Spin-flavor oscillations of Dirac neutrinos in matter under the influence of a plane electromagnetic wave

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We study oscillations of Dirac neutrinos in background matter and a plane electromagnetic wave. First, we find the new exact solution of the Dirac-Pauli equation for a massive neutrino with the anomalous magnetic moment electroweakly interacting with matter under the influence of a plane electromagnetic wave with the circular polarization. We use this result to describe neutrino spin oscillations in the external fields in question. Then we consider several neutrino flavors and study neutrino spin-flavor oscillations in this system. For this purpose we formulate the initial condition problem and solve it accounting for the considered external fields. We derive the analytical expressions for the transition probabilities of spin-flavor oscillations for different types of neutrino magnetic moments. These analytical expressions are compared with the numerical solutions of the effective Schrödinger equation and with the findings of other authors. In particular, we reveal that a resonance in neutrino spin-flavor oscillations in the considered external fields cannot happen contrary to the previous claims. Finally, we briefly discuss some possible astrophysical applications.

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I. INTRODUCTION

Nowadays it is commonly believed that neutrinos possess nonzero masses and mixing between different flavor eigenstates [1]. These properties of neutrinos result in transitions between neutrino flavors, which are called neutrino flavor oscillations [2]. Neutrino flavor oscillations are known to happen even in vacuum, i.e., at the absence of external fields.

As constituents of the standard model, neutrinos can interact with other fermions, which a background matter is made of, by exchanging virtual W and Z bosons. This kind of interaction, although it is quite weak, can significantly influence the process of neutrino flavor oscillations resulting in the resonance enhancement of the transition probability, known as the Mikheyev-Smirnov-Wolfenstein (MSW) effect [3]. The MSW effect is believed to be the most plausible solution to the solar neutrino problem [4].

Despite neutrinos are electrically neutral particles, nothing prevents them to have nonzero magnetic moments [5,6], which are of a pure anomalous origin. Neutrino magnetic moments result in the particle spin precession in an external electromagnetic field. Thus, a left polarized neutrino, which exists in the standard model, can be transformed to a right polarized particle, invisible to the detectors. If this process happens within one neutrino generation, it is called neutrino spin oscillations [5]. There is a possibility for neutrinos to change both flavor and the polarization in an external electromagnetic field. In this situation, these transitions are named neutrino spin-flavor oscillations. Neutrino spin and spin-flavor oscillations were recently reviewed in Ref. [7].

Neutrino spin-flavor oscillations were studied mainly in a constant magnetic field, which is transverse with respect to the neutrino motion. However, other nontrivial configurations of the electromagnetic field, like an electromagnetic wave are of interest. This interest is inspired, e.g., by the suggestion in Refs. [8,9] to explore the neutrino evolution in intense laser pulses. Note that the study on the development of intense lasers in Ref. [10] was recognized by the Nobel committee in 2018.

Neutrino spin and spin-flavor oscillations in matter and an electromagnetic wave were previously discussed in Refs. [11,12] Recently, we demonstrated in Ref. [13] that the results of Ref. [11] are not applicable for the description of spin-flavor oscillations. In the present work, we continue the study of Ref. [13]. However, besides the neutrino interaction only with a plane electromagnetic wave [13], now we account for the electroweak interaction of neutrinos with background matter. As in Ref. [13], here we suppose that neutrinos are Dirac particles. Despite multiple models for the neutrino mass generation predict that neutrinos are

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likely to be Majorana fermions [14], the nature of these particles is still unclear [15].

This paper is organized as follows. We start in Sec. II with the basics of neutrino electrodynamics in background matter. Then, in Sec. III, we find the new exact solution of the wave equation for a single neutrino mass eigenstate interacting with matter and a plane electromagnetic wave with the circular polarization. The obtained results are applied in Sec. IV to describe neutrino spin oscillations in the considered external fields. Then, in Sec. V, we study neutrino spin-flavor oscillations in matter and a plane electromagnetic wave, with the diagonal magnetic moments being greater than the transition one. The opposite situation, when the transition magnetic moment is dominant, is considered in Sec. VI. Some possible astrophysical applications are also briefly discussed in Sec. VI. Finally, in Sec. VII, we summarize our results.

II. NEUTRINO INTERACTION WITH EXTERNAL FIELDS

In this section, we briefly recall how neutrinos can interact with background matter and an external electromagnetic field. We consider these interactions both in flavor and mass eigenstates bases.

Without loss of generality, we shall study the system of two massive neutrinos (ν_α, ν_β) with a nonzero mixing. For example, we can take that $\nu_\alpha \equiv \nu_{\mu,\tau}$ and $\nu_\beta \equiv \nu_e$. These neutrinos can electroweakly interact with background matter consisting of electrons, protons, and neutrons. The background matter is supposed to be nonmoving and unpolarized. Moreover, we shall take that neutrinos have nonzero magnetic moments and can interact with the external electromagnetic field $F_{\mu\nu} = (\mathbf{E}, \mathbf{B})$.

The Lagrangian for the system of these neutrinos has the form,

$$\mathcal{L} = \sum_{\lambda\lambda'=\alpha,\beta} \bar{\nu}_\lambda \left[\delta_{\lambda\lambda'} \mathrm{i}\gamma^\mu \partial_\mu - m_{\lambda\lambda'} - \frac{M_{\lambda\lambda'}}{2} F_{\mu\nu}\sigma^{\mu\nu} - \frac{f_{\lambda\lambda'}}{2}\gamma^0(1-\gamma^5) \right] \nu_{\lambda'}, \tag{2.1}$$

where $\gamma^\mu = (\gamma^0, \boldsymbol{\gamma})$, $\gamma^5 = \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3$, and $\sigma_{\mu\nu} = \frac{\mathrm{i}}{2}[\gamma_\mu, \gamma_\nu]_-$ are the Dirac matrices. The mass matrix $(m_{\lambda\lambda'})$ and the matrix of magnetic moments $(M_{\lambda\lambda'})$ are independent in general. The matrix of the effective potentials of the neutrino interaction with matter is diagonal in the flavor basis: $f_{\lambda\lambda'} = f_\lambda \delta_{\lambda\lambda'}$. The explicit form of f_λ in the electroneutral matter can be obtained on the basis of the results of Ref. [16] as

$$f_{\nu_e} = \sqrt{2} G_\mathrm{F}\left(n_e - \frac{1}{2}n_n\right), \qquad f_{\nu_\mu,\nu_\tau} = -\frac{1}{\sqrt{2}} G_\mathrm{F} n_n, \tag{2.2}$$

where $G_\mathrm{F} = 1.17 \times 10^{-5}\ \mathrm{GeV}^{-2}$ is the Fermi constant and $n_{e,n}$ are the number densities of electrons and neutrons.

The nature of neutrinos can be revealed only if we transform the flavor wave functions ν_λ to the mass eigenstates basis,

$$\nu_\lambda = \sum_{a=1,2} U_{\lambda a}\psi_a, \qquad (U_{\lambda a}) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}, \tag{2.3}$$

where θ is the vacuum mixing angle, which is chosen in such a way to diagonalize the mass matrix: $(U_{\lambda a})^\dagger (m_{\lambda\lambda'})(U_{\lambda' b}) = m_a \delta_{ab}$, where m_a are the neutrino masses. The neutrino mass eigenstates ψ_a, $a = 1, 2$, are taken to be Dirac particles. In general situation, the matrices of magnetic moments $(\mu_{ab}) = (U_{\lambda a})^\dagger (M_{\lambda\lambda'})(U_{\lambda' b})$ and neutrino interaction with background matter $(V_{ab}) = (U_{\lambda a})^\dagger (f_{\lambda\lambda'})(U_{\lambda' b})$ are nondiagonal in the mass eigenstates basis.

Using Eq. (2.3), we can rewrite the Lagrangian in Eq. (2.1) in the following way:

$$\mathcal{L} = \sum_{ab=1,2} \bar{\psi}_a \left[\delta_{ab}(\mathrm{i}\gamma^\mu \partial_\mu - m_a) - \frac{\mu_{ab}}{2} F_{\mu\nu}\sigma^{\mu\nu} - \frac{V_{ab}}{2}\gamma^0(1-\gamma^5) \right] \psi_b. \tag{2.4}$$

One can see that the Dirac equations for different mass eigenstates, resulting from the Lagrangian in Eq. (2.4), are coupled due to the presence of external fields.

III. SOLUTION OF THE DIRAC-PAULI EQUATION

In this section, we study the evolution of a single neutrino mass eigenstate in matter under the influence of a plane electromagnetic wave. We write down the wave equation for a massive neutrino in these external fields and find its exact solution.

In this section, we neglect the mixing between different neutrino types. Thus, using Eq. (2.4) and omitting the index a there, we obtain the wave equation for a Dirac neutrino with the nonzero mass m and the magnetic moment μ, interacting with nonmoving and unpolarized background matter and with the external electromagnetic field, in the form,

$$\left[\mathrm{i}\gamma^\mu \partial_\mu - m - \frac{\mu}{2} F_{\mu\nu}\sigma^{\mu\nu} - \frac{V}{2}\gamma^0(1-\gamma^5) \right] \psi = 0, \tag{3.1}$$

where ψ is the neutrino bispinor. The effective potential V can be obtained basing on Eqs. (2.2) and (2.3). We suppose that a neutrino interacts with a plane electromagnetic wave. Neglecting the dispersion of the wave, one gets that the electric and magnetic fields, $\mathbf{E}$ and $\mathbf{B}$, depend on $t - z$. If the wave propagates along the z-axis, one has that

$\mathbf{B} = (B_x, B_y, 0)$, and $\mathbf{E} = (B_y, -B_x, 0)$, where B_x and B_y are linearly independent components of the magnetic field.

We rewrite Eq. (3.1) in the Hamilton form as

$$\mathrm{i}\partial_t\psi = \left[-\mathrm{i}(\boldsymbol{\alpha}\nabla) + \beta m + \mu(\mathrm{i}\boldsymbol{\gamma}\mathbf{E} - \beta\boldsymbol{\Sigma}\mathbf{B}) + \frac{V}{2}(1-\gamma^5)\right]\psi, \tag{3.2}$$

where $\boldsymbol{\alpha} = \gamma^0\boldsymbol{\gamma}$, $\beta = \gamma^0$, and $\boldsymbol{\Sigma} = \gamma^5\gamma^0\boldsymbol{\gamma}$ are the Dirac matrices. If the density of background matter is constant, we can gauge the term $V/2$ out: $\psi \to \exp(-\mathrm{i}Vt/2)\psi$. It is convenient to introduce the new variables $u_0 = t - z$ and $u_3 = t + z$. Defining the derivatives with respect to u_0 and u_3 as ∂_0 and ∂_3, one gets that Eq. (3.2) has the following integrals:

$$-\mathrm{i}\nabla_\perp\psi = \mathbf{p}_\perp\psi, \qquad 2\mathrm{i}\partial_3\psi = \lambda\psi, \tag{3.3}$$

where $\nabla_\perp = (\partial_x, \partial_y, 0)$ and $\mathbf{p}_\perp = (p_x, p_y, 0)$. Here we assume that the background matter is uniform.

Then we look for the solution of Eq. (3.2) in the form,

$$\psi = \exp(-\mathrm{i}Vt/2 + \mathrm{i}\mathbf{p}_\perp\mathbf{x}_\perp - \mathrm{i}\lambda u_3/2)\psi_0, \tag{3.4}$$

where $\psi_0 = \psi_0(u_0)$. The equation for ψ_0 reads

$$\begin{aligned}\mathrm{i}(1-\alpha_z)\partial_0\psi_0 = \bigg[&(\boldsymbol{\alpha}_\perp\mathbf{p}_\perp) - \frac{\lambda}{2}(1+\alpha_z) + \beta m \\ &+ \mu(\mathrm{i}\boldsymbol{\gamma}\mathbf{E} - \beta\boldsymbol{\Sigma}\mathbf{B}) - \frac{V}{2}\gamma^5\bigg]\psi_0.\end{aligned} \tag{3.5}$$

Note that the matrix $(1-\alpha_z)$ is singular. Thus some of the components of ψ_0 obey the algebraic rather than differential equations.

It is convenient to choose the Dirac matrices in the chiral representation [17]. If we define $\psi_0^{\mathrm{T}} = (\psi_1, \psi_2, \psi_3, \psi_4)$, then only ψ_2 and ψ_3 are independent and satisfy the equations

$$\begin{aligned}\mathrm{i}\partial_0\psi_2 = &\left[\frac{\lambda}{2}\frac{p_\perp^2 + m^2}{\lambda^2 - V^2/4} - \frac{V}{4}\left(1 + \frac{p_\perp^2 - m^2}{\lambda^2 - V^2/4}\right)\right]\psi_2 \\ &+ \left\{\mu(B_x + \mathrm{i}B_y) + \frac{V}{2}\frac{m(p_x + \mathrm{i}p_y)}{\lambda^2 - V^2/4}\right\}\psi_3, \\ \mathrm{i}\partial_0\psi_3 = &\left[\frac{\lambda}{2}\frac{p_\perp^2 + m^2}{\lambda^2 - V^2/4} + \frac{V}{4}\left(1 + \frac{p_\perp^2 - m^2}{\lambda^2 - V^2/4}\right)\right]\psi_3 \\ &+ \left\{\mu(B_x - \mathrm{i}B_y) + \frac{V}{2}\frac{m(p_x - \mathrm{i}p_y)}{\lambda^2 - V^2/4}\right\}\psi_2.\end{aligned} \tag{3.6}$$

After the separation the common factor in $\psi_{2,3}$ as

$$\psi_{2,3} = \exp\left(-\mathrm{i}\frac{\lambda}{2}\frac{p_\perp^2 + m^2}{\lambda^2 - V^2/4}u_0\right)v_{1,2}, \tag{3.7}$$

one gets that the spinor $v = v(u_0) = (v_1, v_2)^{\mathrm{T}}$ obeys the equation,

$$\mathrm{i}\partial_0 v = (\boldsymbol{\sigma}^*\mathbf{R})v, \tag{3.8}$$

where $\boldsymbol{\sigma}$ are the Pauli matrices and

$$\mathbf{R} = \mu\mathbf{B} + \frac{V}{2}\frac{m\mathbf{p}_\perp}{\lambda^2 - V^2/4} - \frac{V}{4}\left(1 + \frac{p_\perp^2 - m^2}{\lambda^2 - V^2/4}\right)\mathbf{e}_z. \tag{3.9}$$

Here $\mathbf{e}_z$ is the unit vector along the wave propagation.

Finally, using Eqs. (3.4) and (3.7), the general solution of the wave Eq. (3.1) has the form,

$$\psi = \exp\left(-\mathrm{i}\frac{V}{2}t + \mathrm{i}\mathbf{p}_\perp\mathbf{x}_\perp - \mathrm{i}\frac{\lambda}{2}u_3 - \mathrm{i}\frac{\lambda}{2}\frac{p_\perp^2 + m^2}{\lambda^2 - V^2/4}u_0\right)u, \tag{3.10}$$

where

$$u = \frac{1}{\sqrt{N}}\begin{pmatrix} \frac{(p_x - \mathrm{i}p_y)v_1 - mv_2}{\lambda + V/2} \\ v_1 \\ v_2 \\ -\frac{(p_x + \mathrm{i}p_y)v_2 + mv_1}{\lambda - V/2} \end{pmatrix}, \tag{3.11}$$

is the basis spinor. The normalization coefficient N is given by the condition $|u|^2 = 1$.

IV. NEUTRINO SPIN OSCILLATIONS

Now we use the general solution of the Dirac-Pauli equation, found in Sec. III, to describe neutrino spin oscillations in a plane wave with the circular polarization. We specify the initial condition and find the transition probability. The obtained results are compared with previous findings of other authors.

Before we proceed, it is convenient to replace the total wave function ψ in Eq. (3.10) by its projection to the subspace of the linearly independent components $\psi_{2,3}$,

$$\psi \to \tilde{\psi} = \frac{1}{2}(1 - \alpha_z)\psi. \tag{4.1}$$

The basis spinor $\tilde{u}$, which is the analogue of u in Eq. (3.11), takes the form, $\tilde{u}^{\mathrm{T}} = (0, v_1, v_2, 0)$. This spinor is automatically normalized to one if $|v_1|^2 + |v_2|^2 = 1$. The last condition results from the unitary dynamics of v implied by Eq. (3.8). The mean value of an operator $\hat{O}$ can be found as $\langle\hat{O}\rangle = \tilde{\psi}^\dagger\hat{O}\tilde{\psi}$.

In order not to encumber the presentation, we discuss the situation of a neutrino propagating along an electromagnetic wave, i.e., $\mathbf{p}_\perp = 0$. This case is implemented if neutrinos and an electromagnetic wave are emitted by the same source. Then we consider a wave with the circular polarization, i.e., $B_x = B_0\cos[g\omega(t-z)]$ and $B_y = B_0\sin[g\omega(t-z)]$, where B_0 is the amplitude of the wave, ω is its frequency, and $g = \pm 1$ is the sign factor corresponding to right or left polarizations.

The solution of Eq. (3.8) for a circularly polarized wave has the form,

$$v = \mathcal{U}_z[\cos(\Omega u_0) - \mathrm{i}(\boldsymbol{\sigma}\mathbf{n})\sin(\Omega u_0)]v_0, \qquad (4.2)$$

where

$$\mathcal{U}_z = \exp(\mathrm{i}\sigma_3 g\omega u_0/2), \qquad \Omega = \sqrt{(R_z + g\omega/2)^2 + (\mu B_0)^2}, \qquad (4.3)$$

and

$$\mathbf{n} = \frac{1}{\Omega}(\mu B_0, 0, R_z + g\omega/2), \qquad (4.4)$$

is the unit vector, R_z is the z-component of the vector $\mathbf{R}$ in Eq. (3.9), and v_0 is the initial spinor corresponding to $u_0 = 0$.

We suppose that, at $u_0 = 0$, only left polarized neutrinos are presented in the space-time region outside the wave propagation. Thus, we impose the condition $\Sigma_z \tilde{u} = -\tilde{u}$ on the basis spinor $\tilde{u}$ at $u_0 = 0$. The components of the spinor v_0 are $v_1(0) = 1$ and $v_2(0) = 0$.

We are interested in the appearance of right polarized particles after neutrinos interact with external fields. It is the situation, which is implemented in neutrino spin oscillations: one looks for right polarized neutrinos in a beam initially consisting of left particles of the same type. Using Eq. (4.2), one obtains that the probability for $L \to R$ transitions has the form,

$$P_{\mathrm{L}\to\mathrm{R}} = \frac{1}{2}\tilde{\psi}^\dagger(1 + \Sigma_z)\tilde{\psi} = |v_2|^2 = \frac{\mu^2 B_0^2}{\Omega^2}\sin^2\Omega u_0. \qquad (4.5)$$

In a general situation, $P_{\mathrm{L}\to\mathrm{R}}$ in Eq. (4.5) depends on $u_0 = t - z$. We also note that this expression contains the dependence on the quantum number λ which does not have a clear physical meaning yet. Hence, one should express λ in terms of the neutrino energy and momentum.

The Hamiltonian of Eq. (3.2) explicitly depends on t and z. Thus the neutrino energy E and the momentum p_z along the wave propagation direction are not defined. Nevertheless we can define the effective E and p_z as

$$E - p_z = 2\mathrm{i}\tilde{\psi}^\dagger\partial_3\tilde{\psi} = \lambda + \frac{V}{2}, \qquad (4.6)$$

$$E + p_z = 2\mathrm{i}\tilde{\psi}^\dagger\partial_0\tilde{\psi} = \lambda\frac{m^2}{\lambda^2 - V^2/4} + \frac{V}{2} + 2v^\dagger(\boldsymbol{\sigma}^*\mathbf{R})v. \qquad (4.7)$$

Using Eq. (4.2), one gets that

$$v^\dagger(\boldsymbol{\sigma}^*\mathbf{R})v = \pm\left(R_z + \frac{\rho}{2}\right),$$
$$\rho(\omega, u_0) = 2g\omega\frac{\mu^2 B_0^2}{\Omega^2}\sin^2\Omega u_0, \qquad (4.8)$$

where the signs $\pm$ stay for initially left and right polarized neutrinos. A right polarized neutrino corresponds to $v_0^\mathrm{T} = (0, 1)$ in Eq. (4.2).

Finally, using Eqs. (4.6)–(4.8), we have the energy of left neutrinos as

$$E_\mathrm{L} = \frac{V + \rho}{2} + \sqrt{m^2 + \left(p_z + \frac{V - \rho}{2}\right)^2} \approx p_z + \frac{m^2}{2p_z} + V, \qquad (4.9)$$

and

$$E_\mathrm{R} = \frac{V - \rho}{2} + \sqrt{m^2 + \left(p_z - \frac{V - \rho}{2}\right)^2} \approx p_z + \frac{m^2}{2p_z}, \qquad (4.10)$$

for right particles. The expansions in Eqs. (4.9) and (4.10) correspond to ultrarelativistic neutrinos with $p_z \gg m$. One can see that, in this case, the energies have the conventional form. However, for nonrelativistic neutrinos, the effective energies become time and z dependent.

The transition probability depends on the quantity R_z given in Eq. (3.9). Using Eqs. (4.6) and (4.9), one gets that R_z has the following form for left neutrinos:

$$R_z = -\frac{V}{4}\left(1 - \frac{m^2}{\lambda^2 - V^2/4}\right) \approx \frac{V}{2}\left(\frac{V}{p_z} + \frac{m^2}{2p_z^2}\right)^{-1}. \qquad (4.11)$$

Basing on Eqs. (4.5) and (4.11), the transition probability can be rewritten as

$$P_{\mathrm{L}\to\mathrm{R}} = \frac{\mu^2 B_0^2}{\Omega^2}\sin^2[\Omega(t - z)],$$
$$\Omega \approx \sqrt{\mu^2 B_0^2 + \left[\frac{V}{2}\left(\frac{V}{p_z} + \frac{m^2}{2p_z^2}\right)^{-1} + \frac{g\omega}{2}\right]^2}, \qquad (4.12)$$

where we explicitly show the dependence on t and z.

Now let us consider the quasiclassical approximation. In this limit, a neutrino moves along a trajectory, which is a straight line $z = \beta t$, where $\beta = p_z/E$ is the neutrino velocity. We can represent the transition probability in Eq. (4.12) in the following way:

$$P_{\mathrm{L}\to\mathrm{R}}(t) = \frac{\mu^2 B_0^2 (1-\beta)^2}{\mu^2 B_0^2 (1-\beta)^2 + [\frac{V}{2} + \frac{g\omega}{2}(1-\beta)]^2} \times \sin^2\left(\sqrt{\mu^2 B_0^2 (1-\beta)^2 + \left[\frac{V}{2} + \frac{g\omega}{2}(1-\beta)\right]^2} t\right), \tag{4.13}$$

since

$$\frac{V}{2}\left(\frac{V}{p_z} + \frac{m^2}{2p_z^2}\right)^{-1}(1-\beta) \approx \frac{V}{2}. \tag{4.14}$$

The expression for $P_{\mathrm{L}\to\mathrm{R}}$ in Eq. (4.13) coincides with the result of Ref. [11], where the neutrino spin evolution in matter under the influence of a plane electromagnetic wave was treated within the quasiclassical approach from the very beginning.

V. SPIN-FLAVOR OSCILLATIONS: GREAT DIAGONAL MAGNETIC MOMENTS

Now we turn to the study of neutrino spin-flavor oscillations. Here we are interested in the situation of great diagonal magnetic moments. Basing on the results of Sec. IV, we derive the analytical transition probability for this type of oscillations.

Using Eq. (2.4), we obtain the system of coupled Dirac equations for the neutrino mass eigenstates ψ_a, $a = 1, 2$,

$$\begin{aligned} \mathrm{i}\partial_t \psi_a &= \mathcal{H}_a \psi_a + \mathcal{V}\psi_b, \qquad a \neq b, \\ \mathcal{H}_a &= -\mathrm{i}(\boldsymbol{\alpha}\nabla) + \beta m_a + \mu_a(\mathrm{i}\boldsymbol{\gamma}\mathbf{E} - \beta\boldsymbol{\Sigma}\mathbf{B}) + \frac{V_a}{2}(1-\gamma^5), \\ \mathcal{V} &= \mu(\mathrm{i}\boldsymbol{\gamma}\mathbf{E} - \beta\boldsymbol{\Sigma}\mathbf{B}) + \frac{V}{2}(1-\gamma^5), \end{aligned} \tag{5.1}$$

where $V_a \equiv V_{aa}$ and $\mu_a \equiv \mu_{aa}$ for $a = 1, 2$, $V \equiv V_{12}$, and $\mu \equiv \mu_{12}$ is the transition magnetic moment.

We shall analyze the system in Eq. (5.1) in the approximation when $\mu_a \gg \mu$. There are multiple models of the neutrino magnetic moments generation. Some of the models predict the diagonal magnetic moments μ_a proportional to the neutrino masses m_a. In these cases, the value of μ is suppressed because of the Glashow-Iliopoulos-Maiani (GIM) mechanism [6].

Moreover, we suppose that $|V_a| \gg V$. If we study $\nu_e \to \nu_{\mu,\tau}$ oscillations, then, using Eqs. (2.2) and (2.3), we get that

$$\begin{aligned} V_1 &= \sqrt{2}G_{\mathrm{F}}\left(n_e \sin^2\theta - \frac{1}{2}n_n\right), \\ V_2 &= \sqrt{2}G_{\mathrm{F}}\left(n_e \cos^2\theta - \frac{1}{2}n_n\right), \\ V &= \frac{G_{\mathrm{F}}}{\sqrt{2}} n_e \sin 2\theta. \end{aligned} \tag{5.2}$$

Basing on Eq. (5.2), one gets that the condition $|V_a| \gg V$ is satisfied if either $n_n \gg n_e$ or $\theta \ll 1$. The former case is implemented in a neutron rich environment like a neutron star. The latter situation takes place if we study $\nu_e \to \nu_\tau$ oscillations since, as found in Ref. [18], $\theta \equiv \theta_{13} = 0.15$ is much less than both $\theta_\odot = 0.6$ [19] and $\theta_{\mathrm{ATM}} = (0.75 \div 0.85)$ [20].

We are interested in spin-flavor oscillations of the type $\nu_{\beta\mathrm{L}} \to \nu_{\alpha\mathrm{R}}$, i.e., when both flavor and the polarization are changed. If the above approximations are satisfied, we can derive the analytical expression for the transition probability for $\nu_{\beta\mathrm{L}} \to \nu_{\alpha\mathrm{R}}$ oscillations. Indeed, if we neglect $\mathcal{V}$ in Eq. (5.1), the neutrino spin evolves independently within each mass eigenstate, as described in Sec. IV. The transitions between different neutrino flavors are solely owing to the vacuum neutrino mixing. As in Sec. IV, here we consider a neutrino beam propagating along the electromagnetic wave.

To describe the evolution of neutrinos, we use the approach developed in Ref. [21], where the initial condition problem is solved. The initial conditions corresponding to $\nu_{\beta\mathrm{L}} \to \nu_{\alpha\mathrm{R}}$ are the following. Since there are no right polarized neutrinos initially, we choose $\nu_{\alpha\mathrm{R}}(z,0) = \nu_{\beta\mathrm{R}}(z,0) = 0$. The wave functions of left polarized neutrinos should be chosen like $\nu_{\alpha\mathrm{L}}(z,0) = 0$ and $\nu_{\beta\mathrm{L}}(z,0) \sim \exp(\mathrm{i}p_z z)$. Such a choice of the initial condition for $\nu_{\beta\mathrm{L}}(z,0)$ corresponds to a broad wave packet. The arbitrary initial wave packets are discussed in Ref. [13]. Here the spin projections are defined using the operators $(1 \pm \Sigma_z)/2$.

The projected wave functions of mass eigenstates, given in Eq. (4.1), which satisfy the system in Eq. (5.1), have the form,

$$\begin{aligned} \tilde{\psi}_a(z,t) &= \sum_{s=\mathrm{L,R}} \exp(-\mathrm{i}E_{as}t + \mathrm{i}p_z z) a_{as} \tilde{u}_{as}, \\ \tilde{u}_{as}^{\mathrm{T}} &= (0, v_{s1}^{(a)}, v_{s2}^{(a)}, 0), \end{aligned} \tag{5.3}$$

where the index $s = \mathrm{L,R}$ corresponds to initially left or right polarized neutrinos and the energies E_{as} are given by Eqs. (4.9) and (4.10) with the replacements $m \to m_a$ and $V \to V_a$. Since we neglect $\mathcal{V}$ in Eq. (5.1), the coefficients a_{as} are constant and entirely fixed by the initial condition. Using Eq. (2.3), we get that $a_{1\mathrm{L}} = \sin\theta$ and $a_{2\mathrm{L}} = \cos\theta$. Moreover $a_{1,2\mathrm{R}} = 0$ since there are no right polarized particles initially.

To describe the evolution of the spinors $v_s^{(a)}$ in Eq. (5.3), we use the quasiclassical approximation from the very beginning, i.e., we suppose that $z = \bar{\beta}t$, where $\bar{\beta} = 2p_z/(E_{1\mathrm{L}} + E_{2\mathrm{L}})$ is the center of inertia velocity. Using Eq. (4.2), we get that the components of $v_{a\mathrm{L}}$ evolve as

$$\begin{aligned} v_{\mathrm{L}1}^{(a)}(t) &= \exp[\mathrm{i}g\omega(1-\bar{\beta})t/2]\{\cos[\Omega_a(1-\bar{\beta})t] \\ &\quad - \mathrm{i}n_z^{(a)} \sin[\Omega_a(1-\bar{\beta})t]\}, \\ v_{\mathrm{L}2}^{(a)}(t) &= -\mathrm{i}n_x^{(a)} \exp[-\mathrm{i}g\omega(1-\bar{\beta})t/2] \\ &\quad \times \sin[\Omega_a(1-\bar{\beta})t], \end{aligned} \tag{5.4}$$

where the quantities Ω_a and $n_{x,z}^{(a)}$ are the natural generalizations of the corresponding parameters given in Eqs. (4.3) and (4.4) with the replacements $m \to m_a$, $\mu \to \mu_a$, and $V \to V_a$ there. The evolution of $v_{\mathrm{R}}^{(a)}$ is not important since there are not right polarized neutrinos initially.

Basing on Eqs. (2.3), (5.3), and (5.4), we derive the right polarized wave function $\nu_{\alpha\mathrm{R}}(z,t) = (1+\Sigma_z)[\cos\theta\psi_1(z,t) - \sin\theta\psi_2(z,t)]/2$ in the following form:

$$\nu_{\alpha\mathrm{R}}^{\mathrm{T}}(z,t) = \sin\theta\cos\theta\exp(\mathrm{i}p_z z)(0,0,\exp(-\mathrm{i}E_{1\mathrm{L}}t)v_{\mathrm{L}2}^{(1)}(t) - \exp(-\mathrm{i}E_{2\mathrm{L}}t)v_{\mathrm{L}2}^{(2)}(t),0). \tag{5.5}$$

Now, using Eq. (5.5), one obtains the probability for transitions $\nu_{\beta\mathrm{L}} \to \nu_{\alpha\mathrm{R}}$ in the form,

$$P_{\beta\mathrm{L}\to\alpha\mathrm{R}}(t) = |\nu_{\alpha\mathrm{R}}(z,t)|^2 = \sin^2(2\theta)\left[\frac{1}{4}(A_1 - A_2)^2 + A_1 A_2 \sin^2(\Phi t)\right], \tag{5.6}$$

where

$$A_a(t) = \frac{\mu_a B_0(1-\bar{\beta})}{\sqrt{\mu_a^2 B_0^2(1-\bar{\beta})^2 + [\frac{V_a}{2} + \frac{g\omega}{2}(1-\bar{\beta})]^2}} \times \sin\left(\sqrt{\mu_a^2 B_0^2(1-\bar{\beta})^2 + \left[\frac{V_a}{2} + \frac{g\omega}{2}(1-\bar{\beta})\right]^2}\, t\right), \tag{5.7}$$

is the amplitude of spin oscillations within one mass eigenstate, $\Phi = \Phi_{\mathrm{vac}} + (V_1 - V_2)/2$ is the phase of neutrino flavor oscillations accounting for the matter contribution, $\Phi_{\mathrm{vac}} = \delta m^2/4p_z$ is the phase of neutrino oscillations in vacuum, and $\delta m^2 = m_1^2 - m_2^2$. To derive Eq. (5.7) we use the analogues of Eqs. (4.11) and (4.14).

One can see that Eqs. (5.6) and (5.7) are the generalization of the corresponding expressions obtained in Ref. [13] for the situation when neutrinos interact not only with a plane electromagnetic wave but also with the background matter.

VI. SPIN-FLAVOR OSCILLATIONS: GREAT TRANSITION MAGNETIC MOMENT

In this section, we continue to study spin-flavor oscillations. However, unlike the case considered in Sec. V, we discuss the situation of the great transition magnetic moment.

If $\mu \gg \mu_a$, we cannot neglect $\mathcal{V}$ in Eq. (5.1). It means that a_{as} in Eq. (5.3) is no longer constant. Analogously to Ref. [13] we suppose that $a_{as} = a_{as}(t-z)$. Our main goal is to find the behavior of a_{as}. Moreover, in the analogue of Eq. (5.3), we shall use the total wave function ψ_{as} rather than the projection $\tilde{\psi}_{as}$. Hence we look for the solution of Eq. (5.1) in the form,

$$\psi_a = \sum_{s=\mathrm{L,R}} \exp(-\mathrm{i}E_{as}t + \mathrm{i}p_z z) a_{as}(t-z) u_{as}. \tag{6.1}$$

We consider neutrinos propagating along the wave in Eq. (6.1). Since a_{as} is time dependent for both $s = \mathrm{L}$ and $s = \mathrm{R}$, we should account for the time evolution of the basis spinors $u_{a\mathrm{L,R}}$. For this purpose we choose two linearly independent initial spinors v_0 for $a = 1, 2$, $v_{0\mathrm{L}} = (1,0)^{\mathrm{T}}$ and $v_{0\mathrm{R}} = (0,1)^{\mathrm{T}}$, which contribute to Eq. (4.2).

Substituting Eq. (6.1) to Eq. (5.1) and taking into account that $\sim\exp(-\mathrm{i}E_{as}t + \mathrm{i}p_z z)u_{as}$ is the solution of the diagonal part of the system in Eq. (5.1), i.e., without $\mathcal{V}$, one gets the equation for the coefficients a_{as} in the form,

$$\mathrm{i}\frac{1}{2}\sum_{s=\mathrm{L,R}} u_{as'}^{\dagger}(1-\alpha_z)u_{as}\partial_0 a_{as} = \frac{1}{2}\sum_{s=\mathrm{L,R}} u_{as'}^{\dagger}\mathcal{V}u_{bs}a_{bs}\exp[\mathrm{i}(E_{as'} - E_{bs})t]. \tag{6.2}$$

Using Eq. (3.11), we obtain the following mean values:

$$\begin{aligned} \frac{1}{2}u_{as'}^{\dagger}(1-\alpha_z)u_{as} &= v_{as'}^{\dagger}v_{as} = v_{0s'}^{\dagger}v_{0s} = \delta_{ss'}, \\ \frac{\mu}{2}u_{as'}^{\dagger}(\mathrm{i}\boldsymbol{\gamma}\mathbf{E} - \beta\boldsymbol{\Sigma}\mathbf{B})u_{bs} &= \mu v_{as'}^{\dagger}(\boldsymbol{\sigma}^*\mathbf{B})v_{bs}, \\ \frac{V}{4}u_{as'}^{\dagger}(1-\gamma^5)u_{bs} &= \frac{V}{2}\left(v_{as'2}^* v_{bs2} + \frac{m_a}{\lambda_a - V_a/2}\frac{m_b}{\lambda_b - V_b/2}v_{as'1}^* v_{bs1}\right). \end{aligned} \tag{6.3}$$

Then we adopt the quasiclassical approximation, in which $\partial_0 = (1-\bar{\beta})^{-1}\partial_t$, where $\bar{\beta}$ is the mean velocity of the neutrino wave packet, defined in Sec. V.

In this section, we consider the situation when $\mu_a \ll \mu$. It means that the components of the vector $\mathbf{n}_a$, which defines the neutrino spin evolution, have the following values: $n_x^{(a)} = 0$ and $n_z^{(a)} = 1$. We can use Eq. (4.2) to compute the mean values of the spinors v_{as} in Eq. (6.3) assuming that the electromagnetic wave has the circular polarization. Then we define the effective wave function $\Psi^{\mathrm{T}} = (a_{1\mathrm{R}}, a_{1\mathrm{L}}, a_{2\mathrm{R}}, a_{2\mathrm{L}})$, which obeys the Schrödinger equation,

$$
\mathrm{i}\frac{\mathrm{d}\Psi}{\mathrm{d}t} = H\Psi,
$$

$$
H = \begin{pmatrix} 0 & 0 & 0 & \mu B_0(1-\bar{\beta})e^{\mathrm{i}\phi_2 t} \\ 0 & 0 & \mu B_0(1-\bar{\beta})e^{\mathrm{i}\phi_1 t} & Ve^{\mathrm{i}\phi' t} \\ 0 & \mu B_0(1-\bar{\beta})e^{-\mathrm{i}\phi_1 t} & 0 & 0 \\ \mu B_0(1-\bar{\beta})e^{-\mathrm{i}\phi_2 t} & Ve^{-\mathrm{i}\phi' t} & 0 & 0 \end{pmatrix}. \tag{6.4}
$$

Here

$$
\phi_2 \approx \frac{\delta m^2}{2p_z} - V_2 - g\omega(1-\bar{\beta}),
$$
$$
\phi_1 \approx \frac{\delta m^2}{2p_z} + V_1 + g\omega(1-\bar{\beta}),
$$
$$
\phi' \approx \frac{\delta m^2}{2p_z} + V_1 - V_2, \tag{6.5}
$$

where we take that neutrinos are ultrarelativistic particles.

Let us introduce the new wave function $\tilde{\Psi}^{\mathrm{T}} = (\tilde{a}_{1\mathrm{R}}, \tilde{a}_{1\mathrm{L}}, \tilde{a}_{2\mathrm{R}}, \tilde{a}_{2\mathrm{L}})$ as $\Psi = \mathcal{U}\tilde{\Psi}$, where

$$
\mathcal{U} = \mathrm{diag}\left\{\exp\left[\mathrm{i}\left(\Phi_- - \frac{V_1+V_2}{4}\right)t\right], \exp\left[\mathrm{i}\left(\Phi_+ + \frac{3V_1-V_2}{4}\right)t\right], \exp\left[-\mathrm{i}\left(\Phi_+ + \frac{V_1+V_2}{4}\right)t\right], \exp\left[-\mathrm{i}\left(\Phi_- - \frac{3V_2-V_1}{4}\right)t\right]\right\}. \tag{6.6}
$$

Here $\Phi_\pm = \Phi_{\mathrm{vac}} \pm (1-\bar{\beta})g\omega/2$. The wave function $\tilde{\Psi}$ obeys the equation

$$
\mathrm{i}\frac{\mathrm{d}\tilde{\Psi}}{\mathrm{d}t} = \tilde{H}\tilde{\Psi},
$$

$$
\tilde{H} = \mathcal{U}^\dagger H \mathcal{U} - i\mathcal{U}^\dagger\dot{\mathcal{U}} = \begin{pmatrix} \Phi_- - \frac{V_1+V_2}{4} & 0 & 0 & \mu B_0(1-\bar{\beta}) \\ 0 & \Phi_+ + \frac{3V_1-V_2}{4} & \mu B_0(1-\bar{\beta}) & V \\ 0 & \mu B_0(1-\bar{\beta}) & -\Phi_+ - \frac{V_1+V_2}{4} & 0 \\ \mu B_0(1-\bar{\beta}) & V & 0 & -\Phi_- + \frac{3V_2-V_1}{4} \end{pmatrix}. \tag{6.7}
$$

One can see that the effective Hamiltonian $\tilde{H}$ in Eq. (6.7) generalizes the analogous effective Hamiltonian derived in Ref. [13] for the nonzero interaction of neutrinos with background matter. Moreover, if set $(1-\bar{\beta}) \to 1$ and $\omega \to 0$ in Eq. (6.7), we reproduce the effective Hamiltonian for neutrino spin-flavor oscillations in matter under the influence of a transverse magnetic field derived in Ref. [22] using the relativistic quantum mechanics approach.

The solution of the Schrödinger equation in Eq. (6.7) results in the algebraic characteristic equation of the forth order, which implies quite cumbersome expressions for eigenvalues and eigenvectors. That is why we again suppose that $V \ll |V_a|$, as in Sec. V, to proceed with the analytical solution. The validity of this approximation will be discussed below.

In this case, the evolution of $\tilde{\Psi}$ can be represented in the form,

$$
\tilde{\Psi}(t) = \sum_{\substack{a=1,2 \\ \zeta=\pm}} \exp(-\mathrm{i}\mathcal{E}_a^{(\zeta)}t)(U_a^{(\zeta)} \otimes U_a^{(\zeta)\dagger})\tilde{\Psi}(0), \tag{6.8}
$$

where

$$
\mathcal{E}_{1,2}^{(\zeta)} = \pm\frac{V_1 - V_2}{4} + \zeta\Omega_{1,2},
$$
$$
\Omega_{1,2} = \sqrt{(\mu B_0)^2(1-\bar{\beta})^2 + \left(\Phi_\pm \pm \frac{V_{1,2}}{2}\right)^2}, \tag{6.9}
$$

are the eigenvalues of the Hamiltonian $\tilde{H}$ in Eq. (6.7) and

$$U_1^+ = \sqrt{\frac{\Omega_1 + \Phi_+ + V_1/2}{2\Omega_1}} \begin{pmatrix} 0 \\ 1 \\ \frac{\mu B_0 (1-\bar{\beta})}{\Omega_1 + \Phi_+ + V_1/2} \\ 0 \end{pmatrix},$$

$$U_1^- = \sqrt{\frac{\Omega_1 + \Phi_+ + V_1/2}{2\Omega_1}} \begin{pmatrix} 0 \\ -\frac{\mu B_0 (1-\bar{\beta})}{\Omega_1 + \Phi_+ + V_1/2} \\ 1 \\ 0 \end{pmatrix},$$

$$U_2^+ = \sqrt{\frac{\Omega_2 + \Phi_- - V_2/2}{2\Omega_2}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ \frac{\mu B_0 (1-\bar{\beta})}{\Omega_2 + \Phi_- - V_2/2} \end{pmatrix},$$

$$U_2^- = \sqrt{\frac{\Omega_2 + \Phi_- - V_2/2}{2\Omega_2}} \begin{pmatrix} -\frac{\mu B_0 (1-\bar{\beta})}{\mathcal{E}_2 + \Phi_- - V_2/2} \\ 0 \\ 0 \\ 1 \end{pmatrix}, \tag{6.10}$$

are the eigenvectors.

Equation (6.8) should be supplied with the initial condition of the form,

$$\tilde{\Psi}^{\mathrm{T}}(0) = (0, \sin\theta, 0, \cos\theta), \tag{6.11}$$

which means that there are only neutrinos of the type $\nu_{\beta\mathrm{L}}$ initially. Using Eqs. (6.8)–(6.11), one gets that the coefficients $a_{1,2\mathrm{R}}$ are expressed in the following way:

$$a_{1\mathrm{R}}(t) = -\mathrm{i}\frac{\mu B_0}{\Omega_2}(1-\bar{\beta}) \exp\left[\mathrm{i}\left(\Phi_- - \frac{V_2}{2}\right)t\right] \cos\theta \sin\Omega_2 t,$$
$$a_{2\mathrm{R}}(t) = -\mathrm{i}\frac{\mu B_0}{\Omega_1}(1-\bar{\beta}) \exp\left[-\mathrm{i}\left(\Phi_+ + \frac{V_1}{2}\right)t\right] \sin\theta \sin\Omega_1 t. \tag{6.12}$$

The values of $a_{1,2\mathrm{L}}$ are not important for our purposes since we are interested in spin-flavor oscillations when both flavor and helicity change.

Basing on Eqs. (2.3), (6.1), and (6.12), the neutrino wave function $\nu_{\alpha\mathrm{R}}(z,t) = \cos\theta\psi_{1\mathrm{R}}(z,t) - \sin\theta\psi_{2\mathrm{R}}(z,t)$ reads

$$\begin{aligned} \nu_{\alpha\mathrm{R}}(z,t) &= \exp(\mathrm{i}p_z z)[\cos\theta \exp(-\mathrm{i}E_{1\mathrm{R}}t) a_{1\mathrm{R}}(t) - \sin\theta \exp(-\mathrm{i}E_{2\mathrm{R}}t) a_{2\mathrm{R}}(t)]\nu_\mathrm{R} \\ &= -\mathrm{i}\exp\left(-\mathrm{i}\left[p_z + \frac{m_1^2 + m_2^2}{4p_z} + \frac{g\omega}{2}(1-\bar{\beta})\right] + \mathrm{i}p_z z\right)\mu B_0 (1-\bar{\beta}) \\ &\quad \times \left[\cos^2\theta \exp\left(-\mathrm{i}\frac{V_2}{2}t\right)\frac{\sin\Omega_2 t}{\Omega_2} - \sin^2\theta \exp\left(-\mathrm{i}\frac{V_1}{2}t\right)\frac{\sin\Omega_1 t}{\Omega_1}\right]\nu_\mathrm{R}, \end{aligned} \tag{6.13}$$

where ν_R is the constant bispinor satisfying $|\nu_\mathrm{R}|^2 = 1$ and $\Sigma_z \nu_\mathrm{R} = \nu_\mathrm{R}$.

The probability for transitions $\nu_{\beta\mathrm{L}} \to \nu_{\alpha\mathrm{R}}$ is derived using Eq. (6.13) as

$$\begin{aligned} P_{\beta\mathrm{L}\to\alpha\mathrm{R}}(t) &= |\nu_{\alpha\mathrm{R}}(z,t)|^2 \\ &= \left\{[\mathcal{A}_2 - \mathcal{A}_1]^2 + 4\mathcal{A}_1\mathcal{A}_2 \sin^2\left(\frac{V_1 - V_2}{4}t\right)\right\}, \end{aligned} \tag{6.14}$$

where

$$\mathcal{A}_1 = \mu B_0 (1-\bar{\beta}) \sin^2\theta \frac{\sin\Omega_1 t}{\Omega_1},$$
$$\mathcal{A}_2 = \mu B_0 (1-\bar{\beta}) \cos^2\theta \frac{\sin\Omega_2 t}{\Omega_2}, \tag{6.15}$$

are the amplitudes of the transitions $\psi_{(1,\mathrm{L}),(2,\mathrm{R})} \leftrightarrow \psi_{(2,\mathrm{R}),(1,\mathrm{L})}$ in matter under the influence of an electromagnetic wave. The analogue of $\mathcal{A}_{1,2}$ for the constant transverse magnetic fields was introduced in Ref. [23].

The behavior of the transition probability in Eq. (6.14) is shown in Fig. 1(a) for $\nu_{e\mathrm{L}} \to \nu_{\tau\mathrm{R}}$ oscillations channel versus the distance $z \approx t$ passed by the neutrino beam. We suppose that the electromagnetic wave has the following characteristics: $B_0 = 10^{18}$ G and $\omega = 10^{13}$ s^{-1}. The neutrino energy and the transition magnetic moment are taken to be $E_\nu \equiv p_z = 1$ keV and $\mu = 10^{-11}\mu_\mathrm{B}$, where μ_B is the Bohr magneton. As mentioned in Ref. [13], these parameters can model neutrino spin-flavor oscillations in the vicinity of a highly magnetized pulsar. To estimate the mean velocity of neutrinos $\bar{\beta}$ we assume that the neutrino masses are on the level of 1 eV [24].

The motivation for the choice of the matter density value in Fig. 1 is the following. We can consider neutrino spin-flavor oscillations in the vicinity of a compact astrophysical object surrounded by an accretion disk. For example, properties of a gamma-ray burst (GRB) can be explained by the matter accretion to a central object. In this model of GRB, the matter density of a hydrogen plasma in the inner part of an accretion disk can reach 10^{26} cm^{-3} [27] or be even higher [28]. Such values of n_e are close to these used

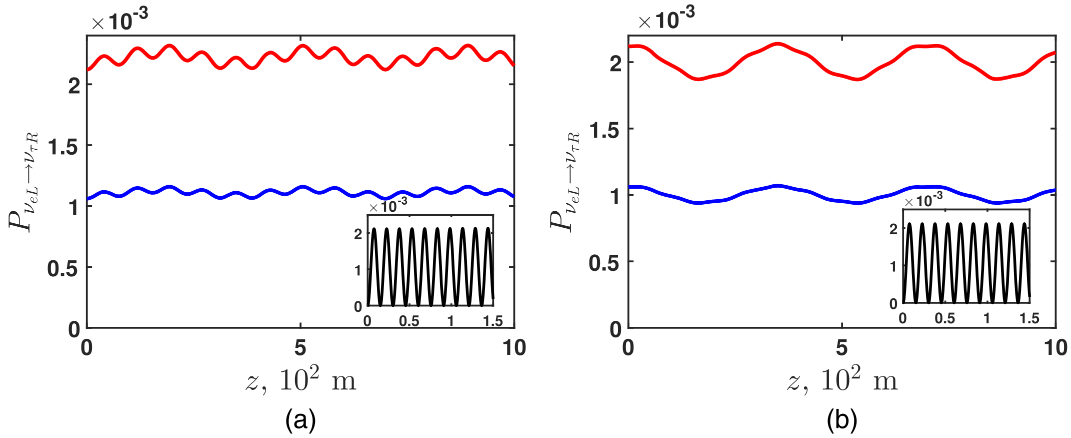

FIG. 1. The transition probabilities for $\nu_{eL} \to \nu_{\tau R}$ oscillations in the electroneutral hydrogen plasma with $n_e = 10^{29}$ cm^{-3} under the influence of the electromagnetic wave with $B_0 = 10^{18}$ G and $\omega = 10^{13}$ s^{-1} versus the distance $z = \bar{\beta} t$ traveled by the neutrino beam. The parameters of neutrinos are $\delta m^2 = 2.5 \times 10^{-3}$ eV^2 [25], $\theta = 0.15$ [18], $p_z = 1$ keV, and $\mu = 10^{-11} \mu_B$ [26]. (a) The approximate transition probability in Eq. (6.14) corresponding to the case $V = 0$ in $\tilde{H}$ in Eq. (6.7). (b) The transition probability in Eq. (6.16) based on the numerical solution of Eq. (6.7) with $V \neq 0$. Red and blue lines are the upper envelope function and the averaged transition probability. The insets in panels (a) and (b) show $P_{\nu_{eL}\to\nu_{\tau R}}(z)$ at $0 < z < 150$ m.

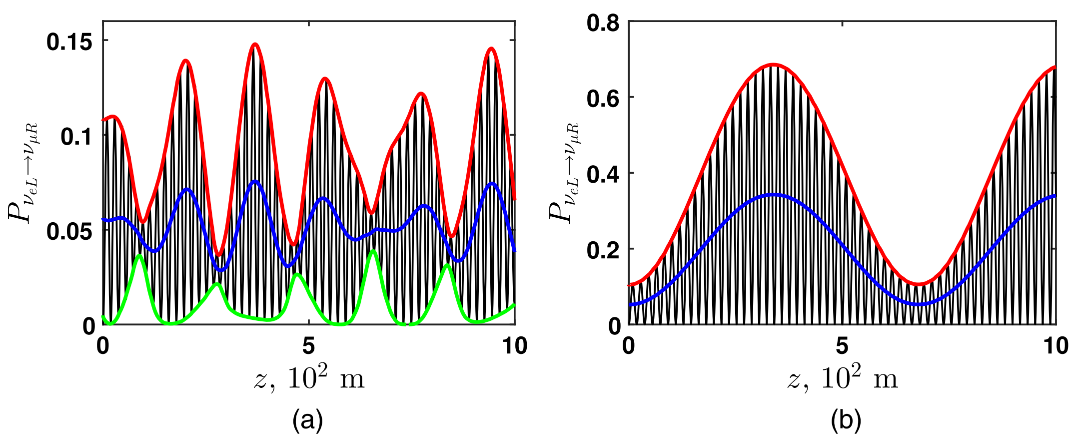

FIG. 2. The transition probabilities for $\nu_{eL} \to \nu_{\mu R}$ oscillations in the electroneutral hydrogen plasma when particles interact with the electromagnetic wave having $B_0 = 10^{18}$ G and $\omega = 10^{13}$ s^{-1} versus the distance $z = \bar{\beta} t$ passed by the neutrino beam. The parameters of neutrinos are $\delta m^2 = 7.59 \times 10^{-5}$ eV^2 [29], $\theta = 0.6$ [19], $p_z = 1$ keV, and $\mu = 10^{-11} \mu_B$. These transition probabilities correspond to Eq. (6.16), which is based on the numerical solution of Eq. (6.7) with $V \neq 0$. (a) $n_e = 10^{29}$ cm^{-3}; and (b) $n_e = 10^{27}$ cm^{-3}. Red and blue lines are the upper envelope functions and the averaged transition probabilities. The green line in panel (a) is the lower envelope function.

in our simulations (especially see Fig. 2 below). Note that this model of GRB predicts a high neutrino emissivity by an accretion disk [27,28].

The function $P_{\nu_{eL}\to\nu_{\tau R}}(z)$ is a rapidly oscillating one. It is the typical feature of a neutrino system which experiences spin-flavor oscillations in matter and an electromagnetic field with different oscillations frequencies induced by matter and an electromagnetic field; cf. Refs. [13,21]. That is why, here, we show only the upper envelope function and the averaged transition probability. The upper envelope function is built using the spline interpolation of the maxima of $P_{\nu_{eL}\to\nu_{\tau R}}(z)$. The transition probability $P_{\nu_{eL}\to\nu_{\tau R}}(z)$ is shown only in the inset in Fig. 1(a) for small z.

One can see in Fig. 1(a) that the transition probability for the considered oscillations channel reaches only a tiny value $\sim 10^{-3}$. This fact can be explained by the great value of Φ_{vac} for $\nu_{eL} \to \nu_{\tau R}$ oscillations, which is about 2 orders of magnitude greater than other entries in $\tilde{H}$ in Eq. (6.7). Hence $\Omega_{1,2} \gg \mu B_0 (1 - \bar{\beta})$ and $\mathcal{A}_{1,2} \ll 1$ in Eq. (6.15).

Now we compare the exact solution, given in Eqs. (6.14) and (6.15), of the approximate effective Schrödinger

Eq. (6.7), where we put $V = 0$, with the numerical solution of the exact Eq. (6.7). Should one have the solution $\tilde{\Psi}^{\mathrm{T}}(t) = (\tilde{\Psi}_1, \tilde{\Psi}_2, \tilde{\Psi}_3, \tilde{\Psi}_4)$ of Eq. (6.7), supplied with the initial condition in Eq. (6.11), the transition probability for $\nu_{\beta\mathrm{L}} \to \nu_{\alpha\mathrm{R}}$ oscillations can be found as

$$P_{\beta\mathrm{L}\to\alpha\mathrm{R}}(t) = |\cos\theta\tilde{\Psi}_1(t) - \sin\theta\tilde{\Psi}_3(t)|^2. \qquad (6.16)$$

Equation (6.16) can be verified with help of Eqs. (6.6) and (6.13).

In Fig. 1(b), we show the transition probability for $\nu_{e\mathrm{L}} \to \nu_{\tau\mathrm{R}}$ oscillations based on Eq. (6.16) calculated using the numerical solution of Eq. (6.7) with $V \neq 0$. The transition probability $P_{\nu_{e\mathrm{L}}\to\nu_{\tau\mathrm{R}}}(z)$ corresponds to the same parameters of the neutrino system and the external fields, which are used in Fig. 1(a). Comparing Figs. 1(a) and 1(b), one can see that the upper envelope function, depicted by the red line, and the averaged transition probability, shown by the blue line, oscillate near the mean values $\approx 2 \times 10^{-3}$ and $\approx 10^{-3}$ respectively. Despite the frequencies of this oscillation are different, the mean values of the upper envelope function and the averaged transition probability are practically the same. Thus the exact solution in Eqs. (6.14) and (6.15) of the approximate Schrödinger Eq. (6.7) with $V = 0$ represents a qualitatively correct description of $\nu_{e\mathrm{L}} \to \nu_{\tau\mathrm{R}}$ oscillations.

Now we consider $\nu_{e\mathrm{L}} \to \nu_{\mu\mathrm{R}}$ oscillations channel. In this situation, we cannot neglect V in Eq. (6.7) since $\theta \equiv \theta_{\odot} = 0.6$ is not small. That is why Eqs. (6.14) and (6.15) are not applicable and we have to use the numerical solution of Eq. (6.7) from the very beginning.

In Fig. 2(a), we show the transition probability $P_{\nu_{e\mathrm{L}}\to\nu_{\mu\mathrm{R}}}(z)$, the upper and lower envelope functions, and the averaged transition probability. The values of the parameters of the external fields and the neutrino system, except δm^2 and θ, are the same as in Fig. 1. One can see in Fig. 2(a) that the averaged transition probability oscillates near 5% value. It is much greater than in Fig. 1(a). This feature can be explained by the fact that all the entries of $\tilde{H}$ in Eq. (6.7) are of the same order of magnitude for $\nu_{e\mathrm{L}} \to \nu_{\mu\mathrm{R}}$ oscillations unlike the $\nu_{e\mathrm{L}} \to \nu_{\tau\mathrm{R}}$ channel, in which Φ_{vac} is dominant.

In Fig. 2(b), we depict $P_{\nu_{e\mathrm{L}}\to\nu_{\mu\mathrm{R}}}(z)$ for lower matter density $n_e = 10^{27}\ \mathrm{cm}^{-3}$, which is very close to the value in the inner part of an accretion disk predicted by the model of GRB in Ref. [28]. The transition probability in this case reproduces the result in Ref. [13], where spin-flavor oscillations $\nu_{e\mathrm{L}} \to \nu_{\mu\mathrm{R}}$ were described at the absence of the matter contribution. Comparing Figs. 2(a) and 2(b), one can see that the lower matter density is, the higher transition probability is. Thus, one does not expect the appearance of a resonance in neutrino spin-flavor oscillations in matter under the influence of a plane electromagnetic wave, as claimed in Ref. [11]. The highest transition probability can be observed when neutrinos do not interact with background matter.

To highlight the difference between our results and the findings of Ref. [11] we present the transition probability for $\nu_{\beta\mathrm{L}} \to \nu_{\alpha\mathrm{R}}$, which can be derived on the basis of Eq. (21) in Ref. [11]. It has the form,

$$P_{\nu_{\beta\mathrm{L}}\to\nu_{\alpha\mathrm{R}}}(t) = \frac{\mu^2 B_0^2(1-\bar{\beta})^2}{\mu^2 B_0^2(1-\bar{\beta})^2 + \Delta^2} \times \sin^2\left(\sqrt{\mu^2 B_0^2(1-\bar{\beta})^2 + \Delta^2}\,t\right),$$
$$\Delta = \frac{\delta m^2}{4p_z}A(\theta) - \frac{G_{\mathrm{F}} n_e}{\sqrt{2}} + \frac{g\omega}{2}(1-\bar{\beta}), \qquad (6.17)$$

where take into account that, for $\nu_{e\mathrm{L}} \to \nu_{\mu,\tau\mathrm{R}}$ oscillations channel, $A(\theta) = (1+\cos 2\theta)/2$ [30] and $f_{\nu_e} - f_{\nu_{\mu,\tau}} = \sqrt{2}G_{\mathrm{F}} n_e$; cf. Eq. (2.2).

One can see in Eq. (6.17) that the amplitude of the transition probability would become ~ 1 if $\Delta = 0$. This fact contradicts to out results both in Eqs. (6.14) and (6.15) and the numerical simulations shown in Figs. 1 and 2. This inconsistency can be accounted for by the incorrect generalization of the Bargmann-Michel-Telegdi equation for the description of neutrino spin-flavor oscillations. In general situation, when one studies spin-flavor oscillations of Dirac neutrinos, an effective Schrödinger equation cannot have a 2×2 Hamiltonian. Typically, in this kind of problems, one deals with the system of 4 differential equations, e.g., as in Eq. (6.4) or Eq. (6.7).

VII. CONCLUSION

In the present work, we have studied neutrino spin and spin-flavor oscillations in matter under the influence of a plane electromagnetic wave with the circular polarization. Neutrinos are supposed to be massive Dirac particles with nonzero mixing between different neutrino flavors, and possessing arbitrary matrix of magnetic moments. We have started in Sec. II with reminding the basic features of neutrino interaction with background matter and an electromagnetic field.

In Sec. III, we have found the new exact solution of the Dirac-Pauli equation for a massive neutrino with a nonzero magnetic moment interacting with matter under the influence a plane electromagnetic wave. Previously, the solution of the wave equation for a Dirac fermion with an anomalous magnetic moment interacting with a plane electromagnetic wave in vacuum, i.e., at the absence of the electroweak background matter, was known (see, e.g., Ref. [31]).

In Sec. IV, we have applied the solution obtained in Sec. III for the description of neutrino spin oscillation in the considered external fields. We have studied the process $\nu_{\mathrm{L}} \to \nu_{\mathrm{R}}$, that is the neutrino spin precession within one neutrino mass eigenstate. The probability $P_{\mathrm{L}\to\mathrm{R}}$ for transitions of this kind has been derived. We have demonstrated that, in the quasiclassical approximation, the expression for $P_{\mathrm{L}\to\mathrm{R}}$ in Eq. (4.13) coincides with the result of Ref. [11], where the neutrino spin evolution in the external fields was studied within the quasiclassical approach from the very beginning.

Then, we have turned to the consideration of spin-flavor oscillations. For this purpose we have formulated the initial condition problem. This approach for the description of neutrino flavor and spin-flavor oscillations in constant external fields has been developed in Ref. [21] earlier.

First, in Sec. V, we have discussed the case of great diagonal magnetic moments. This situation takes place when a transition magnetic moment is suppressed by the GIM mechanism. If one considers the $\nu_e \to \nu_\tau$ oscillations channel, i.e., relatively small vacuum mixing angle, we can find the analytical transition probability for spin-flavor oscillations of neutrinos with great diagonal magnetic moments in matter and an electromagnetic wave; cf. Eqs. (5.6) and (5.7). However, the situation of great diagonal magnetic moments is not very interesting from the point of view of phenomenology since the GIM mechanism is valid if $\mu_a \sim m_a$ [6]. It makes μ_a to be very small for reasonable neutrino masses [24]. Therefore $A_a \ll 1$ in Eq. (5.7) and, hence, $P_{\beta\mathrm{L}\to\alpha\mathrm{R}} \ll 1$ in Eq. (5.6).

We have also considered the case of the great transition magnetic moment in Sec. VI. In this situation, we have derived the effective Schrödinger Eq. (6.7) and have found its exact solution for the $\nu_e \to \nu_\tau$ oscillations channel neglecting V in Eq. (6.7). Comparing Eqs. (6.14) and (6.15), as well as Eqs. (5.6) and (5.7), with the analogous transition probability derived in Ref. [11], one can see that the results of Ref. [11] are not applicable for the description of neutrino spin-flavor oscillations in the considered external fields. The reason for the discrepancy of our results and those in Ref. [11] has been analyzed in Sec. VI. Then, we have examined the numerical solution of Eq. (6.7) and revealed that the obtained exact solution qualitatively describes $\nu_{e\mathrm{L}} \to \nu_{\tau\mathrm{R}}$ oscillations.

Finally, basing on Eqs. (6.7) and (6.16), we have numerically studied $\nu_{e\mathrm{L}} \to \nu_{\mu\mathrm{R}}$ oscillations in matter with different densities. The transition probabilities have been plotted in Fig. 2. One can see in Fig. 2 that, if one accounts for the high matter density in neutrino spin-flavor oscillation in a plane electromagnetic wave, it diminishes the averaged transition probability. Thus one does not expect the appearance of a resonance in spin-flavor oscillations in the considered external fields, predicted in Ref. [11].

At the end of this section, we mention that described neutrino spin-flavor oscillations in background matter and a plane electromagnetic wave can take place in the vicinity of a highly magnetized compact astrophysical object, emitting intense electromagnetic radiation, being surrounded by dense matter, and being a source of neutrinos. It can be, e.g., a pulsar with a dense accretion disk. The estimates of the parameters of the neutrino system and the external fields, corresponding to the implementation of these spin-flavor oscillations in astrophysical media, are given in Ref. [13] and Sec. VI.

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