Effect of vacuum polarization on the magnetic fields around a Schwarzschild black hole

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It is a well-known result that the effect of vacuum polarization in gravitational fields will lead to a nonminimal coupling between gravity and electromagnetism. We investigate this phenomenon further by considering the description of static magnetic field around a Schwarzschild black hole. It is found that close to the Schwarzschild horizon the magnetic fields can be strongly modified with respect to both cases of magnetic fields on flat spacetime and magnetic fields minimally coupled on curved spacetime. Under the proper sign of the nonminimal coupling parameter, q, the effective fields can undergo large amplifications. Furthermore, we discuss the physical meaning of the singularities that arise in the considered problem. We conclude by discussing the potential observational effects of vacuum polarization on the magnetic fields. In the case of astrophysical black holes, depending on the value of the coupling parameter, significant modifications of the magnetic fields near the black hole horizons are possible—which could be used to detect the vacuum polarization effect or at least to put constraints on the values of the coupling parameter. Moreover, we show how the considered effect directly constrains the viability of primordial black holes of sizes smaller than that of the Compton wavelength for the electron and also impacts the distribution of magnetic fields in the early Universe.

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I. INTRODUCTION

Although the description of electrodynamic phenomena in terms of Maxwell's equations represents one of the most successful physical theories, it is known that for high enough electromagnetic fields quantum radiative corrections should become important [1]. Vacuum polarization is an effect of appearance of virtual pairs in a "fermion loop," which changes the propagation properties of photons in vacuum. This process can be understood as a result of quantum fluctuations, in which a photon can fluctuate into a fermion pair, described by some probability amplitude which will in general be different from zero. These effects are theoretically well founded and have been known for a long time, but their observation still remains an open experimental goal [2]. Moreover, the observational aspects of vacuum polarization were mostly focused on the Minkowski (flat) spacetime.

While studying the electromagnetic fields on curved spacetimes in the context of general relativity [3,4], the focus is mostly put on the macroscopic and classical aspects of these fields. In this framework, the electromagnetic field is represented by the electromagnetic tensor, which contributes to the stress-energy tensor, thus leading to spacetime curvature, in the same fashion as other forms of energy distributions. The equations of motion are derived from the Lagrangian density consisting of the Einstein-Hilbert and Maxwell contributions, and the coupling between the electrodynamics and gravitational sector is therefore only minimal.

However, for strong enough electromagnetic and gravitational fields, the analysis of electromagnetism on curved spacetimes should be modified to take into account the effects coming from the vacuum polarization. The dominant process will be given by the transition of a photon into an e^+e^- pair. This fluctuation defines a characteristic length scale for the photon, which is of the order of the Compton wavelength of the electron. Since it now has an effective characteristic length, the photon will be influenced by the change in geometry along this characteristic scale. This means that properties of photons in vacuum will be influenced by the spacetime curvature, coming from tidal effects on its length, which are effectively acquired through the vacuum polarization. This phenomenon was studied in Ref. [5] in the one-loop approach, and it was shown to introduce the nonminimal coupling between the electromagnetic field and curvature. Apart from the standard Maxwell

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part, the Lagrangian for electrodynamics now also consists of the additional contribution, given by the contraction of electromagnetic and curvature tensors, leading to gauge and curvature invariant terms. This effect also has some important implications on the regime of validity of general relativity. One of the central principles of general relativity is the equivalence principle, which assumes the equivalence of gravitational and inertial effects for sufficiently small regions of spacetime, such that tidal effects can be ignored. However, it is obvious that tidal effects cannot be ignored even on the scale of electron Compton wavelength, when the vacuum polarization becomes important.

In Ref. [5], the QED vacuum polarization on the curved spacetime was discussed for the simplest case of a photon propagating in vacuum. It is not simple to generalize these results to some arbitrary field configurations, but it seems very plausible that the same type of coupling between electromagnetic and curvature terms will remain valid in general, at least in the leading order. However, the coupling coefficients should be considered as different, and they will probably depend on the characteristic physical regime. In principle, for strong electromagnetic and gravitational fields, characterized by a complex configuration of photons, the characteristic coupling between the electromagnetic and gravitational fields could then be much stronger than the one-photon correction, which is of the order of the square of the Compton wavelength for the electron. Nonminimal coupling between electrodynamics and gravitational sector was in recent decades investigated in various theoretical settings [6–14].

Although the effects of vacuum polarization on curved spacetime electrodynamics are of fundamental significance for our understanding of physical interactions, and are the first step toward the unification of electromagnetism and gravity, further research on this important issue is limited by the fact that these effects are quantitatively negligible in most of the observational settings. They could, however, become important in the high curvature regimes where electromagnetic fields are present, for instance in the very early Universe and around black holes. Due to the very high conductivity of the Universe, electric fields can be taken as vanishing, while at the same time, magnetic fields are known to be present on all scales of the observable Universe—from planets to galaxy clusters and voids of intergalactic media [15–17]. It therefore seems necessary that any empirical confirmation of vacuum polarization leading to a nonminimal coupling between electrodynamics and gravity will be based on the observations of electromagnetic fields in these settings. Probably the most promising strategy for the potential detection of these effects is the analysis of deformation of the galactic magnetic field near the black hole horizons, which is in principle observable, for instance using the Zeeman splitting method. Conversely, the absence of such effects in measurements could be used to set constraints on the values of coupling coefficients between electromagnetism and gravity. The vacuum polarization could become especially important for primordial black holes, thus changing the evolution of magnetic fields in the early Universe and its large scale distribution. In this work, we propose these directions for further study of nonminimal interaction between electromagnetism and gravity, by analyzing the problem of modification of the galactic field around a black hole that comes as its consequence. This paper is organized as follows. In Sec. II, we briefly present the theory of nonminimal coupling of gravity and electromagnetism. In Sec. III, the relevant equations of motion are derived with the corresponding assumptions. In Sec. IV, the equations of motion are solved, their physical content is analyzed. In Sec. V, some applications on realistic physical systems are presented, and in Sec. VI. we conclude our work.

II. NONMINIMAL COUPLING OF GRAVITY AND ELECTROMAGNETISM

The effects coming from the virtual photon loops, leading to the vacuum polarization in the presence of a gravitational and electromagnetic field, can be described by adding a corresponding quantum correction term, S_{corr}, to the electromagnetic, S_{em}, and gravitational action, S_{grav}, so that the total action is given by $S = S_{\mathrm{em}} + S_{\mathrm{grav}} + S_{\mathrm{corr}}$, where [5]

$$S_{\mathrm{corr}} = \sum_{n,\mathrm{even}} \frac{1}{n!} \int \prod_{i=1}^{n} d^4 x_i A_{\mu_i}(x_i) G^{\mu_1 \cdots \mu_n}(x_1 \ldots x_n). \quad (1)$$

Here, A_μ is the gauge $U(1)$ potential, and $G^{\mu_1 \cdots \mu_n}$ represents the sum over one-particle-irreducible Feynmann diagrams. When this correction is computed to the lowest order in powers of the Compton wavelength, and only the quadratic terms in A_μ are considered, the total action can be written as [5,18]

$$S = \int d^4 x \sqrt{-g} \mathcal{L}, \quad (2)$$

where the Lagrangian density is

$$\mathcal{L} = \frac{R}{\kappa} + \frac{1}{2} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} \mathcal{R}^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} + \mathcal{L}_{\mathrm{matter}}, \quad (3)$$

where $\kappa = 8\pi G/c^4$ (from now on, we will set $c = G = 1$), g is the determinant of the metric tensor, R is the Ricci scalar, and $F^{\mu\nu}$ is the Maxwell tensor obeying $F^{\mu\nu} = \nabla^\mu A^\nu - \nabla^\nu A^\mu$ where ∇_μ is the covariant derivative and $\mathcal{L}_{\mathrm{matter}}$ is the Lagrangian of neutral matter. The impact of the vacuum polarization is given through the tensor defined as

$$
\begin{aligned}
\mathcal{R}^{\mu\nu\rho\sigma} \equiv & \frac{q_1}{2}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho})R \\
& + \frac{q_2}{2}(R^{\nu\rho}g^{\nu\sigma} - R^{\mu\sigma}g^{\nu\rho} + R^{\nu\sigma}g^{\mu\rho} - R^{\nu\rho}g^{\mu\sigma}) \\
& + q_3 R^{\mu\nu\rho\sigma},
\end{aligned} \tag{4}
$$

where q_1, q_2, and q_3 are the coupling constants and as usual $R^{\mu\nu}$ is the Ricci tensor and $R^{\mu\nu\rho\sigma}$ is the Riemann tensor. We can thus clearly see that the considered correction will lead to the nonminimal coupling between electromagnetism and gravity. For the simplest case of a single photon propagation, the coupling constants will naturally be of the order of the Compton wavelength, and their values were computed in Ref. [5]. It is natural to generalize this type of quantum correction, which leads to the nonminimal coupling between gravitational and electromagnetic tensors and to the more general and complex field configurations. Thus, we will assume that the same type of coupling will remain valid in arbitrary settings, at least to the leading order, but with different values of the coupling coefficients. Now, it is straightforward to obtain the equations of motion in the nonminimal coupled gravity and electromagnetism, as they are given by varying the action with respect to the $U(1)$ gauge potential A_μ. By doing so and by rewriting the equations in the familiar form, we get

$$
\nabla_\mu (F^{\mu\nu} + \mathcal{R}^{\mu\nu\rho\sigma}F_{\rho\sigma}) = 0. \tag{5}
$$

By specifying the metric tensor $g^{\mu\nu}$ and the Maxwell tensor $F^{\mu\nu}$, one can solve Eq. (5) and inspect the behavior of the solution in the nonminimal coupled theory of gravity and electromagnetism.

III. EQUATIONS OF MOTION IN THE SCHWARZSCHILD METRIC

As discussed before, we are interested in the influence of the static spherically symmetric spacetime on the electromagnetic field. Since the astrophysical electric fields can be taken as vanishing, as discussed in the Introduction, we set the electric component of the Maxwell tensor to zero, and the remaining fields are only the magnetic fields. We will also assume that the magnetic fields are weak with respect to gravity effects, so the influence of the magnetic fields to the geometry will be negligible. In this framework, the equations are considerably simplified, and the problem consists in finding the solutions for nonminimally coupled magnetic fields on a static spherically symmetric spacetime. The static spherically symmetric spacetime is described by the Schwarzschild metric

$$
\begin{aligned}
ds^2 = & -\left(1 - \frac{2M}{r}\right)dt^2 + \frac{dr^2}{(1 - \frac{2M}{r})} \\
& + r^2(d\theta^2 + \sin^2\theta d\phi^2),
\end{aligned} \tag{6}
$$

and in these coordinates, the Maxwell tensor becomes

$$
F_{\mu\nu} = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & \frac{rB_\phi}{\sqrt{1-\frac{2M}{r}}} & -\frac{r\sin\theta B_\theta}{\sqrt{1-\frac{2M}{r}}} \\
0 & -\frac{rB_\phi}{\sqrt{1-\frac{2M}{r}}} & 0 & r^2\sin\theta B_r \\
0 & \frac{r\sin\theta B_\theta}{\sqrt{1-\frac{2M}{r}}} & -r^2\sin\theta B_r & 0
\end{pmatrix}; \tag{7}
$$

now, we can solve Eq. (5). The Schwarzschild metric is a vacuum solution, and it follows that $R = 0$ and $R^{\mu\nu} = 0$; the only remaining part of $\mathcal{R}^{\mu\nu\rho\sigma}$ is the Riemann tensor,

$$
\mathcal{R}^{\mu\nu\rho\sigma} = q_3 R^{\mu\nu\rho\sigma}, \tag{8}
$$

and Eq. (5) becomes

$$
\nabla_\mu F^{\mu\nu} + q_3 \nabla_\mu (R^{\mu\nu\rho\sigma}F_{\rho\sigma}) = 0. \tag{9}
$$

In order to inspect the magnetic fields in the vicinity of the Schwarzschild metric, it is convenient to consider a realistic configuration of magnetic fields, which is of physical interest. Since the Schwarzschild metric is asymptotically flat, at distances which are considerably larger than the Schwarzchild horizon, the black hole spacetime will approach the Minkowski spacetime, and the solution of (5) will be asymptotically given by the problem of magnetic fields on the flat spacetime, which is the same as in the minimally coupled case. Due to its relevance as the model for the galactic magnetic field distribution, we will focus on the field configuration, which leads to the magnetic dipole on the flat spacetime.

In this way, we can further simplify the problem and set $B_\phi = 0$, $B_r(r,\theta,\phi) = B_r(r,\theta)$, $B_\theta(r,\theta,\phi) = B_\theta(r,\theta)$, and $B_\theta(r,\theta) = \tan\theta B_r(r,\theta)/2$. By putting all this assumptions in Eq. (5), we get

$$
\begin{aligned}
& r(r-2M)(r^3 - 2Mq_3)\frac{dB_r(r,\theta)}{dr} \\
& \quad - (10M^2 q_3 - r^4 + M(r^3 - 4q_3 r))B_r(r,\theta) \\
& = \frac{2}{\tan\theta}\sqrt{1 - \frac{2M}{r}}\, r(4Mq_3 + r^3)\frac{dB_r(r,\theta)}{d\theta}.
\end{aligned} \tag{10}
$$

Equation (10) can be separated by using $B_r(r,\theta) = B_{\mathrm{Rad}}(r)\Theta(\theta)$, and by doing so, we get two equations,

$$
\frac{d\Theta(\theta)}{d\theta} - C\tan\theta\Theta(\theta) = 0, \tag{11}
$$

$$
\frac{dB_{\mathrm{Rad}}(r)}{dr} - \frac{A_1(r) + CA_2(r)}{r(r-2M)(r^3 - 2Mq_3)}B_{\mathrm{Rad}}(r) = 0, \tag{12}
$$

where

$$A_1(r) = (10M^2 q_3 - r^4 + M(r^3 - 4q_3 r)), \tag{13}$$

$$A_2(r) = 2\sqrt{1 - \frac{2M}{r}}r(4Mq_3 + r^3), \tag{14}$$

and C is the separation constant. Equation (11) can be easily solved, and the solution is

$$\Theta(\theta) = \Theta_0 \cos^{-C}\theta, \tag{15}$$

where Θ_0 is the integration constant. When we are far away from the black hole horizon, $2M \ll r$, Eq. (15) remains the same, and the dipole magnetic field configuration requires the choice $C = -1$, while (12) simplifies, leading to a well-known dipole solution: $B_{RAD} = \text{constant}/r^3$. The considered system thus reduces to the case of the magnetic dipole on a flat spacetime in the asymptotically flat limit. In the next section, we will consider the numerical solutions of (12) around the black hole horizon.

IV. RESULTS

First, we will rescale Eq. (12) to work in dimensionless variables,

$$r \to \tilde{r} = \frac{r}{r_0}, \quad M \to \tilde{M} = \frac{M}{r_0}, \quad q_3 \to \tilde{q} = \frac{q_3}{r_0^2}, \tag{16}$$

where r_0 is the free scaling parameter and in our case we set $r_0 = 2M$—the Schwarzschild radius. In order to numerically solve Eq. (12), we need one boundary condition for which we choose $B_{\text{Rad}}(r = 100r_0) = 10^{-10}$ T. This value was motivated by the typical value of the magnetic field in our Galaxy, which we assume to be valid on distances much larger than r_0 [19]. As the obtained solutions will be scaled with respect to the magnetic dipole on flat spacetime with the same boundary condition, this numerical assumption will not influence our results. The solution to Eq. (12) for different parameters is depicted in Fig. 1. It is also interesting to plot the magnetic field as a function of $\tilde{q}$ for a fixed radial value, as shown in Fig. 2. In this fashion the dependence of the relative change of the magnetic field on the parameter $\tilde{q}$ can be seen. In the vicinity of the Schwarzschild radius, the relative change of magnitude of the magnetic field is drastically increased by increasing $\tilde{q}$, but farther away from the Schwarzschild radius, this dependence starts to be less pronounced. This means that at sufficiently large r the nonminimal coupling effect of gravity and electromagnetism approximately disappears. It can be observed that Eq. (12), apart from two known singularities (the coordinate singularity at the horizon $r = 2M$ and the physical singularity at $r = 0$), leads to an additional singularity at $r = (2Mq_3)^{1/3}$. It is important to emphasize that the singularity at $r = 0$ is no longer only a spacetime singularity but is now also a field source singularity. The geometrical nature of this singularity can be shown by using the standard analysis of divergence of the Kretschmann scalar, $R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$, at $r = 0$. The additional existence of field source singularity at the same spacetime point can be simply observed by taking the limit of Minkowski spacetime where the magnetic dipole solution, $B_{RAD} = \text{constant}/r^3$, is singular at $r = 0$. On the other hand, the new singularity at $r = (2Mq_3)^{1/3}$ is obviously not related to the curvature singularity on the spacetime described by Eq. (6) but is a consequence of introducing

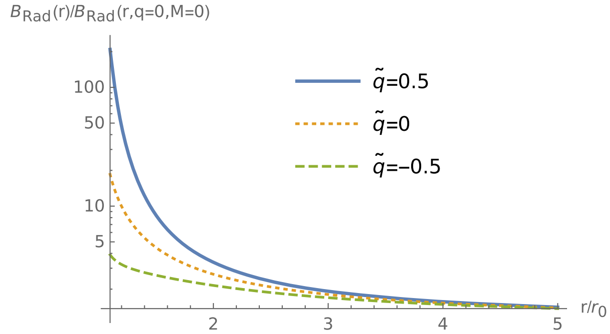

FIG. 1. The radial component of the magnetic field $B_r(r)$ scaled with respect to the magnetic dipole field as a function of r/r_0 for different values of $\tilde{q}$. The full blue line represents the $\tilde{q} = 0.5$, the orange dotted line is the general relativity (minimal coupling) case, while the green dashed line is the negative case with $\tilde{q} = -0.5$. For positive values of $\tilde{q}$, the magnetic field experiences a significant amplification near the horizon; on the other hand, for negative $\tilde{q}$, the field slowly diminishes as it approaches the horizon. It is also visible that by going farther away from the horizon the fields soon approach the general relativity (minimal coupling) case.

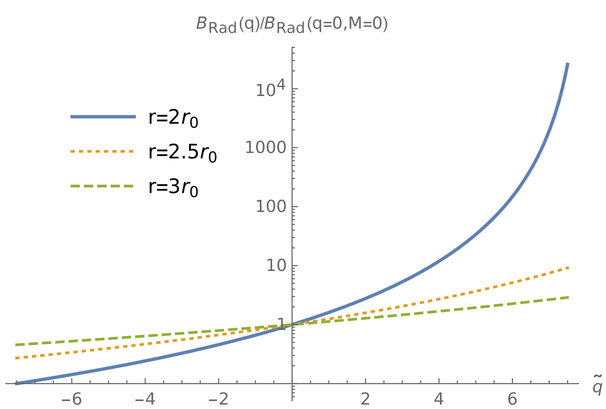

FIG. 2. The radial component of the magnetic field $B_r(q)$ scaled with respect to the magnetic dipole field as a function of $\tilde{q}$ for fixed radial value. The full blue line represents the magnetic field as a function of $\tilde{q}$ for the fixed radial value $r = 2r_0$, the orange dotted line is for the fixed radial value $r = 2.5r_0$, while the green dashed line is the $r = 3r_0$ case. Here, again, it is visible that near the horizon the magnetic fields experience a high amplification; by going farther away from the horizon, the impact of $\tilde{q}$ becomes suppressed. By increasing $\tilde{q}$, the magnetic field blows up at the effective source singularity $r = (2Mq_3)^{1/3}$.

the vacuum polarization. This can be understood as a result of the additional terms which enter in the electromagnetic field equation due to the vacuum polarization effect. These additional terms can be viewed as a new effective field configuration by defining $F^{\mu\nu}_{\rm eff} \equiv F^{\mu\nu} + \mathcal{R}^{\mu\nu\rho\sigma}F_{\rho\sigma}$. In this sense, the new singularity can be seen as a source singularity of the effective quantum corrected electromagnetic field distribution. We note that under the assumptions taken in this work this singularity will always be hidden within the black hole event horizon. This follows from the fact that under the approximation of negligible feedback on the spacetime, $|q_3| \ll M^2$, the condition $(2|q_3|M)^{1/3} \ll 2M$ needs to be satisfied. The presence of such a singularity could become important in the case of strong electromagnetic fields where the interplay between the field distribution and the spacetime geometry could no longer be neglected. If this singularity were to still persist in the fully self-consistent analysis then it would be necessary to go beyond one-loop correction used in Eq. (4).

V. IMPLICATION FOR PHYSICAL SYSTEMS

As can be seen by the results presented in the previous section, visible in Fig. 1, the positive sign of the vacuum polarization parameter $\tilde{q}$ typically leads to amplification of dipole magnetic fields near the black hole horizon with respect to both the magnetic dipole solution and minimally coupled ($\tilde{q} = 0$) magnetic field on Schwarzschild spacetime. In the opposite case with negative $\tilde{q}$, magnetic fields are suppressed. However, these effects become negligible as soon as we move away from the horizon. From Fig. 2, we conclude that a relevant amplification of magnetic fields requires the coupling coefficient to be at least of the order of magnitude, $\tilde{q} \approx 1$. From definition (16), we see that this implies $q_3 \approx r_0^2$, which is a huge value of a coupling parameter if the usual astrophysical values of r_0 are assumed. It can be seen from Fig. 2 that for $\tilde{q}$ of the same order of magnitude the magnetic fields can be enhanced for more than 4 orders of magnitude, therefore in principle leading to practically observable consequences. On the other hand, in most systems apart from black holes, the gravitational curvature effects will be negligible and thus would also be the effects of vacuum polarization related to this parameter. For instance, in the case of the Earth with $\tilde{q} = 1$ where $R_{\rm Earth}/r_0 \approx 10^9$ and the Sun $R_{\rm Sun}/r_0 \approx 10^5$, the resulting ratios $B_{\rm Rad}(R_{\rm Earth})/B_{\rm Rad}(R_{\rm Earth}, \tilde{q} = 0, M = 0)$ and $B_{\rm Rad}(R_{\rm Sun})/B_{\rm Rad}(R_{\rm Sun}, \tilde{q} = 0, M = 0)$ are practically indistinguishable from the general relativity case with $\tilde{q} = 0$. The difference is more pronounced in the case of neutron stars where $R_{NS}/r_0 \approx 17$, so the resulting variation from the $\tilde{q} = 0$ is around 0.1%. Therefore, even the high values of the coupling parameter, making it of the order of macroscopic lengths (as given by the square of Schwarzschild radius), would lead to practically detectable modifications of magnetic fields only in the vicinity of black holes. The strategy to potentially observe the electromagnetic effects of vacuum polarization in a gravitational field—or at least to put constraints on the coupling parameter q_3—would be therefore to systematically search for the variations of magnetic field around the horizons of black holes.

Although the value of the coupling q_3 should be considered as a free parameter for the systems studied in this work, we know [5] that its value needs to be at least of the order of the square of the Compton wavelength for the electron, λ_c. This most conservative option should thus be considered with a specific attention. In this case, $\tilde{q} = (\lambda_c/r_0)^2$. For all astrophysical systems it clearly follows $\tilde{q} << 1$ and thus no observable effect can be expected. However, it has been speculated [20,21]—and this option has gained a lot of popularity recently—that black holes of microscopic sizes could be created in the conditions of the early Universe. Since the minimal Schwarzschild radius of primordial black holes is expected to be larger only from the Planck length, if primordial black holes exist, one would also expect that at least for some of them $r_0 \le \lambda_c$ and their existence would lead to modifications of primordial magnetic fields, which could not be neglected. In fact, assuming the existence of primordial black holes comparable to Planck length would lead to $\tilde{q} \approx 10^{46}$, which would have a tremendous effect on the strengths and distribution of primordial magnetic fields, but such strong coupling regimes are far beyond the weak field and one-loop approximations used in this work. In any case, the investigations assuming the existence of such primordial black holes and discussing their distribution should consider the effects on the primordial magnetic fields due to the vacuum polarization. Leaving this regime aside for now, let us briefly discuss the potential consequences of primordial black holes for which $r_0 \sim \lambda_c$. From the results presented in the previous section, it follows that such primordial black holes would lead to an effective amplification or suppression of the primordial dipole magnetic field in the close vicinity of their microscopic Schwarzschild radius potentially for many orders of magnitude. The primordial black holes would subsequently evaporate as the Universe evolves, but the change in the local field strengths in the vicinity of regions where they were previously existing would in principle still be present even today. They would be visible as strongly localized significant departures from the average magnetic field strengths. Searching and analyzing such field patterns in the observed astrophysical magnetic fields could be used for testing the discussed vacuum polarization effects and setting the constraints on its characteristic parameters and also the existence and distribution of primordial black holes. Taking into account that the question of magnetogenesis and the subsequent evolution of cosmological magnetic fields represents an interesting and important problem in cosmology

[22–25], these considerations are also important since they could significantly influence it.

VI. CONCLUSION

Motivated by the effect of vacuum polarization for photons in a gravitational field, which leads to the nonminimal coupling between gravity and electromagnetism, we studied its potential consequences on magnetic fields present in the Universe. By assuming that magnetic fields are sufficiently weak so that their backreaction on spacetime can be ignored, which is the case for characteristic strengths of astrophysical magnetic fields, we solve the resulting field equation on the Schwarzschild spacetime. The obtained solutions demonstrate that in the vicinity of the Schwarzschild horizon the magnetic fields can become significantly amplified or damped—with respect to flat spacetime and minimal coupling case—depending on the value and sign of the characteristic coupling constant. Furthermore, by moving away from the Schwarzschild horizon, the obtained magnetic fields rapidly approach the magnetic dipole configuration on flat spacetime. We have also discussed the singularities present in the considered system—where apart from the well-known curvature and field source singularities at $r = 0$ a new singularity at $r = (2Mq_3)^{1/3}$ arises. We have argued that this singularity can be understood as the result of the new effective quantum corrected electromagnetic field distribution $F^{\mu\nu}_{\text{eff}} \equiv F^{\mu\nu} + \mathcal{R}^{\mu\nu\rho\sigma}F_{\rho\sigma}$. Moreover, under the assumption of negligible feedback of the magnetic fields on the spacetime, this singularity will always be hidden inside the event horizon. Finally, we have discussed how such effects can be observable in principle by systematically considering the variations of magnetic fields around the horizons of black holes. Also, this effect could be strongly pronounced in the context of cosmological magnetic fields where the existence of primordial black holes could lead to strong modifications of magnetic field configurations and strengths. These effects would manifest, depending on the sign of the coupling parameter, either as pronounced peaks in the distribution of astrophysical magnetic fields or in the opposite case as localized regions of negligible magnetic field strengths significantly departing from the average values. Therefore, the considered effects can have important consequences on the questions of magnetogenesis and evolution of magnetic fields in the early Universe.

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