Electromagnetic currents for dressed hadrons

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We propose an extension of the minimal-substitution prescription for coupling the electromagnetic field to hadronic systems with internal structure. The resulting rules of extended substitution necessarily distinguish between couplings to scalar and Dirac particles. Moreover, they allow for the incorporation of electromagnetic form factors for virtual photons in an effective phenomenological framework. Applied to pions and nucleons, assumed to be fully dressed to all orders, the resulting dressed currents are shown to be locally gauge invariant. Moreover, half-on-shell expressions of (hadron propagator) × (electromagnetic current) needed in all descriptions of physical processes will lose *all* information about hadronic dressing for real photons. The Ball-Chiu ansatz for the spin-$1/2$ current is seen to suffer from an incomplete coupling procedure where some essential aspects of the Dirac particle are effectively treated as those of a scalar particle. Applied to real Compton scattering on pions and nucleons, we find that *all* dressing information cancels exactly when external hadrons are on shell, leaving only gauge-invariant bare Born-type contributions with physical masses. Hence, nontrivial descriptions necessarily require contact-type two-photon processes obtained by hadrons looping around two photon insertion points.

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I. INTRODUCTION

The photon arguably provides the cleanest probe of hadronic structure available in experiments. As reviewed in Ref. [1], real and virtual photon probes have been and are being used in many experimental facilities around the world—JLab, MAMI, ELSA, SPring-8, GRAAL, and others—to help unravel the internal dynamics of hadronic systems.

On the theoretical side, however, the situation is much less clean because of the effective nature of the hadronic degrees of freedom (d.o.f.) that appear in experiments. While the electromagnetic interaction is understood perfectly well at the elementary level, its application to composite baryonic and mesonic systems of elementary particles is not straightforward. Notwithstanding these problems, elaborate and sophisticated expansion and power-counting schemes have been devised to permit the extraction of meaningful model-independent results from the experimental data (see Ref. [2–4] and references therein). However, given the nature of such schemes, while they work well at low energies, their application becomes increasingly difficult away from threshold. At intermediate energies, in particular, where most baryonic states with nontrivial structure are found, one oftentimes must resort to effective Lagrangian formulations because they provide a more direct access to the actual hadronic d.o.f.—mesons and baryons—as they manifest themselves in experiments.

The present work addresses the question of how to implement the electromagnetic interaction in an effective Lagrangian formulation where the descriptive d.o.f. are mesons and baryons, however, assuming that they are fully dressed.

Minimal substitution is the standard way of coupling the electromagnetic field A^μ to a charged particle with momentum p. The corresponding replacement rule,

$$p^\mu \to p^\mu - QA^\mu, \tag{1}$$

where Q is the particle's charge operator, is based on the usual covariant derivative of quantum field theory [5,6]. One of the especially attractive features of minimal substitution is that it will preserve local gauge invariance as a matter of course if implemented consistently. However, while well defined in limited circumstances, it cannot be relied upon to produce correct or consistent results in all situations [7]. In effective theories, in particular, that subsume elementary QCD d.o.f. in terms of hadronic ones, many, if not most, of the shortcomings of minimal substitution can be traced back to the incorrect or incomplete implementation of magnetic and polarization properties that arise from the interaction of the electromagnetic field with the extended charge structure of the hadrons. The coupling operators for such interactions are transverse and therefore gauge invariance is not affected by such

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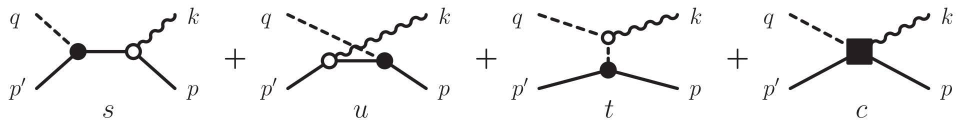

FIG. 1. Basic topology of pion photoproduction off the nucleon, with associated four-momenta. Here, and in all other diagrams, time runs from right to left. The first three diagrams depict s-, u-, and t-channel, respectively, according to the respective Mandelstam variable of the intermediate off-shell hadron. The last diagram comprises all nonpolar contact-type mechanisms including final-state interactions [11]. The half-on-shell contributions for the pion current in the t-channel and for the nucleon currents in the s- and u-channels are discussed in Secs. III B and IV C, respectively.

incomplete implementations, but the correct description of the physics at hand may suffer.

In the minimal-substitution framework, the currents J^μ describing the first-order interaction of a system with an external electromagnetic field are defined by implementing the substitution (1) for the (connected) Green's function of the system, performing an expansion in A_μ, and taking the functional derivative $\delta/\delta A_\mu$ for $A_\mu = 0$, and then truncating all external single-particle propagators according to the LSZ prescription [5,6,8], which isolates the current J^μ as the coefficient operator of the first-order interaction term $J^\mu A_\mu$. It should be obvious that this procedure cannot be expected to correctly produce anomalous magnetic moments because those may be present even if the system as a whole is uncharged. As a case in point, if one considers the neutron, for example, as a single effective hadronic system, the corresponding current vanishes and there is no anomalous contribution at all. To produce anomalous contributions and polarization effects, one needs to explicitly consider the substructure of the nucleon in terms of meson-loop dressings and explicitly incorporate all possible interaction terms in a gauge-invariant manner [2]. This can be largely understood as an implementation of minimal substitution at a more microscopic level.

In general, however, minimal substitution seems to be incapable of producing electromagnetic form factors for virtual photons directly. As has been pointed out [9], the dressing effects due to minimal substitution—even when applied to fully dressed hadronic entities—will depend on the squared hadronic four-momenta going into and coming out of the electromagnetic vertex, but they cannot produce the clean k^2 dependence required to produce the electromagnetic form factors $F(k^2)$, where k is the photon four-momentum. Hence, the k^2 dependence of form factors—usually simply stated as a requirement based on Lorentz invariance—must result from other dynamical effects. Rather than discussing the possible nature of such effects, we point out that we cannot expect to be able to describe form-factor effects in any standard application of minimal substitution in an effective dynamical framework.

Given this situation, we propose here to turn the question around and ask how one can extend minimal substitution to incorporate known experimental information—masses, charges, anomalous moments, and k^2 dependence of form factors—into a framework that assumes that *all* of the hadronic dressing effects of system are known to all orders. How such dressing effects are obtained is then a problem secondary to constructing current operators that are consistent with the assumed hadronic dressing.

We will show that this consistency requirement means that the proposed *extended substitution* must distinguish in essential aspects between how the field couples to scalar (spin-0) particles and Dirac (spin-1/2) particles. We will construct the fully dressed electromagnetic currents for pions and for nucleons within the proposed framework which will incorporate known experimental information about these hadrons. For the nucleon current, this will remedy a shortcoming of the spin-1/2 Ball-Chiu ansatz [10]. Specifically, we will show that the Ball-Chiu current results from treating some essential aspects of the nucleon dressing as being that of a scalar particle, which clearly is inconsistent.

We will also show that products of (hadron propagator)× (electromagnetic current)—both fully dressed—will lose *all* information about their detailed dressing mechanisms when taken half-on-shell on the current side, with the *physical* hadron mass being the only remnant of the dressing and additional off-shell information only entering for virtual photons. For meson photoproduction processes with real photon, in particular, this means that for the usual s-, u-, and t-channel Born terms depicted in Fig. 1 only the meson-production vertex carries any structure information. Complete cancellations of off-shell effects also extend to the respective real Compton scattering reactions for both pions and nucleons, leaving only Born-type expressions. It is argued that nontrivial Compton-scattering contributions require genuine two-photon processes where the respective (off-shell) Compton tensor is dressed by hadron loops.

The paper is organized as follows. In Sec. II, we will present the basic rules for the proposed extension of minimal substitution. We will do so using the device of the gauge derivative of Ref. [11] which provides a convenient shorthand notation for the implementation of minimal substitution. (For completeness, some pertinent details of the gauge-derivative procedure are recapitulated in the Appendix.) In Secs. III and IV, respectively, we apply the proposed extended substitution rules to the fully dressed pion and nucleon propagators as examples for spin-0 and spin-1/2 particles and construct their corresponding fully dressed current operators. For the nucleon case, we show that the Ball-Chiu current ansatz [10] suffers from an incomplete coupling procedure. We also derive the aforementioned cancellation of dressing effects for half-on-shell combinations of propagator and current for real photons, which extends to the on-shell Compton tensors. The final Sec. V provides a summary and discussion of the present findings.

II. RULES OF EXTENDED SUBSTITUTION

Following Ref. [11], we will use the device of the gauge derivative as a shorthand tool for describing how minimal substitution affects the reaction dynamics of a particle with momentum p and associated charge operator Q. As the examples in Refs. [11,12] demonstrate, the gauge derivative may be used to consistently link all topological elements of reaction mechanisms that contribute to the interaction with the external electromagnetic field, in a procedure sometimes referred to as "gauging of equations." However, for the present purpose, we only need to consider the 'last step', when the gauge derivative is applied to the functions—propagators, vertices—that describe elements of the reaction at hand to provide the actual coupling mechanisms J^μ to the external field A_μ.

With the rules given in Ref. [11], briefly summarized here in the Appendix for completeness, for a spin-0 scalar particle of momentum p and charge Q, the current operator results from

$$\{p^2\}^\mu = Q(p'+p)^\mu, \tag{2}$$

where $p' = p + k$ for an (incoming) photon with four-momentum k. The gauge-derivative braces $\{\cdots\}^\mu$ here indicate coupling of the photon four-momentum k^μ to the functional dependence p^2. The result (2) immediately follows from Eq. (A7) in the Appendix and the fact that the inverse propagator of a scalar particle is a function of p^2 alone. For a spin-$1/2$ Dirac particle, we also find the usual coupling mechanism

$$\{\not{p}\}^\mu = Q\gamma^\mu \tag{3}$$

because its inverse propagator is a function of $\not{p}$.

Considering now the gauge derivative of an invariant scalar functions $f(p^2)$ of the particle's squared four-momentum, clearly, we have

$$\begin{aligned}\{f(p^2)\}^\mu_S &= \{p^2\}^\mu \frac{f(p'^2)-f(p^2)}{p'^2-p^2}\\ &= Q(p'+p)^\mu \frac{f(p'^2)-f(p^2)}{p'^2-p^2},\end{aligned} \tag{4}$$

as expected, where the index S indicates that $f(p^2)$ results from the dynamics of a scalar particle. The proof is easily found by expanding $f(p^2)$ in powers of p^2, applying the product rule to every term in the expansion, and then resumming. (This assumes that the expansion is well defined at least at the formal level. In general, nonanalytic functions may require special treatments.) The function ratio on the right-hand side of Eq. (4) presents a well-defined $\frac{0}{0}$ situation at $p'^2 = p^2$ providing the derivative of f. Because of this result, such finite-difference derivatives (FDDs) will be ubiquitous in the present investigation.

For a Dirac particle, we may write $\not{p}^2 = p^2$, which is crucial to expressing the corresponding Feynman propagator as

$$\frac{1}{\not{p}-m} = \frac{\not{p}+m}{p^2-m^2} \tag{5}$$

to establish that, in addition to the Dirac equation, it also solves the Klein-Gordon equation for the same mass m. One finds this equivalence would be destroyed if one applied the scalar gauge-derivative result (2) to the p^2 dependence on the right-hand side of this equation. Instead, as explained in the context of Eq. (A9) in the Appendix, equivalence demands that with

$$\begin{aligned}\{\not{p}^2\}^\mu &= \not{p}'\{\not{p}\}^\mu + \{\not{p}\}^\mu\not{p}\\ &= Q(\not{p}'\gamma^\mu + \gamma^\mu\not{p}),\end{aligned} \tag{6}$$

which follows from the product rule (A4), one needs to introduce the Dirac version

$$\{f(p^2)\}^\mu_D = \{\not{p}^2\}^\mu \frac{f(p'^2)-f(p^2)}{p'^2-p^2} \tag{7}$$

to replace Eq. (4) for Dirac particles. The relationship between $\{f\}^\mu_D$ and $\{f\}^\mu_S$ is given by the Gordon identity,

$$\not{p}'\gamma^\mu + \gamma^\mu\not{p} = (p'+p)^\mu + i\sigma^{\mu\nu}k_\nu, \tag{8}$$

i.e., the two derivatives differ by a manifestly transverse term. Their respective four-divergences,

$$k_\mu\{f(p^2)\}^\mu_S = k_\mu\{f(p^2)\}^\mu_D = Q[f(p'^2)-f(p^2)], \tag{9}$$

therefore, are unaffected which is crucial for maintaining gauge invariance.

A. Extending minimal substitution

The basic coupling mechanisms for scalar and Dirac particles described so far, if implemented consistently, are sufficient to provide a current for any system that maintains local gauge invariance as expressed in terms of Ward-Takahashi identities (WTI) for three-point functions [13] and generalized WTIs for N-point functions [11,12,14]. However, as discussed in the Introduction, they cannot account for any structure that results from electromagnetic form factors. Such effects, therefore, must come from manifestly transverse coupling mechanisms.

For definiteness, we will here consider the pion as an example of a scalar particle, identified by index π, and the nucleon as a spin-1/2 Dirac particle, with two charge states $N = p, n$, where

$$Q_p = e\frac{1+\tau_3}{2} \quad \text{and} \quad Q_n = e\frac{1-\tau_3}{2} \tag{10}$$

are the respective isospin operators for proton (p) and neutron (n), respectively, with τ_3 being the usual Pauli matrix; e is the fundamental charge unit.

For the pion with four-momentum q, we amend the elementary scalar coupling rule (2) and allow for

$$\{q^2\}^\mu = Q_\pi(q'+q)^\mu + T^\mu_\pi(q',q), \quad \text{with} \quad k_\mu T^\mu_\pi \equiv 0, \tag{11}$$

where the transverse current is given by

$$T^\mu_\pi(q',q) = Q_\pi t^\mu(q',q) f_\pi(q',q), \tag{12}$$

where

$$t^\mu(q',q) = (q'+q)^\mu - k^\mu \frac{q'^2-q^2}{k^2} \tag{13}$$

is the only manifestly transverse operator one can construct with two independent four-momenta q and $q' = k+q$. The scalar (and symmetric) form-factor function $f_\pi(q',q) = f_\pi(q'^2,q^2;k^2)$ here must vanish at $k^2 = 0$ to make the current nonsingular. More details about its relationship to the physical (on-shell) pion form factor $F_\pi(k^2)$ will be given in the subsequent Sec. III. Q_π here describes the charges of the pions in units of e, with f_π being the same for $\pi^\pm$; π^0 has no form factor because it is its own antiparticle.

For the nucleon N with four-momentum p and mass m, we amend the basic Dirac-particle rule (3) by

$$\{\not{p}\}^\mu = \Gamma^\mu_N(p',p) \equiv Q_p\gamma^\mu + T^\mu_N(p',p), \tag{14}$$

where T^μ_N is a manifestly transverse current,

$$k_\mu T^\mu_N \equiv 0, \quad \text{for } N = p, n, \tag{15}$$

that can be expressed in terms of the two transverse operators

$$\sigma^\mu_{\mathrm{T}} = \frac{i\sigma^{\mu\nu}k_\nu}{2m} \quad \text{and} \quad \gamma^\mu_{\mathrm{T}} = \gamma^\mu - k^\mu\frac{\not{p}'-\not{p}}{k^2}, \tag{16}$$

where the latter will require a coefficient function that vanishes at $k^2 = 0$ to render the corresponding current well defined. The transverse currents then may be written as

$$T^\mu_N(p',p) = Q_N(\gamma^\mu_{\mathrm{T}} f^N_1 + \sigma^\mu_{\mathrm{T}} f^N_2), \quad \text{for } N = p, n, \tag{17}$$

where the four scalar (and symmetric) coefficient functions $f^N_i = f^N_i(p'^2, p^2; k^2)$ ($N = p$, n; $i = 1$, 2) are to be constrained by the Dirac and Pauli form factors of the proton and neutron. More details will be discussed in the nucleon section IV below.

Note here that the k^μ contributions in (13) and (16) are necessary for formal reasons to verify the respective transversality conditions. For practical purposes, however, we may drop such terms from any physically relevant current since $\epsilon_\mu k^\mu = 0$ for covariant physical photon polarization ϵ_μ, irrespective of whether the photon is real or virtual. Moreover, in view of the numerator expressions $q'^2 - q^2$ and $\not{p}' - \not{p}$ in the respective transverse couplings, these terms do not contribute anyway for on-shell hadrons. Nevertheless, one might be well advised to drop these terms only at the very end, when physical matrix elements are to be calculated because doing so at the very start may lead to erroneous conclusions.

To see the effect of the extended substitution rules (11) and (14) over the respective basic rules (2) and (3) in the following considerations, one only needs to put the respective current T^μ_π and T^μ_N to zero. Obviously, since the extensions only add transverse currents, they have no effect on gauge invariance at all.

III. PION: SPIN 0

The most general propagator for a fully dressed pion with four-momentum q and mass μ can be written as

$$\Delta_\pi(q^2) = \frac{1}{(q^2-\mu^2)\Pi(q^2)}, \tag{18}$$

where the scalar dressing function $\Pi(q^2)$ is normalized as

$$\Pi(\mu^2) = 1, \tag{19}$$

which provides the required unit residue for the propagator.

We may expand the dressing functions around the pole by writing

$$\Pi(q^2) = 1 + \frac{q^2-\mu^2}{\mu^2} D_\pi(q^2) \tag{20}$$

where

$$D_\pi(q^2) = \mu^2\frac{\Pi(q^2)-1}{q^2-\mu^2} \tag{21}$$

is a well-defined nonsingular finite-difference derivative (FDD) that is proportional to the derivative of the dressing function in the limit $q^2 \to \mu^2$ at the pole. (The proportionality factor μ^2 is only introduced to make D_π dimensionless.) We thus have

$$\Delta_\pi(q^2) = \frac{1}{q^2-\mu^2} - \frac{D_\pi(q^2)}{\Pi(q^2)\mu^2}, \tag{22}$$

where the only remnant of the dressing in the pole term is the physical mass μ; all other dressing effects sit in the nonpolar contact-type counterterm.

A. Pion current

Using the extended substitution (11), the pion current for an incoming photon with momentum $k = q' - q$ is obtained from gauging the inverse propagator according to

$$\begin{aligned} J^\mu_\pi(q',q) &= \{\Delta^{-1}_\pi(q^2)\}^\mu_S \\ &= \left\{\frac{q^2\Pi(q^2)+\Pi(q^2)q^2}{2} - \mu^2\Pi(q^2)\right\}^\mu_S, \end{aligned} \tag{23}$$

where the scalar-particle gauge derivative is applied to the symmetrized expressions as explained in Appendix. With the scalar gauge-derivative rule (4), using the extended substitution (11), we then obtain the fully dressed current as

$$J^\mu_\pi(q',q) = [Q_\pi(q'+q)^\mu + T^\mu_\pi]\frac{\Delta^{-1}_\pi(q'^2) - \Delta^{-1}_\pi(q^2)}{q'^2-q^2}. \tag{24}$$

For $T^\mu_\pi \equiv 0$, this result was given in Ref. [9]. This current trivially satisfies the appropriate WTI for the pion,

$$k_\mu J^\mu_\pi(q',q) = Q_\pi[\Delta^{-1}_\pi(q'^2) - \Delta^{-1}_\pi(q^2)], \tag{25}$$

and thus is manifestly locally gauge invariant. We emphasize that this finding does not result from a condition that needs to be imposed on the current, but that it is a straightforward consequence of the construction procedure in terms of the gauge derivative (23). Note that the WTI does not involve any form-factor information even if $T^\mu_\pi \neq 0$.

We emphasize that the current (24) is constructed here to be consistent with the propagator (18) such that the WTI follows as a matter of course. The procedure, however, makes no assumption about the details of the dressing function $\Pi(q^2)$ other than stipulating that it produces a unit residue. Therefore, any (nonpathological) determination of $\Pi(q^2)$, whether sophisticated or not, that produces $\Pi(\mu^2) = 1$ will fit the present framework.

With nonzero transverse contributions T^μ_π, the current (24) actually comprises the most general expression one can write down for the scalar current if one allows for arbitrary *nonsingular* symmetric expressions for the form factor, $f_\pi(q',q) = f_\pi(q'^2,q^2;k^2) = f_\pi(q^2,q'^2;k^2)$. Since *any* additional dressing effect must be transverse and off-shell, they may always be assumed to be already subsumed in these form factors as a matter of course. Including on- and off-shell d.o.f. in these form factors, therefore, the current (24) comprises all possibilities. For further discussion of off-shell freedom, see the summarizing Sec. V.

Half on shell, the current reduces to

$$J^\mu_\pi(q',q) = [Q_\pi(q'+q)^\mu + T^\mu_\pi(q',q)]\Pi(q^2_{\rm off}), \tag{26}$$

where $q^2_{\rm off}$ is either q'^2 or q^2 depending on which leg is off shell. The fully on-shell current thus reads

$$\begin{aligned} J^\mu_\pi(\underline{q}',\underline{q}) &= Q_\pi(\underline{q}'+\underline{q})^\mu + T^\mu_\pi(\underline{q}',\underline{q}) \\ &= Q_\pi(\underline{q}'+\underline{q})^\mu[1+f_\pi(\mu^2,\mu^2;k^2)], \end{aligned} \tag{27}$$

where the underlining indicates on-shell momenta. Hence, in view of the unit result for the physical form factor $F_\pi(k^2)$ for real photons and to ensure that the current (24) is nonsingular both on and off shell, we may write

$$f_\pi(q'^2,q^2;k^2) = \frac{k^2}{\mu^2}H_\pi(q'^2,q^2;k^2), \tag{28}$$

where the nonsingular symmetric function H_π is defined by this equation, and determine the physical (on-shell) form factor as

$$F_\pi(k^2) = 1 + \frac{k^2}{\mu^2}H_\pi(\mu^2,\mu^2;k^2). \tag{29}$$

Hence, the coupling operator of the pion current (24) may be written as

$$\begin{aligned} Q_\pi(q'+q)^\mu + T^\mu_\pi &= Q_\pi(q'+q)^\mu\left[1+\frac{k^2}{\mu^2}H_\pi(q'^2,q^2;k^2)\right] \\ &\quad - Q_\pi k^\mu\frac{q'^2-q^2}{\mu^2}H_\pi(q'^2,q^2;k^2). \end{aligned} \tag{30}$$

B. Half-on-shell contribution

Let us consider the half-on-shell situation of the current for the outgoing t-channel pion with the off-shell intermediate propagator, as depicted in the photoproduction diagrams of Fig. 1. Using the kinematics of the figure, with the outgoing pion on shell at $q^2 = \mu^2$ and $t = (q-k)^2$ for the intermediate pion, we have

$$J^\mu_\pi(\underline{q},q-k)\Delta_\pi(t) = [Q_\pi(2q-k)^\mu + T^\mu_\pi]\frac{1}{t-\mu^2}, \tag{31}$$

where the propagator dressing completely cancels. Using (30) and the FDD

$$D_H(t;k^2) = \mu^2\frac{H_\pi(\mu^2,t;k^2) - H_\pi(\mu^2,\mu^2;k^2)}{t-\mu^2}, \tag{32}$$

we obtain

$$J_\pi^\mu(\underline{q}, q-k)\Delta_\pi(t) = Q_\pi \frac{(2q-k)^\mu}{t-\mu^2} F_\pi(k^2) + Q_\pi \frac{(2q-k)^\mu}{\mu^2}\frac{k^2}{\mu^2} D_H(t;k^2) + Q_\pi \frac{k^\mu}{\mu^2} H_\pi(\mu^2,t;k^2). \tag{33}$$

This result is exact for the dressed current (24). All hadronic dressing effects fully cancel here and we are left with the usual expression resulting from elementary Feynman rules for the pole term. The contact term depending on the FDD $D_H(t;k^2)$ contributes only for electroproduction. Again, the contact-type k^μ term can be ignored for physical matrix elements, however, it is necessary to provide the correct four-divergence of this half-on-shell expression,

$$k_\mu J_\pi^\mu(\underline{q}, q-k)\Delta_\pi(t) = -Q_\pi, \tag{34}$$

where the minus sign signifies that the on-shell pion is outgoing. This result is identical to

$$k_\mu Q_\pi (2q-k)^\mu \frac{1}{t-\mu^2} = -Q_\pi \tag{35}$$

for the "bare" situation where the only dressing effect is the physical mass μ. Without the k^μ term in (33), this equivalence cannot be established.

We add here that in the four-point-function context of the t-channel graph in Fig. 1, the cancellation of dressing effects does not extend to the second vertex; in other words, the purely hadronic πNN vertex here retains the dressing that accounts for the intermediate pion being off-shell.

C. Real Compton scattering on the pion

A particularly straightforward application is given by making the second vertex an electromagnetic one as well, resulting in Compton scattering on a charged pion as depicted in Fig. 2. For real Compton scattering the situation is particularly simple because there is no electromagnetic form factor dependence and the only structure is provided by the propagator dressing function $\Pi(q^2)$. With the half-on-shell result (33) taken for $k^2=0$, dropping the irrelevant k^μ contribution, and with Eq. (26), we easily see that the Compton tensors for the s- and u-channel diagrams of Fig. 2, where $s=(q+k)^2$ and $u=(q-k')^2$ are the Mandelstam variables for the respective intermediate off-shell particle, read

$$M_s^{\nu\mu}(\underline{q}', \underline{q}) = e^2(2q'+k')^\nu \frac{\Pi(s)}{s-\mu^2}(2q+k)^\mu = e^2\frac{(2q'+k')^\nu(2q+k)^\mu}{s-\mu^2} + e^2(2q'+k')^\nu \frac{D_\pi(s)}{\mu^2}(2q+k)^\mu, \tag{36a}$$

$$M_u^{\nu\mu}(\underline{q}', \underline{q}) = e^2(2q'-k)^\mu \frac{\Pi(u)}{u-\mu^2}(2q-k')^\nu = e^2\frac{(2q'-k)^\mu(2q-k')^\nu}{u-\mu^2} + e^2(2q'-k)^\mu \frac{D_\pi(u)}{\mu^2}(2q-k')^\nu, \tag{36b}$$

where the respective second equalities provide the decomposition into pure (undressed) pole contributions and contact terms depending on the FDDs of the propagator dressing function according to Eq. (21). The squared charge operator Q_π^2 for $\pi^\pm$ is simply written here as e^2.

The contact term in Fig. 2 can be obtained explicitly by coupling a second photon into the current (24). Rather than doing this fully off-shell, it is easier to employ the half-on-shell expressions (26), and then symmetrize and apply the gauge derivative to entities depending on the off-shell momentum $q_{\rm off}$. With an overall minus sign resulting from the relationship of the gauge derivative to the functional derivative, as seen from Eq. (A1), one finds

$$M_c^{\nu\mu}(\underline{q}', \underline{q}) = -\frac{1}{2}[\{J_\pi^\mu(q_{\rm off}, \underline{q})\}^\nu + \{J_\pi^\nu(q_{\rm off}, \underline{q}\}^\mu] - \frac{1}{2}[\{J_\pi^\mu(\underline{q}', q_{\rm off})\}^\nu + \{J_\pi^\nu(\underline{q}', q_{\rm off}\}^\mu] = -2e^2 g^{\mu\nu} - e^2(2q'+k')^\nu\frac{D_\pi(s)}{\mu^2}(2q+k)^\mu - e^2(2q'-k)^\mu\frac{D_\pi(u)}{\mu^2}(2q-k')^\nu. \tag{37}$$

The last two terms here cancel the FDD contributions in the s- and u-channel terms in (36). Hence, summing up all three contributions, the entire Compton tensor for all diagrams in Fig. 2 for real photons scattering off the pion is given as

$$M_\pi^{\nu\mu}(\underline{q}', \underline{q}) = e^2\frac{(2q'+k')^\nu(2q+k)^\mu}{s-\mu^2} + e^2\frac{(2q'-k)^\mu(2q-k')^\nu}{u-\mu^2} - 2e^2 g^{\mu\nu}, \tag{38}$$

where the only remnant of the dressing structure is the physical pion mass. All other effects completely cancel and what remains is just what is usually referred to as the Born

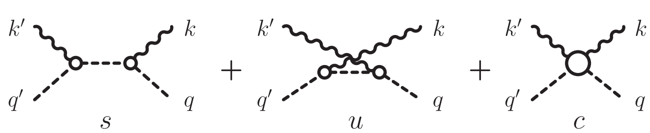

FIG. 2. Diagrams for Compton scattering on a charged pion. In Eqs. (36)–(38), the incoming and outgoing four-momenta are k^μ and k'^ν, respectively.

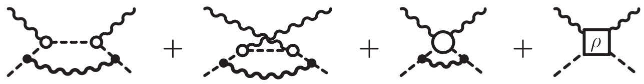

FIG. 3. Example of genuine two-photon contributions to Compton scattering on the pion: Crossing-symmetric dressing of the amplitude depicted in Fig. 2 by a vector-meson loop. The contact-type square labeled ρ in the last diagram subsumes necessary mechanisms that make the entire contribution transverse.

amplitude for real Compton scattering on the pion, with undressed Feynman-type s- and u-channel pole terms and the $-2g^{\mu\nu}$ coupling for the contact term. This on-shell tensor is trivially gauge invariant because of four-momentum conservation, $q' + k' - q - k = 0$.

The present finding of all dressing effects canceling here seems to corroborate the conjecture of Kaloshin [15] that this would be the case. This was disputed in Ref. [9]. Kaloshin's conjecture was based on the limited evidence of an s-wave one-loop calculation with intermediate $\pi\sigma$ states. The derivation of Eq. (38) shows that this is indeed true at any order, for any dressing mechanism of the pion. However, the present construction of the Compton tensor is limited to the part of Compton scattering that can be understood as two sequential one-photon processes. Genuine two-photon processes like, for example, the vector-meson dressing mechanism depicted in Fig. 3 are not taken into account. Contact-type two-photon currents of this kind, therefore, can be expected to resolve the discrepancy between Refs. [15,9].

We emphasize that the complete cancellation of all dressing effects found here is true only for real photons and when the external pions are on shell. The latter requirement is not true for the dressed Compton tensor in Fig. 3, which therefore retains its off-shell structure. We add that on-shell cancellations are not particular to the pion; in Sec. IV D below, we will find a similar result for real Compton scattering on the nucleon.

IV. NUCLEON: SPIN 1/2

Without lack of generality, the dressed spin-1/2 propagator for the nucleon with physical mass m and four-momentum p may be written as

$$S(p) = \frac{1}{\not{p}A(p^2) - mB(p^2)}. \tag{39}$$

The two independent scalar dressing functions A and B are constrained by

$$A(m^2) = B(m^2), \tag{40}$$

which ensures the propagator has a pole at $\not{p} \to m$, and

$$A(m^2) + 2m^2 \frac{d[A(p^2) - B(p^2)]}{dp^2}\bigg|_{p^2=m^2} = 1, \tag{41}$$

which provides a unit residue for this pole.

Even though we will not make use of it here, we note that, without lack of generality, we may write

$$A(p^2) = \frac{p^2 + (2a-1)m^2}{2m^2} F_A(p^2), \tag{42a}$$

$$B(p^2) = \frac{a(p^2 + m^2)}{2m^2} F_B(p^2), \tag{42b}$$

where the abbreviation

$$a = A(m^2) = B(m^2) \tag{43}$$

was used. To provide the residue conditions (40) and (41), at the pole both functions F_A and F_B must have unit values and vanishing first derivatives. Also, they may have simple poles in the spacelike region, at $p^2 = -(2a-1)m^2$ for F_A and at $p^2 = -m^2$ for F_B, and their combined effect must not produce an additional pole for S. Determination of additional properties requires a detailed microscopic derivation of the dressed propagator outside of the scope of the present work; for more details of the corresponding nonlinear Dyson-Schwinger-type framework, see Ref. [11].

A. Isolating dressing contributions

The dressed propagator may also be written equivalently as

$$S(p) = \frac{Z(p)}{\not{p} - m} \tag{44}$$

where the residual function,

$$Z(p) = \frac{\not{p} - m}{\not{p}A(p^2) - mB(p^2)}, \tag{45}$$

goes to unity at the pole. Writing $Z = 1 + (Z-1)$, the term $Z - 1$ will vanish at the pole and thus

$$S(p) = \frac{1}{\not{p} - m} + \frac{Z(p) - 1}{\not{p} - m} \tag{46}$$

will give rise to contact-type nonpolar contributions according to

$$\frac{Z(p) - 1}{\not{p} - m} = \frac{\not{p}C_1(p^2) + mC_0(p^2)}{2m^2}, \tag{47}$$

with scalar, nonpolar functions C_0 and C_1.

These functions can be easily expressed in terms of the dressing functions A and B, however, since there is no need to do this here, we will forego a more detailed discussion of their structure.

For later purposes, we will need finite-difference derivatives of the dressing functions A and B defined as

$$D_A(p^2) = 2m^2 \frac{A(p^2) - a}{p^2 - m^2}, \tag{48a}$$

$$D_B(p^2) = 2m^2 \frac{B(p^2) - a}{p^2 - m^2}, \tag{48b}$$

with well-defined $\frac{0}{0}$ expressions at the pole, and their difference,

$$\begin{aligned} D_{AB}(p^2) &= 2m^2 \frac{A(p^2) - B(p^2)}{p^2 - m^2} \\ &= D_A(p^2) - D_B(p^2), \end{aligned} \tag{49}$$

which is related to the residue condition (41) by

$$D_{AB}(m^2) = 1 - a. \tag{50}$$

B. Nucleon current

Symmetrizing the inverse propagator, the nucleon current is obtained as

$$\begin{aligned} J_N^\mu(p', p) &= \left\{ \frac{\not{p} A(p^2) + A(p^2)\not{p}}{2} - mB(p^2) \right\}_D^\mu \\ &= \Gamma_N^\mu \frac{A(p'^2) + A(p^2)}{2} \\ &\quad + \frac{(p'^2 + p^2)\Gamma_N^\mu + 2\not{p}'\Gamma_N^\mu \not{p}}{4m^2} D_A(p'^2, p^2) \\ &\quad - \frac{\not{p}'\Gamma_N^\mu + \Gamma_N^\mu \not{p}}{2m} D_B(p'^2, p^2), \end{aligned} \tag{51}$$

where the scalar dependence was evaluated according to the Dirac-particle rule (7), employing the extended substitution rule (14). The FDDs

$$D_f(p'^2, p^2) = 2m^2 \frac{f(p'^2) - f(p^2)}{p'^2 - p^2}, \quad \text{for } f = A, B, \tag{52}$$

here are obvious off-shell versions of Eqs. (48). Hence, the on-shell form of the current is given by

$$J_N^\mu(\underline{p}', \underline{p}) = Q_p \gamma^\mu + T_N^\mu(\underline{p}', \underline{p}), \tag{53}$$

where the underlining indicates on-shell momenta; the corresponding incoming and outgoing spinors have been suppressed for clarity. The form-factor functions contained in T_N^μ according to (17) will be constrained by their relation to Dirac and Pauli form factors, as given in Sec. IV B 2.

Pulling out off-shell factors $(\not{p}' - m)$ and $(\not{p} - m)$ on the left and right, respectively, the whole current may be written as

$$\begin{aligned} J_N^\mu(p', p) &= \Gamma_N^\mu(p', p) R(p'^2, p^2) \\ &\quad + \left[\frac{\not{p}' - m}{2m} \Gamma_N^\mu(p', p) + \Gamma_N^\mu(p', p) \frac{\not{p} - m}{2m} \right] \\ &\quad \times [D_A(p'^2, p^2) - D_B(p'^2, p^2)] \\ &\quad + 2 \frac{\not{p}' - m}{2m} \Gamma_N^\mu(p', p) \frac{\not{p} - m}{2m} D_A(p'^2, p^2), \end{aligned} \tag{54}$$

where

$$R(p'^2, p^2) = \hat{A}(p'^2, p^2) + D_A(p'^2, p^2) - D_B(p'^2, p^2), \tag{55}$$

with

$$\hat{A}(p'^2, p^2) = \frac{(p'^2 - m^2)A(p'^2) - (p^2 - m^2)A(p^2)}{p'^2 - p^2}. \tag{56}$$

The latter is a well-defined FDD with half-on-shell limits

$$\hat{A}(p'^2, p^2) = \begin{cases} A(p'^2) & \text{for } p^2 = m^2, \\ A(p^2) & \text{for } p'^2 = m^2. \end{cases} \tag{57}$$

The entire function $R(p'^2, p^2)$, therefore, goes to unity on shell because of the residue condition (41) producing the on-shell result (53).

Equation (54) cleanly separates on-shell, half-off-shell, and fully off-shell contributions here in a manner that is useful for further applications. Even though it may not be immediately obvious from its appearance, it does satisfy the WTI [13] with the fully dressed propagator of Eq. (39),

$$k_\mu J_N^\mu(p', p) = Q_N[S^{-1}(p') - S^{-1}(p)], \tag{58}$$

as mandated by local gauge invariance. Only the $Q_p \gamma^\mu$ part of the elementary current Γ^μ contributes to this result, of course. Any information about electromagnetic structure is transverse and does not enter the WTI.

The current as written here in (54) is similar in operator structure to the most general ansatz discussed in Ref. [16]. Since it reproduces the WTI (58) for the fully dressed propagator, its structure clearly exhausts the necessary dependence on dressing functions A and B. The possibility of additional terms—which would necessarily have to be off shell and transverse—will be discussed in the summarizing Sec. V.

1. Relationship to Ball-Chiu current

If we switch off the additional transverse piece T_N^μ for the moment, producing $J_N^\mu \to J_0^\mu$, the remaining current for the proton may be written as

$$J_0^\mu(p',p) = J_{\rm BC}^\mu(p',p) + Q_p \frac{\not p' i\sigma^{\mu\nu}k_\nu + i\sigma^{\mu\nu}k_\nu \not p}{4m^2} D_A(p'^2,p^2) - Q_p \frac{i\sigma^{\mu\nu}k_\nu}{2m} D_B(p'^2,p^2), \tag{59}$$

where, suppressing arguments of D_A and D_B for simplicity,

$$\begin{aligned} J_{\rm BC}^\mu(p',p) &= Q_p\gamma^\mu \frac{A(p'^2)+A(p^2)}{2} + Q_p \frac{(p'+p)^\mu}{2m}\left[\frac{\not p'+\not p}{2m}D_A - D_B\right] \\ &= Q_p(p'+p)^\mu \frac{S^{-1}(p') - S^{-1}(p)}{p'^2-p^2} + Q_p\left[\gamma^\mu - \frac{(p'+p)^\mu}{p'^2-p^2}\not k\right]\frac{A(p'^2)+A(p^2)}{2}. \end{aligned} \tag{60}$$

This current was proposed by Ball and Chiu [10] as one of the simplest nonsingular symmetric expressions whose four-divergence provides the WTI for the fully dressed propagator (39). The additional transverse term in the last expression is needed to cancel the singularity of the first term at $p'^2 = p^2$.

These relations shows most clearly that all three currents—$J_{\rm BC}^\mu$, J_0^μ, and J_N^μ—satisfy the WTI (58) because they differ from each other by manifestly transverse contributions.

We see here that the Ball-Chiu current is obtained as

$$J_{\rm BC}^\mu(p',p) = \{S^{-1}(p)\}_S^\mu, \tag{61}$$

whereas

$$J_0^\mu(p',p) = \{S^{-1}(p)\}_D^\mu, \tag{62}$$

i.e., they differ by how the electromagnetic field is being coupled to their scalar parts. If we consider their respective on-shell forms, we find

$$J_{\rm BC}^\mu(\underline{p}',\underline{p}) = Q_p\gamma^\mu + Q_p\frac{i\sigma^{\mu\nu}k_\nu}{2m}(a-1) \tag{63}$$

and

$$J_0^\mu(\underline{p}',\underline{p}) = Q_p\gamma^\mu, \tag{64}$$

where the additional transverse $\sigma^{\mu\nu}k_\nu$ contributions in (59) that originate from the proper Dirac treatment cancel the $(a-1)$ term in (63). Clearly, Eq. (64) provides the correct on-shell limit if one switches off all transverse pieces in the substitution rule (14). (This result is also obtained if $a = 1$, of course, but this value is *not* required by the residue conditions.)

2. Determining Dirac and Pauli form factors

Writing out the specific on-shell forms of J_N^μ for proton and neutron,

$$J_N^\mu(\underline{p}',\underline{p}) = \begin{cases} e\gamma^\mu(1+f_1^p) + e\sigma_{\rm T}^\mu f_2^p & \text{(proton)}, \\ e\gamma^\mu f_1^n + e\sigma_{\rm T}^\mu f_2^n & \text{(neutron)}, \end{cases} \tag{65}$$

and comparing this with the usual expressions,

$$J_p^\mu(\underline{p}',\underline{p}) = e\gamma^\mu F_1^p(k^2) + e\sigma_{\rm T}^\mu\kappa_p F_2^p(k^2), \tag{66a}$$

$$J_n^\mu(\underline{p}',\underline{p}) = e\gamma^\mu F_1^n(k^2) + e\sigma_{\rm T}^\mu\kappa_n F_2^n(k^2), \tag{66b}$$

where $F_1^N(k^2)$ and $F_2^N(k^2)$ are the Dirac and Pauli form factors, with κ_p and κ_n being the anomalous magnetic moments of proton and neutron, respectively, this produces the identifications

$$f_1^p(m^2,m^2;k^2) = F_1^p(k^2) - 1, \tag{67a}$$

$$f_2^p(m^2,m^2;k^2) = \kappa_p F_2^p(k^2), \tag{67b}$$

$$f_1^n(m^2,m^2;k^2) = F_1^n(k^2), \tag{67c}$$

$$f_2^n(m^2,m^2;k^2) = \kappa_n F_2^n(k^2). \tag{67d}$$

In view of the normalizations

$$F_1^n(0) = 0, \qquad F_2^n(0) = 1, \tag{68a}$$

$$F_1^p(0) = 1, \qquad F_2^p(0) = 1, \tag{68b}$$

the two Dirac coefficient functions f_1^N vanish at $k^2 = 0$, cancelling the $1/k^2$ singularity in $\gamma_{\rm T}^\mu$; the two Pauli coefficient functions f_2^n produce the respective anomalous magnetic moment at $k^2 = 0$.

With the on-shell identifications (67), the nucleon current (51) thus reproduces the correct experimental information by construction.

Allowing for off-shell nucleons, the form factors f_i^N $(i = 1, 2)$ may be written without lack of generality as

$$f_1^N(p'^2,p^2;k^2) = \frac{k^2}{m^2}H_1^N(p'^2,p^2;k^2), \tag{69a}$$

$$f_2^N(p'^2,p^2;k^2) = 1 + \frac{k^2}{m^2}H_2^N(p'^2,p^2;k^2), \tag{69b}$$

for $N = p, n$, with nonsingular symmetric scalar functions H_i^N, for $i = 1, 2$. The current operator Γ_N^μ of Eq. (14) then reads in full detail

$$\Gamma_N^\mu(p',p) = \gamma^\mu\left[Q_p + Q_N\frac{k^2}{m^2}H_1^N(p'^2,p^2;k^2)\right] + Q_N\kappa_N\frac{i\sigma^{\mu\nu}k_\nu}{2m}\left[1+\frac{k^2}{m^2}H_2^N(p'^2,p^2;k^2)\right] + Q_N k^\mu\frac{\not{p}'-\not{p}}{m^2}H_1^N(p'^2,p^2;k^2). \quad (70)$$

We emphasize here that electromagnetic structure information enters this expression only in manifestly transverse terms. We also note that this is not obvious if one drops the k^μ term prematurely.

C. Half-on-shell contribution

Let us consider the half-on-shell version of the current (54) where the incoming nucleon is on shell. The kinematics then are the same as for the s-channel diagram in Fig. 1. One finds

$$J_N^\mu(p_s,\underline{p}) = \left[R(s) + \frac{\not{p}_s - m}{2m}D_{AB}(s)\right]\Gamma_N^\mu(p_s,\underline{p}), \quad (71)$$

where $p_s = p+k$ and $s = p_s^2$ and $R(s)$ is shorthand for $R(s,m^2)$. Multiplying this by the s-channel propagator $S(p_s)$ for the fully dressed intermediate nucleon, one obtains, after tedious, but straightforward, algebra,

$$S(p_s)J_N^\mu(p_s,\underline{p}) = \frac{1}{\not{p}_s - m}\Gamma_N^\mu(p_s,\underline{p}) = \frac{Q_p}{\not{p}_s - m}\gamma^\mu + \frac{1}{\not{p}_s - m}T_N^\mu(p_s,\underline{p}). \quad (72)$$

All dressing effects here cancel fully. This result is exact for the current (54) constructed here. The only dressing effects left here are electromagnetic in nature, in the two half-on-shell form factors $f_i^N(s,m^2;k^2)$ $(i = 1, 2)$ within T_N^μ.

As was the case for the pion treatment in Sec. III, structure information enters the half-on-shell element only via manifestly transverse terms, and the four-divergence here indeed produces

$$k_\mu S(p_s)J_N^\mu(p_s,\underline{p}) = Q_p, \quad (73)$$

as demanded by local gauge invariance.

The form factors that enter the transverse currents Γ_N^μ are only needed here half on shell. Using the physical limits (67), we may expand the s-dependent off-shell side of the equation in terms of well-defined FDDs and write the half-on-shell contribution as

$$S(p_s)J_N^\mu(p_s,\underline{p}) = \frac{Q_N}{\not{p}_s - m}\Gamma_{N,0}^\mu(k) + \frac{k^2}{m^3}Q_N C_N^\mu(p_s;k), \quad (74)$$

where the polar term comprises the usual on-shell nucleon current,

$$\Gamma_{N,0}^\mu(k) = \gamma^\mu F_1^N(k^2) + \kappa_N\frac{i\sigma^{\mu\nu}k_\nu}{2m}F_2^N(k^2), \quad (75)$$

and the contact term reads

$$C_N^\mu(p_s;k) = \frac{\not{p}_s + m}{2m}\left[\gamma^\mu D_1^N(s;k^2) + \frac{i\sigma^{\mu\nu}k_\nu}{2m}D_2^N(s;k^2)\right], \quad (76)$$

with FDDs

$$D_i^N(s;k^2) = 2m^2\frac{H_i^N(s;k^2) - H_i^N(m^2;k^2)}{s-m^2}, \quad (77)$$

for $i = 1, 2$, where the incoming on-shell variable $(p^2 = m^2)$ is suppressed. The unphysical k^μ term was dropped in Eq. (74), i.e., this equation cannot be used to verify the gauge-invariance condition (73).

Structurally, the half-on-shell result (74) is exactly the same as Eq. (33) for the pion and—as for the pion—the nucleon structure information only enters for virtual photons for the current determined here. Hence, any other possible structure can only come from off-shell electromagnetic form factors $H_i^N(s;k^2)$ (for $N = p$, n; $i = 1$, 2) that contribute only for electroproduction, when $k^2 \neq 0$.

It should be obvious that the analogous half-on-shell cancellations can also easily be verified for the u-channel process in Fig. 1. Hence, for all three Born-type contributions to the pion photoproduction process, only the respective hadronic πNN vertices carry structure information. The detailed dynamics of the problem, therefore, is hidden in the contact-type current of the last diagram in Fig. 1.

D. Real Compton scattering on the nucleon

Let us now consider real Compton scattering on the proton using the current (54). We may then replace Q_N by e and put $k^2 = k'^2 = 0$ everywhere. Moreover, taking k^μ and k'^ν to be the four-momenta for outgoing and incoming photons, respectively, we may use

$$\Gamma_p^\mu \equiv \Gamma_{p,0}^\mu(k) \quad \text{and} \quad \Gamma_p^\nu \equiv \Gamma_{p,0}^\nu(k') \quad (78)$$

as shorthand notations for the corresponding incoming and outgoing currents. Further, denoting the intermediate four-momenta in the s- and u-channel diagrams of Fig. 4 by

$$p_s = p + k = p' + k' \quad \text{and} \quad p_u = p' - k = p - k', \quad (79)$$

with Mandelstam variables $s = p_s^2$ and $u = p_u^2$, and using the half-on-shell results (71) and (74), the s- and u-channel parts of the Compton tensor read

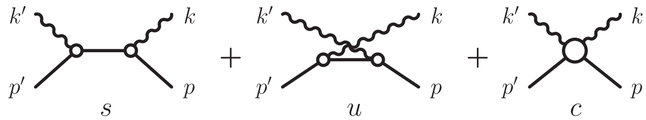

FIG. 4. Diagrams contributing to Compton scattering on the nucleon (ignoring t-channel exchange). In Eqs. (80)–(83), the incoming and outgoing four-momenta are k^μ and k'^ν, respectively.

$$M_s^{\nu\mu}(p',p) = \Gamma_p^\nu \frac{R(s)}{\not{p}_s - m}\Gamma_p^\mu + \Gamma_p^\nu \frac{D_{AB}(s)}{2m}\Gamma_p^\mu$$
$$= \Gamma_p^\nu \frac{1}{\not{p}_s - m}\Gamma_p^\mu$$
$$+ \Gamma_p^\nu \left[\frac{\not{p}_s + m}{2m^2} D_R(s) + \frac{D_{AB}(s)}{2m}\right]\Gamma_p^\mu, \quad (80a)$$

$$M_u^{\nu\mu}(p',p) = \Gamma_p^\mu \frac{R(u)}{\not{p}_u - m}\Gamma_p^\nu + \Gamma_p^\mu \frac{D_{AB}(u)}{2m}\Gamma_p^\nu$$
$$= \Gamma_p^\mu \frac{1}{\not{p}_u - m}\Gamma_p^\nu$$
$$+ \Gamma_p^\mu \left[\frac{\not{p}_u + m}{2m^2} D_R(u) + \frac{D_{AB}(u)}{2m}\right]\Gamma_p^\nu, \quad (80b)$$

where the FDD

$$D_R(x) = 2m^2 \frac{R(x) - 1}{x - m^2}, \quad \text{for } x = s, u, \quad (81)$$

was introduced. The respective second equalities in (80) separate pole terms from contact terms. The contact current is obtained by the same construction used already for the pion contact term (37). One obtains

$$M_c^{\nu\mu} = -\frac{1}{2}[\{J_N^\mu(p_{\rm off}, \underline{p})\}^\nu + \{J_N^\nu(p_{\rm off}, \underline{p})\}^\mu]$$
$$-\frac{1}{2}[\{J_N^\mu(\underline{p}', p_{\rm off})\}^\nu + \{J_N^\nu(\underline{p}', p_{\rm off})\}^\mu]$$
$$= -\Gamma_p^\nu \left[\frac{\not{p}_s + m}{2m^2} D_R(s) + \frac{D_{AB}(s)}{2m}\right]\Gamma_p^\mu$$
$$- \Gamma_p^\mu \left[\frac{\not{p}_u + m}{2m^2} D_R(u) + \frac{D_{AB}(u)}{2m}\right]\Gamma_p^\nu, \quad (82)$$

which exactly cancels the contact terms in the s- and u-channel contributions. The Compton tensor for the diagrams in Fig. 4 then simply reads

$$M_N^{\nu\mu} = M_s^{\nu\mu} + M_u^{\nu\mu} + M_c^{\nu\mu}$$
$$= \Gamma_p^\nu \frac{1}{\not{p}_s - m}\Gamma_p^\mu + \Gamma_p^\mu \frac{1}{\not{p}_u - m}\Gamma_p^\nu, \quad (83)$$

leaving only undressed s- and u-channel terms which, taken together, are trivially gauge invariant.

This bare expression provides the Powell cross section [17], but it does not describe more complex experimental data like electric and magnetic polarizibilities (see Refs. [18–20] and references therein). However, as with the real Compton scattering tensor for the pion discussed in Sec. III C, the present construction corresponds to sequential single-photon processes. Genuine two-photon processes like those given by the examples of dressing mechanisms depicted in Fig. 5, or processes (not shown) with other intermediate resonant baryonic states (Δ etc.), are not taken into account. It is well known that such mechanisms will indeed describe polarizabilities. Note also that the Compton tensors dressed by such loop mechanisms are *not* the undressed ones. Their external hadron lines are off-shell, thus making them fully dependent on the propagator dressing functions Π, for the pion, and A and B, for the nucleon.

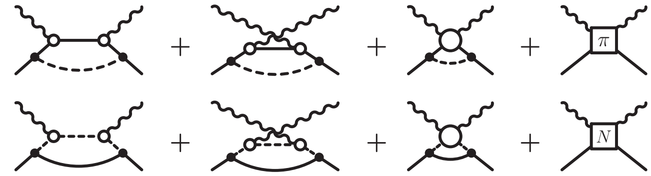

FIG. 5. Examples of genuine two-photon processes contributing to Compton scattering on the nucleon. The first line of diagrams provides the crossing-symmetric dressing of the amplitude depicted in Fig. 4 by a meson loop and the second line corresponds to intermediate Compton scattering on the pion as depicted in Fig. 2. The contact-type squares labeled π and N subsume necessary mechanisms to make the contribution of each respective line transverse.

V. SUMMARY AND DISCUSSION

We have presented here an extension of the usual minimal substitution procedure that provides a straightforward inclusion of electromagnetic form factors into hadronic current operators. The important starting point here is the—known, but oftentimes forgotten—fact that electromagnetic structure information *always* is limited to manifestly transverse current contributions. This follows simply from the fact that the only electromagnetic structure information that enters the Ward-Takahashi identities for electromagnetic currents are the respective charges—electromagnetic form factors do not enter [cf. Eq. (A8)].

The resulting extended substitution ansätze proposed in Sec. II, therefore, concern only manifestly transverse additions to the respective basic currents for spin-0 and spin-1/2 particles that result from Feynman rules. Phenomenological additions of electromagnetic form factors are nothing new and have been undertaken before. The novel aspect of the present approach is the consequent application of this extended electromagnetic substitution with dressed hadronic propagators, providing currents that incorporate all dressing effects in a consistent manner.

The gauge-derivative procedure of Ref. [11] used in determining the consistent currents also means, as shown in Sec. II, that coupling the electromagnetic field to scalar dressing contributions needs to be treated differently for spin-0 and spin-$1/2$ particles, lest one ignores the fact that spin-$1/2$ solutions also must solve the Klein-Gordon equation. As a consequence, we showed in Sec. IV B 1 that the Ball-Chiu current [10] lacks transverse contributions that, when added, provide the correct on-shell limit.

Regarding the consequences of dressing, we emphasize that, although we started out by assuming that particle propagators are fully dressed, the current expressions obtained by taking their gauge derivatives do *not* make any assumptions about how the corresponding dressing functions are obtained. The only features that matter are the respective residue conditions. Therefore, *any* determination of hadronic dressing effects that meets these conditions, whether by simple single-loop models or sophisticated self-energy expansions to all orders, can be accommodated, thus making the cancelation effects between dressed propagators and correspondingly determined currents found here true for any of such dressings. The cancelations are also independent of the detailed extended substitution features. In other words, they are also true if one simply applies minimal substitutions, without any extensions, to the respective dressed propagators.

Note that the procedure for the current construction used here, while always producing a fully locally gauge-invariant current, in general does not preclude the possibility of additional *transverse off-shell* contributions, where transversality and off-shellness are both essential requirements. Such currents would spoil the perfect cancelations, of course, but they cannot be excluded on general grounds. However, as discussed in Refs. [18,21–23], *any* reaction-dynamical description is subject to representation-dependent ambiguities because one can *always* trade off the off-shell dependence of pole terms against associated contact-type contributions by using appropriate finite-difference derivatives. The ubiquitous use of FDDs in the present formulation is testament to this ambiguity.

In any formulation of the reaction dynamics of a photoprocess, the question, therefore, is *not* whether a particular off-shell or transverse current has been included for the photoprocess at hand, but whether all effects—whether of the polar-type or contact-type—have been included *consistently.* In this respect, therefore, their is no need to consider additional transverse contributions for the currents derived here consistently with their fully dressed hadron propagators because any additional contribution deemed necessary for the description of the physics at hand can be expressed in terms of contact terms. The hadron loops around the (off-shell) Compton tensors in Figs. 3 and 5 provide examples of this kind.

In view of the obvious representation dependence of off-shell polar *vs* contact-term contributions discussed above, perhaps the most surprising aspect of the present findings is that—for real photons at least—the dressing cancellations found here *force* a natural split into pure pole terms and residual contact-type contributions in certain dynamical on-shell situation, even if one starts out assuming fully dressed hadrons. The key to this result is the consistency of the currents derived in the present framework with whatever dressing goes into the hadron propagators.

We add here that for processes like pion production off the nucleon, as depicted in Fig. 1, the dressing cancellations found here mean that for the usual s-, u-, and t-channel terms, the only relevant dressing contributions are those stemming from the hadronic πNN vertex. This may at least partially explain the relative success of phenomenological approaches to photo- and electroproduction of mesons that model the hadronic vertex by a simple cutoff function and completely ignore any other dressing effects. Furthermore, as a consequence of the present findings, this means that, other than adding baryonic resonance content, most of the effort in describing the detailed dynamics of meson-production reactions needs to go into the determination of *transverse* contact-type currents that arise from final-state interactions. This is indeed the approach advocated and executed in Refs. [24–27], where it was based on gauge-invariance considerations alone, without the benefit of the present insights.

Finally, we have not yet considered electromagnetic processes involving particles with higher spins beyond $1/2$ along the lines presented here, but we expect this should be possible as well.

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APPENDIX: GAUGE DERIVATIVE AND MINIMAL SUBSTITUTION

To make the present paper self-contained, we recapitulate here some basic features of the gauge-derivative device introducing in Ref. [11], where full details can be found. The gauge derivative provides a convenient shorthand procedure for implementing minimal substitution in settings where the external electromagnetic field A^μ interacts with larger systems of strongly interacting particles. The properties of such systems are fully described by their connected Green's functions. As described in the Introduction, the coupling of the electromagnetic field is usually effected by an LSZ-type reduction of the gauged Green's function [5,6,8]. However, to maintain full local gauge invariance, in principle, this should be done at every level of all details that enter the microscopic description of the reaction at hand. The gauge derivative is designed to make bookkeeping easier, by—loosely speaking—allowing attaching a photon to every particle with charge Q and injecting the photon's four-momentum such that it is available to every reaction mechanism "downstream" of the initial photon interaction.

The basic definition of the gauge-derivative braces $\{\cdots\}^\mu$ is given by the functional derivative of the minimal-substitution rule (1) for a particle of four-momentum p with charge operator Q,

$$\{p^\nu\}^\mu = -\frac{\delta}{\delta A_\mu}(p^\nu - QA^\nu) = Qg^{\mu\nu}. \tag{A1}$$

Since it is a derivative, the product rule applies providing

$$\begin{aligned}\{p^2\}^\mu &= g_{\lambda\nu}\{p^\lambda p^\nu\}^\mu = g_{\lambda\nu}[\{p^\lambda\}^\mu p^\nu + p'^\lambda\{p^\nu\}^\mu] \\ &= Q(p'+p)^\mu,\end{aligned} \tag{A2}$$

where the four-momentum downstream of where the gauge derivative is applied is increased by the photon's four-momentum $k = p' - p$. Moreover, since the gauge derivative acts only on momenta, one has

$$\{\not p\}^\mu = \gamma_\nu\{p^\nu\}^\mu = Q\gamma^\mu. \tag{A3}$$

These two results provide the basic coupling mechanisms for scalar and Dirac particles, respectively, given in Eqs. (2) and (3) of Sec. II.

The product rule also applies to any functions $f(p)$ and $g(p)$ of the four-momentum, i.e.,

$$\{f(p)g(p)\}^\mu = \{f(p)\}^\mu g(p) + f(p')\{g(p)\}^\mu, \tag{A4}$$

and if the functions commute, symmetrization is required to prevent ambiguities,

$$\begin{aligned}\{f(p)g(p)\}^\mu &\to \left\{\frac{f(p)g(p)+g(p)f(p)}{2}\right\}^\mu \\ &= \{f(p)\}^\mu\frac{g(p')+g(p)}{2} \\ &\quad + \{g(p)\}^\mu\frac{f(p')+f(p)}{2}.\end{aligned} \tag{A5}$$

Generally, unsymmetrized results differ by transverse terms from symmetrized ones.

Applying the product rule to

$$\{t(p)t^{-1}(p)\}^\mu = \{1\}^\mu = 0, \tag{A6}$$

where $t(p)$ is a generic propagator for a particle with four-momentum p, one immediately finds that the electromagnetic current for this particle is determined by the generic expression

$$J^\mu = -t^{-1}(p')\{t(p)\}^\mu t^{-1}(p) = \{t^{-1}(p)\}^\mu, \tag{A7}$$

where the current definition follows from the LSZ procedure [8] since $t(p)$ is the two-point Green's function for single-particle propagation. For scalar and Dirac particles, this respectively provides Eqs. (A2) and (A3) as the basic coupling mechanisms since their inverse undressed propagators are given by $(p^2 - m^2)$ and $(\not p - m)$, respectively. The generic Ward-Takahashi identity [13] for the current (A7) is given by

$$k_\mu J^\mu = Q[t^{-1}(p') - t^{-1}(p)], \tag{A8}$$

which in addition to the static charge Q only retains the hadronic dressing information that resides in the propagator. No other electromagnetic information enters here. The WTI (A8) is the necessary and sufficient condition for the current J^μ to be *locally* gauge invariant. Global gauge invariance—i.e., current conservation—follows trivially in the on-shell limit.

Moreover, as explained in Sec. II in the context of Eq. (5), it is crucially important that this equality extends to the respective current expressions. This requires that the corresponding gauge derivatives of both sides of Eq. (5) must be identical, leading to the condition

$$\left\{\frac{1}{\not p - m}\right\}^\mu \overset{!}{=} \frac{1}{2}\left\{\frac{1}{p^2-m^2}(\not p + m) + (\not p + m)\frac{1}{p^2-m^2}\right\}^\mu, \tag{A9}$$

where the right-hand side was symmetrized. Straightforward algebra shows then that with (A3) given, the coupling associated with p^2 on the right-hand side must be evaluated according to the Dirac particle rule

$$\{p^2\}^\mu_D \equiv \{\not p^2\}^\mu = Q(\not p'\gamma^\mu + \gamma^\mu\not p), \tag{A10}$$

as stated in Eq. (7).

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