$Z \to \pi^+\pi^-, K^+K^-$: A touchstone of the perturbative QCD approach

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We study two rare decays, $Z \to \pi^+\pi^-$ and K^+K^-, in the perturbative QCD approach up to the next-to-leading order of the strong coupling and the leading power of $1/m_Z$, m_Z being the Z boson mass. The branching ratios $\mathcal{B}(Z \to \pi^+\pi^-) = (0.83 \pm 0.02 \pm 0.02 \pm 0.04) \times 10^{-12}$ and $\mathcal{B}(Z \to K^+K^-) = (1.74^{+0.03}_{-0.05} \pm 0.04 \pm 0.02) \times 10^{-12}$ are obtained and can be measured at a Tera-Z factory. Because the subleading-power contributions to the branching ratios are negligible, and the leading one does not depend on any free parameter, the two channels can serve as a touchstone for the applicability of the perturbative QCD approach.

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I. INTRODUCTION

Two-body nonleptonic B meson decays play an essential role in particle physics and help us understand the QCD and the charge conjugation parity violation in the Standard Model. They have inspired the development of many theoretical frameworks or approaches, including the QCD-improved factorization approach [1], the soft-collinear effective theory [2,3], the light-cone sum rules [4], the perturbative QCD (PQCD) approach [5] based on the k_T factorization theorem [6–9], and the factorization-assisted topological-amplitude approach [10] proposed recently. Among them, the PQCD approach is the most predictive one, in which a high-energy hadronic process is factorized into universal distribution amplitudes of hadrons and a perturbatively calculable hard kernel. However, it is also this unique feature of PQCD that has been questioned. The power counting analyses of the $B \to \pi$ form factor and the timelike pion form factor [1,11] imply that both the nonperturbative small-x (x is the momentum fraction of a constitute quark in a pion) region and the perturbative $x \sim 1/2$ region contribute at the leading power of $1/m_B$, m_B being the B meson mass. On the other hand, a PQCD calculation shows that the small-x region is practically suppressed by the Sudakov factor from the k_T resummation, and thus the form factors are dominated by perturbative contributions [12–14]. To test which argument is valid, we propose the $Z^0 \to \pi^+\pi^-$ (K^+K^-) channel as a touchstone here. In the PQCD approach to the $Z \to \pi^+\pi^-$ decay rate, power corrections in $1/m_Z$, m_Z being the Z boson mass, are so small that they can be neglected safely. It hints that we need to consider only the twist-2 light-cone distribution amplitude (LCDA) of the pion and that its simple asymptotic form may be justified. As a result, the calculation is free of arbitrariness, since the nonperturbative pion LCDA has been fixed [15–17]. The two channels are expected to be observed or strictly constrained at a future Tera-Z factory like the FCC-ee, formerly known as TLEP [18], and/or the Circular Electron-Positron Collider [19], which can be used not only to precisely study the Higgs and Z properties (e.g., see Ref. [20]) and discover new particles (e.g., see Ref. [21]) but also to improve our understanding of QCD as elaborated in this paper.

The $Z \to \pi^+\pi^-$ decay amplitude is proportional to the timelike pion form factor, which can be investigated in several different methods in principle. One is the partial wave analysis, in which elastic and inelastic scatterings as well as effects of resonances are handled [22,23]. Another one, the light-cone sum-rule approach, is powerful for spacelike form factors, while dispersion relations and some resonance models are inevitable for the timelike region [24]. Both the above approaches work well only in the low-energy region and are model dependent. To access the form factor with the dipion invariant mass at order of m_Z, the PQCD approach is more appropriate [25–28]. In this paper, we will evaluate the $Z^0 \to \pi^+\pi^-$ (K^+K^-) decay rate up to the next-to-leading order (NLO) of the strong coupling α_s and at the leading power of $1/m_Z$ in the PQCD formalism [5]. We obtain the branching ratio about $0.83(1.74) \times 10^{-12}$, which

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is likely to be measured at a Tera-Z factory. Whichever of them is found, it will be the first observation of an exclusive hadronic Z decay and serve as a touchstone to verify the PQCD approach.

The rest of the paper is organized as follows. In Sec. II, the PQCD calculation of the timelike pion form factor is performed up to NLO, and the analytical formulas are given. In Sec. III, we present the numerical results for the $Z \to \pi^+\pi^-, K^+K^-$ branching ratios. Section IV is the conclusion.

II. PERTURBATIVE CALCULATION

In the Standard Model, the $Z\bar{q}q$ interaction is described by $-J_\mu^{(Z)}Z^\mu$ in the Lagrangian with the current

$$J_\mu^{(Z)} = \frac{g}{2\cos\theta_w}\sum_q [(T_q - 2Q_q \sin^2\theta_w)\bar{q}\gamma_\mu q - T_q\bar{q}\gamma_\mu\gamma_5 q], \tag{1}$$

where g is the SU(2) gauge coupling, θ_w is the weak mixing angle, and the hypercharges and electric charges of the quarks are $T_{u,d} = \pm 1/2$ and $Q_u = 2/3$ and $Q_d = -1/3$, respectively. We then write the $Z \to \pi^+\pi^-$ decay amplitude as

$$i\mathcal{M}(Z \to \pi^+\pi^-) = \langle \pi^+\pi^-|J_\mu^{(Z)}|0\rangle \epsilon_Z^\mu, \tag{2}$$

with the polarization vector ϵ_Z of the decaying Z boson. Only the vector components in $J_\mu^{(Z)}$ contribute, because hadronic matrix elements induced by the axial-vector currents are forbidden by parity. The timelike pion form factor $\mathcal{G}(Q^2)$ is defined via

$$\begin{aligned}
\langle \pi^+\pi^-|\bar{u}\gamma^\mu u|0\rangle &= (p_1^\mu - p_2^\mu)\mathcal{G}(Q^2),\\
\langle \pi^+\pi^-|\bar{d}\gamma^\mu d|0\rangle &= -(p_1^\mu - p_2^\mu)\mathcal{G}(Q^2),
\end{aligned} \tag{3}$$

with p_1 and p_2 being the momenta of π^+ and π^-, respectively; $q = p_1 + p_2$; and $Q^2 = q^2$. The above two definitions are equivalent due to the isospin symmetry. Performing the phase space integral, we obtain the spin-averaged decay width

$$\begin{aligned}
\Gamma(Z \to \pi^+\pi^-) &= \frac{1}{3}\frac{1}{16\pi m_Z}\sum_s |\mathcal{M}(Z \to \pi^+\pi^-)|^2,\\
&= \frac{1}{3}\frac{1}{16\pi m_Z}(g_V^u - g_V^d)^2|\mathcal{G}(m_Z^2)|^2(p_1^\mu - p_2^\mu)\\
&\quad \times (p_1^\nu - p_2^\nu)\sum_s \epsilon_\mu^*(P_Z)\epsilon_\nu(P_Z),\\
&= \frac{m_Z}{48\pi}(g_V^u - g_V^d)^2|\mathcal{G}(m_Z^2)|^2,
\end{aligned} \tag{4}$$

where $g_V^q = g/(2\cos\theta_w) \times (T_q - 2Q_q \sin^2\theta_w)$ [29,30], and the pion mass effect has been neglected. The factor $(g_V^q - g_V^{q'})^2$ indicates that the $Z \to \pi^0\pi^0$ and $K^0\bar{K}^0$ decays are forbidden at leading power. Below, we focus on the evaluation of the form factor $\mathcal{G}(Q^2)$ at $Q^2 = m_Z^2$ in the PQCD approach.

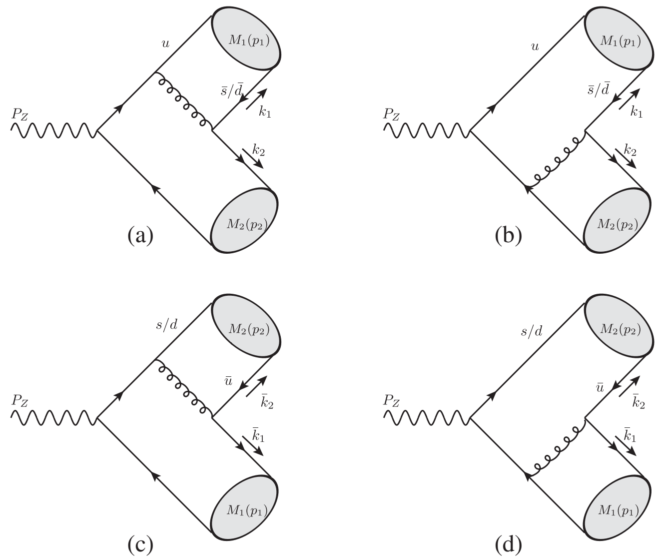

FIG. 1. Feynman diagrams for $Z \to M_1M_2$ decays at leading order with $M_1 = \pi^+, K^+$ and $M_2 = \pi^-, K^-$.

A. Kinematics and the LO form factor

As depicted in Fig. 1, the two upper and lower diagrams contribute to the timelike form factors from $\langle \pi^+\pi^-|\bar{u}\gamma^\mu u|0\rangle$ and $\langle \pi^+\pi^-|\bar{d}\gamma^\mu d|0\rangle$, respectively, at leading order (LO) of QCD. We choose the following kinematics for the initial- and the final-state particles expressed in terms of light-cone coordinates,

$$\begin{aligned}
p_Z &= \frac{m_Z}{\sqrt{2}}(1,1,\mathbf{0}), \qquad p_1 = \frac{m_Z}{\sqrt{2}}(1,0,\mathbf{0}),\\
p_2 &= \frac{m_Z}{\sqrt{2}}(0,1,\mathbf{0}),
\end{aligned} \tag{5}$$

where p_Z is the momentum of the Z boson. The Z boson is at rest in this frame, and the two pion momenta are collimated to the two light-cone directions, with the pion masses being ignored. The momenta of the constitute quarks and antiquarks in Fig. 1 are parametrized as

$$\begin{aligned}
k_1 &= \left(x_1\frac{m_Z}{\sqrt{2}}, 0, \mathbf{k}_{1T}\right), \qquad k_2 = \left(0, x_2\frac{m_Z}{\sqrt{2}}, \mathbf{k}_{2T}\right),\\
\bar{k}_1 &= p_1 - k_1, \qquad \bar{k}_2 = p_2 - k_2.
\end{aligned} \tag{6}$$

We can get the pion form factor at leading power[1] by computing any diagram in Fig. 1,

[1]Details of the calculation and the factorization formula for the contribution from higher-twist LCDAs are given in Appendix.

$$\begin{aligned}\mathcal{G}_{\rm II}(Q^2)_{\rm LO} = -16\pi C_F Q^2 \int_0^1 dx_1 dx_2 \\ \times \int db_1 db_2 b_1 b_2 \alpha_s(\mu) x_2 \phi_\pi(x_1)\phi_\pi(x_2) \\ \times h_{\rm II}(x_1,b_1,x_2,b_2,Q) \\ \times {\rm Exp}[-S_{\rm II}(x_1,b_1,x_2,b_2,\mu)],\end{aligned} \tag{7}$$

where $C_F = 4/3$, b_i are the conjugate variables of the transverse momenta $\mathbf{k}_{iT}$ and $\phi_\pi(x)$ is the twist-2 pion LCDA. The factorization scale μ is set to $\max(1/b_1, 1/b_2, \sqrt{x_2}Q)$. The Sudakov factor derived from the k_T resummation up to the next-to-leading-logarithm accuracy is written as

$$S_{\rm II}(x_i,b_i,\mu) = \sum_{i=1,2}\left[s\left(x_i\frac{m_Z}{\sqrt{2}},b_i\right) + s\left((1-x_i)\frac{m_Z}{\sqrt{2}},b_i\right) + s_q(b_i,\mu)\right], \tag{8}$$

where the terms $s(Q_i,b_i)$ collect the double and single logarithms in the vertex correction associated with an energetic light quark (see Eq. (10) of Ref. [8]), and the term

$$\begin{aligned}s_q(b,\mu) &= -2\int_{1/b}^{\mu}\frac{d\bar{\mu}}{\bar{\mu}}\frac{\alpha_s(\bar{\mu})}{\pi} \\ &= -\frac{1}{\beta_1}\log\left(\frac{\log(t/\Lambda^{(5)})}{-\log(b\Lambda^{(5)})}\right) \\ &\quad -\frac{\beta_2}{2\beta_1^3}\left(\frac{\log[2\log(t/\Lambda^{(5)})]+1}{\log(t/\Lambda^{(5)})} - \frac{\log[-2\log(b\Lambda^{(5)})]+1}{-\log(b\Lambda^{(5)}))}\right)\end{aligned} \tag{9}$$

resums the single logarithms in the quark self-energy correction [5]. We adopt the two-loop expression for the strong coupling,

$$\alpha_s(\mu) = \frac{\pi}{2\beta_1\log(\mu/\Lambda^{(5)})}\left(1 - \frac{\beta_2}{\beta_1^2}\frac{\log(2\log(\mu/\Lambda^{(5)}))}{2\log(\mu/\Lambda^{(5)})}\right), \tag{10}$$

with $\beta_1 = (33-2n_f)/12$, $\beta_2 = (153-19n_f)/24$ and the flavor number $n_f = 5$. We do not take into account the threshold resummation factor [12–14] for the hard kernel, which is important only for subleading contributions from higher-twist LCDAs.

The hard function $h_{\rm II}(x_1,b_1,x_2,b_2,Q)$ in the form factor contains the internal propagators expressed in the coordinate space conjugate to the transverse momenta,

$$\begin{aligned}&\int\frac{d^2\mathbf{b}_1 d^2\mathbf{k}_{1T}}{(2\pi)^2}\int\frac{d^2\mathbf{b}_2 d^2\mathbf{k}_{2T}}{(2\pi)^2}e^{-i(\mathbf{k}_{1T}\cdot\mathbf{b}_1+\mathbf{k}_{2T}\cdot\mathbf{b}_2)} \\ &\times\frac{1}{x_2Q^2-\mathbf{k}_{2T}^2+i\epsilon}\frac{1}{x_1x_2Q^2-(\mathbf{k}_{1T}+\mathbf{k}_{2T})^2+i\epsilon} \\ &= \int_0^\infty db_1db_2b_1b_2\left(\frac{i\pi}{2}\right)^2 H_0^{(1)}(\sqrt{x_1x_2}Qb_1) \\ &\times[\theta(b_1-b_2)J_0(\sqrt{x_2}Qb_2)H_0^{(1)}(\sqrt{x_2}Qb_1)+(b_1\leftrightarrow b_2)] \\ &\equiv \int_0^\infty db_1db_2b_1b_2 h_{\rm II}(x_1,b_1,x_2,b_2,Q),\end{aligned} \tag{11}$$

in which J_0 is the Bessel function of the first kind and $H_0^{(1)}$ is the Hankel function of the first kind. We notice that Eq. (11) oscillates violently as Q^2 goes beyond 50 GeV^2, resulting from the large hierarchy between the two scales, Q^2 and k_T^2. The strong oscillation causes difficulty in obtaining the convergent multiple integral in (7) numerically.[2] To overcome this difficulty, we assume the hierarchy ansatz $x_iQ^2 \gg x_1x_2Q^2 \sim k_T^2$ according to the power counting in the PQCD approach, dropping the transverse momentum in the quark propagator but retaining the transverse momentum in the propagator of the hard gluon. As a consequence, the double-b hard function in (11) is reduced to a single-b one,

$$\begin{aligned}&\int\frac{d^2\mathbf{b}_1 d^2\mathbf{k}_{1T}}{(2\pi)^2}\int\frac{d^2\mathbf{b}_2 d^2\mathbf{k}_{2T}}{(2\pi)^2}e^{-i(\mathbf{k}_{1T}\cdot\mathbf{b}_1+\mathbf{k}_{2T}\cdot\mathbf{b}_2)} \\ &\times\frac{1}{x_2Q^2+i\epsilon}\frac{1}{x_1x_2Q^2-(\mathbf{k}_{1T}+\mathbf{k}_{2T})^2+i\epsilon} \\ &= \int_0^\infty dbb\frac{1}{x_2Q^2}\left(-\frac{i\pi}{2}\right)H_0^{(1)}(\sqrt{x_1x_2}Qb) \\ &\equiv \int_0^\infty dbb h_{\rm I}(x_1,x_2,b,Q),\end{aligned} \tag{12}$$

with $b = b_1 = b_2$ read off the above derivation. This approximation simplifies the computational task and also extends the numerically manageable range in Q^2 from dozens to thousands of GeV^2. The form factor at LO is then modified to

$$\begin{aligned}\mathcal{G}_{\rm I}(Q^2)_{\rm LO} &= -16\pi C_F Q^2\int_0^1 dx_1dx_2\int dbb\alpha_s(\mu)x_2\phi_\pi(x_1) \\ &\quad\times\phi_\pi(x_2)h_{\rm I}(x_1,x_2,b,Q){\rm Exp}[-S_{\rm I}(x_1,x_2,b,\mu)] \\ &= i8\pi^2C_F\int_0^1 dx_1dx_2\int dbb\alpha_s(\mu)\phi_\pi(x_1)\phi_\pi(x_2) \\ &\quad\times H_0^{(1)}(\sqrt{x_1x_2}Qb){\rm Exp}[-S_{\rm I}(x_1,x_2,b,\mu)],\end{aligned} \tag{13}$$

[2] This hierarchy is less obvious in B meson decays because of $Q^2 = m_B^2$, and the numerical integrals converge quickly.

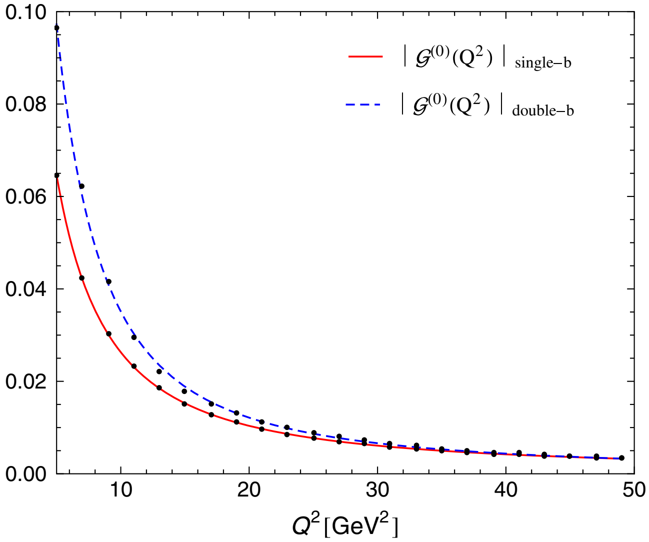

FIG. 2. Magnitude of the LO timelike pion form factor $\mathcal{G}(Q^2)$ derived in the double-b and single-b formulations.

with $S_{\rm I}(x_1, x_2, b, \mu) = S_{\rm II}(x_1, b, x_2, b, \mu)$. As will be observed in Fig. 2, in which the double-b and single-b results are compared, the single-b approximation works very well in the high-Q^2 region.

B. Next-to-leading-order QCD correction

The NLO correction to the timelike pion form factor has been explored in the PQCD approach with the single-b convolution [31],

$$\begin{aligned}\mathcal{G}_{\rm I}(Q^2)_{\rm NLO} = {}& i2\pi C_F^2 \int_0^1 dx_1 dx_2 \int db b \alpha_s^2(\mu)\phi_\pi(x_1)\phi_\pi(x_2) \\ & \times \mathrm{Exp}[-S_{\rm I}(x_1, x_2, b, \mu)] \\ & \times [\tilde{h}(x_1, x_2, b, Q, \mu) H_0^{(1)}(\sqrt{x_1 x_2} Q b) \\ & + H_0^{(1)\prime\prime}(\sqrt{x_1 x_2} Q b)],\end{aligned} \tag{14}$$

where the explicit expression of the NLO function $\tilde{h}(x_1, x_2, b, Q, \mu)$ is referred to Eq. (18) of Ref. [25]. For the second derivative of the Hankel function on the order parameter

$$H_0^{(1)\prime\prime}(x) \equiv \left[\frac{d^2}{d\alpha^2} H_\alpha^{(1)}(x)\right]_{\alpha=0}, \tag{15}$$

we take the following fit function in practice:

$$\mathrm{Re}[H_0^{(1)\prime\prime}(x)] = \begin{cases} 0.798 + 0.454x - 0.0603x^2 + 0.00590x^3 - 0.00021x^4 - 1.35\log x & \\ +J_0(x)(-0.581 + 1.48\log x - 0.497\log^2 x) & \\ +Y_0(x)(-3.62 - 0.194x + 0.665\log x + 0.331\log^2 x), & x \geq 10, \\ (-0.0870x^{-5/2} + 1.05x^{-3/2})\cos(\frac{\pi}{4} - x) & \\ +(-0.624x^{-5/2} - 0.000588x^{-3/2} + 1.97x^{-1/2})\sin(\frac{\pi}{4} - x), & x < 10, \end{cases} \tag{16}$$

$$\mathrm{Im}[H_0^{(1)\prime\prime}(x)] = \begin{cases} -4.58 + 0.720x - 0.151x^2 + 0.00643x^3 + 2.57\log x & \\ +J_0(x)(3.16 - 0.794x + 0.0179x^2 - 5.65\log x + 2.26\log^2 x) & \\ +Y_0(x)(4.10 - 2.03\log x - 0.00708\log^2 x), & x \geq 10, \\ (0.610x^{-5/2} + 0.00182x^{-3/2} - 1.97x^{-1/2})\cos(\frac{\pi}{4} - x) & \\ +(-0.0897x^{-5/2} + 1.05x^{-3/2})\sin(\frac{\pi}{4} - x), & x < 10. \end{cases} \tag{17}$$

The NLO correction (14) was applied recently to analyze the B_c pair production at electron-positron colliders with the nonrelativistic-QCD B_c meson distribution amplitudes [32], in which only the small argument limit for $H_0^{(1)\prime\prime}(x)$ was considered.

III. NUMERICS

The asymptotic form of the twist-2 pion LCDA is employed here,

$$\phi_\pi(x) = \frac{f_\pi}{2\sqrt{2N_c}} 6x(1-x), \tag{18}$$

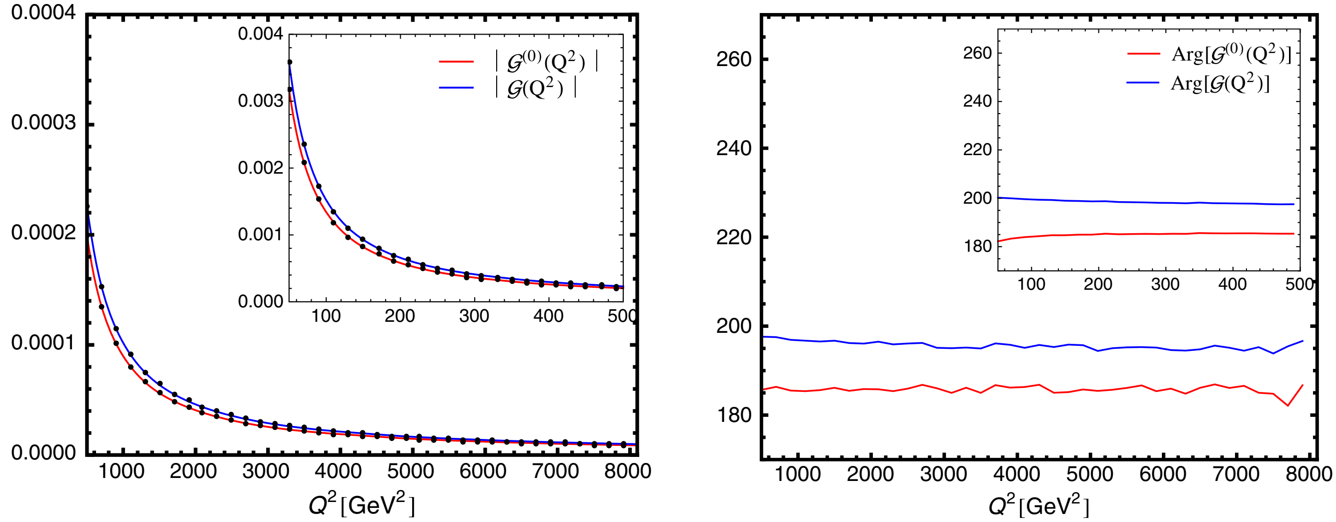

FIG. 3. LO ($\mathcal{G}^{(0)}$) and NLO ($\mathcal{G}$) predictions for the magnitude (left) and the phase (right) of the pion form factor for Q^2 between 50 GeV^2 and m_Z^2.

with the pion decay constant $f_\pi = 130.2 \pm 1.4$ GeV [16]. The other numerical inputs include [33] the width $\Gamma_Z = 2.4952 \pm 0.0023$ GeV and

$$\sin^2\theta_w(m_Z) = 0.23129 \pm 0.00005,$$
$$\alpha_s(m_Z) = 0.1182 \pm 0.0012,$$
$$\alpha(m_Z)^{-1} = 127.950 \pm 0.017, \tag{19}$$

defined at the m_Z scale under the modified minimal subtraction ($\overline{\mathrm{MS}}$) scheme. To reproduce the central value of $\alpha_s(m_Z)$ with the two-loop accuracy, the scale $\Lambda^{(5)}_{\overline{\mathrm{MS}}} = 0.2327$ GeV is chosen. Using a Monte Carlo integration strategy with the Vegas [34] algorithm from the GNU Scientific Library [35], we estimate the integral with 500,000,000 sampling points for the real and imaginary parts of Eqs. (13) and (14), which achieves a relative precision better than permillage level. The central values of the LO pion form factor and the NLO correction at $Q^2 = m_Z^2$ are

$$\mathcal{G}(m_Z^2)_{\mathrm{LO}} = (-8.29 - i0.771) \times 10^{-6},$$
$$\mathcal{G}(m_Z^2)_{\mathrm{NLO}} = (-0.764 - i1.58) \times 10^{-6}, \tag{20}$$

from which we see that the NLO correction enhances the LO result reasonably by about 10%. The PQCD prediction up to NLO for the $Z \to \pi^+\pi^-$ branching ratio is given by

$$\mathcal{B}(Z \to \pi^+\pi^-) = (0.83 \pm 0.02 \pm 0.02 \pm 0.04) \times 10^{-12}, \tag{21}$$

with the three uncertainties coming from the scale variation from $\mu/2$ to 2μ, the strong coupling constant and the pion decay constant, respectively. Replacing the pion decay constant in the calculation with the kaon decay constant $f_K = 155.6 \pm 0.4$ GeV [16], we have the corresponding $Z \to K^+K^-$ branching ratio

$$\mathcal{B}(Z \to K^+K^-) = (1.74^{+0.03}_{-0.05} \pm 0.04 \pm 0.02) \times 10^{-12}. \tag{22}$$

According to Ref. [19], the Circular Electron-Positron Collider is expected to collect 7×10^{11} Z^0 bosons in two years with the instantaneous luminosity of 32×10^{34} cm^{-2} s^{-1} and two interaction points. The FCC-ee [18], with the instantaneous luminosity of 56×10^{34} cm^{-2} s^{-1} and four interaction points, will quadruple this number roughly. If the two channels are combined, observations at the two Tera-Z factories will be quite promising, owing to almost 100% detection efficiencies of charged pions and kaons. On the other hand, if the contributions from the "small-x" region are actually dominant in the pion and kaon timelike form factors as postulated in Refs. [1,11], more events will be expected.

To confirm the validity of the single-b configuration, we compute the LO timelike pion form factor in the region $Q^2 \in [1, 50]$ GeV^2 using both the double-b and double-b formulas (7) and (13) and display them in Fig. 2. The discrepancy between the two results is visible in the low-Q^2 region, while starting from ~40 GeV^2, we can safely omit the transverse momentum effect in the internal quark propagator and adopt the single-b approximation.

The magnitude and the strong phase of the pion form factor at LO and NLO for Q^2 between 50 GeV^2 and m_Z^2 are shown in Fig. 3. The NLO correction to the magnitude is found to be around 11% in the whole considered Q^2 range. For the strong phase, the LO prediction is about 180°, and the NLO correction yields an increase not more than 20°.[3] We suggest a parametrization formula for the form factor far away from the resonance region with Q in units of GeV,

[3]The NLO correction brings a large enhancement to the imaginary part, but it is still considerably smaller than the LO real part.

$$|\mathcal{G}(Q^2)| = \frac{A + Q^2 B}{Q^4 + Q^2 C + A}, \quad (23)$$

which is inspired by the parametrization with the reciprocal of the square polynomial [36]. Here, we have added another Q^2 term in the numerator to relieve a sudden drop at Q^2 around several hundred squared giga-electron-volts. The equality of the constant terms in the numerator and the denominator is motivated by the normalization condition of the pion form factor $\mathcal{G}_\pi(0) = 1$ (for references, see, e.g., Ref. [37]). For the LO timelike pion form factor, the parameters $A^{(0)} = 0.0879, B^{(0)} = 46.1$, and $C^{(0)} = 10.9$ are determined. Including the NLO correction, we have $A = 0.0996$, $B = 48.2$, and $C = 12.6$.

IV. CONCLUSION

We have studied the $Z \to \pi^+\pi^-, K^+K^-$ decays in the PQCD formalism, the branching ratios of which are governed by the timelike form factors of the corresponding mesons. With a high $Q^2 = m_Z^2$, we can safely neglect the power corrections in the PQCD evaluation of the form factors, which then do not depend on any unknown nonperturbative parameters and can be predicted precisely. Our predictions up to NLO for the branching ratios of the two channels are $\mathcal{B}(Z \to \pi^+\pi^-) = (0.83 \pm 0.02 \pm 0.02 \pm 0.04) \times 10^{-12}$ and $\mathcal{B}(Z \to K^+K^-) = (1.74^{+0.03}_{-0.05} \pm 0.04 \pm 0.02) \times 10^{-12}$. They can be accessed at a future Tera-Z factory, and the measurements will represent a touchstone of the PQCD approach.

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APPENDIX: PION FORM FACTOR UP TO SUBLEADING TWIST

The pion transverse-momentum-dependent wave function has been proposed in Refs. [38,39] and regularizes both the rapidity and self-energy divergences. Compared to Ref. [38], the form in Ref. [39] is simpler and compatible with the k_T factorization. In the limit of vanishing infrared regulators, they both approach the naive definition in Ref. [40]. Here, we assume that the dependence on the parton transverse momentum has been organized into the Sudakov factor and consider only the dependence on the longitudinal momentum fraction, which can be formulated as [41]

$$\begin{aligned}
&\langle \pi^-(p_2)|\bar{d}_j(z)u(0)_l|0\rangle \\
&= \frac{i}{\sqrt{2N_c}}\int_0^1 e^{ix_2 p_2\cdot z}\gamma_5 \\
&\quad \times \{\not{p}_2\phi_\pi(x_2) + m_0^\pi \phi_\pi^P(x_2) + m_0^\pi(\not{n}\not{v} - 1)\phi_\pi^T(x_2)\}_{lj}.
\end{aligned} \quad (A1)$$

Keeping the chiral mass m_0^π, which is expected to contribute the dominant subleading-power correction, we find that the LO result of the pion form factor (7) is modified to

$$\begin{aligned}
\mathcal{G}(Q^2) = &-16\pi C_F Q^2 \int_0^1 dx_1 dx_2 \int b_1 db_1 b_2 db_2 \alpha_s(\mu) h_{\mathrm{II}} \\
&\times (x_1, b_1, x_2, b_2, Q)\mathrm{Exp}[-S_{\mathrm{II}}(x_1, b_1, x_2, b_2, \mu)] \\
&\times \{x_2\phi_\pi(x_1)\phi_\pi(x_2) + 2r_\pi^2\phi_\pi^P(x_1)(\phi_\pi^P(x_2) - \phi_\pi^T(x_2)) \\
&+ 2x_2 r_\pi^2 \phi_\pi^P(x_1)(\phi_\pi^P(x_2) + \phi_\pi^T(x_2))\},
\end{aligned} \quad (A2)$$

with $r_\pi \equiv m_0^\pi/Q$. It is easy to confirm that the relative size of the power correction is of order of $r_\pi^2 \sim 10^{-4}$ at $Q = m_Z$.

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