Scouring meson decays for true muonium

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Rare meson decay experiments promise to measure branching ratios as small as 10^{-13}. This presents an opportunity to discover the $\mu^+\mu^-$ bound state true muonium. We consider a set of possible channels, all with branching ratios above $\sim 10^{-11}$. For the electromagnetic decays $\eta/\eta' \to (\mu^+\mu^-)\gamma$, theoretical and phenomenological form factors $F_{\eta/\eta'\gamma\gamma^*}(Q^2)$ allow predictions of $\mathcal{BR}(\eta \to (\mu^+\mu^-)\gamma) \sim 4.8 \times 10^{-10}$ and $\mathcal{BR}(\eta' \to (\mu^+\mu^-)\gamma) \sim 3.7 \times 10^{-11}$ at the 5% level. Discussion of experimental prospects and potential backgrounds are made.

I. INTRODUCTION

Within the Standard Model, only the Higgs interaction breaks "lepton universality," but the discovery of neutrino masses implies that at least one beyond-Standard Model modification is required. Many precision physics searches have been undertaken in the charged lepton sector to detect additional lepton universality violations. Measurements of $(g - 2)_\ell$ [1–3], charge radii [4,5], and B meson decays [6–16] have each shown hints of discrepancy. The bound state of $(\mu^+\mu^-)$, "true muonium" (TM), presents another avenue for investigating lepton universality [17,18]. To facilitate these studies, efforts are ongoing to improve theoretical predictions [19–26]. Alas, true muonium remains undetected today. There are two categories of $(\mu^+\mu^-)$ production methods discussed within the literature: particle collisions (fixed-target and collider) [27–34] or through rare decays of mesons [35–41]. Both are challenging due to the low production rates. Currently, the HPS [42] experiment is searching for true muonium [43] via $e^-Z \to (\mu^+\mu^-)X$. Another fixed-target experiment, DIRAC [44], could look for $(\mu^+\mu^-)$ in an upgraded run [45]. The existing KOTO experiment [46] and proposed NA62-KLEVER [47] hope to achieve sensitivities to K_L decays with $\mathcal{BR} \sim 10^{-13}$, which would also present an opportunity to detect true muonium [40].

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In this work, we present predictions for $\mathcal{BR}(\eta/\eta' \to (\mu^+\mu^-)\gamma)$ where true muonium is accompanied by a monochromatic photon (which in the η/η' rest frame are 233.2 and 455.6 MeV, respectively) including $\mathcal{O}(\alpha)$ radiative corrections. These calculations improve upon previous constituent quark model calculations which estimated $\mathcal{BR}(\eta \to (\mu^+\mu^-)\gamma) \approx 10^{-9}$ [35,37]. Numerous studies of $F_{\eta/\eta'\gamma\gamma^*}(Q^2)$ are available to estimate potential systematics for this process. Other discovery channels involving hadronic final states with $\mathcal{BR} \geq 10^{-12}$ are discussed and comments on backgrounds are made.

II. $\mathcal{BR}(X \to Y(\ell^+\ell^-))$

Before proceeding to the specific decays of interest, it is useful to consider where the scaling comes from. The order of magnitude of $\mathcal{BR}(X \to Y(\ell^+\ell^-))$ can be estimated by multiplying $\mathcal{BR}(X \to Y\gamma)$ by α^4, which arises from $|\mathcal{M}(\gamma^* \to \ell^{+*}\ell^{-*})\mathcal{M}(\ell^{+*}\ell^{-*} \to (\ell^+\ell^-))|^2 \propto \alpha|\psi(0)|^2$. This implies that $\mathcal{BR}(X \to Y(\mu^+\mu^-)) \lesssim 10^{-9}$. In Table I, we have included a list of channels which led to branching ratios to true muonium of $\gtrsim 10^{-12}$. There are two processes with $\mathcal{BR} > 10\%$: $\eta' \to \rho\gamma$ and the electromagnetic $\eta \to \gamma\gamma$, the latter alongside the percent level $\eta' \to \gamma\gamma$ we will discuss in more detail. The large number of J/ψ events being collected in the near future also presents the possibility of subpercent channels like $J/\psi \to \eta'\gamma$, as well as the percent level but more complicated inclusive $J/\psi \to Y_h\gamma$ [48]. Predictions of $Y_h + (\mu^+\mu^-)$ decays require knowledge of hadronic transition form factors and are more limited in precision compared to processes like $\eta/\eta' \to (\mu^+\mu^-)\gamma$ where the better studied electromagnetic form factors are available. These electromagnetic decays are the main focus of the rest of the paper.

TABLE I. Meson decay branching ratios involving photons from Particle Data Group [49] considered in this work. The first two are electromagnetic decays, while the others are strong decays. Branching ratios to true muonium can be estimated by multiplying by $\alpha^4 \approx 2.8 \times 10^{-9}$.

Channel	$\mathcal{BR}$
$\eta \to \gamma\gamma$	$3.941(20) \times 10^{-1}$
$\eta' \to \gamma\gamma$	$2.22(8) \times 10^{-2}$
$\eta' \to \rho\gamma$	$2.89(5) \times 10^{-1}$
$\omega \to \pi^0\gamma$	$8.40(22) \times 10^{-2}$
$\eta' \to \omega\gamma$	$2.62(13) \times 10^{-2}$
$J/\psi \to \eta'\gamma$	$5.13(17) \times 10^{-3}$
$J/\psi \to Y_h\gamma$	$7.1(1.2) \times 10^{-2}$

Following previous calculations for the electromagnetic decay of mesons to atoms [35–40], the leading-order (LO) decay rate is

$$\frac{\Gamma_{\text{LO}}(\eta/\eta' \to (\mu^+\mu^-)\gamma)}{\Gamma(\eta/\eta' \to \gamma\gamma)}$$
$$= \frac{\alpha^4\zeta(3)}{2}(1 - z_{\text{TM}})^3 |f_{\eta/\eta'}(z_{\text{TM}})|^2, \qquad (1)$$

where $\zeta(3) = \sum_n 1/n^3$ arising from the sum over all allowed $(\mu^+\mu^-)$ states, $z_{\text{TM}} = M_{\text{TM}}^2/M_{\eta/\eta'}^2 \approx 4M_\mu^2/M_{\eta/\eta'}^2$, and $f(z) = F_{\eta/\eta'\gamma\gamma^*}(z)/F_{\eta/\eta'\gamma\gamma}(0)$. Including the next-to-leading-order (NLO) contributions, the squared matrix element is

$$|\mathcal{M}|^2 = |\mathcal{M}_{\text{LO}} + \mathcal{M}_{\text{NLO}}|^2$$
$$= |\mathcal{M}_{\text{LO}}|^2 + \mathcal{M}_{\text{LO}}\mathcal{M}_{\text{NLO}}^* + \mathcal{M}_{\text{LO}}^*\mathcal{M}_{\text{NLO}}$$
$$+ |\mathcal{M}_{\text{NLO}}|^2. \qquad (2)$$

The final $|\mathcal{M}_{\text{NLO}}|^2$ is $\mathcal{O}(\alpha^2)$ and therefore can be neglected for our purpose. The remaining $\mathcal{O}(\alpha)$ radiative corrections can be decomposed into six contributions: vacuum polarizations for the three lepton flavors and hadrons previously computed [35–37,40], the leptonic vertex correction [36], and the initial-meson-dependent double-virtual-photon contribution of Fig. 1. With the exception of the double-virtual-photon contribution, all contributions only depend on $|F_{\eta/\eta'\gamma\gamma^*}(z)|^2$ and therefore modify Eq. (1) by a multiplicative factor $1 + C\frac{\alpha}{\pi}$. In contrast, the double-virtual-photon contribution gives a contribution

$$\Gamma_{\gamma^*\gamma^*} \sim F_{\eta/\eta'\gamma\gamma^*}^*(z_{\text{TM}})\mathcal{N}_{\text{LO}}^*$$
$$\times \int \frac{d^4k}{(2\pi)^4} F_{\eta/\eta'\gamma^*\gamma^*}(k^2, (P_{\eta/\eta'} - k)^2)\mathcal{N}_{\text{NLO}}$$
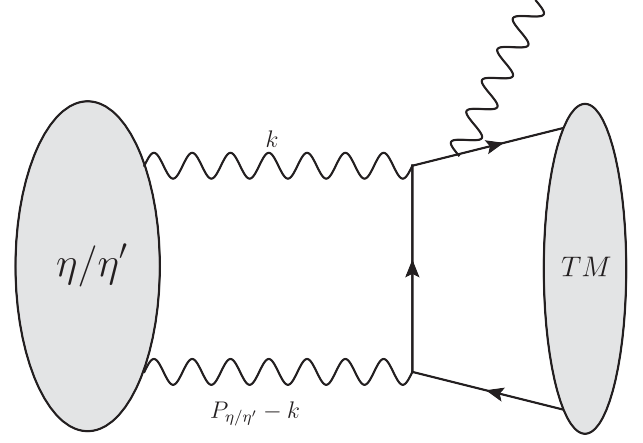$$+ \text{c.c.}, \qquad (3)$$

FIG. 1. One of the Feynman diagrams of $\eta/\eta' \to \gamma^*\gamma^* \to (\mu^+\mu^-)\gamma$ which contribute to the branching ratio at $\mathcal{O}(\alpha^5)$ and is proportional to $F_{\gamma^*\gamma^*}(z_1, z_2)$.

where $\mathcal{N}_i$ represent the remaining matrix elements after the form factor has been factored out. The convolution depends upon the initial state and involves the double-virtual-photon form factor. Formally, this prevents writing the radiative corrections into the form of a coefficient C multiplying Eq. (1). Following previous works [50–52] in three- and four-body electromagnetic decays of mesons, we approximate the double-virtual-photon form factor as a constant $F_{\gamma\gamma^*}(0, z_{\text{TM}})$. This allows it to be moved out of the integral, and therefore this contribution also provides only a multiplicative correction to the leading-order term. After introducing a counterterm in the $\overline{\text{MS}}$ scheme to cancel out the $\frac{1}{\epsilon}$ divergence of the integral, we are able to find the double-virtual-photon contribution. This approximation has been compared in three- and four-body pseudoscalar meson decays (e.g., $\pi \to e^+e^-\gamma$ and $\eta \to e^+e^-\mu^+\mu^-$) where the virtual momentum is not fixed [50] and Eq. (3) is evaluated directly using various models of the double-virtual-photon form factors. From these studies, the approximation introduces systematic errors less than 10% [51,52] where the largest (~10%) was for $\eta \to 4\mu$, which is very distinct from $\eta/\eta' \to (\mu^+\mu^-)\gamma$ and the smallest (<1%) for the $\pi \to e^+e^-\gamma$. We take 4% as a conservative estimate for our particular process, given that unlike the three- and four-body processes we do not integrate over $F_{\eta/\eta'\gamma\gamma^*}$ afterward to arrive at the final decay rate due to our two-body nature.

Within this approximation the branching ratios ($\Gamma/\Gamma_{\text{total}}$) normalized to the two-photon decay can be expressed as

$$\frac{\mathcal{BR}(\eta/\eta' \to (\mu^+\mu^-)\gamma)}{\mathcal{BR}(\eta/\eta' \to \gamma\gamma)}$$
$$= \frac{\alpha^4\zeta(3)}{2}(1 - z_{\text{TM}})^3\left[1 + C_{\eta/\eta'}\frac{\alpha}{\pi}\right]|f_{\eta/\eta'}(z_{\text{TM}})|^2, \qquad (4)$$

where $C_\eta = -0.35(6)$ and $C_{\eta'} = 0.35(3)$ are the total radiative corrections, which we discuss in detail in the next section.

III. COMPUTING $C_{\eta/\eta'}$

To reiterate, the six contributions at $\mathcal{O}(\alpha)$ are vacuum polarizations for the three lepton flavors and hadrons, the vertex correction, and the double-virtual-photon contribution. Together their sum is

$$C_{\eta/\eta'} = [C_{eVP} + C_{\mu VP} + C_{\tau VP} + C_{hVP} + C_{ver}] + C_{\gamma^*\gamma^*}$$
$$= \begin{cases} \left[\frac{8.526(4)}{9}\right] - \frac{11.7(5)}{9} & \text{for } \eta \\ \left[\frac{8.526(4)}{9}\right] - \frac{5.4(3)}{9} & \text{for } \eta' \end{cases}, \quad (5)$$

where the bracketed terms are independent of the initial meson and have been computed in the literature [35–37,40]. C_{iVP} indicate vacuum polarization contributions from $i = e$, μ, and hadrons (where as pointed out in [19] the electron vacuum polarization contributions to true muonium are substantial), C_{ver} is the vertex correction term of [36], while $C_{\gamma^*\gamma^*}$ is the contribution from the diagram in Fig. 1 and alike. A similar calculation for positronium, where the τ, μ and hadronic vacuum polarization and the double-virtual-photon corrections are negligible, finds the $\frac{\alpha}{\pi}$ coefficient to be $C_0 = C_{VP} + C_{ver} = -52/9$ [36].

C_{iVP} are found by computing

$$C_{iVP} = 4m_\mu^2 \int_{4m_i^2}^\infty dt \, \frac{\text{Im}\Pi(t)}{t(4m_\mu^2 - t)} \quad (6)$$

from the spectral functions $\text{Im}\Pi(t)$. This standard QED vacuum polarization function is known to leading order analytically for the leptons and is derived from the experiment for the hadronic distribution.

To derive $C_{\gamma^*\gamma^*}$, we computed the double-virtual-photon matrix element using FEYNCALC [53,54] and obtained its interference with the leading-order matrix element. This reproduces the result of [50] where it is called $\delta^{1\gamma IR;\text{loop}}(x, y)$ [Eq. (3.16)]. We then evaluate it for our process at $x = z_{\text{TM}}$ and $y = 0$ to obtain $C_{\gamma^*\gamma^*}$ by subtracting the $\frac{1}{\epsilon}$ pole in the $\overline{\text{MS}}$ scheme.

IV. FORM FACTOR DEPENDENCE

Evaluating Eq. (4) with our computed radiative corrections and the experimentally measured $\mathcal{BR}(\eta/\eta' \to (\mu^+\mu^-)\gamma)$ [49]

$$\mathcal{BR}(\eta \to \gamma\gamma) = 39.41(20)\%,$$
$$\mathcal{BR}(\eta' \to \gamma\gamma) = 2.22(8)\%, \quad (7)$$

we arrive at predictions for the branching ratios that depend only on the value of the η/η' electromagnetic form factors

$$\mathcal{BR}(\eta \to (\mu^+\mu^-)\gamma) = 4.14(3) \times 10^{-10}|f(z_{\text{TM}})|^2,$$
$$\mathcal{BR}(\eta' \to (\mu^+\mu^-)\gamma) = 3.26(12) \times 10^{-11}|f(z_{\text{TM}})|^2, \quad (8)$$

where the dominant error is from $\mathcal{BR}(\eta/\eta' \to \gamma\gamma)$. The radiative corrections correspond to 0.07% corrections to the branching ratios, and therefore the statistical uncertainty dominating $\mathcal{BR}(\eta/\eta' \to \gamma\gamma)$ prevents their measurement via this normalization. This error could be reduced, yielding an ~0.07% precision level measurement, from a 10^4 times larger η/η' sample. This is within reach of near-term experimental proposals.

Results for the form factors $F_{\eta/\eta'\gamma\gamma^*}(Q^2)$ can be broadly classified into three groups: theoretical predictions [55–58], experimental extractions [59–64], and dispersion analyses [65–68]. The standard parametrization for $f(z)$ is a series expansion in z

$$f(z) = 1 + b_{\eta/\eta'}z + c_{\eta/\eta'}z^2 + d_{\eta/\eta'}z^3 + \cdots, \quad (9)$$

which for all but the dispersion analyses of [67,68] truncate at first order. The theoretical predictions make vastly different assumptions about the coupling between η/η' and the photon, as well as different modeling of the mixing between the two mesons. The experimental results rely upon integrating the functional form of Eq. (9) in Q^2 bins, and then nonlinearly fitting $b_{\eta/\eta'}$. The dispersive analyses rely upon connecting experimental data for multiple processes to the virtual-photon form factors through analyticity and crossing symmetry. We have tabulated all of the form factors considered in this work in Table II. With the exception of the Brodsky-Lepage interpolation predictions of η [56], the branching ratios predicted for true muonium agree within the uncertainties.

V. LEPTON UNIVERSALITY TESTS

If this discovery channel for true muonium is utilized, one should also be able to measure the positronium (e^+e^-) branching ratio ~10^{-10}. This would allow for the construction of a lepton universality test ratio,

$$R = \frac{\mathcal{BR}(\eta/\eta' \to (\mu^+\mu^-)\gamma)}{\mathcal{BR}(\eta/\eta' \to (e^+e^-)\gamma)}$$
$$= \frac{(1 - z_{\text{TM}})^3(1 + C_{\eta/\eta'}\frac{\alpha}{\pi})|f(z_{\text{TM}})|^2}{(1 - z_{Ps})^3(1 - \frac{52}{9}\frac{\alpha}{\pi})|f(z_{Ps})|^2}$$
$$= K_{\eta/\eta'}\left|\frac{f(z_{\text{TM}})}{f(z_{Ps})}\right|^2, \quad (10)$$

where $K_\eta = 0.62469(8)$ and $K_{\eta'} = 0.87340(5)$. The dominant error in $K_{\eta/\eta'}$ is from the approximation of the double-virtual-photon contribution to $C_{\eta/\eta'}$ and would require an unreasonably precise measurement of the ratio to probe. R could be used in two ways. First, if form factors are taken as inputs, R tests lepton universality at the 5% level due to the

TABLE II. Form factor coefficients and $\mathcal{BR}(\eta/\eta' \to (\mu^+\mu^-)\gamma)$.

	η	Coefficients	$\mathcal{BR}(\eta \to (\mu^+\mu^-)\gamma) \times 10^{10}$	η'	Coefficients	$\mathcal{BR}(\eta' \to (\mu^+\mu^-)\gamma) \times 10^{11}$	Ref.
χPT	b_η	0.51	4.79(3)	$b_{\eta'}$	1.47	3.74(14)	[55]
VMD	b_η	0.53	4.82(3)	$b_{\eta'}$	1.33	3.70(14)	[55]
CQ loops	b_η	0.51	4.79(3)	$b_{\eta'}$	1.30	3.69(14)	[55]
BL Interp.	b_η	0.36	4.59(3)	$b_{\eta'}$	2.11	3.96(15)	[56]
RχT—1 Octet	b_η	0.546(9)	4.84(3)	$b_{\eta'}$	1.384(3)	3.71(14)	[57]
RχT—2 Octets	b_η	0.521(2)	4.81(3)	$b_{\eta'}$	1.384(3)	3.71(14)	[57]
Anomaly SR	b_η	0.51	4.79(3)	$b_{\eta'}$	1.06	3.61(13)	[58]
Anomaly SR	b_η	0.54	4.83(3)	$b_{\eta'}$	1.16	3.64(14)	[58]
CELLO	b_η	0.428(89)	4.68(12)	$b_{\eta'}$	1.46(23)	3.74(16)	[59]
CLEO	b_η	0.501(38)	4.78(6)	$b_{\eta'}$	1.24(8)	3.67(14)	[60]
Lepton-G	b_η	0.57(12)	4.87(17)	$b_{\eta'}$	1.6(4)	3.79(20)	[61,69]
NA60	b_η	0.585(51)	4.89(8)	$\cdots$	$\cdots$	$\cdots$	[62]
WASA	b_η	0.68(26)	5.0(4)	$\cdots$	$\cdots$	$\cdots$	[63]
A2	b_η	0.59(5)	4.90(8)	$\cdots$	$\cdots$	$\cdots$	[64]
DA	b_η	$0.62^{+0.07}_{-0.03}$	4.94(10)	$b_{\eta'}$	$1.45^{+0.17}_{-0.12}$	3.74(15)	[65]
DA	b_η	$0.57^{+0.06}_{-0.03}$	4.87(9)	$\cdots$	$\cdots$	$\cdots$	[66]
RA	b_η	$0.576(11)_{st}(4)_{sy}$			$b_{\eta'}$	$1.31(4)_{st}(1)_{sy}$	
	c_η	$0.339(15)_{st}(5)_{sy}$			$c_{\eta'}$	$1.74(9)_{st}(3)_{sy}$	
	d_η	$0.200(14)_{st}(18)_{sy}$	$4.953(30)_{st}(6)_{sy}$		$d_{\eta'}$	$2.30(19)_{st}(21)_{sy}$	$3.720(140)_{st}(4)_{sy}$	[67,68]

double-virtual-photon approximation and $\mathcal{O}(\alpha^2)$ contributions and would be competitive with other lepton universality tests in the light meson sector. The other use would be to constrain the form factors themselves within the uncertainties via R's fixed Q^2 value dependence allowed by leptonic atom production, unlike other decays that require integration over Q^2. This would partially break the degeneracy between coefficients in Eq. (9) because it does not require integrating the differential cross section over Q^2 bins.

VI. EXPERIMENTAL OUTLOOK

Compared to this theoretical precision, the current and near-future experimental outlooks are less optimistic. While K_L experiments are reaching sensitivities of 10^{-13}, the sensitivity of J/ψ and η/η' searches is worse. At present, the BESIII experiments have the largest J/ψ and η/η' datasets. From the 1.3×10^9 J/ψ events collected and using the two largest branching ratios, $\gamma\eta/\eta'$ and $\phi\eta/\eta'$, one anticipates 2.0×10^6 η events and 7.7×10^6 η' events with a factor to 10 increase expected in the next decade [70]. This 10^{10} J/ψ production capability is right at the edge of what is necessary for an inclusive search. Another similar sized dataset exists from the A2 collaboration in Mainz where η is produced through $\gamma p \to \eta p$ and have 6.2×10^6 η events [71]. The recently approved JLab Eta Factory experiment anticipates collecting 1.3×10^8 η and 9.8×10^7 η' events with 200 days of beam time [72], and beyond that can be run in parallel with the GlueX

experiment if the latter is extended beyond 2023 [72]. Further into the future, proposals like REDTOP at FermiLab discussed methods to accrue $10^{13}\eta$ and $10^{11}\eta'$ events [73].

Clearly the present $10^6\eta/\eta'$ BESIII and A2 data are insufficient for observing true muonium. From these, one would expect to place upper limits on the branching ratios $\mathcal{BR} \lesssim 10^{-5}$ which is 10^4–10^5 times larger than the predicted rates. This should be compared with the situation for $K_L \to (\mu^+\mu^-)\gamma$ where upper bounds of $\mathcal{BR} \lesssim 10^{-9}$ are possible at KTEV [74,75], which are superior limits but still 10^3 times as large as predicted [40]. In the next decade, BESIII's larger dataset is still inadequate for even single-event detection through the η/η' processes, although the inclusive J/ψ channel is potentially viable. The JLab Eta Factory experiment would be competitive with the possible bounds from KTEV based on the Standard Model predictions. What is required is a proposal like REDTOP at FermiLab which would be sufficient for a discovery of true muonium through the decay of η with $N_s > 100$ events and potentially an observation of the η' decay.

The most promising signal channel for discovering true muonium in mesonic decays is $e^+e^-\gamma$, with a large background from the free decays $\eta/\eta' \to e^+e^-\gamma$. This background can be computed by integrating the differential cross section in an invariant mass bin M_{bin}, centered around the $(\mu^+\mu^-)$ peak defined as $[2m_\mu - M_{bin}/2, 2m_\mu + M_{bin}/2]$ since $\frac{p_{TM}^2 - 4m_\mu^2}{4m_\mu^2} \ll 1$. For bin sizes similar to BESIII (20 MeV), the values are $\mathcal{BR}(\eta \to e^+e^-\gamma)_{bin} = 4 \times 10^{-6}M_{bin}$, and

$\mathcal{BR}(\eta' \to e^+ e^- \gamma)_{\text{bin}} = 3 \times 10^{-7} M_{\text{bin}}$ where M_{bin} is in MeV. This large raw background, which corresponds to $N_b \sim 10^4 \times N_s$, must be reduced or even a REDTOP-like proposal is insufficient. If we estimate the significance of an observation by counting statistics, $\sigma \approx N_s / \sqrt{N_b}$, then at least $N_s > 10^4$ is required for an observation. Luckily, several distinct features of the three-body decay compared to true muonium decays can plausibly reduce the background by at least an order of magnitude, as we will explain in what follows.

The two two-body decay topology suggests cuts on momenta and angular distribution would be powerful in background suppression, but exact values of the suppression will be highly detector efficiency dependent. As an example, for radiative Dalitz decay the angle θ_e between the $e^+ e^-$ can be arbitrary, but from the true muonium decay $\theta_e \sim m_{\text{TM}} / E_{\text{TM}} \sim 50° \times \frac{\text{GeV}}{E_{\eta/\eta'}}$. In BESIII, where the typical η/η' is produced from the decay of J/Ψ, one finds $\theta_e \sim 30°$. This correlation can be more precisely measured than the invariant mass and can yield a factor of 10 in background suppression. Full reconstruction of the η/η' allows for cuts on the γ energy, where the bin resolution is $\mathcal{O}(M_{\text{bin}})$. The antiparallel correlation between the γ and the true muonium from the η/η' decay yields further background suppression. To estimate this suppression, we compute the three-body decay rate where the angle between e^+ and γ is restricted to be greater than 120° (a wider range than the 165° suggested by θ_e). These cuts result in an additional factor of 3 reduction in background. The highly collimated nature of true muonium production in both angle and invariant mass means these cuts will have minimum effect on signal events. With these, we find $N_b \sim 300 \times N_s$, sufficient for an observation with a REDTOP-like experiment where hundreds of signal events occur.

Further reduction of background is possible with more aggressive or detector-dependent cuts. If the vertex resolution can be made much better than 0.5 mm, cuts can be made using the proper lifetime of true muonium ground state $c\tau = 0.5$ mm which can strongly remove backgrounds but exponentially reduces the signal as well. Otherwise nearly all the $e^+ e^-$ will be insufficiently separated from the primary $\eta/\eta' \to e^+ e^- \gamma$ vertex to distinguish. The advent of picosecond timing in experiments also presents opportunities for clean discrimination from $e^+ e^-$ backgrounds based on time of flight, but without severe degradation of signal.

VII. CONCLUSIONS

In this paper, we have considered a set of channels for discovery of true muonium, all with branching ratios above $\sim 10^{-11}$. For the electromagnetic decays $\eta/\eta' \to (\mu^+ \mu^-)\gamma$, theoretical and phenomenological form factors $F_{\eta/\eta'\gamma\gamma^*}(Q^2)$ allow predictions of $\mathcal{BR}(\eta' \to (\mu^+ \mu^-)\gamma) \sim 4.8 \times 10^{-10}$ and $\mathcal{BR}(\eta' \to (\mu^+ \mu^-)\gamma) \sim 3.7 \times 10^{-11}$ at the 5% level. While present experimental luminosities are inadequate, there is reason for hope in the near future. We have shown how some of the potential backgrounds can be reduced given the special topology of the decay, but future work is necessary to rigorously model the realistic detector backgrounds. Given that rare meson decays with $\mathcal{BR} \sim 10^{-11}$ are presently being measured, there seems no fundamental obstacle to discovering true muonium in this way, although it will be a similar challenge.

ACKNOWLEDGMENTS

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