Supersymmetry breaking and ghost Goldstino in modulated vacua

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We discuss spontaneous supersymmetry (SUSY) breaking mechanisms by means of modulated vacua in four-dimensional $\mathcal{N} = 1$ supersymmetric field theories. The SUSY breaking due to spatially modulated vacua is extended to the cases of temporally and lightlike modulated vacua, using a higher-derivative model with a chiral superfield, free from the Ostrogradsky instability and the auxiliary field problem. For all the kinds of modulated vacua, SUSY is spontaneously broken and the fermion in the chiral superfield becomes a Goldstino. We further investigate the stability of the modulated vacua. The vacua are (meta)stable if the vacuum energy density is non-negative. However, the vacua become unstable due to the presence of the ghost Goldstino if the vacuum energy density is negative. Finally, we derive the relation between the presence of the ghost Goldstino and the negative vacuum energy density in the modulated vacua using the SUSY algebra.

I. INTRODUCTION

Understanding the vacuum structure of the quantum field theory under study is the starting point for any analysis. Toy models may have a simple vacuum structure in which the fields are energetically preferred to sit at the origin of the field space, retaining the symmetries present in the Lagrangian formulation.

Another example of vacua is that of QCD, which is expected to be far from trivial and dynamically generate a mass gap giving mass to the lightest glue state—the glueball—and to the hadrons of the theory when coupled to fermions.

In a recent series of papers, we have studied nontrivial vacua in which the vacuum expectation value (VEV) is not a constant but has a phase that winds along a spatial direction [1–3], along a temporal direction [3], or along the direction of the light cone [3]. Conceptually, we can think of this construction as an intermediate situation between the trivial vacua with a vanishing VEV and the enormously complicated vacuum of QCD. Our construction is inspired by the so-called Fulde-Ferrel (FF) state [4], which is the

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lowest-energy state in certain condensed matter systems, such as a superconducting ring with a magnetic field applied perpendicularly [5] (for a review see, e.g., Ref. [6]). The FF state also exists in QCD itself (Nambu-Jona–Lasino model) at finite density and temperature and is called a dual chiral density wave or a chiral spiral [7,8]. In these cases, Lorentz invariance is absent due to the finite temperature and/or density. In contrast, our construction works in a Lorentz invariant theory at vanishing density; however, it relies on the use of higher-derivative operators and the imposition of shift symmetry. In Ref. [2], we have shown that the global stability of this class of model dictates that the highest-derivative term must have $2(2\ell + 1)$ derivatives with $\ell \in \mathbb{Z}_{>0}$. The simplest class of models that contains a phase-modulated (FF-type) vacuum in the spatial, temporal, and lightlike directions has a sixth-order derivative term as the highest-derivative term. This model interestingly has, as a submodel, a supersymmetric extension [2].

It is well known that terms in the Lagrangian with more than one spacetime derivatives on one field cause an instability of the system. This is called the Ostrogradsky instability [9], which substantially results in ghost states in the quantum regime. At the classical level, it implies the loss of a lowest-energy state. We therefore focus on models where only higher-dimension operators given by the first-order derivatives of fields appear such as $(\partial\varphi)^{2n}$ for a scalar field φ.

Higher-derivative terms in supersymmetric field theories are quite nontrivial as they generically suffer from a problem called the auxiliary field problem. More precisely, in generic supersymmetric higher-derivative models, the

equation of motion (EOM) for the auxiliary field F is not necessarily algebraic [10,11]. The result is that eliminating F and finding the on-shell Lagrangian is essentially impossible. Usually, this auxiliary-field problem comes with an Ostrogradsky ghost [12,13], but there is also an exception [14,15].

We therefore look for supersymmetry (SUSY) models that do not suffer from the Ostrogradsky instability nor from the auxiliary field problem. A natural candidate for such a model is the higher-derivative chiral SUSY model studied in Refs. [2,16–22]. This latter model canonically gives a supersymmetric fourth-order derivative term, multiplied by a function of the superfield, $\Lambda(\Phi)$. Because of the fourth-order term being saturated in the nilpotent series of Grassmann numbers, the Grassmannian integral only picks up the bosonic component of the function Λ, and hence it is straightforward to construct a sixth-order derivative model this way. The model constructed this way turns out to be exactly a submodel of the phase-modulated higher-derivative scalar field theory models that we constructed in Refs. [1,3]. One of the interesting features is that SUSY is spontaneously broken due to derivatives of the field, $\partial\varphi$, in contrast to the conventional cases in which a nonzero SUSY auxiliary field, F term or D term, breaks SUSY.

In this paper, we study the SUSY breaking in all kinds of the phase-modulated (FF-type) vacua, i.e., spatially, temporally, and lightlike modulated vacua. First, we will review the construction of the phase-modulated vacuum solutions of Refs. [1–3] in the cases of spatially, temporally, and lightlike modulated vacua. Then we will discuss the fluctuations about these modulated vacuum solutions, for both the scalar field and the fermion. The main new result in this paper is that we derive a relation between the Goldstino and the vacuum energy density of the vacua in the models. After the discussion of the modulated vacua and the ghost Goldstinos in a concrete model, we rederive the same result in a model-independent way by using only the SUSY algebra and the knowledge of the broken/unbroken symmetries of the vacuum.

The plan of the paper is as follows. In Sec. II we review the type of higher-derivative chiral SUSY model that is a unique candidate for avoiding the Ostrogradsky problem and auxiliary field problem and lies in the class of models that can possess modulated vacua of Refs. [1–3] whose construction we review in Sec. III. Section IV reviews the bosonic fluctuation spectra and introduces the main new result, which comes from studying the fermionic fluctuations and finding the relation to the vacuum energy density. This latter relation is then studied using the SUSY algebra in Sec. V. Finally, Sec. VI concludes with a summary and a discussion of the open problems.

II. HIGHER-DERIVATIVE SUSY MODEL

In this section, we introduce a supersymmetric model in which modulated vacua of the FF-type is allowed. Since modulated vacua are characterized by a nonzero VEV of spacetime derivatives of a scalar field $\partial_m\varphi$, it is necessary to introduce a "potential" of the derivative term $\partial_m\varphi$ for it to develop a nonzero VEV.[1] This inevitably results in models where the bosonic part of the Lagrangian consists of terms with polynomials of $\partial_m\varphi$, i.e., higher-derivative SUSY models. In order to consider higher-derivative models, it is convenient to work in the off-shell superfield formalism. The four-dimensional $\mathcal{N} = 1$ superspace is characterized by the bosonic spacetime coordinates (x^m) $(m = 0, 1, 2, 3)$ and the fermionic coordinates given by Grassmann numbers $(\theta^\alpha, \bar{\theta}_{\dot{\alpha}})$. Here, the Greek letters beginning with $\alpha, \beta, \ldots$ and $\dot{\alpha}, \dot{\beta}, \ldots$ denote undotted and dotted spinors, respectively. We use the notation and conventions of Ref. [24] throughout this paper.

We introduce a chiral superfield, Φ, which contains a complex scalar field φ. This is utilized to describe VEVs in modulated vacua. The component fields of the chiral superfield, Φ, are defined as

$$\varphi = \Phi|, \qquad \psi_\alpha = \frac{1}{\sqrt{2}}D_\alpha\Phi|, \qquad F = -\frac{1}{4}D^2\Phi|, \quad (1)$$

where D_α is the supercovariant derivative and the symbol | denotes that the values are evaluated at $\theta^\alpha = \bar{\theta}_{\dot{\alpha}} = 0$. The field ψ_α is a Weyl fermion and F is an auxiliary field.

We now discuss the supersymmetric higher-derivative chiral models. As already mentioned in the Introduction, we here focus on models that only depend on the first derivative of the fields. This will sidestep the issue of the Ostrogradsky instability. Furthermore, to avoid the auxiliary field problem, we work in the higher-derivative chiral SUSY model of Refs. [2,16–22,25,26], which is known to be free from this problem as the EOM for F remains algebraic. The Lagrangian is given by[2]

$$\mathcal{L} = \int d^4\theta K(\Phi, \bar{\Phi}) + \left(\int d^2\theta W(\Phi) + \text{H.c.}\right)$$
$$+ \frac{1}{16}\int d^4\theta \Lambda(\Phi, \Phi, \partial_m\Phi, \partial_m\bar{\Phi}, D^2\Phi, \bar{D}^2\bar{\Phi})$$
$$\times (D^\alpha\Phi)(D_\alpha\Phi)(\bar{D}_{\dot{\alpha}}\bar{\Phi})(\bar{D}^{\dot{\alpha}}\bar{\Phi}). \quad (2)$$

Here, K, W, and Λ are a Kähler potential, a superpotential, and a real scalar function, respectively. The right-hand side of the first line gives us a quadratic kinetic term and a

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[1]A similar mechanism for nonzero VEVs of $\partial_m\varphi$ is discussed in the context of ghost condensation [23], in which case, due to the wrong sign of the canonical kinetic term, $\dot{\varphi}$ develops a nonzero VEV.

[2]The case of a constant Λ was first found long ago [27,28], the dependence of Λ on Φ, $\bar{\Phi}$ was introduced in Refs. [16–19], the dependence on $\partial_m\Phi$, $\partial_m\bar{\Phi}$ was found in Refs. [20–22], and finally the dependence on $D^2\Phi$, $\bar{D}^2\bar{\Phi}$ was found in Refs. [25,26].

potential term for φ, while the second line leads to higher-derivative terms.

As discussed in Ref. [3], in order to realize modulated vacua in our construction, it is necessary to introduce at least sixth-order derivative terms $(\partial\varphi)^6$. This is based on the global stability of the vacua. For simplicity, we also assume that the model possesses shift symmetry $\varphi \to \varphi + c$ where c is a complex constant. The simplest model that accommodates these conditions is

$$K = k\Phi\bar{\Phi}, \qquad W = 0, \qquad \Lambda = \lambda + \alpha\partial^m\Phi\partial_m\bar{\Phi}, \quad (3)$$

where k, λ, and α are real constants. Therefore, the model is given by

$$\mathcal{L} = \int d^4\theta k\Phi\bar{\Phi}$$
$$+ \frac{1}{16}\int d^4\theta(\lambda + \alpha\partial^m\Phi\partial_m\bar{\Phi})(D^\alpha\Phi)(D_\alpha\Phi)(\bar{D}_{\dot{\alpha}}\bar{\Phi})(\bar{D}^{\dot{\alpha}}\bar{\Phi}). \tag{4}$$

The above model was proposed in Ref. [2] where SUSY breaking in a spatially modulated vacuum is discussed.

In the Lagrangian, there is an auxiliary field F that does not have physical degrees of freedom. We eliminate the auxiliary field by the EOM. In order to obtain the EOM, it is convenient to write out the component Lagrangian from Eq. (4). The bosonic part of the Lagrangian is

$$\mathcal{L}_{\text{boson}} = -k\partial^m\varphi\partial_m\bar{\varphi} + kF\bar{F}$$
$$+ (\lambda + \alpha\partial^m\varphi\partial_m\bar{\varphi})((\partial^n\varphi\partial_n\varphi)(\partial^p\bar{\varphi}\partial_p\bar{\varphi})$$
$$- 2F\bar{F}\partial^n\varphi\partial_n\bar{\varphi} + F^2\bar{F}^2), \tag{5}$$

where we have omitted the fermions, since the fermions will be irrelevant to find the modulated vacua. Note that the fermionic part will be used when we discuss the fluctuation of the Goldstino. The EOM for the auxiliary field is

$$kF - 2F(\lambda + 2\alpha\partial^m\varphi\partial_m\bar{\varphi})(\partial^n\varphi\partial_n\bar{\varphi} - |F|^2) = 0. \tag{6}$$

As advertised above, the equation is algebraic; i.e., it does not involve terms with spacetime derivatives of F. We can therefore easily find solutions to this equation, and they are

$$F = 0, \qquad |F|^2 = -\frac{k}{2(\lambda + \alpha\partial^m\varphi\partial_m\bar{\varphi})} + \partial^m\varphi\partial_m\bar{\varphi}. \tag{7}$$

We note that these are exact analytic solutions in the bosonic sector, but including fermions is somewhat cumbersome. They can be incorporated in the solutions perturbatively as we will see in Sec. IV (see also Ref. [2] for the detailed analysis). There are two distinct on-shell branches corresponding to these solutions. For the first solution, the on-shell Lagrangian is

$$\mathcal{L}_{\text{boson}} = -k\partial^m\varphi\partial_m\bar{\varphi} + \lambda(\partial^m\varphi\partial_m\varphi)(\partial^n\bar{\varphi}\partial_n\bar{\varphi})$$
$$+ \alpha(\partial^m\varphi\partial_m\bar{\varphi})(\partial^n\varphi\partial_n\varphi)(\partial^p\bar{\varphi}\partial_p\bar{\varphi}). \tag{8}$$

Thus, the model contains a fourth- and a sixth-order derivative of the complex scalar field φ, as expected. On this branch, the higher-derivative terms are introduced perturbatively in addition to the canonical (quadratic) kinetic term. This is the so-called canonical branch. Several supersymmetric higher-derivative models are constructed using this branch including Dirac-Born-Infeld models [29,30], supersymmetric $P(X, \phi)$ models [16–18], higher-derivative corrections to a low-energy effective theory [21], and so on.

For the second solution in Eq. (8), the on-shell Lagrangian is

$$\mathcal{L}_{\text{boson,nc}} = (|\partial_m\varphi\partial^m\varphi|^2 - (\partial_m\varphi\partial^m\bar{\varphi})^2)(\lambda + \alpha\partial^n\varphi\partial_n\bar{\varphi})$$
$$- \frac{k^2}{\lambda + \alpha\partial^m\varphi\partial_m\bar{\varphi}}. \tag{9}$$

In this Lagrangian, the canonical (quadratic) kinetic term vanishes. This branch is the so-called noncanonical branch [16,18,20]. On this branch, the higher-derivative terms are not introduced perturbatively because we cannot take the limit $\lambda \to 0$ or $\alpha \to 0$. On the noncanonical branch, supersymmetric (baby-)Skyrme models have been discussed in Refs. [19,31–34]. Since the model that allows modulated vacua has a quadratic kinetic term, we use the first solution and its on-shell Lagrangian in Eq. (8) rather than that of Eq. (9).

III. MODULATED VACUA IN SUSY THEORIES

In this section, we examine modulated vacua in the model (8). First, we will discuss the general arguments for the modulated vacua. Second, we find the modulated vacua as solutions to the EOMs and energy-extremum conditions. We also calculate the energy density in the modulated vacua, which will be used in the later discussion. This section is mostly a review of the results in Ref. [3]. In the following, we give a brief overview of the general discussion of modulated vacua in Lorentz-invariant field theories. For more details, see Ref. [3].

A. General discussion

In the ordinary situation where VEVs are constants, they are determined by the extremal condition of the energy density. In this case, the VEVs solve the EOM automatically. On the other hand, the latter condition is not trivial for modulated vacua since in that case the VEVs depend on spacetime coordinates. In the following, we will write down the conditions for modulated vacua to solve both the energy-extremal condition and the EOM.

To find vacua, we solve the EOM and the energy-extremum condition for the complex scalar field φ using the Ansatz $\langle\psi_\alpha\rangle = \langle\bar{\psi}_{\dot{\alpha}}\rangle = 0$. The EOM for φ is generically

$$0 = \partial_m \frac{\partial \mathcal{L}_{\text{boson}}}{\partial \partial_m \varphi}$$

$$= \frac{\partial^2 \mathcal{L}_{\text{boson}}}{\partial(\partial_m \varphi)\partial(\partial_n \varphi)} \partial_m \partial_n \varphi + \frac{\partial^2 \mathcal{L}_{\text{boson}}}{\partial(\partial_m \varphi)\partial(\partial_n \bar{\varphi})} \partial_m \partial_n \bar{\varphi}, \quad (10)$$

and the EOM for $\bar{\varphi}$ is the complex conjugate of the above equation. The EOM together with its complex conjugate can be written in matrix form as follows:

$$0 = \mathbf{L}^{mn} \begin{pmatrix} \partial_m \partial_n \varphi \\ \partial_m \partial_n \bar{\varphi} \end{pmatrix}$$

$$= \mathbf{L}^{00} \begin{pmatrix} \ddot{\varphi} \\ \ddot{\bar{\varphi}} \end{pmatrix} + (\mathbf{L}^{0i} + \mathbf{L}^{i0}) \begin{pmatrix} \partial_i \dot{\varphi} \\ \partial_i \dot{\bar{\varphi}} \end{pmatrix} + \mathbf{L}^{ij} \begin{pmatrix} \partial_i \partial_j \varphi \\ \partial_i \partial_j \bar{\varphi} \end{pmatrix}. \quad (11)$$

Here, $\mathbf{L}^{mn}$ is defined as

$$\mathbf{L}^{mn} := \begin{pmatrix} \frac{\partial^2 \mathcal{L}}{\partial(\partial_m \bar{\varphi})\partial(\partial_n \varphi)} & \frac{\partial^2 \mathcal{L}}{\partial(\partial_m \bar{\varphi})\partial(\partial_n \bar{\varphi})} \\ \frac{\partial^2 \mathcal{L}}{\partial(\partial_m \varphi)\partial(\partial_n \varphi)} & \frac{\partial^2 \mathcal{L}}{\partial(\partial_m \varphi)\partial(\partial_n \bar{\varphi})} \end{pmatrix}. \quad (12)$$

In Eq. (11), we have split the spacetime derivative $\partial_m \varphi$ into the temporal direction $\partial_0 \varphi = \dot{\varphi}$ and the spatial directions $\partial_i \varphi$ ($i = 1, 2, 3$), since we will discuss the temporally, spatially, and lightlike modulated vacua. Vacua in field theories are characterized by (local) minima of the energy functional. The energy density (Hamiltonian) is defined as

$$\mathcal{H} := \frac{\partial \mathcal{L}}{\partial \dot{\varphi}} \dot{\varphi} + \frac{\partial \mathcal{L}}{\partial \dot{\bar{\varphi}}} \dot{\bar{\varphi}} - \mathcal{L}. \quad (13)$$

Since the energy density is written in terms of $\partial_m \varphi$ and its conjugate, the minima of the energy satisfy the following conditions:

$$0 = \frac{\partial \mathcal{H}}{\partial \partial_m \varphi} \quad (14)$$

and its complex conjugate. The conditions can be rewritten in terms of $\mathbf{L}^{mn}$ as

$$0 = \mathbf{L}^{00} \begin{pmatrix} \dot{\varphi} \\ \dot{\bar{\varphi}} \end{pmatrix}, \qquad 0 = \mathbf{L}^{i0} \begin{pmatrix} \dot{\varphi} \\ \dot{\bar{\varphi}} \end{pmatrix} - \begin{pmatrix} \frac{\partial \mathcal{L}}{\partial \partial_i \bar{\varphi}} \\ \frac{\partial \mathcal{L}}{\partial \partial_i \varphi} \end{pmatrix}. \quad (15)$$

The modulated vacua are characterized by the solutions to Eqs. (11) and (15).

In the vacua, spacetime or internal symmetries are generally broken. The vacua can be classified by the broken translational generators $P^{\hat{m}}$. If $P^{\hat{m}}$ is spacelike, timelike, or lightlike (null), the vacua are called spatially, temporally, or lightlike modulated vacua, respectively. In Ref. [3], the conditions for the presence of the spatially, temporally, or lightlike modulated vacua are studied systematically. Since

parts of the symmetries in the theory are broken, it is natural to study the Nambu-Goldstone (NG) modes in the vacuum. In the ordinary cases where VEVs are constants, the NG modes correspond to the flat directions of the potential term. The zero modes of the Hessian matrix associated with the curvature of the potential correspond to the NG modes. This implies that the quadratic term of the NG modes—the mass term—vanishes, and hence they are massless modes. In our setup, however, there are no ordinary potential terms but instead a "potential" for the derivatives of the fields. Therefore, we found that it is useful to consider the notion of the *generalized Nambu-Goldstone modes* to examine the flat directions in the modulated vacuum [1,3]. Similar to ordinary NG modes, the generalized NG modes correspond to zero modes of the Hessian matrix (the generalized mass matrix):

$$\mathcal{M} = \begin{pmatrix} \mathbf{M}^{00} & \mathbf{M}^{01} & \cdots & \mathbf{M}^{03} \\ \mathbf{M}^{10} & \mathbf{M}^{11} & \cdots & \\ \vdots & & \ddots & \\ \mathbf{M}^{30} & \cdots & & \mathbf{M}^{33} \end{pmatrix}, \quad (16)$$

where $\mathbf{M}^{mn}$ are given by

$$\mathbf{M}^{mn} = \begin{pmatrix} \frac{\partial^2 \mathcal{H}}{\partial(\partial_m \bar{\varphi})\partial(\partial_n \varphi)} & \frac{\partial^2 \mathcal{H}}{\partial(\partial_m \bar{\varphi})\partial(\partial_n \bar{\varphi})} \\ \frac{\partial^2 \mathcal{H}}{\partial(\partial_m \varphi)\partial(\partial_n \varphi)} & \frac{\partial^2 \mathcal{H}}{\partial(\partial_m \varphi)\partial(\partial_n \bar{\varphi})} \end{pmatrix}. \quad (17)$$

Now that we have the general conditions and material to analyze modulated vacua, we will in the next subsection solve the conditions Eqs. (11) and (15) in the cases of temporally, spatially, or lightlike modulated vacua.

B. Spatially modulated vacua

First, we consider the spatially modulated vacua. We employ the following Ansatz for solving Eqs. (11) and (15):

$$\langle \varphi \rangle = \varphi_0 e^{icx^1},$$

$$\langle \dot{\varphi} \rangle = \langle \partial_{2,3} \varphi \rangle = \langle \psi_\alpha \rangle = \langle \bar{\psi}_{\dot{\alpha}} \rangle = \langle F \rangle = \langle \bar{F} \rangle = 0. \quad (18)$$

Here, φ_0 and c are complex and real constants, respectively. Within the Ansatz, the energy-extremum condition in Eq. (15) becomes

$$0 = \partial_1 \bar{\varphi}(-k + 2\lambda|\partial_1 \varphi|^2 + 3\alpha|\partial_1 \varphi|^4). \quad (19)$$

When $\lambda^2 + 3\alpha k > 0$, there is a local minimum in the energy potential for which φ_0 is nonzero. In this case, the above condition determines the amplitude of the VEV:

$$|\partial_1 \varphi|^2 = c^2 |\varphi_0|^2 = \frac{-\lambda \pm \sqrt{\lambda^2 + 3\alpha k}}{3\alpha}. \quad (20)$$

Since $|\partial_1\varphi|^2$ is positive, the parameter α should be negative. As discussed in Ref. [3], for the Ansatz (18), we have the relation $\mathcal{H} = -\mathcal{L}$, and this implies that the extremal condition of $\mathcal{H}$ is equivalent to the EOM given by the first variation of the Lagrangian: $\delta\mathcal{L} = 0$. Therefore, solutions to Eq. (11) automatically satisfy the condition (15). Indeed, in the Ansatz (18), the right-hand side of the EOM in Eq. (11) reduces to

$$\mathbf{L}^{11}\begin{pmatrix} \partial_1^2\varphi \\ \partial_1^2\bar\varphi \end{pmatrix} = ic\begin{pmatrix} \partial_1\varphi(-k + 2\lambda|\partial_1\varphi|^2 + 3\alpha|\partial_1\varphi|^4) \\ -\partial_1\bar\varphi(-k + 2\lambda|\partial_1\varphi|^2 + 3\alpha|\partial_1\varphi|^4) \end{pmatrix},$$
(21)

which automatically vanishes if the energy-extremum condition is satisfied. In this vacuum, the VEV of the vacuum-energy density $\mathcal{E}_{sp}$ is

$$\mathcal{E}_{sp} = k|\partial_1\varphi|^2 - \lambda|\partial_1\varphi|^4 - \alpha|\partial_1\varphi|^6$$
$$= -\frac{1}{27\alpha^2}(\lambda + \sqrt{\lambda^2 + 3\alpha k})(6\alpha k + \lambda(\lambda + \sqrt{\lambda^2 + 3\alpha k})).$$
(22)

The energy density can be positive, zero, or negative depending on the parameters. If $\lambda^2 < -4\alpha k$, the energy density is positive. In this case, the modulated vacuum is metastable. If $\lambda^2 = -4\alpha k$, the energy density is zero. In this case, the trivial vacuum $\partial_1\varphi = 0$ and the modulated vacuum has the same energy density. If $\lambda^2 > -4\alpha k$, the energy density is negative. In this case, the modulated vacuum is energetically favored.

Finally, we discuss the spontaneous breaking of the symmetries in the spatially modulated vacuum. Because of the nonzero VEV of $\varphi = \varphi_0 e^{icx^1}$, the translational transformation P^1, the global $U(1)$ transformation, shift transformation S, and the Lorentz transformations M^{1m} become broken generators. However, since the simultaneous transformation of P^1 and the global $U(1)$ transformation is preserved, the symmetry breaking pattern is $\mathrm{ISO}(3,1) \times U(1) \times S \to \mathrm{ISO}(2,1) \times [U(1) \times \mathcal{P}^1]_{\mathrm{diag.}}$. Here, ISO denotes the Poincaré group, and $\mathcal{P}^m$ denotes the spacetime translational group. We also note that, in the vacuum (18), the SUSY variation of the fermion is nonvanishing:

$$\delta\psi_\alpha = i\sqrt{2}(\sigma^m)_{\alpha\dot\alpha}\bar\xi^{\dot\alpha}\partial_m\varphi + \sqrt{2}\xi_\alpha F = i\sqrt{2}(\bar\sigma^1)_{\alpha\dot\alpha}\bar\xi^{\dot\alpha}\partial_1\varphi.$$
(23)

Here $\xi, \bar\xi$ are SUSY transformation parameters. Therefore, SUSY is spontaneously broken in the spatially modulated vacuum. We note that the condition (23) holds for any values of the energy density. Namely, SUSY can be broken even for zero vacuum energy density in supersymmetric higher-derivative models. This is in contradistinction to the ordinary situation in which vacua are given by the extrema of potentials and VEVs are constants.

C. Temporally modulated vacua

For the temporally modulated vacua, we will solve Eqs. (11) and (15) by using the following Ansatz:

$$\langle\varphi\rangle = \varphi_0 e^{i\omega x^0},$$
$$\langle\partial_i\varphi\rangle = \langle\psi_\alpha\rangle = \langle\bar\psi_{\dot\alpha}\rangle = \langle F\rangle = \langle\bar F\rangle = 0. \quad (24)$$

Here, φ_0 and ω are complex and real constants, respectively. By the Ansatz, the condition for the extremum of the energy in Eq. (15) is reduced to

$$\mathbf{L}^{00}\begin{pmatrix} \dot\varphi \\ \dot{\bar\varphi} \end{pmatrix} = 0,$$
(25)

while the EOM in Eq. (10) is

$$\mathbf{L}^{00}\begin{pmatrix} \ddot\varphi \\ \ddot{\bar\varphi} \end{pmatrix} = 0.$$
(26)

Here, the matrix $\mathbf{L}^{00}$ is calculated as

$$\mathbf{L}^{00} = \begin{pmatrix} k + 4\lambda|\dot\varphi|^2 - 9\alpha|\varphi|^4 & 2\dot\varphi^2(-3\alpha|\dot\varphi|^2 + \lambda) \\ 2\dot{\bar\varphi}^2(-3\alpha|\dot\varphi|^2 + \lambda) & k + 4\lambda|\dot\varphi|^2 - 9\alpha|\varphi|^4 \end{pmatrix}.$$
(27)

These conditions are satisfied by $\mathbf{L}^{00} = 0$. The conditions lead to the temporally modulated vacua

$$|\langle\dot\varphi\rangle|^2 = \frac{\lambda}{3\alpha},$$
(28)

with the condition on the parameter

$$k = -\frac{\lambda^2}{3\alpha}.$$
(29)

If we further assume $k > 0$, both of the parameters λ and α must be negative.

Now we calculate the vacuum energy of the temporally modulated vacua. Since the parameters are restricted by Eq. (29), in contrast to the spatially modulated vacua, the energy density of the vacuum is determined to be the following negative value:

$$\mathcal{E}_{temp} = \frac{\lambda^3}{27\alpha^2} < 0.$$
(30)

Finally, we discuss the symmetry breaking pattern. Within the Ansatz (24), the broken symmetries are the temporal translation P^0, the Lorentz boost M^{0m}, the global $U(1)$ transformation, and the shift transformation. However, the

simultaneous transformation of the temporal translation and the global $U(1)$ transformation remains unbroken. Thus, the symmetry breaking pattern is $\mathrm{ISO}(3,1) \times U(1) \times S \to \mathrm{ISO}(2,1) \times [U(1) \times \mathcal{P}^0]_{\mathrm{diag.}}$.

The SUSY variation of the fermion in the vacuum reads

$$\delta\psi_\alpha = \sqrt{2}i(\bar{\sigma}^0)_{\alpha\dot{\alpha}}\bar{\xi}^{\dot{\alpha}}\dot{\varphi}. \tag{31}$$

We find again that SUSY is spontaneously broken in the temporally modulated vacuum, and in this case, the energy density is not positive but negative.

D. Lightlike modulated vacua

For the lightlike modulation, we assume the following Ansatz:

$$\langle\varphi\rangle = \varphi_0 e^{i\omega(x^0+x^1)},$$
$$\langle\partial_2\varphi\rangle = \langle\partial_3\varphi\rangle = \langle\psi_\alpha\rangle = \langle\bar{\psi}_{\dot{\alpha}}\rangle = \langle F\rangle = \langle\bar{F}\rangle = 0. \tag{32}$$

The Ansatz implies $\partial^m\varphi\partial_m\varphi = \partial^m\varphi\partial_m\bar{\varphi} = 0$. The conditions (11) and (15) are satisfied if we demand that $\mathbf{L}^{10} = 0$. Note that this condition is too strong, but it is a sufficient condition for the lightlike modulated vacua. Under the condition $\mathbf{L}^{10} = 0$, the energy-extremum condition implies

$$\frac{\partial\mathcal{L}}{\partial\partial_1\varphi} = -k\partial_1\bar{\varphi} = 0, \qquad \frac{\partial\mathcal{L}}{\partial\partial_1\bar{\varphi}} = -k\partial_1\varphi = 0. \tag{33}$$

These equations are satisfied if

$$k = 0. \tag{34}$$

Since $k = 0$, the condition $\mathbf{L}^{10} = 0$ leads to $4\lambda|\dot{\varphi}|^2 = 0$. Therefore, the parameter λ vanishes:

$$\lambda = 0. \tag{35}$$

With the conditions, $k = \lambda = 0$, the EOM is automatically satisfied. We can now calculate the vacuum energy, and it vanishes identically,

$$\mathcal{E}_{\mathrm{LL}} = 0, \tag{36}$$

since $k = \lambda = 0$ and $\partial^m\varphi\partial_m\varphi = \partial^m\varphi\partial_m\bar{\varphi} = 0$.

The SUSY variation of the fermion in the vacuum reads

$$\delta\psi_\alpha = \sqrt{2}i(\bar{\sigma}^+)_{\alpha\dot{\alpha}}\bar{\xi}^{\dot{\alpha}}\partial_+\varphi, \tag{37}$$

where $\sigma^+ = \sigma^0 + \sigma^1$, $x^+ = x^0 + x^1$, and $\partial_+ = \frac{\partial}{\partial x^+}$. Thus, SUSY is spontaneously broken in the lightlike modulated vacuum.

IV. FLUCTUATIONS AROUND THE MODULATED VACUA

In this section, we consider fluctuations of both the complex scalar field and the fermion around the modulated vacua. In the previous section, we have studied the modulated vacua that are configurations satisfying the EOM and the energy-extremum condition. Here, we discuss the local stability of the modulated vacua by calculating the quadratic fluctuations of the dynamical fields about the modulated vacua. First, we review the bosonic fluctuations in the modulated vacua [3]. Second, we consider the fermionic fluctuations. In the modulated vacua, the fermion becomes a Goldstino since SUSY is spontaneously broken. We will see that the Goldstino becomes a ghost if the vacuum energy is negative in the modulated vacua.

A. Fluctuation of the complex scalar field

Here, we recapitulate the fluctuation of the complex scalar field and its stability [3]. The fluctuation of the complex scalar field ϕ is characterized by the value of the complex scalar field around the VEV $\langle\varphi\rangle$,

$$\varphi \to \langle\varphi\rangle + \phi. \tag{38}$$

In the previous section, the vacua have been characterized by the energy-extremum conditions. In order to find physical vacua, we should consider the stability of the vacua. The local stability of the vacua can be seen from the stability of the fluctuation spectrum at the second order. Hence, we expand the energy density as follows:

$$\mathcal{H}(\partial_m\varphi, \partial_m\bar{\varphi}) = \mathcal{H}|_0 + \frac{\partial\mathcal{H}}{\partial\partial_m\varphi}\bigg|_0 \partial_m\phi + \frac{\partial\mathcal{H}}{\partial\partial_m\bar{\varphi}}\bigg|_0 \partial_m\bar{\phi}$$
$$+ \frac{1}{2}(\partial_m\bar{\phi}\,\partial_m\phi)\mathbf{M}^{mn}|_0 \begin{pmatrix} \partial_n\phi \\ \partial_n\bar{\phi} \end{pmatrix} + \cdots. \tag{39}$$

Here, the symbol $|_0$ denotes the value at the vacuum, the ellipses $\cdots$ mean the terms at the third order of the fluctuation field or higher. The matrices $\mathbf{M}^{mn}$ are the second-order derivatives of the energy density, defined in Eq. (17). Note that $\mathbf{M}^{\dagger 00} = \mathbf{M}^{00}$, $\mathbf{M}^{\dagger 0i} = \mathbf{M}^{i0}$, and $\mathbf{M}^{\dagger ij} = \mathbf{M}^{ji}$. In the modulated vacua, the energy-extremum condition implies

$$\frac{\partial\mathcal{H}}{\partial\partial_m\varphi}\bigg|_0 = \frac{\partial\mathcal{H}}{\partial\partial_m\bar{\varphi}}\bigg|_0 = 0. \tag{40}$$

Thus, the energy density can be rewritten as

$$\mathcal{H}(\partial_m\varphi, \partial_m\bar{\varphi}) = \mathcal{H}|_0 + \frac{1}{2}(\partial_m\bar{\phi}\,\partial_m\phi)\mathbf{M}^{mn}|_0 \begin{pmatrix} \partial_n\phi \\ \partial_n\bar{\phi} \end{pmatrix} + \cdots. \tag{41}$$

The local stability depends on the eigenvalues of $\mathbf{M}^{mn}$. If all the eigenvalues are non-negative, the vacua are locally stable.

The dynamics of the fluctuations is determined by the effective Lagrangian for the fluctuation fields, which is found by expanding the original Lagrangian around the vacua to second order:

$$\mathcal{L} = \mathcal{L}|_0 + \frac{1}{2}(\partial_m\bar{\phi}\,\partial_m\phi)\mathbf{L}^{mn}|_0 \begin{pmatrix} \partial_n\phi \\ \partial_n\bar{\phi} \end{pmatrix} + \cdots. \quad (42)$$

$$\mathbf{M}^{00} = -\mathbf{M}^{22} = -\mathbf{M}^{33} = \begin{pmatrix} k - \alpha|\partial_1\varphi|^4 & -2(\partial_1\varphi)^2(\lambda + \alpha|\partial_1\varphi|^2) \\ -2(\partial_1\bar{\varphi})^2(\lambda + \alpha|\partial_1\varphi|^2) & k - \alpha|\partial_1\varphi|^4 \end{pmatrix}, \quad (43)$$

$$\mathbf{M}^{11} = \begin{pmatrix} k - 4\lambda|\partial_1\varphi|^2 - 9\alpha|\partial_1\varphi|^4 & -2(\partial_1\varphi)^2(\lambda + 3\alpha|\partial_1\varphi|^2) \\ -2(\partial_1\bar{\varphi})^2(\lambda + 3\alpha|\partial_1\varphi|^2) & k - 4\lambda|\partial_1\varphi|^2 - 9\alpha|\partial_1\varphi|^4 \end{pmatrix}, \quad (44)$$

whereas the remaining components of $\mathbf{M}^{mn}$ vanish. Since the generalized mass matrix $\mathcal{M}$ is totally block diagonal, the local stability of the modulated vacua is determined from the spectrum of eigenvalues of the matrices $\mathbf{M}^{mn}$. For the matrices $\mathbf{M}^{00}$, $\mathbf{M}^{22}$, and $\mathbf{M}^{33}$, the eigenvalues A_1, A_2 are

$$A_1 = 0, \qquad A_2 = \frac{12\alpha k - 4\alpha\lambda(\lambda + \sqrt{\lambda^2 + 3k\alpha})}{9\alpha^2}, \quad (45)$$

whereas the eigenvalues B_1, B_2 of the matrix $\mathbf{M}^{11}$ are

$$B_1 = 0, \qquad B_2 = -\frac{4}{3\alpha}(\lambda^2 + 3\alpha k + \lambda\sqrt{\lambda^2 + 3\alpha k}). \quad (46)$$

The zero eigenvalue of $\mathbf{M}^{11}$ corresponds to the generalized NG mode due to the broken translational symmetry. The zero eigenvalues of $\mathbf{M}^{22}$ and $\mathbf{M}^{33}$ originate from the rotational symmetry $SO(3) \subset SO(1,3)$ of the original Lagrangian. In the region where $\alpha < 0$, $\lambda > 0$, and $\lambda^2 + 3\alpha k > 0$, the nonzero eigenvalues are positive. Since there are no negative eigenvalues, the modulated vacua are locally stable.

The fluctuations in the Lagrangian can also be computed. The eigenvectors corresponding to the zero eigenvalues of the matrix $\mathbf{M}^{00}$ are nondynamical up to the second order of the fluctuations. Explicitly, the Lagrangian is

$$\mathcal{L} = \mathcal{L}|_0 + \frac{1}{2}(\dot{\bar{\phi}} \quad \dot{\phi})\mathbf{M}^{00}|_0 \begin{pmatrix} \dot{\phi} \\ \dot{\bar{\phi}} \end{pmatrix} + \cdots. \quad (47)$$

Here, we have used $\mathbf{M}^{00}|_0 = \mathbf{L}^{00}|_0$ in the spatially modulated vacua. Therefore, the eigenvector associated with the zero eigenvalue is not dynamical. Since the zero modes of the generalized mass matrix and those of the matrices $\mathbf{L}^{mn}$

Note that we have used that the first order variation vanishes by the EOM: $\frac{\partial\mathcal{L}}{\partial\partial_m\varphi}|_0 = \frac{\partial\mathcal{L}}{\partial\partial_m\bar{\varphi}}|_0 = 0$. In the following, we will consider the fluctuation Lagrangian's corresponding stability in each of the cases of spatially, temporally, and lightlike modulated vacua in turn.

1. Spatially modulated vacua

For the spatially modulated vacua, the components of the $\mathbf{M}^{mn}$ are

coincide in the spatially modulated vacua in general, the canonical kinetic terms for the generalized NG modes disappear in those vacua [1]. It has been shown that this, however, is not the case for temporal and lightlike modulated vacua [3] as we will see below.

2. Temporally modulated vacua

For the temporally modulated vacua, the matrices $\mathbf{M}^{mn}$ are calculated as

$$\mathbf{M}^{00} = \begin{pmatrix} k + 12\lambda|\dot{\varphi}|^2 - 45\alpha|\dot{\varphi}|^4 & 6\dot{\varphi}^2(\lambda - 5\alpha|\dot{\varphi}|^2) \\ 6\dot{\bar{\varphi}}^2(\lambda - 5\alpha|\dot{\varphi}|^2) & k + 12\lambda|\dot{\varphi}|^2 - 45\alpha|\dot{\varphi}|^4 \end{pmatrix}, \quad (48)$$

$$\mathbf{M}^{11} = \mathbf{M}^{22} = \mathbf{M}^{33}$$
$$= \begin{pmatrix} k + 3\alpha|\dot{\varphi}|^4 & 2\dot{\varphi}^2(-\lambda + 3\alpha|\dot{\varphi}|^2) \\ 2\dot{\bar{\varphi}}^2(-\lambda + 3\alpha|\dot{\varphi}|^2) & k + 3\alpha|\dot{\varphi}|^4 \end{pmatrix}, \quad (49)$$

and the others vanish. Again, the generalized mass matrix $\mathcal{M}$ is block diagonal. The eigenvalues of the matrices can be calculated as follows. For the matrix $\mathbf{M}^{00}$, the eigenvalues A_1 and A_2 are

$$A_1 = 0, \qquad A_2 = -\frac{8\lambda^2}{3\alpha}. \quad (50)$$

The nonzero eigenvalue, A_2, is positive because α must be negative in temporally modulated vacua (see Sec. III C). The zero mode, A_1, is interpreted as a generalized NG mode for the temporally modulated vacua.

For the matrices $\mathbf{M}^{11}$, $\mathbf{M}^{22}$, and $\mathbf{M}^{33}$, the eigenvalues B_1 and B_2 vanish,

$$B_1 = B_2 = 0. \tag{51}$$

These zero modes are expected to be accidental, since translational invariance along the spatial directions is not broken in the temporally modulated vacua. Hence, there are no unstable modes in the bosonic fluctuation spectrum up to the second order in the fluctuations.

The dynamics of the bosonic fluctuations is determined by the effective Lagrangian for the fluctuation. Since the matrix $\mathbf{L}^{00}$ vanishes in the temporally modulated vacua, the fluctuations are not dynamical up to second order in derivatives. However, the fluctuations have a nonvanishing spatial dispersion relation because there is a nonzero eigenvalue in $\mathbf{L}^{11} = \mathbf{L}^{22} = \mathbf{L}^{33}$:

$$\mathbf{L}^{11} = \begin{pmatrix} -k + \alpha|\dot{\varphi}|^4 & -2\dot{\varphi}^2(\lambda - 2\alpha|\dot{\varphi}|^2) \\ -2\dot{\bar{\varphi}}^2(\lambda - 2\alpha|\dot{\varphi}|^2) & -k + \alpha|\dot{\varphi}|^4 \end{pmatrix}. \tag{52}$$

The eigenvalues s_1 and s_2 are

$$s_1 = 0, \qquad s_2 = \frac{8\lambda^2}{9\alpha}. \tag{53}$$

Since s_2 is positive, there are no unstable modes in the spatial-derivative sector.

3. Lightlike modulated vacua

For the lightlike modulated vacua, the matrices $\mathbf{M}^{mn}$ are

$$\mathbf{M}^{00} = \mathbf{M}^{11} = -\mathbf{M}^{01} = -\mathbf{M}^{10}$$
$$= \begin{pmatrix} -16\alpha|\dot{\varphi}|^4 & -8\alpha\dot{\varphi}^2|\dot{\varphi}|^2 \\ -8\alpha\dot{\bar{\varphi}}^2|\dot{\varphi}|^2 & -16\alpha|\dot{\varphi}|^4 \end{pmatrix}, \tag{54}$$

and the remaining matrices, including $\mathbf{M}^{22}$ and $\mathbf{M}^{33}$, vanish. Here, we have already used the condition $k = \lambda = 0$ which is necessary for the existence of the lightlike modulated vacua. Since the temporal and spatial modulations are mixed in the lightlike modulated vacua, it is convenient to switch to light-cone coordinates

$$\begin{pmatrix} x^+ \\ x^- \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \end{pmatrix}. \tag{55}$$

In these coordinates, the matrices $\mathbf{M}^{mn}$ are simply expressed as

$$\mathbf{M}^{--} = \mathbf{M}^{00} - \mathbf{M}^{01} - \mathbf{M}^{10} + \mathbf{M}^{11}$$
$$= 4\begin{pmatrix} -16\alpha|\dot{\varphi}|^4 & -8\alpha\dot{\varphi}^2|\dot{\varphi}|^2 \\ -8\alpha\dot{\bar{\varphi}}^2|\dot{\varphi}|^2 & -16\alpha|\dot{\varphi}|^4 \end{pmatrix}, \tag{56}$$

$$\mathbf{M}^{++} = \mathbf{M}^{+-} = \mathbf{M}^{-+} = 0. \tag{57}$$

The eigenvalues A_1, A_2 of $\mathbf{M}^{--}$ are

$$A_1 = -32\alpha\omega^4|\varphi_0|^4, \qquad A_2 = -96\alpha\omega^4|\varphi_0|^4. \tag{58}$$

In order for all the eigenvalues to be positive, we require that

$$\alpha < 0. \tag{59}$$

With this condition, the lightlike modulated vacua are locally stable.

Up to the second order in the fluctuation, there is no dynamics of the fluctuations in the lightlike modulated vacua. This is because all the matrices $\mathbf{L}^{mn}$ vanish in the lightlike modulated vacua. For example, the matrices along x^0 or x^1 directions are

$$\mathbf{L}^{00} = \mathbf{L}^{11} = -\mathbf{L}^{01} = -\mathbf{L}^{10} = 2\lambda\begin{pmatrix} |\dot{\varphi}|^2 & \dot{\varphi}^2 \\ \dot{\bar{\varphi}}^2 & |\dot{\varphi}|^2 \end{pmatrix} = 0. \tag{60}$$

Note that the fluctuations may become dynamical due to higher-order terms than the quadratic ones.

B. Fluctuation of the fermion

Since we consider a supersymmetric theory, we should consider the stability in the fermionic sector in the spatially, temporally, and lightlike modulated vacua as well. We will thus consider each case in turn in the following.

1. General arguments

We will now discuss the general arguments for the fluctuations of the fermion in spatially, temporally, or lightlike modulated vacua. The first observation is that the fermion becomes a Goldstino in the modulated vacua. This is due to the nonvanishing SUSY transformation of the fermion in the modulated vacua:

$$\langle \delta\psi_\alpha \rangle = \sqrt{2}i(\sigma^m)_{\alpha\dot{\beta}}\bar{\xi}^{\dot{\beta}}\langle \partial_m\varphi \rangle \neq 0. \tag{61}$$

Here, $\bar{\xi}^{\dot{\beta}}$ is a SUSY transformation parameter, and we have used that $\langle F \rangle = 0$ in the modulated vacua. The nonvanishing SUSY transformation of the fermion implies the existence of the Goldstino.

The kinetic term for the Goldstino can be found by expanding the Lagrangian around the modulated vacua, which up to the second order in the fluctuations reads

$$\mathcal{L}_{\text{f.kin}} = \int d^4\theta \left(k\Phi\bar{\Phi} + \frac{1}{16}(\lambda + \alpha\partial_m\Phi\partial^m\bar{\Phi})(D\Phi)^2(\bar{D}\bar{\Phi})^2 \right)$$
$$= -ik\bar{\psi}\bar{\sigma}^m\partial_m\psi + (\lambda + \alpha|\partial_m\varphi|^2)\Omega$$
$$\quad - i\alpha(\partial^m\psi\sigma^p\bar{\psi})(\partial_m\varphi)^2(\partial_n\varphi)^2(\partial_p\bar{\varphi})$$
$$\quad - i\alpha(\partial^m\bar{\psi}\bar{\sigma}^p\psi)(\partial_m\varphi)(\partial_n\bar{\varphi})^2(\partial_p\varphi) + \cdots. \tag{62}$$

Here, the ellipses $\cdots$ denote higher order terms in the fluctuations and are hence irrelevant for the kinetic term of the Goldstino. Ω is defined by

$$\Omega \equiv -\frac{i}{2}(\psi\sigma^m\bar{\sigma}^n\sigma^p\partial_p\bar{\psi})(\partial_m\varphi\partial_n\bar{\varphi})$$

$$+\frac{i}{2}(\partial_p\psi\sigma^p\bar{\sigma}^m\sigma^n\bar{\psi})(\partial_m\varphi\partial_n\bar{\varphi})$$

$$+i(\psi\sigma^m\partial^n\bar{\psi})(\partial_m\varphi\partial_n\bar{\varphi}) - i(\partial^m\psi\sigma^n\bar{\psi})(\partial_m\varphi\partial_n\bar{\varphi}). \quad (63)$$

In this expansion, we have used the fact that the fermion appears only at second order in the auxiliary field: $F = 0 + \mathcal{O}(\psi^2)$ on the canonical branch [2].

In the following, we will explicitly show the stability of the fluctuation of the Goldstino in the modulated vacua up to second order. The stability of the Goldstino in the modulated vacua depends on the sign of its kinetic term. In the Lagrangian, the sign of the time-derivative term $i\psi\bar{\sigma}^0\partial_0\psi$ of the Goldstino is given by that of the parameter k, which we assume to be positive: $k \geq 0$. However, the sign of the kinetic term can be altered by the presence of the VEV of the complex scalar field in the modulated vacua. If the kinetic term has the correct (wrong) sign, the fluctuation of the Goldstino is stable (unstable). For the spatially modulated vacua, there are metastable, degenerate, and unstable vacua. For the temporally modulated vacua, the Goldstino becomes a ghost, and the vacua become unstable. For the lightlike modulated vacua, the Goldstino is not dynamical. In the following, we will study the stability explicitly for each case in turn.

2. Spatially modulated vacua

For the spatially modulated vacua, the sign of the kinetic term depends on the model parameters as well as the vacuum solution as follows:

$$\mathcal{L}_{\text{f.kin}} = i(-k + \lambda|\partial_1\varphi|^2 + \alpha|\partial_1\varphi|^4)\bar{\psi}_{\dot{\alpha}}(\bar{\sigma}^0)^{\dot{\alpha}\beta}\partial_0\psi_\alpha$$

$$= -i\frac{\mathcal{E}_{\text{sp}}}{|\langle\partial_1\varphi\rangle|^2}\bar{\psi}_{\dot{\alpha}}(\bar{\sigma}^0)^{\dot{\alpha}\beta}\partial_0\psi_\alpha. \quad (64)$$

The sign of the kinetic term is thus related to that of the vacuum energy density (22). We can see that if the energy density is positive (negative), the Goldstino has the correct (wrong) sign for its kinetic term. This property was clarified in Ref. [2]. In Sec. V we will see that this relation is consistent with the analysis using the SUSY algebra.

3. Temporally modulated vacua

For the temporally modulated vacua, the kinetic term of the Goldstino becomes

$$\mathcal{L}_{\text{f.kin}} = i(-k - 3\omega^2|\varphi_0|^2\lambda + 5\alpha\omega^4|\varphi_0|^4)\bar{\psi}_{\dot{\alpha}}(\bar{\sigma}^0)^{\dot{\alpha}\alpha}\partial_0\psi_\alpha + \cdots$$

$$= -i\frac{\mathcal{E}_{\text{temp}}}{|\langle\dot{\varphi}\rangle|^2}\bar{\psi}_{\dot{\alpha}}(\bar{\sigma}^0)^{\dot{\alpha}\alpha}\partial_0\psi_\alpha, \quad (65)$$

where we have used the relations $k = -\frac{\lambda^2}{3\alpha}$, $\omega^2|\varphi_0|^2 = \frac{\lambda}{3\alpha}$, and $\mathcal{E}_{\text{temp}} = \frac{\lambda^3}{27\alpha^2}$. Since the energy density is negative,

$\mathcal{E}_{\text{temp}} < 0$, we can conclude that the Goldstino is a ghost Goldstino in the temporally modulated vacua.

4. Lightlike modulated vacua

For the lightlike modulated vacua, however, the quadratic kinetic term in Eq. (62) vanishes. This can be shown as follows. The existence of the lightlike modulated vacua requires $k = \lambda = 0$. Therefore, the Lagrangian up to second order in fermionic fluctuations becomes

$$\mathcal{L}_{\text{f.kin}} = \alpha|\partial_m\varphi|^2\Omega - i\alpha(\partial^m\psi\sigma^p\bar{\psi})(\partial_m\bar{\varphi})(\partial_n\varphi)^2\partial_p\bar{\varphi}$$

$$- i\alpha(\partial^m\bar{\psi}\bar{\sigma}^p\psi)(\partial_m\varphi)(\partial_n\bar{\varphi})^2\partial_p\varphi + \cdots. \quad (66)$$

However, the VEVs of $\partial^m\varphi\partial_m\varphi$ and $\partial^m\varphi\partial_m\bar{\varphi}$ vanish in the lightlike modulated vacua

$$\langle\partial^m\varphi\partial_m\varphi\rangle = \langle\partial^m\varphi\partial_m\bar{\varphi}\rangle = 0, \quad (67)$$

and thus the kinetic term of the Goldstino vanishes, too. Thus, the Goldstino is not dynamical. This property is consistent with the fact that the vacuum energy density vanishes in the lightlike modulated vacua.

V. VACUUM ENERGY DENSITY VS STABILITY OF GOLDSTINO

In this section, we will derive the relation between the sign of the kinetic term of the Goldstino and that of the vacuum energy density in the modulated vacua. In the previous sections, we have used a specific model for the modulated vacua, whereas the relation that we will demonstrate in this section is model independent as it is based entirely on the SUSY algebra and the preserved symmetries of the model (and corresponding modulated vacuum) at hand.

Since the fermion becomes the Goldstino, the dynamics of the fluctuation of the fermion can be kinematically discussed by using the SUSY algebra, i.e., the relation between SUSY and the Hamiltonian,

$$H = \frac{1}{4}(Q_1\bar{Q}_{\dot{1}} + Q_2\bar{Q}_{\dot{2}} + \bar{Q}_{\dot{1}}Q_1 + \bar{Q}_{\dot{2}}Q_2) =: \frac{1}{4}\sum_{\alpha:\text{spinors}}\bar{Q}_\alpha Q_\alpha.$$

$$(68)$$

By considering the vacuum expectation value of both sides, one can show that there is a ghost Goldstino when the vacuum energy is negative. We thus apply this to the discussion of the modulated vacua.

However, the translational generators along spatial or temporal directions may not be well-defined operators in the modulated vacua. This problem is caused by the divergence of the spatial integration of a charge operator. Therefore, we should discuss the relation between SUSY and the Hamiltonian in a system with finite (but large)

volume V with periodic boundary conditions to preserve translational invariance along spatial directions. The following discussion is similar to the one in Ref. [35].

A. Temporally modulated vacua

We will now show that the Goldstino is a ghost in the temporally modulated vacua in the case where the model has negative vacuum energy. First, we consider the vacuum expectation value of the relation between the Hamiltonian and the supercharges due to the SUSY algebra. For the temporally modulated vacua, we should discuss the relation in a finite volume. The vacuum expectation values read

$$\langle \text{vac}, \text{box} | H | \text{vac}, \text{box} \rangle = \frac{1}{4} \sum_{\alpha: \text{spinors}} \langle \text{vac}, \text{box} | \bar{Q}_\alpha Q_\alpha | \text{vac}, \text{box} \rangle,$$

(69)

where the ket $|\text{vac}, \text{box}\rangle$ denotes the vacuum state of the system in a box with the above discussed periodic boundary conditions. We will show that the right-hand side of Eq. (69) is the norm of the Goldstino one-particle state. We expand the right-hand side by inserting multiparticle states normalized by the finite volume $|X, \text{box}\rangle$ as follows:

$$\langle \text{vac}, \text{box} | H | \text{vac}, \text{box} \rangle = \frac{1}{4} \sum_{X, \alpha: \text{spinors}} |\langle X, \text{box} | Q_\alpha | \text{vac}, \text{box} \rangle|^2.$$

(70)

Now, we consider a case where the energy density has the vacuum expectation value $\mathcal{E}$:

$$\langle \text{vac}, \text{box} | H | \text{vac}, \text{box} \rangle = V \mathcal{E}.$$

(71)

Relating the ket in a finite system to that of an infinite system, we get

$$|X, \text{box}\rangle \rightarrow \left(\sqrt{\frac{(2\pi)^3}{V}} \right)^{N_X} |X\rangle,$$

(72)

where N_X denotes the number of particles in the state X. By this replacement, Eq. (70) can be rewritten as

$$V \mathcal{E} = \frac{1}{4} \sum_{X, \alpha: \text{spinors}} \frac{(2\pi)^{3N_X}}{V^{N_X}} |\langle X | Q_\alpha | \text{vac} \rangle|^2.$$

(73)

We argue that only the zero-momentum state in $|X, \text{box}\rangle$ contributes on the right-hand side. This is because $Q_\alpha |\text{vac}, \text{box}\rangle$ belongs to the same eigenstate of the three-momentum as $|\text{vac}, \text{box}\rangle$, since $[Q, P_i] = 0$ for $i = 1, 2, 3$ holds. Therefore, only the eigenstates with zero three-momentum $|X(\mathbf{p} = \mathbf{0})\rangle$ in $|X\rangle$ contribute to the right-hand side of Eq. (70). Therefore, $Q_\alpha = \int d^3 x S_\alpha^{m=0}(x^0, \mathbf{x})$ can be

reduced into the product of the volume V and the supercurrent $S_\alpha^m(x^0, \mathbf{x})$ at $\mathbf{x} = \mathbf{0}$:

$$|\langle X | Q_\alpha | \text{vac} \rangle|^2$$

$$= \left| \int d^3 x \langle X(\mathbf{p} = \mathbf{0}) | S_\alpha^0(x^0, \mathbf{x}) | \text{vac} \rangle \right|^2$$

$$= \left| \int d^3 x \langle X(\mathbf{p} = \mathbf{0}) | e^{i\mathbf{P} \cdot \mathbf{x}} S_\alpha^0(x^0, \mathbf{x} = \mathbf{0}) e^{-i\mathbf{P} \cdot \mathbf{x}} | \text{vac} \rangle \right|^2$$

$$= \left| \int d^3 x \langle X(\mathbf{p} = \mathbf{0}) | S_\alpha^0(x^0, \mathbf{x} = \mathbf{0}) | \text{vac} \rangle \right|^2$$

$$= V^2 |\langle X(\mathbf{p} = \mathbf{0}) | S_\alpha^0(x^0, \mathbf{x} = \mathbf{0}) | \text{vac} \rangle|^2,$$

(74)

where we have used that both the vacuum and the state X are zero-momentum states and hence the integral, finally, is independent of $\mathbf{x}$ and hence proportional to the volume. By using the above equation, Eq. (73) can be written as

$$\mathcal{E} = \frac{1}{4} \sum_{X, \alpha: \text{spinors}} \frac{(2\pi)^{3N_X}}{V^{N_X-1}} |\langle X(\mathbf{p} = \mathbf{0}) | S_\alpha^0(x^0, \mathbf{0}) | \text{vac} \rangle|^2. \quad (75)$$

In the limit $V \rightarrow \infty$, the dominant but finite contributions to the right-hand side come from the states with $N_X = 1$. Note, that the contribution from the zero-particle state should vanish since such a contribution diverges in the limit $V \rightarrow \infty$ while the left-hand side is finite. For the states with $N_X = 1$, we find the relation

$$\mathcal{E} = \frac{1}{4} \sum_{X, \alpha: \text{spinors}} (2\pi)^3 |\langle X(\mathbf{p} = \mathbf{0}, N_X = 1) | S_\alpha^0(x^0, \mathbf{0}) | \text{vac} \rangle|^2.$$

(76)

If the vacuum energy density is nonzero $\mathcal{E} \neq 0$, the state $S_\alpha^0(x^0, \mathbf{0}) | \text{vac} \rangle$ carries one particle state with $\mathbf{p} = \mathbf{0}$, which is identified as the Goldstino. Further, Eq. (76) can be seen as a norm of the one particle state $S_\alpha^0(x^0, \mathbf{0}) | \text{vac} \rangle$. Therefore, the norm of the Goldstino is negative if the vacuum energy density is negative.

B. Spatially modulated vacua

We will now show the relation between the negative vacuum energy and the ghost Goldstino in the spatially modulated vacua. In the spatially modulated vacua, the discussion is almost the same as in Sec. V A, except for the fact that the spatial translation (P_1) is broken, in spatially modulated vacua.

If we assume the phase of the vacuum expectation value of the complex scalar field is modulated as $\langle \varphi \rangle = \varphi_0 e^{icx^1}$, we can argue that the simultaneous transformation with P_1 and the global $U(1)$ is preserved. Here, we assume that $A\varphi = q\varphi$, where A is the Hermitian generator of the $U(1)$ transformation, and q is the charge of the complex scalar field. The unbroken operator is then given by

$$P_1^S := P_1 - \frac{c}{q}A, \qquad (77)$$

where P_1 is the Hermitian generator of the translation along the x^1 direction: $P_1\varphi = -i\partial_1\varphi$. Thus, we should try to use P_1^S instead of P_1. The only fact we need is that the $U(1)$ generator A commutes with the SUSY charge Q_α,

$$[A, Q_\alpha] = 0. \qquad (78)$$

The state X that contributes in Eq. (74) is the one with $p_2 = p_3 = 0$, but finite momentum and finite $U(1)$ charge q: $P_1|X\rangle = c|X\rangle$ and $\frac{c}{q}A|X\rangle = c|X\rangle$, respectively. This is because the conserved quantity is P_1^S, which corresponds to a translation and simultaneous local $U(1)$ transformation. Therefore, the relation between the supercurrent and the vacuum energy becomes

$$
\begin{aligned}
|\langle X|Q_\alpha|\text{vac}\rangle|^2 &= \left| \int d^3\boldsymbol{x} \langle X(p_2 = p_3 = 0, p_1 = c)|S_\alpha^0(x^0, \boldsymbol{x})|\text{vac}\rangle \right|^2 \\
&= \left| \int d^3x \langle X(p_2 = p_3 = 0, p_1 = c)|e^{ix\cdot P}e^{\frac{-ix^1 cA}{q}}e^{\frac{ix^1 cA}{q}}S_\alpha^0(x^0, \boldsymbol{x} = \boldsymbol{0})e^{-ix\cdot P}|\text{vac}\rangle \right|^2 \\
&= \left| \int d^3x \langle X(p_2 = p_3 = 0, p_1 = c)|e^{ix\cdot P}e^{\frac{-ix^1 cA}{q}}S_\alpha^0(x^0, \boldsymbol{x} = \boldsymbol{0})e^{\frac{ix^1 cA}{q}}e^{-ix\cdot P}|\text{vac}\rangle \right|^2 \\
&= \left| \int d^3x \langle X(p_2 = p_3 = 0, p_1 = c)|e^{ix\cdot P^S}S_\alpha^0(x^0, \boldsymbol{x} = \boldsymbol{0})e^{-ix\cdot P^S}|\text{vac}\rangle \right|^2 \\
&= \left| \int d^3x \langle X(p_2 = p_3 = 0, p_1 = c)|S_\alpha^0(x^0, \boldsymbol{x} = \boldsymbol{0})|\text{vac}\rangle \right|^2 \\
&= V^2 |\langle X(p_2 = p_3 = 0, p_1 = c)|S_\alpha^0(x^0, \boldsymbol{x} = \boldsymbol{0})|\text{vac}\rangle|^2, \qquad (79)
\end{aligned}
$$

where we have inserted a $U(1)$ transformation together with its inverse on the left-hand side of the supercurrent and commuted the inverse transformation to the other side of the latter. The resulting vector $\boldsymbol{P}^S = (P_1^S, P_2, P_3)$ is a set of operators for the unbroken symmetries.

By the same argument as in the case of the temporally modulated vacua, we conclude that the one-particle state $|\langle X(p_2 = p_3 = 0, p_1 = c, N_X = 1)|S_\alpha^0(x^0, \boldsymbol{0})|\text{vac}\rangle|^2$ becomes a ghost if the vacuum energy is negative.

C. Lightlike modulated vacua

For the lightlike modulated vacua, it will be convenient to use the light-cone coordinates. The symmetry breaking pattern in this case is $U(1) \times \mathcal{P}^0 \times \mathcal{P}^1 \to [U(1) \times \mathcal{P}^\pm]_{\text{diag}} \times \mathcal{P}^\mp$, where the $\mathcal{P}^\pm$ represents the translational symmetry group along the light-cone directions $x^\pm = x^0 \pm x^1$. For the VEV $\langle\varphi\rangle = \varphi_0 e^{i\omega(x^0 + x^1)}$, the symmetry breaking pattern becomes $U(1) \times \mathcal{P}^0 \times \mathcal{P}^1 \to [U(1) \times \mathcal{P}^+]_{\text{diag}} \times \mathcal{P}^-$. Note that we define the Hermitian generator of the translational group $\mathcal{P}^+$ and $\mathcal{P}^-$ as

$$P_+ := P_0 + P_1, \qquad P_- := P_0 - P_1, \qquad (80)$$

respectively. We again assume that the $U(1)$ charge of the complex scalar field is q: $A\varphi = q\varphi$. With this assumption, the unbroken generator corresponding to the unbroken group $[U(1) \times \mathcal{P}^+]_{\text{diag}}$ can explicitly be written as

$$P_+^L := P_+ - \frac{2\omega}{q}A, \qquad (81)$$

where we have used $P_+\varphi = 2\omega\varphi$. Thus, the unbroken translational operator along the x^1 direction P_1^L can be written in terms of unbroken generators as

$$P_1^L = \frac{1}{2}(P_+^L - P_-) = P_1 - \frac{\omega}{q}A, \qquad (82)$$

which is also an unbroken operator. Since A commutes with the SUSY generator, we can repeat the argument of Sec. V B by replacing c with ω. Thus, the relation between the sign of the vacuum energy density and the norm of the Goldstino also holds in the lightlike modulated vacua. In particular, the Goldstino becomes a zero-norm state in the vacua where the vacuum energy density vanishes, which agrees with our result in Sec. IV B 4.

VI. SUMMARY AND DISCUSSION

In this paper, we have explored a new spontaneous SUSY-breaking mechanism with spatially, temporally, or lightlike modulated vacua. We have used a ghost-free SUSY higher-derivative model with a chiral superfield, which is a supersymmetric extension [2] of the model used in Refs. [1,3].

In this model, all the spatially, temporally, or lightlike modulated vacua are realized as the energy-extremum state

and the solution to the EOM within the Ansatz of phase modulation. We have calculated the vacuum energy density of each of the modulated vacua. For the spatially modulated vacua, the vacuum energy can be positive, zero, or negative, depending on the choice of the parameter of the model. For the temporally modulated vacua, the vacuum energy density is always negative in our model. For the lightlike modulated vacua, the vacuum energy density vanishes.

We have then investigated the stability of the fluctuation around the modulated vacua. For the bosonic fluctuation given by a complex scalar field, there are stable and nondynamical fluctuations while there are no unstable modes in any of the modulated vacua. This property coincides with the non-SUSY case [3]. However, for the fermionic fluctuations, there are unstable ghost modes in the spatially or temporally modulated vacua. We have argued that the ghost can be related to the negative vacuum energy density of the modulated vacuum, using the SUSY algebra.

There are several possible directions for future work. One is to discuss the instability of the temporally modulated vacua in supersymmetric theories. The temporally modulated vacuum is unstable due to the negative vacuum energy density in our SUSY model, in contrast to the non-SUSY case [3], where the temporally modulated vacuum is stable. It is plausible that this instability is model dependent; however, future investigations are needed for such a conclusion. It may also be possible that the vacuum energy density is uplifted by higher-order terms such as $(\partial\phi)^8$ and the Goldstino becomes a physical fluctuation. As another possibility, SUSY might forbid stable temporally modulated vacua. In such a case, we should discuss the instability in a more model independent way. The application of our model to more realistic phenomenological models with metastable SUSY breaking modulated vacua would be interesting. We have studied SUSY breaking in a higher-derivative chiral superfield in this paper. An extension to a vector superfield would also be possible, since the most general higher-derivative vector superfield action, free from ghosts and the auxiliary field problem, is available [36]. We will leave these questions for future work.

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