U-spin sum rules for CP asymmetries of three-body charmed baryon decays

Yuval Grossman* and Stefan Schacht†

Department of Physics, LEPP, Cornell University, Ithaca, New York 14853, USA

(Received 6 December 2018; published 12 February 2019)

Triggered by a recent LHCb measurement and prospects for Belle II, we derive U-spin symmetry relations between integrated CP asymmetries of three-body Λ_c^+ and Ξ_c^+ decays. The sum rules read $A_{CP}(\Lambda_c^+ \to pK^-K^+) + A_{CP}(\Xi_c^+ \to \Sigma^+\pi^-\pi^+) = 0$, $A_{CP}(\Lambda_c^+ \to p\pi^-\pi^+) + A_{CP}(\Xi_c^+ \to \Sigma^+ K^-K^+) = 0$, and $A_{CP}(\Lambda_c^+ \to \Sigma^+\pi^-K^+) + A_{CP}(\Xi_c^+ \to pK^-\pi^+) = 0$. No U-spin sum rule exists between $A_{CP}(\Lambda_c^+ \to pK^-K^+)$ and $A_{CP}(\Lambda_c^+ \to p\pi^-\pi^+)$. All of these sum rules are associated with a complete interchange of d and s quarks. Furthermore, there are no U-spin CP asymmetry sum rules which hold to first order U-spin breaking.

DOI: 10.1103/PhysRevD.99.033005

I. INTRODUCTION

Recently, after establishing the first evidence for CP violation in beauty baryon decays [1], LHCb measured the difference of CP asymmetries of the three-body singly Cabibbo-suppressed (SCS) Λ_c^+ decays [2],

$$A_{CP}(\Lambda_c^+ \to pK^-K^+) - A_{CP}(\Lambda_c^+ \to p\pi^-\pi^+) = (0.30 \pm 0.91 \pm 0.61)\%. \quad (1)$$

Here, A_{CP} is the CP asymmetry of the rates integrated over the whole phase space (for details see Ref. [2]), and we give a formal definition in Eq. (22). Prospects for future improvements are bright [3], and there is also a rich physics program with charmed baryons at Belle II [4,5].

For charmed meson decays, sum rules between direct CP asymmetries are known. In the U-spin limit we have for the direct CP asymmetries (see, e.g., Refs. [6–9])

$$a_{CP}^{\mathrm{dir}}(D^0 \to K^+K^-) + a_{CP}^{\mathrm{dir}}(D^0 \to \pi^+\pi^-) = 0, \quad (2)$$

$$a_{CP}^{\mathrm{dir}}(D^+ \to K_S K^+) + a_{CP}^{\mathrm{dir}}(D_s^+ \to K_S \pi^+) = 0. \quad (3)$$

Generalizations including $\mathrm{SU}(3)_F$ breaking effects have also been discussed in the literature [9–11]. Consequently, in view of the measurement, Eq. (1), the question arises if similar U-spin symmetry relations also exist between the decays involved therein. In this article we address this question. We focus therefore only on SCS three-body charmed baryon decays which are related to $\Lambda_c^+ \to pK^-K^+$ and $\Lambda_c^+ \to p\pi^-\pi^+$ by U-spin. These are the decays $\Xi_c^+ \to \Sigma^+\pi^-\pi^+$, $\Lambda_c^+ \to \Sigma^+\pi^-K^+$, $\Xi_c^+ \to pK^-\pi^+$, and $\Xi_c^+ \to \Sigma^+K^-K^+$, i.e., altogether six decay channels connected by U-spin.

Naively, one could expect that replacing the D^0 by a Λ_c^+ and adding a proton in the final states in Eq. (2) would also give a valid sum rule. As we show, however, the presence of the spectator quark has nontrivial implications as the three-body decay allows more combinatorial possibilities for the flavor-flow diagrams. The d spectator quark can end in the proton or the pion, but not in the kaon. Therefore, it turns out that $\Lambda_c^+ \to p\pi^-\pi^+$ has additional independent topological diagrams which are not present in the case of $\Lambda_c^+ \to pK^-K^+$ and there is no U-spin sum rule between the two respective CP asymmetries. However, we find that analogs of Eq. (2) still exist and correlate Λ_c^+ and Ξ_c^+ decays. These sum rules share with Eqs. (2) and (3) the feature that they come from interchanging all d and s quarks of a given process [12–14].

The symmetries of charm decay amplitudes which lead to correlations between different CP asymmetries can be expressed in the form of topological diagrams or reduced matrix elements from group theory. After reviewing the available literature on charmed baryon decays in Sec. II, we introduce both parametrizations in Sec. III. We show that both approaches result in equivalent decompositions. In Sec. IV we discuss how the pointwise CP asymmetries are connected to the integrated ones and conclude in Sec. V. In Appendix we give the U-spin breaking contributions which show that no CP asymmetry sum rules exist at first order U-spin breaking.

*yg73@cornell.edu
†ss3843@cornell.edu

II. LITERATURE REVIEW

A variety of methods has been applied to charm baryon decays in the literature. Most promising are $SU(3)_F$ methods; however, large corrections of $\mathcal{O}(30\%)$ are expected from $SU(3)_F$ breaking. Those symmetry-based methods have been used for two-body charmed baryon decays for a long time, including discussions of CP violation [15–23]; for general reviews see Refs. [24–27]. The connection with the diagrammatic approach for two-body decays [28–31] and $SU(3)_F$ breaking [32] has also been discussed, and even $SU(4)_F$ has been applied [33,34]. More recent works, which, however, do not discuss CP violation are Refs. [35–38]. Besides $SU(3)_F$ there have also been several other approaches to two-body charmed baryon decays, such as (covariant) quark models [39–42], pole and factorization models [43–45], heavy quark effective theory (HQET) [25,46], and a light-front approach [47]. A comparison of several model-dependent approaches is provided in Ref. [48]. The CP violating effect from the interference of charm and neutral kaon decays in two-body charmed baryon decays has been discussed in Ref. [49]. CP violation in $\Lambda_c \to BP$ and $\Lambda_c \to BV$ (where B is a baryon, P a pseudoscalar meson, and V a vector meson) has been discussed in Ref. [50], however, not in the context of $SU(3)_F$ sum rules. Prospects for decay asymmetry parameter measurements at BESIII are given in Ref. [51]. There is also literature on using $SU(3)_F$ for decays of baryons with more than one charm quark; see Refs. [52–57].

Three-body charmed baryon decays have been covered in the $SU(3)_F$ approach in Refs. [19,58–60]. A general analysis of the new physics (NP) sensitivity of different baryonic decay channels can be found in Ref. [61]. In Ref. [62] a statistical isospin model has been applied. However, the Cabibbo-Kobayashi-Maskawa (CKM)–subleading parts which are essential for CP asymmetries are not studied in these references. Moreover, we were unable to find sum rules for CP asymmetries of three-body charmed baryon decays in the literature, and this is what we do next.

III. U-SPIN DECOMPOSITION

In this paper we consider only the Standard Model (SM). Then, the CKM structure of amplitudes of SCS charm decays can be written as

$$\mathcal{A} = \Sigma(A_\Sigma^s - A_\Sigma^d) + \Delta A_\Delta, \tag{4}$$

where A_Σ^s, A_Σ^d, and A_Δ carry a strong phase only. The CKM matrix elements appear in the combinations

$$\Sigma \equiv \frac{V_{cs}^* V_{us} - V_{cd}^* V_{ud}}{2},$$
$$\Delta \equiv \frac{V_{cs}^* V_{us} + V_{cd}^* V_{ud}}{2} = -\frac{V_{cb}^* V_{ub}}{2}, \tag{5}$$

where we used CKM unitarity for Δ.

A_Σ^s (A_Σ^d) contains $c \to s$ ($c \to d$) quark-level transitions. Note that for some decays both A_Σ^s and A_Σ^d are nonzero; see Table I. We have $\Delta \ll \Sigma$; thus, $A_\Sigma \equiv A_\Sigma^s - A_\Sigma^d$ is the CKM-leading part, whereas A_Δ is CKM subleading. Actually, the contribution of ΔA_Δ is negligible for the current and near-future experimental precision of branching ratio measurements. However, the interference of ΔA_Δ with ΣA_Σ is essential for nonvanishing direct charm CP asymmetries.

For deriving the diagrammatic and group-theoretical parametrizations we use the following conventions for the quark flavor states of the relevant baryon and meson states, which are compatible with Refs. [63–66],

$$|\Lambda_c^+\rangle \equiv |udc\rangle = \left|\frac{1}{2},\frac{1}{2}\right\rangle, \qquad |\Xi_c^+\rangle \equiv |usc\rangle = \left|\frac{1}{2},-\frac{1}{2}\right\rangle,$$
$$|p\rangle \equiv |uud\rangle = \left|\frac{1}{2},\frac{1}{2}\right\rangle, \qquad |\Sigma^+\rangle \equiv |uus\rangle = \left|\frac{1}{2},-\frac{1}{2}\right\rangle,$$
$$|K^+\rangle \equiv |u\bar{s}\rangle = \left|\frac{1}{2},\frac{1}{2}\right\rangle, \qquad |\pi^+\rangle \equiv |u\bar{d}\rangle = -\left|\frac{1}{2},-\frac{1}{2}\right\rangle,$$
$$-|\pi^-\rangle \equiv |d\bar{u}\rangle = \left|\frac{1}{2},\frac{1}{2}\right\rangle, \qquad -|K^-\rangle \equiv |s\bar{u}\rangle = \left|\frac{1}{2},-\frac{1}{2}\right\rangle,$$

and where we write the states as U-spin doublets. The operators of the effective Hamiltonian for SCS decays can be written as a sum of spurions with $\Delta U = 1$ and $\Delta U = 0$,

$$\mathcal{H}_{\mathrm{eff}} \sim \Sigma(1,0) + \Delta(0,0), \tag{6}$$

where $(i,j) \equiv \mathcal{O}_{\Delta U_3=j}^{\Delta U=i}$; see the discussion in Ref. [67]. We show here the flavor structure with respect to U-spin only, absorbing any overall factors into the group representations.

The group-theoretical decomposition is obtained by applying the Wigner-Eckart theorem. For the final states, we use the order $(B \otimes P^-) \otimes P^+$; i.e., we calculate first the tensor product of the baryon with the negatively charged pseudoscalar, and then we calculate the tensor product of the result with the positively charged pseudoscalar. For the final state $\langle\frac{1}{2}|$ we put a subscript "0" or "1" depending on whether it comes from the tensor product $0 \times \frac{1}{2}$ or $1 \times \frac{1}{2}$,

TABLE I. SCS decays connected to $\Lambda_c \to p\pi^+\pi^-$ by U-spin and their underlying quark level transitions in the nonpenguin diagrams.

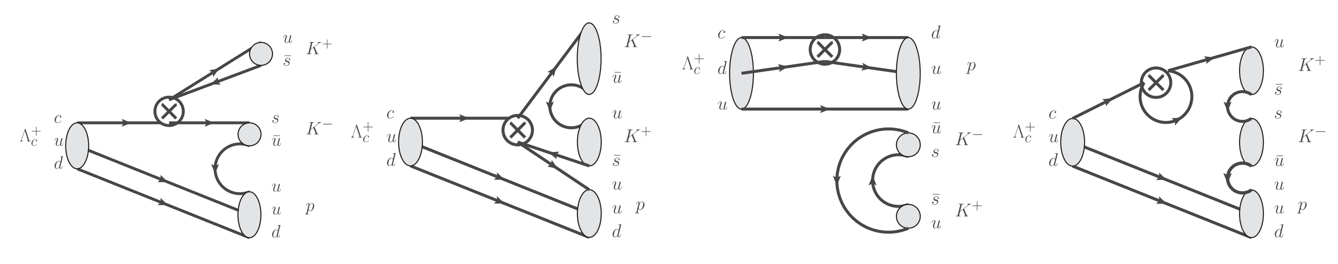
Decay ampl. $\mathcal{A}$	$c \to s$	$c \to d$
$\mathcal{A}(\Lambda_c^+ \to pK^-K^+)$	✓	✓
$\mathcal{A}(\Xi_c^+ \to \Sigma^+\pi^-\pi^+)$	✓	✓
$\mathcal{A}(\Lambda_c^+ \to \Sigma^+\pi^-K^+)$	×	✓
$\mathcal{A}(\Xi_c^+ \to pK^-\pi^+)$	✓	×
$\mathcal{A}(\Lambda_c^+ \to p\pi^-\pi^+)$	×	✓
$\mathcal{A}(\Xi_c^+ \to \Sigma^+K^-K^+)$	✓	×

TABLE II. SCS decays connected to $\Lambda_c \to p\pi^+\pi^-$ by U-spin and their group-theoretical decomposition.

Decay ampl. $\mathcal{A}$	$\Sigma\langle\frac{1}{2}\vert_0 1\vert\frac{1}{2}\rangle$	$\Sigma\langle\frac{1}{2}\vert_1 1\vert\frac{1}{2}\rangle$	$\Sigma\langle\frac{3}{2}\vert 1\vert\frac{1}{2}\rangle$	$\Delta\langle\frac{1}{2}\vert_0 0\vert\frac{1}{2}\rangle$	$\Delta\langle\frac{1}{2}\vert_1 0\vert\frac{1}{2}\rangle$
$\mathcal{A}(\Lambda_c^+ \to pK^-K^+)$	$-\frac{1}{\sqrt{6}}$	$\frac{1}{3\sqrt{2}}$	$-\frac{\sqrt{2}}{3}$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{6}}$
$\mathcal{A}(\Xi_c^+ \to \Sigma^+\pi^-\pi^+)$	$\frac{1}{\sqrt{6}}$	$-\frac{1}{3\sqrt{2}}$	$\frac{\sqrt{2}}{3}$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{6}}$
$\mathcal{A}(\Lambda_c^+ \to \Sigma^+\pi^-K^+)$	$\frac{1}{\sqrt{6}}$	$\frac{1}{3\sqrt{2}}$	$-\frac{\sqrt{2}}{3}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{6}}$
$\mathcal{A}(\Xi_c^+ \to pK^-\pi^+)$	$-\frac{1}{\sqrt{6}}$	$-\frac{1}{3\sqrt{2}}$	$\frac{\sqrt{2}}{3}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{6}}$
$\mathcal{A}(\Lambda_c^+ \to p\pi^-\pi^+)$	0	$\frac{\sqrt{2}}{3}$	$\frac{\sqrt{2}}{3}$	0	$\sqrt{\frac{2}{3}}$
$\mathcal{A}(\Xi_c^+ \to \Sigma^+K^-K^+)$	0	$-\frac{\sqrt{2}}{3}$	$-\frac{\sqrt{2}}{3}$	0	$\sqrt{\frac{2}{3}}$

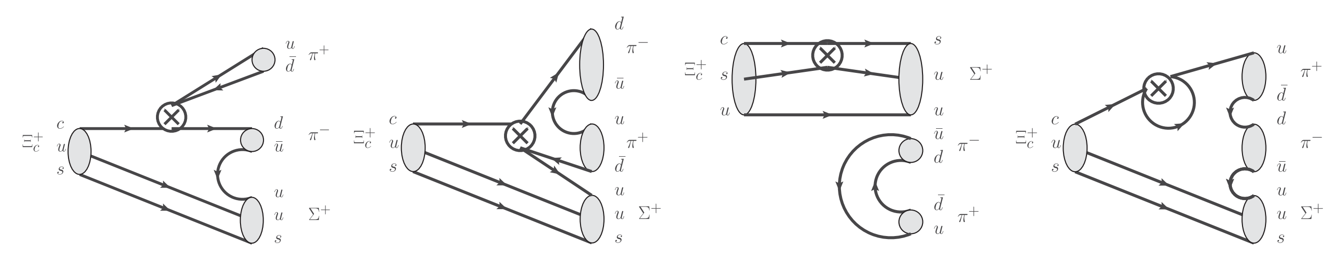
respectively, and we distinguish the corresponding reduced matrix elements. Our result is shown in Table II.

For the diagrammatic approach, the topological diagrams are shown in Figs. 1–6. The topological diagrams are all-order QCD diagrams which capture the flavor flow only. In each diagram we imply the sum over all possible combinations to connect the final state up quarks. As we consider U-spin partners only here, these are the same for



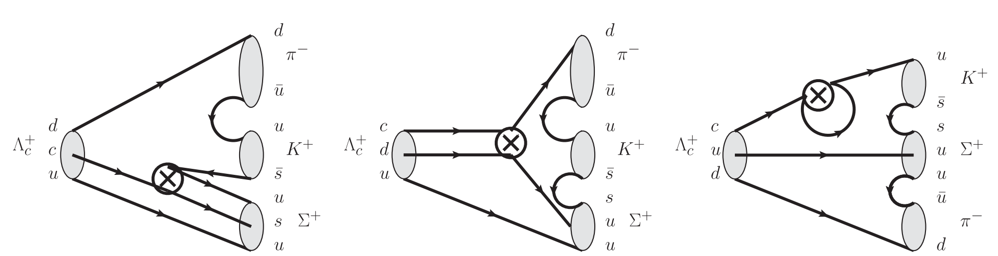
(a) T (b) C_1 (c) E_2 (d) P_1

FIG. 1. Diagrams in $\mathcal{A}(\Lambda_c^+ \to pK^-K^+) = (\Sigma + \Delta)(-T - C_1) + (-\Sigma + \Delta)(-E_2) + \Delta(-P_1)$. All diagrams have been drawn using JaxoDraw [69,70].



(a) T (b) C_1 (c) E_2 (d) P_1

FIG. 2. Diagrams in $\mathcal{A}(\Xi_c^+ \to \Sigma^+\pi^-\pi^+) = (-\Sigma + \Delta)(-T - C_1) + (\Sigma + \Delta)(-E_2) + \Delta(-P_1)$.



(a) C_2 (b) E_1 (c) P_2

FIG. 3. Diagrams in $\mathcal{A}(\Lambda_c^+ \to \Sigma^+\pi^-K^+) = (\Sigma + \Delta)(-C_2) + (-\Sigma + \Delta)(-E_1) + \Delta(-P_2)$.

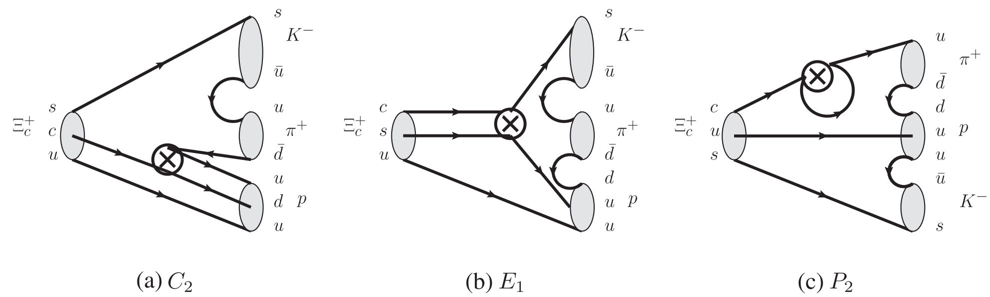


FIG. 4. Diagrams in $\mathcal{A}(\Xi_c^+ \to pK^-\pi^+) = (-\Sigma + \Delta)(-C_2) + (\Sigma + \Delta)(-E_1) + \Delta(-P_2)$.

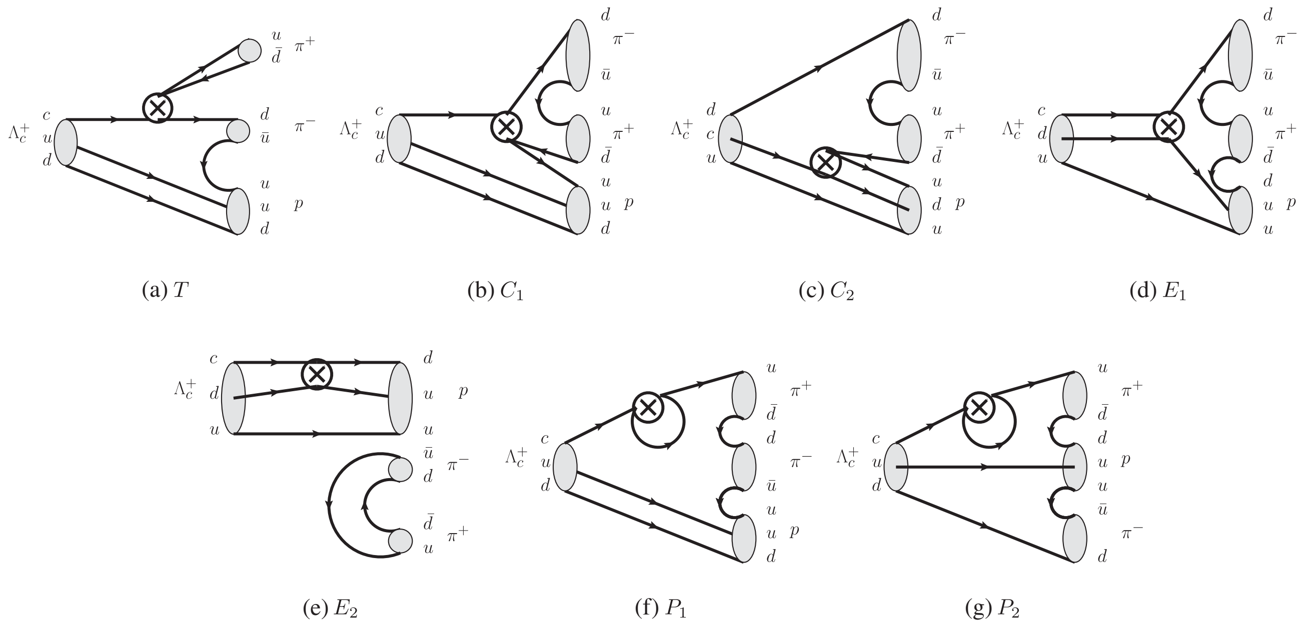


FIG. 5. Diagrams in $\mathcal{A}(\Lambda_c^+ \to p\pi^-\pi^+) = (-\Sigma + \Delta)(-T - C_1 - C_2 - E_1 - E_2) + \Delta(-P_1 - P_2)$.

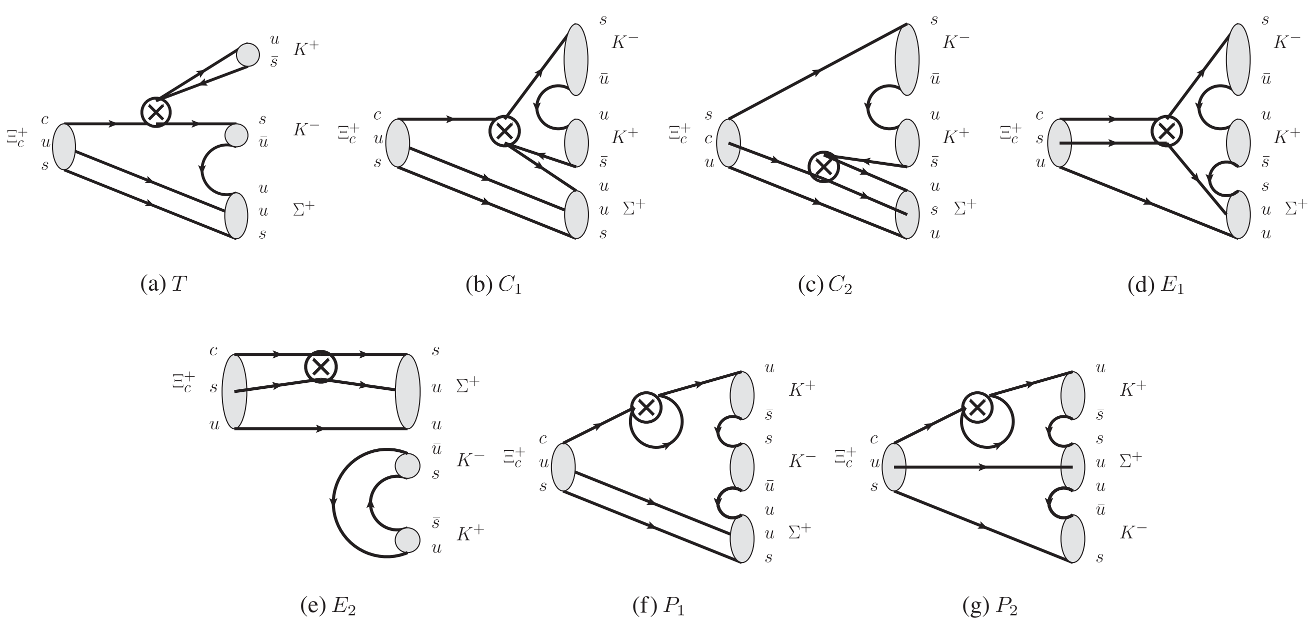


FIG. 6. Diagrams in $\mathcal{A}(\Xi_c^+ \to \Sigma^+K^-K^+) = (\Sigma + \Delta)(-T - C_1 - C_2 - E_1 - E_2) + \Delta(-P_1 - P_2)$.

TABLE III. SCS decays connected to $\Lambda_c \to p\pi^+\pi^-$ by U-spin and their diagrammatical decomposition.

Decay ampl. $\mathcal{A}$	$\Sigma(T+C_1-E_2)$	$\Sigma(C_2+E_2)$	$\Sigma(E_1+E_2)$	$\Delta(T+C_1+E_2+P_1)$	$\Delta(C_2+E_1+P_2)$
$\mathcal{A}(\Lambda_c^+ \to pK^-K^+)$	−1	0	0	−1	0
$\mathcal{A}(\Xi_c^+ \to \Sigma^+\pi^-\pi^+)$	1	0	0	−1	0
$\mathcal{A}(\Lambda_c^+ \to \Sigma^+\pi^-K^+)$	0	−1	1	0	−1
$\mathcal{A}(\Xi_c^+ \to pK^-\pi^+)$	0	1	−1	0	−1
$\mathcal{A}(\Lambda_c^+ \to p\pi^-\pi^+)$	1	1	1	−1	−1
$\mathcal{A}(\Xi_c^+ \to \Sigma^+K^-K^+)$	−1	−1	−1	−1	−1

all decay channels. Furthermore, in the case of the penguin diagram the shown topology is defined as

$$P \equiv P_s + P_d - 2P_b, \tag{7}$$

where P_q is the penguin diagram with the down-type quark q running in the loop; see Eq. (5) and Ref. [11]. Annihilation diagrams with antiquarks from the sea of the initial state do not play a role here. Our result for the diagrammatical decomposition is given in Table III, where we form combinations of the topologies which give linear independent contributions. Note that the parametrizations in Tables II and III are equivalent; see also Refs. [63,66,68] for the same observation for two-body meson decays. Both of the shown parametrization matrices have rank five, and we have a one-to-one matching of the independent parameter combinations of the two parametrizations on each other. Explicitly, the mapping of the two parametrizations reads

$$\begin{pmatrix} \left\langle \tfrac{1}{2}\Big|_0 1 \Big| \tfrac{1}{2}\right\rangle \\ \left\langle \tfrac{1}{2}\Big|_1 1 \Big| \tfrac{1}{2}\right\rangle \\ \left\langle \tfrac{3}{2}\Big| 1 \Big| \tfrac{1}{2}\right\rangle \\ \left\langle \tfrac{1}{2}\Big|_0 0 \Big| \tfrac{1}{2}\right\rangle \\ \left\langle \tfrac{1}{2}\Big|_1 0 \Big| \tfrac{1}{2}\right\rangle \end{pmatrix} = \begin{pmatrix} \sqrt{\tfrac{3}{2}} & -\sqrt{\tfrac{3}{2}} & \sqrt{\tfrac{3}{2}} & 0 & 0 \\ \tfrac{1}{\sqrt{2}} & \tfrac{1}{\sqrt{2}} & \tfrac{3}{\sqrt{2}} & 0 & 0 \\ \sqrt{2} & \sqrt{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \tfrac{1}{\sqrt{2}} & -\tfrac{1}{\sqrt{2}} \\ 0 & 0 & 0 & -\sqrt{\tfrac{3}{2}} & -\sqrt{\tfrac{3}{2}} \end{pmatrix} \times \begin{pmatrix} T+C_1-E_2 \\ C_2+E_2 \\ E_1+E_2 \\ T+C_1+E_2+P_1 \\ C_2+E_1+P_2 \end{pmatrix}. \tag{8}$$

Herein, the first three reduced matrix elements correspond to the CKM-leading part and the last two to the CKM-subleading part, which is why the translation matrix is block diagonal. As the coefficient submatrix of the CKM-leading part has matrix rank three, there are three U-spin sum rules for the A_Σ part, which one can read off directly as

$$A_\Sigma(\Lambda_c^+ \to pK^-K^+) = -A_\Sigma(\Xi_c^+ \to \Sigma^+\pi^-\pi^+), \tag{9}$$

$$A_\Sigma(\Lambda_c^+ \to \Sigma^+\pi^-K^+) = -A_\Sigma(\Xi_c^+ \to pK^-\pi^+), \tag{10}$$

$$A_\Sigma(\Lambda_c^+ \to p\pi^-\pi^+) = -A_\Sigma(\Xi_c^+ \to \Sigma^+K^-K^+). \tag{11}$$

The sum rules Eqs. (9)–(11) agree with the ones that can be read off Table XI in Ref. [19]. The CKM-subleading coefficient submatrix has rank two, so there are four corresponding U-spin sum rules. Three of them correspond to the ones for the CKM-leading part, namely

$$A_\Delta(\Lambda_c^+ \to pK^-K^+) = A_\Delta(\Xi_c^+ \to \Sigma^+\pi^-\pi^+), \tag{12}$$

$$A_\Delta(\Lambda_c^+ \to \Sigma^+\pi^-K^+) = A_\Delta(\Xi_c^+ \to pK^-\pi^+), \tag{13}$$

$$A_\Delta(\Lambda_c^+ \to p\pi^-\pi^+) = A_\Delta(\Xi_c^+ \to \Sigma^+K^-K^+). \tag{14}$$

The additional one is given as

$$A_\Delta(\Lambda_c^+ \to pK^-K^+) + A_\Delta(\Lambda_c^+ \to \Sigma^+\pi^-K^+) = A_\Delta(\Lambda_c^+ \to p\pi^-\pi^+). \tag{15}$$

Finally, the full U-spin limit coefficient matrix has rank five; therefore there is one sum rule for the full amplitudes

$$\mathcal{A}(\Lambda_c^+ \to pK^-K^+) + \mathcal{A}(\Xi_c^+ \to \Sigma^+\pi^-\pi^+) + \mathcal{A}(\Lambda_c^+ \to \Sigma^+\pi^-K^+) + \mathcal{A}(\Xi_c^+ \to pK^-\pi^+) - \mathcal{A}(\Lambda_c^+ \to p\pi^-\pi^+) - \mathcal{A}(\Xi_c^+ \to \Sigma^+K^-K^+) = 0. \tag{16}$$

IV. *CP* ASYMMETRY SUM RULES

We start our discussion in the U-spin limit (later we consider also U-spin breaking). Furthermore, as [71]

$$\mathrm{Im}(-2\Delta/\Sigma) \approx -6 \times 10^{-4}, \tag{17}$$

disregarding powers of $\mathcal{O}(\Delta^2/\Sigma^2)$ is an excellent approximation. Within this approximation, the CP asymmetry at a certain point in the Dalitz plot can be written as (see, e.g., Refs. [7,72,73])

$$a_{CP} \equiv \frac{|\mathcal{A}|^2 - |\bar{\mathcal{A}}|^2}{|\mathcal{A}|^2 + |\bar{\mathcal{A}}|^2} = \mathrm{Im}\left(\frac{-2\Delta}{\Sigma}\right)\mathrm{Im}\left(\frac{A_\Delta}{A_\Sigma}\right). \tag{18}$$

Inserting the amplitude sum rules Eqs. (9)–(14) into Eq. (18) we obtain the pointwise CP asymmetry sum rules

$$a_{CP}(\Lambda_c^+ \to pK^-K^+) + a_{CP}(\Xi_c^+ \to \Sigma^+\pi^-\pi^+) = 0, \qquad (19)$$

$$a_{CP}(\Lambda_c^+ \to \Sigma^+\pi^-K^+) + a_{CP}(\Xi_c^+ \to pK^-\pi^+) = 0, \qquad (20)$$

$$a_{CP}(\Lambda_c^+ \to p\pi^-\pi^+) + a_{CP}(\Xi_c^+ \to \Sigma^+K^-K^+) = 0. \qquad (21)$$

Next, we move to the discussion of the phase space integrated CP asymmetry. In the U-spin limit and to linear order in Δ/Σ it is given as

$$A_{CP} \equiv \frac{\int |\mathcal{A}|^2 dp - \int |\bar{\mathcal{A}}|^2 dp}{\int |\mathcal{A}|^2 dp + \int |\bar{\mathcal{A}}|^2 dp} = \mathrm{Im}\left(\frac{-2\Delta}{\Sigma}\right) I_p, \qquad (22)$$

with

$$I_p = \frac{\int \mathrm{Im}(A_\Sigma^* A_\Delta) dp}{\int |A_\Sigma|^2 dp}. \qquad (23)$$

Here, the dp integration denotes the integration over all phase space variables.

In the case of two-body charm meson decays to pseudoscalars, Eq. (22) gives a trivial integral, and we have $A_{CP} = a_{CP}$ as it must be. Note that for D^0 decays the CP asymmetries have additional contributions from indirect CP violation due to charm mixing. This additional complication is not present for baryon decay.

In order to promote a sum rule which is valid for pointwise CP asymmetries, a_{CP}, to a sum rule between CP asymmetries of integrated rates, A_{CP}, it is necessary that $|I_p|$ agrees for the involved CP asymmetries. From Eqs. (9)–(14) it is clear that this criterion is fulfilled by all three pairs of decays in Eqs. (19)–(21). Thus, the pointwise sum rules can be promoted to ones for CP asymmetries of the integrated rates

$$A_{CP}(\Lambda_c^+ \to pK^-K^+) + A_{CP}(\Xi_c^+ \to \Sigma^+\pi^-\pi^+) = 0, \qquad (24)$$

$$A_{CP}(\Lambda_c^+ \to \Sigma^+\pi^-K^+) + A_{CP}(\Xi_c^+ \to pK^-\pi^+) = 0, \qquad (25)$$

$$A_{CP}(\Lambda_c^+ \to p\pi^-\pi^+) + A_{CP}(\Xi_c^+ \to \Sigma^+K^-K^+) = 0. \qquad (26)$$

Moreover, from Tables II and III it is clear that no such sum rule connects $A_{CP}(\Lambda_c^+ \to pK^-K^+)$ and $A_{CP}(\Lambda_c^+ \to p\pi^-\pi^+)$. Additionally, as we discuss in Appendix, there are not even pointwise CP asymmetry sum rules at first order U-spin breaking. This means that Eqs. (19)–(21) and Eqs. (24)–(26) are expected to get corrections of $\mathcal{O}(30\%)$ [8,11,67].

V. CONCLUSIONS

We construct U-spin CP asymmetry sum rules between SCS three-body charmed baryon decays, which we give in Eqs. (19)–(21) and Eqs. (24)–(26). The sum rules are valid both pointwise at any point in the Dalitz plot and for the integrated CP asymmetries. There are no U-spin CP asymmetry sum rules besides the trivial ones due to the interchange of all d and s quarks. Furthermore, there is no U-spin CP asymmetry sum rule which is valid beyond the U-spin limit. Also, there is no U-spin sum rule connecting $A_{CP}(\Lambda_c^+ \to pK^-K^+)$ and $A_{CP}(\Lambda_c^+ \to p\pi^-\pi^+)$ whose difference recently has been measured by LHCb [2]. The dynamic reason for the latter is that the presence of the spectator quark and the additional combinatorial possibilities due to the three-body decay lead eventually to more possible topological combinations for $\Lambda_c^+ \to p\pi^-\pi^+$ than for the $\Lambda_c^+ \to pK^-K^+$ in both the CKM-leading and the CKM-subleading parts of the amplitudes. These additional contributions remain in the sum of the two CP asymmetries and do not cancel out.

There are more opportunities for studying U-spin sum rules and their breaking in three-body charm decays by including also the branching ratios of Cabibbo-favored and doubly Cabibbo-suppressed decays in the discussion, which we leave for future work.

ACKNOWLEDGMENTS

We thank Alan Schwartz for asking the question which led to this work. The work of Y. G. is supported in part by NSF Grant No. PHY1316222. S. S. is supported by a DFG Forschungsstipendium under Contract No. SCHA 2125/1-1.

APPENDIX: *U*-SPIN BREAKING

The U-spin breaking from the difference of d and s quark masses gives rise to a triplet spurion operator. For implications for meson decays see, e.g., Refs. [67,74–76]. In order to include these corrections within perturbation theory we perform the tensor products with the unperturbed Hamiltonian. We have

$$(1,0) \otimes (1,0) = \sqrt{\frac{2}{3}}(2,0) - \sqrt{\frac{1}{3}}(0,0). \qquad (A1)$$

TABLE IV. Decomposition of the CKM-leading U-spin breaking part of the SCS decays which are connected to $\Lambda_c \to p\pi^+\pi^-$ by U-spin.

Decay ampl. $\mathcal{A}_X$	$\Sigma\langle\frac{3}{2}\lvert 2\rvert\frac{1}{2}\rangle$	$\Sigma\langle\frac{1}{2}\lvert_0 0\rvert\frac{1}{2}\rangle$	$\Sigma\langle\frac{1}{2}\lvert_1 0\rvert\frac{1}{2}\rangle$
$\mathcal{A}_X(\Lambda_c^+ \to pK^-K^+)$	$-\frac{2}{3\sqrt{5}}$	$\frac{1}{\sqrt{6}}$	$-\frac{1}{3\sqrt{2}}$
$\mathcal{A}_X(\Xi_c^+ \to \Sigma^+\pi^-\pi^+)$	$-\frac{2}{3\sqrt{5}}$	$\frac{1}{\sqrt{6}}$	$-\frac{1}{3\sqrt{2}}$
$\mathcal{A}_X(\Lambda_c^+ \to \Sigma^+\pi^-K^+)$	$-\frac{2}{3\sqrt{5}}$	$-\frac{1}{\sqrt{6}}$	$-\frac{1}{3\sqrt{2}}$
$\mathcal{A}_X(\Xi_c^+ \to pK^-\pi^+)$	$-\frac{2}{3\sqrt{5}}$	$-\frac{1}{\sqrt{6}}$	$-\frac{1}{3\sqrt{2}}$
$\mathcal{A}_X(\Lambda_c^+ \to p\pi^-\pi^+)$	$\frac{2}{3\sqrt{5}}$	0	$-\frac{\sqrt{2}}{3}$
$\mathcal{A}_X(\Xi_c^+ \to \Sigma^+K^-K^+)$	$\frac{2}{3\sqrt{5}}$	0	$-\frac{\sqrt{2}}{3}$

Note that there is no triplet present on the right-hand side in Eq. (A1) as $\Delta U_3 = 0$ for both $\Delta U = 1$ operators on the left-hand side, and the (1,0) in the corresponding product comes with a vanishing Clebsch-Gordan coefficient. Our result for the parametrization of the CKM-leading U-spin breaking contribution $\mathcal{A}_X$ to the decay amplitudes is given in Table IV. Combining this result with the CKM-leading part of the parametrization given in Table II we obtain a matrix with rank six. That means there are no U-spin sum rules valid at this order between the SCS decays—neither for the full amplitudes nor for the CKM-leading part only. Furthermore, at this order there are not even pointwise CP asymmetry sum rules, not to mention ones for CP asymmetries of integrated rates.

[1] R. Aaij *et al.* (LHCb Collaboration), Nat. Phys. **13**, 391 (2017).
[2] R. Aaij *et al.* (LHCb Collaboration), J. High Energy Phys. 03 (2018) 182.
[3] I. Bediaga *et al.* (LHCb Collaboration), arXiv:1808.08865.
[4] A. J. Schwartz, Acta Phys. Pol. B **49**, 1335 (2018).
[5] E. Kou *et al.* (Belle II Collaboration), arXiv:1808.10567.
[6] Y. Grossman, A. L. Kagan, and Y. Nir, Phys. Rev. D **75**, 036008 (2007).
[7] D. Pirtskhalava and P. Uttayarat, Phys. Lett. B **712**, 81 (2012).
[8] G. Hiller, M. Jung, and S. Schacht, Phys. Rev. D **87**, 014024 (2013).
[9] Y. Grossman and D. J. Robinson, J. High Energy Phys. 04 (2013) 067.
[10] Y. Grossman, Z. Ligeti, and D. J. Robinson, J. High Energy Phys. 01 (2014) 066.
[11] S. Müller, U. Nierste, and S. Schacht, Phys. Rev. Lett. **115**, 251802 (2015).
[12] M. Gronau, Phys. Lett. B **492**, 297 (2000).
[13] R. Fleischer, Phys. Lett. B **459**, 306 (1999).
[14] M. Gronau and J. L. Rosner, Phys. Lett. B **482**, 71 (2000).
[15] G. Altarelli, N. Cabibbo, and L. Maiani, Phys. Lett. B **57B**, 277 (1975).
[16] R. L. Kingsley, S. B. Treiman, F. Wilczek, and A. Zee, Phys. Rev. D **11**, 1919 (1975).
[17] J. G. Korner, G. Kramer, and J. Willrodt, Z. Phys. C **2**, 117 (1979).
[18] M. Matsuda, M. Nakagawa, K. Odaka, S. Ogawa, and M. Shin-Mura, Prog. Theor. Phys. **59**, 666 (1978).
[19] M. J. Savage and R. P. Springer, Phys. Rev. D **42**, 1527 (1990).
[20] S. Pakvasa, S. P. Rosen, and S. F. Tuan, Phys. Rev. D **42**, 3746 (1990).
[21] M. J. Savage, Phys. Lett. B **259**, 135 (1991).
[22] R. C. Verma and M. P. Khanna, Phys. Rev. D **53**, 3723 (1996).
[23] K. K. Sharma and R. C. Verma, Phys. Rev. D **55**, 7067 (1997).
[24] J. G. Korner and H. W. Siebert, Annu. Rev. Nucl. Part. Sci. **41**, 511 (1991).
[25] J. G. Korner, M. Kramer, and D. Pirjol, Prog. Part. Nucl. Phys. **33**, 787 (1994).
[26] J. G. Korner, in *Baryons '95. Proceedings, 7th International Conference on the Structure of Baryons, Santa Fe, USA, October 3-7, 1995* (World Scientific, Singapore, 1996) pp. 221–230.
[27] H.-Y. Cheng, Front. Phys. **10**, 101406 (2015).
[28] Y. Kohara, Phys. Rev. D **44**, 2799 (1991).
[29] L.-L. Chau, H.-Y. Cheng, and B. Tseng, Phys. Rev. D **54**, 2132 (1996).
[30] Y. Kohara, arXiv:hep-ph/9701287.
[31] Fayyazuddin and Riazuddin, Phys. Rev. D **55**, 255 (1997); **56**, 531(E) (1997).
[32] M. J. Savage, Phys. Lett. B **257**, 414 (1991).
[33] S. M. Sheikholeslami, M. P. Khanna, and R. C. Verma, Phys. Rev. D **43**, 170 (1991).
[34] M. P. Khanna, Phys. Rev. D **49**, 5921 (1994).
[35] C. Q. Geng, Y. K. Hsiao, Y.-H. Lin, and L.-L. Liu, Phys. Lett. B **776**, 265 (2018).
[36] C. Q. Geng, Y. K. Hsiao, C.-W. Liu, and T.-H. Tsai, J. High Energy Phys. 11 (2017) 147.
[37] H.-Y. Jiang and F.-S. Yu, Eur. Phys. J. C **78**, 224 (2018).
[38] C. Q. Geng, Y. K. Hsiao, C.-W. Liu, and T.-H. Tsai, Eur. Phys. J. C **78**, 593 (2018).
[39] J. G. Korner and M. Kramer, Z. Phys. C **55**, 659 (1992).
[40] P. Zenczykowski, Phys. Rev. D **50**, 402 (1994).
[41] T. Uppal, R. C. Verma, and M. P. Khanna, Phys. Rev. D **49**, 3417 (1994).
[42] T. Gutsche, M. A. Ivanov, J. G. Körner, and V. E. Lyubovitskij, Phys. Rev. D **98**, 074011 (2018).
[43] H.-Y. Cheng and B. Tseng, Phys. Rev. D **46**, 1042 (1992); **55**, 1697(E) (1997).
[44] A. Sharma and R. C. Verma, Phys. Rev. D **80**, 094001 (2009).
[45] A. Sharma and R. C. Verma, J. Phys. G **36**, 075005 (2009).
[46] K. K. Sharma and R. C. Verma, Eur. Phys. J. C **7**, 217 (1999).
[47] Z.-X. Zhao, Chin. Phys. C **42**, 093101 (2018).
[48] H.-Y. Cheng, X.-W. Kang, and F. Xu, Phys. Rev. D **97**, 074028 (2018).
[49] D. Wang, P.-F. Guo, W.-H. Long, and F.-S. Yu, J. High Energy Phys. 03 (2018) 066.
[50] X.-W. Kang, H.-B. Li, G.-R. Lu, and A. Datta, Int. J. Mod. Phys. A **26**, 2523 (2011).
[51] D. Wang, R.-G. Ping, L. Li, X.-R. Lyu, and Y.-H. Zheng, Chin. Phys. C **41**, 023106 (2017).
[52] M. J. Savage and R. P. Springer, Int. J. Mod. Phys. A **06**, 1701 (1991).
[53] D. A. Egolf, R. P. Springer, and J. Urban, Phys. Rev. D **68**, 013003 (2003).

[54] J.-M. Richard, in Workshop on Future Physics at COMPASS Geneva, Switzerland, 2002 (CERN yellow report, 2002).
[55] W. Wang, Z.-P. Xing, and J. Xu, Eur. Phys. J. C **77**, 800 (2017).
[56] Y.-J. Shi, W. Wang, Y. Xing, and J. Xu, Eur. Phys. J. C **78**, 56 (2018).
[57] W. Wang and J. Xu, Phys. Rev. D **97**, 093007 (2018).
[58] C.-D. Lü, W. Wang, and F.-S. Yu, Phys. Rev. D **93**, 056008 (2016).
[59] C. Q. Geng, Y. K. Hsiao, C.-W. Liu, and T.-H. Tsai, Phys. Rev. D **97**, 073006 (2018).
[60] C. Q. Geng, Y. K. Hsiao, C.-W. Liu, and T.-H. Tsai, arXiv: 1810.01079.
[61] I. I. Bigi, arXiv:1206.4554.
[62] M. Gronau and J. L. Rosner, Phys. Rev. D **97**, 116015 (2018).
[63] M. Gronau, O. F. Hernandez, D. London, and J. L. Rosner, Phys. Rev. D **50**, 4529 (1994).
[64] M. Gronau, O. F. Hernandez, D. London, and J. L. Rosner, Phys. Rev. D **52**, 6356 (1995).
[65] A. Soni and D. A. Suprun, Phys. Rev. D **75**, 054006 (2007).
[66] S. Müller, U. Nierste, and S. Schacht, Phys. Rev. D **92**, 014004 (2015).
[67] J. Brod, Y. Grossman, A. L. Kagan, and J. Zupan, J. High Energy Phys. 10 (2012) 161.
[68] D. Zeppenfeld, Z. Phys. C **8**, 77 (1981).
[69] D. Binosi and L. Theussl, Comput. Phys. Commun. **161**, 76 (2004).
[70] J. A. M. Vermaseren, Comput. Phys. Commun. **83**, 45 (1994).
[71] M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018).
[72] M. Golden and B. Grinstein, Phys. Lett. B **222**, 501 (1989).
[73] U. Nierste and S. Schacht, Phys. Rev. Lett. **119**, 251801 (2017).
[74] T. Feldmann, S. Nandi, and A. Soni, J. High Energy Phys. 06 (2012) 007.
[75] M. Jung and T. Mannel, Phys. Rev. D **80**, 116002 (2009).
[76] R. Fleischer, R. Jaarsma, and K. K. Vos, J. High Energy Phys. 03 (2017) 055.