Dimensional reduction of a finite-size scalar field model at finite temperature

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We investigate the process of dimensional reduction of one spatial dimension in a thermal scalar field model defined in D dimensions (inverse temperature and $D-1$ spatial dimensions). We obtain that a thermal model in D dimensions with one of the spatial dimensions having a finite size L is related to the finite-temperature model with just $D-1$ spatial dimensions and no finite size. Our results are obtained for one-loop calculations and for any dimension D. For example, in $D=4$ we have a relationship between a thin film with thickness L at finite temperature and a surface at finite temperature. We show that, although a strict dimensional reduction is not allowed, it is possible to define a valid prescription for this procedure.

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I. INTRODUCTION

Dimensional reduction states that by considering a model in D dimensions we can obtain—following some prescription—a theory with fewer dimensions. One approach is to take a model with a compactified dimension and investigate its behavior when the size of the compactified dimension is reduced to zero.

Let us consider the case of a finite-temperature field theory using the imaginary-time formalism. Temperature is introduced by compactifying the imaginary-time variable using periodic or antiperiodic boundary conditions, respectively, for bosons or fermions. In this context, dimensional reduction would be equivalent to a high-temperature limit. This idea was proposed long ago by Appelquist and Pisarski [1] and later formulated in more detail by Landsman [2]. This idea is currently accepted and understood [3,4]. However, up to now, we think that there is a lack of investigation on this topic when one is interested in a greater number of compactifications.

We might ask how a system in arbitrary D dimensions with two compactified dimensions—one corresponding to the inverse temperature $\beta = 1/T$ and the other with a finite size L—behave when $L \to 0$. However, strictly speaking, in the context of both first-order and second-order phase transitions, it can be shown that the limit of $L \to 0$ cannot be fully attained [5]. There have been many works in the context of phase transitions in thin films using different models (Ginzburg-Landau, Nambu-Jona-Lasinio, Gross-Neveu, and others; see Refs. [6–16]) that indicate the existence of a minimal thickness below which no phase transitions occur. Reference [6] also strengthened this indication by comparing a phenomenological model for superconducting thin films using a Ginzburg-Landau model with experimental results. Therefore, this is a direct indication that the physics of surfaces and thin films are different and one cannot achieve a surface from a thin film.

In this article, we investigate the problem of dimensional reduction from a mathematical physics perspective, e.g., to investigate the relationship between systems in the form of "films" and "surfaces." Here this is generalized so that we investigate the relation between D-dimensional and $D-1$-dimensional scenarios.

To study a quantum field theory at finite temperature and finite size, we use the formalism of quantum field theory in spaces with toroidal topologies [5,17]. As a first investigation, we consider a scalar field model at the one-loop level. We obtain a remarkably simple relationship between the following situations:

(1) Starting with a space in D dimensions, we consider two compactifications: one of the dimensions corresponds to the inverse temperature $1/T$ and another corresponds to a finite size L, while the other $D-2$ dimensions are of infinite size.
(2) Another possibility is to eliminate one spatial dimension from the beginning; starting in a space with $D-1$ dimensions, we consider one compactification corresponding to the inverse temperature $1/T$.

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This should, e.g., provide us with a relation between surfaces and thin films for $D = 4$. Furthermore, from a mathematical physics perspective, we also investigate the possibility of fractal dimensions.

II. THE MODEL

In this article, we take a scalar field theory with a quartic interaction in D dimensions with Euclidean action,

$$S_E = \int d^D x \left\{ \frac{1}{2}(\partial\phi)^2 + \frac{M^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4 \right\}. \tag{1}$$

We shall only discuss the one-loop Feynman amplitude $\mathcal{I}$ with ρ propagators and zero external momenta,

$$\mathcal{I}_\rho^D(M^2) = \int \frac{d^D p}{(2\pi)^D} \frac{1}{(p^2 + M^2)^\rho}. \tag{2}$$

In particular, $\rho = 1$ corresponds to the tadpole contribution to the effective mass and $\rho = 2$ corresponds to the first-order correction to the coupling constant. We introduce periodic boundary conditions on $d < D$ coordinates. The compactification of the imaginary time introduces the inverse temperature $\beta = 1/T \equiv L_0$ and the compactification of the spatial coordinates introduces the characteristic lengths L_i. Thus, the amplitude becomes

$$\mathcal{I}_\rho^{D,d}(M^2; L_\alpha) = \frac{1}{\prod_{\alpha=0}^{d-1} L_\alpha} \sum_{n_0,\ldots,n_{d-1}=-\infty}^{\infty} \int \frac{d^{D-d} q}{(2\pi)^{D-d}} \frac{1}{[q^2 + M^2 + \sum_{\alpha=0}^{d-1} (\frac{2\pi n_\alpha}{L_\alpha})^2]^\rho}. \tag{3}$$

We compute the remaining integrals on the $(D - d)$-dimensional subspace using dimensional regularization. The remaining infinite sum can be identified as an Epstein-Hurwitz zeta function [18] and leads—after an analytic continuation—to the sum over modified Bessel functions of the second kind $K_\nu(x)$; see Refs. [5,19] for further details. The function $\mathcal{I}_\rho^{D,d}(M^2; L_\alpha)$ in the case of $d = 2$ reads

$$\mathcal{I}_\rho^{D,2}(M^2; \beta, L) = \frac{(M^2)^{-\rho+\frac{D}{2}} \Gamma[\rho - \frac{D}{2}]}{(4\pi)^{\frac{D}{2}} \Gamma[\rho]} + \frac{\mathcal{W}_{\frac{D}{2}-\rho}(M^2; \beta, L)}{(2\pi)^{\frac{D}{2}} 2^{\rho-2} \Gamma[\rho]}, \tag{4}$$

where

$$\mathcal{W}_\nu(M^2; \beta, L) = \sum_{n=1}^{\infty} \left(\frac{M}{n\beta}\right)^\nu K_\nu(n\beta M) + \sum_{n=1}^{\infty} \left(\frac{M}{nL}\right)^\nu K_\nu(nLM) + 2 \sum_{n_0,n_1=1}^{\infty} \left(\frac{M}{\sqrt{n_0^2\beta^2 + n_1^2 L^2}}\right)^\nu K_\nu\left(M\sqrt{n_0^2\beta^2 + n_1^2 L^2}\right), \tag{5}$$

with $\nu = \frac{D}{2} - \rho$.

As in Ref. [20], we investigate these infinite sums using a representation of the modified Bessel function in the complex plane,

$$K_\nu(X) = \frac{1}{4\pi i} \int_{c-i\infty}^{c+i\infty} dt\, \Gamma(t)\Gamma(t-\nu) \left(\frac{X}{2}\right)^{-2t+\nu}. \tag{6}$$

We remark that c is a point located on the positive real axis that has a greater value than all of the poles of the gamma function. With this definition, it is clear that $c > \max[0, \nu]$. However, we extend this definition so that we are allowed to interchange the integral over t and the summation over n. Therefore, c must be chosen in such a way that there is no pole located to the right of it. Substituting Eq. (6) into Eq. (5), we obtain

$$\begin{aligned}\mathcal{W}_\nu(M^2; \beta, L) &= \frac{1}{4\pi i} \int_{c-i\infty}^{c+i\infty} dt\, \Gamma(t)\Gamma(t-\nu)\zeta(2t) \left(\frac{M^2}{2}\right)^\nu \left[\left(\frac{M\beta}{2}\right)^{-2t} + \left(\frac{ML}{2}\right)^{-2t}\right] \\ &\quad + \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dt\, \Gamma(t)\Gamma(t-\nu) \left(\frac{M^2}{2}\right)^\nu \left(\frac{M}{2}\right)^{-2t} \sum_{n_0,n_1=1}^{\infty} (n_0^2\beta^2 + n_1^2 L^2)^{-t}.\end{aligned} \tag{7}$$

The double sum in Eq. (7) is known and has the analytical extension [18]

$$\sum_{n_0,n_1=1}^{\infty}(n_0^2\beta^2+n_1^2L^2)^{-t}=-\frac{\zeta(2t)}{2L^{2t}}+\frac{\sqrt{\pi}L}{2\beta}\frac{\Gamma(t-\frac{1}{2})\zeta(2t-1)}{\Gamma(t)L^{2t}}+\frac{2\pi^t}{\Gamma(t)}\sum_{n_0,n_1=1}^{\infty}\left(\frac{n_0}{n_1}\right)^{t-\frac{1}{2}}\sqrt{\frac{L}{\beta}}\frac{1}{(\beta L)^t}K_{t-\frac{1}{2}}\left(2\pi n_0n_1\frac{L}{\beta}\right). \quad (8)$$

After substituting Eq. (8) into Eq. (7) and taking into account Eq. (6), we finally obtain the integral representation in the complex plane for the function $\mathcal{W}_\nu$ as the sum of these terms:

$$\begin{aligned}
\left(\frac{2}{M^2}\right)^{\nu}\mathcal{W}_\nu &= \mathcal{W}_\nu^{(1)}+\mathcal{W}_\nu^{(2)}+\mathcal{W}_\nu^{(3)}, \quad \text{where}\\
\mathcal{W}_\nu^{(1)} &= \frac{1}{4\pi i}\int_{c-i\infty}^{c+i\infty}dt\,\Gamma(t)\Gamma(t-\nu)\zeta(2t)\left(\frac{M\beta}{2}\right)^{-2t},\\
\mathcal{W}_\nu^{(2)} &= \frac{\sqrt{\pi}L}{4\pi i\beta}\int_{c-i\infty}^{c+i\infty}dt\,\Gamma(t-\nu)\Gamma\left(t-\frac{1}{2}\right)\zeta(2t-1)\left(\frac{ML}{2}\right)^{-2t},\\
\mathcal{W}_\nu^{(3)} &= \frac{1}{2\pi i}\sum_{k=0}^{\infty}\frac{(-1)^k}{\Gamma(k+1)}\left(\frac{M\beta}{2\pi}\right)^{2k-2\nu}\int_{c-i\infty}^{c+i\infty}\frac{dt}{\sqrt{\pi}}\Gamma(t)\zeta(2t)\Gamma\left(t-\nu+k+\frac{1}{2}\right)\zeta(2t-2\nu+2k+1)\left(\frac{\pi L}{\beta}\right)^{-2t}. \quad (9)
\end{aligned}$$

The next step is to determine the positions of the poles of the functions in the complex-plane integrals and compute their residues. We see that the positions of the poles depend on the value of ν. We compute the integrals for the following specific cases:

(1) Integer ν, related to an even number of dimensions D.
(2) Half-integer ν, related to an odd number of dimensions D.
(3) Other real values of ν (for completeness), which can be thought of as related to fractal dimensions D.

In this article we consider two different scenarios. The first scenario involves a model in D dimensions with two compactified dimensions: one related to the inverse temperature $\beta=1/T$ and the other with a finite size L. For this case we need the function $\mathcal{W}_\nu$ [as defined in Eq. (9)] and Eq. (4). The second scenario involves a model in $D-1$ dimensions with just one compactified dimension which is related to the inverse temperature. We see that the amplitude only requires the knowledge of $\mathcal{W}_\nu^{(1)}$,

$$\mathcal{I}_\rho^{D-1,1}(M^2;\beta)=\frac{(\frac{M^2}{2})^{\frac{D}{2}-\rho}}{(2\pi)^{\frac{D}{2}}2^{\rho-2}\Gamma[\rho]}\left\{\frac{\sqrt{\pi}}{2M}\Gamma\left(\rho-\frac{D}{2}+\frac{1}{2}\right)+\frac{2\sqrt{\pi}}{M}\mathcal{W}^{(1)}_{\frac{D}{2}-\frac{1}{2}-\rho}(M^2;\beta)\right\}. \quad (10)$$

To avoid a lengthy exposition, we only show the final expressions for each case. For integer ν the function $\mathcal{W}_\nu^{(1)}$ reads

$$\mathcal{W}_\nu^{(1)}=\frac{\sqrt{\pi}}{2M\beta}\Gamma\left(\frac{1}{2}-\nu\right)+\frac{1}{2}\mathcal{S}_0^{(3)}\left(\nu;\frac{M\beta}{2}\right)+\frac{1}{2}\mathcal{S}^{(2)}\left(\nu;\frac{M\beta}{4\pi}\right)+\begin{cases}-\frac{1}{4}\Gamma(-\nu) & \nu<0,\\ -\frac{(-1)^\nu}{4\Gamma(\nu+1)^2}\begin{bmatrix}\nu+1\\ 2\end{bmatrix}+\frac{(-1)^\nu}{2\Gamma(\nu+1)}\left(\gamma+\ln\frac{M\beta}{4\pi}\right) & \nu\geq 0,\end{cases} \quad (11)$$

and $\mathcal{W}_\nu$ reads

$$\left(\frac{2}{M^2}\right)^\nu \mathcal{W}_\nu = +\frac{1}{2}\mathcal{S}^{(2)}\left(\nu;\frac{ML}{4\pi}\right) + \frac{1}{2}\mathcal{S}_1^{(3)}\left(\nu;\frac{ML}{2}\right) - \frac{2\pi}{M^2\beta L}\left\{\mathcal{S}^{(1)}\left(\nu;\frac{M\beta}{2\pi}\right) + \mathcal{S}^{(4)}(\nu;M\beta)\right\}$$
$$+\begin{cases} -\frac{1}{4}\Gamma(-\nu) & \nu < 0,\\ +\frac{(-1)^\nu}{2\Gamma(\nu+1)}\left\{-\frac{1}{2\Gamma(\nu+1)}\begin{bmatrix}\nu+1\\2\end{bmatrix} + \gamma + \ln\frac{ML}{4\pi} + \frac{\pi}{6}\frac{\beta}{L}\right\} & \nu \geq 0,\\ +\frac{\pi}{M^2\beta L}\Gamma(-\nu+1) & \nu < 1,\\ +\frac{\pi}{M^2\beta L}\frac{(-1)^\nu}{\Gamma(\nu)}\left\{-\frac{1}{\Gamma(\nu)}\begin{bmatrix}\nu\\2\end{bmatrix} + 2\ln M\beta - \frac{\pi}{3}\frac{\beta}{L}\right\} & \nu \geq 1. \end{cases} \tag{12}$$

Here the notation $\left[{a \atop 2}\right]$ indicates the unsigned Stirling number of the first kind:

$$\begin{bmatrix}1\\2\end{bmatrix} = 0, \qquad \begin{bmatrix}2\\2\end{bmatrix} = 1, \qquad \begin{bmatrix}3\\2\end{bmatrix} = 3, \qquad \begin{bmatrix}4\\2\end{bmatrix} = 11, \qquad \begin{bmatrix}5\\2\end{bmatrix} = 50.$$

In the above expressions, the functions $\mathcal{S}^{(1)}$, $\mathcal{S}^{(2)}$, $\mathcal{S}^{(3)}$, and $\mathcal{S}^{(4)}$ are defined by

$$\mathcal{S}_i^{(1)}(\nu;\alpha) = \sum_{k=1+i}^{\infty}(-1)^{k+\nu}\frac{\Gamma(k)\zeta(2k)}{\Gamma(k+\nu)}\alpha^{2k}, \tag{13}$$

$$\mathcal{S}^{(2)}(\nu;\alpha) = \sum_{k=1}^{\infty}(-1)^{k+\nu}\frac{\Gamma(2k+1)\zeta(2k+1)}{\Gamma(k+1)\Gamma(k+\nu+1)}\alpha^{2k}, \tag{14}$$

$$\mathcal{S}_i^{(3)}(\nu;\alpha) = \sum_{k=1+i}^{\nu}(-1)^{\nu-k}\frac{\Gamma(k)\zeta(2k)}{\Gamma(\nu-k+1)}\alpha^{-2k}, \tag{15}$$

$$\mathcal{S}^{(4)}(\nu;\alpha) = \sum_{k=1}^{\nu-1}(-1)^{\nu-k}\frac{\Gamma(2k+1)\zeta(2k+1)}{\Gamma(k+1)\Gamma(\nu-k)}\alpha^{-2k}. \tag{16}$$

For half-integer values of ν ($\nu = \mu + 1/2$, for integer μ), we obtain

$$\mathcal{W}_\nu^{(1)} = -\frac{1}{4}\Gamma\left(-\frac{1}{2}-\mu\right) - \frac{\sqrt{\pi}}{M\beta}\mathcal{S}^{(4)}(\mu+1;M\beta) - \frac{\sqrt{\pi}}{M\beta}\mathcal{S}_0^{(1)}\left(\mu+1;\frac{M\beta}{2\pi}\right)$$
$$+\begin{cases} +\frac{\sqrt{\pi}}{2M\beta}\Gamma(-\mu) & \mu < 0,\\ +\frac{\sqrt{\pi}}{2M\beta}\frac{(-1)^\mu}{\Gamma(\mu+1)}\left(\begin{bmatrix}1+\mu\\2\end{bmatrix}\frac{1}{\Gamma(\mu+1)} - \ln M\beta\right) & \mu \geq 0, \end{cases} \tag{17}$$

and

$$\mathcal{W}_\nu\left(\frac{2}{M^2}\right)^\nu = -\frac{1}{4}\Gamma\left(-\frac{1}{2}-\mu\right) + \frac{\pi}{M^2\beta L}\Gamma\left(\frac{1}{2}-\mu\right) + \frac{\sqrt{\pi}}{ML}\frac{(-1)^\mu}{\Gamma(\mu+1)}\left(\gamma + \ln\frac{\beta}{4\pi L}\right)$$
$$-\frac{\sqrt{\pi}}{ML}\mathcal{S}_0^{(1)}\left(\mu+1;\frac{ML}{2\pi}\right) + \frac{\sqrt{\pi}}{ML}\mathcal{S}^{(2)}\left(\mu;\frac{M\beta}{4\pi}\right) - \frac{\sqrt{\pi}}{ML}\mathcal{S}^{(4)}(\mu+1;ML) + \frac{\sqrt{\pi}}{ML}\mathcal{S}_0^{(3)}\left(\mu;\frac{M\beta}{2}\right). \tag{18}$$

Finally, for other real values of the index ν that are neither integer nor half-integer, we have

$$\mathcal{W}_\nu^{(1)} = \frac{1}{2}\left[\frac{\sqrt{\pi}}{M\beta}\Gamma\left(\frac{1}{2}-\nu\right) - \frac{1}{2}\Gamma(-\nu) + \sum_{k=0}^{\infty}\frac{(-1)^k}{\Gamma(k+1)}\Gamma(\nu-k)\zeta(2\nu-2k)\left(\frac{M\beta}{2}\right)^{-2\nu+2k}\right] \tag{19}$$

and

$$\mathcal{W}_\nu\left(\frac{2}{M^2}\right)^\nu = -\frac{1}{4}\Gamma(-\nu) + \frac{\pi}{M^2\beta L}\Gamma(1-\nu) + \frac{1}{2}\sum_{k=0}^{\infty}\frac{(-1)^k}{\Gamma(k+1)}\Gamma(\nu-k)\zeta(2\nu-2k)\left[\left(\frac{M\beta}{2}\right)^{-2\nu+2k} + \left(\frac{ML}{2}\right)^{-2\nu+2k}\right]$$
$$+\frac{\sqrt{\pi}L}{2\beta}\sum_{k=0}^{\infty}\frac{(-1)^k}{\Gamma(k+1)}\Gamma\left(\nu-k-\frac{1}{2}\right)\zeta(2\nu-2k-1)\left[\left(\frac{M\beta}{2}\right)^{2k-2\nu} - \left(\frac{ML}{2}\right)^{2k-2\nu}\right]$$
$$+\sum_{k=0}^{\infty}\frac{(-1)^k}{\Gamma(k+1)}\left(\frac{M\beta}{2\pi}\right)^{2k-2\nu}\left[\frac{\beta}{2\pi L}\Gamma(1-\nu+k)\zeta(2-2\nu+2k) - \frac{1}{2\sqrt{\pi}}\Gamma\left(\frac{1}{2}-\nu+k\right)\zeta(1-2\nu+2k)\right]. \quad (20)$$

So far we have managed to take both situations into account, i.e., one or two compactified dimensions. Let us remember that we are interested in the one-loop Feynman diagram with ρ internal lines and at zero external momentum. The amplitude in the scenario with two compactifications (one related to the inverse temperature $\beta = 1/T$ and another to a finite size L) is given by Eq. (4), and the amplitude in the scenario in which there are $D-1$ dimensions and just one compactification related to the inverse temperature β is given by Eq. (10).

In the following, we investigate, for any value of ν, the relationship between both scenarios. The contribution from $\Gamma(-\nu)$ in the amplitude is divergent for integer values of $\nu \geq 0$. To avoid the presence of poles in physical quantities we employ the modified minimal subtraction scheme [21], and the function $\Gamma(-\nu)$ is replaced by $\bar{\Gamma}(-\nu)$ such that

$$\bar{\Gamma}(-\nu) = -\frac{(-1)^{\nu+1}}{\Gamma(\nu+1)^2}\begin{bmatrix}\nu+1\\2\end{bmatrix}, \qquad \nu \geq 0. \quad (21)$$

III. DIMENSIONAL REDUCTION

We have considered a class of one-loop Feynman diagrams with ρ internal lines in D dimensions. Their contributions involve the above-defined functions $\mathcal{W}_\nu$, where the index ν is given by $\nu = D/2-\rho$. In the previous section we managed to obtain the final version of $\mathcal{W}_\nu$ in terms of some analytical functions and sums over the Riemann zeta function in the argument. This was done for the specific cases of integer values of ν (useful for even dimensions), half-integer values of ν (for odd dimensions), and other real values of ν (for completeness, and which can be considered for models with fractal dimensions).

Taking the situation with D dimensions and letting two of them be compactified, which introduces the temperature $1/\beta$ and a finite length L, one might ask how the function behaves as one takes the limit $L \to 0$. This can be interpreted as a "dimensional reduction." In general, what happens is that the function $\mathcal{W}_\nu(\beta, L)$ diverges as $L \to 0$. However, if we interpret the procedure of "dimensional reduction" as taking the dominant contribution[1] in β in the limit $L \to 0$ and ignore the remaining dependence on the finite length, we obtain the relation

$$L\mathcal{I}_\rho^{D,2}(M^2;\beta,L)|_{L\to 0} = \mathcal{I}_\rho^{D-1,1}(M^2;\beta) + \text{divergent terms.} \quad (22)$$

This relation holds for any real value of D, which will be shown in the following subsections. The divergent behavior of $\mathcal{I}$ in Eq. (22) as L goes to zero depends on the quantity $D/2-\rho$.

A. Integer ν, even D

To investigate the so-called dimensional reduction we first consider the case with integer values of ν, which corresponds to even dimensions D. The amplitude of a one-loop Feynman diagram in a scenario with both finite temperature and finite size is given by Eq. (4). To study its behavior we substitute Eq. (12) into Eq. (4) and split its contributions coming from the three functions $\mathcal{F}_1$, $\mathcal{G}_1$, and $\mathcal{H}_1$, such that

$$L\mathcal{I}_\rho^{D,2}(M^2;\beta,L) = \frac{L}{(2\pi)^{\frac{D}{2}}2^{\rho-2}\Gamma(\rho)} \times\left[\left(\frac{M^2}{2}\right)^{\frac{D}{2}-\rho}\frac{\Gamma(\rho-\frac{D}{2})}{4} + \mathcal{W}_{\frac{D}{2}-\rho}\right] = \left(\frac{M^2}{2}\right)^{\frac{D}{2}-\rho}\frac{\mathcal{F}_1+\mathcal{G}_1+\mathcal{H}_1}{(2\pi)^{\frac{D}{2}}2^{\rho-2}\Gamma(\rho)}. \quad (23)$$

The function $\mathcal{H}_1$ is the contribution that vanishes in the $L \to 0$ limit:

$$\mathcal{H}_1 = \frac{L}{2}\mathcal{S}^{(2)}\left(\frac{D}{2}-\rho;\frac{ML}{4\pi}\right) + \frac{(-1)^{\frac{D}{2}-\rho}L}{2\Gamma(\frac{D}{2}-\rho+1)}\left(\gamma+\ln\frac{ML}{4\pi}\right). \quad (24)$$

[1]This is a stronger result for integer dimension D as the divergent terms do not depend on β. However, this is not the case for a noninteger dimension D as it can depend on the temperature.

Note that the zero-temperature contribution was consistently subtracted. The divergent behavior as L goes to zero is given by $\mathcal{G}_1$,

$$\mathcal{G}_1 = -\frac{(-1)^{\frac{D}{2}-\rho}}{\Gamma(\frac{D}{2}-\rho)}\frac{\pi^2}{3M^2L} + \frac{L}{2}\mathcal{S}_1^{(3)}\left(\frac{D}{2}-\rho;\frac{ML}{2}\right). \tag{25}$$

Finally, the function $\mathcal{F}_1$ is the contribution that survives during the *dimensional reduction* and does not diverge,

$$\begin{aligned}\mathcal{F}_1 = &-\frac{2\pi}{M^2\beta}\left[\mathcal{S}_1^{(1)}\left(\frac{D}{2}-\rho;\frac{M\beta}{2\pi}\right) + \mathcal{S}^{(4)}\left(\frac{D}{2}-\rho;M\beta\right)\right] + \frac{\pi\beta}{12}\frac{(-1)^{\frac{D}{2}-\rho}}{\Gamma(\frac{D}{2}-\rho+1)} \\ &+\begin{cases}\frac{\pi}{M^2\beta}\Gamma\left(1-\frac{D}{2}+\rho\right), & \frac{D}{2}\leq\rho, \\ \frac{\pi}{M^2\beta}\frac{(-1)^{\frac{D}{2}-\rho}}{\Gamma(\frac{D}{2}-\rho)}\left(-\frac{1}{\Gamma(\frac{D}{2}-\rho)}\begin{bmatrix}\frac{D}{2}-\rho \\ 2\end{bmatrix} + 2\ln M\beta\right), & \frac{D}{2}>\rho.\end{cases}\end{aligned} \tag{26}$$

Therefore, as the $L\to 0$ limit is taken the contribution $\mathcal{H}_1$ vanishes and the divergent behavior in L is given by the function $\mathcal{G}_1$:

$$L\mathcal{I}_\rho^{D,2}(M^2;\beta,L)|_{L\to 0} = \frac{1}{(2\pi)^{\frac{D}{2}}2^{\rho-2}\Gamma(\rho)}L\mathcal{W}_{\frac{D}{2}-\rho}|_{L\to 0} = \left(\frac{M^2}{2}\right)^{\frac{D}{2}-\rho}\frac{\mathcal{F}_1+\mathcal{G}_1}{(2\pi)^{\frac{D}{2}}2^{\rho-2}\Gamma(\rho)}. \tag{27}$$

On the other hand, let us consider the second case in which we start from a scenario with one less dimension, $D-1$. The amplitude of the one-loop Feynman diagram is simply Eq. (10), written here as

$$\mathcal{I}_\rho^{D-1,1}(M^2;\beta) = \frac{(\frac{M^2}{2})^{\frac{D}{2}-\rho}}{(2\pi)^{\frac{D}{2}}2^{\rho-2}\Gamma[\rho]}\left\{\frac{\sqrt{\pi}}{2M}\Gamma\left(\rho-\frac{D}{2}+\frac{1}{2}\right) + \frac{2\sqrt{\pi}}{M}\mathcal{W}^{(1)}_{\frac{D}{2}-\frac{1}{2}-\rho}(M^2;\beta)\right\}. \tag{28}$$

And, for even dimensions D, we get that $\frac{D}{2}-\frac{1}{2}-\rho$ is a half-integer. So, we use Eq. (17) with $\mu=\frac{D}{2}-\rho-1$ and obtain

$$\begin{aligned}\mathcal{W}^{(1)}_{\frac{D}{2}-\frac{1}{2}-\rho} = &-\frac{1}{4}\Gamma\left(\frac{1}{2}-\frac{D}{2}+\rho\right) + \frac{\sqrt{\pi}}{2M\beta}\Gamma\left(-\frac{D}{2}+\rho+1\right) + \frac{\sqrt{\pi}}{2M\beta}\frac{(-1)^{\frac{D}{2}-\rho-1}}{\Gamma(\frac{D}{2}-\rho)}\left(\begin{bmatrix}\frac{D}{2}-\rho \\ 2\end{bmatrix}\frac{1}{\Gamma(\frac{D}{2}-\rho)} - 2\ln M\beta\right) \\ &-\frac{\sqrt{\pi}}{M\beta}\mathcal{S}^{(4)}\left(\frac{D}{2}-\rho;M\beta\right) - \frac{\sqrt{\pi}}{M\beta}\mathcal{S}_0^{(1)}\left(\frac{D}{2}-\rho;\frac{M\beta}{2\pi}\right).\end{aligned} \tag{29}$$

Therefore, Eq. (28) becomes

$$\begin{aligned}\mathcal{I}_\rho^{D-1,1}(M^2;\beta) = \frac{(\frac{M^2}{2})^{\frac{D}{2}-\rho}}{(2\pi)^{\frac{D}{2}}2^{\rho-2}\Gamma[\rho]}&\left\{\frac{\pi}{M^2\beta}\Gamma\left(\rho-\frac{D}{2}+1\right) + \frac{\pi}{M^2\beta}\frac{(-1)^{\frac{D}{2}-\rho-1}}{\Gamma(\frac{D}{2}-\rho)}\left(\begin{bmatrix}\frac{D}{2}-\rho \\ 2\end{bmatrix}\frac{1}{\Gamma(\frac{D}{2}-\rho)} - 2\ln M\beta\right)\right. \\ &\left.-\frac{2\pi}{M^2\beta}\mathcal{S}^{(4)}\left(\frac{D}{2}-\rho;M\beta\right) - \frac{2\pi}{M^2\beta}\mathcal{S}_0^{(1)}\left(\frac{D}{2}-\rho;\frac{M\beta}{2\pi}\right)\right\}.\end{aligned} \tag{30}$$

Furthermore, since $\frac{2\pi}{M^2\beta}\mathcal{S}_0^{(1)}(\frac{D}{2}-\rho;\frac{M\beta}{2\pi}) = \frac{2\pi}{M^2\beta}\mathcal{S}_1^{(1)}(\frac{D}{2}-\rho;\frac{M\beta}{2\pi}) - \frac{\pi\beta}{12}\frac{(-1)^{\frac{D}{2}-\rho}}{\Gamma(\frac{D}{2}-\rho+1)}$, by direct comparison with $\mathcal{F}_1$ we get

$$\mathcal{I}_\rho^{D-1,1}(M^2;\beta) = \frac{(\frac{M^2}{2})^{\frac{D}{2}-\rho}}{(2\pi)^{\frac{D}{2}}2^{\rho-2}\Gamma[\rho]}\mathcal{F}_1. \tag{31}$$

Then we find the relation

$$L\mathcal{I}_\rho^{D,2}(M^2;\beta,L)|_{L\to 0} = \mathcal{I}_\rho^{D-1,1}(M^2;\beta) + \left(\frac{M^2}{2}\right)^{\frac{D}{2}-\rho}\frac{\mathcal{G}_1}{(2\pi)^{\frac{D}{2}}2^{\rho-2}\Gamma(\rho)} = \mathcal{I}_\rho^{D-1,1}(M^2;\beta) + \mathcal{O}(L^{-1}). \tag{32}$$

The asymptotic behavior of the function $\mathcal{G}_1$ as $L \to 0$ is

$$\mathcal{G}_1 \sim \begin{cases} L^{-1}, & D-2\rho < 4, \\ L^{-(D-2\rho-1)}, & D-2\rho \geq 4. \end{cases}$$

B. Half-integer ν, odd D

Now we consider odd dimensions D, which implies half-integer values of $\nu = \mu + 1/2$. For reference, $\mu = \frac{D-1}{2} - \rho$. Again, we split the contributions into three functions $\mathcal{F}_2$, $\mathcal{G}_2$, and $\mathcal{H}_2$, and after substituting Eq. (18) into Eq. (4) the amplitude is given by

$$L\mathcal{I}_\rho^{D,2}(M^2;\beta,L) = \left(\frac{M^2}{2}\right)^{\frac{D}{2}-\rho} \frac{1}{(2\pi)^{\frac{D}{2}} 2^{\rho-2}\Gamma(\rho)} (\mathcal{F}_2 + \mathcal{G}_2 + \mathcal{H}_2), \tag{33}$$

with

$$\mathcal{F}_2 = \frac{\pi}{M^2\beta}\Gamma\left(1-\frac{D}{2}+\rho\right) + \frac{\sqrt{\pi}}{M}\frac{(-1)^{\frac{D-1}{2}-\rho}}{\Gamma(\frac{D+1}{2}-\rho)}\left(\gamma + \ln\frac{M\beta}{4\pi}\right) + \frac{\sqrt{\pi}}{M}\mathcal{S}^{(2)}\left(\frac{D-1}{2}-\rho;\frac{M\beta}{4\pi}\right) + \frac{\sqrt{\pi}}{M}\mathcal{S}_0^{(3)}\left(\frac{D-1}{2}-\rho;\frac{M\beta}{2}\right), \tag{34}$$

$$\mathcal{G}_2 = -\frac{\sqrt{\pi}}{M}\frac{(-1)^{\frac{D-1}{2}-\rho}}{\Gamma(\frac{D+1}{2}-\rho)}\ln ML - \frac{\sqrt{\pi}}{M}\mathcal{S}^{(4)}\left(\frac{D+1}{2}-\rho;ML\right), \tag{35}$$

$$\mathcal{H}_2 = -\frac{\sqrt{\pi}}{M}\mathcal{S}_0^{(1)}\left(\frac{D+1}{2}-\rho;\frac{ML}{2\pi}\right). \tag{36}$$

The second scenario, with a reduced number of dimensions, is given by Eq. (10):

$$\mathcal{I}_\rho^{D-1,1}(M^2;\beta) = \frac{(\frac{M^2}{2})^{\frac{D}{2}-\rho}}{(2\pi)^{\frac{D}{2}}2^{\rho-2}\Gamma[\rho]}\left\{\frac{\sqrt{\pi}}{2M}\Gamma\left(\rho+\frac{1-D}{2}\right) + \frac{2\sqrt{\pi}}{M}\mathcal{W}^{(1)}_{\frac{D-1}{2}-\rho}(M^2;\beta)\right\}, \tag{37}$$

with $\mathcal{W}^{(1)}$ given by Eq. (11), that is,

$$\mathcal{W}^{(1)}_{\frac{D-1}{2}-\rho} = -\frac{1}{4}\Gamma\left(\rho+\frac{1-D}{2}\right) - \frac{(-1)^{\frac{D-1}{2}-\rho}}{4\Gamma(\frac{D+1}{2}-\rho)^2}\begin{bmatrix}\frac{D+1}{2}-\rho \\ 2\end{bmatrix} + \frac{\sqrt{\pi}}{2M\beta}\Gamma\left(1+\rho-\frac{D}{2}\right) + \frac{(-1)^{\frac{D-1}{2}-\rho}}{2\Gamma(\frac{D+1}{2}-\rho)}\left(\gamma+\ln\frac{M\beta}{4\pi}\right) + \frac{1}{2}\mathcal{S}_0^{(3)}\left(\frac{D-1}{2}-\rho;\frac{M\beta}{2}\right) + \frac{1}{2}\mathcal{S}^{(2)}\left(\frac{D-1}{2}-\rho;\frac{M\beta}{4\pi}\right). \tag{38}$$

Once again, we must be careful with the zero-temperature contribution, as a modified minimal subtraction scheme is assumed [see Eq. (21)]. We obtain, after the cancellation with the zero-temperature contribution,

$$\mathcal{I}_\rho^{D-1,1}(M^2;\beta) = \frac{(\frac{M^2}{2})^{\frac{D}{2}-\rho}}{(2\pi)^{\frac{D}{2}}2^{\rho-2}\Gamma[\rho]}\mathcal{F}_2. \tag{39}$$

Therefore, we get the relation

$$L\mathcal{I}_\rho^{D,2}(M^2;\beta,L)|_{L\to 0} = \mathcal{I}_\rho^{D-1,1}(M^2;\beta) + \left(\frac{M^2}{2}\right)^{\frac{D}{2}-\rho}\frac{\mathcal{G}_2}{(2\pi)^{\frac{D}{2}}2^{\rho-2}\Gamma(\rho)} = \mathcal{I}_\rho^{D-1,1}(M^2;\beta) + \mathcal{O}(\ln L). \tag{40}$$

The only difference compared to the scenario with integer values of ν is the divergent behavior of $\mathcal{G}_2$; in this case, we have

$$\mathcal{G}_2 \sim \begin{cases} \ln L, & D-2\rho < 3, \\ L^{-(D-2\rho-1)}, & D-2\rho \geq 3. \end{cases}$$

C. Other real ν, noninteger D

To complete the analysis, we consider other real values of ν that allow to take into account noninteger values of the dimension D. We follow the same procedure and define three functions $\mathcal{F}_3$, $\mathcal{G}_3$, and $\mathcal{H}_3$; after substituting Eq. (20) into Eq. (4), we get

$$L\mathcal{I}_\rho^{D,2}(M^2;\beta,L) = \left(\frac{M^2}{2}\right)^{\frac{D}{2}-\rho}\frac{1}{(2\pi)^{\frac{D}{2}}2^{\rho-2}\Gamma(\rho)}(\mathcal{F}_3+\mathcal{G}_3+\mathcal{H}_3), \tag{41}$$

$$\mathcal{F}_3 = \frac{\pi}{M^2\beta}\Gamma(1-\nu) + \sum_{k=0}^{\infty}\frac{(-1)^k}{\Gamma(k+1)}\left(\frac{M\beta}{2\pi}\right)^{2k-2\nu}\frac{\beta}{2\pi}\Gamma(1-\nu+k)\zeta(2-2\nu+2k), \tag{42}$$

$$\begin{aligned}\mathcal{G}_3 = {}& \frac{L}{2}\sum_{k=0}^{\lfloor\frac{\nu-1}{2}\rfloor}\frac{(-1)^k}{\Gamma(k+1)}\Gamma(\nu-k)\zeta(2\nu-2k)\left(\frac{ML}{2}\right)^{-2\nu+2k}\\ &-\frac{\sqrt{\pi}L^2}{2\beta}\sum_{k=0}^{\lfloor\nu-1\rfloor}\frac{(-1)^k}{\Gamma(k+1)}\Gamma\left(\nu-k-\frac{1}{2}\right)\zeta(2\nu-2k-1)\left(\frac{ML}{2}\right)^{2k-2\nu},\end{aligned} \tag{43}$$

$$\begin{aligned}\mathcal{H}_3 = {}& \frac{L}{2}\sum_{k=0}^{\infty}\frac{(-1)^k}{\Gamma(k+1)}\Gamma(\nu-k)\zeta(2\nu-2k)\left(\frac{M\beta}{2}\right)^{-2\nu+2k}\\ &+\frac{L}{2}\sum_{k=\max(0,\lfloor\frac{\nu-1}{2}\rfloor)}^{\infty}\frac{(-1)^k}{\Gamma(k+1)}\Gamma(\nu-k)\zeta(2\nu-2k)\left(\frac{ML}{2}\right)^{-2\nu+2k}\\ &+\frac{\sqrt{\pi}L^2}{2\beta}\sum_{k=0}^{\infty}\frac{(-1)^k}{\Gamma(k+1)}\Gamma\left(\nu-k-\frac{1}{2}\right)\zeta(2\nu-2k-1)\left(\frac{M\beta}{2}\right)^{2k-2\nu}\\ &-\frac{\sqrt{\pi}L^2}{2\beta}\sum_{k=\max(0,\lfloor\nu-1\rfloor)}^{\infty}\frac{(-1)^k}{\Gamma(k+1)}\Gamma\left(\nu-k-\frac{1}{2}\right)\zeta(2\nu-2k-1)\left(\frac{ML}{2}\right)^{2k-2\nu}\\ &-\frac{L}{2\sqrt{\pi}}\sum_{k=0}^{\infty}\frac{(-1)^k}{\Gamma(k+1)}\left(\frac{M\beta}{2\pi}\right)^{2k-2\nu}\Gamma\left(\frac{1}{2}-\nu+k\right)\zeta(1-2\nu+2k).\end{aligned} \tag{44}$$

In this case, the second scenario is obtained after substituting Eq. (19) into Eq. (10),

$$\mathcal{I}_\rho^{D-1,1}(M^2;\beta) = \frac{(\frac{M^2}{2})^{\frac{D}{2}-\rho}}{(2\pi)^{\frac{D}{2}}2^{\rho-2}\Gamma(\rho)}\left[\frac{\pi}{M^2\beta}\Gamma(1-\nu)+\frac{\sqrt{\pi}}{M}\sum_{k=0}^{\infty}\frac{(-1)^k}{\Gamma(k+1)}\Gamma\left(\nu-k-\frac{1}{2}\right)\zeta(2\nu-2k-1)\left(\frac{M\beta}{2}\right)^{-2\nu+2k+1}\right].$$

The product of the gamma and zeta functions can be rewritten, and we obtain

$$\mathcal{I}_\rho^{D-1,1}(M^2;\beta) = \left(\frac{M^2}{2}\right)^{\frac{D}{2}-\rho}\frac{1}{(2\pi)^{\frac{D}{2}}2^{\rho-2}\Gamma(\rho)}\mathcal{F}_3. \tag{45}$$

Finally, we obtain the simple relation

$$L\mathcal{I}_\rho^{D,2}(M^2;\beta,L)|_{L\to 0} = \mathcal{I}_\rho^{D-1,1}(M^2;\beta) + \left(\frac{M^2}{2}\right)^{\frac{D}{2}-\rho}\frac{\mathcal{G}_3}{(2\pi)^{\frac{D}{2}}2^{\rho-2}\Gamma(\rho)} = \mathcal{I}_\rho^{D-1,1}(M^2;\beta)+\mathcal{O}(L). \tag{46}$$

The leading-L behavior depends on the structure of $\mathcal{G}_3$ which has the asymptotic behavior $\mathcal{G}_3 \sim L^{-(D-2\rho-1)}$, $D-2\rho \geq 2$ as $L \to 0$.

IV. CONCLUSIONS

We obtained that, at least for the class of one-loop diagrams, it is possible to consistently reduce the dimension of the system if one proceeds carefully.

We are not allowed to perform a strict dimensional reduction by taking a model with a finite length L and suppressing it continuously to zero. The first obstruction is the function $\mathcal{G}$, which carries the divergent behavior as L goes to zero. Of course, strictly speaking, $\mathcal{I}_\rho^{D,2}(M^2;\beta,L)$ cannot be evaluated at $L=0$ due to this divergence. However, we can also obtain some specific possibilities where $\mathcal{G}=0$, which occurs for $D=1$, 2 or any noninteger dimension $D<4$. Of course, $D=1$ is inconsistent (as there are two compactified dimensions) and must be discarded, and $D=2$ is the case where all dimensions are compactified. Anyway, assuming $\mathcal{G}=0$, the procedure of dimensional reduction for $L\to 0$ is

$$L\mathcal{I}_\rho^{D,2}(M^2;\beta,L)|_{L\to 0}=\mathcal{I}_\rho^{D-1,1}(M^2;\beta) \qquad \left(\text{for } \frac{D}{2}-\rho<1\right). \tag{47}$$

The identification in Eq. (47) is also valid for any D if we choose the prescription to ignore the divergent function $\mathcal{G}$. This prescription is what we defined as "dimensional reduction": we take the limit of a function as the length L goes to zero and remove its divergent components.

We remark that the L on the right-hand side of the above equation simply indicates that $\mathcal{I}_\rho^{D,2}$ also diverges as L goes to zero despite the existence of $\mathcal{G}$. The result that a continuous approach to the dimensional reduction is not possible is not a surprise. Indeed, in previous articles (in the context of phase transitions) a *minimal length* has been found below which the phase transition does not occur. Moreover, this result agrees with experimental observations about the existence of a minimal length.

However, now that we have made it clear that a strictly dimensional reduction is not attainable, we are allowed to discuss the existence of a prescription to do so. The idea is that there is a relationship between both situations: one with a small system length L and another where this dimension is ignored from the beginning.

Moreover, we could consider an N-component scalar model with a quartic interaction in $D=3$ with a tree-level coupling λ and mass m: this describes a heated surface. Taking the large-N limit and using a formal resummation, the one-loop corrections to the coupling constant g and squared mass M^2 are

$$g_3=\frac{\lambda_3}{1-\lambda_3\mathcal{I}_2^{3,1}(M^2;\beta)}, \tag{48a}$$

$$M^2=m^2+\lambda_3\mathcal{I}_1^{3,1}(M^2;\beta). \tag{48b}$$

Assuming that the surface is indeed a "dimensionally reduced" case of a heated film with thickness L, we can ignore the divergent component $\mathcal{G}$ and use the identification in Eq. (47) to write Eqs. (48a)–(48b) as

$$g_3=\frac{\lambda_3}{1-(\lambda_3 L)\mathcal{I}_2^{4,2}(M^2;\beta,L)}, \tag{49a}$$

$$M^2=m^2+(\lambda_3 L)\mathcal{I}_1^{4,2}(M^2;\beta,L). \tag{49b}$$

On the other hand, if we simply consider a heated film in $D=4$ with N scalar fields and explore its behavior for very small thickness, we get

$$g_4=\frac{\lambda_4}{1-\lambda_4\mathcal{I}_2^{4,2}(M^2;\beta,L)}, \tag{50a}$$

$$M^2=m^2+\lambda_4\mathcal{I}_1^{4,2}(M^2;\beta,L). \tag{50b}$$

Comparing both scenarios in Eqs. (49a)–(49b) and (50a)–(50b), we see that the coupling constant from the planar scenario (both the free λ_3 and corrected g_3) is related to the coupling constant from the thin film scenario by

$$\lambda_3 L=\lambda_4. \tag{51}$$

We can extract from this simple relation some important conclusions:

(1) A strict dimensional reduction, once again, is not allowed. It would require that the coupling constant for the reduced scenario goes to infinity.
(2) This is a direct indication that both scenarios are physically different.
(3) In the context of phase transitions or some other situation where the existence of a minimal thickness $L_{\min}$ can be observed, we can consider that the effective coupling constant in the planar scenario is $\lambda_3=\lambda_4/L_{\min}$.

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