Anti-evaporation and evaporation of an *n*-dimensional Reissner-Nordström black hole

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We generalize the $f(R)$ theory of anti-evaporation and evaporation for a Reissner-Nordström black hole in n-dimensional spacetime. We consider nonlinear conformally invariant Maxwell field. By perturbing the fields over Nariai-like spacetime associated with degenerate horizon, we describe the dynamic behavior of the horizon. We show that $f(R)$ gravity can offer both anti-evaporation and evaporation in an n-dimensional Reissner-Nordström black hole depending on the dimension n and the functional form of $f(R)$. Furthermore, we argue that, in one class of a nonoscillatory solution, stable and unstable anti-evaporation and evaporation exist. In the other class of oscillatory solutions, anti-evaporation and evaporation exist only with instability. The first class of solutions may explain a long-lived black hole.

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I. INTRODUCTION

One possible candidate which facilitates in exploring the current universe and delving into the early universe is primordial black hole. In the early universe, black hole can be formed due to nonlinear metric perturbations [1–3], density perturbations [4,5], the evolution of gravitational bound objects [6] etc. The observation of such black holes depends on its mass and anti-evaporation or evaporation properties. Primordial black hole could be .a possible component of dark matter [7–10]. Primordial black hole could explain current dark matter density better if one considers a wide range of mass of such black hole not limited only in a particular range. Dark matter may be explained by considering a lower mass range ($10^6 M_p$–$10^{11} M_p$) [7], where M_p is the Planck mass. So far, not much attention was given for primordial black hole with masses smaller than 10^{15} g in explaining dark matter since they are considered evaporated. The phenomenon of anti-evaporation offers a wider range of mass, since primordial black hole with lower masses may exist in the current universe. The black holes with low masses may survive and hence contribute to dark matter density today. Contrary to that, evaporation reduces the chance of the presence of primordial black hole in the current universe and hence its contribution to dark matter density. However, whether anti-evaporation exists is debatable.

The existence of evaporation of a black hole was first proposed by Hawking [11]. Later on, in contrast to that, Hawking and Bousso introduced anti-evaporation which is due to quantum correction [12] and appears for Nariai spacetime [13,14] where cosmological horizon and event horizon coincide. In the evaporation process, the black hole reduces its horizon size by emitting radiation through the quantum effect. The phenomenon of anti-evaporation, as its name, has the properties reverse to that of evaporation [15,16]. Grand unified theory is also a theory which explains anti-evaporation [17,18]. However, we will follow $f(R)$ theory in higher dimensions for a black hole with multihorizons, in particular, where these horizons become degenerate.

In Refs. [12,19,20], anti-evaporation due to quantum correction is studied by considering two-dimensional one-loop effective action. Here, the calculation is done in s-wave approximation. In addition to that, the appearance of conformal anomaly in four dimensions may provide anti-evaporation [15,20,21]. However, anti-evaporation is also possible at the classical level in $f(R)$ gravity [22–25]. $f(R)$ gravity may prevent a primordial black hole to be evaporated and assist to be long-lived even having small masses. Some recent efforts are made in Refs. [26–28]. Despite the fact that anti-evaporation is associated with instability at the classical level in some theories, e.g., in Ref. [23], attention should be given at the classical level in addition to the quantum level to the search for a stable solution. In this paper, we generalize the possibility of anti-evaporation and evaporation in $f(R)$ gravity in n dimensions at the classical level.

Kaluza-Klein theory was the first theory, where higher dimensions were introduced first [29–31] to unify gravity and electromagnetism. Later on, the idea of higher

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dimensions became a platform in supergravity [32] and string theory [33,34] in constructing a unified theory of gravity and other fundamental forces. Inspiring from these higher-dimensional theories, a lot of progress towards black hole physics have been made [35,36]. Some of the interesting results were found if one studies black hole physics in higher dimensions. For example, there is some possibility of creation of a mini higher-dimensional black hole at LHC [37]. String theory can calculate the black hole entropy statistically [38]. Furthermore, Schwarzschild, Reissner-Nordström and Kerr solutions were found in higher dimensions [39]. Higher dimensions were later considered in charged black holes [40], charged black holes in (a)dS spaces [41], the Banados-Teitelboim-Zaneli black hole [42,43], the radiating black hole [44], etc. In this paper, we explore the possibility of evaporation and anti-evaporation of a black hole in $f(R)$ gravity in higher dimensions. We generalize the $f(R)$-theory of anti-evaporation for Reissner Nordström black hole [23]. To work with exact analytical solution, we consider a conformally invariant Maxwell action in n dimensions which constrains the dimension n [45]. The electric field is obtained in this case the same as in four dimensions.

Section II briefly discusses the realization of anti-evaporation in $f(R)$ gravity. We discuss analytical solutions of the Reissner-Nordström black hole in $f(R)$ gravity in higher dimensions in Sec. III, and we also mention solutions for extreme black holes. In Sec. IV, we write modified equations up to the first order of perturbations and obtain solutions in n dimensions. We explain the anti-evaporation and evaporation for different values of n and other parameters of the theory. Finally, we conclude in Sec. V.

II. ANTI-EVAPORATION IN $F(R)$ GRAVITY

Generalizing the theory of Bousso and Hawking [12] for anti-evaporation, Odintsov and Nojiri showed its possibility even at the classical level [22]. In this construction, the authors considered $f(R)$ gravity, which is the basic requirement in explaining anti-evaporation. We can obtain a Nariai-like solution, where cosmological horizon and event horizon coincide, and this solution is associated with the solution for anti-evaporation. In the following, we present this formalism briefly in the same way as that of Ref. [27]. The action corresponding to $f(R)$ gravity and matter with gravitational constant G and Ricci scalar R can be written as

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}f(R) + S_m, \tag{1}$$

and the corresponding field equation of the metric to the action (1) is

$$f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - \nabla_\mu\nabla_\nu f'(R) + g_{\mu\nu}\Box f'(R) = 8\pi G T_{\mu\nu}, \tag{2}$$

where $T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta L_m}{\delta g^{\mu\nu}}$ is the energy momentum tensor. One can consider the energy-momentum of Maxwell field. However, to show the mechanism in this section, we assume no matter ($T_{\mu\nu} = 0$) and covariantly constant Ricci tensor, i.e., Ricci tensor is proportional to the metric $g_{\mu\nu}$. The field equation in this case reduces to

$$f(R) - \frac{1}{2}Rf'(R) = 0. \tag{3}$$

Eq. (3) provides a solution,

$$A(r) = 1 - \frac{R_0 r^2}{12} - \frac{M}{r}, \tag{4}$$

for the spacetime given by

$$ds^2 = -A(r)dt^2 + \frac{1}{A(r)}dr^2 + r^2 d\Omega^2, \tag{5}$$

where R_0 is the constant Ricci scalar and M is the mass of the black hole. This spacetime can be written similar as Nariai spacetime,

$$ds^2 = \frac{1}{\Lambda^2}\frac{1}{\cosh^2 x}(d\tau^2 - dx^2) + \frac{1}{\Lambda'^2}d\Omega^2, \tag{6}$$

where we defined new coordinates τ and x related to t and r via $t = 2r_0^2\tau/[(1 - R_0 r_0^2/2)\epsilon]$ and $r = r_0 + \frac{\epsilon}{2}(1 + \tanh x)$ with $\Lambda^2 = \frac{1 - r_0^2 R_0/2}{r_0^2}$, $\Lambda' = \frac{1}{r_0}$ (Λ^2 can be positive and negative). Here, in Nariai spacetime, two horizons are separated by a small distance $\epsilon \to 0$ ($r_1 = r_0 + \epsilon$) at r_0 and r_1. Λ becomes $1/r_0$ for $R_0 = 0$. To understand the behavior of horizon we consider a more general spacetime in terms of perturbations as follows,

$$ds^2 = \frac{e^{2\rho(x,\tau)}}{\Lambda^2}(d\tau^2 - dx^2) + \frac{e^{-2\phi(x,\tau)}}{\Lambda'^2}d\Omega^2, \tag{7}$$

where $\rho(x,\tau)$ and $\phi(x,\tau)$ are given by

$$\rho = -\ln(\cosh x) + \delta\rho, \tag{8}$$

$$\phi = \delta\phi. \tag{9}$$

We perturb the modified Einstein equations up to the first order. By considering $F''(R_0) \neq 0$, these first order equations offer a solution which is given by

$$\delta\rho = \rho_0 \cosh\omega\tau \cosh^\beta x, \qquad \delta\phi = \phi_0 \cosh\omega\tau \cosh^\beta x, \tag{10}$$

where ω and β are constants determined by the field equations. The horizon radius r_h is defined by

$$g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi = 0. \tag{11}$$

Discussion of the above equation, which defines the dynamic horizon, can be found in Refs. [12,23]. Here we give a more physically intuitive explanation. With spherical symmetric perturbations, we can replace the coordinates (x,τ) with "physical coordinates" (t,r) where r is defined by $r \equiv \frac{e^{-\phi(x,\tau)}}{\Lambda'}$, and t is chosen such that the metric has the form $ds^2 = g_{tt}(t,r)dt^2 + g_{rr}(t,r)dr^2 + r^2 d\Omega^2$. If the metric component g^{rr} has different signs inside and outside a surface $r = r_h$, no timelike trajectory can cross the surface, thus defining a horizon. The change of signature of g^{rr} at $r = r_h$ then implies $g^{rr}(r_h) = 0$. Note that in this coordinate system, $\phi = -\ln(\Lambda' r)$ and on the horizon $g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = g^{rr}\partial_r\phi\partial_r\phi = 0$ trivially holds. From the solution $\delta\phi = \phi_0 \cosh\omega\tau \cosh^\beta x$ and horizon condition (11), we obtain $\tanh^2\omega\tau = \tanh^2 x$ because of $\omega^2 = \beta^2$ [22]. It further simplifies the perturbation as $\delta\phi = \delta\phi_h = \phi_0(\cosh\beta\tau)^{\beta+1}$ at the horizon and hence the horizon radius $r_h = \frac{e^{-\delta\phi_h}}{\Lambda'}$ turns out to be

$$r_h = \frac{e^{-\phi_0(\cosh\beta\tau)^{\beta+1}}}{\Lambda'}. \tag{12}$$

The horizon radius in Eq. (12) can be increasing, decreasing or oscillatory depending on the parameters β or ω and ϕ_0. For real positive values of $\beta+1$, we have increasing or decreasing horizon for ϕ_0 negative or positive respectively. For $\phi_0 < 0$, the anti-evaporation occurs for $\beta+1$ positive. However, instability occurs in this case of anti-evaporation. For the other case $\phi_0 > 0$ with $\beta+1$ negative, we obtain stable anti-evaporation. For β imaginary, we can get oscillatory solution. Using this formalism, we study the anti-evaporation problem in n-dimensional spacetime. It is possible to include quantum corrections, however, we will consider only the classical phenomenon in this paper. In Sec. III, we consider the Maxwell field and obtain a solution in n dimensions. The analytical solution may not be obtained as long as we consider nonconformal invariant action. In order to get analytical solutions for perturbations, we require an analytical form of background solution. Therefore, we consider conformal invariant action for Maxwell field [45].

III. FIELD EQUATIONS IN N-DIMENSIONAL SPACETIME

As long as we assume conformally symmetric action for Maxwell field, the analytical solution can be obtained. We choose nonlinear form of action corresponding to Maxwell field to achieve such possibility. In this section we present the field equations and discuss the solution for $f(R)$ gravity with conformally invariant Maxwell field in n-dimensional spacetime. We consider the following action,

$$S = \int d^n x\sqrt{-g}[f(R) - (F_{\mu\nu}F^{\mu\nu})^p]. \tag{13}$$

Here R is Ricci scalar, $F_{\mu\nu}$ is electromagnetic field tensor and p is a positive integer. By varying the action with respect to the metric $g_{\mu\nu}$ and the Maxwell field A_μ respectively, we obtain

$$f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} + g_{\mu\nu}\nabla^\rho\nabla_\rho f'(R) - \nabla_\mu\nabla_\nu f'(R) = \frac{T_{\mu\nu}}{2}, \tag{14}$$

$$\nabla_\mu(F^{p-1}F^{\mu\nu}) = 0, \tag{15}$$

where $f'(R)$ is the derivative of $f(R)$ with respect to R and the energy momentum tensor may be written as

$$T_{\mu\nu} = 4\left(pF^{p-1}F_{\mu\lambda}F^\lambda_\nu - \frac{1}{4}g_{\mu\nu}(F)^p\right). \tag{16}$$

We seek for a constant curvature solution, i.e., $R = R_0 =$ constant. For such a case, the trace of the energy-momentum tensor should be zero. Under this condition, we find $n = 4p$. In addition, from Eq. (14), we also have

$$R_0 f'(R_0) - \frac{n}{2}f(R_0) = 0. \tag{17}$$

Eq. (17) simplifies Eq. (14) as

$$f'(R_0)\left(R_{\mu\nu} - \frac{g_{\mu\nu}}{n}R_0\right) = \frac{T_{\mu\nu}}{2}. \tag{18}$$

In n-dimensional spacetime, we consider the following line element,

$$ds^2 = -N(r)dt^2 + \frac{1}{N(r)}dr^2 + r^2 d\Omega^2_{n-2}, \tag{19}$$

where $d\Omega^2_{n-2}$ is the metric of an unit $(n-2)$-sphere and $N(r)$ is a static spherically symmetric function. In n-dimensional spacetime, the Ricci scalar turns out to be

$$R = -N''(r) - \frac{2(n-2)N'(r)}{r} + \frac{(n-2)(n-3)}{r^2} - \frac{(n-2)(n-3)N(r)}{r^2} = R_0. \tag{20}$$

The solution for $N(r)$ corresponding to Eq. (20) can be written as

$$N(r) = 1 - \frac{2m}{r^{n-3}} + \frac{C_1}{r^{n-2}} - C_2 r^2, \tag{21}$$

where $C_1 = \frac{q^2(-2q^2)^{(n-4)/4}}{f'(R_0)}$, $C_2 = \frac{R_0}{n(n-1)}$, m and q are constants associated to the mass and the charge of black hole respectively. It is noted that in this framework, the electric field behaves as in its standard form [45]. The electric field in this case is given by $E = \frac{q}{r^{\frac{n-2}{2p-1}}}$ and takes its standard form for $n = 4p$. In the following subsection, we obtain the conditions for degenerate horizon.

A. Extreme black hole

To investigate the instabilities and the evaporation and anti-evaporation, we consider a spacetime near the degenerate horizon. In general, the black hole has n horizons in this theory. Depending on the values of parameters, $f(R)$ black hole may have degenerate horizons where two or more horizons coincide. For such degenerate horizon, we have $N(r_0) = N'(r_0) = 0$ which provides the following equations,

$$N(r_0) = \frac{r_0^{n-2} - 2mr_0 + C_1 - C_2 r_0^n}{r_0^{n-2}} = 0, \tag{22}$$

or,

$$r_0^{n-2} - 2mr_0 + C_1 - C_2 r_0^n = 0, \tag{23}$$

and

$$N'(r_0) = -2m(3-n)r_0^{2-n} + C_1(2-n)r_0^{1-n} - 2C_2 r_0 = 0, \tag{24}$$

or

$$-2(3-n)mr_0 + (2-n)C_1 - 2C_2 r_0^n = 0. \tag{25}$$

We choose the value of C_2 from Eq. (23),

$$C_2 = \frac{r_0^{(n-2)} - 2mr_0 + C_1}{r_0^n}, \tag{26}$$

and we use this in Eq. (25) to obtain the value of m,

$$m = \frac{r_0^{(n-2)} + nC_1/2}{(n-1)r_0}. \tag{27}$$

Substituting Eq. (27) in Eq. (26), we obtain

$$C_2 = \frac{1}{(n-1)}\left[\frac{n-3}{r_0^2} - \frac{C_1}{r_0^n}\right], \tag{28}$$

which leads to

$$R_0 = n\left[\frac{n-3}{r_0^2} - \frac{C_1}{r_0^n}\right]. \tag{29}$$

To define a nearly extreme black hole, we transform r and t in terms of x and τ as

$$r = r_0 + \epsilon\cos(x), \tag{30}$$

$$t = \frac{2\tau}{\epsilon N''(r_0)}, \tag{31}$$

where ϵ is very small. A nearly extreme black hole will have the following form of function $N(r)$ [46,47],

$$N(r) \approx \frac{N''(r_0)}{2}(r - r_c)(r - r_h), \tag{32}$$

where $r_c = r_0 + \epsilon$ with $x = 0$, $r_h = r_0 - \epsilon$ with $x = \pi$. For such an extreme black hole, we can write the metric in the following form,

$$ds^2 = \frac{2}{N''(r_0)}(\sin^2 x d\tau^2 - dx^2) + r_0^2 d\Omega_{n-2}^2. \tag{33}$$

In Fig. 1, we plot $N(r)$ where we set parameters such that we get the degenerate horizon $N(r_0) = N'(r_0) = 0$. In Sec. IV, we study the perturbations around the solution for the extreme black hole. These two cases with positive

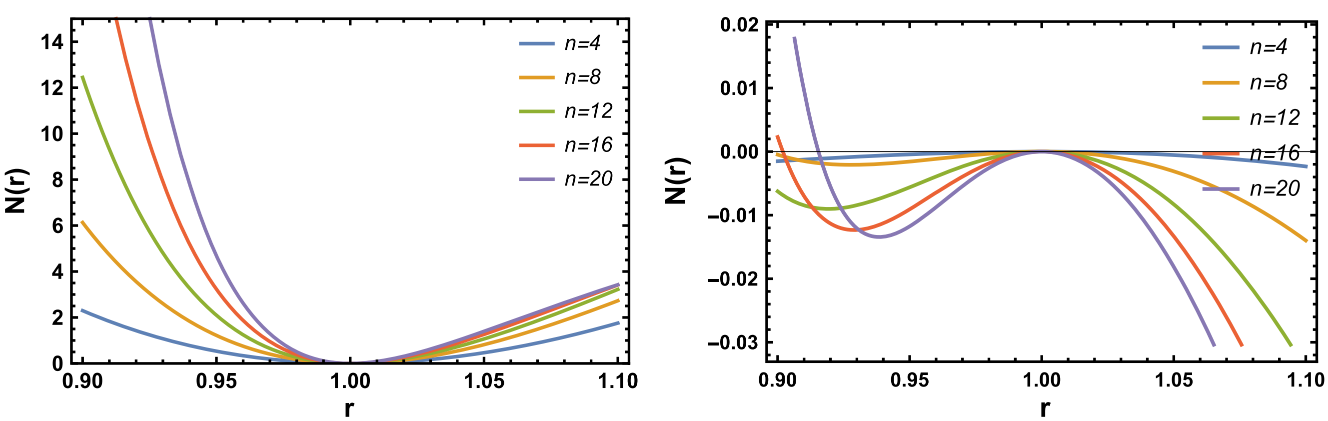

FIG. 1. Left and right plots are for $N(r)$ with respect to r for positive and negative $N''(r_0)$ respectively with a degenerate horizon at $r_0 = 1$.

and negative $N''(r_0)$ will be considered while discussing the solution of perturbations in Sec. IV.

IV. ANTI-EVAPORATION AND EVAPORATION

In Fig. 1, the plots for $N(r)$ are given for different values of $N''(r_0)$ and dimension n with a degenerate horizon at $r_0 = 1$. The analytical definition of $N(r)$ near the degenerate horizon is given by Eq. (32). Near the degenerate horizon, the spacetime is written as in Eq. (33) in n dimensions in different coordinate system. In this section, we study the anti-evaporation of an n-dimensional black hole near the degenerate horizon. Here we implement perturbation analysis up to the linear order. It will be shown that $f(R)$ theory is relevant for the dynamic nature of the perturbation $\delta\phi$. Setting $f''(R_0) = 0$, one obtains $\delta\phi = 0$, which indicates the constant horizon. Working with $f(R)$, the perturbation $\delta\phi$ becomes proportional to δR and it gives a possibility of dynamic $\delta\phi$. In simplifying constants, we use the background equations. Maxwell field tensor is defined by $\nabla_\mu(F^{p-1}F^{\mu\nu}) = 0$ and is modified in n dimensions.

A. Perturbation

To know the behavior of the horizon, we introduce the fields $\rho(\tau, x)$ and $\phi(\tau, x)$ in the spacetime around the extreme black hole solution as follows,

$$ds^2 = (\sin^2 x d\tau^2 - dx^2)\frac{e^{2\rho(\tau,x)}}{\Lambda^2} + \frac{e^{-2\phi(\tau,x)}}{\Lambda'^2} d\Omega_{n-2}^2, \quad (34)$$

where $\frac{1}{\Lambda^2} = \frac{2}{N''(r_0)}$ and $\frac{1}{\Lambda'^2} = r_0^2$. With this metric, the term $\nabla^\rho\nabla_\rho f'(R)$ in Eq. (14) turns out to be

$$\nabla^\rho\nabla_\rho f'(R) = e^{-2\rho}\Lambda^2\left[\frac{1}{\sin^2 x}\frac{\partial^2 f'(R)}{\partial\tau^2} - \cot x\frac{\partial f'(R)}{\partial x} - \frac{\partial^2 f'(R)}{\partial x^2} + (n-2)\left(-\frac{\dot\phi}{\sin^2 x}\frac{\partial f'(R)}{\partial\tau} + \phi'\frac{\partial f'(R)}{\partial x}\right)\right] \quad (35)$$

and the components of the Ricci tensor take the forms as

$$R_{\tau\tau} = -\ddot\rho + \rho''\sin^2 x + \sin x\cos x\rho' - \sin^2 x + (n-2)(\ddot\phi - \dot\phi^2) - (n-2)\dot\phi\dot\rho \\ - (n-2)\sin x\cos x\phi' - (n-2)\sin^2 x\rho'\phi', \quad (36)$$

$$R_{xx} = \frac{\ddot\rho}{\sin^2 x} - \rho'' + 1 + (n-2)(\phi'' - \phi'^2) - (n-2)\left(\frac{\dot\rho\dot\phi}{\sin^2 x} + \phi'\rho'\right) - \rho'\cot x, \quad (37)$$

$$R_{\tau x} = (n-2)[\dot\phi' - \dot\phi\rho' - \phi'\dot\rho - \dot\phi\phi' - \dot\phi\cot x], \quad (38)$$

$$R_{\theta_1\theta_1} = \frac{\Lambda^2}{\Lambda'^2}e^{-2(\rho+\phi)}\left[\frac{1}{\sin^2 x}(\ddot\phi + (2-n)\dot\phi^2) - \phi'' - \phi'\cot x + (n-2)\phi'^2\right] + (n-3), \quad (39)$$

$$R_{\theta_2\theta_2} = \sin^2\theta_1 R_{\theta_1\theta_1} \quad (40)$$

$$R_{\theta_3\theta_3} = \sin^2\theta_1\sin^2\theta_2 R_{\theta_1\theta_1} \text{….so on.} \quad (41)$$

Ricci scalar in n-dimensional spacetime may be written as

$$R = \frac{\Lambda^2 e^{-2\rho}}{\sin^2 x}[-2\ddot\rho + 2\rho''\sin^2 x + 2(n-2)\ddot\phi - 2(n-2)\sin^2 x\phi'' + 2\rho'\sin x\cos x - 2(n-2)\phi'\sin x\cos x \\ - (n-1)(n-2)\dot\phi^2 + (n-1)(n-2)\sin^2 x\phi'^2 - 2\sin^2 x] + (n-2)(n-3)\Lambda'^2 e^{2\phi} \quad (42)$$

Here, primes and dots over ϕ or ρ are derivatives with respect to "x" and "τ", respectively. In this spacetime, from Eq. (15), the electric field is given by

$$F_{x\tau} = Cq^{\frac{1}{2p-1}}e^{\frac{(n-2)\phi}{2p-1}}e^{2\rho}\sin x, \quad (43)$$

where C is a constant defined by $C \equiv \Lambda^{-2}[(-1)^p 2^{1-p}\Lambda'^{n-2}]^{\frac{1}{2p-1}}$, and q is the charge of black hole. The components of energy-momentum tensor can be written as

$$T_{\tau\tau} = 2\left(p - \frac{1}{2}\right)(-2q^2\Lambda'^{2(n-2)}e^{2(n-2)\phi})^{\frac{p}{2p-1}}\frac{\sin^2 x e^{2\rho}}{\Lambda^2}, \tag{44}$$

$$T_{xx} = 2\left(p - \frac{1}{2}\right)(-2q^2\Lambda'^{2(n-2)}e^{2(n-2)\phi})^{\frac{p}{2p-1}}\left(-\frac{e^{2\rho}}{\Lambda^2}\right), \tag{45}$$

$$T_{x\tau} = 0, \tag{46}$$

$$T_{\theta_1\theta_1} = -(-2q^2\Lambda'^{2(n-2)}e^{2(n-2)\phi})^{\frac{p}{2p-1}}\left(\frac{e^{-2\phi}}{\Lambda'^2}\right), \tag{47}$$

$$T_{\theta_2\theta_2} = \sin^2\theta_1 T_{\theta_1\theta_1} \tag{48}$$

$$T_{\theta_3\theta_3} = \sin^2\theta_1 \sin^2\theta_2 T_{\theta_1\theta_1} \ldots\text{so on.} \tag{49}$$

This leads the components of Eq. (14) to

$$\begin{aligned}
&\frac{\Lambda^2}{\sin^2 x}e^{-2\rho}f'(R)[-\ddot{\rho} + \rho''\sin^2 x + \sin x\cos x\rho' - \sin^2 x + (n-2)(\ddot{\phi} - \dot{\phi}^2) - (n-2)\dot{\phi}\dot{\rho} - (n-2)\sin x\cos x\phi' \\
&\quad - (n-2)\sin^2 x\phi'\rho'] - \frac{1}{2}f(R) + e^{-2\rho}\Lambda^2\left[-\frac{\partial^2 f'(R)}{\partial x^2} + (n-2)\left(-\frac{\dot{\phi}}{\sin^2 x}\frac{\partial f'(R)}{\partial\tau} + \phi'\frac{\partial f'(R)}{\partial x}\right)\right] \\
&\quad + \frac{\Lambda^2}{\sin^2 x}e^{-2\rho}\left[\dot{\rho}\frac{\partial f'(R)}{\partial\tau} + \sin^2 x\rho'\frac{\partial f'(R)}{\partial x}\right] = \left(p - \frac{1}{2}\right)(-2q^2\Lambda'^{2(n-2)}e^{2(n-2)\phi})^{\frac{p}{2p-1}},
\end{aligned} \tag{50}$$

$$\begin{aligned}
&-\frac{\Lambda^2}{\sin^2 x}e^{-2\rho}f'(R)[\ddot{\rho} - \rho''\sin^2 x + \sin^2 x + (n-2)\sin^2 x(\phi'' - \phi'^2) - (n-2)(\dot{\phi}\dot{\rho} + \phi'\rho'\sin^2 x) - \rho'\sin x\cos x] \\
&\quad - \frac{1}{2}f(R) + e^{-2\rho}\Lambda^2\left[\frac{1}{\sin^2 x}\frac{\partial^2 f'(R)}{\partial\tau^2} - \cot x\frac{\partial f'(R)}{\partial x} + (n-2)\left(-\frac{\dot{\phi}}{\sin^2 x}\frac{\partial f'(R)}{\partial\tau} + \phi'\frac{\partial f'(R)}{\partial x}\right)\right] \\
&\quad - \frac{\Lambda^2}{\sin^2 x}e^{-2\rho}\left[\dot{\rho}\frac{\partial f'(R)}{\partial\tau} + \sin^2 x\rho'\frac{\partial f'(R)}{\partial x}\right] = \left(p - \frac{1}{2}\right)(-2q^2\Lambda'^{2(n-2)}e^{2(n-2)\phi})^{\frac{p}{2p-1}},
\end{aligned} \tag{51}$$

$$(n-2)f'(R)[\dot{\phi}' - \dot{\phi}\rho' - \phi'\dot{\rho} - \dot{\phi}\phi' - \dot{\phi}\cot x] - \left[\frac{\partial^2 f'(R)}{\partial\tau\partial x} - (\rho' + \cot x)\frac{\partial f'(R)}{\partial\tau} - \dot{\rho}\frac{\partial f'(R)}{\partial x}\right] = 0, \tag{52}$$

$$\begin{aligned}
&\frac{\Lambda^2}{\sin^2 x}e^{-2\rho}f'(R)[\ddot{\phi} + (2-n)\dot{\phi}^2 - \sin^2 x\phi'' - \phi'\sin x\cos x + (n-2)\sin^2 x\phi'^2] + (n-3)e^{2\phi}\Lambda'^2 f'(R) - \frac{1}{2}f(R) \\
&\quad + \Lambda^2 e^{-2\rho}\left[\frac{1}{\sin^2 x}\frac{\partial^2 f'(R)}{\partial\tau^2} - \cot x\frac{\partial f'(R)}{\partial x} - \frac{\partial^2 f'(R)}{\partial x^2} + (n-2)\left(-\frac{\dot{\phi}}{\sin^2 x}\frac{\partial f'(R)}{\partial\tau} + \phi'\frac{\partial f'(R)}{\partial x}\right)\right] \\
&\quad - e^{-2\rho}\Lambda^2\left(-\frac{\dot{\phi}}{\sin^2 x}\frac{\partial f'(R)}{\partial\tau} + \phi'\frac{\partial f'(R)}{\partial x}\right) = -\frac{1}{2}(-2q^2\Lambda'^{2(n-2)}e^{2(n-2)\phi})^{\frac{p}{2p-1}}.
\end{aligned} \tag{53}$$

We now perturb the fields $\phi = \delta\phi(\tau, x)$ and $\rho = \delta\rho(\tau, x)$ around Nariai-like spacetime. We also perturb Ricci scalar R around its constant background R_0. From Eq. (52), we obtain

$$\delta R = \frac{(n-2)f'(R_0)}{f''(R_0)}\delta\phi. \tag{54}$$

Here, we note that the perturbation $\delta\phi$ vanishes if $f''(R_0) = 0$, indicating no possibility of evaporation or anti-evaporation even in n dimensions. Equation (53) provides a differential equation of the perturbation $\delta\phi(\tau, x)$,

$$(n-1)\Lambda^2\left[\frac{\ddot{\delta\phi}}{\sin^2 x}-\delta\phi''-\cot x\delta\phi'\right]+\left[n(n-3)\Lambda'^2-\frac{(n-2)f'(R_0)}{2f''(R_0)}+\frac{p(n-2)}{f'(R_0)(2p-1)}q'\right]\delta\phi=0, \tag{55}$$

and Eq. (50) or (51) can be written as

$$-\frac{\ddot{\delta\rho}}{\sin^2 x}+\delta\rho''+\cot x\delta\rho'+2\delta\rho+(n-2)\left[\frac{\ddot{\delta\phi}}{\sin^2 x}-\delta\phi''-\cot x\delta\phi'\right]-(n-2)\left[1+\frac{f'(R_0)}{2f''(R_0)\Lambda^2}+\frac{pq'}{f'(R_0)\Lambda^2}\right]\delta\phi=0, \tag{56}$$

where $q'=[-2q^2\Lambda'^{2(n-2)})]^{\frac{p}{2p-1}}$.

Under the coordinate transformation $dx=\sin x du$, Eq. (55) becomes

$$\ddot{\delta\phi}-\delta\phi_{,uu}+\alpha\cosh^{-2}(u+c)\delta\phi=0, \tag{57}$$

where "$_{,u}$" denotes $\partial/\partial u$, c is an integral constant and α is a constant given by

$$\alpha=\frac{1}{(n-1)\Lambda^2}\left[n(n-3)\Lambda'^2-\frac{(n-2)f'(R_0)}{2f''(R_0)}+\frac{p(n-2)}{f'(R_0)(2p-1)}q'\right]. \tag{58}$$

A solution for Eq. (57) is $\delta\phi=(Ae^{\omega\tau}+Be^{-\omega\tau})\times\cosh^{\beta}(u+c)$, with

$$\omega^2-\beta^2=0, \qquad 0=\alpha+\beta(\beta-1), \tag{59}$$

which gives

$$\omega=\pm\beta, \qquad \beta=\beta_{\pm}\equiv\frac{1}{2}(1\pm\sqrt{1-4\alpha}). \tag{60}$$

Using the horizon condition from Eq. (11), we have

$$\tanh(u+c)=\frac{Ae^{\omega\tau}-Be^{-\omega\tau}}{Ae^{\omega\tau}+Be^{-\omega\tau}}. \tag{61}$$

Then we find that

$$\delta\phi=\frac{(Ae^{\beta\tau}+Be^{-\beta\tau})^{\beta+1}}{(2\sqrt{AB})^{\beta}}. \tag{62}$$

To know the behavior of $\delta\phi$, we consider a case where $A=B$, which leads to the expression of $\delta\phi$ as

$$\delta\phi=2A[\cosh(\beta\tau)]^{\beta+1}, \tag{63}$$

and according to the description given in Sec. II, which is also valid for n-dimensional spacetime, the radius of the horizon is given by

$$r_h=\frac{e^{-2A[\cosh(\beta\tau)]^{\beta+1}}}{\Lambda'}. \tag{64}$$

The negative value of $\beta+1$ can be obtained with negative root of β. It is noted that if $\beta+1$ is negative which we can see in case (a) discussed below (where Λ^2 is negative) by setting, e.g., $n=8$ and $n'=-1$, the perturbation $\delta\phi$ decreases and thus the horizon size increases for positive values of A, which is the case in anti-evaporation. After a certain time, the horizon size becomes a constant, $r_h=\frac{1}{\Lambda'}$. For positive $\beta+1$, e.g., $n=4$ and $n'=-1$ with positive root in case (a), evaporation occurs. The positive value of $\beta+1$ can also explain anti-evaporation with negative A. However, in this case of positive $\beta+1$, instability occurs. We can also have solution for $\delta\rho$ in terms of $\delta\phi$, given by $\delta\rho=\gamma\delta\phi$ satisfying Eqs. (55) and (56), where $\gamma=\frac{(n-2)(C_3+\alpha)}{(2+\alpha)}$ and C_3 is given by

$$C_3=1+\frac{f'(R_0)}{2f''(R_0)\Lambda^2}+\frac{pq'}{f'(R_0)\Lambda^2}. \tag{65}$$

The perturbation $\delta\rho$ evolves in the same way as $\delta\phi$. One can eliminate q' from the background equation. From the background Einstein equation, one can find

$$pq'=-f'(R_0)(\Lambda^2+(n-3)\Lambda'^2), \tag{66}$$

which results in

$$\alpha=-\frac{2}{n-1}-\frac{(n-2)f'(R_0)}{2(n-1)f''(R_0)\Lambda^2}+\frac{(n-2)(n-3)\Lambda'^2}{(n-1)\Lambda^2}, \tag{67}$$

where we used $n=4p$. Here the constants can be computed at an extreme horizon,

$$\Lambda^2=-\frac{(n-3)}{r_0^2}+\frac{C_1 n}{2r_0^n}, \tag{68}$$

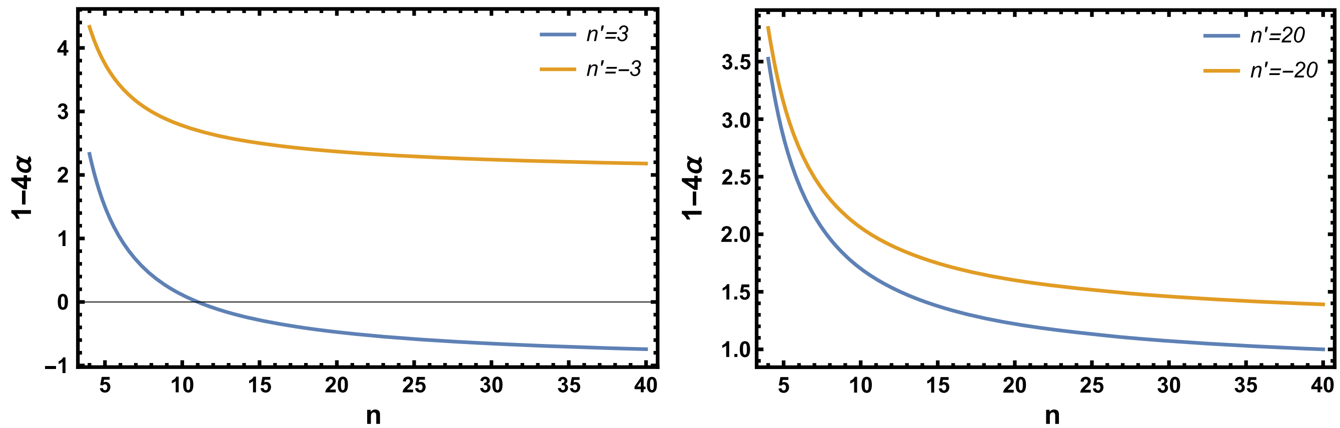

FIG. 2. The plot of $1 - 4\alpha$ with dimension n of spacetime for $r_0 = 0.1$ and $C_1 = 10$.

$$\Lambda'^2 = \frac{1}{r_0^2}. \tag{69}$$

Now we consider two cases (a) and (b). In the first case, we assume the condition $\frac{(n-3)}{r_0^2} \gg \frac{C_1 n}{2 r_0^n}$, and in the second case we consider the opposite way.

1. Case (a)

In this case, we have $\Lambda^2 \approx -\frac{(n-3)}{r_0^2}$. If one considers a theory $f(R) \sim R^{n'}$, then n' and the dimension of spacetime n can determine the value of constant α as follows,

$$\alpha = -\frac{n}{(n-1)} + \frac{n(n-2)}{2(n-1)(n'-1)}, \tag{70}$$

and the term $1 - 4\alpha$ in Eq. (60) can be positive if

$$n' \geq \frac{(2n-1)(n+1)}{(5n-1)} \quad \text{and} \quad n' \leq 1. \tag{71}$$

For $n = 4$, we get $n' \geq 1.84$ for real value of β. We can get positive and negative values of $1 + \beta$ with different choices of n and n'. For example, as already mentioned, $n' = -1$ can explain both anti-evaporation and evaporation for negative and positive values of $\beta + 1$ with positive A. For imaginary values of β, we have an oscillatory solution which we will discuss bellow.

2. Case (b)

In this case where $\frac{C_1}{2 r_0^n} \gg \frac{(n-3)}{r_0^2}$, under similar theories, we find

$$\alpha = -\frac{2}{(n-1)} + \frac{(n-2)}{(n-1)} \left[\frac{1}{(n'-1)} + \frac{2(n-3) r_0^{(n-2)}}{n C_1} \right]. \tag{72}$$

For $n = 4$ and $n' = 2$, we obtain very small positive value of α resulting only decreasing horizon for positive and negative roots with positive A. We plot $1 - 4\alpha$ with respect to dimension n in the left panel in Fig. 2 for $n' = \pm 3$. It is observed on the left panel of Fig. 2 that initially, for lower n, the first negative term is dominant and becomes smaller after a certain value of n, making the whole term $1 - 4\alpha$ nearly constant for both the positive and negative value of n'. However, for the negative value of n', the term $1 - 4\alpha$ remains positive for all large values of n. For large value n', both curves approach to each other for both positive and negative values and converge to nearly unity since α becomes very small as shown in the right panel of Fig. 2.

For negative value of $1 - 4\alpha$, i.e., imaginary value of β, we observe the oscillatory solution. Let us consider $\sqrt{1 - 4\alpha} = iy$, where y takes positive and negative values. We can write the real solution of $\delta\phi$ as

$$\delta\phi = 2A e^{\frac{3\gamma}{2} - \frac{y\theta}{2}} \cos\left(\frac{y\gamma}{2} + \frac{3\theta}{2}\right), \tag{73}$$

where

$$\gamma = \ln \sqrt{\cos^2 \frac{y\tau}{2} \cosh^2 \frac{\tau}{2} + \sin^2 \frac{y\tau}{2} \sinh^2 \frac{\tau}{2}}, \tag{74}$$

and θ is given by

$$\theta = \tan^{-1}\left[\tan \frac{y\tau}{2} \tanh \frac{\tau}{2}\right] + \theta_1, \tag{75}$$

where θ_1 is the phase term given by

$$\theta_1 = \pi, \quad \text{for } \cos\frac{y\tau}{2}\cosh\frac{\tau}{2} < 0 \quad \text{and} \quad \sin\frac{y\tau}{2}\sinh\frac{\tau}{2} \geq 0,$$
$$\theta_1 = -\pi, \quad \text{for } \cos\frac{y\tau}{2}\cosh\frac{\tau}{2} < 0 \quad \text{and} \quad \sin\frac{y\tau}{2}\sinh\frac{\tau}{2} < 0,$$
$$\theta_1 = 0 \quad \text{for } \cos\frac{y\tau}{2}\cosh\frac{\tau}{2} > 0. \tag{76}$$

However, we can remove the phase term θ_1 as it is a constant term in the solution of $\delta\phi$. The solution given in Eq. (73) is oscillatory. The amplitude of the oscillation increases exponentially in this case, thus exhibiting instability. We can see that the given solution is independent of the sign of y and τ. The same solution can be obtained with the negative root, i.e., with $-y$. We note that the instability can not be controlled by parameter β (or y).

We have discussed above the nonoscillatory and oscillatory solutions. Both solutions can explain anti-evaporation and evaporation. However, nonoscillatory solutions are stable and unstable; on the other hand, oscillatory solutions are only unstable. In the nonoscillatory case, the phenomena of anti-evaporation and evaporation can survive for a long time with some specific values of parameters, e.g., for very small $\beta + 1$.

V. CONCLUSION

General relativity predicts a constant horizon around the Nariai-like spacetime for the Reissner-Nordström black hole during its evolution at the classical level. In contrast, $f(R)$ gravity offers a possibility of dynamic behavior of the degenerate horizon in this black hole which could even be possible in the Schwarzschild black hole [23]. In this work, we generalize the theory in n dimensions to broaden the implications of $f(R)$ gravity. First, it was found that general relativity with n dimensions still does not explain anti-evaporation and evaporation. This can be realized if one sets $f''(R_0) = 0$ in the perturbation equations. Therefore, despite n dimensions being richer, it does not help in anti-evaporation and evaporation unless we replace general relativity with $f(R)$ gravity. Considering $f(R)$ gravity, we obtain the dynamic equation for the degenerate horizon, and we categorize solutions in three types—one in which we obtain an increasing solution for the horizon with positive constant A and negative $\beta + 1$, which indicates anti-evaporation and is stable. After a certain time, the horizon size becomes a constant. Anti-evaporation also occurs with the negative value of A with positive $\beta + 1$; however, instability is associated with anti-evaporation in this case. The second type, where evaporation occurs, is a decreasing solution for the horizon. The stable evaporation can be explained with negative A and $\beta + 1$. As with anti-evaporation, evaporation can also occur for positive A and positive $\beta + 1$ with instability. The last type is an oscillating solution with increasing amplitude. Both the degenerate horizon and Ricci scalar oscillate in the same way, since the perturbation of the degenerate horizon is proportional the perturbation field of the Ricci scalar. In this paper, it is noted that a black hole can have anti-evaporation at the classical level, and this effect can remain for a long time. This offers the possibility of a long-lived primordial black hole even with smaller mass.

[1] P. Ivanov, Nonlinear metric perturbations and production of primordial black holes, Phys. Rev. D **57**, 7145 (1998).

[2] J. S. Bullock and J. R. Primack, Comments on nonGaussian density perturbations and the production of primordial black holes, arXiv:astro-ph/9806301.

[3] J. S. Bullock and J. R. Primack, NonGaussian fluctuations and primordial black holes from inflation, Phys. Rev. D **55**, 7423 (1997).

[4] S. Hawking, Gravitationally collapsed objects of very low mass, Mon. Not. R. Astron. Soc. **152**, 75 (1971).

[5] B. J. Carr and S. W. Hawking, Black holes in the early Universe, Mon. Not. R. Astron. Soc. **168**, 399 (1974).

[6] O. K. Kalashnikov and M. Y. Khlopov, On the possibility of checking the cosmology of asymptotically free SU(5) theory, Phys. Lett. **127B**, 407 (1983).

[7] B. Carr, F. Kuhnel, and M. Sandstad, Primordial black holes as dark matter, Phys. Rev. D **94**, 083504 (2016).

[8] P. Ivanov, P. Naselsky, and I. Novikov, Inflation and primordial black holes as dark matter, Phys. Rev. D **50**, 7173 (1994).

[9] D. Blais, C. Kiefer, and D. Polarski, Can primordial black holes be a significant part of dark matter?, Phys. Lett. B **535**, 11 (2002).

[10] L. K. Chavda and A. L. Chavda, Dark matter and stable bound states of primordial black holes, Classical Quantum Gravity **19**, 2927 (2002).

[11] S. W. Hawking, Particle creation by black holes, Commun. Math. Phys. **43**, 199 (1975); Erratum, Commun. Math. Phys. **46**, 206(E) (1976).

[12] R. Bousso and S. W. Hawking, (Anti)evaporation of Schwarzschild-de Sitter black holes, Phys. Rev. D **57**, 2436 (1998).

[13] H. Nariai, On some static solutions of Einstein's gravitational field equations in a spherically symmetric case, Sci. Rep. Tohoku Univ. **34**, 160 (1950).

[14] H. Nariai, On a new cosmological solution of Einstein's field equations of gravitation, Sci. Rep. Tohoku Univ. **35**, 62 (1951).

[15] S. Nojiri and S. D. Odintsov, Quantum evolution of Schwarzschild-de Sitter (Nariai) black holes, Phys. Rev. D **59**, 044026 (1999).

[16] M. Buric and V. Radovanovic, Quantum corrections for anti-evaporating black hole, Phys. Rev. D **63**, 044020 (2001).

[17] E. Elizalde, S. Nojiri, and S. D. Odintsov, Possible quantum instability of primordial black holes, Phys. Rev. D **59**, 061501 (1999).

[18] A. A. Bytsenko, S. Nojiri, and S. D. Odintsov, Quantum generation of Schwarzschild-de Sitter (Nariai) black holes in effective Dilaton—Maxwell gravity, Phys. Lett. B **443**, 121 (1998).

[19] J. C. Niemeyer and R. Bousso, The Nonlinear evolution of de Sitter space instabilities, Phys. Rev. D **62**, 023503 (2000).

[20] S. Nojiri and S. D. Odintsov, Effective action for conformal scalars and anti-evaporation of black holes, Int. J. Mod. Phys. A **14**, 1293 (1999).
[21] S. Nojiri and S. D. Odintsov, Quantum dilatonic gravity in ($D = 2$)-dimensions, ($D = 4$)-dimensions and ($D = 5$)-dimensions, Int. J. Mod. Phys. A **16**, 1015 (2001).
[22] S. Nojiri and S. D. Odintsov, Anti evaporation of Schwarzschild-de Sitter black holes in $f(R)$ gravity, Classical Quantum Gravity **30**, 125003 (2013).
[23] S. Nojiri and S. D. Odintsov, Instabilities and anti-evaporation of Reissner-Nordström black holes in modified $F(R)$ gravity, Phys. Lett. B **735**, 376 (2014).
[24] A. Addazi, (Anti)evaporation of Dyonic Black Holes in string-inspired dilaton $f(R)$-gravity, Int. J. Mod. Phys. A **32**, 1750102 (2017).
[25] V. K. Oikonomou, Reissner-Nordström anti-de Sitter black holes in mimetic $F(R)$ gravity, Universe **2**, 10 (2016).
[26] T. Katsuragawa and S. Nojiri, Stability and antievaporation of the Schwarzschildde Sitter black holes in bigravity, Phys. Rev. D **91**, 084001 (2015).
[27] D. V. Singh and N. K. Singh, Anti-evaporation of Bardeen de-Sitter black holes, Ann. Phys. (Amsterdam) **383**, 600 (2017).
[28] A. Addazi, S. Nojiri, and S. Odintsov, Evaporation and anti-evaporation instability of a Schwarzschild-de Sitter braneworld: The case of five-dimensional F(R) gravity, Phys. Rev. D **95**, 124020 (2017).
[29] T. Kaluza, Zum Unitätsproblem der physik, Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.) **1921**, 966 (1921) [On the unification problem in physics, Int. J. Mod. Phys. D **27**, 1870001 (2018)].
[30] O. Klein, The atomicity of electricity as a quantum theory law, Nature (London) **118**, 516 (1926).
[31] T. Appelquist, A. Chodos, and P. G. O. Freund, *Modern Kaluza-Klein Theories*, Frontiers in Physics (Addison-Wesley, Reading, MA, 1987), p. 619.
[32] M. J. Duff, B. E. W. Nilsson, and C. N. Pope, Kaluza-Klein supergravity, Phys. Rep. **130**, 1 (1986).
[33] M. B. Green, J. H. Schwarz, and E. Witten, *Superstring Theory: 25th Anniversary Edition* (Cambridge University Press, Cambridge, England, 1987), Vol. 1.
[34] M. B. Green, J. H. Schwarz, and E. Witten, *Superstring Theory: 25th Anniversary Edition* (Cambridge University Press, Cambridge, England, 1987), Vol. 2.
[35] R. Emparan and H. S. Reall, Black holes in higher dimensions, Living Rev. Relativity **11**, 6 (2008).
[36] H. S. Reall, Higher dimensional black holes, Int. J. Mod. Phys. D **21**, 1230001 (2012).
[37] P. Kanti, Black holes at the LHC, Lect. Notes Phys. **769**, 387 (2009).
[38] A. Strominger and C. Vafa, Microscopic origin of the Bekenstein-Hawking entropy, Phys. Lett. B **379**, 99 (1996).
[39] R. C. Myers and M. J. Perry, Black holes in higher dimensional space-times, Ann. Phys. (N.Y.) **172**, 304 (1986).
[40] X. Dianyan, Exact solutions of Einstein and Einstein-maxwell equations in higher dimensional space-time, Classical Quantum Gravity **5**, 871 (1988).
[41] J. T. Liu and W. A. Sabra, Charged configurations in (A)dS spaces, Nucl. Phys. **B679**, 329 (2004).
[42] S. G. Ghosh, Nonstatic charged BTZ-like black holes in $N + 1$ dimensions, Int. J. Mod. Phys. D **21**, 1250022 (2012).
[43] S. H. Hendi, Charged BTZ-like black holes in higher dimensions, Eur. Phys. J. C **71**, 1551 (2011).
[44] S. G. Ghosh and A. K. Dawood, Radiating black hole solutions in arbitrary dimensions, Gen. Relativ. Gravit. **40**, 9 (2008).
[45] A. Sheykhi, Higher-dimensional charged $f(R)$ black holes, Phys. Rev. D **86**, 024013 (2012).
[46] J. Matyjasek, P. Sadurski, and D. Tryniecki, Inside the degenerate horizons of regular black holes, Phys. Rev. D **87**, 124025 (2013).
[47] S. Fernando, Bardeen-de Sitter black holes, Int. J. Mod. Phys. D **26**, 1750071 (2017).