Next-to-leading order corrections to $B \to \rho$ transition in the k_T factorization

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In this paper, we investigate the factorization hypothesis step by step for the exclusive process $B \to \rho$ at next-to-leading order (NLO), and then we extend our results to the k_T factorization frame. We show that the soft divergence from the specific NLO diagrams will cancel each other at the quark level, while the remaining collinear divergence can be absorbed into the NLO wave functions of the mesons involved. The full NLO amplitudes can be factorized into two parts: the B meson and the ρ meson NLO wave functions containing the collinear divergence and the finite leading order hard kernels. We give the general expressions of the nonlocal hadron matrix for the NLO B meson and ρ meson wave functions and the analytical results of factorization for the combinations of different twist parts.

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I. INTRODUCTION

As the foundation of the perturbative QCD formalism, the factorization theorem [1–3] claims that the hard part of the QCD interactions is finite and can be calculated perturbatively, meanwhile the nonperturbative part can be factorized into the universal wave functions defined in an infinite momentum frame. The perturbative QCD (PQCD) approach [4] based on the k_T factorization theorem is proposed to eliminate the endpoint singularity by picking up the previously dropped transversal momentum of the propagators and taking into account the further suppression from the Sudakov resummation [5,6].

In recent years, several exclusive processes $\pi\gamma^* \to \gamma(\pi)$, $\rho\gamma^* \to \pi$ and $B \to \gamma(\pi) l\bar{\nu}$ have been investigated by many authors for example in Refs. [5–9] at the leading order (LO) and next-to-leading order (NLO). The calculations of the space like and the time like form factors [10] play an important role for high precision studies for B meson decays.

In this paper, taking the ρ meson as one example for those light vector mesons, we consider the $B \to \rho$ transition process, show explicitly the structure of the infrared divergences at the NLO level for the combinations of the leading twist parts of the wave functions. The Fierz identity and the Eikonal approximation will be taken into account to factorize the fermion flow and the momentum flow respectively. The power counting for various gamma matrices discussed in Ref. [11] are also considered in order to give the right color factors with triple gluon vertex diagrams. By summing up the contributions from all subdiagrams, we can prove that the soft divergence will be canceled each other between the quark diagrams, and the remaining collinear divergence can also be absorbed into the NLO meson wave functions. The convolutions of the NLO wave function and hard kernel have two forms: the one has additional gluon emitted from the external quarks flowing into hard kernel; another has no additional gluon flowing into the hard kernel. We will finally write down the hadronic matrix elements for B and ρ meson wave functions with its collinear parts in the k_T factorization.

This paper is organized as follows. In Sec. II, we show the dynamical analysis and the leading order amplitudes for $B \to \rho$ transition. In Sec. III, we present the investigations for $B \to \rho$ transition process at the NLO step by step, and give the hadronic matrix elements for the wave functions in k_T factorization. In Sec. IV, finally, a brief summary and some discussions are given.

II. LEADING ORDER HARD KERNEL

In this section, we calculate the leading order $O(\alpha_s)$ hard kernel for $B \to \rho$ transition, and the topological diagrams of

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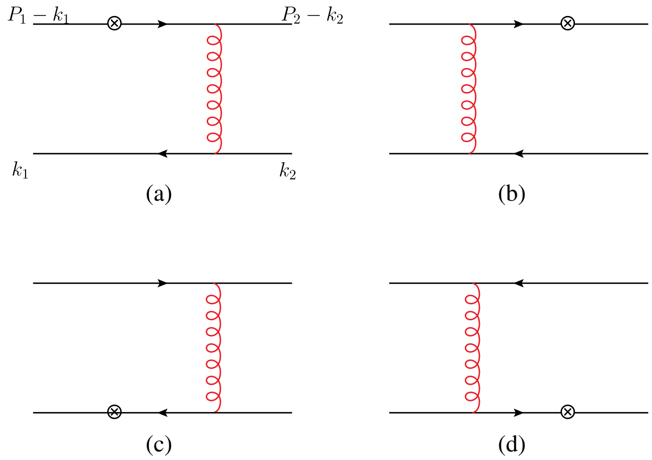

FIG. 1. The leading-order quark diagrams for the $B \to \rho$ transition form factors with the symbol $\otimes$ representing the weak vertex.

leading-order transitions are displayed in Fig. 1. The definitions of the kinematics in light cone coordinate are of the following form:

$$p_{1\mu} = \frac{m_B}{\sqrt{2}}(1, 1, \mathbf{0_T}), \qquad k_{1\mu} = \frac{m_B}{\sqrt{2}}(x_1, 0, \mathbf{0_T});$$
$$\epsilon_{1\mu}(L) = \frac{1}{\sqrt{2}}(1, 0, \mathbf{0_T}), \qquad \epsilon_{1\mu}(T) = (0, 0, \mathbf{1_T}); \tag{1}$$

$$p_{2\mu} = \frac{\eta m_B}{\sqrt{2}}(0, 1, \mathbf{0_T}), \qquad k_{2\mu} = \frac{\eta m_B}{\sqrt{2}}(0, x_2, \mathbf{0_T});$$
$$\epsilon_{2\mu}(L) = \frac{1}{\sqrt{2}\gamma_\rho}(-\gamma_\rho^2, 1, \mathbf{0_T}), \qquad \epsilon_{2\mu}(T) = (0, 0, \mathbf{1_T}), \tag{2}$$

where m_B is the mass of B-meson. The energy fraction η of the ρ meson in large recoil region is of order 1, and the polarization vector with the definition $\gamma_\rho = m_\rho/Q$ is defined by condition $\epsilon_i^2(L/T) = -1$. The momentum transfer squared $Q^2 = -(p_1 - p_2)^2 (Q^2 > 0)$ is taken to describe the evolution behaviors of the form factors.

The leading twist (twist-2) wave functions of the B meson and ρ meson are chosen in the same form as those in Refs. [12,13]:

$$\Phi_B(x_1, p_1) = \frac{1}{\sqrt{6}}(\not{p}_1 + M_B)\left[\frac{\not{n}_+}{\sqrt{2}}\phi_B^+(x_1) + \frac{1}{\sqrt{2}}\left(\not{n}_- - k_1^+\gamma_\perp^\nu \frac{\partial}{\partial \mathbf{k}_{1T}^\nu}\right)\phi_B^-(x_1)\right],$$
$$\Phi_\rho(p_2, \epsilon_{2L}) = \frac{i}{\sqrt{6}}[M_\rho \not{\epsilon}_{2L}\phi_\rho(x_2)],$$
$$\Phi_\rho(p_2, \epsilon_{2T}) = \frac{i}{\sqrt{6}}[M_\rho \not{\epsilon}_{2T}\phi_\rho^v(x_2)], \tag{3}$$

where $n_+ = (1, 0, 0)$ and $n_- = (0, 1, 0)$ are the unit vector in the light cone coordinate. The wave functions of the vector ρ meson contain both the longitudinal and the transversal component $\Phi_\rho(p_2, \epsilon_{2L})$ and $\Phi_\rho(p_2, \epsilon_{2T})$. To insure the gauge invariance and the accuracy of the analytic calculations, both contributions from ϕ_B^+ and ϕ_B^- will be taken into account in this work.

Because of the symmetry relations, the contributions from Figs. 1(c) and 1(d) can be simply derived from those of Figs. 1(a) and 1(b) by exchange of momentum fraction for up (down) quarks. The LO transition amplitude of Figs. 1(a) and 1(b) can be simplified to the following forms:

$$H_{aL}^{(0)} = -\frac{ieg^2C_F}{2}\mathbf{Tr}\left\{\frac{\gamma^\alpha[M_\rho\not{\epsilon}_{2L}\phi_\rho(x_2)]\gamma_\alpha(\not{p}_2 - \not{k}_1)\gamma_\mu(\not{p}_1 + m_B)\gamma_5[\frac{\not{n}_+}{\sqrt{2}}\phi_B^+(x_1) + \frac{\not{n}_-}{\sqrt{2}}\phi_B^-(x_1)]}{(p_2 - k_1)^2(k_1 - k_2)^2}\right\}$$
$$= -ieg^2C_F m_B\mathbf{Tr}\left[\frac{\not{\epsilon}_{2L}\not{k}_1\gamma_\mu\gamma_5\frac{\not{n}_+}{\sqrt{2}}\phi_B^+(x_1)}{(p_2 - k_1)^2(k_1 - k_2)^2}\right], \tag{4}$$

$$H_{bL}^{(0)} = -ieg^2C_F m_B\mathbf{Tr}\left[\frac{M_\rho\not{\epsilon}_{2L}\phi_\rho(x_2)\gamma_\mu\not{k}_2\frac{\not{n}_+}{\sqrt{2}}\phi_B^+(x_1)\gamma_5 + M_\rho\not{\epsilon}_{2L}\phi_\rho(x_2)\gamma_\mu m_b\phi_B^-(x_1)\gamma_5}{[(p_1 - k_2)^2 - m_b^2](k_1 - k_2)^2}\right], \tag{5}$$

$$H_{bT}^{(0)} = -ieg^2C_F m_B\mathbf{Tr}\left[\frac{[\not{\epsilon}_{2T}\not{p}_2\phi_\rho^T(x_2)]\gamma_\mu\not{p}_1[\phi_B^+(x_1) + \phi_B^-(x_1)]\gamma_5 - [\not{\epsilon}_{2T}\not{p}_2\phi_\rho^T(x_2)]\gamma_\mu m_b\frac{\not{n}_+}{\sqrt{2}}\phi_B^+(x_1)\gamma_5}{[(p_1 - k_2)^2 - m_b^2](k_1 - k_2)^2}\right], \tag{6}$$

where $C_F = 4/3$ is the color factor. Since $H_{aT}^{(0)} = 0$ for Fig. 1(a), we do not show $H_{aT}^{(0)}$ explicitly in Eq. (4). For $H_{aL}^{(0)}$, only the $\not{k}_1$ in quark propagator and m_B component in B wave function have contributions, which also requires γ^α and γ_μ should be $\gamma_\perp$ and γ^+. $H_{bL}^{(0)}$ contains two components corresponding to the two parts of the B wave function ϕ_B^+ and ϕ_B^- respectively, which leads to different restrictions on the vertex with γ_α and γ_μ. Similarly, $H_{bT}^{(0)}$ contains three components, two for ϕ_B^+ and one for ϕ_B^-. Every part of the combinations requires unique choice for P_1 momentum (P_1^-/P_1^+) and gamma matrix γ_μ and γ_α. We should consider each part independently.

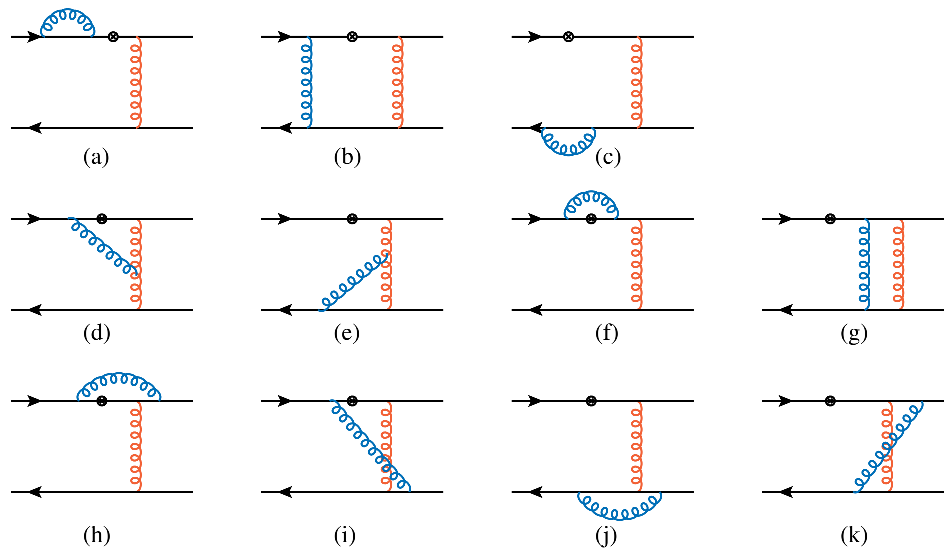

FIG. 2. The quark diagrams for NLO corrections to Fig. 1(a) with additional gluon emitted from the quark(antiquark) lines of initial B meson.

III. FACTORIZATION OF $B \to \rho$ AT NEXT-TO-LEADING ORDER

In this section, we will study the exclusive process $B \to \rho$ at NLO level, consider both momentum space and color space factorization but neglecting the transverse momentum first. At the end of this section, we will pick up again the transverse momentum and discuss how to deal with the infrared contribution carrying by k_T-related part.

In momentum space, the Fermion flow can be factorized by inserting the Fierz identity

$$I_{ij}I_{lk} = \frac{1}{4}I_{ik}I_{lj} + \frac{1}{4}(\gamma_5)_{ik}(\gamma_5)_{lj} + \frac{1}{4}(\gamma^\alpha)_{ik}(\gamma_\alpha)_{lj} + \frac{1}{4}(\gamma_5\gamma^\alpha)_{ik}(\gamma_\alpha\gamma_5)_{lj} + \frac{1}{8}(\sigma^{\alpha\beta})_{ik}(\sigma_{\alpha\beta})_{lj}, \qquad (7)$$

into the proper place of the matrix elements, here $\sigma^{\alpha\beta} = i[\gamma^\alpha, \gamma^\beta]/2$. The different terms on the right hand side correspond to the contributions from different twists of $B(\rho)$ meson. The Eikonal approximation is taken before inserting the Fierz identity to reformulate the singularity propagator into a simplified form and reduce the gamma matrix for convenience meanwhile. All the possible diagrams for a gluon radiation should be considered and finally be resumed to collect the color factors and maintain the gauge invariance.

A. The factorization of the NLO($\mathcal{O}(\alpha_s^2)$) corrections to $H_{aL}^{(0)}$

We will only show the factorization of the NLO corrections for Figs. 1(a) and 1(b), for the reason that Figs. 1(c) and 1(d) can be derived from Figs. 1(a) and 1(b) by symmetry. Taking into account the different twists' combinations, there exist six different structures from the decay amplitudes as listed Eqs. (4)–(6). Each structure has two terms corresponding to the additional gluon lines emitted from the initial B meson or from the finial ρ meson.

The topological diagrams for NLO corrections to Fig. 1(a) with additional gluon emitted from initial B meson are showed in Fig. 2. Generally, there exists two kinds of infrared divergence: (a) the soft divergence when the momentum of additional gluon is small at $l = (l^+, l^-, l_\perp) \sim (\lambda, \lambda, \lambda)$ for $\lambda \sim \Lambda_{\mathrm{QCD}}$; and (b) the collinear divergence when the additional gluon is parallel with one longitudinal direction $l = (l^+, l^-, l_\perp) \sim (\lambda^2/Q^2, Q, \lambda^2)$ [14,15]. The third possible infrared divergence appeared in the Glauber region of NLO spectator amplitudes of B meson two-body nonleptonic decays [16,17] does not exist here. We rewrite the Eq. (4) explicitly in the following form:

$$H_{aL}^{(0)} = -\frac{ieg^2C_F}{2}\mathbf{Tr} \times \left\{\frac{\gamma^\alpha[M_\rho \not\epsilon_{2L}\phi_\rho(x_2)]\gamma_\alpha(-\not k_1)\gamma_\mu(m_B)\gamma_5[\frac{\not n_+}{\sqrt{2}}\phi_B^+(x_1)]}{(p_2-k_1)^2(k_1-k_2)^2}\right\}, \qquad (8)$$

where we can see that only $\not k_1$ in internal quark propagator and m_B part in B meson wave function can contribute in this case, and γ_μ and γ^α should be γ^+ and $\gamma_\perp$.

We can write down the NLO amplitudes of Figs. 2(a)–2(c) since no radiated gluon lines attached to internal line, and therefore no momentum will flow into the structure of LO hard kernel. So factorization can be derived simply by inserting the Fierz identity, neatly cutting on the external quark (antiquark) line.

$$G^{(1)}_{aL,2a} = \frac{1}{2}\frac{eg^4C_F^2}{2}\mathbf{Tr}\left\{\frac{\gamma^\alpha[M_\rho \not{\epsilon}_{2L}\phi_\rho(x_2)]\gamma_\alpha(-\not{k}_1)\gamma_\mu(\not{p}_1-\not{k}_1+m_b)\gamma^\nu(\not{p}_1-\not{k}_1-\not{l}+m_b)\gamma_\nu(m_B)\gamma_5[\frac{\not{n}_+}{\sqrt{2}}\phi_B^+(x_1)]}{(p_2-k_1)^2(k_1-k_2)^2[(p_1-k_1)^2-m_b^2][(p_1-k_1-l)^2-m_b^2]l^2}\right\}$$
$$= \frac{1}{2}\phi^{(1)}_{B,a}\otimes G^{(0)}_{aL}(x_1,x_2), \tag{9}$$

$$G^{(1)}_{aL,2b} = -\frac{eg^4C_F^2}{2}\mathbf{Tr}\left\{\frac{\gamma^\nu(\not{k}_1-\not{l})\gamma^\alpha[M_\rho \not{\epsilon}_{2L}\phi_\rho(x_2)]\gamma_\alpha(-\not{k}_1+\not{l})\gamma_\mu(\not{p}_1-\not{k}_1+\not{l}+m_b)\gamma_\nu(m_B)\gamma_5[\frac{\not{n}_+}{\sqrt{2}}\phi_B^+(x_1)]}{(p_2-k_1+l)^2(k_1-k_2-l)^2(k_1-l)^2[(p_1-k_1+l)^2-m_b^2]l^2}\right\}$$
$$= \phi^{(1)}_{B,b}\otimes G^{(0)}_{aL}(\xi_1,x_2), \tag{10}$$

$$G^{(1)}_{aL,2c} = \frac{1}{2}\frac{eg^4C_F^2}{2}\mathbf{Tr}\left\{\frac{\gamma^\nu(\not{k}_1-\not{l})\gamma_\nu(\not{k}_1)\gamma^\alpha[M_\rho \not{\epsilon}_{2L}\phi_\rho(x_2)]\gamma_\alpha(-\not{k}_1)\gamma_\mu(m_B)\gamma_5[\frac{\not{n}_+}{\sqrt{2}}\phi_B^+(x_1)]}{(p_2-k_1)^2(k_1-k_2)^2(k_1-l)^2(k_1)l^2}\right\}$$
$$= \frac{1}{2}\phi^{(1)}_{B,c}\otimes G^{(0)}_{aL}(x_1,x_2), \tag{11}$$

with

$$\phi^{(1)}_{B,a} = \frac{-ig^2C_F}{4}\frac{\gamma_5\gamma^\rho\gamma_\rho\gamma_5(\not{p}_1-\not{k}_1+m_b)\gamma^\nu(\not{p}_1-\not{k}_1-\not{l}+m_b)\gamma_\nu}{[(p_1-k_1)^2-m_b^2][(p_1-k_1-l)^2-m_b^2]l^2}, \tag{12}$$

$$\phi^{(1)}_{B,b} = \frac{ig^2C_F}{4}\frac{\gamma_5\gamma^\rho(\not{p}_1-\not{k}_1+\not{l}+m_b)\gamma^\nu\gamma_5\gamma_\rho\gamma_\nu(\not{k}_1-\not{l})}{(k_1-l)^2[(p_1-k_1+l)^2-m_b^2]l^2}, \tag{13}$$

$$\phi^{(1)}_{B,c} = \frac{-ig^2C_F}{4}\frac{\gamma_5\gamma_\rho\gamma^\nu(\not{k}_1-\not{l})\gamma_\nu(\not{k}_1)\gamma^\rho\gamma_5}{(k_1-l)^2(k_1)l^2}. \tag{14}$$

The factor $1/2$ appeared in Eqs. (9) and (11) comes from the symmetry of Figs. 2(a) and 2(c). From the QCD diagrams and dynamics, no independent soft divergences exist in reducible diagrams, then the collinear divergences can be absorbed into NLO wave functions concisely. These infrared singularities from amplitudes of Figs. 2(a)–2(c) are absorbed into NLO wave functions $\phi^{(1)}_{B,i}(i=a,b,c)$ and can be reexpressed as in Eqs. (12)–(14). For the reason that the infrared divergence in reducible diagrams without mixing can be simply matched one by one between QCD diagrams and effective diagrams [8], these results are independent of those irreducible diagrams. So, we just pay more attention to irreducible diagrams.

For Figs. 2(d) and 2(e) we have

$$G^{(1)}_{aL,2d} = \frac{ieg^4\mathrm{Tr}[T^aT^bT^c]f_{abc}}{2N_c}\mathbf{Tr}\left\{\frac{\gamma^\alpha[M_\rho \not{\epsilon}_{2L}\phi_\rho(x_2)]\gamma^\beta(\not{p}_2-\not{k}_1+\not{l})\gamma_\mu(\not{p}_1-\not{k}_1+\not{l}+m_b)\gamma^\gamma(m_B)\gamma_5[\frac{\not{n}_+}{\sqrt{2}}\phi_B^+(x_1)]F_{\alpha\beta\gamma}}{(k_1-k_2)^2(k_1-k_2-l)^2(p_2-k_1+l)^2[(p_1-k_1+l)^2-m_b^2]l^2}\right\}$$
$$= \frac{9}{8}\phi^{(1)}_{B,d}\otimes[G^{(0)}_{aL}(\xi_1,x_1,x_2)-G^{(0)}_{aL}(\xi_1,x_2)],$$
$$G^{(1)}_{aL,2e} = \frac{ieg^4\mathrm{Tr}[T^aT^bT^c]f_{abc}}{2N_c}\mathbf{Tr}\left\{\frac{\gamma^\alpha(\not{k}_1-\not{l})\gamma^\beta[M_\rho \not{\epsilon}_{2L}\phi_\rho(x_2)]\gamma^\gamma(\not{p}_2-\not{k}_1)\gamma_\mu(m_B)\gamma_5[\frac{\not{n}_+}{\sqrt{2}}\phi_B^+(x_1)]F_{\alpha\beta\gamma}}{(k_1-k_2)^2(k_1-k_2-l)^2(p_2-k_1)^2(k_1-l)^2l^2}\right\}$$
$$= \frac{9}{8}\phi^{(1)}_{B,e}\otimes[G^{(0)}_{aL}(x_1,x_2)-G^{(0)}_{aL}(x_1,\xi_1,x_2)], \tag{15}$$

with

$$\phi^{(1)}_{B,d} = \frac{-ig^2C_F}{4}\frac{\gamma^+\gamma_5(\not{p}_1-\not{k}_1+\not{l}+m_b)\gamma^\rho\gamma_5\gamma^-}{[(p_1-k_1+l)^2-m_b^2]l^2}\frac{\nu_\rho}{\nu\cdot l},$$
$$\phi^{(1)}_{B,e} = \frac{ig^2C_F}{4}\frac{\gamma_5\gamma^-\gamma^\rho(\not{k}_1-\not{l})\gamma^+\gamma_5}{(k_1-l)^2l^2}\frac{\nu_\rho}{\nu\cdot l}. \tag{16}$$

For Fig. 2(d) where the triple gluon vertex appeared, as we examined in Ref. [11], only the terms in $G^{(1)}_{aL,2d}$ proportional to $g_{\alpha\beta}$ in $F_{\alpha\beta\gamma} = g_{\alpha\beta}(2k_2 - 2k_1 + l)_\gamma + g_{\beta\gamma}(k_1 - k_2 - 2l)_\alpha + g_{\gamma\alpha}(k_1 - k_2 + l)_\beta$ give the right NLO corrections to the LO hard kernel. The other terms are power suppressed by power counting of matrix elements and also unphysical in topological diagrams. For Fig. 2(e), similarly, only the terms proportional to $g_{\beta\gamma}$ in $F_{\alpha\beta\gamma} = g_{\alpha\beta}(k_1 - k_2 - 2l)_\gamma + g_{\beta\gamma}(2k_2 - 2k_1 + l)_\alpha + g_{\gamma\alpha}(k_1 - k_2 + l)_\beta$ contribute effectively. Then the NLO B meson wave function containing infrared divergence with "l" emitted from quark (antiquark) line can be represented as in Eq. (16). Because of the large scale of m_b, the $\phi^{(1)}_{B,d}$ contains the m_b term in propagator when "l" emits from the b quark.

The factorization of amplitudes for remaining diagrams Figs. 2(f)–2(k) can be expressed as follows:

$$G^{(1)}_{aL,2f} = -\frac{eg^4 \mathrm{Tr}[T^aT^aT^cT^c]}{2N_c} \mathbf{Tr}\left\{\frac{\gamma^\alpha[M_\rho \not\epsilon_{2L}\phi_\rho(x_2)]\gamma_\alpha(\not p_2 - \not k_1)\gamma^\nu(\not p_2 - \not k_1 + \not l)\gamma_\mu(\not p_1 - \not k_1 + \not l + m_b)\gamma_\nu(m_B)\gamma_5[\frac{\not n_+}{\sqrt{2}}\phi^+_B(x_1)]}{(p_2 - k_1)^2(k_1 - k_2)^2(p_2 - k_1 + l)^2[(p_1 - k_1 + l)^2 - m_b^2]l^2}\right\}$$
$$\sim 0, \tag{17}$$

$$G^{(1)}_{aL,2g} = \frac{eg^4 \mathrm{Tr}[T^aT^aT^cT^c]}{2N_c} \mathbf{Tr}\left\{\frac{\gamma^\nu(\not k_1 - \not l)\gamma^\alpha[M_\rho \not\epsilon_{2L}\phi_\rho(x_2)]\gamma_\alpha(\not p_2 - \not k_1 + \not l)\gamma_\nu(\not p_2 - \not k_1)\gamma_\mu(m_B)\gamma_5[\frac{\not n_+}{\sqrt{2}}\phi^+_B(x_1)]}{(p_2 - k_1)^2(k_1 - k_2 - l)^2(p_2 - k_1 + l)^2(k_1 - l)^2 l^2}\right\} \sim 0, \tag{18}$$

$$G^{(1)}_{aL,2h} = -\frac{eg^4 \mathrm{Tr}[T^aT^cT^aT^c]}{2N_c}$$
$$\times \mathbf{Tr}\left\{\frac{\gamma^\alpha[M_\rho \not\epsilon_{2L}\phi_\rho(x_2)]\gamma^\nu(\not p_2 - \not k_2 + \not l)\gamma_\alpha(\not p_2 - \not k_1 + \not l)\gamma_\mu(\not p_1 - \not k_1 + \not l + m_b)\gamma_\nu(m_B)\gamma_5[\frac{\not n_+}{\sqrt{2}}\phi^+_B(x_1)]}{(p_2 - k_1 + l)^2(k_1 - k_2)^2(p_2 - k_2 + l)^2[(p_1 - k_1 + l)^2 - m_b^2]l^2}\right\}$$
$$= -\frac{1}{8}\phi^{(1)}_{B,d} \otimes G^{(0)}_{aL}(\xi_1, x_1, x_2), \tag{19}$$

$$G^{(1)}_{aL,2i} = \frac{eg^4 \mathrm{Tr}[T^aT^cT^aT^c]}{2N_c} \mathbf{Tr}\left\{\frac{\gamma^\alpha(\not k_2 + \not l)\gamma^\nu[M_\rho \not\epsilon_{2L}\phi_\rho(x_2)]\gamma_\alpha(\not p_2 - \not k_1 + \not l)\gamma_\mu(\not p_1 - \not k_1 + \not l + m_b)\gamma_\nu(m_B)\gamma_5[\frac{\not n_+}{\sqrt{2}}\phi^+_B(x_1)]}{(p_2 - k_1 + l)^2(k_1 - k_2 - l)^2(k_2 + l)^2[(p_1 - k_1 + l)^2 - m_b^2]l^2}\right\}$$
$$= \frac{1}{8}\phi^{(1)}_{B,d} \otimes G^{(0)}_{aL}(\xi_1, x_2), \tag{20}$$

$$G^{(1)}_{aL,2j} = -\frac{eg^4 \mathrm{Tr}[T^aT^cT^aT^c]}{2N_c} \mathbf{Tr}\left\{\frac{\gamma^\nu(\not k_1 - \not l)\gamma^\alpha(\not k_2 - \not l)\gamma_\nu[M_\rho \not\epsilon_{2L}\phi_\rho(x_2)]\gamma_\alpha(\not p_2 - \not k_1)\gamma_\mu(m_B)\gamma_5[\frac{\not n_+}{\sqrt{2}}\phi^+_B(x_1)]}{(p_2 - k_1 + l)^2(k_1 - k_2)^2(k_1 - l)^2(k_2 - l)^2 l^2}\right\}$$
$$= -\frac{1}{8}\phi^{(1)}_{B,e} \otimes G^{(0)}_{aL}(x_1, x_2), \tag{21}$$

$$G^{(1)}_{aL,2k} = \frac{eg^4 \mathrm{Tr}[T^aT^cT^aT^c]}{2N_c} \mathbf{Tr}\left\{\frac{\gamma^\nu(\not k_1 - \not l)\gamma^\alpha[M_\rho \not\epsilon_{2L}\phi_\rho(x_2)]\gamma_\nu(\not p_2 - \not k_2 - \not l)\gamma_\alpha(\not p_2 - \not k_1)\gamma_\mu(m_B)\gamma_5[\frac{\not n_+}{\sqrt{2}}\phi^+_B(x_1)]}{(p_2 - k_1)^2(k_1 - k_2 - l)^2(p_2 - k_2 - l)^2(k_1 - l)^2 l^2}\right\}$$
$$= \frac{1}{8}\phi^{(1)}_{B,e} \otimes G^{(0)}_{aL}(x_1, \xi_1, x_2), \tag{22}$$

where the infrared divergence in Figs. 2(f) and 2(g) are suppressed by kinematics, and we briefly set them to zero for we concentrate on infrared divergence only in factorization theorem here. All the diagrams Figs. 2(d) and 2(g) generate no soft divergence, since the soft divergence only occurs when the additional gluon bridge the initial and final quarks as shown in Figs. 2(h) and 2(k). Then these diagrams Figs. 2(h) and 2(k) will generate the soft divergence as well as the collinear divergence. But fortunately, we know that these soft divergence will cancel between Figs. 2(h)–2(k) by a simple deformation of propagators. This conclusion is also supported by analytic calculations for quark diagrams as shown in Refs. [7–9].

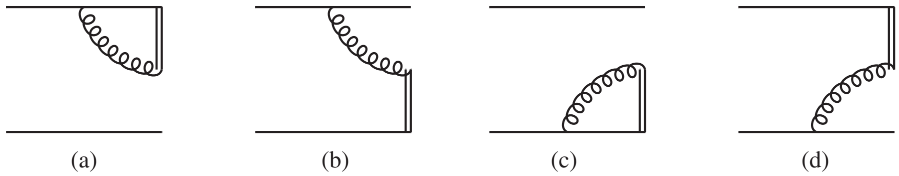

FIG. 3. The effective diagrams for the NLO initial B meson.

We will sum up all the irreducible diagrams to correct the color factors and ensure the gauge invariance. All the diagrams Figs. 2(d) and 2(i) with the additional gluon emitted from the up-quark lines of B meson together give the result:

$$G^{(1)}_{\mathrm{up},aL}(x_1,x_2)=\phi^{(1)}_{B,d}(x_1,\xi_1)\otimes[G^{(0)}_{aL}(\xi_1,x_1,x_2)-G^{(0)}_{aL}(\xi_1,x_2)]. \tag{23}$$

The diagrams Figs. 2(e) and 2(k) with the additional gluon emitted from the down-quark lines of B meson together give the result:

$$G^{(1)}_{\mathrm{down},aL}(x_1,x_2)=\phi^{(1)}_{B,e}(x_1,\xi_1)\otimes[G^{(0)}_{aL}(x_1,x_2)-G^{(0)}_{aL}(x_1,\xi_1,x_2)]. \tag{24}$$

We take the similar representation as our previous work [6], the function $G^{(0)}_{aL}(\xi_1,x_1,x_2)$, $G^{(0)}_{aL}(x_1,\xi_1,x_2)$ and $G^{(0)}_{aL}(x_1,\xi_1,x_2)$ in Eqs. (23) and (24) represent the different forms of LO hard kernel with gluon momentum. The different location of ξ_1 in vertex represents tiny difference of types as gluon momentum flowing into hard kernel.

$$G^{(0)}_{aL}(\xi_1,x_1,x_2)=-\frac{ieg^2C_F}{2}\mathbf{Tr}\left\{\frac{\gamma^\alpha[M_\rho\not{\epsilon}_{2L}\phi_\rho(x_2)]\gamma_\alpha(\not{p}_2-\not{k}_1+\not{l})\gamma_\mu(m_B)\gamma_5[\frac{\not{n}_+}{\sqrt{2}}\phi^+_B(x_1)]}{(p_2-k_1+l)^2(k_1-k_2)^2}\right\}, \tag{25}$$

$$G^{(0)}_{aL}(x_1,\xi_1,x_2)=-\frac{ieg^2C_F}{2}\mathbf{Tr}\left\{\frac{\gamma^\alpha[M_\rho\not{\epsilon}_{2L}\phi_\rho(x_2)]\gamma_\alpha(\not{p}_2-\not{k}_1)\gamma_\mu(m_B)\gamma_5[\frac{\not{n}_+}{\sqrt{2}}\phi^+_B(x_1)]}{(p_2-k_1)^2(k_1-k_2-l)^2}\right\}, \tag{26}$$

$$G^{(0)}_{aL}(\xi_1,x_2)=-\frac{ieg^2C_F}{2}\mathbf{Tr}\left\{\frac{\gamma^\alpha[M_\rho\not{\epsilon}_{2L}\phi_\rho(x_2)]\gamma_\alpha(\not{p}_2-\not{k}_1+\not{l})\gamma_\mu(m_B)\gamma_5[\frac{\not{n}_+}{\sqrt{2}}\phi^+_B(x_1)]}{(p_2-k_1+l)^2(k_1-k_2-l)^2}\right\}. \tag{27}$$

After the cancellation of soft divergence, the remaining collinear divergence will be absorbed into those effective wave functions of the mesons involved in the transition processes. The effective diagrams for the NLO initial B meson are shown as Fig. 3, where the Figs. 3(a) and 3(c) represent diagrams with no gluon momentum flows into the LO hard kernel, while the Figs. 3(b) and 3(d) are the subdiagrams with the gluon momentum flowing into the LO hard kernel. The nonlocal hadronic matrix element for the B meson wave function can be written as:

$$\Phi^{(1)}_B=\frac{1}{6P_1^+}\int\frac{dy^-}{2\pi}e^{-ixp_1^+y^-}\langle 0|\bar{q}(y^-)\Gamma(-ig_s)\times\int_0^{y^-}dzvA(zv)q(0)|h_v\bar{q}(p_1)\rangle\rangle, \tag{28}$$

where $A(zv)$ and h_v represent the gauge field and the effective heavy-quark field respectively, and Γ represents the gamma matrix decided by specific twist parts, which can be chosen as $\gamma_5\gamma^-/2$ here for this case.

Now we consider the NLO corrections to one of the structures of Fig. 1(b). The first part of the LO amplitude $H^{(0)}_{bL}$ as given in Eq. (5) can be written in the form of

$$G^{(0)}_{bL,1}=\frac{-ieg^2C_Fm_B}{2}\times\mathbf{Tr}\left[\frac{\gamma^\alpha M_\rho\not{\epsilon}_{2L}\phi_\rho(x_2)\gamma_\mu\not{k}_2\gamma_\alpha\gamma_5\frac{\not{n}_+}{\sqrt{2}}\phi^+_B(x_1)}{[(p_1-k_2)^2-m_b^2](k_1-k_2)^2}\right]. \tag{29}$$

We retain the γ^α matrix which should be $\gamma_\perp$ in Eq. (29) for convenience. The factorization about the NLO corrections to this part of the LO hard kernel gives the results as follows. The NLO B meson wave function containing infrared divergence is of the form as defined in Eq. (33). Each sub-diagram of this channel contributes without dynamical forbidden. While, the change of the weak vertex's location in the diagrams leads to different color structure for Figs. 4(f) and 4(g) comparing with those in Fig. 2.

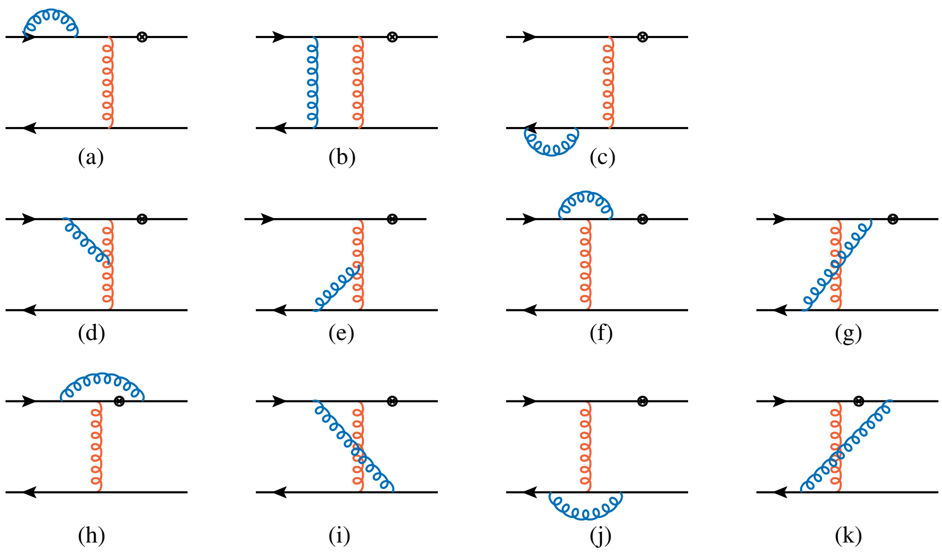

FIG. 4. The quark diagrams for NLO corrections to Fig. 1(b) with additional gluon emitted from the quark(antiquark) lines of the initial B meson.

$$G^{(1)}_{bL1,2d} = eg^4 \mathbf{Tr}\left\{\frac{\gamma^\alpha[M_\rho \not{\epsilon}_{2L}\phi_\rho(x_2)]\gamma_\mu \not{k}_2\gamma^\beta(\not{p}_1 - \not{k}_1 + \not{l} + m_b)\gamma^\gamma(m_B)\gamma_5[\frac{\not{n}_+}{\sqrt{2}}\phi^+_B(x_1)]F_{\alpha\beta\gamma}}{(k_1-k_2)^2(k_1-k_2-l)^2[(p_1-k_2)^2-m_b^2][(p_1-k_1+l)^2-m_b^2]l^2}\right\}$$
$$= -\frac{9}{8}\phi^{(1)}_{B,d} \otimes [G^{(0)}_{bL,1}(x_1,x_2) - G^{(0)}_{bL,1}(x_1,\xi_1,x_2)], \quad (30)$$

$$G^{(1)}_{bL1,2e} = -eg^4 \mathbf{Tr}\left\{\frac{\gamma^\alpha(\not{k}_1 - \not{l})\gamma^\beta[M_\rho \not{\epsilon}_{2L}\phi_\rho(x_2)]\gamma_\mu \not{k}_2\gamma^\gamma(m_B)\gamma_5[\frac{\not{n}_+}{\sqrt{2}}\phi^+_B(x_1)]F_{\alpha\beta\gamma}}{(k_1-k_2)^2(k_1-k_2-l)^2[(p_1-k_2)^2-m_b^2](k_1-l)^2 l^2}\right\}$$
$$= -\frac{9}{8}\phi^{(1)}_{B,e} \otimes [G^{(0)}_{bL,1}(x_1,x_2) - G^{(0)}_{bL,1}(x_1,\xi_1,x_2)], \quad (31)$$

with

$$\phi^{(1)}_{B,d} = \frac{-ig^2 C_F}{4}\frac{\gamma^+\gamma_5(\not{p}_1 - \not{k}_1 + \not{l} + m_b)\gamma^\rho\gamma_5\gamma^-}{[(p_1-k_1+l)^2-m_b^2]l^2}\frac{\nu_\rho}{\nu\cdot l}, \quad (32)$$

$$\phi^{(1)}_{B,e} = \frac{ig^2 C_F}{4}\frac{\gamma_5\gamma^-\gamma^\rho(\not{k}_1 - \not{l})\gamma^+\gamma_5}{(k_1-l)^2 l^2}\frac{\nu_\rho}{\nu\cdot l}. \quad (33)$$

and

$$G^{(1)}_{bL1,2f} = \frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\alpha[M_\rho \not{\epsilon}_{2L}\phi_\rho(x_2)]\gamma_\mu \not{k}_2\gamma^\nu(\not{p}_1 - \not{k}_2 + \not{l} + m_b)\gamma_\alpha(\not{p}_1 - \not{k}_1 + \not{l} + m_b)\gamma_\nu(m_B)\gamma_5[\frac{\not{n}_+}{\sqrt{2}}\phi^+_B(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2)^2[(p_1-k_2+l)^2-m_b^2][(p_1-k_1+l)^2-m_b^2]l^2}\right\}$$
$$= \frac{1}{8}\phi^{(1)}_{B,d} \otimes [G^{(0)}_{bL,1}(x_1,x_2) - G^{(0)}_{bL,1}(\xi_1,x_1,x_2)], \quad (34)$$

$$G^{(1)}_{bL1,2g} = -\frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\nu(\not{k}_1 - \not{l})\gamma^\alpha[M_\rho \not{\epsilon}_{2L}\phi_\rho(x_2)]\gamma_\mu \not{k}_2\gamma_\nu(\not{p}_1 - \not{k}_2 - \not{l} + m_b)\gamma_\alpha(m_B)\gamma_5[\frac{\not{n}_+}{\sqrt{2}}\phi^+_B(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2-l)^2[(p_1-k_2-l)^2-m_b^2](k_1-l)^2 l^2}\right\}$$
$$= -\frac{1}{8}\phi^{(1)}_{B,e} \otimes [G^{(0)}_{bL,1}(x_1,\xi_1,x_2) - G^{(0)}_{bL,1}(\xi_1,x_2)], \quad (35)$$

$$G^{(1)}_{bL1,2h} = \frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^{\alpha}[M_\rho \not\epsilon_{2L}\phi_\rho(x_2)]\gamma^{\nu}(\not p_2 - \not k_2 + \not l)\gamma_\mu(\not k_2 - \not l)\gamma_\alpha(\not p_1 - \not k_1 + \not l + m_b)\gamma_\nu(m_B)\gamma_5[\frac{\not n_+}{\sqrt{2}}\phi_B^+(x_1)]}{[(p_1-k_2+l)^2-m_b^2](k_1-k_2)^2(p_2-k_2+l)^2[(p_1-k_1+l)^2-m_b^2]l^2}\right\}$$
$$= \frac{1}{8}\phi^{(1)}_{B,d} \otimes G^{(0)}_{bL,1}(\xi_1, x_1, x_2), \tag{36}$$

$$G^{(1)}_{bL1,2i} = -\frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^{\alpha}(\not k_2 + \not l)\gamma^{\nu}[M_\rho \not\epsilon_{2L}\phi_\rho(x_2)]\gamma_\mu \not k_2\gamma_\alpha(\not p_1 - \not k_1 + \not l + m_b)\gamma_\nu(m_B)\gamma_5[\frac{\not n_+}{\sqrt{2}}\phi_B^+(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2+l)^2(k_2+l)^2[(p_1-k_1+l)^2-m_b^2]l^2}\right\}$$
$$= -\frac{1}{8}\phi^{(1)}_{B,d} \otimes G^{(0)}_{bL,1}(x_1, \xi_1, x_2), \tag{37}$$

$$G^{(1)}_{bL1,2j} = -\frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^{\nu}(\not k_1 - \not l)\gamma^{\alpha}(\not k_2 - \not l)\gamma_\nu[M_\rho \not\epsilon_{2L}\phi_\rho(x_2)]\gamma_\mu \not k_2\gamma_\alpha(m_B)\gamma_5[\frac{\not n_+}{\sqrt{2}}\phi_B^+(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2)^2(k_1-l)^2(k_2-l)^2l^2}\right\}$$
$$= \frac{1}{8}\phi^{(1)}_{B,e} \otimes G^{(0)}_{bL,1}(x_1, x_2), \tag{38}$$

$$G^{(1)}_{bL1,2k} = \frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^{\nu}(\not k_1 - \not l)\gamma^{\alpha}[M_\rho \not\epsilon_{2L}\phi_\rho(x_2)]\gamma_\nu(\not p_2 - \not k_2 - \not l)\gamma_\mu(\not k_2 + \not l)\gamma_\alpha(m_B)\gamma_5[\frac{\not n_+}{\sqrt{2}}\phi_B^+(x_1)]}{[(p_1-k_2-l)^2-m_b^2](k_1-k_2-l)^2(p_2-k_2-l)^2(k_1-l)^2l^2}\right\}$$
$$= -\frac{1}{8}\phi^{(1)}_{B,e} \otimes G^{(0)}_{bL,1}(\xi_1, x_2). \tag{39}$$

We also sum up all the contributions from the diagrams with additional gluon emitted from up-quark (down-quark) lines of the initial B meson of this channel and the results are of the following form:

$$G^{(1)}_{\mathrm{up},bL1}(x_1,x_2) = -\phi^{(1)}_{B,d}(x_1,\xi_1) \otimes [G^{(0)}_{aL}(x_1,x_2) - G^{(0)}_{aL}(x_1,\xi_1,x_2)],$$
$$G^{(1)}_{\mathrm{down},bL1}(x_1,x_2) = -\phi^{(1)}_{B,e}(x_1,\xi_1) \otimes [G^{(0)}_{aL}(x_1,x_2) - G^{(0)}_{aL}(x_1,\xi_1,x_2)]. \tag{40}$$

The minus sign here is induced by the sign of the term $-\not k_2$ in the gluon propagator at LO. Although the Figs. 4(f) and 4(g) now give the different color structures, the summation of all the sub-diagrams still give the right color factor in full agreement with those from the effective diagrams directly. As we did for G^1_{aL} or other terms, after the cancellation of the soft divergence, the remaining collinear divergence can be factorized into the NLO B meson wave function without any other theoretical problems.

B. The k_T factorization of NLO corrections to $B \to \rho$ transition

We pick up the transversal momentum "k_T" in k_T factorization frame to remove the endpoint singularity emerged in the small x region. The Sudakov factor $e^{-S(t)}$ is introduced [18] in order to suppress the collinear divergence by resummation of the large logarithmic terms which k_T is concerned.

In k_T factorization frame, the k_T works only in the $x \to 0$ region, which indicates that we have the hierarchy $k^2_{iT} \ll k_1 \cdot k_2$. Comparing with the expressions as shown in Eqs. (4) and (6), we can also keep the same forms of the LO hard kernel in the k_T factorization frame. After the inclusion of the NLO corrections, the only difference we should pay attention to is the transverse momentum $\mathbf{l_T}$ in propagators which can be understood as a small momentum shift.

The ratio term $v_\rho/v \cdot l$ in Eq. (16) represents the Feynman rule related to Wilson line. Considering the transversal momentum $\mathbf{l_T}$, we rewrite the hadronic matrix element in Eq. (28) containing Wilson line through Fourier transformation for the gauge field form from $A(zv)$ to $\tilde{A}(l)$:

$$\int_0^\infty dzv \cdot A(zv) \to \int_0^\infty dz \int dl e^{iz(v\cdot l + i\epsilon)} v \cdot \tilde{A}(l) \to i \int dl \frac{v_\rho}{v \cdot l}\tilde{A}^\rho(l), \tag{41}$$

$$\int_\infty^{y^-} dzv \cdot A(zv) \to \int_0^{y^-} dz \int dl e^{[iz(v\cdot l + i\epsilon) - i\mathbf{l}_T\cdot\mathbf{b}]} v \cdot \tilde{A}(l) \to -i \int dl \frac{v_\rho}{v \cdot l} e^{[il^+y^- - i\mathbf{l}_T\cdot\mathbf{b}]}\tilde{A}^\rho(l), \tag{42}$$

The factor $e^{il^+y^-}$ will generate a delta function $\delta(\xi_1 - x_1 + \frac{l^+}{p_1})$ which describes a momentum shift when the l flowing into hard kernel. The second term $e^{-i\mathbf{l}_T\cdot\mathbf{b}}$ in Eq. (42) represents a small transversal part flowing into the hard kernel. The NLO B and ρ meson wave functions can be finally written as a general form:

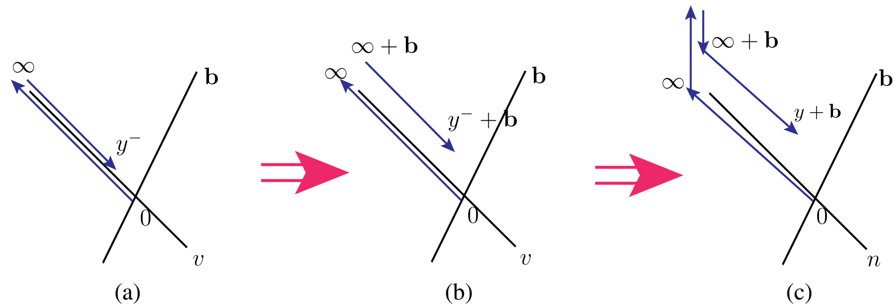

FIG. 5. The graph for the deviation of the integral (Wilson line) by a shift $\mathbf{b}$ in light cone coordinate for the two-parton meson wave function.

$$\Phi_B^{(1)}(x_1,\xi_1;\mathbf{b_1}) = \frac{1}{2N_c P_1^+}\int \frac{dy^-}{2\pi}\frac{d\mathbf{b_1}}{(2\pi)^2} e^{-ixp_1^+y^- + i\mathbf{k_{1T}}\cdot\mathbf{b_1}} \times \cdot\langle 0|\bar{q}(y^-)\Gamma\cdot(-ig_s) \times \int_0^y dzn\cdot A(zn)q(0)|h_\nu B(p_1)\rangle, \quad (43)$$

$$\Phi_\rho^{(1)}(x_2,\xi_2;\mathbf{b_2}) = \frac{1}{2N_c P_2^-}\int \frac{dy^-}{2\pi}\frac{d\mathbf{b_2}}{(2\pi)^2} e^{-ixp_2^+y^- + i\mathbf{k_{2T}}\cdot\mathbf{b_2}} \cdot\langle 0|\bar{q}(y^-)\Gamma'\cdot(-ig_s) \times \int_0^y dzn\cdot A(zn)q(0)|\rho(p_1)\rangle. \quad (44)$$

where Γ is the gamma matrix depending on the twist of the B and ρ meson wave functions.

The whole process of the integration and modification in Eqs. (41) and (42) can be understood by graphs in Fig. 5. Both the Wilson lines in Figs. 5(a) and 5(b) parallel to the light cone coordinate will generate light cone singularity. To avoid this singularity, we can slightly rotate the direction of Wilson line as in Fig. 5(c), then such a singularity can be regularized by $n^2(n^2 \neq 0)$. The choice of number n^2 is scheme dependent, which has been demonstrated to be small [19] and we can simply set $n^2 = 1$ for convenience in calculation. While this rotation and the choose of n^2 will generate a pinched singularity during the integration for Wilson lines. To solve this problem, the transverse-momentum-dependent(TMD) wave function with soft substraction factor with a square root is introduced to the unsubtracted wave function [20], and a more elegant TMD wave function involves two pieces of nonlightlike Wilson links is proposed in Ref. [21]. We pay more attention to the factorization of the collinear divergence in this paper, more discussions on these TMD wave functions will be given in future work.

IV. SUMMARY

In this paper, we investigate the $B \to \rho$ transition process and give the proof of the factorization at the NLO level in the k_T factorization frame. Because of the different structures in the initial B meson and the final ρ meson state, all three amplitudes with six parts of twists' forms for LO diagrams as shown in Figs. 1(a) and 1(b) should be considered. Also, both kinds of the corrections to the initial state and the finial state meson should be considered. The calculations of all these six channels will be very complex, which suggests that we are better to make estimation first for proportion of every channel in LO level before the numerical calculations done at the NLO level.

We have verified that with the right power counting for triple gluon vertex, the amplitudes of the NLO corrections to the LO hard kernel with an additional gluon emitted from the initial B meson or the final ρ meson can be separated into a convolution of the NLO wave function with the LO hard kernel. The LO hard kernel here can be distinguished as two forms, with gluon momentum flowing into the hard kernel, or with no gluon momentum flowing into the hard kernel. We then extend these results to k_T factorization frame and the conclusion can still be stable by a momentum shift. The form of nonlocal two quarks hadron matrices of NLO $B(\rho)$ meson are given in Sec. III. This work will also play an essential role for future numerical calculations of $B \to \rho$ transition form factors.

ACKNOWLEDGMENTS

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APPENDIX: FACTORIZATION OF THE NLO AMPLITUDES

In this Appendix, we present the explicit expressions for NLO corrections to the remaining LO amplitude H_{bL}^0 and H_{bT}^0. The function $G_{bL2}^{(0)}$ corresponds to the second part of H_{bL}^0 as given in Eq. (5). The function $G_{bT1}^{(0)}$, $G_{bT2}^{(0)}$, and $G_{bT3}^{(0)}$ come from the three components of H_{bT}^0 as defined in Eq. (6): two for ϕ_B^+ and one for ϕ_B^- component. For the sake of clearness, we write these four functions in their original nonreduced form:

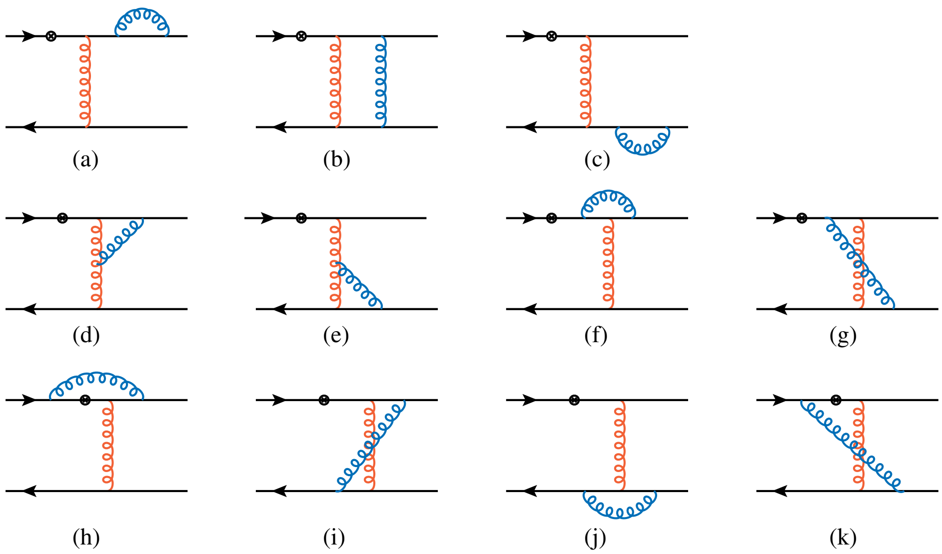

FIG. 6. The Feynman diagrams for NLO corrections to Fig. 1(a) with the additional gluon emitted from the quark (antiquark) lines of the finial ρ meson.

$$G^{(0)}_{bL2} = -\frac{ieg^2C_F}{2}\mathbf{Tr}\left\{\frac{\gamma^\alpha[M_\rho \not\epsilon_{2L}\phi_\rho(x_2)]\gamma_\mu(m_b)\gamma_\alpha(\not p_1)\gamma_5[\frac{\not n_-}{\sqrt{2}}\phi_B^-(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2)^2}\right\}, \tag{A1}$$

$$G^{(0)}_{bT1} = -\frac{ieg^2C_F}{2}\mathbf{Tr}\left\{\frac{\gamma^\alpha[\not\epsilon_{2T}\not p_2\phi_\rho^T(x_2)]\gamma_\mu(\not p_1)\gamma_\alpha(\not p_1)\gamma_5[\frac{\not n_+}{\sqrt{2}}\phi_B^+(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2)^2}\right\}, \tag{A2}$$

$$G^{(0)}_{bT2} = -\frac{ieg^2C_F}{2}\mathbf{Tr}\left\{\frac{\gamma^\alpha[\not\epsilon_{2T}\not p_2\phi_\rho^T(x_2)]\gamma_\mu(\not p_1)\gamma_\alpha(\not p_1)\gamma_5[\frac{\not n_-}{\sqrt{2}}\phi_B^-(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2)^2}\right\}, \tag{A3}$$

$$G^{(0)}_{bT3} = -\frac{ieg^2C_F}{2}\mathbf{Tr}\left\{\frac{\gamma^\alpha[\not\epsilon_{2T}\not p_2\phi_\rho^T(x_2)]\gamma_\mu(m_b)\gamma_\alpha(m_B)\gamma_5[\frac{\not n_+}{\sqrt{2}}\phi_B^+(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2)^2}\right\}. \tag{A4}$$

The NLO corrections with additional gluon emitted from the final ρ meson are also supplied. In Figs. 6 and 7, we show the Feynman diagrams for the NLO corrections to Figs. 1(a) and 1(b) respectively, where the second blue gluon are emitted from the quark and antiquark lines of the finial ρ meson. Notice that the $G^{(0)}_{aL}$ and $G^{(0)}_{bL1}$ with the gluon emitted from the final ρ meson have the similar structure with the ones for $\rho \to \rho$ decays as defined in Ref. [6], so we here do not present them explicitly.

1. The NLO amplitudes for $G^{(0)}_{bL2}$

The amplitudes for the NLO contributions from the irreducible diagrams to $G^{(0)}_{bL2}$ are the following:

$$\begin{aligned} G^{(1)}_{bL2,2d} &= -eg^4\mathbf{Tr}\left\{\frac{\gamma^\alpha[M_\rho \not\epsilon_{2L}\phi_\rho(x_2)]\gamma_\mu m_b\gamma^\beta(\not p_1-\not k_1+\not l+m_b)\gamma^\gamma(\not p_1)\gamma_5[\frac{\not n_-}{\sqrt{2}}\phi_B^-(x_1)]F_{\alpha\beta\gamma}}{(k_1-k_2)^2(k_1-k_2-l)^2[(p_1-k_2)^2-m_b^2][(p_1-k_1+l)^2-m_b^2]l^2}\right\} \\ &= \frac{9}{8}\phi^{(1)}_{B,d}\otimes[G^{(0)}_{bL,2}(x_1,x_2)-G^{(0)}_{bL,2}(x_1,\xi_1,x_2)], \end{aligned} \tag{A5}$$

$$G^{(1)}_{bL2,2e} = -eg^4\times\mathbf{Tr}\left\{\frac{\gamma^\alpha(\not k_1-\not l)\gamma^\beta[M_\rho \not\epsilon_{2L}\phi_\rho(x_2)]\gamma_\mu m_b\gamma^\gamma(\not p_1)\gamma_5[\frac{\not n_-}{\sqrt{2}}\phi_B^-(x_1)]F_{\alpha\beta\gamma}}{(k_1-k_2)^2(k_1-k_2-l)^2[(p_1-k_2)^2-m_b^2](k_1-l)^2l^2}\right\}\sim 0. \tag{A6}$$

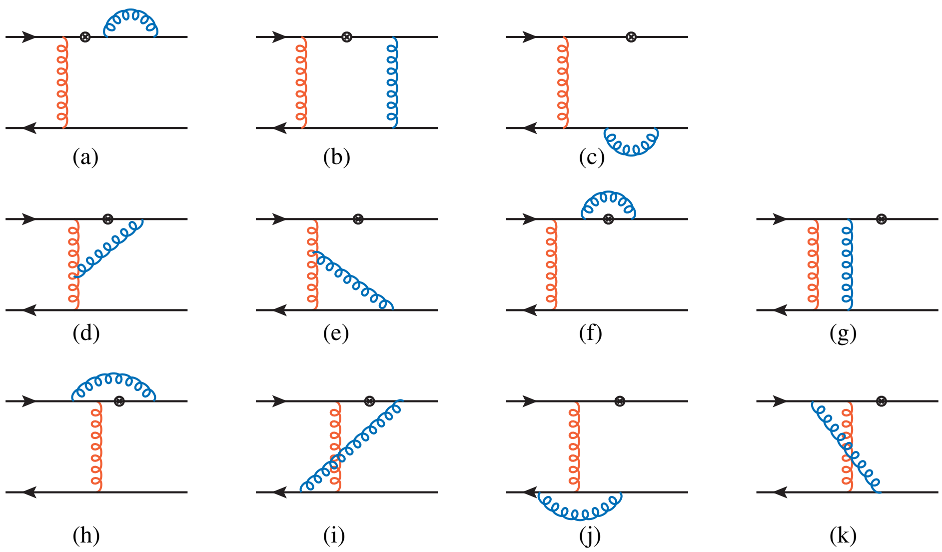

FIG. 7. The Feynman diagrams for NLO corrections to Fig. 1(b) with the additional gluon emitted from the quark (antiquark) lines of the finial ρ meson.

The contribution $G^{(1)}_{bL2,2e}$ is suppressed by dynamics and we simply set it 0 here. For other contributions we have:

$$G^{(1)}_{bL2,2f} = \frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\alpha[M_\rho \not{\epsilon}_{2L}\phi_\rho(x_2)]\gamma_\mu m_b \gamma^\nu(\not{p}_1-\not{k}_2+\not{l}+m_b)\gamma_\alpha(\not{p}_1-\not{k}_1+\not{l}+m_b)\gamma_\nu(\not{p}_1)\gamma_5[\frac{\not{n}_-}{\sqrt{2}}\phi_B^-(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2)^2[(p_1-k_2+l)^2-m_b^2][(p_1-k_1+l)^2-m_b^2]l^2}\right\} \sim 0, \quad \text{(A7)}$$

$$G^{(1)}_{bL2,2g} = -\frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\nu(\not{k}_1-\not{l})\gamma^\alpha[M_\rho \not{\epsilon}_{2L}\phi_\rho(x_2)]\gamma_\mu m_b \gamma_\nu(\not{p}_1-\not{k}_2+\not{l}+m_b)\gamma_\alpha(\not{p}_1)\gamma_5[\frac{\not{n}_-}{\sqrt{2}}\phi_B^-(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2-l)^2[(p_1-k_2+l)^2-m_b^2](k_1-l)^2l^2}\right\} \sim 0, \quad \text{(A8)}$$

$$G^{(1)}_{bL2,2h} = \frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\alpha[M_\rho \not{\epsilon}_{2L}\phi_\rho(x_2)]\gamma^\nu(\not{p}_2-\not{k}_2+\not{l})\gamma_\mu(m_b+\not{l})\gamma_\alpha(\not{p}_1-\not{k}_1+\not{l}+m_b)\gamma_\nu(\not{p}_1)\gamma_5[\frac{\not{n}_-}{\sqrt{2}}\phi_B^-(x_1)]}{[(p_1-k_2+l)^2-m_b^2](k_1-k_2)^2(p_2-k_2+l)^2[(p_1-k_1+l)^2-m_b^2]l^2}\right\}$$
$$= -\frac{1}{8}\phi^{(1)}_{B,d} \otimes G^{(0)}_{bL,1}(x_1,x_2), \quad \text{(A9)}$$

$$G^{(1)}_{bL2,2i} = -\frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\alpha(\not{k}_2+\not{l})\gamma^\nu[M_\rho \not{\epsilon}_{2L}\phi_\rho(x_2)]\gamma_\mu m_b \gamma_\alpha(\not{p}_1-\not{k}_1+\not{l}+m_b)\gamma_\nu(\not{p}_1)\gamma_5[\frac{\not{n}_-}{\sqrt{2}}\phi_B^-(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2+l)^2(k_2+l)^2[(p_1-k_1+l)^2-m_b^2]l^2}\right\}$$
$$= \frac{1}{8}\phi^{(1)}_{B,d} \otimes G^{(0)}_{bL,1}(x_1,\xi_1,x_2), \quad \text{(A10)}$$

$$G^{(1)}_{bL2,2j} = \frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\nu(\not{k}_1-\not{l})\gamma^\alpha(\not{k}_2-\not{l})\gamma_\nu[M_\rho \not{\epsilon}_{2L}\phi_\rho(x_2)]\gamma_\mu m_b \gamma_\alpha(\not{p}_1)\gamma_5[\frac{\not{n}_-}{\sqrt{2}}\phi_B^-(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2)^2(k_1-l)^2(k_2-l)^2l^2}\right\} \sim 0, \quad \text{(A11)}$$

$$G^{(1)}_{bL2,2k} = -\frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\nu(\not{k}_1-\not{l})\gamma^\alpha[M_\rho \not{\epsilon}_{2L}\phi_\rho(x_2)]\gamma_\nu(\not{p}_2-\not{k}_2-\not{l})\gamma_\mu(m_b-\not{l})\gamma_\alpha(\not{p}_1)\gamma_5[\frac{\not{n}_-}{\sqrt{2}}\phi_B^-(x_1)]}{[(p_1-k_2-l)^2-m_b^2](k_1-k_2-l)^2(p_2-k_2-l)^2(k_1-l)^2l^2}\right\} \sim 0, \quad \text{(A12)}$$

with

$$\phi_{B,d}^{(1)} = \frac{-ig^2 C_F}{8} \frac{\gamma^- \gamma_5 \gamma^+ (\not{p}_1 - \not{k}_1 + \not{l} + m_b)\gamma^\rho \gamma^- \gamma_5 \gamma^+}{[(p_1 - k_1 + l)^2 - m_b^2] l^2} \frac{\nu_\rho}{\nu \cdot l}. \tag{A13}$$

The summation over above contributions gives the total result:

$$G^{(1)}_{\mathrm{up},bL2}(x_1, x_2) = \phi^{(1)}_{B,d}(x_1, \xi_1) \otimes [G^{(0)}_{aL}(x_1, x_2) - G^{(0)}_{aL}(x_1, \xi_1, x_2)]. \tag{A14}$$

2. The NLO amplitudes for $G^{(0)}_{bT1}$

The amplitudes for the NLO contributions from the irreducible diagrams to $G^{(0)}_{bT1}$ are the following:

$$\begin{aligned} G^{(1)}_{bT1,2d} &= -eg^4 \mathbf{Tr}\left\{ \frac{\gamma^\alpha [\not{\epsilon}_{2T} \not{p}_2 \phi^T_\rho(x_2)] \gamma_\mu \not{p}_1 \gamma^\beta (\not{p}_1 - \not{k}_1 + \not{l} + m_b)\gamma^\gamma (\not{p}_1)\gamma_5 [\frac{\not{n}_+}{\sqrt{2}} \phi^+_B(x_1)] F_{\alpha\beta\gamma}}{(k_1 - k_2)^2 (k_1 - k_2 - l)^2 [(p_1 - k_2)^2 - m_b^2][(p_1 - k_1 + l)^2 - m_b^2] l^2} \right\} \\ &= \frac{9}{8} \phi^{(1)}_{B,d} \otimes [G^{(0)}_{bT,1}(x_1, x_2) - G^{(0)}_{bT,1}(x_1, \xi_1, x_2)], \end{aligned} \tag{A15}$$

$$\begin{aligned} G^{(1)}_{bT1,2e} &= eg^4 \mathbf{Tr}\left\{ \frac{\gamma^\alpha (\not{k}_1 - \not{l})\gamma^\beta [\not{\epsilon}_{2T} \not{p}_2 \phi^T_\rho(x_2)] \gamma_\mu \not{p}_1 \gamma^\gamma (\not{p}_1)\gamma_5 [\frac{\not{n}_+}{\sqrt{2}} \phi^+_B(x_1)] F_{\alpha\beta\gamma}}{(k_1 - k_2)^2 (k_1 - k_2 - l)^2 [(p_1 - k_2)^2 - m_b^2](k_1 - l)^2 l^2} \right\} \\ &= \frac{9}{8} \phi^{(1)}_{B,e} \otimes [G^{(0)}_{bT,1}(x_1, x_2) - G^{(0)}_{bT,1}(x_1, \xi_1, x_2)], \end{aligned} \tag{A16}$$

$$\begin{aligned} G^{(1)}_{bT1,2f} &= \frac{eg^4}{9} \mathbf{Tr}\left\{ \frac{\gamma^\alpha [\not{\epsilon}_{2T} \not{p}_2 \phi^T_\rho(x_2)] \gamma_\mu \not{p}_1 \gamma^\nu (\not{p}_1 - \not{k}_2 + \not{l} + m_b)\gamma_\alpha (\not{p}_1 - \not{k}_1 + \not{l} + m_b)\gamma_\nu (\not{p}_1)\gamma_5 [\frac{\not{n}_+}{\sqrt{2}} \phi^+_B(x_1)]}{[(p_1 - k_2)^2 - m_b^2](k_1 - k_2)^2 [(p_1 - k_2 + l)^2 - m_b^2][(p_1 - k_1 + l)^2 - m_b^2] l^2} \right\} \\ &= -\frac{1}{8} \phi^{(1)}_{B,d} \otimes [G^{(0)}_{bT,1}(x_1, x_2) - G^{(0)}_{bT,1}(\xi_1, x_1, x_2)], \end{aligned} \tag{A17}$$

$$G^{(1)}_{bT1,2g} = -\frac{eg^4}{9} \mathbf{Tr}\left\{ \frac{\gamma^\nu (\not{k}_1 - \not{l})\gamma^\alpha [\not{\epsilon}_{2T} \not{p}_2 \phi^T_\rho(x_2)] \gamma_\mu \not{p}_1 \gamma_\nu (\not{p}_1 - \not{k}_2 - \not{l} + m_b)\gamma_\alpha (\not{p}_1)\gamma_5 [\frac{\not{n}_+}{\sqrt{2}} \phi^+_B(x_1)]}{[(p_1 - k_2)^2 - m_b^2](k_1 - k_2 - l)^2 [(p_1 - k_2 - l)^2 - m_b^2](k_1 - l)^2 l^2} \right\} \sim 0, \tag{A18}$$

$$G^{(1)}_{bT1,2h} = \frac{eg^4}{9} \mathbf{Tr}\left\{ \frac{\gamma^\alpha [\not{\epsilon}_{2T} \not{p}_2 \phi^T_\rho(x_2)] \gamma^\nu (\not{p}_2 - \not{k}_2 + \not{l})\gamma_\mu (\not{p}_1 + \not{l})\gamma_\alpha (\not{p}_1 - \not{k}_1 + \not{l} + m_b)\gamma_\nu (\not{p}_1)\gamma_5 [\frac{\not{n}_+}{\sqrt{2}} \phi^+_B(x_1)]}{[(p_1 - k_2 + l)^2 - m_b^2](k_1 - k_2)^2 (p_2 - k_2 + l)^2 [(p_1 - k_1 + l)^2 - m_b^2] l^2} \right\} \sim 0, \tag{A19}$$

$$G^{(1)}_{bT1,2i} = -\frac{eg^4}{9} \mathbf{Tr}\left\{ \frac{\gamma^\alpha (\not{k}_2 + \not{l})\gamma^\nu [\not{\epsilon}_{2T} \not{p}_2 \phi^T_\rho(x_2)] \gamma_\mu \not{p}_1 \gamma_\alpha (\not{p}_1 - \not{k}_1 + \not{l} + m_b)\gamma_\nu (\not{p}_1)\gamma_5 [\frac{\not{n}_+}{\sqrt{2}} \phi^+_B(x_1)]}{[(p_1 - k_2)^2 - m_b^2](k_1 - k_2 + l)^2 (k_2 + l)^2 [(p_1 - k_1 + l)^2 - m_b^2] l^2} \right\} \sim 0, \tag{A20}$$

$$\begin{aligned} G^{(1)}_{bT1,2j} &= \frac{eg^4}{9} \mathbf{Tr}\left\{ \frac{\gamma^\nu (\not{k}_1 - \not{l})\gamma^\alpha (\not{k}_2 - \not{l})\gamma_\nu [\not{\epsilon}_{2T} \not{p}_2 \phi^T_\rho(x_2)] \gamma_\mu \not{p}_1 \gamma_\alpha (\not{p}_1)\gamma_5 [\frac{\not{n}_+}{\sqrt{2}} \phi^+_B(x_1)]}{[(p_1 - k_2)^2 - m_b^2](k_1 - k_2)^2 (k_1 - l)^2 (k_2 - l)^2 l^2} \right\} \\ &= \frac{1}{8} \phi^{(1)}_{B,e} \otimes G^{(0)}_{bT,1}(x_1, x_2), \end{aligned} \tag{A21}$$

$$G^{(1)}_{bT1,2k} = \frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\nu(\not{k}_1-\not{l})\gamma^\alpha[\not{\epsilon}_{2T}\not{p}_2\phi^T_\rho(x_2)]\gamma_\nu(\not{p}_2-\not{k}_2-\not{l})\gamma_\mu(\not{p}_1-\not{l})\gamma_\alpha(\not{p}_1)\gamma_5[\frac{\not{n}_+}{\sqrt{2}}\phi^+_B(x_1)]}{[(p_1-k_2-l)^2-m_b^2](k_1-k_2-l)^2(p_2-k_2-l)^2(k_1-l)^2l^2}\right\}$$
$$= -\frac{1}{8}\phi^{(1)}_{B,e}\otimes G^{(0)}_{bT,1}(\xi_1,x_2), \tag{A22}$$

with the wave functions $\phi^{(1)}_{B,d}$ and $\phi^{(1)}_{B,e}$ are of the following form:

$$\phi^{(1)}_{B,d} = \frac{-ig^2C_F}{8}\frac{\gamma^+\gamma_5\gamma^-(\not{p}_1-\not{k}_1+\not{l}+m_b)\gamma^\rho\gamma^+\gamma_5\gamma^-}{[(p_1-k_1+l)^2-m_b^2]l^2}\frac{\nu_\rho}{\nu\cdot l}, \tag{A23}$$

$$\phi^{(1)}_{B,e} = \frac{ig^2C_F}{8}\frac{\gamma^+\gamma_5\gamma^-\gamma^\rho(\not{k}_1-\not{l})\gamma^+\gamma_5\gamma^-}{(k_1-l)^2l^2}\frac{\nu_\rho}{\nu\cdot l}. \tag{A24}$$

The direct summation of above contributions gives the following result:

$$G^{(1)}_{\mathrm{up},bT3}(x_1,x_2) = \phi^{(1)}_{B,d}(x_1,\xi_1)\otimes[G^{(0)}_{bT}(x_1,x_2)-G^{(0)}_{bT}(x_1,\xi_1,x_2)]$$
$$+\frac{1}{8}\phi^{(1)}_{B,d}(x_1,\xi_1)\otimes[G^{(0)}_{bT}(\xi_1,x_1,x_2)-G^{(0)}_{bT}(x_1,\xi_1,x_2)], \tag{A25}$$

$$G^{(1)}_{\mathrm{down},aL}(x_1,x_2) = \phi^{(1)}_{B,e}(x_1,\xi_1)\otimes[G^{(0)}_{aL}(x_1,x_2)-G^{(0)}_{aL}(x_1,\xi_1,x_2)], \tag{A26}$$

where the $1/8$ term seems abnormal here. Fortunately, as we demonstrated in Eqs. (25)–(27), the difference between $G^{(0)}_{bT}(\xi_1,x_1,x_2)$ and $G^{(0)}_{bT}(x_1,\xi_1,x_2)$ is the form of the propagator in denominator. The flow l here can be recognized as a momentum shift for $x_1' = x_1 - l^+/p_1$ and with the symmetry by choice of P_2 and k_2, the infrared structure between the two terms will be canceled each other.

3. The NLO amplitudes for $G^{(0)}_{bT2}$

The amplitudes for the NLO contributions from the irreducible diagrams to $G^{(0)}_{bT2}$ are the following:

$$G^{(1)}_{bT2,2d} = -eg^4\mathbf{Tr}\left\{\frac{\gamma^\alpha[\not{\epsilon}_{2T}\not{p}_2\phi^T_\rho(x_2)]\gamma_\mu\not{p}_1\gamma^\beta(\not{p}_1-\not{k}_1+\not{l}+m_b)\gamma^\gamma(\not{p}_1)\gamma_5[\frac{\not{n}_-}{\sqrt{2}}\phi^-_B(x_1)]F_{\alpha\beta\gamma}}{(k_1-k_2)^2(k_1-k_2-l)^2[(p_1-k_2)^2-m_b^2][(p_1-k_1+l)^2-m_b^2]l^2}\right\}$$
$$= \frac{9}{8}\phi^{(1)}_{B,d}\otimes[G^{(0)}_{bT,2}(x_1,x_2)-G^{(0)}_{bT,2}(x_1,\xi_1,x_2)], \tag{A27}$$

$$G^{(1)}_{bT2,2e} = eg^4\mathbf{Tr}\left\{\frac{\gamma^\alpha(\not{k}_1-\not{l})\gamma^\beta[\not{\epsilon}_{2T}\not{p}_2\phi^T_\rho(x_2)]\gamma_\mu\not{p}_1\gamma^\gamma(\not{p}_1)\gamma_5[\frac{\not{n}_-}{\sqrt{2}}\phi^-_B(x_1)]F_{\alpha\beta\gamma}}{(k_1-k_2)^2(k_1-k_2-l)^2[(p_1-k_2)^2-m_b^2](k_1-l)^2l^2}\right\}\sim 0, \tag{A28}$$

$$G^{(1)}_{bT2,2f} = \frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\alpha[\not{\epsilon}_{2T}\not{p}_2\phi^T_\rho(x_2)]\gamma_\mu\not{p}_1\gamma^\nu(\not{p}_1-\not{k}_2+\not{l}+m_b)\gamma_\alpha(\not{p}_1-\not{k}_1+\not{l}+m_b)\gamma_\nu(\not{p}_1)\gamma_5[\frac{\not{n}_-}{\sqrt{2}}\phi^-_B(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2)^2[(p_1-k_2+l)^2-m_b^2][(p_1-k_1+l)^2-m_b^2]l^2}\right\}$$
$$= -\frac{1}{8}\phi^{(1)}_{B,d}\otimes[G^{(0)}_{bT,2}(x_1,x_2)-G^{(0)}_{bT,2}(\xi_1,x_1,x_2)], \tag{A29}$$

$$G^{(1)}_{bT2,2g} = -\frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\nu(\not{k}_1-\not{l})\gamma^\alpha[\not{\epsilon}_{2T}\not{p}_2\phi^T_\rho(x_2)]\gamma_\mu\not{p}_1\gamma_\nu(\not{p}_1-\not{k}_2-\not{l}+m_b)\gamma_\alpha(\not{p}_1)\gamma_5[\frac{\not{n}_-}{\sqrt{2}}\phi^-_B(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2-l)^2[(p_1-k_2-l)^2-m_b^2](k_1-l)^2l^2}\right\}\sim 0, \tag{A30}$$

$$G^{(1)}_{bT2,2h} = \frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\alpha[\not\epsilon_{2T}\not p_2\phi^T_\rho(x_2)]\gamma^\nu(\not p_2-\not k_2+\not l)\gamma_\mu(\not p_1+\not l)\gamma_\alpha(\not p_1-\not k_1+\not l+m_b)\gamma_\nu(\not p_1)\gamma_5[\frac{\not n_-}{\sqrt{2}}\phi^-_B(x_1)]}{[(p_1-k_2+l)^2-m_b^2](k_1-k_2)^2(p_2-k_2+l)^2[(p_1-k_1+l)^2-m_b^2]l^2}\right\}$$
$$= -\frac{1}{8}\phi^{(1)}_{B,d}\otimes G^{(0)}_{bT,2}(\xi_1,x_1,x_2), \tag{A31}$$

$$G^{(1)}_{bT2,2i} = -\frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\alpha(\not k_2+\not l)\gamma^\nu[\not\epsilon_{2T}\not p_2\phi^T_\rho(x_2)]\gamma_\mu\not p_1\gamma_\alpha(\not p_1-\not k_1+\not l+m_b)\gamma_\nu(\not p_1)\gamma_5[\frac{\not n_-}{\sqrt{2}}\phi^-_B(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2+l)^2(k_2+l)^2[(p_1-k_1+l)^2-m_b^2]l^2}\right\}$$
$$= \frac{1}{8}\phi^{(1)}_{B,d}\otimes G^{(0)}_{bT,2}(x_1,\xi_1,x_2), \tag{A32}$$

$$G^{(1)}_{bT2,2j} = -\frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\nu(\not k_1-\not l)\gamma^\alpha(\not k_2-\not l)\gamma_\nu[\not\epsilon_{2T}\not p_2\phi^T_\rho(x_2)]\gamma_\mu\not p_1\gamma_\alpha(\not p_1)\gamma_5[\frac{\not n_-}{\sqrt{2}}\phi^-_B(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2)^2(k_1-l)^2(k_2-l)^2l^2}\right\}\sim 0, \tag{A33}$$

$$G^{(1)}_{bT2,2k} = \frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\nu(\not k_1-\not l)\gamma^\alpha[\not\epsilon_{2T}\not p_2\phi^T_\rho(x_2)]\gamma_\nu(\not p_2-\not k_2-\not l)\gamma_\mu(\not p_1-\not l)\gamma_\alpha(\not p_1)\gamma_5[\frac{\not n_-}{\sqrt{2}}\phi^-_B(x_1)]}{[(p_1-k_2-l)^2-m_b^2](k_1-k_2-l)^2(p_2-k_2-l)^2(k_1-l)^2l^2}\right\}\sim 0, \tag{A34}$$

with the wave function $\phi^{(1)}_{B,d}$ as the form of

$$\phi^{(1)}_{B,d} = \frac{-ig^2C_F}{8}\frac{\gamma^-\gamma_5\gamma^+(\not p_1-\not k_1+\not l+m_b)\gamma^\rho\gamma^-\gamma_5\gamma^+}{[(p_1-k_1+l)^2-m_b^2]l^2}\frac{\nu_\rho}{\nu\cdot l}. \tag{A35}$$

The summation of the contributions from these irreducible subdiagrams gives the following result:

$$G^{(1)}_{\mathrm{up},bT2}(x_1,x_2) = \phi^{(1)}_{B,d}(x_1,\xi_1)\otimes[G^{(0)}_{bT}(x_1,x_2)-G^{(0)}_{bT}(x_1,\xi_1,x_2)]. \tag{A36}$$

4. The NLO amplitudes for $G^{(0)}_{bT3}$

The amplitudes for the NLO contributions from the irreducible diagrams to $G^{(0)}_{bT3}$ are the following:

$$G^{(1)}_{bT3,2d} = -eg^4\mathbf{Tr}\left\{\frac{\gamma^\alpha[\not\epsilon_{2T}\not p_2\phi^T_\rho(x_2)]\gamma_\mu m_b\gamma^\beta(\not p_1-\not k_1+\not l+m_b)\gamma^\gamma(m_B)\gamma_5[\frac{\not n_+}{\sqrt{2}}\phi^+_B(x_1)]F_{\alpha\beta\gamma}}{(k_1-k_2)^2(k_1-k_2-l)^2[(p_1-k_2)^2-m_b^2][(p_1-k_1+l)^2-m_b^2]l^2}\right\}$$
$$= \frac{9}{8}\phi^{(1)}_{B,d}\otimes[G^{(0)}_{bT,3}(x_1,x_2)-G^{(0)}_{bT,3}(x_1,\xi_1,x_2)], \tag{A37}$$

$$G^{(1)}_{bT3,2e} = eg^4\mathbf{Tr}\left\{\frac{\gamma^\alpha(\not k_1-\not l)\gamma^\beta[\not\epsilon_{2T}\not p_2\phi^T_\rho(x_2)]\gamma_\mu m_b\gamma^\gamma(m_B)\gamma_5[\frac{\not n_+}{\sqrt{2}}\phi^+_B(x_1)]F_{\alpha\beta\gamma}}{(k_1-k_2)^2(k_1-k_2-l)^2[(p_1-k_2)^2-m_b^2](k_1-l)^2l^2}\right\}$$
$$= \frac{9}{8}\phi^{(1)}_{B,e}\otimes[G^{(0)}_{bT,3}(x_1,x_2)-G^{(0)}_{bT,3}(x_1,\xi_1,x_2)], \tag{A38}$$

$$G^{(1)}_{bT3,2f} = \frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\alpha[\not\epsilon_{2T}\not p_2\phi^T_\rho(x_2)]\gamma_\mu m_b\gamma^\nu(\not p_1-\not k_2+\not l+m_b)\gamma_\alpha(\not p_1-\not k_1+\not l+m_b)\gamma_\nu(m_B)\gamma_5[\frac{\not n_+}{\sqrt{2}}\phi^+_B(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2)^2[(p_1-k_2+l)^2-m_b^2][(p_1-k_1+l)^2-m_b^2]l^2}\right\}$$
$$= -\frac{1}{8}\phi^{(1)}_{B,d}\otimes[G^{(0)}_{bT,3}(x_1,x_2)-G^{(0)}_{bT,3}(\xi_1,x_1,x_2)], \tag{A39}$$

$$G^{(1)}_{bT3,2g} = -\frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\nu(\not{k}_1-\not{l})\gamma^\alpha[\not{\epsilon}_{2T}\not{p}_2\phi^T_\rho(x_2)]\gamma_\mu m_b\gamma_\nu(\not{p}_1-\not{k}_2-\not{l}+m_b)\gamma_\alpha(m_B)\gamma_5[\frac{\not{n}_+}{\sqrt{2}}\phi^+_B(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2-l)^2[(p_1-k_2-l)^2-m_b^2](k_1-l)^2l^2}\right\}$$
$$= -\frac{1}{8}\phi^{(1)}_{B,e}\otimes[G^{(0)}_{bT,3}(x_1,\xi_1,x_2)-G^{(0)}_{bT,3}(\xi_1,x_2)], \tag{A40}$$

$$G^{(1)}_{bT3,2h} = \frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\alpha[\not{\epsilon}_{2T}\not{p}_2\phi^T_\rho(x_2)]\gamma^\nu(\not{p}_2-\not{k}_2+\not{l})\gamma_\mu(m_b+\not{l})\gamma_\alpha(\not{p}_1-\not{k}_1+\not{l}+m_b)\gamma_\nu(m_B)\gamma_5[\frac{\not{n}_+}{\sqrt{2}}\phi^+_B(x_1)]}{[(p_1-k_2+l)^2-m_b^2](k_1-k_2)^2(p_2-k_2+l)^2[(p_1-k_1+l)^2-m_b^2]l^2}\right\}$$
$$= -\frac{1}{8}\phi^{(1)}_{B,d}\otimes G^{(0)}_{bT,3}(\xi_1,x_1,x_2), \tag{A41}$$

$$G^{(1)}_{bT3,2i} = -\frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\alpha(\not{k}_2+\not{l})\gamma^\nu[\not{\epsilon}_{2T}\not{p}_2\phi^T_\rho(x_2)]\gamma_\mu m_b\gamma_\alpha(\not{p}_1-\not{k}_1+\not{l}+m_b)\gamma_\nu(m_B)\gamma_5[\frac{\not{n}_+}{\sqrt{2}}\phi^+_B(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2+l)^2(k_2+l)^2[(p_1-k_1+l)^2-m_b^2]l^2}\right\}$$
$$= \frac{1}{8}\phi^{(1)}_{B,d}\otimes G^{(0)}_{bT,3}(x_1,\xi_1,x_2), \tag{A42}$$

$$G^{(1)}_{bT3,2j} = \frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\nu(\not{k}_1-\not{l})\gamma^\alpha(\not{k}_2-\not{l})\gamma_\nu[\not{\epsilon}_{2T}\not{p}_2\phi^T_\rho(x_2)]\gamma_\mu m_b\gamma_\alpha(m_B)\gamma_5[\frac{\not{n}_+}{\sqrt{2}}\phi^+_B(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2)^2(k_1-l)^2(k_2-l)^2l^2}\right\}$$
$$= \frac{1}{8}\phi^{(1)}_{B,e}\otimes G^{(0)}_{bT,3}(x_1,x_2), \tag{A43}$$

$$G^{(1)}_{bT3,2k} = -\frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\nu(\not{k}_1-\not{l})\gamma^\alpha[\not{\epsilon}_{2T}\not{p}_2\phi^T_\rho(x_2)]\gamma_\nu(\not{p}_2-\not{k}_2-\not{l})\gamma_\mu(m_b-\not{l})\gamma_\alpha(m_B)\gamma_5[\frac{\not{n}_+}{\sqrt{2}}\phi^+_B(x_1)]}{[(p_1-k_2-l)^2-m_b^2](k_1-k_2-l)^2(p_2-k_2-l)^2(k_1-l)^2l^2}\right\}$$
$$= -\frac{1}{8}\phi^{(1)}_{B,e}\otimes G^{(0)}_{bT,3}(\xi_1,x_2), \tag{A44}$$

with the wave functions $\phi^{(1)}_{B,d}$ and $\phi^{(1)}_{B,e}$ in the form of

$$\phi^{(1)}_{B,d} = \frac{-ig^2C_F}{4}\frac{\gamma^+\gamma_5(\not{p}_1-\not{k}_1+\not{l}+m_b)\gamma^\rho\gamma_5\gamma^-}{[(p_1-k_1+l)^2-m_b^2]l^2}\frac{\nu_\rho}{\nu\cdot l},$$
$$\phi^{(1)}_{B,e} = \frac{ig^2C_F}{4}\frac{\gamma_5\gamma^-\gamma^\rho(\not{k}_1-\not{l})\gamma^+\gamma_5}{(k_1-l)^2l^2}\frac{\nu_\rho}{\nu\cdot l}. \tag{A45}$$

The summation of above terms from the irreducible subdiagrams gives the following result:

$$G^{(1)}_{\mathrm{up},bT3}(x_1,x_2) = \phi^{(1)}_{B,d}(x_1,\xi_1)\otimes[G^{(0)}_{bT}(x_1,x_2)-G^{(0)}_{bT}(x_1,\xi_1,x_2)], \tag{A46}$$

$$G^{(1)}_{\mathrm{down},aL}(x_1,x_2) = \phi^{(1)}_{B,e}(x_1,\xi_1)\otimes[G^{(0)}_{aL}(x_1,x_2)-G^{(0)}_{aL}(x_1,\xi_1,x_2)]. \tag{A47}$$

5. The NLO amplitudes for $G^{(0')}_{bL2}$

The amplitudes for the NLO contributions from the irreducible diagrams to $G^{(0')}_{bL2}$ with the additional gluon emitted from the final ρ meson are the following:

$$G^{(1')}_{bL2,2d} = -eg^4\mathbf{Tr}\left\{\frac{\gamma^\alpha[M_\rho\not{\epsilon}_{2L}\phi_\rho(x_2)]\gamma^\beta(\not{p}_2-\not{k}_2-\not{l})\gamma_\mu m_b\gamma^\gamma(\not{p}_1)\gamma_5[\frac{\not{n}_-}{\sqrt{2}}\phi^-_B(x_1)]F_{\alpha\beta\gamma}}{(k_1-k_2)^2(k_1-k_2-l)^2[(p_1-k_2-l)^2-m_b^2](p_2-k_2-l)^2l^2}\right\}$$
$$= \frac{9}{8}\phi^{(1)}_{\rho,d}\otimes[G^{(0)}_{bL,2}(x_1,\xi_2,x_2)-G^{(0)}_{bL,2}(x_1,\xi_2)], \tag{A48}$$

$$G^{(1')}_{bL2,2e} = eg^4 \mathbf{Tr}\left\{\frac{\gamma^\alpha(\not{k}_2+\not{l})\gamma^\beta[M_\rho \not{\epsilon}_{2L}\phi_\rho(x_2)]\gamma_\mu m_b \gamma^\gamma(\not{p}_1)\gamma_5[\frac{\not{n}_-}{\sqrt{2}}\phi_B^-(x_1)]F_{\alpha\beta\gamma}}{(k_1-k_2)^2(k_1-k_2-l)^2[(p_1-k_2)^2-m_b^2](k_2+l)^2 l^2}\right\}$$
$$= \frac{9}{8}\phi^{(1)}_{\rho,e}\otimes[-G^{(0)}_{bL,2}(x_1,x_2)+G^{(0)}_{bL,2}(x_1,x_2,\xi_2)], \tag{A49}$$

$$G^{(1')}_{bL2,2f} = -\frac{eg^4 C_F^2}{2}\mathbf{Tr}\left\{\frac{\gamma^\alpha[M_\rho \not{\epsilon}_{2L}\phi_\rho(x_2)]\gamma^\nu(\not{p}_2-\not{k}_2-\not{l})\gamma_\mu(\not{p}_1-\not{k}_2-\not{l}+m_b)\gamma_\nu m_b\gamma_\alpha(\not{p}_1)\gamma_5[\frac{\not{n}_-}{\sqrt{2}}\phi_B^-(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2)^2[(p_1-k_2-l)^2-m_b^2](p_2-k_2-l)^2 l^2}\right\}\sim 0, \tag{A50}$$

$$G^{(1')}_{bL2,2g} = \frac{eg^4 C_F^2}{2}\mathbf{Tr}\left\{\frac{\gamma^\alpha(\not{k}_2+\not{l})\gamma^\nu[M_\rho \not{\epsilon}_{2L}\phi_\rho(x_2)]\gamma_\mu m_b\gamma_\nu(\not{p}_1-\not{k}_2-\not{l}+m_b)\gamma_\alpha(\not{p}_1)\gamma_5[\frac{\not{n}_-}{\sqrt{2}}\phi_B^-(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2-l)^2[(p_1-k_2-l)^2-m_b^2](k_2+l)^2 l^2}\right\}$$
$$= \phi^{(1)}_{\rho,e}\otimes[-G^{(0)}_{bL,2}(x_1,x_2,\xi_2)+G^{(0)}_{bL,2}(x_1,\xi_2)], \tag{A51}$$

$$G^{(1')}_{bL2,2h} = \frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\alpha[M_\rho \not{\epsilon}_{2L}\phi_\rho(x_2)]\gamma^\nu(\not{p}_2-\not{k}_2-\not{l})\gamma_\mu(m_b-\not{l})\gamma_\alpha(\not{p}_1-\not{k}_1-\not{l}+m_b)\gamma_\nu(\not{p}_1)\gamma_5[\frac{\not{n}_-}{\sqrt{2}}\phi_B^-(x_1)]}{[(p_1-k_2-l)^2-m_b^2](k_1-k_2)^2(p_2-k_2-l)^2[(p_1-k_1-l)^2-m_b^2]l^2}\right\}$$
$$= -\frac{1}{8}\phi^{(1)}_{\rho,d}\otimes G^{(0)}_{bL,2}(x_1,\xi_2,x_2), \tag{A52}$$

$$G^{(1')}_{bL2,2i} = -\frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\nu(\not{k}_1-\not{l})\gamma^\alpha[M_\rho \not{\epsilon}_{2L}\phi_\rho(x_2)]\gamma_\nu(\not{p}_2-\not{k}_2-\not{l})\gamma_\mu(m_b-\not{l})\gamma_\alpha(\not{p}_1)\gamma_5[\frac{\not{n}_-}{\sqrt{2}}\phi_B^-(x_1)]}{[(p_1-k_2-l)^2-m_b^2](k_1-k_2-l)^2(p_2-k_2-l)^2(k_1-l)^2 l^2}\right\}\sim 0, \tag{A53}$$

$$G^{(1')}_{bL2,2j} = \frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\nu(\not{k}_1+\not{l})\gamma^\alpha(\not{k}_2+\not{l})\gamma_\nu[M_\rho \not{\epsilon}_{2L}\phi_\rho(x_2)]\gamma_\mu m_b\gamma_\alpha(\not{p}_1)\gamma_5[\frac{\not{n}_-}{\sqrt{2}}\phi_B^-(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2)^2(k_1+l)^2(k_2+l)^2 l^2}\right\}\sim 0, \tag{A54}$$

$$G^{(1')}_{bL2,2k} = -\frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\alpha(\not{k}_2+\not{l})\gamma^\nu[M_\rho \not{\epsilon}_{2L}\phi_\rho(x_2)]\gamma_\mu m_b\gamma_\alpha(\not{p}_1-\not{k}_1+\not{l}+m_b)\gamma_\nu(\not{p}_1)\gamma_5[\frac{\not{n}_-}{\sqrt{2}}\phi_B^-(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2-l)^2[(p_1-k_1+l)^2-m_b^2](k_2+l)^2 l^2}\right\}$$
$$= -\frac{1}{8}\phi^{(1)}_{\rho,e}\otimes G^{(0)}_{bL,2}(x_1,x_2,\xi_2), \tag{A55}$$

with the wave functions $\phi^{(1)}_{\rho,d}$ and $\phi^{(1)}_{\rho,e}$ in the form of

$$\phi^{(1)}_{\rho,d} = \frac{-ig^2 C_F}{4}\frac{\gamma^+\gamma^\rho(\not{p}_2-\not{k}_2-\not{l})\gamma^-}{(p_2-k_2-l)^2 l^2}\frac{n_\rho}{n\cdot l},$$
$$\phi^{(1)}_{\rho,e} = \frac{ig^2 C_F}{4}\frac{\gamma^-(\not{k}_2+\not{l})\gamma^\rho\gamma^+}{(k_2+l)^2 l^2}\frac{n_\rho}{n\cdot l}. \tag{A56}$$

Analogous to the ones in Eq. (A25), the $1/8$ terms also appear in this channel. After the summation of the contributions from these irreducible subdiagrams, one finds the following result:

$$G^{(1')}_{\mathrm{up},bL2}(x_1,x_2) = \phi^{(1)}_{\rho,d}\otimes[G^{(0)}_{bL,2}(x_1,\xi_2,x_2)-G^{(0)}_{bL,2}(x_1,\xi_2)], \tag{A57}$$

$$G^{(1')}_{\mathrm{down},bL2}(x_1,x_2) = \phi^{(1)}_{\rho,e}\otimes[-G^{(0)}_{bL,2}(x_1,x_2)+G^{(0)}_{bL,2}(x_1,x_2,\xi_2)]. \tag{A58}$$

6. The NLO amplitudes for $G_{bT1}^{(0')}$

The amplitudes for the NLO contributions from the irreducible diagrams to $G_{bT1}^{(0')}$ with the additional gluon emitted from the final ρ meson are the following:

$$G_{bT1,2d}^{(1')} = -eg^4 \mathbf{Tr}\left\{\frac{\gamma^\alpha[\not\epsilon_{2T}\not p_2 \phi_\rho^T(x_2)]\gamma^\beta(\not p_2 - \not k_2 - \not l)\gamma_\mu \not p_1 \gamma^\gamma(\not p_1)\gamma_5[\frac{\not n_+}{\sqrt{2}}\phi_B^+(x_1)]F_{\alpha\beta\gamma}}{(k_1-k_2)^2(k_1-k_2-l)^2[(p_1-k_2-l)^2-m_b^2](p_2-k_2-l)^2 l^2}\right\}$$
$$= \frac{9}{8}\phi_{\rho,d}^{(1)} \otimes [G_{bT,1}^{(0)}(x_1,\xi_2,x_2) - G_{bT,1}^{(0)}(x_1,\xi_2)], \tag{A59}$$

$$G_{bT1,2e}^{(1')} = eg^4 \mathbf{Tr}\left\{\frac{\gamma^\alpha(\not k_2 + \not l)\gamma^\beta[\not\epsilon_{2T}\not p_2 \phi_\rho^T(x_2)]\gamma_\mu \not p_1 \gamma^\gamma(\not p_1)\gamma_5[\frac{\not n_+}{\sqrt{2}}\phi_B^+(x_1)]F_{\alpha\beta\gamma}}{(k_1-k_2)^2(k_1-k_2-l)^2[(p_1-k_2)^2-m_b^2](k_2+l)^2 l^2}\right\}$$
$$= \frac{9}{8}\phi_{\rho,e}^{(1)} \otimes [-G_{bT,1}^{(0)}(x_1,x_2) + G_{bT,1}^{(0)}(x_1,x_2,\xi_2)], \tag{A60}$$

$$G_{bT1,2f}^{(1')} = -\frac{eg^4 C_F^2}{2}\mathbf{Tr}\left\{\frac{\gamma^\alpha[\not\epsilon_{2T}\not p_2 \phi_\rho^T(x_2)]\gamma^\nu(\not p_2 - \not k_2 - \not l)\gamma_\mu(\not p_1 - \not k_2 - \not l + m_b)\gamma_\nu \not p_1 \gamma_\alpha(\not p_1)\gamma_5[\frac{\not n_+}{\sqrt{2}}\phi_B^+(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2)^2[(p_1-k_2-l)^2-m_b^2](p_2-k_2-l)^2 l^2}\right\}$$
$$= \phi_{\rho,d}^{(1)} \otimes [G_{bT,1}^{(0)}(x_1,x_2) - G_{bT,1}^{(0)}(x_1,\xi_2,x_2)], \tag{A61}$$

$$G_{bT1,2g}^{(1')} = \frac{eg^4 C_F^2}{2}\mathbf{Tr}\left\{\frac{\gamma^\alpha(\not k_2 + \not l)\gamma^\nu[\not\epsilon_{2T}\not p_2 \phi_\rho^T(x_2)]\gamma_\mu \not p_1 \gamma_\nu(\not p_1 - \not k_2 - \not l + m_b)\gamma_\alpha(\not p_1)\gamma_5[\frac{\not n_+}{\sqrt{2}}\phi_B^+(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2-l)^2[(p_1-k_2-l)^2-m_b^2](k_2+l)^2 l^2}\right\}$$
$$= \phi_{\rho,e}^{(1)} \otimes [-G_{bT,1}^{(0)}(x_1,x_2,\xi_2) + G_{bT,1}^{(0)}(x_1,\xi_2)], \tag{A62}$$

$$G_{bT1,2h}^{(1')} = \frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\alpha[\not\epsilon_{2T}\not p_2 \phi_\rho^T(x_2)]\gamma^\nu(\not p_2 - \not k_2 + \not l)\gamma_\mu(\not p_1 + \not l)\gamma_\alpha(\not p_1 - \not k_1 + \not l + m_b)\gamma_\nu(\not p_1)\gamma_5[\frac{\not n_+}{\sqrt{2}}\phi_B^+(x_1)]}{[(p_1-k_2+l)^2-m_b^2](k_1-k_2)^2(p_2-k_2+l)^2[(p_1-k_1+l)^2-m_b^2] l^2}\right\} \sim 0, \tag{A63}$$

$$G_{bT1,2i}^{(1')} = -\frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\nu(\not k_1 - \not l)\gamma^\alpha[\not\epsilon_{2T}\not p_2 \phi_\rho^T(x_2)]\gamma_\nu(\not p_2 - \not k_2 - \not l)\gamma_\mu(\not p_1 - \not l)\gamma_\alpha(\not p_1)\gamma_5[\frac{\not n_+}{\sqrt{2}}\phi_B^+(x_1)]}{[(p_1-k_2-l)^2-m_b^2](k_1-k_2-l)^2(p_2-k_2-l)^2(k_1-l)^2 l^2}\right\}$$
$$= \frac{1}{8}\phi_{\rho,d}^{(1)} \otimes G_{bT,1}^{(0)}(x_1,\xi_2), \tag{A64}$$

$$G_{bT1,2j}^{(1')} = \frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\nu(\not k_1 + \not l)\gamma^\alpha(\not k_2 + \not l)\gamma_\nu[\not\epsilon_{2T}\not p_2 \phi_\rho^T(x_2)]\gamma_\mu \not p_1 \gamma_\alpha(\not p_1)\gamma_5[\frac{\not n_+}{\sqrt{2}}\phi_B^+(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2)^2(k_1+l)^2(k_2+l)^2 l^2}\right\}$$
$$= \frac{1}{8}\phi_{\rho,e}^{(1)} \otimes G_{bT,2}^{(0)}(x_1,x_2), \tag{A65}$$

$$G_{bT1,2k}^{(1')} = -\frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\alpha(\not k_2 + \not l)\gamma^\nu[\not\epsilon_{2T}\not p_2 \phi_\rho^T(x_2)]\gamma_\mu \not p_1 \gamma_\alpha(\not p_1 - \not k_1 + \not l + m_b)\gamma_\nu(\not p_1)\gamma_5[\frac{\not n_+}{\sqrt{2}}\phi_B^+(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2-l)^2[(p_1-k_1+l)^2-m_b^2](k_2+l)^2 l^2}\right\} \sim 0, \tag{A66}$$

with the wave functions $\phi_{\rho,d}^{(1)}$ and $\phi_{\rho,e}^{(1)}$ in the form of

$$\phi^{(1)}_{\rho,d} = \frac{-ig^2C_F}{4}\frac{\gamma_\perp\gamma^+\gamma^\rho(\not{p}_2-\not{k}_2-\not{l})\gamma^-\gamma_\perp}{(p_2-k_2-l)^2l^2}\frac{n_\rho}{n\cdot l}$$
$$\phi^{(1)}_{\rho,e} = \frac{ig^2C_F}{4}\frac{\gamma^-\gamma_\perp(\not{k}_2+\not{l})\gamma^\rho\gamma_\perp\gamma^+}{(k_2+l)^2l^2}\frac{n_\rho}{n\cdot l}. \quad \text{(A67)}$$

The summation of the contributions from above irreducible subdiagrams gives the following result:

$$G^{(1')}_{\mathrm{up},bT1}(x_1,x_2) = \phi^{(1)}_{\rho,d}\otimes[G^{(0)}_{bT,1}(x_1,\xi_2,x_2) - G^{(0)}_{bT,1}(x_1,\xi_2)], \quad \text{(A68)}$$

$$G^{(1')}_{\mathrm{down},bT1}(x_1,x_2) = \phi^{(1)}_{\rho,e}\otimes[-G^{(0)}_{bT,1}(x_1,x_2) + G^{(0)}_{bT,1}(x_1,x_2,\xi_2)]. \quad \text{(A69)}$$

7. The NLO amplitudes for $G^{(0')}_{bT2}$

The NLO amplitudes for $G^{(0')}_{bT2}$ with the additional gluon emitted from the final state ρ meson are the following:

$$G^{(1')}_{bT2,2d} = -eg^4\mathbf{Tr}\left\{\frac{\gamma^\alpha[\not{\epsilon}_{2T}\not{p}_2\phi^T_\rho(x_2)]\gamma^\beta(\not{p}_2-\not{k}_2-\not{l})\gamma_\mu\not{p}_1\gamma^\gamma(\not{p}_1)\gamma_5[\frac{\not{n}_-}{\sqrt{2}}\phi^-_B(x_1)]F_{\alpha\beta\gamma}}{(k_1-k_2)^2(k_1-k_2-l)^2[(p_1-k_2-l)^2-m_b^2](p_2-k_2-l)^2l^2}\right\}$$
$$= \frac{9}{8}\phi^{(1)}_{\rho,d}\otimes[G^{(0)}_{bT,2}(x_1,\xi_2,x_2) - G^{(0)}_{bT,2}(x_1,\xi_2)], \quad \text{(A70)}$$

$$G^{(1')}_{bT2,2e} = eg^4\mathbf{Tr}\left\{\frac{\gamma^\alpha(\not{k}_2+\not{l})\gamma^\beta[\not{\epsilon}_{2T}\not{p}_2\phi^T_\rho(x_2)]\gamma_\mu\not{p}_1\gamma^\gamma(\not{p}_1)\gamma_5[\frac{\not{n}_-}{\sqrt{2}}\phi^-_B(x_1)]F_{\alpha\beta\gamma}}{(k_1-k_2)^2(k_1-k_2-l)^2[(p_1-k_2)^2-m_b^2](k_2+l)^2l^2}\right\}$$
$$= \frac{9}{8}\phi^{(1)}_{\rho,e}\otimes[-G^{(0)}_{bT,2}(x_1,x_2) + G^{(0)}_{bT,2}(x_1,x_2,\xi_2)], \quad \text{(A71)}$$

$$G^{(1')}_{bT2,2f} = -\frac{eg^4C_F^2}{2}\mathbf{Tr}\left\{\frac{\gamma^\alpha[\not{\epsilon}_{2T}\not{p}_2\phi^T_\rho(x_2)]\gamma^\nu(\not{p}_2-\not{k}_2-\not{l})\gamma_\mu(\not{p}_1-\not{k}_2-\not{l}+m_b)\gamma_\nu\not{p}_1\gamma_\alpha(\not{p}_1)\gamma_5[\frac{\not{n}_-}{\sqrt{2}}\phi^-_B(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2)^2[(p_1-k_2-l)^2-m_b^2](p_2-k_2-l)^2l^2}\right\}$$
$$= \phi^{(1)}_{\rho,d}\otimes[G^{(0)}_{bT,2}(x_1,x_2) - G^{(0)}_{bT,2}(x_1,\xi_2,x_2)], \quad \text{(A72)}$$

$$G^{(1')}_{bT2,2g} = \frac{eg^4C_F^2}{2}\mathbf{Tr}\left\{\frac{\gamma^\alpha(\not{k}_2+\not{l})\gamma^\nu[\not{\epsilon}_{2T}\not{p}_2\phi^T_\rho(x_2)]\gamma_\mu\not{p}_1\gamma_\nu(\not{p}_1-\not{k}_2-\not{l}+m_b)\gamma_\alpha(\not{p}_1)\gamma_5[\frac{\not{n}_-}{\sqrt{2}}\phi^-_B(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2-l)^2[(p_1-k_2-l)^2-m_b^2](k_2+l)^2l^2}\right\}$$
$$= \phi^{(1)}_{\rho,e}\otimes[-G^{(0)}_{bT,2}(x_1,x_2,\xi_2) + G^{(0)}_{bT,2}(x_1,\xi_2)], \quad \text{(A73)}$$

$$G^{(1')}_{bT2,2h} = \frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\alpha[\not{\epsilon}_{2T}\not{p}_2\phi^T_\rho(x_2)]\gamma^\nu(\not{p}_2-\not{k}_2-\not{l})\gamma_\mu(\not{p}_1+\not{l})\gamma_\alpha(\not{p}_1-\not{k}_1-\not{l}+m_b)\gamma_\nu(\not{p}_1)\gamma_5[\frac{\not{n}_-}{\sqrt{2}}\phi^-_B(x_1)]}{[(p_1-k_2+l)^2-m_b^2](k_1-k_2)^2(p_2-k_2+l)^2[(p_1-k_1-l)^2-m_b^2]l^2}\right\}$$
$$= -\frac{1}{8}\phi^{(1)}_{\rho,d}\otimes G^{(0)}_{bT,2}(x_1,\xi_2,x_2), \quad \text{(A74)}$$

$$G^{(1')}_{bT2,2i} = -\frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\nu(\not{k}_1-\not{l})\gamma^\alpha[\not{\epsilon}_{2T}\not{p}_2\phi^T_\rho(x_2)]\gamma_\nu(\not{p}_2-\not{k}_2-\not{l})\gamma_\mu(\not{p}_1-\not{l})\gamma_\alpha(\not{p}_1)\gamma_5[\frac{\not{n}_-}{\sqrt{2}}\phi^-_B(x_1)]}{[(p_1-k_2-l)^2-m_b^2](k_1-k_2-l)^2(p_2-k_2-l)^2(k_1-l)^2l^2}\right\}\sim 0, \quad \text{(A75)}$$

$$G^{(1')}_{bT2,2j} = \frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\nu(\not{k}_1+\not{l})\gamma^\alpha(\not{k}_2+\not{l})\gamma_\nu[\not{\epsilon}_{2T}\not{p}_2\phi^T_\rho(x_2)]\gamma_\mu\not{p}_1\gamma_\alpha(\not{p}_1)\gamma_5[\frac{\not{n}_-}{\sqrt{2}}\phi^-_B(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2)^2(k_1+l)^2(k_2+l)^2l^2}\right\}\sim 0, \quad \text{(A76)}$$

$$G^{(1')}_{bT2,2k} = -\frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\alpha(\not{k}_2+\not{l})\gamma^\nu[\not{\epsilon}_{2T}\not{p}_2\phi^T_\rho(x_2)]\gamma_\mu\not{p}_1\gamma_\alpha(\not{p}_1-\not{k}_1+\not{l}+m_b)\gamma_\nu(\not{p}_1)\gamma_5[\frac{\not{n}_-}{\sqrt{2}}\phi^-_B(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2-l)^2[(p_1-k_1+l)^2-m_b^2](k_2+l)^2l^2}\right\}$$
$$= -\frac{1}{8}\phi^{(1)}_{\rho,e}\otimes G^{(0)}_{bT,2}(x_1,x_2,\xi_2), \tag{A77}$$

with the wave functions $\phi^{(1)}_{\rho,d}$ and $\phi^{(1)}_{\rho,e}$ in the form of

$$\phi^{(1)}_{\rho,d} = \frac{-ig^2C_F}{4}\frac{\gamma_\perp\gamma^+\gamma^\rho(\not{p}_2-\not{k}_2-\not{l})\gamma^-\gamma_\perp}{(p_2-k_2-l)^2l^2}\frac{n_\rho}{n\cdot l},$$
$$\phi^{(1)}_{\rho,e} = \frac{ig^2C_F}{4}\frac{\gamma^-\gamma_\perp(\not{k}_2+\not{l})\gamma^\rho\gamma_\perp\gamma^+}{(k_2+l)^2l^2}\frac{n_\rho}{n\cdot l}. \tag{A78}$$

The summation over the contributions from above irreducible subdiagrams gives the following result:

$$G^{(1')}_{\mathrm{up},bT2}(x_1,x_2) = \phi^{(1)}_{\rho,d}\otimes[G^{(0)}_{bT,2}(x_1,\xi_2,x_2) - G^{(0)}_{bT,2}(x_1,\xi_2)], \tag{A79}$$

$$G^{(1')}_{\mathrm{down},bT1}(x_1,x_2) = \phi^{(1)}_{\rho,e}\otimes[-G^{(0)}_{bT,2}(x_1,x_2) + G^{(0)}_{bT,2}(x_1,x_2,\xi_2)]. \tag{A80}$$

8. The NLO amplitudes for $G^{(0')}_{bT3}$

The NLO amplitudes for $G^{(0')}_{bT3}$ with the additional gluon emitted from the final ρ meson are the following:

$$G^{(1')}_{bT3,2d} = -eg^4\mathbf{Tr}\left\{\frac{\gamma^\alpha[\not{\epsilon}_{2T}\not{p}_2\phi^T_\rho(x_2)]\gamma^\beta(\not{p}_2-\not{k}_2-\not{l})\gamma_\mu m_b\gamma^\gamma(m_B)\gamma_5[\frac{\not{n}_+}{\sqrt{2}}\phi^+_B(x_1)]F_{\alpha\beta\gamma}}{(k_1-k_2)^2(k_1-k_2-l)^2[(p_1-k_2-l)^2-m_b^2](p_2-k_2-l)^2l^2}\right\}$$
$$= \frac{9}{8}\phi^{(1)}_{\rho,d}\otimes[G^{(0)}_{bT,3}(x_1,\xi_2,x_2) - G^{(0)}_{bT,3}(x_1,\xi_2)], \tag{A81}$$

$$G^{(1')}_{bT3,2e} = eg^4\mathbf{Tr}\left\{\frac{\gamma^\alpha(\not{k}_2+\not{l})\gamma^\beta[\not{\epsilon}_{2T}\not{p}_2\phi^T_\rho(x_2)]\gamma_\mu m_b\gamma^\gamma(m_B)\gamma_5[\frac{\not{n}_+}{\sqrt{2}}\phi^+_B(x_1)]F_{\alpha\beta\gamma}}{(k_1-k_2)^2(k_1-k_2-l)^2[(p_1-k_2)^2-m_b^2](k_2+l)^2l^2}\right\}$$
$$= \frac{9}{8}\phi^{(1)}_{\rho,e}\otimes[-G^{(0)}_{bT,3}(x_1,x_2) + G^{(0)}_{bT,3}(x_1,x_2,\xi_2)], \tag{A82}$$

$$G^{(1')}_{bT3,2f} = -\frac{eg^4C_F^2}{2}\mathbf{Tr}\left\{\frac{\gamma^\alpha[\not{\epsilon}_{2T}\not{p}_2\phi^T_\rho(x_2)]\gamma^\nu(\not{p}_2-\not{k}_2-\not{l})\gamma_\mu(\not{p}_1-\not{k}_2-\not{l}+m_b)\gamma_\nu m_b\gamma_\alpha(m_B)\gamma_5[\frac{\not{n}_+}{\sqrt{2}}\phi^+_B(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2)^2[(p_1-k_2-l)^2-m_b^2](p_2-k_2-l)^2l^2}\right\}$$
$$= \phi^{(1)}_{\rho,d}\otimes[G^{(0)}_{bT,3}(x_1,x_2) - G^{(0)}_{bT,3}(x_1,\xi_2,x_2)], \tag{A83}$$

$$G^{(1')}_{bT3,2g} = \frac{eg^4C_F^2}{2}\mathbf{Tr}\left\{\frac{\gamma^\alpha(\not{k}_2+\not{l})\gamma^\nu[\not{\epsilon}_{2T}\not{p}_2\phi^T_\rho(x_2)]\gamma_\mu m_b\gamma_\nu(\not{p}_1-\not{k}_2-\not{l}+m_b)\gamma_\alpha(m_B)\gamma_5[\frac{\not{n}_+}{\sqrt{2}}\phi^+_B(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2-l)^2[(p_1-k_2-l)^2-m_b^2](k_2+l)^2l^2}\right\}\sim 0, \tag{A84}$$

$$G^{(1')}_{bT3,2h} = \frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\alpha[\not{\epsilon}_{2T}\not{p}_2\phi^T_\rho(x_2)]\gamma^\nu(\not{p}_2-\not{k}_2-\not{l})\gamma_\mu(m_b-\not{l})\gamma_\alpha(\not{p}_1-\not{k}_1-\not{l}+m_b)\gamma_\nu(m_B)\gamma_5[\frac{\not{n}_+}{\sqrt{2}}\phi^+_B(x_1)]}{[(p_1-k_2-l)^2-m_b^2](k_1-k_2)^2(p_2-k_2-l)^2[(p_1-k_1-l)^2-m_b^2]l^2}\right\}$$
$$= -\phi^{(1)}_{\rho,d}\otimes G^{(0)}_{bT,3}(x_1,\xi_2,x_2), \tag{A85}$$

$$G^{(1')}_{bT3,2i} = -\frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\nu(\not{k}_1-\not{l})\gamma^\alpha[\not{\epsilon}_{2T}\not{p}_2\phi^T_\rho(x_2)]\gamma_\nu(\not{p}_2-\not{k}_2-\not{l})\gamma_\mu(m_b-\not{l})\gamma_\alpha(m_B)\gamma_5[\frac{\not{n}_+}{\sqrt{2}}\phi^+_B(x_1)]}{[(p_1-k_2-l)^2-m_b^2](k_1-k_2-l)^2(p_2-k_2-l)^2(k_1-l)^2l^2}\right\}$$
$$= \frac{1}{8}\phi^{(1)}_{\rho,d}\otimes G^{(0)}_{bT,3}(x_1,\xi_2), \tag{A86}$$

$$G^{(1')}_{bT3,2j} = \frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\nu(\not{k}_1+\not{l})\gamma^\alpha(\not{k}_2+\not{l})\gamma_\nu[\not{\epsilon}_{2T}\not{p}_2\phi^T_\rho(x_2)]\gamma_\mu m_b\gamma_\alpha(m_B)\gamma_5[\frac{\not{n}_+}{\sqrt{2}}\phi^+_B(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2)^2(k_1+l)^2(k_2+l)^2l^2}\right\}$$
$$= \frac{1}{8}\phi^{(1)}_{\rho,e}\otimes G^{(0)}_{bT,3}(x_1,x_2), \tag{A87}$$

$$G^{(1')}_{bT3,2k} = -\frac{eg^4}{9}\mathbf{Tr}\left\{\frac{\gamma^\alpha(\not{k}_2+\not{l})\gamma^\nu[\not{\epsilon}_{2T}\not{p}_2\phi^T_\rho(x_2)]\gamma_\mu m_b\gamma_\alpha(\not{p}_1-\not{k}_1+\not{l}+m_b)\gamma_\nu(m_B)\gamma_5[\frac{\not{n}_+}{\sqrt{2}}\phi^+_B(x_1)]}{[(p_1-k_2)^2-m_b^2](k_1-k_2-l)^2[(p_1-k_1+l)^2-m_b^2](k_2+l)^2l^2}\right\}$$
$$= -\frac{1}{8}\phi^{(1)}_{\rho,e}\otimes G^{(0)}_{bT,3}(x_1,x_2,\xi_2), \tag{A88}$$

with the wave functions $\phi^{(1)}_{\rho,d}$ and $\phi^{(1)}_{\rho,e}$ in the form of

$$\phi^{(1)}_{\rho,d} = \frac{-ig^2C_F}{4}\frac{\gamma_\perp\gamma^+\gamma^\rho(\not{p}_2-\not{k}_2-\not{l})\gamma^-\gamma_\perp}{(p_2-k_2-l)^2l^2}\frac{n_\rho}{n\cdot l},$$
$$\phi^{(1)}_{\rho,e} = \frac{ig^2C_F}{4}\frac{\gamma^-\gamma_\perp(\not{k}_2+\not{l})\gamma^\rho\gamma_\perp\gamma^+}{(k_2+l)^2l^2}\frac{n_\rho}{n\cdot l}. \tag{A89}$$

The summation of the contributions from above irreducible subdiagrams gives the following result:

$$G^{(1')}_{\mathrm{up},bT3}(x_1,x_2) = \phi^{(1)}_{\rho,d}\otimes[G^{(0)}_{bT,3}(x_1,\xi_2,x_2)-G^{(0)}_{bT,3}(x_1,\xi_2)], \tag{A90}$$

$$G^{(1')}_{\mathrm{down},bT3}(x_1,x_2) = \phi^{(1)}_{\rho,e}\otimes[-G^{(0)}_{bT,3}(x_1,x_2)+G^{(0)}_{bT,3}(x_1,x_2,\xi_2)]. \tag{A91}$$

[1] G. P. Lepage and S. J. Brodsky, Phys. Lett. **87B**, 359 (1979); Phys. Rev. Lett. **43**, 545 (1979); **43**, 1625(E) (1979); Phys. Rev. D **22**, 2157 (1980).

[2] A. G. Grozin and M. Neubert, Phys. Rev. D **55**, 272 (1997).

[3] V. L. Chernyak and A. R. Zhitnitsky, Phys. Rep. **112**, 173 (1984).

[4] J. C. Collins and R. K. Ellis, Nucl. Phys. **B360**, 3 (1991); J. Botts and G. Sterman, Nucl. Phys. **B325**, 62 (1989); H. N. Li and G. Sterman, Nucl. Phys. **B381**, 129 (1992); T. Huang and Q. X. Shen, Z. Phys. C **50**, 139 (1991); S. Cantani, M. Ciafaloni, and F. Hautmann, Nucl. Phys. **B366**, 135 (1991).

[5] H. N. Li, Phys. Rev. D **64**, 014019 (2001); T. Kurimoto, H. N. Li, and A. I. Sanda, Phys. Rev. D **65**, 014007 (2001); M. Nagashima and H. N. Li, Phys. Rev. D **67**, 034001 (2003); M. Nagashima and H. N. Li, Eur. Phys. J. C **40**, 395 (2005).

[6] S. Cheng and Z. J. Xiao, Phys. Rev. D **90**, 076001 (2014); Y. L. Zhang, S. Cheng, J. Hua, and Z. J. Xiao, Phys. Rev. D **93**, 036002 (2016).

[7] Y. L. Zhang, S. Cheng, J. Hua, and Z. J. Xiao, Phys. Rev. D **92**, 094031 (2015); **93**, 099901(A) (2016); J. Hua, S. Cheng, Y. L. Zhang, and Z. J. Xiao, Phys. Rev. D **97**, 113002 (2018).

[8] S. Nandi and H. N. Li, Phys. Rev. D **76**, 034008 (2007); H. N. Li, Y. L. Shen, Y. M. Wang, and H. Zou, Phys. Rev. D **83**, 054029 (2011); H. N. Li, Y. L. Shen, and Y. M. Wang, Phys. Rev. D **85**, 074004 (2012).

[9] S. Cheng, Y. Y. Fan, and Z. J. Xiao, Phys. Rev. D **89**, 054015 (2014); S. Cheng, Y. Y. Fan, X. Yu, C. D. Lü, and Z. J. Xiao, Phys. Rev. D **89**, 094004 (2014).

[10] H. C. Hu and H. N. Li, Phys. Lett. B **718**, 1351 (2013); S. Cheng and Z. J. Xiao, Phys. Lett. B **749**, 1 (2015).

[11] S. Cheng, Y. L. Zhang, J. Hua, H. N. Li, and Z. J. Xiao, Phys. Rev. D **95**, 076005 (2017).
[12] M. Beneke and Th. Feldmann, Nucl. Phys. **B592**, 3 (2001).
[13] P. Ball, J. High Energy Phys. 01 (1999) 010; P. Ball, V. M. Braun, Y. Koike, and K. Tanaka, Nucl. Phys. B, Proc. Suppl. **529**, 323 (1998); P. Ball, V. M. Braun, and A. Lenz, J. High Energy Phys. 05 (2006) 004; P. Ball and R. Zwicky, Phys. Rev. D **71**, 014015 (2005).
[14] R. K. Ellis and G. Zanderighi, J. High Energy Phys. 02 (2008) 002.
[15] A. Denner and S. Dittmaier, Nucl. Phys. **B844**, 199 (2011).
[16] H. N. Li and S. Mishima, Phys. Rev. D **90**, 074018 (2014).
[17] X. Liu, H. N. Li, and Z. J. Xiao, Phys. Rev. D **91**, 114019 (2015); **93**, 014024 (2016).
[18] H. N. Li, Prog. Part. Nucl. Phys. **51**, 85 (2003), and references therein.
[19] H. N. Li, Y. L. Shen, and Y. M. Wang, J. High Energy Phys. 02 (2013) 008; 01 (2014) 004; H. N. Li, Phys. Part. Nucl. **45**, 756 (2014).
[20] J. Collins, Int. J. Mod. Phys. Conf. Ser. **04**, 85 (2011).
[21] H. N. Li and Y. M. Wang, J. High Energy Phys. 06 (2015) 013.