{55}$$

In the large momentum limit, we have $p_k = yP$, and $p_l = (1 - y)P$ as discussed above. The first term is nonzero if

$0 < x < y$, and the second term is always zero for $0 < x < 1$ since $(x+y) > y$. Thus by shifting $y \to y + x$ with $0 < y < 1 - x$, and taking care of factors of P, this contribution matches the correction to the parton distribution function in the light-cone gauge (45).

Second, the $mm + m^\dagger m^\dagger$ term comes with at least one ϕ^-, and is always zero in the large P limit as discussed above. It follows, that the order $1/N_c$ contribution to the quasiparton distribution matches the parton distribution in the large momentum limit without renormalization in two-dimensional spinor QCD. We have explicitly checked that the same holds for two-dimensional scalar QCD.

VIII. CONCLUSIONS

Using a bosonized form of two-dimensional scalar and spinor QCD, we have analyzed the quasiparton distribution of a meson state. In the infinite momentum limit, the quasidistribution matches the parton distribution on the light cone both in leading and subleading order without further renormalization, but the limit is subtle at the parton fractions $x = 0$, 1. This provides a nonperturbative check on the proposal put forth by one of us [1] for extracting the QCD light-cone partonic distributions from their quasidistribution counterparts using pertinent equal-time Euclidean correlators through suitable matching at large momentum.

ACKNOWLEDGMENTS

This work was supported by the U.S. Department of Energy under Contract No. DE-FG02-93ER-40762 for X. J. and No. DE-FG-88ER40388 for Y. L. and I. Z.

APPENDIX: TWO-DIMENSIONAL SPINOR QCD IN THE LIGHT-CONE AND AXIAL GAUGE

Here and for convenience we briefly summarize some of the changes needed to recover spinor QCD from scalar QCD as developed in the main text. Both in the light-cone and axial gauge, the mesonic operators M and N are defined as in Sec. II A with $s = -1$. The fermionic fields in terms of creation-annihilation operators are defined as

$$\psi(x) = \int_0^\infty \frac{dp^+}{2\pi}(a(p^+)e^{-ip^+x^-} + b^\dagger(p^+)e^{-ip^+x^-})$$
$$\psi(x) = \int \frac{dp}{2\pi} e^{ipx}(a(p)u(p) + b^\dagger(-p)v(-p)), \tag{A1}$$

in the light-cone and axial gauge, respectively, with the two-dimensional spinors

$$u(p) = e^{-\frac{1}{2}\theta(p)\gamma^1}(1,0)^T$$
$$v(-p) = e^{-\frac{1}{2}\theta(p)\gamma^1}(0,1)^T. \tag{A2}$$

The angle $\theta(p)$ solves the transcendental equation [12]

$$p\cos(\theta(p)) - m\sin(\theta(p)) = \frac{\lambda}{2}\mathrm{PV}\int dk \frac{\sin(\theta(p)-\theta(k))}{(p-k)^2}. \tag{A3}$$

The mode decomposition in the light-cone gauge is given in (40), and in the axial gauge it is

$$M(k_1, P-k_1) = \frac{1}{\sqrt{|P|}} \times \sum_n (\phi_n^+(k_1,P)m_n(P) - \phi_n^-(-k_2,-P)m_n^\dagger(-P)). \tag{A4}$$

The bosonized Hamiltonian is still of the form (39), with the relevant $M^\dagger MM$ term given in the main text.

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