Bounds on lepton flavor violating physics and decays of neutral mesons from $\tau(\mu) \to 3\ell$, $\ell\gamma\gamma$-decays

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We study two- and three-body lepton flavor violating (LFV) decays involving leptons and neutral vector bosons $V = \rho^0$, ω, ϕ, J/ψ, Υ, Z^0, as well as pseudoscalar $P = \pi^0$, η, η', η_c and scalar $S = f_0(500)$, $f_0(980)$, $a_0(980)$, $\chi_{c0}(1P)$ mesons, without referring to a specific mechanism of LFV realization. In particular, we relate the rates of the three-body LFV decays $\tau(\mu) \to 3\ell$, where $\ell = \mu$ or e, to the two-body LFV decays $(V, P) \to \tau\mu(\tau e, \mu e)$, where V and P play the role of intermediate resonances in the decay process $\tau(\mu) \to 3\ell$. From the experimental upper bounds for the branching ratios of $\tau(\mu) \to 3\ell$ decays, we derive upper limits for the branching ratios of $(V, P) \to \tau\mu(\tau e, \mu e)$. We compare our results to the available experimental data and known theoretical upper limits from previous studies of LFV processes and find that some of our limits are several orders of magnitude more stringent. Using the idea of quark-hadron duality, we extract limits on various quark-lepton dimension-six LFV operators from data on lepton decays. Some of these limits are either new or stronger than those existing in the literature.

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I. INTRODUCTION

Search for lepton flavor violation (LFV) is an important probe of the possible physics beyond the Standard Model (SM). At present, LFV is an established fact, since it has been already observed in neutrino oscillations, and therefore it is natural to expect that LFV is also going to manifest itself in the sector of charged leptons.

A search strategy for LFV should consider those processes which have the best prospect for discovery, both from the viewpoint of their possible experimental identification and from theoretical limitations on the corresponding rates. The latter should incorporate the study of model independent relations between different processes, some of which are already strongly limited by experimental data.

The three-body purely leptonic decays of μ and τ are among the most stringently constrained LFV processes, with the following current limits on their branching ratios [1]

$$\mathrm{Br}(\mu^- \to e^- e^+ e^-) < 1.0 \times 10^{-12}, \tag{1}$$

$$\mathrm{Br}(\tau^- \to e^- e^+ e^-) < 2.7 \times 10^{-8}, \tag{2}$$

$$\mathrm{Br}(\tau^- \to \mu^- e^+ e^-) < 1.8 \times 10^{-8}, \tag{3}$$

$$\mathrm{Br}(\mu^- \to e^- \gamma\gamma) < 7.2 \times 10^{-11}. \tag{4}$$

The purpose of the present paper is to relate the three-body lepton and lepton-photon decays of μ and τ (see Fig. 1) to the two-body LFV decays of neutral vector bosons and pseudoscalar mesons, and to give upper limits for these two-body branching ratios in a model independent way. We also study the LFV dimension-six quark-lepton effective operators underlying these processes and derive limits on their scales from the limits (1)–(3).

There already exist in the literature similar studies of limits on the two-body LFV decays of vector mesons/bosons $V = \rho^0$, ω, ϕ, J/ψ, Υ, $Z^0 \to \mu^\pm e^\mp$, which use the constraint given in (1) and unitarity-inspired arguments [2].

The idea of using effective quark-lepton and hadron-lepton Lagrangians for studying LFV processes (lepton-flavor changing decays, lepton-flavor conversion, double beta decay) have been proposed and developed in Refs. [3–9] and further used in a series of papers (see, e.g., Refs. [10–22]). In particular, in Ref. [7], the on-mass-shell matching

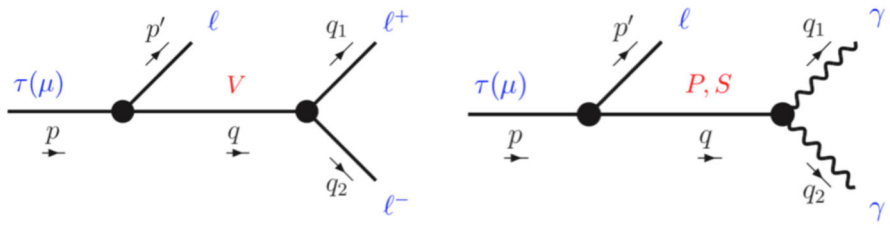

FIG. 1. Three-body LFV decays: (a) $\tau(\mu) \to 3\ell$ and (b) $\tau(\mu) \to \ell\gamma\gamma$.

condition between the quark-level effective Lagrangian and the effective hadronic-level (e.g., nucleon) Lagrangian was proposed, which sets the relations between the couplings at the quark level to those at the hadronic level. In a series of papers [10–14], $\mu^- - e^-$ conversion in nuclei was studied in the framework of an effective Lagrangian approach, without referring to any specific realization of the physics beyond the SM responsible for LFV. Limits on various LFV couplings of vector and scalar mesons to the $\mu - e$ current were derived from the existing experimental data on $\mu^- - e^-$ conversion in nuclei. Here, we extend the application of these techniques, in order to extract limits on two-body LFV decays of vector and pseudoscalar mesons by searching for LFV three-lepton decays of tau leptons and muons.

The paper is organized as follows. In Sec. II, we introduce the relevant effective quark-lepton and meson-lepton LFV operators, without referring to specific mechanisms of LFV. In Sec. III, we derive the relations between three-body lepton LFV decays and two-body LFV meson decays, which is done by taking into account the contribution of neutral vector and pseudoscalar mesons in the three-body lepton LFV process. With these relations, we set the limits on the two-body LFV meson decays. In Sec. IV, we derive the relations between branching ratios of two-body LFV decays of the same quark content and examine the limits on the effective quark-lepton operators from purely leptonic processes. Section V contains our summary and conclusions.

II. EFFECTIVE QUARK-LEPTON AND MESON-LEPTON LFV OPERATORS

Let us assume generic LFV sources, leading to $\tau \to \mu(e)ee$ and $\mu \to 3e$ decays, in the form of effective operators as the low-energy limit of a renormalizable "fundamental" LFV theory at a scale Λ. The leading-order operators have been proposed in Refs. [3–9]. The set of these operators can be written as

$$\text{4-lepton}:\ \mathcal{L}_{4\ell} = \frac{1}{\Lambda^2}\sum_{(IJ)} C^{\Gamma_I\Gamma_J}_{\ell_1\ell_2}[\bar{\ell}_1\Gamma_I\ell_2]\cdot[\bar{e}\Gamma_J e] + \text{H.c.} \tag{5}$$

$$\text{Magnetic}:\ \mathcal{L}_M = \frac{1}{\Lambda}(\bar{C}^T_{\ell_1\ell_2}[\bar{\ell}_1\sigma_{\mu\nu}\ell_2] + \bar{C}^{T5}_{\ell_1\ell_2}[\bar{\ell}_1\sigma_{\mu\nu}\gamma_5\ell_2])F^{\mu\nu} + \text{H.c.}, \tag{6}$$

$$\begin{aligned}\text{Quark-Lepton}:\ \mathcal{L}_{\ell q} &= \frac{1}{\Lambda^2}\sum_{(IJ)} C^{\Gamma_I\Gamma_J}_{if,\ell_1\ell_2}[\bar{\ell}_1\Gamma_I\ell_2]\cdot[\bar{q}_f\Gamma_J q_i] + \text{H.c.}\\ &= \frac{1}{\Lambda^2}(C^{SS}_{if,\ell_1\ell_2}[\bar{\ell}_1\ell_2]\cdot[\bar{q}_f q_i] + C^{PS}_{if,\ell_1\ell_2}[\bar{\ell}_1\gamma^5\ell_2]\cdot[\bar{q}_f q_i] + C^{SP}_{if,\ell_1\ell_2}[\bar{\ell}_1\ell_2]\cdot[\bar{q}_f\gamma_5 q_i]\\ &\quad + C^{PP}_{if,\ell_1\ell_2}[\bar{\ell}_1\gamma^5\ell_2]\cdot[\bar{q}_f\gamma_5 q_i] + C^{VV}_{if,\ell_1\ell_2}[\bar{\ell}_1\gamma^\mu\ell_2]\cdot[\bar{q}_f\gamma_\mu q_i] + C^{AV}_{if,\ell_1\ell_2}[\bar{\ell}_1\gamma^\mu\gamma^5\ell_2]\cdot[\bar{q}_f\gamma_\mu q_i]\\ &\quad + C^{VA}_{if,\ell_1\ell_2}[\bar{\ell}_1\gamma^\mu\ell_2]\cdot[\bar{q}_f\gamma_\mu\gamma_5 q_i] + C^{AA}_{if,\ell_1\ell_2}[\bar{\ell}_1\gamma^\mu\gamma^5\ell_2]\cdot[\bar{q}_f\gamma_\mu\gamma_5 q_i] + C^{TT}_{if,\ell_1\ell_2}[\bar{\ell}_1\sigma^{\mu\nu}\ell_2]\cdot[\bar{q}_f\sigma_{\mu\nu} q_i]) + \text{H.c.},\end{aligned} \tag{7}$$

where $\ell = \mu, e$ and $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ is the electromagnetic field tensor. In (5) and (7), we use $I, J = S, P, V, A, T$ and $\Gamma_{I,J} = 1, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}$, so that the summation runs over $(IJ) = (SS), (PS), (SP), (PP), (AV), (VV), (VA), (AA), (TT)$. In Eq. (7), we displayed the terms in the sum explicitly. After specifying all possible Lorentz structures in Eqs. (5) and (7), we used the identity $\bar{a}\sigma^{\mu\nu}\gamma_5 b\cdot\bar{c}\sigma_{\mu\nu}\gamma_5 d = \bar{a}\sigma^{\mu\nu}b\cdot\bar{c}\sigma_{\mu\nu}d$. Here, we denoted the LFV scale by Λ.

The operators (5) and (6) lead to tree-level contributions to $\tau \to \ell ee$, while the dipole-type operator (6) directly contributes to $\tau \to \ell\gamma$. Limits on the scales of these

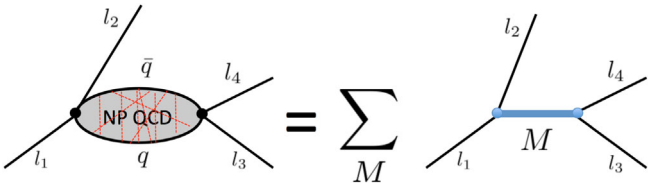

FIG. 2. Quark-lepton contact interaction contribution to $l_1 \to l_2 l_3 l_4$ via meson exchange according to quark-hadron duality.

operators are readily extracted from data [1] and can be found in the literature (see, for instance, Ref. [15]). The quark-lepton LFV operators (7) have been studied by many authors, which consider the two-body decays $\tau \to \ell M$, $M \to \ell_1 \ell_2$, deep inelastic conversion $\tau(\mu) q \to \ell q$ [21] as well as nuclear $\mu - e$- conversion (for a recent review see, for instance, Ref. [23]). The existing data on the rates of these processes allowed extraction of rather stringent limits on the scale of the corresponding operators (7), which also contribute to leptonic LFV decays of mesons $M \to \ell_1 \ell_2$ at tree level. At one-loop level they contribute to purely leptonic LFV processes $\tau^- \to \mu^- e^+ e^-$ and $\mu^- \to e^- e^+ e^-$. However, quark-hadron duality [2] relates these loop contributions, taking into account nonperturbative QCD effects, with the sum over the tree-level contributions (Fig. 2) of all the intermediate meson states with the allowed quantum numbers.

Therefore, effectively the operators in Eq. (7) trigger tree-level contributions to $\ell_1 \to \ell_2 e^+ e^-$ via intermediate meson states. The relevant meson-lepton vertices involving vector V, axial A, pseudoscalar P, and scalar S mesons with quantum numbers $J^{PC} = 1^{--}$, 1^{++} (1^{+-}), 0^{-+}, and 0^{++}, respectively, are

$$\begin{aligned}\mathcal{L}_{\ell M} = {} & V_\mu (g^{(V)}_{V\ell_1\ell_2}[\bar{\ell}_1\gamma^\mu\ell_2] + g^{(A)}_{V\ell_1\ell_2}[\bar{\ell}_1\gamma^\mu\gamma_5\ell_2]) + A_\mu (g^{(V)}_{A\ell_1\ell_2}[\bar{\ell}_1\gamma^\mu\ell_2] + g^{(A)}_{A\ell_1\ell_2}[\bar{\ell}_1\gamma^\mu\gamma_5\ell_2]) + \frac{g^{(T)}_{V\ell_1\ell_2}}{M_V} F^V_{\mu\nu}[\bar{\ell}_1\sigma^{\mu\nu}\ell_2] \\ & + \frac{g^{(T)}_{A\ell_1\ell_2}}{M_A} F^A_{\mu\nu}[\bar{\ell}_1\sigma^{\mu\nu}\gamma^5\ell_2] + S(g^{(S)}_{S\ell_1\ell_2}[\bar{\ell}_1\ell_2] + g^{(P)}_{S\ell_1\ell_2}[\bar{\ell}_1\gamma_5\ell_2]) + P(ig^{(S)}_{P\ell_1\ell_2}[\bar{\ell}_1\ell_2] + ig^{(P)}_{P\ell_1\ell_2}[\bar{\ell}_1\gamma_5\ell_2]) \\ & + \frac{\partial_\mu P}{M_P}(g^{(V)}_{P\ell_1\ell_2}[\bar{\ell}_1\gamma^\mu\ell_2] + g^{(A)}_{P\ell_1\ell_2}[\bar{\ell}_1\gamma^\mu\gamma^5\ell_2]) + \text{H.c.} \end{aligned} \tag{8}$$

Here we introduced the notation $F^M_{\mu\nu} = \partial_\mu M_\nu - \partial_\nu M_\mu$ (with $M = V$, A) for the field tensors of the vector and axial mesons, respectively. Obviously, the lightest mesons dominate in the diagram in Figs. 1(a) and 1(b), because the contributions of meson resonances to the three-body LFV decays scale as $1/M^4$, where M is the mass of intermediate meson.

In the next sections, we shall use the effective hadronic-level Lagrangian of Eq. (8) in order to constrain the quark-lepton operators of Eq. (7), using the bounds given in Eqs. (1)–(3). This is done by applying an appropriate matching condition at the hadronization scale. In this way, we shall constrain the vector and tensor operators related to the corresponding vector boson contribution ($V = \rho^0, \omega, \phi, J/\psi, \Upsilon, Z^0$) to the processes $\tau(\mu) \to 3\ell$, and also constrain the pseudoscalar and scalar operators from the contribution of the pseudoscalar ($P = \pi^0$, η, η', $\eta_c(1S)$) and scalar meson states ($S = f_0(500)$, $f_0(980)$, $a_0(980)$, $\chi_{c0}(1P)$) to the processes $\tau(\mu) \to \ell\gamma\gamma$. Expressions for the LFV two-body decay widths of different meson states are shown in Appendix B.

Let us recall a key point of the present study: nonperturbative QCD effects leading to the formation of the M meson bound states in the intermediate state of $\ell_1 \to \ell_2 e^+ e^-$ are taken into account according to the quark-hadron duality, via two parameters: the meson masses M_M and their leptonic decay constants f_M. Numerical values of these parameters are known either from direct experimental measurements, from lattice simulations or some reliable models. The list of these parameters are given in Appendix A. We shall study these meson exchange mechanisms in the next sections.

III. RELATIONS BETWEEN THREE- AND TWO-BODY LFV DECAYS

Here we derive unitarity-inspired relations between the three-body lepton decays and the two-body vector, scalar, and pseudoscalar meson decays. Unitarity implies the contribution of all intermediate meson states to τ, $\mu \to \ell ee$, $\ell\gamma\gamma$. Following Ref. [2], we retain as a good approximation only the lightest mesons, so that their contributions are described by the meson exchange diagrams in Fig. 1, with the LFV vertices given by the Lagrangian (8). We shall not consider flavored mesons, because their decay rates $M \to e^+ e^-, \gamma\gamma$, which enter in the above-mentioned relations, are GIM-suppressed, and this does not allow us to derive significant limits for their LFV decays.

A. Vector mesons

Let us consider the vector mesons $V = \rho^0, \omega, \phi, J/\psi, \Upsilon, Z^0$. Our goal is to analyze their contribution to $\mu, \tau \to 3\ell$

decays. For the case of $\mu^- \to e^- e^+ e^-$ and vector mesons, this was done in Ref. [2].

Neglecting the final lepton masses, for the muon decay rates we have

$$\Gamma(\mu^- \to e^- e^+ e^-) = \kappa \frac{\tilde{g}^2_{V\mu e} \tilde{g}^2_{Vee}}{M_V^4} \tag{9}$$

$$\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu) = \Gamma(\mu \to \text{All}) = \kappa \frac{g_W^4}{M_W^4}, \tag{10}$$

where M_V is the vector meson/boson mass, $\kappa = M_\mu^5/(384\pi^3)$ is a kinematic-spin factor common to all decay modes involving vector mesons in the intermediate state, while g_W and M_W are the electroweak coupling and the W boson mass, respectively (here g_W is normalized so that the Fermi coupling is $G_F/\sqrt{2} = g_W^2/(2M_W^2)$). By definition $\tilde{g}^2_{V\mu e} = |g^{(V)}_{V\mu e}|^2 + |g^{(A)}_{V\mu e}|^2$. Then one finds for the LFV branching ratio

$$\text{Br}(\mu^- \to e^- e^+ e^-) = \frac{\tilde{g}^2_{V\mu e} \tilde{g}^2_{Vee}}{M_V^4} \frac{M_W^4}{g_W^4}. \tag{11}$$

Formulae for the meson two-body decay rates are given in Appendix B. Neglecting the final lepton masses they can be written as

$$\Gamma(V \to e^+ e^-) = a \tilde{g}^2_{Vee} M_V, \tag{12}$$

$$\Gamma(V \to \mu^\pm e^\mp) = a \tilde{g}^2_{V\mu e} M_V, \tag{13}$$

$$\Gamma(W \to e\bar{\nu}_e) = a g_W^2 M_W, \tag{14}$$

where $a = 1/(12\pi)$ is a kinematic factor common to all these processes.

The branching ratio of Eq. (11) can then be written in terms of the two-body decay rates as:

$$\text{Br}(\mu \to 3e) = \frac{\Gamma(V \to \mu e)\Gamma(V \to e^+ e^-)}{\Gamma(W \to e\bar{\nu}_e)^2} \left(\frac{M_W}{M_V}\right)^6. \tag{15}$$

For the case of $\tau^- \to e^-(\mu^-) e^+ e^-$ there are two main differences with respect to the muon decays: (i) due to the large mass of the τ lepton, there are some on-mass-shell meson contributions to this process; (ii) the τ decay width is not purely an electroweak quantity, i.e., $\Gamma(\tau \to \text{All}) \neq \Gamma(\tau \to \ell\bar{\nu}\nu)$, since it contains hadronic channels. The latter suffer from considerable theoretical uncertainties. However, the tau decay width is an experimentally well measured observable [1]. Combining the above formulae (9), (10) and (12)–(14) with the corresponding replacements, for $M_V > M_\tau$ we find:

$$\text{Br}(\tau^- \to \ell^- e^+ e^-) = \frac{\Gamma(V \to \tau\ell)\Gamma(V \to e^+ e^-)}{\Gamma(W \to e\bar{\nu}_e)^2} \frac{\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu)}{\Gamma(\tau \to \text{All})} \times \left(\frac{M_W}{M_V}\right)^6 \left(\frac{M_\tau}{M_\mu}\right)^5, \tag{16}$$

and for $M_V < M_\tau$

$$\text{Br}(\tau^- \to \ell^- e^+ e^-) = \text{Br}(\tau \to V\ell)\text{Br}(V \to e^+ e^-) \tag{17}$$

where $\ell = \mu$, e. The latter case is not interesting for our analysis, which is related to constraints on $\tau \to V\ell$.

B. Unflavored pseudoscalar and scalar mesons

The unflavored pseudoscalar and scalar mesons contribute to $\mu^- \to e^-\gamma\gamma$ and $\tau^- \to \ell^-\gamma\gamma$, according to the diagram in Fig. 1(b), with the LFV vertex $P(S)\bar{\ell}_1\ell_2$ given in Eq. (8), and

$$\mathcal{L}_{P\gamma\gamma} = \frac{e^2}{4} g_{P\gamma\gamma} P F_{\mu\nu} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}, \tag{18}$$

$$\mathcal{L}_{S\gamma\gamma} = \frac{e^2}{4} g_{S\gamma\gamma} S F_{\mu\nu} F^{\mu\nu}, \tag{19}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic stress tensor, $\epsilon^{\mu\nu\alpha\beta}$ is the Levi-Cevita tensor, and $g_{I\gamma\gamma}$ ($I = P$, S) are the effective couplings of the $I \to \gamma\gamma$ decay widths:

$$\Gamma(I \to \gamma\gamma) = \frac{\pi\alpha^2}{4} g^2_{I\gamma\gamma} M_I^3, \tag{20}$$

where $\alpha \simeq 1/137.036$ is the fine structure constant. In the case of π^0, the coupling $g_{\pi\gamma\gamma}$ is related to the pion decay constant $F_\pi \simeq 92.4$ MeV as

$$g_{\pi\gamma\gamma} = \frac{1}{4\pi^2 F_\pi}. \tag{21}$$

The pion contribution to the decay $\mu \to e\gamma\gamma$ was discussed in Ref. [2]. Extending this analysis to include other scalar and pseudoscalar mesons we can write

$$\text{Br}(\mu^- \to e^-\gamma\gamma) \approx \frac{\Gamma(I \to \mu e)\Gamma(I \to \gamma\gamma)}{\Gamma^2(W \to e\bar{\nu}_e)} \left(\frac{M_W}{M_I}\right)^6 \left(\frac{M_\mu}{2M_I}\right)^4. \tag{22}$$

For the τ lepton decay we find, in analogy to Eq. (16), and for $M_I > M_\tau$:

$$\text{Br}(\tau^- \to \ell^-\gamma\gamma) \approx \frac{\Gamma(I \to \tau\ell)\Gamma(I \to \gamma\gamma)}{\Gamma^2(W \to e\bar{\nu}_e)} \frac{\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu)}{\Gamma(\tau \to \text{All})} \times \left(\frac{M_W}{M_I}\right)^6 \left(\frac{M_\tau}{M_\mu}\right)^5 \left(\frac{M_\tau}{2M_I}\right)^4. \tag{23}$$

The case $M_I < M_\tau$ with on-mass-shell mesons is not interesting for our analysis.

TABLE I. Upper limits for the branching ratios of two-body LFV decays of neutral vector, pseudoscalar, and scalar mesons, and Z-boson, extracted from the bound on the indicated three-body μ and τ decays. "EO-improved" are limits obtained from Eq. (27) relating different LFV processes with the same underlying effective operators (EO).

Mode	Our results	Existing limits	Data
from $\mu^- \to e^-\gamma\gamma$ process			
$\pi^0 \to \mu^\pm e^\mp$	5.8×10^{-11} (3.2×10^{-11})	10^{-10} [2]	3.8×10^{-10} [1]
$\eta \to \mu^\pm e^\mp$	6.2×10^{-9}	10^{-8} [2]	3.0×10^{-6} [1]
$\eta' \to \mu^\pm e^\mp$	1.3×10^{-9}	4.7×10^{-4} [1]	
$\eta_c \to \mu^\pm e^\mp$	5.9×10^{-7}	1.57×10^{-4} [1]	
$f_0(500) \to \mu^\pm e^\mp$	1.6×10^{-15}		
$f_0(980) \to \mu^\pm e^\mp$	1.0×10^{-10}		
$a_0(980) \to \mu^\pm e^\mp$	6.2×10^{-11}		
$\chi_{c0}(1P) \to \mu^\pm e^\mp$	1.5×10^{-5}		

Mode	Our results	EO-improved	Existing limits	Data
from $\mu^- \to e^-e^+e^-$ process				
$\rho^0 \to \mu^\pm e^\mp$	5.8×10^{-21}		3.5×10^{-24} [13]	
$\omega \to \mu^\pm e^\mp$	6.8×10^{-20}	9.1×10^{-21}	6.2×10^{-27} [13]	
$\phi \to \mu^\pm e^\mp$	1.6×10^{-19}	1.1×10^{-19}	4×10^{-17} [2]; $(1.1 \times 10^{-25} - 5.6 \times 10^{-22})$ [13]	2.0×10^{-6} [1]
$J/\psi \to \mu^\pm e^\mp$	2.9×10^{-17}	2.6×10^{-18}	4×10^{-13} [2]; 5.4×10^{-14} [13]	1.6×10^{-7} [1]
$\Upsilon \to \mu^\pm e^\mp$	1.0×10^{-13}	2.5×10^{-16}	2×10^{-9} [2]; 1.1×10^{-6} [13]	
$Z^0 \to \mu^\pm e^\mp$	1.3×10^{-12}		5×10^{-13} [2]; 4.2×10^{-6} [13]	7.5×10^{-7} [1]
from $\tau^- \to e^-e^+e^-$ process				
$J/\psi \to \tau^\pm e^\mp$	4.5×10^{-12} (2.8×10^{-12})		6×10^{-7} [2]	8.3×10^{-6} [1]
$\Upsilon \to \tau^\pm e^\mp$	1.6×10^{-8}	7.3×10^{-10}	1×10^{-2} [2]	
$Z^0 \to \tau^\pm e^\mp$	1.9×10^{-7}		3×10^{-6} [2]	9.8×10^{-6} [1]
from $\tau^- \to \mu^-e^+e^-$ process				
$J/\psi \to \tau^\pm \mu^\mp$	3.0×10^{-12} (1.9×10^{-12})			2.0×10^{-6} [1]
$\Upsilon \to \tau^\pm \mu^\mp$	1.0×10^{-8}	4.9×10^{-10}	No limits	6.0×10^{-6} [1]
$Z^0 \to \tau^\pm \mu^\mp$	1.3×10^{-7}			1.2×10^{-5} [1]

In our numerical analysis, we use the central values of the decay widths of pseudoscalar and scalar mesons quoted from the Particle Data Group [1]:

$$\begin{aligned} \Gamma(\pi^0 \to \gamma\gamma) &= 7.64 \text{ eV}, & \Gamma(\eta \to \gamma\gamma) &= 0.52 \text{ keV}, \\ \Gamma(\eta' \to \gamma\gamma) &= 4.35 \text{ keV}, & \Gamma(\eta_c \to \gamma\gamma) &= 5.02 \text{ keV}, \\ \Gamma(f_0(500) \to \gamma\gamma) &= 2.05 \text{ keV}, & \Gamma(f_0(980) \to \gamma\gamma) &= 0.31 \text{ keV}, \\ \Gamma(a_0(980) \to \gamma\gamma) &= 0.30 \text{ keV}, & \Gamma(\chi_{c0}(1P) \to \gamma\gamma) &= 2.20 \text{ keV}. \end{aligned} \tag{24}$$

Note that up to now there are no experimental constraints on $\tau \to \ell\gamma\gamma$ decay rates. Therefore, in the present paper we present only theoretical formula (23) relating three-body LFV decay of τ with two-body LFV decays $P(S) \to \tau\ell$, which could be useful in future searches of these processes.

C. Limits on two-body LFV meson decays

From Eqs. (15), (16), (22) and (23), we deduce upper limits for the branching ratios of the two-body LFV decays $M(Z) \to \ell_1\ell_2$ of neutral vector and pseudoscalar mesons and Z-boson, using the existing data (1)–(3) for three-body LFV decays $\tau(\mu) \to 3\ell$. We present our results in the second column of Table I and compare them with the limits derived from the study of lepton conversion [2] and available experimental data [1].

In the case of the π^0 and J/ψ contributions, we also show in parenthesis our results for the constraints which take into account the Q^2-dependence of the meson propagator and the form factor $\tilde{g}_{M\ell_1\ell_2}(Q^2)$, when this last effect is significant. For other meson contributions the effect of the Q^2-dependence is negligible. A detailed discussion and estimation of this effect is presented in Appendix C.

One can see from Table I that in most cases we get more stringent constraints on the branching ratios of the

TABLE II. Lower limits on the individual mass scales, $\Lambda_{\ell_1\ell_2}$, of the effective operators (7). "Existing limits" are taken from Ref. [10]. All the limits are derived assuming that only one operator contributes to $\tau \to \mu(e)ee$, $\mu \to 3e$ at a time.

$\Lambda_{\ell_1\ell_2}$	Our limits [TeV]	Existing limits [TeV]	$\Lambda_{\ell_1\ell_2}$	Our limits [TeV]	Existing limits [TeV]	$\Lambda_{\ell_1\ell_2}$	Our limits [TeV]	Existing limits [TeV]
$\Lambda_{\mu e}^{(3)VV,AV}$	86	10^3	$\Lambda_{\mu e}^{(3)PP,SP}$	8.0	None	$\Lambda_{\tau e}^{(c)VV,AV}$	13	None
$\Lambda_{\mu e}^{(3)AA,VA}$	7.1	none	$\Lambda_{\mu e}^{(s)SS,PS}$	None	3×10^3	$\Lambda_{\tau e}^{(b)VV,AV}$	7	None
$\Lambda_{\mu e}^{(0)VV,AV}$	89	4.7×10^3	$\Lambda_{\mu e}^{(s)PP,SP}$	1.3	none	$\Lambda_{\mu e}^{(c)TT}$	19	None
$\Lambda_{\mu e}^{(0)AA,VA}$	2.4	None	$\Lambda_{\mu e}^{(c)SS,PS}$	None	950	$\Lambda_{\mu e}^{(b)TT}$	8.4	None
$\Lambda_{\mu e}^{(s)VV,AV}$	134	770	$\Lambda_{\mu e}^{(b)SS,PS}$	None	540	$\Lambda_{\tau\mu}^{(c)VV,AV}$	14.5	None
$\Lambda_{\mu e}^{(s)AA,VA}$	0.6	None	$\Lambda_{\mu e}^{(t)SS,PS}$	None	90	$\Lambda_{\tau\mu}^{(b)VV,AV}$	7.7	None
$\Lambda_{\mu e}^{(c)VV,AV}$	300	54	$\Lambda_{\mu e}^{(3)TT}$	103	None	$\Lambda_{\tau\mu}^{(c)TT}$	19	None
$\Lambda_{\mu e}^{(b)VV,AV}$	138	3	$\Lambda_{\mu e}^{(0)TT}$	107	None	$\Lambda_{\tau\mu}^{(b)TT}$	9.1	None
$\Lambda_{\mu e}^{(3)SS,PS}$	0.5	1.8×10^3	$\Lambda_{\mu e}^{(s)TT}$	160	None			
$\Lambda_{\mu e}^{(0)SS,PS}$	0.6	6.8×10^3	$\Lambda_{\mu e}^{(c)TT}$	355	None			
$\Lambda_{\mu e}^{(0)PP,SP}$	6.0	None	$\Lambda_{\mu e}^{(b)TT}$	164	None			

two-body LFV decays. In particular, our limits are 3–4 orders of magnitude better than the existing ones for J/Ψ, $\Upsilon \to \mu e$, while for J/Ψ, $\Upsilon \to \tau e$ the improvement is 5 orders of magnitude. To the best of our knowledge, in the literature there are no phenomenological limits for J/Ψ, Υ, $Z \to \tau\mu$, and our limits are significantly more stringent than the existing experimental bounds [1]. In Table I, we also displayed for completeness the LFV decays of f_0, a_0, $\chi_{c0} \to \mu e$, which are unrealistic for experimental observations. We recall that these mesonic states, together with other mesons, are needed for the implementation of the quark-hadron duality and the derivation of the limits on the quark-lepton operators (7).

IV. QUARK-LEPTON EFFECTIVE OPERATORS IN LFV DECAYS OF μ, τ

A. Indirect contribution to $\ell_1 \to \ell_2 ee$

Here we examine the limits on the effective quark-lepton operators (7) from the purely leptonic processes $\tau^- \to \mu^-(e^-)e^+e^-$, $\mu^- \to e^-e^+e^-$ or $\tau^- \to \mu^-(e^-)\gamma\gamma$, $\mu^- \to e^-\gamma\gamma$. The operators (7) contribute to $\tau \to \ell ee$ at one-loop level. However, as we discussed in Sec. II, quark-hadron duality identifies these loop contributions with the tree-level contribution of the mesons states with the corresponding quantum numbers, as shown in Fig. 1. In order to constrain the quark-lepton operators (7), we match them to the corresponding meson-lepton operators in Eq. (8), using the on-mass-shell matching condition [10,11]:

$$\langle \ell_1^+ \ell_2^- | \mathcal{L}_{\rm eff}^{lq} | M \rangle \approx \langle \ell_1^+ \ell_2^- | \mathcal{L}_{\rm eff}^{lM} | M \rangle, \tag{25}$$

where M are the corresponding mass-shell meson states. This equation can be solved using the well-known quark current meson matrix elements shown in Appendix A, and we find relations between the quark-lepton scaled Wilson coefficients, $\mathcal{C}/\Lambda^2$ in Eq. (7), and the meson-lepton couplings, g_M, from (8), which are shown in Appendix D. Using these relations in the decay rate formulas for $\Gamma(M \to l_1 l_2)$ from Appendix A and substituting them into Eqs. (15), (16), (22) and (23), we set upper limits on the coefficients $\mathcal{C}/\Lambda^2$ of the effective operators (7) from the experimental data on $\tau(\mu) \to 3\ell$. There are several operators contributing simultaneously to each of these processes, and therefore the data impose upper limits on linear combinations of the corresponding Wilson coefficients shown in Appendix E. In practice, it is useful to have individual upper limits for these coefficients under certain reasonable assumptions. In the literature, it is conventional to assume that there is no strong cancellation between terms of different origin in the amplitudes and therefore extract limits on each term as if it was present alone. We apply this "one-at-a-time" approach to Eqs. (E1)–(E11). The corresponding results are displayed in Table II in the form of lower limits on the individual mass scales, $\Lambda_{\mu e}^{ij}$, of the operators in Eq. (7). In the conventional definition (see, for instance, Ref. [14]), these scales are related to our notation as

$$|C_a^{XY}| \left(\frac{1\ \text{GeV}}{\Lambda}\right)^2 = 4\pi \left(\frac{1\ \text{GeV}}{\Lambda_a^{XY}}\right)^2 \tag{26}$$

with $a = 0, 3, s, c, b, t$ and $z = hV, rS$, where $h = A, V$ and $r = P, S$ as defined before.

B. Relations between LFV decays of different mesons

Notice that the operators in Eq. (7), either individually or in certain linear combination of them, underly LFV

leptonic decay modes of all the mesons with the same quark content and J^{PC}.

Using the decay rate formulae, the meson matrix elements and the expressions for the LFV meson couplings from Appendixes A, B and D, we find, in the limit of massless final leptons, the following approximate relation between the branching ratios of different mesons $\mathcal{M} = V, P$:

$$\mathrm{Br}(\mathcal{M}_a \to \ell_1 \ell_2) \approx \left(\frac{f_a}{f_b}\right)^2 \left(\frac{M_a}{M_b}\right)^5 \frac{\Gamma(\mathcal{M}_b \to \mathrm{All})}{\Gamma(\mathcal{M}_a \to \mathrm{All})} \cdot \mathrm{Br}(\mathcal{M}_b \to \ell_1 \ell_2). \quad (27)$$

Using this relation and the upper limits in Table I on the branching ratios for one particular meson, we can set limits for the other ones. These "cross-limits", shown in the column "EO improved" of Table I, are in some cases significantly more stringent than the limits derived directly from the contribution of the corresponding meson to $\tau \to \ell ee$.

V. SUMMARY

We derived unitarity-inspired bounds on the two-body LFV decays of unflavored neutral vector and pseudoscalar mesons as well as of the Z-boson, from the experimental bounds on the leptonic LFV decays $\tau(\mu) \to \ell e^+ e^-$, $\ell\gamma\gamma$. Many of our limits are better than those existing to date in the literature. We also derived still nonexistent in the literature theoretical limits for J/Ψ, Υ, $Z \to \tau\mu$, which are significantly more stringent that the experimental bounds. Using the fact that the LFV decays of the mesons with the same quark content and J^{PC} originate from the same linear combination of quark-lepton operators, Eq. (7), we derived improved limits on the decay rate of one meson from the more stringent limit of the decay rate of another meson. In some cases, this improvement approaches 3 orders of magnitude.

We analyzed the contribution of quark-lepton operators (7) to purely leptonic processes $\tau(\mu) \to \ell e^+ e^-$, $\ell\gamma\gamma$, on the basis of the quark-hadron duality, which takes into account these contributions as coming from intermediate meson states. In this approach, the nonperturbative QCD effects in the quark loops are effectively considered by the meson masses and their leptonic decay constants. In order to realize this approach, we matched at the hadronization scale the quark-lepton and meson-lepton effective Lagrangians and derived relations between the quark- and meson-level effective LFV couplings. With this at hand, we extracted lower limits on the individual scales of many LFV operators from (7), which are shown in Table II. The limits for the scales of the tensor, axial-vector, pseudoscalar operators, as well as for $(\bar{q}\Gamma q)(\bar{e}\Gamma\tau)$, $(\bar{q}\Gamma q)(\bar{\mu}\Gamma\tau)$ are new, nonexisting in the literature. These limits can be useful for LFV phenomenology, allowing model independent predictions for the LFV processes induced by the generic set of quark-lepton operators (7).

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APPENDIX A: MESON MATRIX ELEMENTS

Here we show the meson matrix elements needed for the matching between the quark and hadron levels of the effective theory used in our analysis. In the case of vector and scalar operators, these are

$$\langle 0|\bar{u}\,\gamma_\mu\,u|\rho^0(p,\epsilon)\rangle = -\langle 0|\bar{d}\,\gamma_\mu\,d|\rho^0(p,\epsilon)\rangle = M_\rho^2 f_\rho \epsilon_\mu(p), \quad (A1)$$

$$\langle 0|\bar{u}\,\gamma_\mu\,u|\omega(p,\epsilon)\rangle = \langle 0|\bar{d}\,\gamma_\mu\,d|\omega(p,\epsilon)\rangle = 3M_\omega^2 f_\omega \epsilon_\mu(p), \quad (A2)$$

$$\langle 0|\bar{s}\,\gamma_\mu\,s|\phi(p,\epsilon)\rangle = -3M_\phi^2 f_\phi \epsilon_\mu(p), \quad (A3)$$

$$\langle 0|\bar{c}\,\gamma_\mu\,c|J/\psi(p,\epsilon)\rangle = M_{J/\Psi}^2 f_{J/\psi} \epsilon_\mu(p), \quad (A4)$$

$$\langle 0|\bar{b}\,\gamma_\mu\,b|\Upsilon(p,\epsilon)\rangle = M_\Upsilon^2 f_\Upsilon \epsilon_\mu(p), \quad (A5)$$

$$\langle 0|\bar{u}\,u|f_0(p)\rangle = \langle 0|\bar{d}\,d|f_0(p)\rangle = M_{f_0}^2 f_{f_0}, \quad (A6)$$

$$\langle 0|\bar{u}\,u|a_0(p)\rangle = -\langle 0|\bar{d}\,d|a_0(p)\rangle = M_{a_0}^2 f_{a_0}. \quad (A7)$$

Here, p, m_M and f_M are the 4-momentum, mass and dimensionless decay constant of the meson M, respectively, and ϵ_μ is the vector meson polarization state vector.

The current central values of the meson decay constants f_V and masses m_V are [1]

$$f_\rho = 0.2, \quad f_\omega = 0.059, \quad f_\phi = 0.074, \quad f_{J/\psi} = 0.134, \quad f_\Upsilon = 0.08, \quad f_{f_0} = 0.28, \quad f_{a_0} = 0.19, \quad (A8)$$

$$M_\rho = 771.1\ \mathrm{MeV}, \quad M_\omega = 782.6\ \mathrm{MeV}, \quad M_\phi = 1019.5\ \mathrm{MeV}, \quad (A9)$$

$$M_{J/\psi} = 3097 \text{ MeV}, \qquad M_{\Upsilon} = 9460 \text{ MeV},$$
$$M_{f_0} = 500 \text{ MeV}, \qquad M_{a_0} = 980 \text{ MeV}. \tag{A10}$$

The decay constants f_{f_0} and f_{a_0} in Eqs. (A6) and (A7) are not yet known experimentally. The value f_{f_0} was evaluated in Ref. [11] in the linear σ-model, using the approach of Refs. [24,25] and the value a_{f_0} was estimated using QCD sum rules [26].

In the evaluation of tensor operators, we use the identity

$$\sigma^{\mu\nu}\gamma^5 = \frac{i}{2}\epsilon^{\mu\nu\alpha\beta}\sigma_{\alpha\beta}, \tag{A11}$$

which simplifies/constrains the structure of effective Lagrangians with tensor spin structure as

$$\bar{\ell}_1\sigma^{\mu\nu}P_{L/R}\ell_2\bar{q}_f\sigma_{\mu\nu}P_{L/R}q_i = \frac{1}{2}\bar{\ell}_1\sigma^{\mu\nu}\ell_2\bar{q}_f\sigma_{\mu\nu}q_i, \tag{A12}$$

$$\bar{\ell}_1\sigma^{\mu\nu}P_{L/R}\ell_2\bar{q}_f\sigma_{\mu\nu}P_{R/L}q_i \equiv 0. \tag{A13}$$

The matrix element of the tensor quark operator is calculated according to

$$\langle 0|\bar{q}_f\sigma_{\mu\nu}q_i|V(p,\epsilon)\rangle = i(m_i + m_f)(\epsilon_\mu(p)p_\nu - \epsilon_\nu(p)p_\mu)f_V. \tag{A14}$$

In deriving effective Lagrangians with derivates acting on meson fields, we use the convention that the meson is described by an incoming plane wave of the form e^{-ipx}. Therefore, the correspondence between the Lorentz structure $\epsilon_\mu(p)p_\nu - \epsilon_\nu(p)p_\mu$ and the field tensor of a vector meson in coordinate space is set as $i(\epsilon_\mu(p)p_\nu - \epsilon_\nu(p)p_\mu) \to F_{\mu\nu}(x)$.

In the calculation of matrix elements of pseudoscalar, axial, and pseudotensor quark operators, we use the well-known relations [27–29]

$$\langle 0|\bar{q}_f\gamma^\mu\gamma^5 q_i|P(p)\rangle = ip^\mu F_P, \tag{A15}$$

$$\langle 0|\bar{q}_f i\gamma^5 q_i|P(p)\rangle = \frac{M_P^2}{m_i + m_f}F_P, \tag{A16}$$

where the P meson has flavor structure $P = (q_i\bar{q}_f)$, f_P is the pseudoscalar meson coupling constants. In the case of pseudoscalar mesons, we introduce singlet-octet mixing, with a mixing angle of $\theta_P = -13.34°$ [30]

$$\eta \to -\frac{1}{\sqrt{2}}\sin\delta(\bar{u}u + \bar{d}d) - \cos\delta\bar{s}s,$$
$$\eta' \to +\frac{1}{\sqrt{2}}\cos\delta(\bar{u}u + \bar{d}d) - \sin\delta\bar{s}s,$$
$$\delta = \theta_P - \theta_I, \qquad \theta_I = \arctan\frac{1}{\sqrt{2}}. \tag{A17}$$

The masses of the pseudoscalar mesons used in our calculations are [1]

$$M_{\pi^0} = 134.977 \pm 0.0005 \text{ MeV},$$
$$M_{\eta} = 547.862 \pm 0.017 \text{ MeV},$$
$$M_{\eta'} = 957.78 \pm 0.06 \text{ MeV},$$
$$M_{\eta_c} = 2983.9 \pm 0.5 \text{ MeV}. \tag{A18}$$

For the pseudoscalar decay constants of π^0, η, and η' mesons we use the universal value identified with the pion coupling $F_\pi = 92.4$ MeV. For the η_c coupling we take the averaged value of theoretical predictions $F_{\eta_c} = 285$ MeV from Ref. [31].

Therefore, the matrix elements of specific pseudoscalar and axial operators between vacuum and pseudoscalar states are:

$$\langle 0|\bar{u}\gamma^\mu\gamma^5 u|\pi^0(p)\rangle = -\langle 0|\bar{d}\gamma^\mu\gamma^5 d|\pi^0(p)\rangle = ip^\mu F_\pi, \tag{A19}$$

$$\langle 0|\bar{u}\gamma^\mu\gamma^5 u|\eta(p)\rangle = \langle 0|\bar{d}\gamma^\mu\gamma^5 d|\eta(p)\rangle = -ip^\mu F_\pi \sin\delta, \tag{A20}$$

$$\langle 0|\bar{u}\gamma^\mu\gamma^5 u|\eta'(p)\rangle = \langle 0|\bar{d}\gamma^\mu\gamma^5 d|\eta'(p)\rangle = ip^\mu F_\pi \cos\delta, \tag{A21}$$

$$\langle 0|\bar{s}\gamma^\mu\gamma^5 s|\eta(p)\rangle = -ip^\mu F_\pi \cos\delta\sqrt{2}, \tag{A22}$$

$$\langle 0|\bar{s}\gamma^\mu\gamma^5 s|\eta'(p)\rangle = -ip^\mu F_\pi \sin\delta\sqrt{2}, \tag{A23}$$

$$\langle 0|\bar{c}\gamma^\mu\gamma^5 c|\eta_c(p)\rangle = ip^\mu F_{\eta_c}, \tag{A24}$$

$$\langle 0|\bar{u}i\gamma^5 u|\pi^0(p)\rangle = -\langle 0|\bar{d}i\gamma^5 d|\pi^0(p)\rangle = \frac{M_\pi^2}{2\hat{m}}F_\pi, \tag{A25}$$

$$\langle 0|\bar{u}i\gamma^5 u|\eta(p)\rangle = \langle 0|\bar{d}i\gamma^5 d|\eta(p)\rangle = -\frac{M_\eta^2}{2\hat{m}}F_\pi \sin\delta, \tag{A26}$$

$$\langle 0|\bar{u}i\gamma^5 u|\eta'(p)\rangle = \langle 0|\bar{d}i\gamma^5 d|\eta'(p)\rangle = \frac{M_{\eta'}^2}{2\hat{m}}F_\pi \cos\delta, \tag{A27}$$

$$\langle 0|\bar{s}i\gamma^5 s|\eta(p)\rangle = -\frac{M_\eta^2}{2\hat{m}}F_\pi \cos\delta\sqrt{2}, \tag{A28}$$

$$\langle 0|\bar{s}i\gamma^5 s|\eta'(p)\rangle = -\frac{M_{\eta'}^2}{2\hat{m}}F_\pi \sin\delta\sqrt{2}, \tag{A29}$$

$$\langle 0|\bar{c}i\gamma^5 c|\eta_c(p)\rangle = \frac{M_{\eta_c}^2}{2m_c}F_{\eta_c}, \tag{A30}$$

where $\hat{m} = m_u = m_d = 7$ MeV is the mass of u and d quarks in the isospin limit, $m_s = 25\hat{m}$ is the strange quark mass [29], $m_c = 1.275$ GeV and $m_b = 4.18$ GeV are the masses of charm and bottom quarks [1].

APPENDIX B: LFV RATES OF MESONS DECAYING INTO A LEPTONIC PAIR

Here, we present analytical results for the LFV rates of mesons decaying into a leptonic pair governed by the effective Lagrangian (8) and including effects of finite lepton masses,

$V \to \ell_1^+ \ell_2^-$ decays

$$\Gamma(V \to \ell_1^+ \ell_2^-) = \frac{P^*}{6\pi}\left[(g^{(V)}_{V\ell_1\ell_2})^2\left(1-\frac{M_-^2}{M_V^2}\right)\left(1+\frac{M_+^2}{2M_V^2}\right)+(g^{(A)}_{V\ell_1\ell_2})^2\left(1-\frac{M_+^2}{M_V^2}\right)\left(1+\frac{M_-^2}{2M_V^2}\right)\right.$$
$$\left.+2(g^{(T)}_{V\ell_1\ell_2})^2\left(1-\frac{M_-^2}{M_V^2}\right)\left(1+\frac{2M_+^2}{M_V^2}\right)-6g^{(V)}_{V\ell_1\ell_2}g^{(A)}_{V\ell_1\ell_2}\frac{M_+}{M_V}\left(1-\frac{M_-^2}{M_V^2}\right)\right] \tag{B1}$$

$S \to \ell_1^+ \ell_2^-$ decays

$$\Gamma(S \to \ell_1^+ \ell_2^-) = \frac{P^*}{4\pi}\left[(g^{(S)}_{S\ell_1\ell_2})^2\left(1-\frac{M_-^2}{M_P^2}\right)+(g^{(P)}_{S\ell_1\ell_2})^2\left(1-\frac{M_+^2}{M_P^2}\right)\right] \tag{B2}$$

$P \to \ell_1^+ \ell_2^-$ decays

$$\Gamma(P \to \ell_1^+ \ell_2^-) = \frac{P^*}{4\pi}\left[\left(g^{(P)}_{P\ell_1\ell_2}+g^{(A)}_{P\ell_1\ell_2}\frac{M_+}{M_P}\right)^2\left(1-\frac{M_-^2}{M_P^2}\right)+\left(g^{(S)}_{P\ell_1\ell_2}+g^{(V)}_{P\ell_1\ell_2}\frac{M_-}{M_P}\right)^2\left(1-\frac{M_+^2}{M_P^2}\right)\right], \tag{B3}$$

where $M_\pm = M_{\ell_1} \pm M_{\ell_2}$, $P^* = \lambda^{1/2}(M_H^2, M_{\ell_1}^2, M_{\ell_2}^2)/(2M_H)$ is the magnitude of the three momentum of leptons in the rest frame of decaying hadron H and $\lambda(x,y,z) = x^2+y^2+z^2-2xy-2yz-2xz$ is the kinematical triangle Källen function.

APPENDIX C: Q^2 DEPENDENCE OF MESON PROPAGATORS AND FORM FACTORS

Let us note that in Eqs. (9), (10), (16), (22), and (23), we neglected the squared momentum transfer Q^2-dependence of the meson propagator and the form factors $\tilde{g}_{M\ell_1\ell_2}(Q^2)$. For most of the processes of our current interest, this Q^2 dependence results in a less than 5% deviation from the approximate formulae that we use, which for our purposes is more than sufficient. Nevertheless, for two specific states (the intermediate pion in the process $\mu \to e\gamma\gamma$ and the intermediate J/ψ in the processes $\tau \to e(\mu)ee$) the Q^2 dependence of the meson propagator and of the form factors give contributions up to 80%.

Here, we present details of the Q^2 dependent contribution calculation of the meson form factors $\tilde{g}_{M\ell_1\ell_2}(Q^2)$ and propagators $D_M(Q^2)$ to the branchings of the three-body LFV decays of leptons. The meson form factors $\tilde{g}_{M\ell_1\ell_2}(Q^2)$ can be found using a covariant confined quark model [32]. Their Q^2 dependence can be parametrized as

$$\tilde{g}_{M\ell_1\ell_2}(Q^2) = 1/(1-Q^2/\Lambda_M^2), \tag{C1}$$

where Λ_M is the set of cutoff parameters given by

$$\Lambda_\pi = 0.90\ \text{GeV}, \quad \Lambda_\eta = 0.94\ \text{GeV}, \quad \Lambda_{\eta'} = 1.02\ \text{GeV}, \quad \Lambda_{\eta_c} = 4.16\ \text{GeV},$$
$$\Lambda_{f_0(500)} = 1.02\ \text{GeV}, \quad \Lambda_{f_0(980)} = 1.04\ \text{GeV}, \quad \Lambda_{a_0(980)} = 1.06\ \text{GeV}, \quad \Lambda_{\chi_{c0}} = 5.95\ \text{GeV},$$
$$\Lambda_\rho = 0.84\ \text{GeV}, \quad \Lambda_\omega = 0.83\ \text{GeV}, \quad \Lambda_\phi = 1.13\ \text{GeV}, \quad \Lambda_{J/\psi} = 3.54\ \text{GeV}, \quad \Lambda_\Upsilon = 10.07\ \text{GeV}. \tag{C2}$$

In particular, we parametrize this effect by a factor R, which is defined as the ratio of the three LFV decay branching with the complete Q^2 dependence (full result) and the branching without that dependence:

$$R = \frac{\text{Br}_{\text{full}}(\ell_1 \to \ell_2 + X)}{\text{Br}(\ell_1 \to \ell_2 + X)}. \tag{C3}$$

The coefficient R is simply the ratio of the phase space integrals for three-body LFV decays of leptons including the form factors I_{full} and without such effects I,

$$R = \frac{I^{\text{phase}}_{\text{full}}}{I^{\text{phase}}}, \tag{C4}$$

where

TABLE III. Factors R, $R_{\rm pr}$, and $R_{\rm ff}$ representing Q^2 dependence.

	$\mu^- \to e^-\gamma\gamma$ process		
Meson	$R_{\rm pr}$	$R_{\rm ff}$	R
π^0	1.788	1.009	1.808
η	1.025	1.008	1.034
η'	1.008	1.007	1.015
η_c	1.0008	1.0004	1.0013
$f_0(500)$	1.031	1.007	1.038
$f_0(980)$	1.008	1.007	1.015
$a_0(980)$	1.008	1.007	1.015
$\chi_{c0}(1P)$	1.0006	1.0002	1.0013
	$\mu^- \to e^-e^+e^-$ process		
Meson	$R_{\rm pr}$	$R_{\rm ff}$	R
ρ^0	1.013	1.011	1.023
ω	1.012	1.011	1.023
ϕ	1.007	1.006	1.013
J/ψ	1.0008	1.0006	1.001
Υ	1.0001	1.0001	1.0002
	$\tau^- \to e^-e^+e^-$ process		
Meson	$R_{\rm pr}$	$R_{\rm ff}$	R
J/ψ	1.293	1.208	1.605
Υ	1.024	1.021	1.045
	$\tau^- \to \mu^-e^+e^-$ process		
Meson	$R_{\rm pr}$	$R_{\rm ff}$	R
J/ψ	1.273	1.195	1.555
Υ	1.023	1.020	1.044

$$I^{\rm phase}_{\rm full} = \frac{\pi^2}{4M_1^2}\int_{s_2^-}^{s_2^+} ds_2 \lambda^{1/2}(M_1^2, s_2, M_2^2)\tilde{g}^2_{M\ell_1\ell_2}(s_2) D_M^2(s_2), \tag{C5}$$

$$I^{\rm phase} = \frac{\pi^2}{4M_1^2}\frac{1}{M^4}\int_{s_2^-}^{s_2^+} ds_2 \lambda^{1/2}(M_1^2, s_2, M_2^2). \tag{C6}$$

Here, $D_M(s_2) = 1/(M^2 - s_2)$ is the scalar part of meson propagator, s_2 is the Mandelstam variable (invariant mass of two-lepton or two-photon pair in the final state). The upper (s_2^+) and lower (s_2^-) limits of the s_2 variation are defined in terms of the initial lepton masses (M_1), final lepton masses (M_2) and masses of the leptonic pair (M_3, M_4) produced by the intermediate meson, as $s_2^+ = (M_1 - M_2)^2$ and $s_2^- = (M_3 + M_4)^2$. In the case of two-photon processes $s_2^- = 0$. In the evaluation of $R_{\rm pr}$ and $R_{\rm ff}$, we drop the Q^2 dependence of the meson propagator $D_M(s_2) \to 1/M^2$ or the meson form factor $\tilde{g}^2_{M\ell_1\ell_2}(s_2) \to 1$, respectively.

In Table III, we explicitly demonstrate the effect on the three-body LFV decay rates of the Q^2-dependence of the meson propagator and form factors. In particular, we parametrize this effect by the factor R, which is defined as the ratio of the three-body LFV decay taking into account the Q^2 dependences (full result) and the decay without that dependence. We present separate results coming from the Q^2 dependence in the meson propagators (factor $R_{\rm pr}$) and in the form factors (factor $R_{\rm ff}$) and also the total results (factor R) combining these two contributions. From Table III, one can see that effects of form factors are suppressed for all processes and mesons and less 2% except τ decays with J/ψ meson in the intermediate state giving about 20% contribution. Q^2 dependence of meson propagators is less than 3% for most cases except ∼80% contribution of π^0 to the $\mathrm{Br}(\mu^- \to e^-\gamma\gamma)$ and ∼30% contribution of J/ψ to the $\mathrm{Br}(\tau^- \to \ell^-e^+e^-)$. It is clear that the sizeable factors R due to the Q^2 dependence in case of mentioned mesons and modes give more stringent constraints on two-body LFV meson decays.

APPENDIX D: RELATIONS OF MESON-LEPTON TO QUARK-LEPTON COUPLINGS

Here we show the relation between quark-lepton, $\mathcal{C}_{qq}$, and meson-lepton, g_M, couplings from Eqs. (7) and (8) derived as solutions of the matching conditions (25). They are as follows

$$g^{(V/A)}_{\rho^0\ell_1\ell_2} = \frac{M_\rho^2}{\Lambda^2} f_\rho \mathcal{C}^{(3)VV/AV}_{\ell_1\ell_2}, \quad g^{(V/A)}_{\omega\ell_1\ell_2} = \frac{3M_\omega^2}{\Lambda^2} f_\omega \mathcal{C}^{(0)VV/AV}_{\ell_1\ell_2}, \quad g^{(V/A)}_{\phi\ell_1\ell_2} = -\frac{3M_\phi^2}{\Lambda^2} f_\phi \mathcal{C}^{(s)VV/AV}_{\ell_1\ell_2},$$
$$g^{(V/A)}_{J/\psi\ell_1\ell_2} = \frac{M_{J/\psi}^2}{\Lambda^2} f_{J/\psi} \mathcal{C}^{(c)VV/AV}_{\ell_1\ell_2}, \quad g^{(V/A)}_{\Upsilon\ell_1\ell_2} = \frac{M_\Upsilon^2}{\Lambda^2} f_\Upsilon \mathcal{C}^{(b)VV/AV}_{\ell_1\ell_2},$$
$$g^{(T)}_{\rho\ell_1\ell_2} = \frac{\hat{m}M_\rho}{\Lambda^2} f_\rho \mathcal{C}^{(3)TT}_{\ell_1\ell_2}, \quad g^{(T)}_{\omega\ell_1\ell_2} = \frac{3\hat{m}M_\omega}{\Lambda^2} f_\omega \mathcal{C}^{(0)TT}_{\ell_1\ell_2}, \quad g^{(T)}_{\phi\ell_1\ell_2} = -\frac{3m_s M_\phi}{\Lambda^2} f_\phi \mathcal{C}^{(s)TT}_{\ell_1\ell_2},$$
$$g^{(T)}_{J/\psi\ell_1\ell_2} = \frac{m_c M_{J/\psi}}{\Lambda^2} f_{J/\psi} \mathcal{C}^{(c)TT}_{\ell_1\ell_2}, \quad g^{(T)}_{\Upsilon\ell_1\ell_2} = \frac{m_b M_\Upsilon}{\Lambda^2} f_\Upsilon \mathcal{C}^{(b)TT}_{\ell_1\ell_2},$$
$$g^{(S/P)}_{a_0\ell_1\ell_2} = \frac{M_{a_0}^2}{\Lambda^2} f_{a_0} \mathcal{C}^{(3)SS/PS}_{\ell_1\ell_2}, \quad g^{(S/P)}_{f_0\ell_1\ell_2} = \frac{M_{f_0}^2}{\Lambda^2} f_{f_0} \mathcal{C}^{(0)SS/PS}_{\ell_1\ell_2}, \quad g^{(S/P)}_{\chi_{c0}\ell_1\ell_2} = \frac{M_{\chi_{c0}}^2}{\Lambda^2} f_{f_0} \mathcal{C}^{(c)SS/PS}_{\ell_1\ell_2},$$

$$g^{(P/S)}_{\pi\ell_1\ell_2} = \frac{M_\pi^2}{2\hat{m}\Lambda^2} F_\pi \mathcal{C}^{(3)PP/SP}_{\ell_1\ell_2}, \qquad g^{(P/S)}_{\eta_c\ell_1\ell_2} = \frac{M_{\eta_c}^2}{2m_c\Lambda^2} F_{\eta_c} \mathcal{C}^{(c)PP/SP}_{\ell_1\ell_2},$$
$$g^{(P/S)}_{\eta\ell_1\ell_2} = -\frac{M_\eta^2}{2\hat{m}\Lambda^2} F_\pi \left(\sin\delta \mathcal{C}^{(0)PP/SP}_{\ell_1\ell_2} + \frac{\hat{m}}{m_s}\cos\delta\sqrt{2}\mathcal{C}^{(s)PP/SP}_{\ell_1\ell_2} \right), \quad g^{(P/S)}_{\eta'\ell_1\ell_2} \frac{M_{\eta'}^2}{2\hat{m}\Lambda^2} F_\pi \left(\cos\delta \mathcal{C}^{(0)PP/SP}_{qq^+,\ell_1\ell_2} - \frac{\hat{m}}{m_s}\sin\delta\sqrt{2}\mathcal{C}^{(s)PP/SP}_{\ell_1\ell_2} \right),$$
$$g^{(A/V)}_{\pi\ell_1\ell_2} = -\frac{M_\pi}{\Lambda^2} F_\pi \mathcal{C}^{(3)AA/VA}_{\ell_1\ell_2}, \qquad g^{(A/V)}_{\eta_c\ell_1\ell_2} = -\frac{M_{\eta_c}}{\Lambda^2} F_{\eta_c} \mathcal{C}^{(c)AA/VA}_{\ell_1\ell_2}, \quad g^{(A/V)}_{\eta\ell_1\ell_2} = \frac{M_\eta}{\Lambda^2} F_\pi \left(\sin\delta \mathcal{C}^{(0)AA/VA}_{\ell_1\ell_2} + \cos\delta\sqrt{2}\mathcal{C}^{(s)AA/VA}_{\ell_1\ell_2} \right),$$
$$g^{(A/V)}_{\eta'\ell_1\ell_2} = -\frac{M_{\eta'}}{\Lambda^2} F_\pi \left(\cos\delta \mathcal{C}^{(0)AA/VA}_{\ell_1\ell_2} - \sin\delta\sqrt{2}\mathcal{C}^{(0)AA/VA}_{\ell_1\ell_2} \right), \tag{D1}$$

where

$$\mathcal{C}^{(0/3)\Gamma_i\Gamma_J}_{\ell_1\ell_2} = \mathcal{C}^{(u)\Gamma_i\Gamma_J}_{\ell_1\ell_2} \pm \mathcal{C}^{(d)\Gamma_i\Gamma_J}_{\ell_1\ell_2} \tag{D2}$$

is the strong isospin singlet $\mathcal{C}^{(0)}$ and $\mathcal{C}^{(3)}$ triplet combinations.

APPENDIX E: LIMITS ON LINEAR COMBINATIONS OF THE WILSON COEFFICIENTS

Here, we show the limits on the combinations of the quark-lepton couplings $\bar{\mathcal{C}} = \mathcal{C} \cdot (1\ \mathrm{TeV}/\Lambda)^2$ are

$$\left|\mathcal{C}^{(3)VV/AV}_{\mu e}\right| < 1.7 \times 10^{-3}, \qquad \left|\mathcal{C}^{(0)VV/AV}_{\mu e}\right| < 1.6 \times 10^{-3}, \qquad \left|\mathcal{C}^{(s)VV/AV}_{\mu e}\right| < 0.7 \times 10^{-3}, \tag{E1}$$

$$\left|\mathcal{C}^{(c)VV/AV}_{\mu e}\right| < 1.4 \times 10^{-4}, \qquad \left|\mathcal{C}^{(c)VV/AV}_{\tau e}\right| < 7.6 \times 10^{-2}, \qquad \left|\mathcal{C}^{(c)VV/AV}_{\tau\mu}\right| < 6.0 \times 10^{-2}, \tag{E2}$$

$$\left|\mathcal{C}^{(b)VV/AV}_{\mu e}\right| < 6.6 \times 10^{-4}, \qquad \left|\mathcal{C}^{(b)VV/AV}_{\tau e}\right| < 2.6 \times 10^{-1}, \qquad \left|\mathcal{C}^{(b)VV/AV}_{\tau\mu}\right| < 2.1 \times 10^{-1}, \tag{E3}$$

$$\left|\mathcal{C}^{(3)TT}_{\mu e}\right| < 1.2 \times 10^{-3}, \qquad \left|\mathcal{C}^{(0)TT}_{\mu e}\right| < 1.1 \times 10^{-3}, \qquad \left|\mathcal{C}^{(s)TT}_{\mu e}\right| < 0.5 \times 10^{-3}, \tag{E4}$$

$$\left|\mathcal{C}^{(c)TT}_{\mu e}\right| < 1.0 \times 10^{-4}, \qquad \left|\mathcal{C}^{(c)TT}_{\tau e}\right| < 4.5 \times 10^{-2}, \qquad \left|\mathcal{C}^{(c)TT}_{\tau\mu}\right| < 3.5 \times 10^{-2}, \tag{E5}$$

$$\left|\mathcal{C}^{(b)TT}_{\mu e}\right| < 4.7 \times 10^{-4}, \qquad \left|\mathcal{C}^{(b)TT}_{\tau e}\right| < 1.8 \times 10^{-2}, \qquad \left|\mathcal{C}^{(b)TT}_{\tau\mu}\right| < 1.5 \times 10^{-2}, \tag{E6}$$

$$\left|\mathcal{C}^{(3)PP/SP}_{\mu e}\right| < 0.2, \qquad \left|\mathcal{C}^{(c)PP/SP}_{\mu e}\right| < 4.0 \times 10^{2}, \tag{E7}$$

$$\left|\mathcal{C}^{(0)PP/SP}_{\mu e} - 0.05\mathcal{C}^{(s)PP/SP}_{\mu e}\right| < 0.4, \qquad \left|\mathcal{C}^{(0)PP/SP}_{\mu e} + 0.06\mathcal{C}^{(s)PP/SP}_{\mu e}\right| < 0.7, \tag{E8}$$

$$\left|\mathcal{C}^{(3)AA/AV}_{\mu e}\right| < 0.2, \qquad \left|\mathcal{C}^{(c)AA/AV}_{\mu e}\right| < 1.1 \times 10^{5}, \tag{E9}$$

$$\left|\mathcal{C}^{(0)AA/AV}_{\mu e} - 0.05\mathcal{C}^{(s)AA/AV}_{\mu e}\right| < 2.2, \qquad \left|\mathcal{C}^{(0)AA/AV}_{\mu e} + 0.06\mathcal{C}^{(s)AA/AV}_{\mu e}\right| < 5.9, \tag{E10}$$

$$\left|\mathcal{C}^{(3)SS/PS}_{\mu e}\right| < 60.6, \qquad \left|\mathcal{C}^{(0)AA/AV}_{\mu e}\right| < 41.5. \tag{E11}$$

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