Regular stringy black holes?

Pablo A. Cano,* Samuele Chimento,† Tomás Ortín,‡ and Alejandro Ruipérez§
Instituto de Física Teórica UAM/CSIC, C/ Nicolás Cabrera, 13-15, Campus Universitario Cantoblanco, 28049 Madrid, Spain

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We study the first-order α' corrections to the singular four-dimensional massless stringy black holes studied in the 1990s in the context of the heterotic superstring. We show that the α' corrections not only induce a nonvanishing mass and give rise to an event horizon but also eliminate the singularity, giving rise to a regular spacetime whose global structure includes further asymptotically flat regions in which the mass of the spacetime is positive or negative. We study the timelike and null geodesics and their effective potential, showing that the spacetime is geodesically complete. We discuss the validity of this solution, arguing that the very interesting and peculiar properties of the solution are associated to the negative energy contributions coming from the terms quadratic in the curvature. As a matter of fact, the ten-dimensional configuration is singular. We extract some general lessons on attempts to eliminate black-hole singularities by introducing terms of higher order in the curvature.

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A very well-known class of four-dimensional extremal stringy black holes is characterized by four real functions $\mathcal{Z}_0$, $\mathcal{Z}_+$, $\mathcal{Z}_-$, $\mathcal{H}$ that occur in the metric and real scalar fields ϕ, k, ℓ as follows [1],

$$
\begin{aligned}
&ds^2 = e^{2U}dt^2 - e^{-2U}d\vec{x}^2,\\
&e^{-2U} = \sqrt{\mathcal{Z}_0\mathcal{Z}_+\mathcal{Z}_-\mathcal{H}}, \qquad e^{2\phi} = e^{2\phi_\infty}\frac{\mathcal{Z}_0}{\mathcal{Z}_-},\\
&\ell = \ell_\infty\left(\frac{\mathcal{Z}_0\mathcal{Z}_+\mathcal{Z}_-}{\mathcal{H}^3}\right)^{1/6}, \qquad k = k_\infty\left(\frac{\mathcal{Z}_+^2}{\mathcal{Z}_-\mathcal{Z}_0}\right)^{1/4},
\end{aligned}
\tag{1}
$$

where

$$
\mathcal{Z}_{0,+,-} = 1 + \frac{q_{0,+,-}}{r} \quad \text{and} \quad \mathcal{H} = 1 + \frac{q}{r}. \tag{2}
$$

This configuration is a solution at zeroth order in the parameter $\alpha' = \ell_s^2$ (ℓ_s is the string length; see the Appendix) of the so-called STU model that arises in the compactification on a 6-torus of the ten-dimensional heterotic superstring effective action [2,3].

Reexpressed in ten-dimensional variables, it belongs to the family of solutions considered in Ref. [4] that describe fundamental strings (associated to $\mathcal{Z}_-$), Kaluza-Klein (KK) monopoles (associated to $\mathcal{H}$), solitonic, or Neveu-Schwarz (NS) 5-branes (associated to $\mathcal{Z}_0$), and waves traveling along the fundamental strings (associated to $\mathcal{Z}_+$).

In Ref. [4], we showed that $\mathcal{Z}_-$ and $\mathcal{H}$ do not receive any first-order α' corrections. We also showed that, although $\mathcal{Z}_0$ gets corrections, all of them can be eliminated by choosing appropriate SU(2) instanton fields ("symmetric" solutions). Finally, $\mathcal{Z}_+$ also has first-order α' corrections of the form [5,10]

$$
\begin{aligned}
\mathcal{Z}_+ = 1 + \frac{q_+}{r} &+ \frac{\alpha' q_+}{2qq_0}\frac{r^2 + r(q_0 + q_- + q) + qq_0 + qq_- + q_0q_-}{(r+q)(r+q_0)(r+q_-)}\\
&+ \mathcal{O}(\alpha'^2)
\end{aligned}
\tag{3}
$$

that cannot be canceled using the mechanism mentioned above.

Due to the structure of the T-tensors, it can be argued as in Ref. [11] that the symmetric solution with any number of charges and with just the above first-order α' correction of $\mathcal{Z}_+$ may also be an exact solution to all orders in α', or that, at least, the higher-order corrections should be much smaller so the $\mathcal{O}(\alpha'^2)$ terms can be neglected for all purposes. It is interesting to investigate if these corrected and probably exact solutions satisfy some of the properties that are expected to occur in a UV complete theory and, in particular, the resolution of singularities.

We have already observed that α' corrections might resolve the singular horizon of small black holes (with two

*pablo.cano@uam.es
†samuele.chimento@csic.es
‡tomas.ortin@csic.es
§alejandro.ruiperez@uam.es

or three charges) [10,11] as in the classical example of Ref. [12], yielding a smooth horizon with nonvanishing area, though some divergencies would persist in the KK scalars. In this paper, we are going to study a particularly interesting set of singular solutions that have four nonvanishing charges: the so-called massless black holes of Refs. [6,13] (referred to as massless *quadruholes* in Ref. [14]). Research on massless black holes was originally motivated by Strominger's description of the conifold transition in Ref. [15]. Although his description was based on type II string theory and black holes with Ramond-Ramond charge, the solutions may be related by duality, and the metrics are indeed identical.

The massless quadruholes are a particular case of the solutions in Eq. (1) corresponding to the choice

$$q_0 = q_- = -q = -q_+ = Q \geq 0, \tag{4}$$

which is possible if the string coupling constant g_s and the radii of the compactification circles at infinity satisfy

$$g_s = \sqrt{\frac{N_{S5}}{N_{F1}}}, \qquad \frac{R_5}{\ell_s} = \sqrt{\frac{-N_W}{N_{F1}}}, \qquad \frac{R_4}{\ell_s} = \sqrt{\frac{-N_{S5}}{N_{KK}}}. \tag{5}$$

Here, N_{S5}, N_{F1}, N_W, and N_{KK} are integer numbers associated to the stringy objects of the ten-dimensional configuration. The usual requirements $g_s \ll 1$, $R_{4,5} > \ell_s$ are satisfied if these numbers fulfill the hierarchy

$$|N_W| > N_{F1} \gg N_{S5} > |N_{KK}|. \tag{6}$$

At zeroth order in α', the metric of the massless quadruholes reads

$$ds^2 = \left(1 - \frac{Q^2}{r^2}\right)^{-1} dt^2 - \left(1 - \frac{Q^2}{r^2}\right)(dr^2 + r^2 d\Omega^2_{(2)}). \tag{7}$$

The metric has an obvious naked singularity at $r = Q$, where the curvature as well as some scalars diverge. It has some interesting properties, though, such as the fact that this solution is massless and that the dilaton takes a constant value $e^{2\phi} = e^{2\phi_\infty}$. The repulsive behavior noticed in Ref. [13] is characteristic of timelike singularities such as those of the Reissner-Nordström or negative-mass Schwarzschild solutions.

Taking into account the α' corrections given by the general formula (3), the metric function e^{-2U} reads

$$e^{-2U} = \sqrt{\left(1 - \frac{Q^2}{r^2}\right)^2 + \frac{\alpha'}{2Q}\left(\frac{1}{r} + \frac{Q}{r^2} - \frac{Q^2}{r^3}\right)}, \tag{8}$$

and many interesting things start happening:

(1) Now, this solution has a mass

$$M = \frac{\alpha'}{8 Q G_N^{(4)}}. \tag{9}$$

(2) The geometry [16] is now regular at $r = Q$. Indeed, for $Q^2 > \alpha'/8$, the solution can be extended up to $r = 0$, where a smooth $\mathrm{AdS}_2 \times \mathrm{S}^2$ near-horizon geometry arises. The area of the horizon is given by the α'-independent expression [17]

$$A = 4\pi Q^2. \tag{10}$$

This is the standard expression, in terms of the charges, for the entropy of an extremal 4-charge black hole up to α' corrections. However, the relation between the entropy and the mass is very unconventional: A grows with Q, while M goes to zero.

(3) The most striking property of the metric (1) when α' corrections are taken into account (8) is that, if $Q^2 > \frac{\alpha'}{8}$, which corresponds to masses $M < \sqrt{\frac{\alpha'}{8}}/G_N^{(4)}$, it does not contain any singularity behind the horizon. In order to extend the solution beyond $r = 0$, let us introduce the *tortoise coordinate* r_* such that $dr_* \equiv e^{-2U} dr$. We define the ingoing Eddington-Finkelstein coordinate

$$v \equiv t + r_*, \tag{11}$$

which is constant along ingoing null radial geodesics. In terms of v, the metric reads

$$ds^2 = e^{2U} dv^2 - 2 dv dr - e^{-2U} r^2 d\Omega^2_{(2)}. \tag{12}$$

The metric is clearly regular at $r = 0$, and it can be extended to $r < 0$. A singularity would appear whenever $e^{-2U} = 0$, but looking at (8), we see that this function is strictly positive for all values of r if $Q^2 > \alpha'/8$. Hence, this spacetime contains no singularity, and we can extend it up to $r \to -\infty$, where it describes another asymptotically flat region.

(4) Without loss of generality, we can consider the motion of a test particle in the equatorial plane $\theta = \pi/2$. Associated to the Killing vectors ∂_v and ∂_φ, there are two constants of motion ϵ and L, which are given by

$$\epsilon \equiv e^{2U} \dot{v} - \dot{r}, \tag{13}$$

$$L \equiv r^2 e^{-2U} \dot{\varphi}. \tag{14}$$

Then, we can write the mass-shell condition as

$$\dot{r}^2 + V_{\text{eff}}(r) = \epsilon^2, \tag{15}$$

where the radial effective potential for massless and massive particles ($\kappa = 0$, 1 resp.) is given by

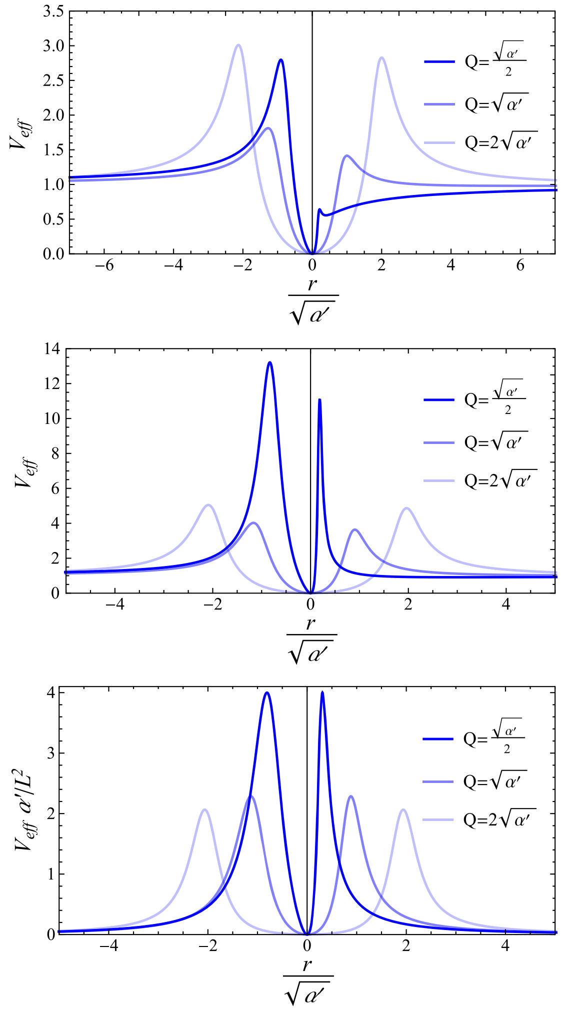

FIG. 1. Effective potential for different types of geodesics and for several values of the charge Q. Top: Massive particle moving along radial geodesics ($L = 0$). Middle: Massive particle in a nonradial geodesic with $L = \alpha'$. Bottom: Massless particle in a general geodesic (for $L = 0$, $V_{\rm eff} = 0$).

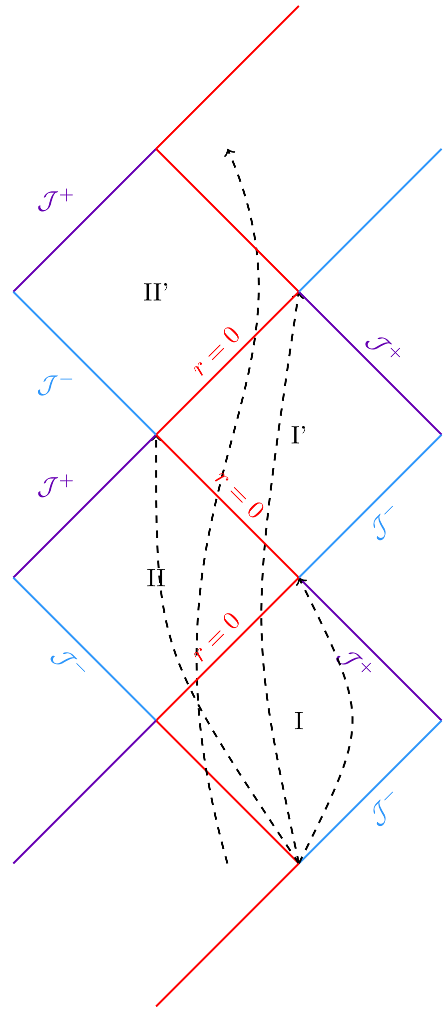

FIG. 2. Penrose diagram of the α'-corrected massless black holes. We have represented four kinds of possible timelike geodesics (described in the main text), including one corresponding to motion confined in the central valley of the effective potential.

$$V_{\rm eff}(r) = e^{2U}\left(\kappa + e^{2U}\frac{L^2}{r^2}\right). \qquad (16)$$

The qualitative behavior of the geodesics can be found by studying this effective potential, which we have plotted for several values of Q for timelike and null geodesics in Fig. 1.

The effective potential has a smoother form for larger values of Q [20]. In all cases, it presents two hills and a valley separating them which contains the black-hole horizon. Let us consider the geodesic of a massive particle. Coming from large positive values of r, the first hill represents a repulsive behavior that can be overcome if the particle has enough energy. If the particle passes over the first, rightmost, hill, it will cross the horizon at $r = 0$ and it will meet the second hill, which is always taller than the first. If the total energy is lower than the tip of the hill, which lies at some negative value of r the particle will bounce towards larger values of r, but, since it is not allowed to cross the horizon (by the very definition of event horizon) it will go into a different region of the spacetime for which $r = 0$ is a past horizon. Such a particle can reach the infinity of the new asymptotically-flat region unless some force pushes it towards the future event horizon of that region. The same discussion can be repeated and it is clear that an infinite number of asymptotically-flat regions connected as in Figure 2 exist.

If the energy of the particle is higher than the summit of the second, leftmost, hill of the effective potential, it will cross over it toward the $r \to -\infty$ of the type II region in the Penrose diagram, pushed by a repulsive force. This repulsive force is, now, associated to the negativity of the mass of the central object as seen from the type II regions. In some sense, it can be said that the mass of the α'-corrected solution remains zero because it has opposite values in contiguous type I and type II regions of the Penrose diagram.

(5) The effective potential for null geodesics, plotted in Fig. 1, has two hills of the same height; if a light ray has small enough impact parameter L/ϵ, then it always goes from type I to type II regions in the Penrose diagram. The maxima of the potential are exactly

$$V_{\rm eff}^{\rm max} = \frac{16L^2Q^2}{\alpha'(8Q^2 - \alpha')}. \tag{17}$$

(6) In the central valleys of the effective potentials, it is possible to have geodesics that never reach infinity and are confined between the hills. The particles cross the horizons (future and past) of different regions an arbitrary number of times and for an arbitrary number of regions as shown in the Penrose diagram.

(7) Finally, observe that, in the timelike case, the effective potential has another minimum in the right-hand side of the diagram (type I region) that becomes shallower the larger Q is. This happens even for $L = 0$ (radial motion), which means that there can be massive particles with purely radial motion confined between two radii.

Summarizing, we have found that α' corrections transform the singular massless black holes (7) into a geodesically complete spacetime which represents a regular black hole with no singularity. We focused on the particular example of the massless black holes for convenience, but the same result is found for more general values of the charges, provided that their signs are chosen as in (4).

Being extremal and supersymmetric, these solutions might evade some of the stability issues related to regular black holes, like the ones reported in Ref. [21]. In particular, we note that our black holes have a different structure as compared to the models analyzed there: they do not contain a de Sitter core, and the instability associated to it does not directly apply.

Although we have argued that the α'-corrected solutions may receive no further corrections, it is not clear to us how seriously they should be taken from the string theory point of view [22] because they are singular in $d = 10$ dimensions. The singularity is to be expected because some compactification circles diverge (shrink to zero radii in the dual theory) at given values of r. This pathology, on the other hand, may be interpreted as a sign of the relation between these solutions and the massless black holes of Ref. [15].

A feature of these solutions that may also be considered as another sign of this relation is the two-sided structure of the solution, which exhibits masses of opposite signs in contiguous type I and type II regions of the Penrose diagram. Some of the states that become massless in the conifold transition could be two-particle states, and, therefore, they should have opposite masses.

Despite the pathological character of the ten-dimensional solution, it should be noted that the cancellation of the black-hole singularity in $d = 4$ is a highly nontrivial effect related to the precise form of the corrections in $\mathcal{Z}_+$ given in (3). This function diverges with the right degree precisely at the points where the functions $\mathcal{H}$, $\mathcal{Z}_0$ and $\mathcal{Z}_-$ vanish, and this is the only way in which the singularity could be removed. Although the compactification is singular, everything conspires to produce a regular four-dimensional geometry.

Finally, since these solutions are also solutions of General Relativity with complicated couplings to matter, an explanation for their completely out of the ordinary features must be proposed. As we mentioned, the mass of the solution has opposite signs in the type I and type II regions. The presence of negative masses is usually associated to that of naked singularities, and the absence of the latter can only be attributed to the lack of positivity of the energy in the theory that we are considering. The terms of higher order in the curvature associated to the α' corrections typically have the wrong sign compared with terms quadratic in Yang-Mills curvatures [23]. Repulsive gravitational behavior associated to these higher-curvature corrections has been observed, for instance, in Ref. [24].

There have been many attempts in the literature to get rid of the singularities at the core of black holes, though the analysis is usually restricted to finding appropriate regular black-hole models [25,26]. The theories that achieve that goal usually introduce higher-derivative terms in the curvature [27–30] or in the matter fields [31,32] which may (or may not) be associated to quantum corrections of a theory of quantum gravity such as string theory. Effectively, many of those terms may introduce negative energy in the theory in a more or less consistent way (nobody really knows) that is ultimately responsible for the removal or softening of the singularities. We believe that this aspect of the higher-derivative terms deserves to be understood in depth if these theories are to be considered internally consistent.

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APPENDIX: THE HETEROTIC SUPERSTRING EFFECTIVE ACTION AT $O(\alpha')$

We work with the effective action of the Heterotic Superstring at the first order in α' derived in [1]

$$S = \frac{g_s^2}{16\pi G_N^{(10)}} \int d^{10}x \sqrt{|g|} e^{-2\phi} \left\{ R - 4(\partial\phi)^2 + \frac{1}{2\cdot 3!} H^2 - \frac{\alpha'}{8} \left(F^A{}_{\mu\nu} F^{A\mu\nu} + R_{(-)\mu\nu}{}^a{}_b R_{(-)}{}^{\mu\nu b}{}_a \right) \right\}, \tag{A1}$$

where F^A is the curvature of the Yang-Mills connection A^A, and $R_{(-)}{}^a{}_b$ is the curvature of the torsionful spin connection $\Omega_{(-)}{}^a{}_b = \omega^a{}_b - \frac{1}{2} H_\mu{}^a{}_b dx^\mu$, i.e.

$$F^A = dA^A + \frac{1}{2} \epsilon^{ABC} A^B \wedge A^C, \tag{A2}$$

$$R_{(-)}{}^a{}_b = d\Omega_{(-)}{}^a{}_b - \Omega_{(-)}{}^a{}_c \wedge \Omega_{(-)}{}^c{}_b. \tag{A3}$$

The 3-form field strength H receives corrections due to the Lorentz and Yang-Mills Chern-Simons terms

$$H = dB + \frac{\alpha'}{4} \left(\omega^{\rm YM} + \omega^{\rm L}_{(-)} \right), \tag{A4}$$

where

$$\omega^{\rm YM} = dA^A \wedge A^A + \frac{1}{3} \epsilon^{ABC} A^A \wedge A^B \wedge A^C, \tag{A5}$$

$$\omega^{\rm L}_{(-)} = d\Omega_{(-)}{}^a{}_b \wedge \Omega_{(-)}{}^b{}_a - \frac{2}{3} \Omega_{(-)}{}^a{}_b \wedge \Omega_{(-)}{}^b{}_c \wedge \Omega_{(-)}{}^c{}_a. \tag{A6}$$

Consequently, the Bianchi identity also gets corrected by

$$dH - \frac{\alpha'}{4} \left(F^A \wedge F^A + R_{(-)}{}^a{}_b \wedge R_{(-)}{}^b{}_a \right) = 0. \tag{A7}$$

Finally, the equations of motion for configurations that already solve the zeroth-order equations of motion are

$$\begin{aligned} R_{\mu\nu} - 2\nabla_\mu \partial_\nu \phi + \frac{1}{4} H_{\mu\rho\sigma} H_\nu{}^{\rho\sigma} - \frac{\alpha'}{4} \left(F^A{}_{\mu\rho} F^A{}_\nu{}^\rho + R_{(-)\mu\rho}{}^a{}_b R_{(-)\nu}{}^{\rho b}{}_a \right) &= 0, \\ (\partial\phi)^2 - \frac{1}{2} \nabla^2 \phi - \frac{1}{4\cdot 3!} H^2 + \frac{\alpha'}{32} \left(F^A{}_{\mu\nu} F^{A\mu\nu} + R_{(-)\mu\nu}{}^a{}_b R_{(-)}{}^{\mu\nu b}{}_a \right) &= 0, \\ d(e^{-2\phi} \star H) &= 0, \\ \alpha' [d(e^{-2\phi} \star F^A) + \epsilon^{ABC} A^B \wedge \star F^C + \star H \wedge F^A] &= 0. \end{aligned} \tag{A8}$$

[1] For the sake of simplicity, we omit the four nonvanishing vector fields associated to the four harmonic functions. They can be derived from the general ten-dimensional solution given in Ref. [4].

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