Bifurcation of fixed points in a O(N)-symmetric (2 + 1)-dimensional gauged Φ^6 theory with a Chern-Simons term

S. Sakhi[1,2] and P. K. Panigrahi[3]

[1]*Department of Physics, College of Arts and Sciences, American University of Sharjah, P.O. Box 26666 Sharjah, United Arab Emirates*
[2]*Materials Science and Engineering Research Institute, American University of Sharjah, P. O. Box 26666, United Arab Emirates*
[3]*Department of Physical Sciences, Indian Institute of Science Education and Research Kolkata, Mohanpur 741246, West Bengal, India*

(Received 18 September 2018; published 22 February 2019)

We examine the phase structure of an Abelian Chern-Simons system with relativistic charged matter fields with a sixth-order potential. Using a large N technique, we compute the quantum effective potential and the renormalization group function of the coupling to the next-to-leading order of the $1/N$ expansion in terms of the Chern-Simons coefficient. The model has a phase which exhibits spontaneous breaking of scale symmetry accompanied by a massless dilaton which is a Goldstone mode. We show that the beta function of the sextic coupling exhibits, at the next order in the $1/N$ expansion, nontrivial running that we analyze explicitly in terms of the Chern-Simons coefficient. Viewed as a dynamical system, the renormalization group (RG) flow exhibits a topological normal form of a generic one-dimensional system having a fold bifurcation. We demonstrate that the corresponding IR and UV fixed points, each describing a conformal phase of the theory, approach each other until they merge, giving rise to a scaling behavior similar to Berezinskii-Kosterlitz-Thouless phase transitions. Our study identifies a window in the parameter space of the Chern-Simons coefficient where the renormalization group flow has a stable infrared fixed point and where scale invariance is recovered. We also find that the Chern-Simons interaction modifies the scaling dimension of the operator crossing marginality at the bifurcation points.

DOI: 10.1103/PhysRevD.99.036017

I. INTRODUCTION

The O(N)-symmetric model of N component scalar fields with sixth-order coupling, $\eta|\Phi|^6$, presents a unique laboratory to study several interesting aspects of quantum field theory, as well as critical and tricritical behavior in condensed matter systems observed, e.g., in liquid helium and in metamagnets [1]. In three dimensions the theory is renormalizable and allows a $1/N$ expansion with an interesting beta function for large N [2], $\beta(\eta) = (3/4\pi^2 N)\eta^2(1 - \eta/192)$. It has an ultraviolet stable fixed point at $\eta = 192$, and an infrared stable fixed point at $\eta = 0$. For small positive values of η, the quadratic term in this beta function shows that the coupling is marginally irrelevant. As η increases, the cubic term becomes important and a perturbative UV fixed point is reached at $\eta^* = 192$. More interestingly, though, a self-consistent nonperturbative UV fixed point was found in [3], in the strict $N = \infty$ limit, at a smaller value $\eta = 16\pi^2 < \eta^*$, whereupon a mass is dynamically generated, resulting in the spontaneous breaking of scale symmetry in a nonperturbative way. This is seen from the effective potential at the tricritical point, $V = (N/3)(\eta^* - \eta)\varphi^3$, which shows that the system has various phases. For $\eta < \eta^*$ there is no spontaneous symmetry breaking ($\varphi = 0$), and the system consists of N massless noninteracting Φ particles. However, for the special value $\eta = \eta^*$, a flat direction in φ opens up and any constant φ is a solution. For a zero expectation value $\langle\varphi\rangle = 0$, the theory continues to consist of N massless Φ fields. When $\langle\varphi\rangle \neq 0$, the theory has N massive Φ particles, which all have the same mass due to the unbroken O(N) symmetry, but in a phase that breaks the scale invariance. The Goldstone boson associated with the spontaneous breaking of scale invariance, the dilaton, is massless and identified as the O(N) singlet field $\varphi - \langle\varphi\rangle$. All the particles are noninteracting in the infinite N limit. For larger values of η the effective potential is unbounded from below, and the system becomes unstable. The analysis of [3] suggests that the instability reflects the inability to define a renormalizable interacting theory. All masses are of the order of the cutoff and there is no mechanism to scale them down to low mass values, i.e., the theory depends strongly on its UV completion. Other aspects of that phenomenon including $1/N$ corrections were analyzed in [4,5]. Models with non-Abelian Chern-Simons gauge fields [6] and the fate of light dilaton under $1/N$ corrections [7] were also investigated.

In this paper we investigate these phenomena in the $U(1)$ gauged $(2+1)$-dimensional theory with a Chem-Simons term. The possibility of describing gauge theories with a Chern-Simons term rather than with a Maxwell term is a special feature of odd-dimensional space-time, and the $2+1$-dimensional case is especially distinguished since the derivative part of the Chern-Simons Lagrangian is quadratic in the gauge fields. This is important from condensed matter point of view since at long distance, the Chern-Simons term having only one derivative is expected to dominate over the Maxwell term which has two derivatives. Another important feature of Chern-Simons theories is that the Chern-Simons term can be induced by radiative quantum effects, even if it is not present as a bare term in the original Lagrangian. The simplest manifestation of this phenomenon occurs in $2+1$-dimensional QED, where a Chern-Simons term is induced in a simple one-loop computation of the fermion effective action [8]. A Chern-Simons term can also be induced in gauge theories without fermions, and in the broken phases of Chern-Simons-Higgs theories [9]. On the other hand, it should be noted that the Chern-Simons gauge field does not have any real dynamics of its own—it is a nonpropagating field which inherits its dynamics from the matter fields to which it is minimally coupled, and so it can be coupled to either relativistic or nonrelativistic matter fields. The implementation of this canonical structure in a quantum theory leads to many interesting features in Chern-Simons theories. For instance, a pure Chern-Simons theory with a symmetry breaking relativistic scalar field potential supports vortex solutions analogue of the Bogomol'nyi self-dual structure of the Abelian-Higgs model [10], provided a suitable sixth-order scalar potential is chosen [11]. Such calculations carried out within the framework of the photonless gauge theory [12] support soliton solutions that minimize the energy within certain constraints of topological nature, characterized by first-order self-dual equations if the Higgs potential takes a special form involving $|\Phi|^6$ potential instead of the standard fourth-order self-interaction scalar potential. Moreover, in Chern-Simons theories the phenomenon of quantum breaking of classical scale invariance takes on more interesting aspects. This occurrence was recognized in Chern-Simons theories with nonrelativistic matter fields with quartic self-interactions in which case one-loop correction to the quantum effective potential break scale invariance, unless the quartic coupling g is chosen to take its self-dual value $g = 1/\kappa$ (which means self-duality condition in the quantum theory is tied to the preservation of scale invariance) [13]. In recent years, Chern-Simons theories have found diverse applications in condensed matter physics. For example, the possibility of anyonic statistics [14] and anyonic theories of superconductivity can be elegantly formulated by using the Chern-Simons term [15]. Chern-Simons theories were also featured in describing the phase transitions between quantum Hall liquids and insulators [16], in the quantum phase transitions of quantum antiferromagnets in two spatial dimensions [17], and in the insulator-superconductor quantum transition in Josephson junction array systems [18].

In this work we uncover another aspect of the tricritical behavior of the gauged $(2+1)$-dimensional $\eta|\Phi|^6$ theory, which will be analyzed with a controlled large N technique combined with a renormalization group procedure. We compute the quantum effective potential from which we obtain information about the phase diagram and the beta functions of the coupling at the next order in the $1/N$ expansion. We invoke bifurcation method of dynamical systems to analyze the renormalization group flow, as was proposed in other models [19]. This paper is organized as follows. In Sec. II, the gauged model is introduced. In Sec. III, the effective potential is derived to leading order of the large N technique. In Sec. IV, the corrections to the effective potential arising from the scalar fields and the fluctuating gauge fields are computed and used to derive the renormalization group flow beta function. In Sec. V, the fixed points of the model are examined analytically and numerically in terms of the Chern-Simons coefficient, and we determine the domains in the parameter space where fixed points exist. Finally Sec. VI summarizes the results.

II. THE MODEL

We consider the model defined by the three-dimensional Euclidean action:

$$S = \int d^3x \left[|(\partial_\mu - i a_\mu)\Phi|^2 + r|\Phi|^2 + \frac{\lambda}{2N}|\Phi|^4 + \frac{\eta}{3N^2}|\Phi|^6 + i\frac{\kappa}{2} N a_\mu \epsilon^{\mu\nu\lambda}\partial_\nu a_\lambda \right]. \tag{1}$$

In order to facilitate the $1/N$ expansion, this theory is enlarged to an O(N)-symmetric model and all couplings are rescaled to produce a meaningful $N \to \infty$ limit. The model at hand is renormalizable in $(2+1)$ dimensions by the standard power counting procedure and possesses the usual UV divergences.

III. EFFECTIVE POTENTIAL AT THE LEADING ORDER

To get the effective potential, we examine the fluctuations in the Euclidean functional integral

$$e^{W(J)} = \int D\Phi D\Phi^\dagger \exp\left[-S + \int d^3x (J^\dagger \cdot \Phi + J \cdot \Phi^\dagger) \right], \tag{2a}$$

where the source J is introduced in order to use the functional integral as a generating functional for the correlators of Φ. For instance the two-point connected correlation functions are

$$\left.\frac{\delta^2 W}{\delta J^\dagger(x_1)\delta J(x_2)}\right|_{J=0} = \langle \Phi(x_1)\Phi^\dagger(x_2)\rangle. \tag{2b}$$

The generating functional of the connected one-particle irreducible correlation functions is obtained by performing a Legendre transformation

$$\Gamma(\Phi) = -W(J) + \int d^3x(J^\dagger \cdot \Phi + J \cdot \Phi^\dagger), \tag{3a}$$

and

$$\Gamma^{(n)}(x_1, x_2, \ldots, x_n) = \frac{\delta^n \Gamma}{\delta\Phi(x_1)\delta\Phi(x_2)\ldots\delta\Phi(x_n)}, \tag{3b}$$

The effective potential is obtained from the action for x-independent Φ,

$$\Gamma(\Phi) = (2\pi)^3\delta^3(p=0)V_{\mathrm{eff}}(\Phi). \tag{3c}$$

In order to facilitate a systematic $1/N$ expansion we introduce a pair of auxiliary fields σ and χ and rewrite the scalar potential Eq. (1) as

$$U = \sigma(|\Phi|^2 - N\chi) + N\left(r\chi + \frac{\lambda}{2}\chi^2 + \frac{\eta}{3}\chi^3\right). \tag{4}$$

The functional integral over σ from $-i\infty$ to $i\infty$ gives a delta function which renders the physical content of Eqs. (4) and (1) identical. The leading order (in $1/N$) effective potential $V_{\mathrm{eff}}(\sigma, \chi)$ is obtained by integrating out the Φ field which appears in Eq. (4) in a quadratic form

$$V_{\mathrm{eff}}(\sigma,\chi) = N\left[-\sigma\chi + r\chi + \frac{\lambda}{2}\chi^2 + \frac{\eta}{3}\chi^3\right] + N\int_p \ln(p^2+\sigma)]. \tag{5}$$

Here we adopt the convention that the Fourier integral $\int_p \equiv \int d^3p/(2\pi)^3$, which is used throughout this paper. We study the region of phase diagram where the O(N) is unbroken, but when the system is in a phase with spontaneously broken scale invariance, characterized by nonzero condensates for χ and σ. In the symmetric phase with zero expectation value of the field Φ, the vacuum structure is characterized by the gap equations:

$$\frac{\partial V}{\partial \sigma} = N\int_p \frac{1}{p^2+\sigma} - N\chi = 0, \tag{6a}$$

whose solution is

$$\varphi = \frac{\sqrt{\sigma}}{4\pi} = \frac{m}{4\pi}, \tag{6b}$$

where m assumes the role of mass and the momentum integral has been cut off at $p = \Lambda \gg m$. The renormalized fields are defined by $\varphi = -\chi + \Lambda/(2\pi^2)$. Eliminating the unphysical fields σ using the above gap equations, we obtain the leading order effective potential:

$$V^{(0)}_{\mathrm{eff}}(\varphi)/N = \frac{1}{3}(16\pi^2 - \eta)\varphi^3 + \frac{\lambda}{2}\varphi^2 - r\varphi. \tag{7}$$

In Eq. (7), the parameters r, λ have been renormalized in order to make the effective potential cutoff independent. At the leading order (in $1/N$), the coupling η remains unrenormalized. Furthermore, for the theory to make sense, we require that the effective potential be bounded from below: $\eta < 16\pi^2$.

IV. EFFECTIVE POTENTIAL AT NEXT-TO-LEADING ORDER

A. Scalar field contribution

To find the effective action to the next-to-leading order in large N, we expand the action to quadratic order in the shifted fields $\delta\sigma$, defined as $\sigma = m^2 + i\delta\sigma$. Differentiating twice the action with respect to these fields, one obtains

$$\frac{\delta^2 V}{\delta\sigma\delta\sigma} = 0, \quad \frac{\delta^2 V}{\delta\sigma\delta\chi} = -i, \quad \frac{\delta^2 V}{\delta\chi\delta\chi} = \lambda - 2\eta\varphi \equiv A. \tag{8}$$

Integrating out the quadratic scalar fluctuations, we obtain the next-to-leading order contribution to the effective potential

$$V_{\mathrm{eff}}/N = \left[-\frac{m^3}{6\pi} + \varphi m^2\right] + V(\varphi) + \frac{1}{2N}\int_p \ln(1 + A\Pi) \tag{9a}$$

where

$$\Pi(p) = \int_k \frac{1}{(k^2+m^2)[(k+p)^2+m^2]} = \frac{1}{4\pi p}\tan^{-1}\left(\frac{p}{2m}\right). \tag{9b}$$

The integral in the next-to-leading order contribution equation (9a) presents new divergences which require renormalization. In order to eliminate divergent terms involving powers of m in the numerator, we express the effective action in terms of the renormalized masses M, which is defined from the self-energy $\Sigma(p, m)$ in diagram in Fig. 1. The two-point function and the renormalized mass M are given by

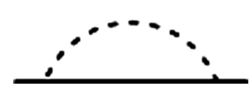

FIG. 1. Scalar field self-energy.

$$\Gamma^{(2)}(p,m) = p^2 + m^2 - \Sigma(p,m) \tag{10a}$$

$$M^2 = m^2 - \Sigma(0,m) + m^2\Sigma' \tag{10b}$$

where $\Sigma' = \left[\frac{\partial\Sigma(p,m)}{\partial p^2}\right]_{p=0}$.

Evaluating diagram Fig. 1 we find

$$\Sigma(p,m) = -\int_q \frac{D_{\sigma\sigma}(q)}{(q+p)^2+m^2} = -\int_q \frac{A}{[(q+p)^2+m^2][1+A\Pi]}. \tag{10c}$$

At this order in the large N expansion, in the pure scalar case, no infinite Φ-field wave-function renormalization is needed as can be seen by inspecting

$$\left[\frac{\partial\Sigma(p,m)}{\partial p^2}\right]_{p=0} = \frac{-1}{3}\int_q \left[\frac{1}{(q^2+m^2)^2} - \frac{4m^2}{(q^2+m^2)^3}\right] D_{\sigma\sigma}(q) \tag{10d}$$

which yields a finite contribution. The divergent terms in the self-energy for m at zero momentum are found by expanding $D_{\sigma\sigma}(q)$ in a Taylor expansion in A:

$$\Sigma^{\mathrm{div}}(0,m) = -\int_q \frac{1}{q^2+m^2}\{A - A^2\Pi(q) + \cdots\} = -A\left(\frac{\Lambda}{2\pi^2} - \frac{m}{4\pi}\right) + \frac{A^2}{16\pi^2}\ln\left(\frac{\Lambda}{m}\right). \tag{10e}$$

Now replacing m in Eq. (9a) by its renormalized quantity

$$m^2 = M^2 + \Sigma(0,M) - M^2\Sigma', \tag{10f}$$

results in the cancellation of all divergent terms proportional to m in the next-to-leading order contribution to the effective potential [these are the first-, second- and third-order terms in a Taylor expansion in A of the last term in Eq. (9a)]. The remaining divergent terms involving φ can be canceled by counterterms added to $V(\varphi)$, which introduce a scale μ, and the effective potential now reads as

$$V_{\mathrm{eff}}/N = V(\varphi) + \left[-\frac{M^3}{6\pi} + \varphi M^2 + \frac{1}{N}\left(\varphi - \frac{M}{4\pi}\right)(\Sigma^{\mathrm{reg}} - M^2\Sigma')\right] + \frac{1}{2N}\int_p^{\mathrm{F.P}} \ln[(1+A\Pi)] + \frac{1}{16N\pi^2}\left[\varphi A^2 + \frac{1}{384}A^3\right]\ln\left(\frac{\mu}{M}\right). \tag{10g}$$

The letters F.P on the integral indicate that its divergent terms have been subtracted out to make the integral finite. The last logarithmic terms in this renormalized effective potential clearly show that scale invariance is indeed violated. Since the effective potential is a physical quantity that should not depend on the renormalization scale [20], we require its couplings to depend on that scale in such a way that the coefficients of V_{eff}, when expanded in powers of the fields φ, do not depend on the scale μ. This yields the following β functions of the ungauged model:

$$\beta_0(r) = \frac{\lambda^2}{16\pi^2 N}\left(1 - \frac{\eta}{64}\right) \tag{11a}$$

$$\beta_0(\lambda) = \frac{\eta\lambda}{2\pi^2 N}\left(1 - \frac{\eta}{128}\right) \tag{11b}$$

$$\beta_0(\eta) = \frac{3\eta^2}{4\pi^2 N}\left(1 - \frac{\eta}{192}\right). \tag{11c}$$

B. Gauge fields contribution

In this section we consider the full gauge invariant action in Eq. (1). The first step is to integrate out the Φ degrees of freedom which leads to an effective action for the gauge fields that has, besides the Chern-Simons term in Eq. (1), induced Maxwell terms from the bosonic functional determinant

$$S_G(a_\mu) = N\mathrm{Tr}\ln[-(\partial_\mu - ia_\mu)^2 + m^2]. \tag{12a}$$

Expanding this nonlocal term about $a_\mu = 0$ and keeping only quadratic terms in the fields gives at an intermediate step

$$S_G(a_\mu) = \frac{N}{2}\int_q a_\mu(-q)\Gamma(q)\delta^T_{\mu\nu}a_\nu(q) - \kappa a_\mu(-q)\epsilon_{\mu\lambda\nu}q_\lambda a_\nu(q) \tag{12b}$$

here $\delta^T_{\mu\nu} = (\delta_{\mu\nu} - q_\mu q_\nu/q^2)$ and where the $\Gamma(q)$ term arises from the usual bosonic one-loop polarization diagrams [10], which are expressed as

$$2\int_k \frac{\delta_{\mu\nu}}{k^2+m^2} - \int_k \frac{[2k_\mu+q_\mu][2k_\nu+q_\nu]}{(k^2+m^2)((k+q)^2+m^2)} = \Gamma(q)\delta^T_{\mu\nu}. \tag{12c}$$

A full analytic evaluation of the integrals is possible using some standard steps, and the result is

$$\Gamma(q) = \frac{q^2+4m^2}{8\pi q}\tan^{-1}\left(\frac{q}{2m}\right) - \frac{m}{4\pi}. \tag{12d}$$

The resulting gauge propagator in Landau gauge is

$$G_{\mu\nu}(q) = \frac{1}{N}\left[\frac{\Gamma(q)}{\Gamma^2(q)+q^2\kappa^2}\delta^T_{\mu\nu} + \frac{\kappa}{\Gamma^2(q)+q^2\kappa^2}\epsilon_{\mu\lambda\nu}q_\lambda\right]. \tag{12e}$$

Next, integrating out the gauge fluctuations yields a new next-to-leading order contribution to the effective potential

$$V^{(1)}_{\text{gauge}} = \frac{1}{2}\int_q \ln[\Gamma^2 + \kappa^2 q^2]. \tag{13a}$$

We find it convenient to subtract out the zero mass terms from this contribution and write it as

$$V^{(1)}_{\text{gauge}}(m) - V^{(1)}_{\text{gauge}}(0,0) = \frac{1}{2}\int_q \ln\left[1+\epsilon\left(\frac{2^8\Gamma^2}{q^2}-1\right)\right] \tag{13b}$$

where the parameter $\epsilon = 1/[1+(16\kappa)^2]$ takes values in the interval [0, 1] with $\epsilon = 0$ corresponding to no gauge coupling (pure scalar model) and $\epsilon = 1$ corresponding to zero Chern-Simons term (gauge model with induced Maxwell term). The gauge fields contribution to the effective potential equation (13b) presents divergences which are contained in the first-, second- and third-order terms in a Taylor expansion in ϵ. To handle such divergences we first express the effective action in terms of the renormalized mass M which involves another self-energy $\Sigma^{\text{gauge}}(p,m)$ in diagram Fig. 2 and by adding counterterms to $V(\varphi)$:

$$\Sigma^{\text{gauge}}(p,m) = \int_q \frac{(2p_\mu+q_\mu)G_{\mu\nu}(q)(2p_\nu+q_\nu)}{(p+q)^2+m^2} = 4\int_q \frac{[p^2-(p\cdot q)^2/q^2)]\Gamma(q)}{(\Gamma^2+\kappa^2q^2)[(p+q)^2+m^2]}. \tag{13c}$$

This is easily evaluated to give

$$\Sigma^{\text{gauge}}(0,m) = 0 \tag{13d}$$

$$\left[\frac{\partial\Sigma^{\text{gauge}}(p,m)}{\partial p^2}\right]_{p=0} = \frac{64\epsilon}{3\pi^2}\ln\left(\frac{\Lambda}{m}\right). \tag{13e}$$

The divergence in Eq. (13e) introduces infinite scalar field wave-function renormalization $\Phi = Z^{1/2}\ \Phi_R$, with $Z = 1+\partial\Sigma^{\text{gauge}}/\partial p^2$. The term introduced by mass renormalization is

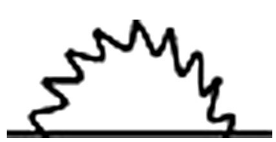

FIG. 2. Gauge fields self-energy.

$$-\left(\varphi-\frac{M}{4\pi}\right)M^2\left[\frac{\partial\Sigma^{\text{gauge}}(p,m)}{\partial p^2}\right]_{p=0}. \tag{13f}$$

These terms are proportional to $(\varphi-\frac{M}{4\pi})$ and vanish on shell. The remaining terms in the gauge field contribution to the effective potential including logarithmic divergent terms are

$$V^{(1)}_{\text{gauge}} = \frac{1}{2}\int_q^{\text{F.P.}} \ln\left[1+\epsilon\left(\frac{2^8\Gamma^2}{q^2}-1\right)\right] - \frac{2^{10}}{3}\epsilon\ln\left(\frac{\mu}{M}\right)\left(1-\left(\frac{3}{2}+\frac{12}{\pi^2}\right)\epsilon+\frac{16}{\pi^2}\epsilon^2\right)\varphi^3. \tag{13g}$$

Collecting the leading order contribution in Eq. (7) and the two next-to-leading order contributions in Eqs. (10g) and (13g) give the complete effective potential to that order which has an explicit dependence on the renormalization scale. Since this scale is unphysical, there must be implicit dependence of V_{eff} on μ through the couplings and the fields so that $\mu\frac{dV}{d\mu} = 0$. This renormalization group equation leads to the new β functions which now take account of wave-function renormalization and are expressed in terms of the ones we already found in the ungauged model as

$$\beta(\eta) = \beta_0(\eta) - \eta\frac{64\varepsilon}{\pi^2N} - \frac{2^{12}}{N}\epsilon\left[1-\left(\frac{3}{2}+\frac{12}{\pi^2}\right)\epsilon+\frac{16}{\pi^2}\epsilon^2\right]. \tag{14}$$

The correction terms with powers of ϵ arise from the gauge fields contribution. The beta function of r is as in Eq. (11a), but with an extra term $-r64\epsilon/(3\pi^2N)$, which accounts for the wave-function renormalization. Similarly, the beta function of λ is as in Eq. (11b) but with an extra term $-\lambda 128\epsilon/(3\pi^2N)$. Consequently, both beta functions for λ and r also vanish as well at $\lambda = r = 0$ in the presence of Chern-Simons term.

V. FIXED POINTS ANALYSIS

We examine the theory at its tricritical point $r = \lambda = 0$ and we redefine $\eta = 192x$. The beta function reads

$$\dot{x} = x^2 - \frac{4\epsilon x}{9} - x^3 - \alpha \tag{15}$$

where $\dot{x}$ stands for $\frac{\pi^2N}{144}dx/d\ln(\mu)$ and $\alpha = \frac{4\pi^2}{27}\epsilon[1-(\frac{3}{2}+\frac{12}{\pi^2})\epsilon+\frac{16}{\pi^2}\epsilon^2]$. The set of fixed points is now obtained from $\dot{x} = 0$ and the general form of them is rather involved. We know, however, that either one or three solutions will be possible. Instead of finding analytically these points, we will use a physically illustrative approach. The fixed points are found at the intersection of the curve

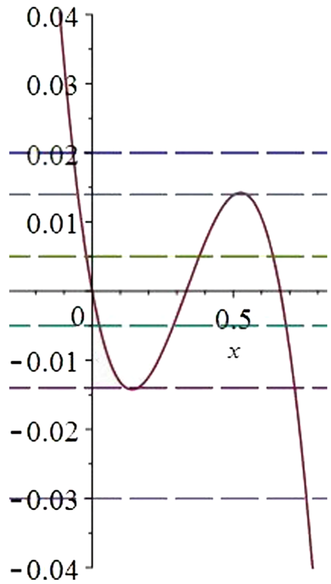

FIG. 3. Plot of the two curves $y_1 = x^2 - x^3 - \frac{4\epsilon}{9}x$ (for $\epsilon = 0.5$) and $y_2 = \alpha$ (dashed lines) whose intersections define the possible fixed points.

$$y_1 = x^2 - x^3 - \frac{4\epsilon x}{9} \tag{16}$$

and the constant line $y_2 = \alpha$. Very different qualitative behaviors are observed depending on the parameter ϵ. For $0.11024 < \epsilon < 0.53965$ or $\epsilon > 0.57192$ a single fixed point is obtained (an example is shown in Fig. 3), whereas for $0.53965 < \epsilon < 0.57192$ three simultaneous fixed points are present. As we move the constant line y_2, we can see that it crosses the y_1 curve in different ways. For some values only one crossing is allowed (giving a single fixed point) whereas for a domain of values three crossings occur.

We show in Fig. 4, the corresponding curve with the location of the fixed points $x^* = x^*(\epsilon)$, as defined by the implicit function $x^2 - x^3 - \frac{4\epsilon}{9}x - \alpha = 0$. We note the shape in the folded curve. A consequence of this folded curve is the presence of a hysteresis effect.

The impact of this folded curve on the system's dynamics is illustrated in Fig. 5, which shows a zoom of the curve $x^*(\epsilon)$ and shows the change in the states as ϵ is continuously tuned. Starting from point 1 on the curve and

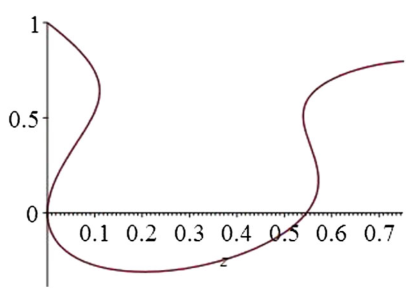

FIG. 4. Folded curve associated with the possible fixed points.

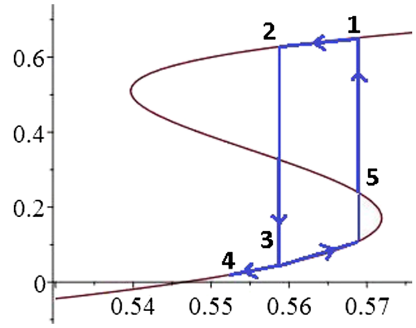

FIG. 5. Folded curve associated with the possible fixed points and hysteresis effect.

continuously tuning ϵ so that the state moves continuously on the curve as the control parameter is varied. After reaching point 2 the state jumps into the lower part of the curve (at 3) and if we keep reducing ϵ a new point is reached under continuous changes (4). On the other hand, starting from 3 and increasing ϵ, the state does not jump back to 2 but instead moves on to the lower branch until the fold is reached again, this time at point 5. A new jump occurs, but with a completely different trajectory. This illustrates that a slow change of the control parameter does not always produce a slow dynamical response.

To show that the beta function exhibits a topological normal form of a generic one-dimensional system having a fold bifurcation (also known as limit point bifurcation or saddle-node bifurcation), we expand the beta function around the turning points up to second-order terms. This gives

$$\beta(x,\epsilon) = \frac{\partial\beta}{\partial\epsilon}(\epsilon - \epsilon_j) + \frac{1}{2}\frac{\partial^2\beta}{\partial x^2}(x - x_j)^2 \tag{17}$$

where ϵ_j, x_j represent the location of the fold, which are given by ($\epsilon_1 = 0.11$, $x_1 = 0.64$), ($\epsilon_2 = 0.54$, $x_2 = 0.51$), and ($\epsilon_3 = 0.57$, $x_3 = 0.17$). In deriving Eq. (17), we used $\beta(x_j, \epsilon_j) = 0$ at the fixed points and $\partial\beta/\partial x = 0$ at the turning points (fold). Furthermore, terms such as $(\epsilon - \epsilon_j)^2, (\epsilon - \epsilon_j)(x - x_j)$ are dropped because we assumed $|x - x_j| \gg |\epsilon - \epsilon_j|$. We note that around (x_1, ϵ_1) both $\frac{\partial\beta}{\partial\epsilon}$ and $\frac{\partial^2\beta}{\partial x^2}$ are negative; around (x_3, ϵ_3) both $\frac{\partial\beta}{\partial\epsilon}$ and $\frac{\partial^2\beta}{\partial x^2}$ are positive; and around (x_2, ϵ_2), $\frac{\partial\beta}{\partial\epsilon}$ is positive and $\frac{\partial^2\beta}{\partial x^2}$ is negative. The corresponding IR and UV fixed points, each describing a conformal phase of the theory approach each other until they merge at $x = x_j$. To see that fixed point merger generically gives rise to Berezinskii-Kosterlitz-Thouless (BKT) scaling, consider for example the case where ϵ is slightly below ϵ_2, and that at a UV scale the coupling takes an initial value $x_{\mathrm{UV}} < x_2$ as seen in Fig. 6.

On scaling to the IR, the coupling then flows to larger values, lingering near x_2 where the beta function is small, and then blowing up quickly, defining an intrinsic IR scale,

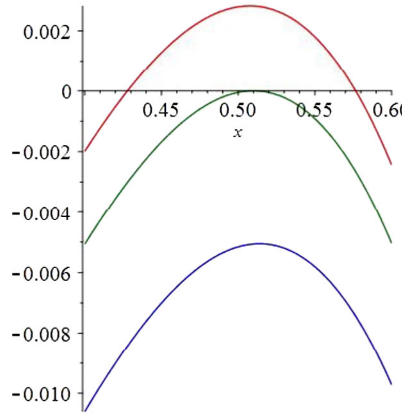

FIG. 6. Beta function. For $\epsilon > \epsilon_2$ (red curve) there are two fixed points at $x_\pm$ which are UV and IR stable; these fixed points merge at x_2 for $\epsilon = \epsilon_2$ (green curve), and disappear for $\epsilon < \epsilon_2$ (blue curve).

which is insensitive to the initial value $x_{\rm UV}$. The scale IR will characterize the longest correlation length in this theory, and can be computed by integrating Eq. (17):

$$\frac{\Lambda_{\rm IR}}{\Lambda_{\rm UV}} = \exp[t_{\rm IR} - t_{\rm UV}] = \exp\left[\int_{x_{\rm UV}}^{x_{\rm IR}} \frac{dx}{\beta(x,\epsilon)}\right] = e^{-c\pi/\sqrt{\epsilon_2-\epsilon}} \tag{18}$$

with c a constant. This indicates that the energy scales exponentially (i.e., Miransky scaling [21]) close to the bifurcation, and also indicates a walking behavior of the mass just below the conformal window. The scaling dimension of the operator crossing marginality at the bifurcations is related to the eigenvalues of the Jacobian of the normal form equation (17), with saddle-node bifurcation at ϵ_j, x_j. This gives the eigenvalue of the operator crossing marginality

$$\pm \frac{\partial^2\beta}{\partial x^2}\sqrt{\frac{-2\frac{\partial\beta}{\partial\epsilon}(\epsilon-\epsilon_j)}{\frac{\partial^2\beta}{\partial x^2}}} \tag{19}$$

which is equal to the anomalous dimension of the sextic term at the bifurcation

$$\gamma = \Delta_6 - 3 = \pm \frac{\partial^2\beta}{\partial x^2}\sqrt{\frac{-2\frac{\partial\beta}{\partial\epsilon}(\epsilon-\epsilon_j)}{\frac{\partial^2\beta}{\partial x^2}}} \tag{20}$$

	$(\epsilon_1=0.11, x_1=0.64)$	$(\epsilon_2=0.54, x_2=0.51)$	$(\epsilon_3=0.57, x_3=0.17)$
γ	$\mp \frac{27.45}{N}\sqrt{\epsilon_1-\epsilon}$	$\mp \frac{15.41}{N}\sqrt{\epsilon-\epsilon_2}$	$\pm \frac{16.77}{N}\sqrt{\epsilon_3-\epsilon}$

This shows another feature of Chern-Simons matter interaction, namely it modifies the scaling dimensions of operators crossing marginality at the bifurcation points as seen in the above table. Before summarizing our results, we make some comments about the model with an added Maxwell term to the action in Eq. (1) with a dimensionful gauge coupling e^2. Accordingly, the gauge propagator in Eq. (12e) and the effective potential in Eqs. (13a) and (13b) have to be modified by shifting Γ into $\Gamma + q^2/e^2$. In such a case the resulting logarithmic divergent terms in the gauge field contribution to the effective potential turn into $\ln(\Lambda/m)[(e^4/16\pi)\varphi + 2\pi e^2\varphi^2]$, instead of those terms in Eq. (13g). These contribute to the beta functions of r and λ but not to the sextic coupling η. Consequently, in the Maxwell-Chern-Simons case the RG flow of the marginal coupling η is unaffected by the gauge fluctuations at least up to first subleading order in $1/N$ expansion. Therefore, our analysis of the phase structure of the fixed points involving the Chern-Simons term should remain valid at energies well below the scale set by the dimensionful gauge coupling e^2.

VI. SUMMARY

We examined a U(1)-symmetric model in three-dimensional space-time with a scalar field coupled to a Chern-Simons term. The investigation was focused on the neighborhood of the tricritical point, where the renormalized quadratic and quartic couplings are set to zero and only the sextic operator is retained. Using a large N technique, we computed the quantum effective potential and the field renormalization (effect of anomalous dimensions) from which we derived the beta function of the scalar coupling. The model has a phase which exhibits spontaneous breaking of scale symmetry accompanied by a massless dilaton which is a Goldstone mode. In this relativistic quantum field theory, the Abelian Chern-Simons term receives no correction from interacting with matter fields. This results in an identically vanishing beta function for the Chern-Simons coupling. The insensibility of the Chern-Simons coupling to energy scale is attributed to the topological nature of the Chern-Simons action. However, the scalar fields in this theory require infinite renormalization and receive anomalous dimensions. We showed that the beta function of the sextic coupling exhibits, at the next order in the $1/N$ expansion, nontrivial running that we analyzed explicitly in terms of the Chern–Simons coefficient. The specific feature of the RG flow, viewed as a dynamical system, exhibited a topological normal form of a generic one-dimensional system having a fold bifurcation. The control parameter is connected with a marginal operator (Chern-Simons). We identified a window in the parameter space where the renormalization group flow has a stable infrared fixed point at which scale invariance is recovered. However, as the control parameter is varied, the IR and UV fixed points in this theory, each describing a conformal phase of the theory, approach each other and ultimately collide, giving rise to a scaling behavior similar to BKT phase transition, although here it happens in a context different from the original BKT phase transition. We also found that the Chern-Simons interaction modifies the scaling dimension of the operator crossing marginality at the bifurcation points.

ACKNOWLEDGMENTS

The work of S. S. was supported by a summer research travel grant from American University of Sharjah Materials Science and Engineering Research Institute (MSERI). Part of this work was conducted during S. S. visiting IISER Kolkata; its hospitality is gratefully acknowledged.

Note added.—Recently, we became aware that the new added Ref. [22] deals with a Φ^6 model coupled to non-Abelian Chern-Simons theory. The flow of the coupling of the sextic term in the potential in that paper is a third-order polynomial similar to our finding. The difference, though, is that in the Abelian model considered here we are able to compute exactly all coefficients of the third-order polynomial, up to first subleading order in $1/N$; while in [22] only the coefficient of the cubic term (x_6^3 in the notation of [22]) is found exactly and the lower coefficients are only known perturbatively but not for all values of the 't Hooft coupling. Furthermore, contrary to the conjecture made in [22] pertaining to the flow always having three fixed points at every nonzero value of the 't Hooft parameter, we demonstrate here that for certain values of the Chern-Simons gauge coupling, the beta function of the sextic term has only one fixed point rather than three. This occurs as shown here when two fixed points collide and annihilate each other leaving only one repulsive fixed point.

[1] M. Blume, V. J. Emery, and R. B. Griffiths, Phys. Rev. A **4**, 1071 (1971).

[2] P. K. Townsend, Nucl. Phys. **B118**, 199 (1977); T. Appelquist and U. Heinz, Phys. Rev. D **25**, 2620 (1982); R. D. Pisarski, Phys. Rev. Lett. **48**, 574 (1982).

[3] W. A. Bardeen, M. Moshe, and M. Bander, Phys. Rev. Lett. **52**, 1188 (1984); W. A. Bardeen and M. Moshe, Phys. Rev. D **28**, 1372 (1983).

[4] F. David, D. A. Kessler, and H. Neuberger, Phys. Rev. Lett. **53**, 2071 (1984); Nucl. Phys. **B257**, 695 (1985).

[5] G. Eyal, M. Moshe, S. Nishigaki, and J. Zinn-Justin, Nucl. Phys. **B470**, 369 (1996).

[6] W. A. Bardeen and M. Moshe, J. High Energy Phys. 06 (2014) 113.

[7] H. Omid, G. W. Semenoff, and L. C. R. Wijewardhana, Phys. Rev. D **94**, 125017 (2016).

[8] A. N. Redlich, Phys. Rev. Lett. **52**, 18 (1984); Phys. Rev. D **29**, 2366 (1984); C. R. Hagen, P. Panigrahi, and S. Ramaswamy, Phys. Rev. Lett. **61**, 389 (1988); S. Coleman and B. Hill, Phys. Lett. B **159**, 184 (1985).

[9] R. Pisarski and S. Rao, Phys. Rev. D **32**, 2081 (1985); A. Khare, R. MacKenzie, P. K. Panigrahi, and M. Paranjape, Phys. Lett. B **355**, 236 (1995); K. S. Babu, P. K. Panigrahi, and S. Ramaswamy, Phys. Rev. D **39**, 1190 (1989).

[10] S. Paul and A. Khare, Phys. Lett. B **174**, 420 (1986); **177**, 453(E) (1986).

[11] J. Hong, Y. Kim, and P.-Y. Pac, Phys. Rev. Lett. **64**, 2230 (1990); R. Jackiw and E. Weinberg, Phys. Rev. Lett. **64**, 2234 (1990).

[12] C. R. Hagen, Ann. Phys. (N.Y.) **157**, 342 (1984); Phys. Rev. D **31**, 848 (1985).

[13] G. Lozano, Phys. Lett. B **283**, 70 (1992).

[14] A. M. Polyakov, Mod. Phys. Lett. A **03**, 325 (1988); G. W. Semenoff, Phys. Rev. Lett. **61**, 517 (1988).

[15] J. Lykken, J. Sonnenschein, and N. Weiss, Int. J. Mod. Phys. A **06**, 5155 (1991); F. Wilczek, *Fractional Statistics and Anyonic Superconductivity* (World Scientific, Singapore, 1990).

[16] X.-G. Wen and Y.-S. Wu, Phys. Rev. Lett. **70**, 1501 (1993); L. Pryadko and S. C. Zhang, Phys. Rev. Lett. **73**, 3282 (1994); G. Murthy and S. Sachdev, Phys. Rev. Lett. **101**, 226801 (2008).

[17] C. Xu and S. Sachdev, Phys. Rev. B **79**, 064405 (2009).

[18] S. Sakhi, Phys. Rev. D **97**, 096015 (2018); **90**, 045028 (2014); Physica (Amsterdam) **392A**, 6255 (2013); J. Phys. A **45**, 175301 (2012); J. Phys. A **41**, 085003 (2008); Europhys. Lett. **87**, 37006 (2009); **73**, 267 (2006); Phys. Rev. B **73**, 132505 (2006).

[19] S. Gukov, Nucl. Phys. **B919**, 583 (2017).

[20] T. J. Hollowood, *Renormalization Group and Fixed Points in Quantum Field Theory* (Springer Heidelberg, New York, 2013).

[21] D. B. Kaplan, J. W. Lee, and D. T. Son, Phys. Rev. D **80**, 125005 (2009); V. A. Miransky, Int. J. Mod. Phys. A **25**, 5105 (2010); K. Jensen, A. Karch, D. T. Son, and E. G. Thompson, Phys. Rev. Lett. **105**, 041601 (2010).

[22] O. Aharony, S. Jain, and S. J. Minwalla, High Energy Phys. Nucl. Phys. **2018**, 58 (2018).